Assignment 1 RM, Group 16

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Present Value in a years' time

In this case study we evaluate the present value in a year's time (FV) with a single factor MC simulation method with different correlation factors.
Thanks to the assumption of homogeneity for the transition matrix Q, we assumed the hazard rate h to be a piece-wise constant function with $h_i = -\ln(1 - q_{i,3})/\Delta t$ (in our case $\Delta t = 1$), for each rating class i and computed the survival probabilities at the corresponding coupon dates as $P_i(1y, t_i) = e^{-h_i*(t_i-1y)}$.
To compute the FV we considered the computation time $1y + \epsilon$ and then we discounted all remaining coupons with the forward defaultable discounts $\bar{B}(1y + \epsilon, t_i)$ and summed the recovery value in case of default. This yielded the following results:

IG	HY	$Default$
100.51	98.43	40

Table 1: Present Values in a years' time

which when weighted by the probability of migration yield $\mathbb{E}[FV] = 99.76$

VaR

We simulate the market factor y and for each obligor i we also simulate the idiosyncratic factor ϵ_i. To then compute the VaR we compute the losses for each scenario and take the 99.9% percentile loss. For this first case we consider 200 obligors.

- For the default only case we compute the loss function as $L(y) = L_D * n_D/I$ where n is the number of defaulted obligors in each scenario and I is the total number of obligors.
- For the migration and default case we compute the loss as $L(y) = L_D * n_D/I + L_d * n_d/I + L_i * n_i/I$.

	Default Only	Default & Migration
VaR	1.49	1.26

Table 2: Credit VaR with correlation $\rho = 0$

It can be seen that the VaR for the Default & Migration case is lower than the Default only case under the assumption that $R = 0$. This is due to the fact that the FV for staying in the same rating class is higher than $\mathbb{E}[FV]$ thus we can mark a profit in our balance sheet. Mathematically this leads to a left shift of the distribution of the loss curve thus, thanks to the monotonic nature of the VaR, we observe a lower quantile.

VaR for Different Correlations

We now pass to evaluate the influence of the correlation on the VaR. In particular we evaluate the following cases for the AVRs correlation: $R = 0.12$, $R = 0.24$ and $R = 0.21$ determined according to the IRB formula for a 1 year probability of default.
Repeating the procedure applied above we land on the following values:

$N = 200$	$R = 0$	$R = 0.12$	$R = 0.24$	R_{IRB}
Default Only	1.49	3.88	7.17	6.27
Default & Migration	1.26	4.21	7.80	6.94
Variation (%)	-15	9	9	11

Table 3: VaR for different AVR as a percentage of the value

Here we can see the relevancy of the correlation factor. Even a moderate value of $R = 0.12$ more than doubles our VaR in both cases with respect to the baseline case. This is due to the fact that there is a higher chance of multiple issuers defaulting together.
Here we can also observe an inversion in the Default Only and Default & Migration VaR. Contrary to what we observed for the $R = 0$ case, here the VaR for the D&M case is always larger.

Concentration risk

Now, to better study the concentration risk we consider only 20 obligors instead of the previous 200. Iterating the procedure employed above yields the following numerical values:

$N = 20$	$R = 0$	$R = 0.12$	$R = 0.24$	R_{IRB}
Default Only	5.98	8.96	8.96	8.96
Default & Migration	5.82	8.74	9.99	9.68
Variation (%)	-3	-2	11	8

Table 4: VaR for different AVR as a percentage of the value

As could be expected, a lower diversification results in a notably higher VaR, this is due to the fact that each default has a proportionally larger impact on the portfolio as a whole. It can be thus argued convincingly that the diversification is a relevant factor in determining a portfolio's VaR.

Discussion

1. (False) Inclusion of migration risk always impacts on the VaR of a portfolio even if there is a high degree of diversification and very high confidence level. Looking at the data (see Table 3), we can observe a variation of about 10% in the VaR's value. This can be explained by looking at the behaviour of the loss distribution curve across the two distinct scenarios. Notably, within the correlation's window of AVR at correlation coefficients ($R = 0.12$,$R = 0.24$) the inclusion of migration risks induces a rightward shift in the curve. Consequently, the quantile corresponding to a confidence level of 99.9% exhibits an increment in value, thus affecting the VaR as well.

2. (True) In a well diversified portfolio the VaR is very sensitive to changes in the AVR correlation in both Default Only and D&M cases. This is due to the fact that a higher correlation makes the market factor more influential and thus issuers tend to default together rather than in an independent fashion. As a consequence the Value at Risk increases in order to cover these interconnected risks.
   On the other hand, if we have a small number of obligors in our portfolio, we can observe a different behaviour in the two cases. In the Default Only case, a saturation effect can be seen, passing from $R = 0.12$ to $R = 0.24$ does not influence the VaR while in the D&M case we can see a slight change, with a positive slope.

3. (False) Inclusion of migration risk causes a decrease of VaR under the rather unrealistic assumption of perfectly uncorrelated issuers ($R = 0$) or a very small number of issuers. However, a well diversified portfolio with a correlation within the IRB window ($R_{min} = 0.12$, $R_{max} = 0.24$) does always show an increase when Migration risk is included. This is true thanks to the monotonicity of VaR as a risk measure, indeed a larger loss always leads to a larger VaR. From a more practical point of view this makes rather sense, in a realistic scenario when correlation between issuers grows not only do we expect them to default together at higher rates but to also be downgraded when related issuers are downgraded or default.

4. (False) A Credit Portfolio Model is sensitive to concentration. Indeed, even when considering a single systematic factor, portfolio diversification consistently reduces risks and subsequently the Value at Risk (VaR). This effect is particularly pronounced when the correlation coefficient (R) approaches zero, indicating minimal interdependence among obligors. As R approaches unity, the disparity between the risk levels of concentrated and diversified portfolios diminishes.

   Analyzing the empirical data provided, we observe a noteworthy increase in VaR of approximately 5% for an Asset-Value Relationship (AVR) correlation of $R = 0.12$, whereas this discrepancy narrows for $R = 0.24$.

   In the limiting case where $R = 1$, the idiosyncratic factor vanishes completely, leaving the model solely influenced by the systematic factor. In this extreme scenario, concentration loses its impact, as the correlation implies that the default of a single obligor precipitates the default of the entire portfolio, resulting in a binary outcome. In a very practical sense, when the correlation factor approaches 1, we are exposing ourselves to the risk associated to a single obligor: the market.

References

- *"A Generalized Single Common Factor Model of Portfolio Credit Risk "* Paul H. Kupiec
- *"An Explanatory Note on the Basel II IRB Risk Weight Functions"*, Bank for International Settlements