

# Homework-4

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## 1 Homework solution

### Task 1

Question: Suppose you are given a sequence (i.e., array) of size  $n$  with each entry holding a distinct number. For some value  $p$  between  $0$  and  $n - 1$ , the values in the array entries increase up to position  $p$  and then decrease on the remainder of the way until position  $n - 1$ . An example of such array would be  $[12, 17, 38, 54, 55, 69, 68, 44, 39, 19, 14, 7]$  where  $p$  and  $n$  could be  $5$  and  $12$ , respectively. Develop an algorithm to find the peak entry  $p$  without having to read the entire array. The algorithm should read as few entries of the array as possible. Also note that  $p$  could be  $0$  or  $n - 1$ , so your algorithm should be able to handle these corner cases as well.

Solution:

```
def peak(arr, left, right):
    if left == right:
        return left

    if arr[left] < arr[right] and right == left + 1:
        return right

    middle = (left + right) // 2

    if arr[left] >= arr[right] and right == left + 1:
        return left

    if arr[middle] > arr[middle + 1] and arr[middle] < arr[middle - 1]:
        return peak(arr, left, middle - 1)

    if arr[middle] > arr[middle + 1] and arr[middle] > arr[middle - 1]:
        return middle

    else:
        return peak(arr, middle + 1, right)

arr = [12, 17, 38, 54, 55, 69, 68, 44, 39, 19, 14, 7]
print("The index of a peak is %d" % peak(arr, 0, len(arr) - 1))
```

The index of a peak is 5

Total cost =  $T(n) = 2T(n/2) + n \leq 2c \cdot n/2 \log(n/2) + n = c \cdot n \log(n) - c \cdot n \log(2) + n = c \cdot n \log(n) - c \cdot n + n \leq c \cdot n \log(n)$   
So, the growth-rate function for this algorithm is  $O(n \log(n))$ .