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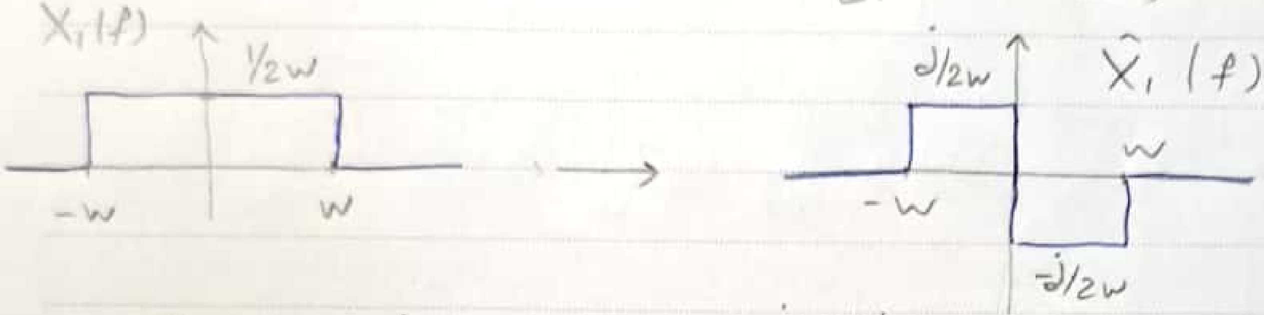
تمرین سری 2 - سیستم های مخابراتی - علی یداللهی - 400102233

سؤال 1)

1) $x_1(t) = \text{sinc}(2Wt) \longrightarrow X_1(f) = \frac{1}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$



$$\longrightarrow \hat{x}_1(t) = \frac{j}{2}\,\text{sinc}(Wt)\left(e^{-jW\pi t} - e^{jW\pi t}\right)$$

$$\longrightarrow \hat{x}_1(t) = \text{sinc}(Wt)\,\frac{\left(e^{j\pi Wt} - e^{-jW\pi t}\right)}{2j} = \text{sinc}(Wt)\sin(W\pi t)$$

2) $$A\cos(2\pi f_0 t + \theta) = \frac{A}{2}\left(e^{j(2\pi f_0 t + \theta)} + e^{-j(2\pi f_0 t + \theta)}\right)$$

$$= \frac{Ae^{j\theta}}{2}e^{j2\pi f_0 t} + \frac{Ae^{-j\theta}}{2}e^{-j2\pi f_0 t}$$

$$\longrightarrow X_2(f) = \frac{Ae^{j\theta}}{2}\delta(f - f_0) + \frac{Ae^{-j\theta}}{2}\delta(f + f_0)$$

$$\longrightarrow \hat{X}_2(f) = -\frac{Ae^{j\theta}}{2}j\,\delta(f - f_0) + \frac{Ae^{-j\theta}}{2}j\,\delta(f + f_0)$$

$$= \frac{Ae^{+j\theta}}{2j}\delta(f - f_0) - \frac{Ae^{-j\theta}}{2j}\delta(f - f_0)$$

$$\longrightarrow \hat{x}_2(t) = \frac{A}{2j}\left(e^{j\theta}e^{2\pi f_0 t} - e^{-j\theta}e^{-2\pi f_0 t}\right)$$

$$= A\,\frac{e^{(2\pi f_0 + \theta)jt} - e^{-(2\pi f_0 t + \theta)jt}}{2j} = A\sin(2\pi f_0 t + \theta)$$

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3) $\hat{x}_3(t) = x_3(t) * \frac{1}{\pi t} = \delta'(t) * \frac{1}{\pi t}$

$\delta'(t) * g(t) = g'(t)$ می دانیم که *

$\longrightarrow \hat{x}_3(t) = -\frac{1}{\pi t^2}$

4) $\hat{x}_4(t) = x_4(t) * \frac{1}{\pi t} = \frac{A}{\pi}\int_{-\infty}^{\infty} \Pi\left(\frac{\tau}{T}\right)\frac{1}{t-\tau}\, d\tau$

$= \frac{A}{\pi}\int_{-T/2}^{T/2} \frac{1}{t-\tau}\, d\tau = \frac{-A}{\pi} \ln(t-\tau)\Big|_{-T/2}^{T/2}$

$= \frac{A}{\pi} \ln\left(\left|\frac{t+T/2}{t-T/2}\right|\right)$

a) $x(t) = c \longrightarrow \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} x(t)\frac{1}{t-\tau}\, d\tau$

$= \frac{1}{\pi}\int_{-\infty}^{\infty} c\,\frac{1}{t-\tau}\, d\tau = \frac{c}{\pi}\int_{-\infty}^{\infty}\frac{1}{t-\tau}\, d\tau$

تغییر متغیر $: t-\tau = u \longrightarrow \frac{-c}{\pi}\int_{-\infty}^{\infty}\frac{1}{u}\, du$

$\frac{1}{u}$ یک تابع فرد است بنابراین : $\int_{-\infty}^{\infty}\frac{1}{u}\, du = 0$

$\longrightarrow \hat{x}(t) = 0$

5) $F\{e^{-a|t|}\} = \frac{2a}{a^2 + 4\pi^2 f^2}$

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تمرین سیگنال (تمرین سیستم)

دوگانی: $F\left\{ \frac{2a}{a^2+4\pi^2 t^2} \right\} = e^{-a|f|}$

$\longrightarrow F\left\{ \frac{1}{a^2+t^2} \right\} = \frac{\pi}{a} e^{-2\pi a|f|}$

$\hat{X}_5(f) = \frac{\pi}{a}\left[-j e^{-2\pi a f} u(f) + j e^{2\pi a f} u(-f) \right]$

$= +j\frac{\pi}{a}\int_{-\infty}^{0} e^{+2\pi a f} e^{j2\pi f t}\, df - j\frac{\pi}{a}\int_{0}^{\infty} e^{-2\pi a f} e^{j2\pi f t}$

$= \frac{\pi}{a} j \left(\frac{1}{2\pi a + 2\pi j t} - \frac{1}{2\pi a - j2\pi t} \right)$

$= \frac{j}{2a}\left(\frac{1}{a+jt} - \frac{1}{a-jt} \right) = \frac{j}{2a}\left(\frac{-2jt}{a^2+t^2} \right)$

$= \frac{t}{a(a^2+t^2)}$

6) $X_6(f) = \delta(f-f_0) \longrightarrow \hat{X}_6(f) = -j\,\delta(f-f_0)$

$\longrightarrow \hat{x}_6(t) = -j e^{2\pi f_0 j t}$

سوال 2)

1) $\hat{X}(f) = -j\,\mathrm{sgn}(f)\, X(f) \xrightarrow{X(0)=0}$

$$\longrightarrow \hat{x}(f) = \begin{cases} jX(f) & f<0 \\ 0 & f=0 \\ -jX(f) & f>0 \end{cases}$$

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$$\hat{\hat{X}}(f) = -j\,\mathrm{sgn}(f)\,\hat{X}(f) = \begin{cases} -X(f) & f<0 \\ 0 & f=0 \\ -X(f) & f>0 \end{cases}$$

$* \ \hat{X}(0) = X(0) = 0$

$$X(0)=0 \longrightarrow -X(0)=0 \longrightarrow \hat{\hat{X}}(f) = -X(f)$$

$$\longrightarrow F\{-x(t)\} = -X(f) \Rightarrow \hat{\hat{X}}(f) = -X(f)$$

$$\longrightarrow \hat{\hat{x}}(t) = -x(t)$$

2)

می دانیم که: $x(t)=x(-t) \longleftrightarrow X(f)=X(-f)$ (زوج)

$x(t)=-x(-t) \longleftrightarrow X(f)=-X(-f)$ (فرد)

اگر $x(t)$ زوج باشد: $x(t)=x(-t) \longrightarrow X(f)=X(-f)$, $\mathrm{sgn}(f)$ فرد است

$$\longrightarrow \hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f) = -\overbrace{\left(-j\,\mathrm{sgn}(-f)\,X(-f)\right)}^{\hat{X}(-f)}$$

$$\longrightarrow \hat{X}(f) = j\,\mathrm{sgn}(-f)\,X(-f) = -\hat{X}(-f)$$

$\longrightarrow \hat{x}(t) = -\hat{x}(-t) \longrightarrow$ فرد است

اگر $x(t)$ فرد باشد: $x(t)=-x(-t) \longrightarrow X(f)=-X(-f)$

$$\longrightarrow \hat{X}(f) = -jX(f)\,\mathrm{sgn}(f) = -j\left(-X(-f)\right)\left(-\mathrm{sgn}(-f)\right)$$

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$= -jX(-f)\,\mathrm{sgn}(-f) = \hat{X}(-f)$

$\hat{X}(f) = \hat{X}(-f) \longrightarrow \hat{x}(t) = \hat{x}(-t) \longrightarrow$ زوج است

3) $y(t) = \dfrac{d}{dt}x(t) \longrightarrow Y(f) = j2\pi f\, X(f)$

$\hat{Y}(f) = -j\,\mathrm{sgn}(f)\left(j2\pi f\, X(f)\right)$

$= j2\pi f\left(-j\,\mathrm{sgn}(f)\, X(f)\right) = j2\pi f\, \hat{X}(f)$

$\rightarrow \hat{Y}(f) = j2\pi f\, \hat{X}(f) \longrightarrow \hat{y}(t) = \dfrac{d}{dt}\hat{x}(t)$

$\longrightarrow \widehat{\dfrac{d}{dt}x(t)} = \dfrac{d}{dt}\hat{x}(t)$

4) $y(t) = \dfrac{dx(t)}{dt} \longrightarrow Y(f) = j2\pi f\, X(f)$

$\rightarrow \hat{Y}(f) = -j\,\mathrm{sgn}(f)\, j2\pi f\, X(f) = 2\pi f\,\mathrm{sgn}(f)\, X(f)$

$f\,\mathrm{sgn}(f):\begin{cases} f<0 & -f \\ f>0 & f \end{cases} \longrightarrow f\,\mathrm{sgn}(f) = |f|$

$\rightarrow F\{\hat{y}(t)\} = F\left\{\widehat{\dfrac{d}{dt}x(t)}\right\} = 2\pi\,|f|\,X(f)$

5) $c(t) = m(t)\,x(t) \longrightarrow C(f) = M(f) * X(f)$

$\rightarrow \hat{C}(f) = -j\,\mathrm{sgn}(f)\left\{M(f) * X(f)\right\}$

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$$\rightarrow \hat{C}(f) = -j\,\mathrm{sgn}(f)\int_{-\infty}^{\infty} M(f')\,X(f-f')\,df'$$

$$= \int_{-\infty}^{\infty} (-j\,\mathrm{sgn}(f))\,M(f')\,X(f-f')\,df'$$

از آنجایی که $X(f)$ میانگذر است و $X(f)$ ، $M(f)$ با هم تداخل ندارند

$$X(f-f')\,\mathrm{sgn}(f) = X(f-f')\,\mathrm{sgn}(f-f')$$

$$\rightarrow \hat{C}(f) = \int_{-\infty}^{\infty} M(f')\left(X(f-f')\,\mathrm{sgn}(f-f')\right)df'$$

$$\rightarrow \hat{C}(f) = \int_{-\infty}^{\infty} M(f')\,\hat{X}(f-f')\,df'$$

$$\rightarrow \hat{C}(f) = M(f) * \hat{X}(f) \rightarrow \hat{c}(t) = m(t)\,\hat{x}(t)$$

سؤال 3:

$$* \quad X_{(dbm)} - Y_{(dbm)} = (X-Y)_{(db)}$$

1) $5_{(dbm)} - (-10)_{(dbm)} \geq 11\left(\frac{db}{km}\right) l_1 \rightarrow l_1 \leq 15\,km$

اگر فرض کنیم نقطهٔ A در $x=0$ است

$$x^2 + (10)^2 = (15)^2 \rightarrow x^2 = 125 \rightarrow x = \sqrt{125} \simeq 11.18\,km$$

محل تقریبی تقویت کننده در فاصلهٔ 11.18^{km} نقطهٔ A است

$$B - x = 50 - 11.18 = 38.82^{km} \rightarrow l_2 = \sqrt{(40)^2 + (38.82)^2}$$

$$\rightarrow l_2 = \sqrt{3107} \simeq 55.74$$

$-10 + g - 55.74(1) \geq 2 \rightarrow g \geq 67.74$

2) $5\,dmb - (-5)\,dbm \geq 1(\frac{dm}{km})\, \ell_1$

$\longrightarrow \ell_1 \leq 10\,km \rightarrow \ell_1 = 10^{km}$ ℓ_1 نمی‌تواند از 10^{km} کمتر باشد

تقویت کننده باید در نقطه A باشد

$\ell_2 = \sqrt{(40)^2 + (50)^2} = \sqrt{4100} \simeq 64$

$-5 + g - 64(1) \geq 2 \longrightarrow g \geq 71$

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سؤال ۹)

$L_{db} = 10 \log_{10} \frac{P_{in}}{P_{out}} = 10 \log_{10} \frac{6.25}{2\times10^{-5}}$

$\simeq 54.95 = 2d \rightarrow d \simeq 27.47$

$n = \frac{500}{27.47} \simeq 18.2 \longrightarrow n_{min} = 19$ کمترین کابل فیبر نوری مورد نیاز

الف) $L_{db} = 10 \log \left(\frac{P_{in}}{P_{out}} \right) = 10 \log \left(\frac{6.25}{10^{-4}} \right)$

$\simeq 47.96$

ب) $L_{db} = (500)(2) - 19\, g_{db(amp)}$

$47.96 = 1000 - 19\, g_{db(amp)}$

$\rightarrow 19 g_{db(amp)} = 952.04 \longrightarrow g_{db(min)} \simeq 50.107$

سؤال 5)

1)

نامساوی کوشی - شوارتز: $\left| \int_{-\infty}^{\infty} g_1(t) g_2^*(t)\, dt \right|^2 \leq \int_{-\infty}^{\infty} |g_1(t)|^2 dt \int_{-\infty}^{\infty} |g_2(t)|^2 dt$

$* \quad c + c^* = 2 \mathrm{Re}\{c\} < 2|c| \rightarrow (c+c^*)^2 = 4(\mathrm{Re}\{c\})^2 \leq 4|c|^2$

$c = \int_{-\infty}^{\infty} g_1(t) g_2^*(t)\, dt \qquad c^* = \int_{-\infty}^{\infty} g_1^*(t) g_2(t)\, dt$

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$$(c+c^*)^2 = \left[\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\,dt + \int_{-\infty}^{\infty} g_1^*(t)g_2(t)\,dt\right]^2$$

$$= \left[\int_{-\infty}^{\infty} g_1(t)g_2^*(t) + g_1^*(t)g_2(t)\,dt\right]^2$$

$$\rightarrow (c+c^*)^2 \leq 4|c|^2 \rightarrow \left[\int_{-\infty}^{\infty} g_1(t)g_2^*(t) + g_1^*(t)g_2(t)\,dt\right]^2 \leq 4\left|\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\right|^2$$

نامساوی کوشی شوارتز : $\left|\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\,dt\right|^2 \leq \int_{-\infty}^{\infty} |g_1(t)|^2 dt \int_{-\infty}^{\infty} |g_2(t)|^2 dt$

$$\rightarrow \left[\int_{-\infty}^{\infty} [g_1^*(t)g_2(t) + g_1(t)g_2^*(t)]\,dt\right]^2 \leq 4\int_{-\infty}^{\infty} |g_1(t)|^2 dt \int_{-\infty}^{\infty} |g_2(t)|^2 dt$$

$$g_1(t) = t\,g(t), \quad g_2(t) = \frac{dg(t)}{dt}$$

$$\rightarrow \left[\int_{-\infty}^{\infty}\left[t\,g^*(t)\frac{dg(t)}{dt} + t\,g(t)\frac{dg^*(t)}{dt}\right]dt\right]^2 \leq$$

$$4\int_{-\infty}^{\infty} t^2|g(t)|^2 dt \int_{-\infty}^{\infty}\left|\frac{dg(t)}{dt}\right|^2 dt$$

$$* \int_{-\infty}^{\infty}\left(t\,g^*(t)\frac{d}{dt}g(t) + t\,g(t)\frac{d\,g^*(t)}{dt}\right)dt$$

$$= \int_{-\infty}^{\infty} t\,\frac{d}{dt}\left(g(t)g^*(t)\right)dt = \int_{-\infty}^{\infty} t\,\frac{d}{dt}|g(t)|^2 dt$$

$\longrightarrow$ انتگرال جزء به جزء : $t|g(t)|^2\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} |g(t)|^2 dt$

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$|t| \to \infty$ ؛ $|g(t)|^2$ سریع‌تر از $\frac{1}{|t|}$ به سمت صفر می‌رود

$$t\,|g(t)|^2 \Big|_{-\infty}^{\infty} = 0$$

$$\rightarrow \left[\int_{-\infty}^{\infty} |g(t)|^2 dt\right]^2 \leq 4\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} \left|\frac{dg(t)}{dt}\right|^2 dt$$

$$* \int_{-\infty}^{\infty} \left|\frac{d}{dt} g(t)\right|^2 dt = \int_{-\infty}^{\infty} |j2\pi f G(f)|^2 df = 4\pi^2 \int_{-\infty}^{\infty} f^2 |G(f)|^2 df$$

$$\rightarrow \left[\int_{-\infty}^{\infty} |g(t)|^2 dt\right]^2 \leq 16\pi^2 \int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} f^2 |G(f)|^2 df$$

$$\xrightarrow{\text{پارسوال}} \int_{-\infty}^{\infty} |g(t)|^2 dt \int_{-\infty}^{\infty} |G(f)|^2 df \leq 16\pi^2 \int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} f^2 |G(f)|^2 df$$

$$\rightarrow \frac{1}{16\pi^2} \leq \underbrace{\frac{\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt}{\int_{-\infty}^{\infty} |g(t)|^2 dt}}_{T_{RMS}^2} \; \underbrace{\frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2 df}{\int_{-\infty}^{\infty} |G(f)|^2 df}}_{W_{RMS}^2}$$

$$\rightarrow \frac{1}{16\pi^2} \leq T_{RMS}^2 W_{RMS}^2 \longrightarrow \frac{1}{4\pi} \leq T_{RMS} W_{RMS}$$

2) $g_1(t) = t e^{-\pi t^2}$ $\quad g_2(t) = \frac{dg(t)}{dt} = -2\pi t e^{-\pi t^2}$

$$\left\{\int_{-\infty}^{\infty} g_1^*(t) g_2(t) + g_1(t) g_2^*(t)\, dt\right\}^2 = \left\{\int_{-\infty}^{\infty} -4\pi t^2 e^{-2\pi t^2}\right\}^2$$

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$$\rightarrow \left\{\int_{-\infty}^{\infty} -4\pi t^2 e^{-2\pi t^2} dt\right\}^2 = \int_{-\infty}^{\infty} -4\pi t^2 e^{-2\pi t^2} dt \int_{-\infty}^{\infty} -4\pi t^2 e^{-2\pi t^2} dt$$

$$= 4\int_{-\infty}^{\infty} t^2 e^{-2\pi t^2} dt \int_{-\infty}^{\infty} -4\pi^2 t^2 e^{-2\pi t^2}$$

$$= 4\int_{-\infty}^{\infty} t^2 e^{-2\pi t^2} dt \int_{-\infty}^{\infty} 4\pi^2 t^2 e^{-2\pi t^2} dt = 4\int_{-\infty}^{\infty} |t e^{-\pi t^2}|^2 dt \int_{-\infty}^{\infty} |-2\pi t e^{-\pi t^2}|^2 dt$$

$$= 4\int_{-\infty}^{\infty} |g_1(t)|^2 dt \int_{-\infty}^{\infty} |g_2(t)|^2 dt$$

در این حالت نامساوی $\left\{\int_{-\infty}^{\infty} [g_1^*(t) g_2(t) + g_1(t) g_2^*(t)]\right\} \le 4\int_{-\infty}^{\infty} |g_1(t)|^2 dt \int_{-\infty}^{\infty} |g_2(t)|^2 dt$

تبدیل به مساوی می‌شود پس نامساوی $\frac{1}{4\pi} \le T_{Rms} W_{Rms}$ هم تبدیل

به مساوی می‌شود و خواهیم داشت $W_{Rms} T_{Rms} = \frac{1}{4\pi}$