In the Name of God

Communication Systems (25751-4)

Problem Set 03

Fall Semester 1402-03

Department of Electrical Engineering

Sharif University of Technology

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Due on Aban 17, 1402 at 17:00

(∗) starred problems are optional and have a bonus mark!

1 Autocorrelation of Morlet Wavelet

The signal $x(t) = e^{-t^2}\cos(2\pi f_0 t)$ known as real-valued Morlet wavelet function is shown in figures bellow:

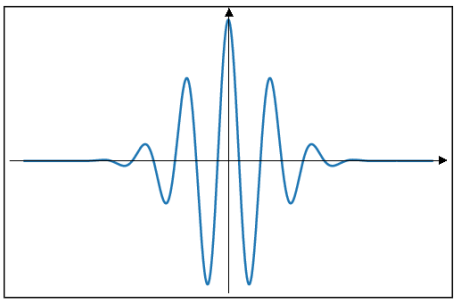

Figure 1: Real-valued Morlet Wavelet

1. Prove that $x(t)$ is an energy signal.
2. Determine the Fourier transform of this signal.
3. Determine the spectral density function of this signal.
4. Determine the autocorrelation function of this signal.

2 Power Spectral Density Estimation

Show that if $x_T(t)$ denotes the truncated signal corresponding to the power-type signal $x(t)$; that is:

$$x_T(t) = \begin{cases} x(t) & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases},$$

and if $\mathcal{S}_{x_T}(f)$ denotes the energy spectral density of $x_T(t)$, then $\mathcal{S}_x(f)$, the power-spectral density of $x(t)$, can be expressed as

$$\mathcal{S}_x(f) = \lim_{T\to\infty} \frac{\mathcal{S}_{x_T}(f)}{T}$$

3 Cross-correlation of Rectangular Pulse and Triplet Pulse

Determine the cross-correlation function of the rectangular pulse and the triplet pulse shown in figures bellow:

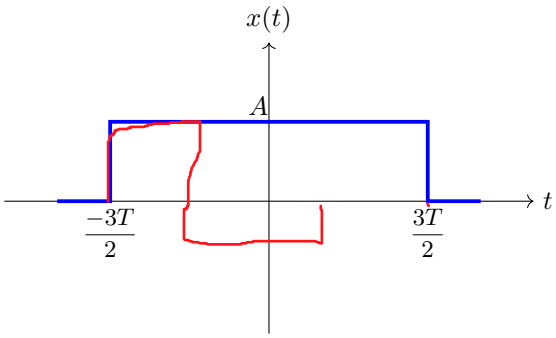

Figure 2: Rectangular Pulse

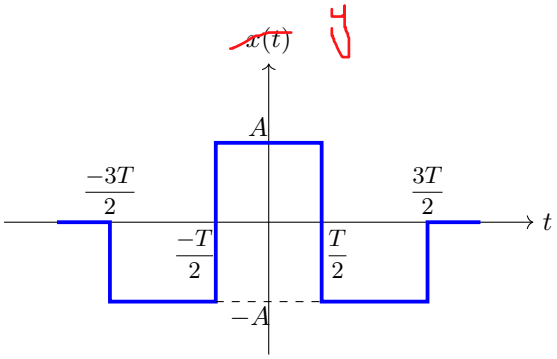

Figure 3: Triplet Pulse

4 Energy and Power Spectral Density

Show that for the power-type signal $x(t)$ the following holds.

$$\mathcal{S}_x(f) = \mathcal{F}\{R_x(\tau)\}$$

Then show that if applying this signal to an LTI system with impulse response $h(t)$ leads to the output $y(t)$, the following holds.

$$\mathcal{S}_y(f) = \mathcal{S}_x(f)|H(f)|^2$$

5 Double Side-Band Modulation

The message signal $m(t) = 2\cos(400t) + 4\sin(500t + \frac{\pi}{3})$ modulates the carrier signal $c(t) = A\cos(8000\pi t)$, using DSB amplitude modulation. Find the time domain and frequency domain representation of the modulated signal and plot the spectrum (Fourier transform) of the modulated signal. What is the power content of the modulated signal?

6 Amplitude Modulation

An AM signal has the form

$$u(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)]\cos(2\pi f_c t)$$

where $f_c = 10^5$ Hz.

1. Sketch the (voltage) spectrum of $u(t)$.
2. Determine the power in each of the frequency components.
3. Determine the modulation index.
4. Determine the power in the sidebands, the total power, and the ratio of the sidebands power to the total power.