# A report on
# "A healthier semi-classical dynamics"

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**Abstract**

This is a report on "A Healthier Semiclassical Dynamics" (Layton–Oppenheim–Weller-Davies, 2023, arXiv:2208.11722), and it reproduces the reference visualizations in the MATLAB environment. My simulations confirm that the decoherence emerges directly from the classical noise kernel $\sigma$, which will be shown explicitly by the new visualization of $\mathbb{P}\left[|\psi_t\rangle\right]$. I finish by outlining potential next steps and ideas: deriving the corrected Einstein equations, applying the framework to gravitational theorems and paradoxes, exploring non-Markovian versions, and probing quantum chaos in stochastic dynamics.

## 1 Motivation

My earlier work reviewing stochastic electrodynamics (my term paper is here), where I showed that a classical oscillator driven by background noise reproduces the quantum ground-state distribution $\rho(x,p) = \frac{1}{\pi\hbar}e^{-m\omega x^2/\hbar - p^2/(m\omega\hbar)}$, convinced me that treating fluctuations like first-class variables can illuminate the quantum–classical boundary. That perspective, together with recent coursework in applied stochastic processes and quantum information, has let me appreciate the importance of your stochastic formalism and how it cures the large-fluctuation failure of traditional semiclassical gravity while building decoherence directly into the dynamics. This report summarizes the central assumptions and results of the paper, recasts them in Hamiltonian form, and presents new MATLAB simulations that visualize the evolving probability density of $|\psi_t\rangle$.

## 2 Stochastic Formalism and Main Results

### 2.1 States as Random Variables

In ordinary quantum mechanics, we already have two kinds of "ignorance." A pure state $|\psi_t\rangle$ encodes quantum uncertainty through superposition, while a density operator $\rho_t$, can also capture classical ignorance about the pure state in which the quantum system is located. The paper adds one more layer: It treats both the quantum state $\rho_t$ and the classical phase-space coordinates Z as random variables on a common probability space. For this new layer, “Collapse” is nothing more than a revealing of the real values of $\rho_t$ or Z; it resembles nothing more than opening a bag to check whether an apple is inside, hence it does not suffer from the usual difficulties measurement problem. It is also a generalization of the usual formalism, since the deterministic $\rho_t$ and Z, along with their evolution, can be recovered by taking their probability distribution functions to concentrate in a specific state at each time moment.

Defining both the density operator $\rho_t$ and the classical phase-space point $Z_t$ in the same probability space opens the possibility for us to correlate them. A classical–quantum bipartite system is therefore described by the pair of correlated random variables $\left(Z_t, \rho_t\right)$ rather than by two independent objects. These correlations are important, as the Hamiltonian interaction $H_{\mathrm{I}}(Z_t)$ would create them even if the quantum and classical initial states are not correlated in the first place.

To obtain a macroscopic observable's A value in classical coordinate $Z_t$ from the hybrid system, we average over every realization of the random pair $\left(Z_t, \rho_t\right)$:

$$\langle A\rangle = \mathbb{E}\Big[\mathrm{Tr}\big\{A\big(Z_t\big)\,\rho_t\big\}\Big].$$

It is reminiscent of calculating expectation values in standard quantum mechanics, but with one extra layer of probability sitting on top of the quantum one. But many different microscopic realizations yield the same macroscopic number—exactly as in statistical mechanics. It is therefore convenient to package those realizations into one macroscopic mixed state:

$$\varrho(z,t) = \mathbb{E}\big[\delta\big(z - Z_t\big)\,\rho_t\big], \qquad \textit{macroscopic state}$$

which is Eq. (3) in the original paper. All macroscopic observables can then be expressed as

$$\langle A\rangle = \int dz\ \mathrm{Tr}\big[A(z)\,\varrho(z,t)\big],$$

making $\varrho(z,t)$ the natural macroscopic state.

## 2.2 Stochastic Dynamics

Under four explicit assumptions, the paper derives the dynamical laws for the random pair $\left(Z_t, \rho_t\right)$:

**Assumption 1.** *Solutions to the dynamics are described by a probability distribution over classical–quantum trajectories, i.e.* $\{(Z_t, \rho_t)\}_{t\geq 0}$. Just like any other dynamical system.

**Assumption 2.** *The dynamics induces an evolution on* $\rho(z,t) = \mathbb{E}[\delta(z - Z_t)\,\rho_t]$ *that is completely positive and linear.* Complete positivity guarantees that a legitimate state of the CQ system and any of its subsystems remains physical under the evolution. Linearity is then required because the evolution of a linear sum of the states (which also is a state) should evolve to the addition of the evolution of each state, as it's a classical ensemble.

**Assumption 3.** *The dynamics is autonomous on the combined classical–quantum system, meaning that the time-evolution of the state $(Z_t, \rho_t)$ depends only on the state at that time and the dynamics is time-independent.* This is the Markov property; I will return to it in the Discussion.

**Assumption 4.** *The classical trajectories $\{Z_t\}_{t\geq 0}$ have continuous sample paths.* Continuity for the classical sector rules out unphysical, instantaneous jumps in the macroscopic degrees of freedom.

With the four assumptions in place, Layton – Oppenheim – Weller-Davies show that the most general Markovian dynamics of the classical–quantum pair is captured by equations (5), (7) and (8) in the original paper, governing the mixed state $\varrho(z,t)$, the classical coordinate $Z_t$, and the conditional quantum state $\rho_t$, respectively. Three matrix coefficients appear in these equations:

$\boldsymbol{D_0(z)}$: The decoherence rate acting solely on the quantum state,

$\boldsymbol{D_1(z)}$: The back-reaction coupling that transfers quantum noise into the classical sector,

$\boldsymbol{D_2(z) = \frac{1}{2}\sigma(z)\sigma^{\mathsf{T}}(z)}$: The classical diffusion tensor induced by that back-reaction.

The complete positivity of the macroscopic state $\varrho(z,t)$ imposes two matrix inequalities:

$$2D_0 \;\geq\; D_1^{\dagger}\big(D_2\big)^{-1} D_1, \qquad \big(\mathbb{I} - D_2 {D_2}^{-1}\big)\, D_1 = 0, \qquad \textit{dyanamical conditions}$$

where "≥" denotes positive semi-definiteness. The first inequality above is called the decoherence–diffusion trade-off. It says that the quantum state must decohere at no less than the rate dictated by the classical diffusion tensor and the back-reaction coupling.

Any dynamics that respects both constraints (2) is a legitimate classical–quantum evolutions. Appendix B shows, however, that the trade-off can always be saturated, meaning that by embedding the system in a larger Hilbert space, one can find dynamics for which

$$D_0 \;=\; D_1^{\dagger}\big(\sigma\sigma^{\mathsf{T}}\big)^{-1} D_1.$$

In this "minimal-decoherence" condition, a trajectory that starts in a pure quantum state remains pure at any time. Analyzing these pure-state dynamics is therefore sufficient, since classical ensembles always evolve linearly.

The paper finally rewrites the coupled SDEs in equations (7) and (9) in the straight Hamiltonian language. By comparing the first terms of these dynamics with the usual classical Poisson bracket and Schrödinger terms, fixing the freedom in the choice of Lindblad operators and the fact that the decoherence–diffusion bound is saturated, one arrives at a compact set of dynamics for $(Z_t, |\psi_t\rangle)$:

$$\begin{aligned} dZ_{t,i} &= \{Z_{t,i}, H_C(Z_t)\}\,dt + \{\{Z_{t,i}, H_j(Z_t)\}\}\,dt + \sigma_{ij}(Z_t)\,dW_t,\\ d|\psi_t\rangle &= -i\,H_j(Z_t)\,|\psi_t\rangle\,dt + \tfrac{1}{2}\sigma^{-1}_{ij}\big(\{Z_j, H_1\} - \{\{Z_j, H_1\}\}\big)\,|\psi_t\rangle\,dW_t\\ &\quad - \tfrac{1}{8}\sigma^{-1}_{ij}\sigma^{-1}_{ik}\,\big(\{Z_j, H_1\} - \{\{Z_j, H_1\}\}\big)\big(\{Z_k, H_1\} - \{\{Z_k, H_1\}\}\big)\,|\psi_t\rangle\,dt. \end{aligned} \qquad \begin{gathered}\textit{final classical–quantum dynamics}\\ \textit{in Hamiltonian formalism}\end{gathered}$$

The three matrix coefficients $(D_0, D_1, D_2)$ are now translated into Hamiltonians plus the noise kernel $\sigma$. The part of the Hamiltonian that acts on the quantum sector will be called $H_I$; because it is a function of phase-space coordinates, it plays the role of interaction. The rest is then called $H_C$, acting on the classical sector. $\sigma$ sets the diffusion strength in the classical sector. It can be any real matrix satisfying the second condition. Crucially, the new additional terms in $d|\psi_t\rangle$ scale with the (pseudo-)inverse $\sigma^{-1}$; meaning that the stronger the noise is in the classical sector, the harder it is for the actual dynamics of the classical sector to back-react on the quantum one. The next section visualises these dynamics through simulations, making these new semi-classical effects explicit.

# 3 Visualisation and Numerical Calculations

With the Hamiltonian representation established, this section presents simulated trajectories and visualisation.

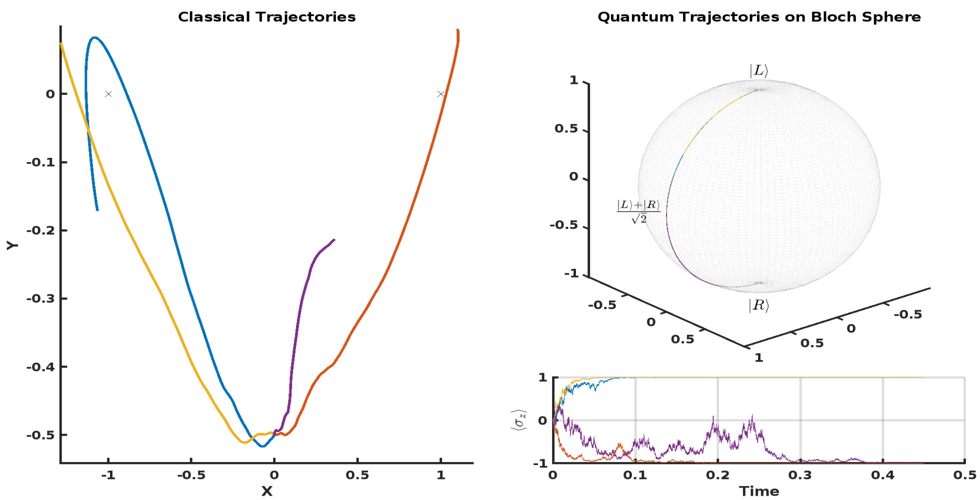


Figure 1: Four realizations of the classical-quantum trajectory described by the system in the exact configurations of FIG. 7 in the original paper (Appendix C.4), except for $\sigma = 2.5m$ and until t=0.45.

I have implemented the toy mass–superposition model from Appendix C.4 to gain more physical intuition. Starting from the reference Python script (Ref. [85]), I rewrote the entire code in MATLAB (the code can be found here). Figure 1 reproduces the similar results given in the paper, but for four different realizations. Each quantum trajectory randomly decoheres to the left or right branch, and the classical coordinate evolves accordingly. You can also clearly see the early-time diffusion: the strong noise initially prevents the quantum state from settling into its preferred eigenstate, while simultaneously kicking the classical path.

I have also constructed the approximate probability distribution of the $\langle\sigma_z\rangle$ component of $|\psi_t\rangle$ across roughly 2000 realizations (Figure 2a), sampling at four time steps up to t=0.15. By averaging over all classical trajectories, these histograms isolate the evolution of the quantum state itself, removing any dependence on the classical path. Then one can see as t increases, the distribution concentrates on the eigenvalues ±1 of the operator pairing $\{Z_i, H_I\}$, with the correct Born–rule weights. In other words, what began as quantum uncertainty between $|R\rangle$ and $|L\rangle$ is converted into classical ignorance over those two outcomes. Figure 2b then makes the decoherence–diffusion trade-off explicit: larger $\sigma$(stronger classical noise) produces slower decoherence, spreading the distribution for longer before it peaks at ±1.

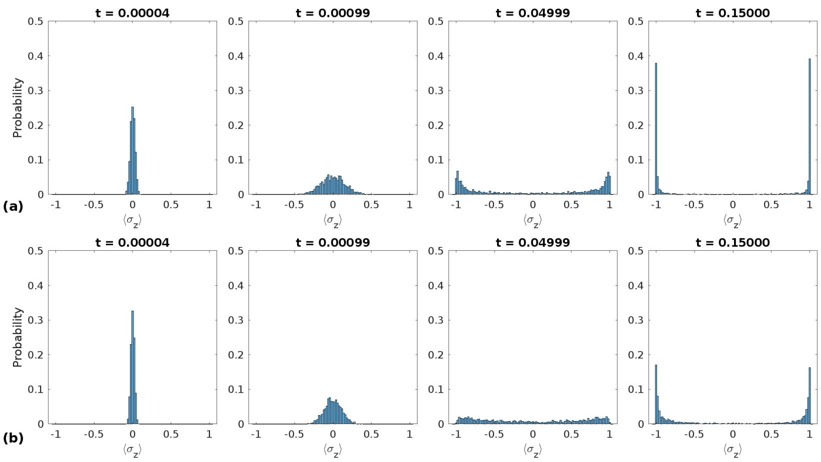


Figure 2: Probability distribution of $\langle\sigma_z\rangle$ (the z-component of $|\psi_t\rangle$) over n=2000 realizations (after discarding ill-conditioned samples), for (a) $\sigma = 1.5\,m$ and (b) $\sigma = 2\,m$ in four time steps with $\Delta t = 10^{-4}$. Since $|\psi_t\rangle$ is on xz plane, the histograms practicaly shows $\mathbb{P}[|\psi_t\rangle]$. They also shows how the equal superposition $\frac{|L\rangle+|R\rangle}{\sqrt{2}}$ ($\langle\sigma_z\rangle = 0$) decoheres into the distinct outcomes $|L\rangle$ ($\langle\sigma_z\rangle = 1$) and $|R\rangle$ ($\langle\sigma_z\rangle = -1$). The difference in decoherence rate between the two noise strengths is clearly visible.

# 4 Conclusion and Discussion

The central achievement of this paper is to show how the random-state formalism provides a clearer description of a hybrid quantum–classical system, both in its dynamics and in the definition of states. By saturating the decoherence–diffusion trade-off, the only free parameter that remains is the noise kernel $\sigma$. Through this framework, one can vividly see how classical fluctuations drive quantum decoherence. This idea has been applied to Einstein equauation in Appendix G of the original paper and in the “post-quantum gravity” paper by Jonathan Oppenheim (arXiv:1811:03116). The next step then can be applying these modified semiclassical equations to long-standing puzzles in gravitational physics, such as the black-hole information paradox and the singularity theorems; to see whether stochastic back-reaction shifts their conventional results.

The quantum–classical transition lies at the heart of quantum foundations, from the measurement problem to the nature of a state. This paper focuses on it by showing how a classical-probabilistic layer can mimic quantum behavior. Where it inevitably breaks down, it offers a clearer roadmap toward a more consistent theory. It is, in my opinion, a very important view.

There are also two extensions that come to my mind: i) One promising suggestion is to relax Assumption 3 (classical–quantum Markovianity). I am keen to dive into Appendix H of the original paper, since non-divisible non-Markovian generalized probabilistic theories (e.g. in arXiv:2302.10778) already reproduce quantum theory. ii) The built-in nonlinearity of the stochastic Schrödinger evolution may hint at a new avenue for studying quantum chaos by developing measures of classical unpredictability in this framework.