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تمرین سری ۱ - سیستم های مخابراتی - علی یداللهی - 400102233

1)

1) $F(e^{-\alpha|t|}) = \dfrac{2\alpha}{\alpha^2+(2\pi f)^2} = G(f)$

$F\{\cos(\beta t)\} = \frac{1}{2}\left[\delta\left(f+\frac{\beta}{2\pi}\right) + \delta\left(f-\frac{\beta}{2\pi}\right)\right]$

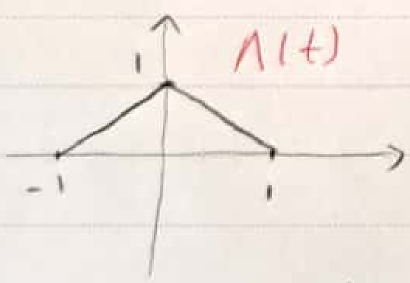
$F\{e^{-\alpha|t|}\cos(\beta t)\} = \frac{1}{2} G(f) * \left[\delta\left(f+\beta/_{2\pi}\right) + \delta\left(f-\beta/_{2\pi}\right)\right]$

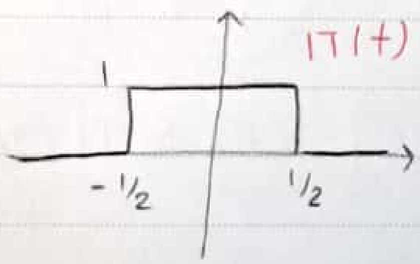
$= \frac{1}{2}\left[G\left(f+\frac{\beta}{2\pi}\right) + G\left(f-\frac{\beta}{2\pi}\right)\right]$

$= \alpha\left[\dfrac{1}{\alpha^2+4\pi^2\left(f+\frac{\beta}{2\pi}\right)^2} + \dfrac{1}{\alpha^2+4\pi^2\left(f-\beta/_{2\pi}\right)^2}\right]$

2) $\Lambda(t) = \Pi(t) * \Pi(t)$: می دانیم که

که :





$F\{\Pi(t)\} = \int_{-\infty}^{\infty} \Pi(t) e^{-j2\pi f t} dt = \int_{-1/2}^{1/2} e^{-j2\pi f t} dt$

$= \dfrac{\sin(\pi f)}{\pi f} = \operatorname{sinc}(f)$

$\Lambda(t) = \Pi(t) * \Pi(t) \longrightarrow F(\Lambda(t)) = F(\Pi(t))\, F(\Pi(t))$

$= \operatorname{sinc}^2(f)$

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3) آکس : $f(t) = -\frac{1}{2} \frac{1}{a^2+t^2} \rightarrow \frac{d}{dt} f(t) = \frac{t}{(a^2+t^2)^2}$

$$e^{-a|t|} \xrightarrow{F} \frac{1}{a^2+4\pi^2 f^2}$$

دوگانی : $\underbrace{\frac{1}{a^2+4\pi^2 t^2}}_{g(t)} \xrightarrow{F} \underbrace{e^{-a|f|}}_{G(f)}$

$$f(t) = -\frac{1}{2} g\left(\frac{t}{2\pi}\right) \rightarrow F(f) = 2\pi\, G(2\pi f)$$

$$\rightarrow F(f) = \frac{2\pi e^{-a|2\pi f|}}{-2} = -\pi e^{-a|2\pi f|}$$

$$F\left(\frac{d}{dt} f(t)\right) = j2\pi f\, (-\pi e^{-a|2\pi f|} = -2j\pi^2 f e^{-a|2\pi f|}$$

4) $f(t) = \frac{d}{dt}\left(\frac{\sin(t)}{t}\right) = \frac{t\cos(t) - \sin(t)}{t^2}$

$$g(t) = \frac{-1}{t} \longrightarrow x_4(t) = f(t)\, g(t)$$

$$F(f) = (j2\pi f)\, \pi \Pi(\pi f) = 2\pi^2 j f\, \Pi(\pi f)$$

$$G(f) = j\pi\, \mathrm{sgn}(f)$$

$$x_4(t) = f(t) g(t) \rightarrow X_4(f) = F(f) * G(f) = \int_{-\infty}^{\infty} F(\alpha)\, G(f-\alpha)\, d\alpha$$

$$= -2\pi^3 \int_{-\infty}^{\infty} \alpha\, \Pi(\pi\alpha)\, \mathrm{sgn}(f-\alpha)\, d\alpha = -2\pi^3 \int_{-1/2\pi}^{1/2\pi} \alpha\, \mathrm{sgn}(f-\alpha)\, d\alpha$$

$$= -2\pi^3\, Y(\alpha)\Big|_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}}$$

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تمرین سیستم ادامه

۴) ادامه:

$$Y_1(\alpha) = \begin{cases} \alpha > f & -\alpha^2/2 \\ \alpha < f & \alpha^2/2 \end{cases} = \frac{\alpha^2}{2}\,\mathrm{sgn}(f-\alpha)$$

$$\rightarrow X_4(f) = -2\pi^3 \left(\frac{\alpha^2}{2}\,\mathrm{sgn}(f-\alpha)\right)\Bigg|_{\alpha=-1/2\pi}^{\alpha=1/2\pi}$$

$$= -\pi^3\left(\frac{1}{4\pi^2}\,\mathrm{sgn}\left(f-\frac{1}{2\pi}\right) - \frac{1}{4\pi^2}\,\mathrm{sgn}\left(f+\frac{1}{2\pi}\right)\right)$$

$$= \frac{\pi}{4}\left[\mathrm{sgn}\left(f+\frac{1}{2\pi}\right) - \mathrm{sgn}\left(f-\frac{1}{2\pi}\right)\right]$$

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تمرین سلیسخ

سؤال 2)

1) $g(t) = e^{-\pi t^2} \rightarrow \frac{d}{dt} g(t) = -2\pi t e^{-\pi t^2} = -2\pi t g(t)$

$\rightarrow j2\pi f\, G(f) = -j\frac{d}{df} G(f)$

$\rightarrow \frac{d}{df} G(f) + 2\pi f\, G(f) = 0$

$\rightarrow \frac{dG(f)}{df} = -2\pi f\, G(f) \rightarrow \frac{dG(f)}{G(f)} = -2\pi f df$

$\rightarrow \ln(G(f)) = -\pi f^2 + c_1$

$\rightarrow G(f) = e^{-\pi f^2} e^{c_1} = k e^{-\pi f^2}$

$G(0) = \int_{-\infty}^{\infty} e^{-\pi t^2} dt = 1 \rightarrow G(f) = e^{-\pi f^2}$

$x(t) = A g(\frac{t}{\tau}) \rightarrow X(f) = A\tau e^{-\pi(\tau f)^2}$

2) در انتهای تمرین شکل ا داده شده است

3) با افزایش دامنه سیگنال در حوزه زمان دامنه آن در حوزه فرکانس کاهش می یابد و برعکس.

3)

1) $\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \right) \left(\int_{-\infty}^{\infty} Y(f') e^{j2\pi f' t} df' \right)^* dt$

$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \right) \left(\int_{-\infty}^{\infty} Y^*(f') e^{-j2\pi f' t} df' \right) dt$

$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f) Y^*(f') \int_{-\infty}^{\infty} e^{2\pi j (f-f')t} dt \; df' df$ (*)

$F(\delta(f-f')) = \int_{-\infty}^{\infty} e^{2\pi j (f-f')} = \delta(f-f')$

(*) $= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f)\, Y^*(f')\, \delta(f-f')\, df' df$

$= \int_{-\infty}^{\infty} X(f) \int_{-\infty}^{\infty} Y^*(f')\, \delta(f-f')\, df' \, df = \int_{-\infty}^{\infty} X(f) Y^*(f)\, df$ $\square$

سؤال 3 - ب)

زوج است

$$\int_0^\infty \frac{\cos^2(\frac{\pi}{2}x)}{(1-x^2)^2}dx = \frac{1}{2}\int_{-\infty}^\infty \frac{\cos^2(\frac{\pi}{2}x)}{(1-x^2)^2}$$

$$g(x) = \frac{\cos(\frac{\pi}{2}x)}{1-x^2} = \frac{1}{2}\cos(\frac{\pi}{2}x)\left[\frac{1}{1-x}+\frac{1}{1+x}\right]$$

$$= \frac{1}{2}\sin(\frac{\pi}{2}-\frac{\pi}{2}x)\left[\frac{1}{1-x}+\frac{1}{1+x}\right]$$

$$= \frac{1}{2}\frac{\sin(\frac{\pi}{2}-\frac{\pi}{2}x)}{1-x}+\frac{1}{2}\frac{\sin(\frac{\pi}{2}-\frac{\pi}{2}x)}{1+x}$$

$$\left.\begin{aligned} &* \sin(\tfrac{\pi}{2}-\tfrac{\pi}{2}x) = -\sin(\tfrac{\pi}{2}x-\tfrac{\pi}{2}) \\ &* \sin(\tfrac{\pi}{2}x-\tfrac{\pi}{2}) = -\sin(\tfrac{\pi}{2}x+\tfrac{\pi}{2}) \end{aligned}\right\} \to \sin(\tfrac{\pi}{2}-\tfrac{\pi}{2}x) = \sin(\tfrac{\pi}{2}+\tfrac{\pi}{2}x)$$

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(سوال 3 ـ ب ـ ادامه)

$$\rightarrow g(x) = \frac{1}{2}\,\frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}x\right)}{1-x} + \frac{1}{2}\,\frac{\sin\left(\frac{\pi}{2} + \frac{\pi}{2}x\right)}{1+x}$$

$$= \frac{1}{2}\,\frac{2}{\pi}\,\frac{\sin\left(\pi\left(\frac{1-x}{2}\right)\right)}{\pi\left(\frac{1-x}{2}\right)} + \frac{1}{2}\,\frac{2}{\pi}\,\frac{\sin\left(\pi\left(\frac{1+x}{2}\right)\right)}{\pi\left(\frac{1+x}{2}\right)}$$

$$= \frac{1}{\pi}\left[\operatorname{sinc}\left(\frac{1-x}{2}\right) + \operatorname{sinc}\left(\frac{1+x}{2}\right)\right]$$

$$* \quad \operatorname{sinc}(t) \xrightarrow{F} \operatorname{rect}(f) \Rightarrow \operatorname{sinc}(t+\tfrac{1}{2}) \xrightarrow{F} \operatorname{rect}(f)\, e^{j\pi f}$$

زوج است

$$\operatorname{sinc}\left(-\tfrac{1}{2}t + \tfrac{1}{2}\right) \xrightarrow{F} -2\operatorname{rect}(-2f)\, e^{-2j\pi f} = -2\operatorname{rect}(2f)\, e^{-2j\pi f}$$

$$\operatorname{sinc}\left(\tfrac{1}{2}t + \tfrac{1}{2}\right) \xrightarrow{F} 2\operatorname{rect}(2f)\, e^{2j\pi f}$$

$$g(t) = \frac{1}{\pi}\left[\operatorname{sinc}\left(\frac{1-t}{2}\right) + \operatorname{sinc}\left(\frac{1+t}{2}\right)\right]$$

$$\rightarrow G(f) = \frac{1}{\pi}\left[2\operatorname{rect}(2f)\, e^{2j\pi f} - 2\operatorname{rect}(2f)\, e^{-2j\pi f}\right]$$

$$= \frac{4j}{\pi}\operatorname{rect}(2f)\sin(2\pi f)$$

صورت سوال:

$$\rightarrow \frac{1}{2}\int_{-\infty}^{\infty} g(t)\, g^*(t)\, dt = \frac{1}{2}\int_{-\infty}^{\infty} G(f)\, G^*(f)\, df$$

$$\rightarrow \frac{1}{2}\int_{-\infty}^{\infty} g^2(t)\, dt = \underbrace{\frac{1}{2}\int_{-\infty}^{\infty} G(f)\, G^*(f)\, df}$$

می توانیم حاصل این انتگرال را حساب کنیم

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$$\frac{1}{2}\int_{-\infty}^{\infty} G(f)\,G^{*}(f)\,df = \frac{1}{2}\int_{-\infty}^{\infty} \frac{16}{\pi^2}\,\text{rect}^2(2f)\sin^2(2\pi f)\,df$$

$$= \frac{8}{\pi^2}\int_{-1/4}^{1/4} \sin^2(2\pi f)\,df = \frac{8}{\pi^2}\int_{-1/4}^{1/4} \frac{1-\cos(4\pi f)}{2}$$

$$= \frac{4}{\pi^2}\int_{-1/4}^{1/4} (1-\cos(4\pi f))\,df = \frac{4}{\pi^2}\left(f\Big|_{-1/4}^{1/4} - \overset{0}{\cancel{\sin(4\pi f)}}\Big|_{-1/4}^{1/4} \right)$$

$$= \frac{4}{\pi^2}\left(\frac{1}{2}\right) = \frac{2}{\pi^2}$$

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4)

1) $H(f)$ تبدیل فوریه $h(t)$ است.

تابع قطار ضربه $s(t)$ را به این صورت تعریف می کنیم: $s(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

ضرایب سری فوریه $s(t)$ عبارت اند از: $\frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{-jk\frac{2\pi t}{T}}\,dt$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jk\frac{2\pi t}{T}}\,dt = 1/T$$

$$\rightarrow s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\frac{2\pi t}{T}}$$ (سری فوریه)

حال دو طرف رابطه $\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T}t}$ را با $h(t)$ کانوالوی کنیم:

$$h(t) * \sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\, h(t) * \sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T}t}$$

$$\rightarrow \sum_{k=-\infty}^{\infty} h(t-kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} F^{-1}\left\{H(f)\,\delta\left(f-\frac{k}{T}\right)\right\}$$

$$\rightarrow x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} F^{-1}\left\{H\left(\frac{k}{T}\right)\delta\left(f-\frac{k}{T}\right)\right\}$$

$$\rightarrow x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right) F^{-1}\left\{\delta\left(f-\frac{k}{T}\right)\right\}$$

$$\rightarrow x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right) e^{jk\frac{2\pi}{T}t} \quad \square$$

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3) اگر در معادلهٔ قسمت (۱) قرار دهیم $t=0$

$$\sum_{k=-\infty}^{\infty} h(t-kT)\Big|_{t=0} = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right) e^{j2\pi\frac{k}{T}t}\Big|_{t=0}$$

$$\longrightarrow \sum_{k=-\infty}^{\infty} h(-kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right)$$

معکوس کردن ترتیب ، $\sum_{k=-\infty}^{\infty} h(kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right)$

4) $$\sum_{k=-\infty}^{\infty} \frac{2\alpha}{\alpha^2+4\pi^2k^2} \xrightarrow{T=1} = \sum_{k=-\infty}^{\infty} e^{-\alpha|k|}$$

$$= 1+2\sum_{k=1}^{\infty} e^{-\alpha k}$$

سری هندسی : جمله اول : $e^{-\alpha}$

قدر نسبت : $e^{-\alpha}$

$$\to \sum_{k=1}^{\infty} e^{-\alpha k} = \frac{e^{-\alpha}}{1-e^{-\alpha}}$$

$$\to 1+\frac{2e^{-\alpha}}{1-e^{-\alpha}} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}} \;\frac{\times e^{\alpha/2}}{\times e^{\alpha/2}}\; = \frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}$$

$$= \coth(\alpha/2)$$

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5)

1) $x_1(t) = A e^{j(2\pi f_0 t + \theta)}$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x_1(t)|\, dt = A^2 \lim_{T\to\infty} \int_{-T}^{T} dt = \lim_{T\to\infty} 2A^2 T$$

$= \infty \longrightarrow$ $x_1(t)$ انرژی نیست

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} (2A^2 T) = A^2$$

$P = A^2 \longleftarrow$ $x_1(t)$ سیگنال توان است

2) $x_2(t) = e^{-\alpha t} \cos(\beta t)\, u(t)$

$$E = \lim_{T\to\infty} \int_{-T}^{T} e^{-2\alpha t} \cos^2(\beta t)\, u(t) = \lim_{T\to\infty} \int_{0}^{T} e^{-2\alpha t} \cos^2(\beta t)\, dt$$

$$= \frac{1}{2} \lim_{T\to\infty} \int_{0}^{T} \left(\frac{1}{2} e^{(j2\beta - 2\alpha)t} + \frac{1}{2} e^{(-j2\beta - 2\alpha)t} + e^{-2\alpha t}\right) dt$$

$$= \left(\frac{e^{(j2\beta - 2\alpha)t}}{j4\beta - 4\alpha} - \frac{e^{-(j2\beta + 2\alpha)t}}{j4\beta + 4\alpha} - \frac{e^{-2\alpha t}}{2\alpha} \right)\Bigg|_0^T$$

$$= \frac{e^{(j\beta - \alpha)2T}}{j4\beta - 4\alpha} - \frac{e^{-(j\beta + \alpha)2T}}{j4\beta + 4\alpha} - \frac{e^{-2\alpha T}}{2\alpha} + \frac{1}{2\alpha} + \frac{8\alpha}{16\alpha^2 + 16\beta^2}$$

$$\xrightarrow{T\to\infty} = \frac{2\alpha^2 + \beta^2}{2\alpha(\alpha^2 + \beta^2)} \xrightarrow{\times 1/2} \frac{2\alpha^2 + \beta^2}{4\alpha(\alpha^2 + \beta^2)} = E$$

سیگنال انرژی است

ادامه سوال 5)

3) $x_3(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-2n)$

$x_3(t)$ تناوبی است ← $E=\infty$: x_3 انرژی نیست ؛ x_3 از نوع توان است

در سیگنال های تناوبی ؛ توان متوسط سیگنال برابر با توان متوسط یک تناوب

سیگنال است .

$$P = \frac{1}{2}\int_{-1}^{1} |x_3(t)|^2 dt = \frac{1}{2}\int_{-1}^{1} (1-|t|)^2$$

$$= \int_{0}^{1} (1-t)^2 = \frac{t^3}{3} - t^2 + t \Big|_0^1 = \frac{1}{3}$$

سیگنال توان $P = \frac{1}{3}$

4)

$$x_4(t) = \begin{cases} k t^{-1/4} & t>0 \\ 0 & t \le 0 \end{cases}$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x_4(t)|^2 dt = k^2 \lim_{T\to\infty} \int_{0}^{T} t^{-1/2} dt$$

$$= \lim_{T\to\infty} 2k\sqrt{t}\Big|_0^T = \lim_{T\to\infty} 2k\sqrt{T} = \infty$$

x_4 انرژی نیست

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_4(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} 2k\sqrt{T} = 0$$

x_4 توان نیست

$x_4(t)$ نه توان است و نه انرژی

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6) $x_1(t) = e^{-\alpha t} u(t) \longrightarrow X_1(f) = \dfrac{1}{\alpha + 2\pi j f}$

$y_1(t) = \delta(t) \longrightarrow Y_1(f) = 1$

$\longrightarrow X_1(f) H(f) = Y_1(f) \longrightarrow H(f) = \alpha + 2\pi j f$

$x_2(t) = e^{-\alpha t} \cos(\beta t) u(t) = \frac{1}{2} e^{-\alpha t} (e^{j\beta t} + e^{-j\beta t}) u(t)$

$$X_2(f) = \frac{1}{2} \frac{1}{\alpha + j2\pi(f - \frac{\beta}{2\pi})} + \frac{1}{2} \frac{1}{\alpha + j2\pi(f + \frac{\beta}{2\pi})}$$

$$= \frac{1}{2} \left[\frac{1}{(\alpha - j\beta) + j2\pi f} + \frac{1}{(\alpha + j\beta) + j2\pi f} \right.$$

$$Y_2(f) = \frac{1}{2} \left[\frac{\alpha + j2\pi f}{(\alpha - j\beta) + j2\pi f} + \frac{\alpha + j2\pi f}{(\alpha + j\beta) + j2\pi f} \right]$$

$$= \frac{1}{2} \left[1 + \frac{j\beta}{(\alpha - j\beta) + j2\pi f} + 1 - \frac{j\beta}{(\alpha + j\beta) + j2\pi f} \right.$$

$$= 1 + \frac{j\beta}{2} \left[\frac{1}{(\alpha - j\beta) + j2\pi f} - \frac{1}{(\alpha + j\beta) + j2\pi f} \right]$$

$$= 1 - \frac{\beta}{2j} \left[\frac{1}{(\alpha - j\beta) + j2\pi f} - \frac{1}{(\alpha + j\beta) + j2\pi f} \right]$$

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سؤال 6 ـ ادامه:

$$\rightarrow y_2(t) = \delta(t) - \frac{\beta}{2j}\left[e^{-\alpha t}\left(e^{j\beta t} - e^{-j\beta t}\right)\right]$$

$$\rightarrow y_2(t) = \delta(t) - \beta e^{-\alpha t} \sin(\beta t)\, u(t)$$

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7)

1)
$$|X_1(f)| = \begin{cases} \frac{A}{W} f & 0 \leq f \leq W \\ -\frac{A}{W} f & -W \leq f \leq 0 \end{cases}$$

$$\angle X_1(f) = \begin{cases} \pi/2 & f > 0 \\ -\pi/2 & f < 0 \end{cases}$$

$$X_1(f) = |X_1(f)| e^{j\angle X_1(f)} = \begin{cases} \frac{A}{W} f e^{j\pi/2} & 0 \leq f \leq W \\ -\frac{A}{W} f e^{-j\pi/2} & -W < f < 0 \end{cases}$$

$e^{j\pi/2} = j , e^{-j\pi/2} = -j$
$$\longrightarrow \quad X_1(f) = \frac{A}{W} j f \quad -W < f < W$$

$$x_1(t) = \int_{-\infty}^{\infty} X_1(f) e^{j2\pi f t} df = \int_{-W}^{W} \frac{A}{W} j f e^{j2\pi f t} df$$

$$= \frac{A}{W} j \int_{-W}^{W} f e^{j2\pi f t} df$$

$$= \frac{A}{W} j \left. \frac{e^{2\pi j f t}(1 - 2\pi j t f)}{4\pi^2 t^2} \right|_{-W}^{W}$$

$$= \frac{Aj}{W} \frac{1}{4\pi^2 t^2} \left[e^{2\pi j W t}(1 - 2\pi j W t) - e^{-2\pi j W t}(1 + 2\pi j W t) \right]$$

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ادامه سوال 7

$$= \frac{-A}{w4\pi^2t^2j}\left[(e^{2\pi jwt}-e^{-2\pi jwt})-2\pi jwt(e^{2\pi jwt}+e^{-2\pi jwt})\right]$$

$$= \frac{-A}{w\pi^2t^2}\left[\sin(2\pi wt)-2\pi jwt\cos(2\pi wt)\right]$$

شکل سیگنال در انتهای تمرین آورده شده است

2)

$$X_2(f):\begin{cases}\frac{A}{w}f & 0\le f\le w\\ -\frac{A}{w}f & -w\le f\le 0\end{cases}$$

$$x_2(t)=\int_{-\infty}^{\infty}X_2(f)e^{j2\pi ft}df=-\frac{A}{w}\int_{-w}^{0}fe^{2\pi jft}df$$

$$+\frac{A}{w}\int_{0}^{w}fe^{2\pi jft}df=\frac{A}{w}\left[\frac{e^{2\pi jft}(1-2\pi jft)}{4\pi^2t^2}\Big|_0^w-\frac{e^{2\pi jft}(1-2\pi jft)}{4\pi^2t^2}\Big|_{-w}^{0}\right]$$

$$=\frac{A}{4\pi^2t^2}\left[e^{2\pi jwt}(1-2\pi jwt)+e^{-2\pi jwt}(1+2\pi jwt)-2\right]$$

$$=\frac{A}{4\pi^2t^2}\left[(e^{2\pi jwt}+e^{-2\pi jwt})-2\pi jwt(e^{2\pi jwt}-e^{-2\pi jwt})-2\right]$$

$$+\frac{2\pi wt}{j}$$

$$=\frac{A}{\pi^2t^2}\left[\cos(2\pi wt)+2\pi wt\sin(2\pi wt)-2\right]$$

شکل سیگنال در انتهای تمرین آورده شده است.

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سوال 7 ادامه:

3) هر دو سیگنال از ضرب یک جمله با درجه 2- ($\frac{1}{t^2}$) در جمع دو جمله سینوسی و کسینوسی
پدید آمده‌اند؛ در $x_2(t)$ علاوه بر ترم های sin , cos یک ترم DC هم داریم که
جمله $\frac{A}{R^2t^2}$ در آن ضرب می‌شود. تفاوت این دو سیگنال در این است
که فاز تبدیل فوریه آن غیر صفر است و یک سیگنال مختلط است در حالی که
$x_2(t)$: فازش صفر است و یک سیگنال حقیقی است.

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8)

1) $I_1 = \int_0^{\infty} e^{-\alpha t} \operatorname{sinc}^2(\beta t)\, dt \quad (\alpha > 0)$

$= \int_{-\infty}^{\infty} e^{-\alpha t} u(t) \operatorname{sinc}^2(\beta t)\, dt$

$F\{\operatorname{sinc}(\beta t)\} = \frac{1}{\beta} \operatorname{rect}\left(\frac{f}{\beta}\right)$

$\rightarrow F\{\operatorname{sinc}^2(\beta t)\} = \frac{1}{\beta} \operatorname{rect}\left(\frac{f}{\beta}\right) * \frac{1}{\beta} \operatorname{rect}\left(\frac{f}{\beta}\right) = \frac{1}{\beta} \Lambda\left(\frac{f}{\beta}\right)$

$F\{e^{-\alpha t} u(t)\} = \frac{1}{\alpha + j2\pi f}$

$* \int_{-\infty}^{\infty} f(t) g^*(t)\, dt = \int_{-\infty}^{\infty} F(f) G(f)\, df \rightarrow$

$I_1 = \int_{-\infty}^{\infty} \frac{1}{\alpha + j2\pi f} \frac{1}{\beta} \Lambda\left(\frac{f}{\beta}\right) = \frac{1}{\beta} \int_{-\beta}^{0} \frac{1 + f/\beta}{\alpha + j2\pi f}\, df$

$+ \frac{1}{\beta} \int_0^{\beta} \frac{1 - f/\beta}{\alpha + j2\pi f}$

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$$\to I_1 = \frac{1}{8\pi^2\beta^2}\Big((2j\pi\beta - \alpha)\ln(4\pi^2\beta^2 + \alpha^2) +$$

$$(4\pi\beta + 2j\alpha)\tan^{-1}\left(\frac{2\pi\beta}{\alpha}\right) + (-4j\pi\ln(\alpha) - 4j\pi)\beta$$

$$+ 2\alpha\ln(\alpha)\Big) + \frac{1}{8\pi^2\beta^2}\Big(-(2j\pi\beta + \alpha)\ln(4\pi^2\beta^2 + \alpha^2) +$$

$$(4\pi\beta - 2j\alpha)\tan^{-1}\left(\frac{2\pi\beta}{\alpha}\right) + (4j\pi\ln(\alpha) + 4j\pi)\beta +$$

$$2\alpha\ln(\alpha)\Big)$$

$$\to I_1 = \frac{1}{8\pi^2\beta^2}\left(-2\alpha\ln(4\pi^2\beta^2 + \alpha^2) + 8\pi\beta\tan^{-1}\left(\frac{2\pi\beta}{\alpha}\right) + 4\alpha\ln(\alpha)\right)$$

$$\to I_1 = \frac{-\alpha}{4\pi^2\beta^2}\ln\left(\frac{4\pi^2\beta^2 + \alpha^2}{\alpha^2}\right) + \frac{1}{\pi\beta}\tan^{-1}\left(\frac{2\pi\beta}{\alpha}\right)$$

2) در صفحات قبل دیدیم : $F(\text{sinc}^2(\beta t)) = \frac{1}{\beta}\Lambda\left(\frac{f}{\beta}\right)$

$$\frac{\sin^2 t}{t^2} = \text{sinc}^2\left(\frac{t}{\pi}\right) \xrightarrow{F} \pi\Lambda(\pi f)$$

قضیه پارسوال : $\displaystyle\int_{-\infty}^{\infty}\frac{\sin^4(t)}{t^4}dt = \int_{-\infty}^{\infty}\pi^2\Lambda^2(\pi f)\,df$

$$= \pi^2\int_0^{1/\pi}(1-\pi f)^2 df + \pi^2\int_{-1/\pi}^{0}(1+\pi f)^2 df$$

$$= \pi^2\left[\frac{\pi^2 f^3}{3} - \pi f^2 + f\right]\Bigg|_0^{1/\pi} + \pi^2\left[\frac{\pi^2 f^3}{3} + \pi f^2 + f\right]\Bigg|_{-1/\pi}^{0}$$

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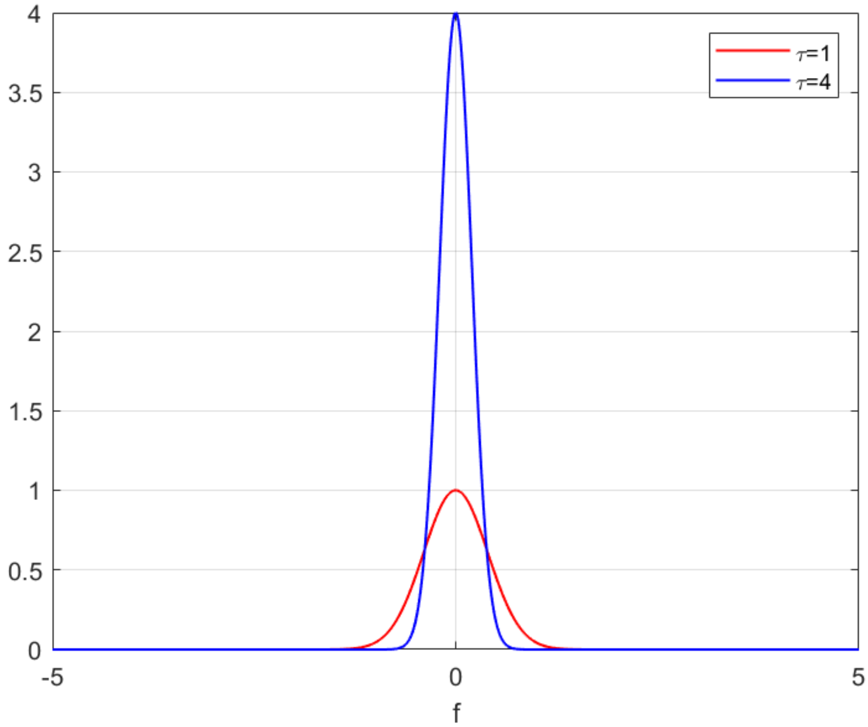
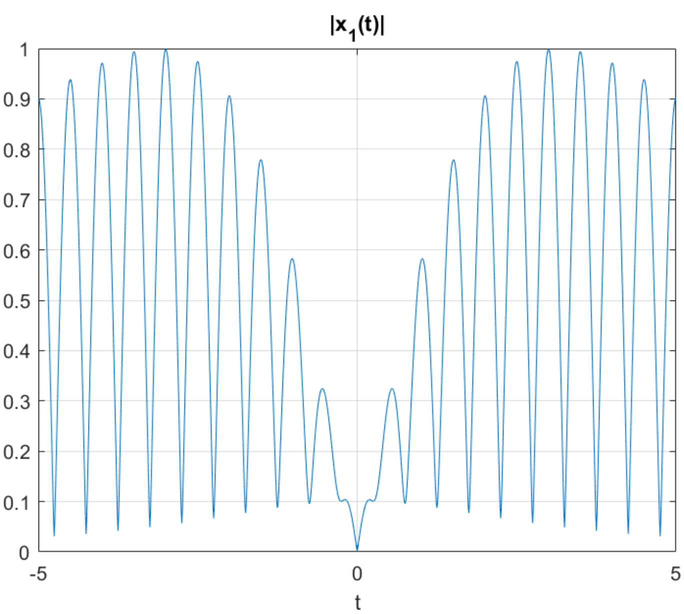
$$= \pi^2 \left[\frac{1}{3\pi} \right] - \pi^2 \left[-\frac{1}{3\pi} \right] = \frac{2\pi}{3}$$

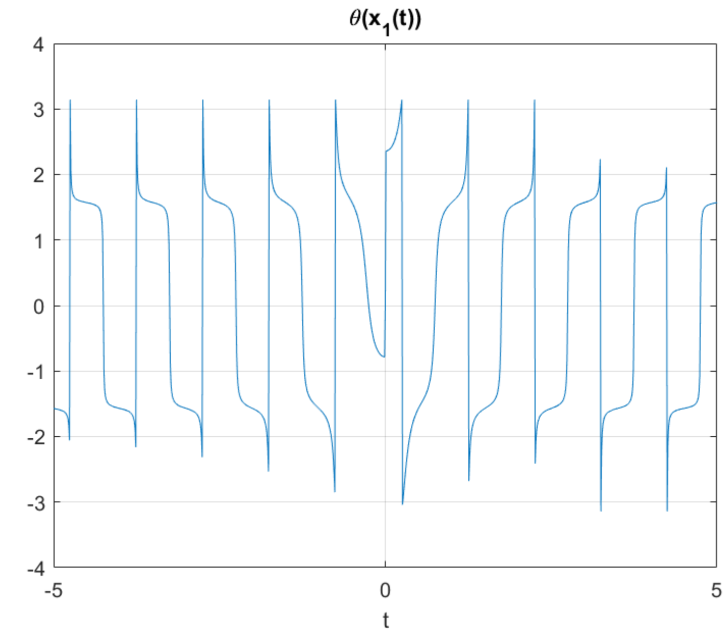
$$\rightarrow I_2 = \int_0^{\infty} \frac{\sin^4(t)}{t^4}\, dt = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\sin^4(t)}{t^4}\, dt = \frac{\pi}{3}$$

سوال دو

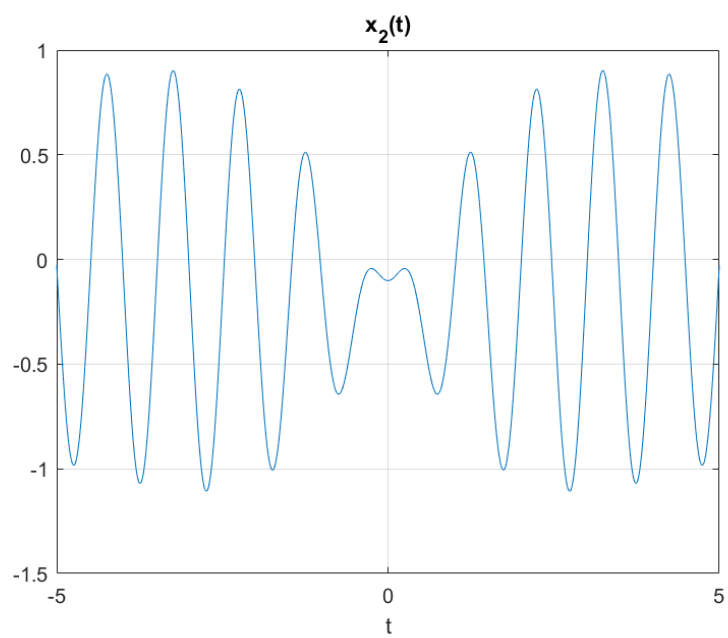


سوال هفت





$\theta(x_1(t))$
4
3
2
1
0
-1
-2
-3
-4
-5
0
5
t



$x_2(t)$
1
0.5
0
-0.5
-1
-1.5
-5
0
5
t