

سیستم های خازنی - قریب سری چهار - علی بدالی - ۲۰۰۱/۲۲۲۲

(سوال 1)

1. سری فوریه  $s(t)$  را می نویسیم.

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_c t}$$

$$c_n = \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} s(t) e^{-j2\pi n f_c t} dt$$

$$= \frac{1}{T_p} \left[ \int_{-\frac{T_p}{2}}^0 (-A) e^{-j2\pi n f_c t} dt + \int_0^{\frac{T_p}{2}} A e^{-j2\pi n f_c t} dt \right]$$

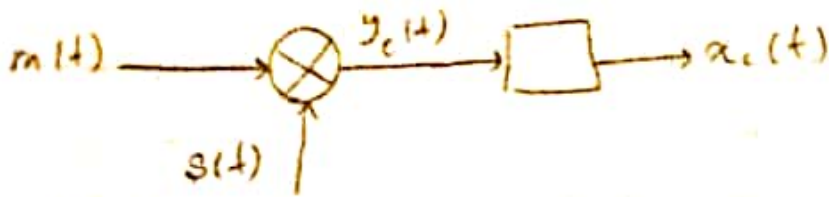
$$= \frac{1}{T_p} \left[ \frac{A}{j2\pi n f_c} e^{-j2\pi n f_c t} \Big|_{-\frac{T_p}{2}}^0 - \frac{A e^{-j2\pi n f_c t}}{j2\pi n f_c} \Big|_0^{\frac{T_p}{2}} \right]$$

$$= \frac{A}{j2\pi n} [1 - e^{j2\pi n f_c \frac{T_p}{2}}] - \frac{A}{j2\pi n} [e^{-j2\pi n f_c \frac{T_p}{2}} - 1]$$

$$= \frac{A}{j2\pi n} (1 - e^{jn\pi}) - \frac{A}{j2\pi n} (e^{-jn\pi} - 1)$$

$$= \frac{A}{2j\pi n} (2 - (e^{jn\pi} + e^{-jn\pi}))$$

$$= \frac{A}{2j\pi n} (1 - \cos(n\pi)) = \frac{A}{j\pi n} (1 - (-1)^n)$$



$$y_c(t) = m(t)s(t) = m(t) \left[ \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{A}{jn\pi} (1 - (-1)^n) e^{jn\pi f_c t} + C_0 \right]$$

$$\rightarrow Y_c(f) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{A}{jn\pi} (1 - (-1)^n) M(f - nf_c) + C_0 M(f)$$

پس از عبور از فیلتر داریم:

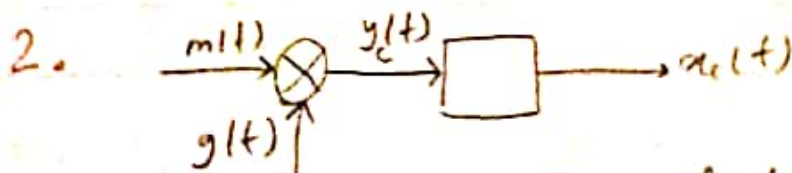
$$X_c(f) = \frac{A}{j(-1)\pi} (1 - (-1)^{(-1)}) M(f + f_c) + \frac{A}{j(1)\pi} (1 - (-1)^{(1)}) M(f - f_c)$$

$$\rightarrow X_c(f) = \frac{2A}{j\pi} [M(f - f_c) - M(f + f_c)]$$

$$\rightarrow x_c(t) = \frac{2A}{j\pi} m(t) [e^{j2\pi f_c t} - e^{-j2\pi f_c t}]$$

$$\rightarrow x_c(t) = \frac{4A}{\pi} m(t) \left( \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j} \right)$$

$$= \underbrace{\frac{4A}{\pi}}_{A_c} m(t) \sin(2\pi f_c t)$$



$$g(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{jn\pi f_c t}$$

Subject .

Date

درتیب سینیغ - سونوب یک ادا نم:

$$y_c(t) = m(t) \left[ c_0 + \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_c t} \right]$$

$$Y_c(f) = (c_0 M(f)) = \sum_{n=-\infty}^{\infty} c_n M(f - n f_c)$$

$$\rightarrow X_c(f) = c_1 M(f - f_c) + c_{-1} M(f + f_c)$$

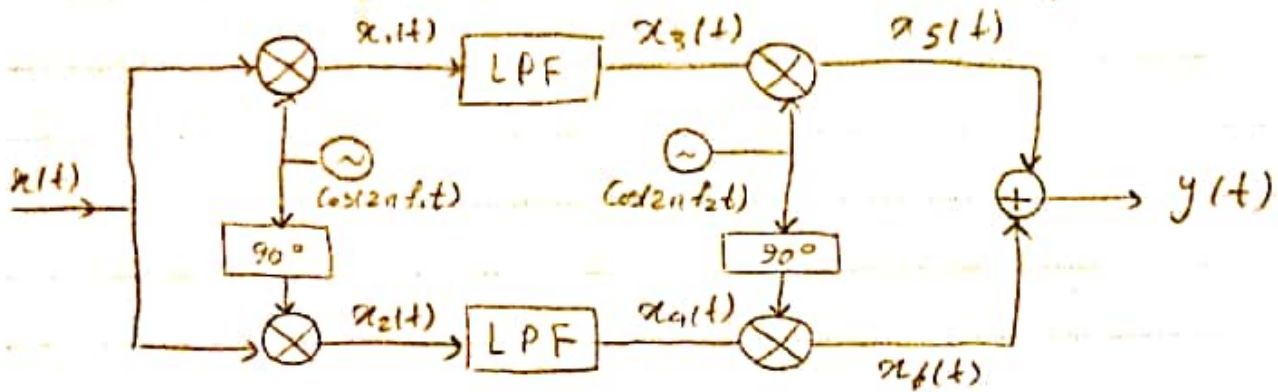
$$\xrightarrow{c_1 = -c_{-1} : \text{فرد}} c_1 (M(f - f_c) - M(f + f_c))$$

$$\rightarrow x_c(t) = c_1 m(t) [e^{j2\pi f_c t} - e^{-j2\pi f_c t}]$$

$$\rightarrow x_c(t) = 2c_1 j m(t) \sin(2\pi f_c t)$$



(سوال ۲)



$$x_1(t) = \cos(2\pi f_m t) \cos(2\pi f_1 t) = \frac{1}{2} \cos(2\pi(f_m + f_1)t) + \frac{1}{2} \cos(2\pi(f_m - f_1)t)$$

$$x_2(t) = \cos(2\pi f_m t) \sin(2\pi f_1 t) = \frac{1}{2} \sin(2\pi(f_m + f_1)t) - \frac{1}{2} \sin(2\pi(f_1 - f_m)t)$$

\* اگر  $|f_1 - f_m| < W$  و  $f_1 + f_m > W$  باشد (مثلاً می‌توان  $f_1 = W$  در نظر گرفت)

$$\rightarrow x_3(t) = \frac{1}{2} \cos(2\pi(f_m - f_1)t) \text{ و } x_4(t) = \frac{1}{2} \sin(2\pi(f_1 - f_m)t)$$

$$x_5(t) = \frac{1}{2} \cos(2\pi(f_m - f_1)t) \cos(2\pi f_2 t)$$

$$x_6(t) = \frac{1}{2} \sin(2\pi(f_1 - f_m)t) \sin(2\pi f_2 t)$$

$$\rightarrow y(t) = x_5(t) + x_6(t) = \frac{1}{2} \left[ \cos(2\pi(f_1 - f_m)t) \cos(2\pi f_2 t) + \sin(2\pi(f_1 - f_m)t) \sin(2\pi f_2 t) \right]$$

$$\rightarrow y(t) = \frac{1}{2} \cos(2\pi(f_1 - f_m - f_2)t) = \frac{1}{2} \cos(2\pi(f_m - f_1 + f_2)t)$$

\* اگر  $x(t) = \cos(2\pi f_m t)$ ,  $A_c = 1$

$$x_{USB}(t) = \frac{1}{2} (\cos(2\pi f_m t) \cos(2\pi f_c t) - \frac{1}{2} \sin(2\pi f_m t) \sin(2\pi f_c t))$$

$$= \frac{1}{2} \cos(2\pi (f_m + f_c) t)$$

$\rightarrow f_m = f_1 + f_2 = f_m + f_c \rightarrow f_2 = f_c + f_1$  USB

\*  $x_{LSB} = \frac{1}{2} (\cos(2\pi f_m t) \cos(2\pi f_c t) + \frac{1}{2} \sin(2\pi f_m t) \sin(2\pi f_c t))$

$$= \frac{1}{2} \cos(2\pi (f_c - f_m) t)$$

LSB

$\rightarrow f_c - f_m = f_m - f_1 + f_2 \rightarrow f_2 = f_c + f_1 - 2f_m$

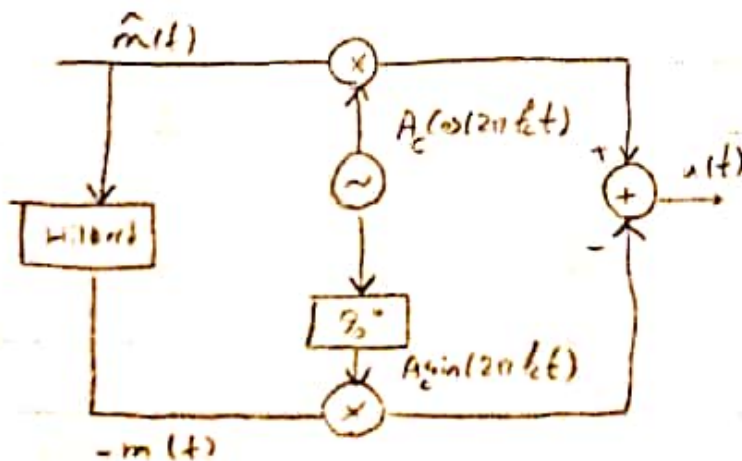
USB:  $f_1 = W$ ,  $f_2 = f_c + W$

LSB:  $f_1 = W$ ,  $f_2 = f_c + W - 2f_m$

سوال (3)

Phase shift method

1.



$$\hat{m}(t) = -m(t)$$



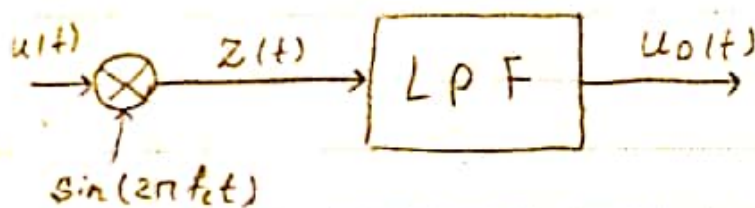
سوال 5 - ادامه

$$u(t) = \frac{A_c}{2} \hat{m}(t) \cos(2\pi f_c t) - \frac{A_c}{2} (-m(t)) \sin(2\pi f_c t)$$

$$= \frac{A_c}{2} \hat{m}(t) \cos(2\pi f_c t) + \frac{A_c}{2} m(t) \sin(2\pi f_c t)$$

2.

Demodulation:



$$z(t) = \frac{A_c}{2} \cos(2\pi f_c t) \sin(2\pi f_c t) \hat{m}(t) + \frac{A_c}{2} \sin^2(2\pi f_c t) m(t)$$

$$= \frac{A_c}{4} \sin(4\pi f_c t) \hat{m}(t) + \frac{A_c}{4} (1 - \cos(4\pi f_c t)) m(t)$$

$$\rightarrow u_0(t) = \frac{A_c}{4} m(t)$$

$$y(t) = y_I(t) \cos(2\pi f_c t) - y_Q(t) \sin(2\pi f_c t) \quad (\text{سوال 4})$$

$$y(t) = \frac{\alpha}{2} [\cos(2\pi (f_c + f_m) t) - \cos(2\pi (f_c - f_m) t)]$$

$$+ \frac{1}{2} \cos(2\pi (f_c - f_m) t) + \cos(2\pi f_c t)$$

$$= -\alpha \sin(2\pi f_m t) \sin(2\pi f_c t) + \frac{1}{2} [\cos(2\pi f_c t) \cos(2\pi f_m t) + \sin(2\pi f_c t) \sin(2\pi f_m t)] + \cos(2\pi f_c t)$$

Subject: \_\_\_\_\_

Date: \_\_\_\_\_

$$= \left( \frac{1}{2} \cos(2\pi f_m t) + 1 \right) \cos(2\pi f_c t) \quad (4 \text{ جمله})$$

$$- \left[ \alpha \sin(2\pi f_m t) - \frac{1}{2} \sin(2\pi f_m t) \right] \sin(2\pi f_c t)$$

$$y_I(t) = 1 + \frac{1}{2} \cos(2\pi f_m t)$$

$$y_Q(t) = \left( \alpha - \frac{1}{2} \right) \sin(2\pi f_m t)$$

$$e(t) = \sqrt{y_I^2(t) + y_Q^2(t)} = \sqrt{\left[ 1 + \frac{1}{2} \cos(2\pi f_m t) \right]^2 + \left[ \left( \frac{1}{2} - \alpha \right) \sin(2\pi f_m t) \right]^2}$$

$$= \left[ 1 + \frac{1}{2} \cos(2\pi f_m t) \right] \sqrt{1 + \left( \frac{\left( \frac{1}{2} - \alpha \right) \sin(2\pi f_m t)}{1 + \frac{1}{2} \cos(2\pi f_m t)} \right)^2}$$

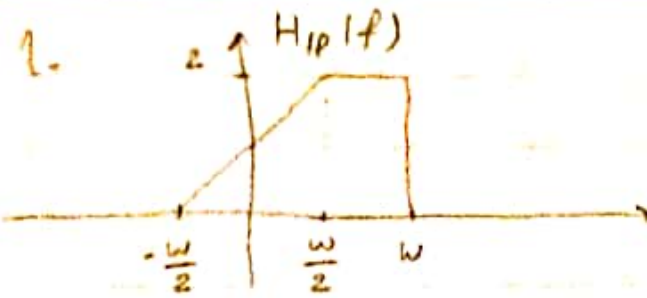
$$= \left[ 1 + \frac{1}{2} \cos(2\pi f_m t) \right] \sqrt{1 + \left( \frac{(1-2\alpha) \sin(2\pi f_m t)}{2 + \cos(2\pi f_m t)} \right)^2}$$

2.

$$0 \leq \alpha < 1 \rightarrow (1-2\alpha)^2 \leq 1 \rightarrow$$

در  $\alpha=0$  مقدار  $d(t)$  متناهی می شود

سؤال (5)



$$h_{lp}(t) = \mathcal{F}^{-1}\{H_{lp}(f)\}$$

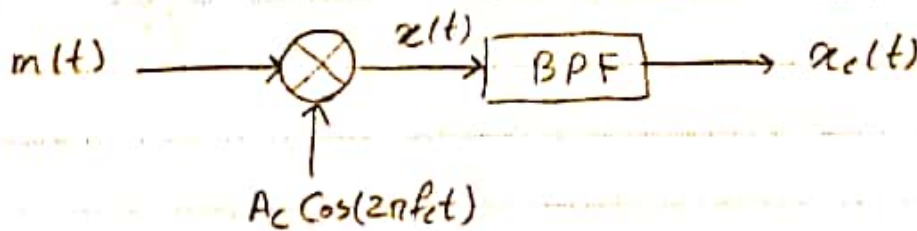
$$= \int_{-\frac{W}{2}}^W H_{lp}(f) e^{j2\pi ft} df = \int_{-\frac{W}{2}}^{\frac{W}{2}} \left(\frac{2f}{W} + 1\right) e^{j2\pi ft} df + \int_{\frac{W}{2}}^W 2 e^{j2\pi ft} df$$

$$= \frac{2}{W} \left[ \frac{1}{j2\pi t} f e^{j2\pi ft} + \frac{1}{4\pi^2 t^2} e^{j2\pi ft} \right]_{-\frac{W}{2}}^{\frac{W}{2}} + \frac{e^{j2\pi ft}}{j2\pi t} \Big|_{\frac{W}{2}}^W$$

$$+ \frac{2}{j2\pi t} e^{j2\pi ft} \Big|_{\frac{W}{2}}^W = \frac{1}{j\pi t} e^{j2\pi Wt} + \frac{j}{\pi^2 t^2 W} \sin(\pi W t)$$

$$= \frac{j}{\pi t} [\text{sinc}(Wt) - e^{j2\pi Wt}]$$

2.



$$z(t) = A_c m(t) \cos(2\pi f_c t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t)$$

$$x_I(t) = A_c m(t) \quad x_Q(t) = 0, \quad x_{lp} = x_I(t) + j x_Q(t) = A_c m(t)$$

$$x_{c,lp}(t) = \frac{1}{2} V_{lp}(t) * h_{lp}(t) = \frac{1}{2} A_c m(t) + \frac{j}{\pi t} [\text{sinc}(Wt) - e^{j2\pi Wt}]$$



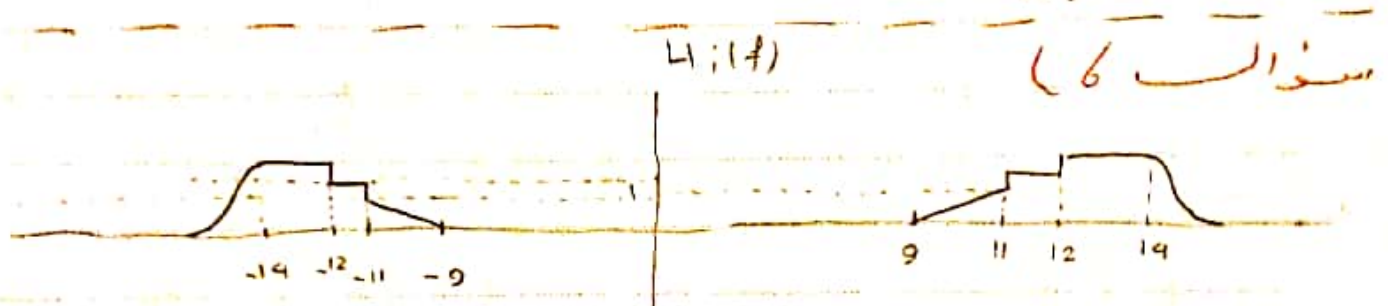
$$= \frac{1}{2} A_c \left[ m(t) * \frac{j \operatorname{sinc}(\omega t)}{\pi t} + m(t) * \frac{e^{j2\pi \omega t}}{j\pi t} \right]$$

$$F \left\{ m(t) * \frac{e^{j2\pi \omega t}}{j\pi t} \right\} = -M(f) \operatorname{sgn}(f - \omega) = M(f)$$

$$x_{c,lp}(t) = \frac{1}{2} A_c \left[ m(t) * \frac{j \operatorname{sinc}(\omega t)}{\pi t} + m(t) \right]$$

$$\rightarrow x_c(t) = \operatorname{Re} \{ x_{c,lp}(t) e^{j2\pi f_c t} \}$$

$$= \frac{1}{2} A_c m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \left[ m(t) * \frac{\operatorname{sinc}(\omega t)}{\pi t} \right] \sin(2\pi f_c t)$$



سؤال 6

قسمت مثبت:

$$H(f) = \begin{cases} 9 < f < 11 : \frac{1}{2} f - \frac{9}{2} \\ 11 < f < 12 : \frac{3}{2} \\ 12 < f < 14 : 1 \end{cases}$$

شيفت بدولت زير

$$x_c(f) = \begin{cases} m(f - 10) \left( \frac{f}{2} - \frac{9}{2} \right) & 9 < f < 11 \\ \frac{3}{2} m(f - 10) & 11 < f < 12 \\ 2 m(f - 10) & 12 < f < 14 \end{cases}$$

$$\rightarrow M'(f) = \begin{cases} m(f) \left( \frac{f}{2} + \frac{1}{2} \right) & -1 < f < 1 \\ \frac{3}{2} m(f) & 1 < f < 2 \\ 2 m(f) & 2 < f < 4 \end{cases}$$

سوال 6 - اداره :

$$H_2(f) : \begin{cases} -11 < f < -9 & -\frac{1}{2} f - \frac{9}{2} \\ -12 < f < -11 & \frac{3}{2} \\ -14 < f < -12 & 2 \end{cases} \quad \text{قسمت منقی:}$$

$$x_{r2}(f) : \begin{cases} M(f+10) \left(-\frac{f}{2} - \frac{9}{2}\right) & -11 < f < -9 \\ \frac{3}{2} M(f+10) & -12 < f < -11 \\ 2 M(f+10) & -14 < f < -12 \end{cases} \quad \text{حول فرکانس صفر}$$

$$m'_2(f) : \begin{cases} M(f) \left(-\frac{f}{2} + \frac{1}{2}\right) & -1 < f < 1 \\ \frac{3}{2} M(f) & -2 < f < -1 \\ 2 M(f) & -4 < f < -2 \end{cases}$$

$$m'(f) \text{ (قبل از اعمال)} : m'_1(f) + m'_2(f)$$

$$= \begin{cases} 2 M(f) & -4 < f < -2 \\ \frac{3}{2} M(f) & -2 < f < -1 \\ M(f) & -1 < f < 1 \\ \frac{3}{2} M(f) & 1 < f < 2 \\ 2 M(f) & 2 < f < 4 \end{cases}$$

Subject: \_\_\_\_\_

Date: \_\_\_\_\_

مسئله ۶-۱۰

$$M'(f) H_0(f) = M(f)$$

$$H_0(f) = \begin{cases} -4 < f < -2 & : 1/2 \\ -2 < f < -1 & : 2/3 \\ -1 < f < 1 & : 1 \\ 1 < f < 2 & : 2/3 \\ 2 < f < 4 & : 1/2 \end{cases}$$

