Sheet 5
Heap - Huffman Coding

1 Question 1

Given an array of integers: 11, 9, 12, 14, 3, 15, 7, 8, 1, do the following.

a) Draw the min heap with the intermediate trees that results from inserting each number.

b) Do a 2 DeleteMin operations on the heap in (a). Show the intermediate steps.

c) What is the minimum and maximum number of comparisons we might have to do when inserting the next value in the heap in (a)?

d) For node k in a 3-ary heap (each node has 3 children), give formulas to get each of the following:

(1) Node k's parent

(2) Node k's three children

2 Question 2

a) Given an array of length n sorted in increasing order, what is the running time of HeapSort? What if the array is sorted in decreasing order? Explain your answer.

b) Show the steps of the operation of buildHeap on the array [5, 3, 17, 10, 84, 19, 6, 22, 9].

c) Show the steps of the operation of maxHeapInsert(A, 10) on the heap A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1].

d) Show the steps of the operation of maxHeapify(A, 3) on the array A = [27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0].

e) Show the steps of the operation of heapSort on the array A = [5, 13, 2, 25, 7, 17, 20, 8, 4].

f) Show the steps of the operation of minHeap() on the array A = [D, A, T, A, S, T, R, U, C, T, U, R, E]

3 Question 3

a) Suppose we are implementing a binary heap (tree-based implementation) and you are a great problem solver. We want to implement a deleteReference(ref) operation which, given a reference to a node in the tree, can delete that node and the item it contains from the heap while maintaining the heap-order property, even if the node isn't the root and its item isn't the minimum. deleteReference(ref) should run in O(log n) time. How can you do it?

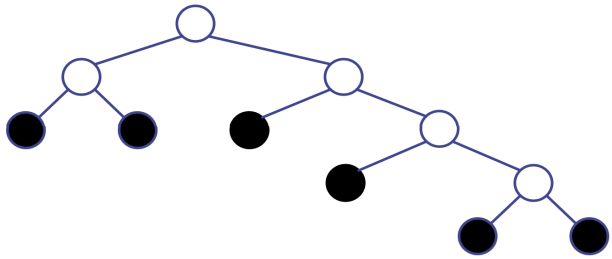
b) Given your answer in (a), explain how to combine a minHeap and maxHeap to yield a data structure that implements insert(), deleteMin(), and deleteMax() in O(log n) time.

c) How can we achieve the same requirements in (a) and (b) if we use array-based heaps?

4 Question 4

a) Assume that the characters from A to H have the set of frequencies: 1, 1, 2, 3, 5, 8, 13, 21 and the characters are represented using a Huffman code. What is the representation of 111110011111101111011110? (Show your steps).

b) Write a frequency list that the Huffman code of this frequency would deterministically create the following structure.



c) How many bits may be required for encoding the message 'noomorrnon'?

5 Notes

- You are required to submit a PDF of your answers and your ID in teams before 11:59 AM.
- You are encouraged to ask any questions on teams, or in person.

Good Luck