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تمرین سری 6 - سیستم های غیرخطی - علی یداللهی - 9006 2233

سؤال 1)

1. $y(t) = g\left(\frac{t}{T_s}\right)$

$$g(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\,\Lambda(t - kT_s)$$

$$g(t) = \Lambda(t) * \left[\sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t - kT_s)\right]$$

$$G(f) = \text{sinc}^2(f)\left[f_s \sum_{k=-\infty}^{\infty} X(f - n f_s)\right]$$

$$y(t) = \Lambda\left(\frac{t}{T_s}\right) * \underbrace{\left[\sum_{k=-\infty}^{\infty} x(kT_s)\,\delta\left(\frac{t - kT_s}{T_s}\right)\right]}_{(*)}$$

$$(*): T_s \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t - kT_s)$$

$$Y(f) = T_s\,\text{sinc}^2(fT_s)\left[\sum_{k=-\infty}^{\infty} X(f - n f_s)\right]$$

2. $f_s - W > W \longrightarrow f_s > 2W \longrightarrow T_s < \frac{1}{2W}$

3. $$H_{eq}(f) = \frac{k e^{-j2\pi f t_d}}{T_s\,\text{sinc}^2(fT_s)}$$

سؤال 2)

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} X(nf_0)\,\delta(f - nf_0)$$

$$\hookrightarrow x_\delta(t) = \sum_{n=-\infty}^{\infty} X(nf_0)\,e^{j2\pi n f_0 t}$$

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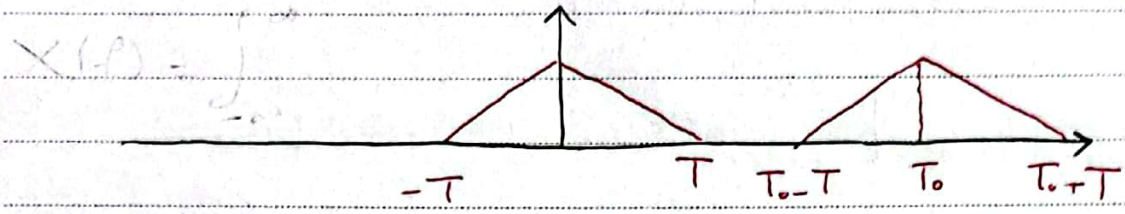
$$X_\delta(f) = \sum_{n=-\infty}^{\infty} X(nf_0)\,\delta(f-nf_0) = \sum_{n=-\infty}^{\infty} X(f)\,\delta(f-nf_0)$$

$$= X(f) \sum_{n=-\infty}^{\infty} \delta(f-nf_0) \longrightarrow x_\delta(t) = x(t) * \left[T_0 \sum_{n=-\infty}^{\infty} \delta(t-nT_0)\right]$$

$$\longrightarrow x_\delta(t) = T_0 \sum_{n=-\infty}^{\infty} x(t-nT_0)$$



$$T \leq T_0 - T \longrightarrow 2T \leq T_0 \longrightarrow \frac{1}{T_0} \leq \frac{1}{2T}$$

$$\longrightarrow f_0 \leq \frac{1}{2T} \quad \checkmark$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\,e^{-j2\pi ft}\,dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sum_{n=-\infty}^{\infty} X(nf_0)\,e^{j2\pi nf_0 t}\,e^{-j2\pi ft}$$

$$= \frac{1}{T_0} \sum_{n=-\infty}^{\infty} X(nf_0) \int_{-T_0/2}^{T_0/2} e^{j2\pi t(nf_0 - f)}\,dt$$

$$= \sum_{n=-\infty}^{\infty} X(nf_0)\,\frac{\sin(\pi T_0(nf_0 - f))}{\pi T_0(nf_0 - f)}$$

$$\longrightarrow X(f) = \sum_{n=-\infty}^{\infty} X(nf_0)\,\mathrm{sinc}(n - fT_0)$$

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سوال 3)

$$SQNR \triangleq \frac{\text{signal Power}}{\text{Q noise power}}$$

$$\sigma_q^2 = \frac{\Delta^2}{12} \quad , \Delta = \frac{2x_m}{2^v} \Rightarrow SQNR = \frac{3P_x}{x_m^2} 2^{2v}$$

$$P_x = \int_{-x_m}^{x_m} x^2 f_X(x)\, dx = \frac{1}{2x_m} \int_{-x_m}^{x_m} x^2 dx$$

$$= \frac{1}{2x_m} \left(\frac{x^3}{3}\right)\Big|_{-x_m}^{x_m} = \frac{1}{2x_m} \frac{2x_m^3}{3} = \frac{1}{3} x_m^2$$

$$\rightarrow SQNR = \frac{3(\frac{1}{3}x_m^2)}{x_m^2} 2^{2v} = 2^{2v}$$

سوال 4)

1) $y(t) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\, \delta(t - nT_s)$ * $f_s = 1/T_s$

$$Y(f) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\, e^{-j2\pi n f T_s} = f_s \sum_{n=-\infty}^{\infty} (-1)^n X(f - nf_s)$$

2) $X(f)$ با قرار دادن یک فیلتر پایین گذر با پهنای باند W می توان $x(t)$ را استخراج کرد و از روی آن $x(t)$ را به دست آورد.

همچنین می توان نوشت: برای $f_s = W$

$$Y(f) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\, e^{-j2\pi f n T_s}$$

$$\rightarrow x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} dt = T_s \int_{-f_s/2}^{f_s/2} X(nT_s)\, e^{-j2\pi n f T_s} e^{j2\pi ft} (-1)^n df$$

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سؤال 4 ـ ادامه

*ـw < f < w برای : $Y(f) = f_s X(f)$

$Y(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s) = f_s X(f)$

$$\rightarrow x(t) = T_s \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s) \int_{-f_s/2}^{f_s/2} e^{j2\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin(-f_s\pi t - \pi n)}{f_s\pi t - \pi n} (-1)^n$$

$$= \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s) \operatorname{sinc}(2f_s t - n)$$

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سؤال 6)

$f_{Nyquist} = 2 \times 3000 = 6000\ Hz$

$f = \frac{4}{3}(6000) = \boxed{8000}$

$\Delta = \frac{2mp}{L} \longrightarrow \frac{\Delta}{2} = \frac{mp}{L} \leq \frac{5}{1000} mp$ L: تعداد سطوح

$\longrightarrow L \geq 200 \longrightarrow L = 256 = 2^n \longrightarrow n = 8$

$n f_s = 64\ kbit/sec$

برای فرستادن 2B bit/sec به کانالی با پهنای باند B احتیاج داریم

$BW = \frac{1}{2} 64k = \boxed{32 kHz}$

$24 \times 64k \times 1.02 = 1566720$

$BW = \frac{1}{2}(1566720) = \boxed{783360\ Hz = 783.36\ kHz}$

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سوال 5

$$f_Y(y) = f_X(y+1.5) + f_X(y+0.5) - f_X(y-0.5) + f_X(y-1.5)$$

$$\rightarrow f_Y(y) = \left(\frac{1}{2} - \frac{1}{4}\left(y+\frac{3}{2}\right)\right) + \left(\frac{1}{2} - \frac{1}{4}\left(y+\frac{1}{2}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\left(y-\frac{1}{2}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\left(y-\frac{3}{2}\right)\right)$$

$$\rightarrow f_Y(y) = 1 \qquad -\frac{1}{2} \le y \le \frac{1}{2}$$