In the Name of God

Communication Systems (25751-4)

Problem Set 01

Fall Semester 1402-03

Department of Electrical Engineering

Sharif University of Technology

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Due on Mehr 28, 1402 at 23:59

($*$) starred problems are optional and have a bonus mark!

1 Fourier Transform

Determine the Fourier transform of each of the following signals:

1. $x_1(t) = e^{-\alpha|t|}\cos(\beta t). \qquad (\alpha > 0)$
2. $x_2(t) = \Lambda(t) = (1-|t|)u(t+1)u(-t+1).$
3. $x_3(t) = \dfrac{t}{(a^2+t^2)^2}.$
4. $x_4(t) = \dfrac{\sin(t) - t\,.\,\cos(t)}{t^3}.$

2 Broadness of the Signal

1. Determine the Fourier transform of the signal below ($X(f)$):

$$x(t) = A\,.\,\exp\left(-\pi(\frac{t}{\tau})^2\right).$$

2. Sketch $|X(f)|^2$ for $\tau = 1$ and $\tau = 4$.
3. Explain how the signal broadness in time and frequency domain are related.

3 Parseval's Theorem

1. Let $x(t)$ and $y(t)$ be two energy-type signals, and let $X(f)$ and $Y(f)$ denote their Fourier transforms, respectively. Show that:

$$\int_{-\infty}^{+\infty} x(t)y^*(t)dt = \int_{-\infty}^{+\infty} X(f)Y^*(f)df.$$

2. Use Parseval's theorem to evaluate:

$$\int_0^{+\infty} \frac{\cos^2\left(\frac{\pi}{2}x\right)}{(1-x^2)^2}dx.$$

4 Fourier Series and Periodic Expansion

1. Assume that $x(t) = \sum_{k=-\infty}^{\infty} h(t - kT)$ where $h(t)$ is an arbitrary function. (Note: $h(t)$ may not be time-limited!) Prove that if:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt,$$

then:

$$x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right)e^{j2\pi\frac{k}{T}t}.$$

2. (*) Use the above theorem to evaluate:

$$\sum_{k=-\infty}^{\infty}\frac{e^{j\pi k}}{a^2+\pi^2k^2}.$$

(Your answer should not contain any infinite summations.)

3. Conclude the following relation known as *Poisson's sum formula*:

$$\sum_{k=-\infty}^{\infty} h(kT_s) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T_s}\right).$$

4. Using Poisson's sum formula, show that:

$$\sum_{k=-\infty}^{\infty}\frac{2\alpha}{\alpha^2+4\pi^2k^2} = \coth(\frac{\alpha}{2}). \qquad (\alpha > 0)$$

5 Types of Signals

Classify the following signals into energy-type, power-type, and neither energy-type nor power-type signals.
For energy-type or power-type signals find the energy or the power contents of the signal.

1. $x_1(t) = Ae^{j(2\pi f_0 t+\theta)}$.
2. $x_2(t) = e^{-\alpha t}\cos(\beta t)u(t) \qquad (\alpha > 0)$.
3. $x_3(t) = \sum_{n=-\infty}^{\infty}\Lambda(t-2n)$.
4. $x_4(t) = \begin{cases} Kt^{-\frac{1}{4}} & t > 0 \\ 0 & t \le 0 \end{cases}$.

6 Output of an LTI System

The response of an LTI system to $x_1(t) = e^{-\alpha t}u(t)$, $(\alpha > 0)$, is $y_1(t) = \delta(t)$. Using frequency domain analysis techniques find the response of the system to $x_2(t) = e^{-\alpha t}\cos(\beta t)u(t)$.

7 A Signal and Its Fourier Transform

1. Determine $x_1(t)$, whose Fourier transform $X_1(f)$ has the following magnitude and angle. Express $x_1(t)$ as a closed-form and sketch this function of time.

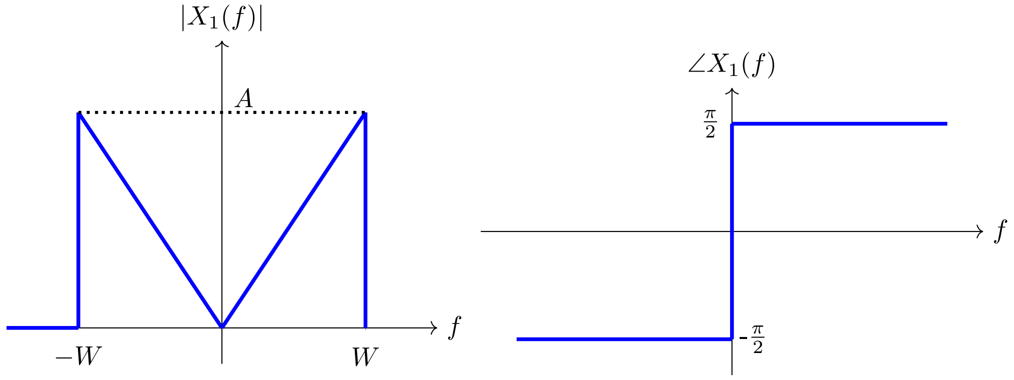

Figure 1: Problem 7, part 1

2. Determine $x_2(t)$, whose Fourier transform $X_2(f)$ has the following magnitude and angle. Express $x_2(t)$ as a closed-form and sketch this function of time.

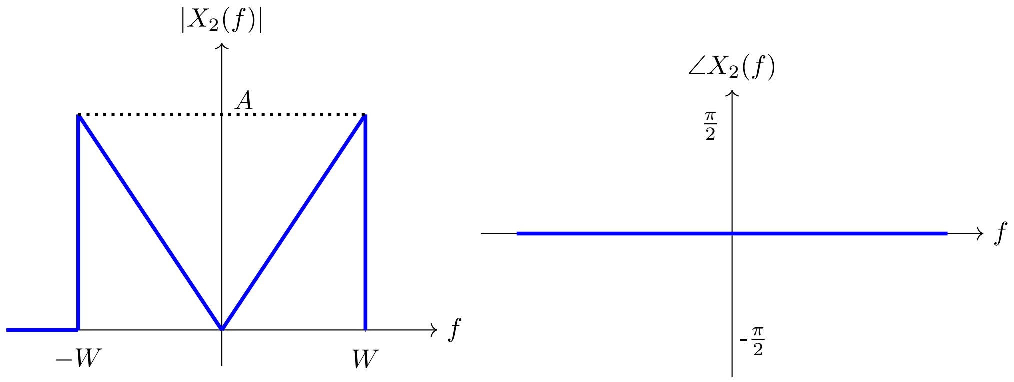

Figure 2: Problem 7, part 2

3. What are important similarities and differences between $x_1(t)$ and $x_2(t)$? how do those similarities and differences manifest in their Fourier transforms?

8 (∗) Fourier Transform and Real Integrals

Use the known properties of the Fourier transform to obtain the following:

1. $I_1 = \int_0^{+\infty} e^{-\alpha t}\text{sinc}^2(\beta t)dt. \qquad (\alpha > 0)$

2. $I_2 = \int_0^{+\infty} \frac{\sin^4(t)}{t^4}dt.$