In the Name of God

Communication Systems (25751-4)

Problem Set 02

Fall Semester 1402-03

Department of Electrical Engineering

Sharif University of Technology

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Due on Aban 5, 1402 at 13:00

(∗) starred problems are optional and have a bonus mark!

1 The Hilbert Transform

Find the Hilbert transform of following signals:

1. $x_1(t) = \text{sinc}(2Wt)$
2. $x_2(t) = A\cos(2\pi f_0 t + \theta)$
3. $x_3(t) = \delta'(t)$
4. $x_4(t) = A\Pi(\frac{t}{T}) = A[u(t + \frac{T}{2}) - u(t - \frac{T}{2})]$
   (a) show that if $v(t) = A$ for all time, then $\hat{v}(t) = 0$
5. $x_5(t) = \frac{1}{a^2+t^2}$
6. $x_6(t) = e^{j2\pi f_0 t}$

2 Hilbert Transform Properties

1. Show that if $X(f)|_{f=0} = 0$, then $\widehat{\hat{x}(t)} = -x(t)$.
2. Show that the Hilbert transform of an even signal is odd and the Hilbert transform of an odd signal is even.
3. Show that the Hilbert transform of the derivative of a signal is equal to the derivative of its Hilbert transform.
4. Show that:
$$\mathcal{F}\{\widehat{\frac{d}{dt}x(t)}\} = 2\pi|f|\mathcal{F}\{x(t)\}.$$
5. Let $x(t)$ represent a bandpass signal and $m(t)$ denote a lowpass signal with nonoverlapping spectra. Show that the Hilbert transform of $c(t) = m(t)x(t)$ is equal to $\hat{c}(t) = m(t)\hat{x}(t)$.

3 Attenuation and Amplification (1)

Consider the following system:

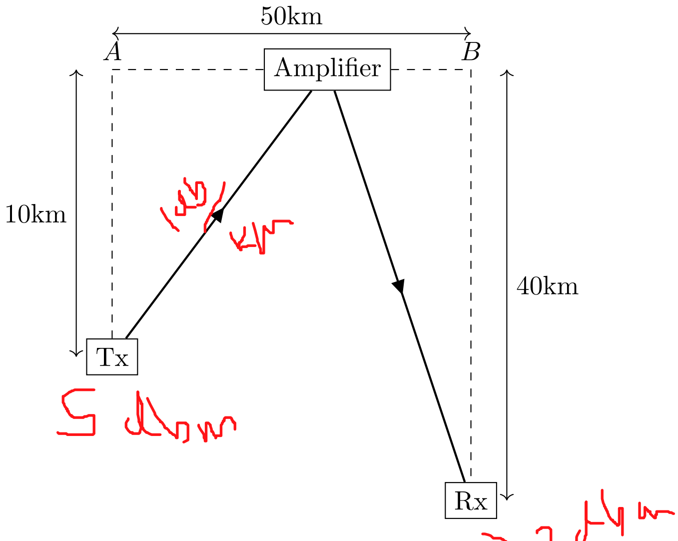

Figure 1: Problem 4

The transmitter sends a message with power $P_{\text{in}} = 5$ dBm. An amplifier amplifies this message on its way. The location of the amplifier must be on the line segment AB, but you can adjust it. Note that the cable connecting the transmitter to the amplifier, same as the cable connecting the amplifier to the receiver, is along the straight line that connects them.The minimum input power of the receiver is $P_{\text{out}} = 2$ dBm. Assume that the cable attenuation is $\alpha = 1\frac{\text{dB}}{\text{km}}$. Find the location and minimum gain of the amplifier in these cases:

1. The minimum input power of amplifier is -10 dBm.
2. The minimum input power of amplifier is -5 dBm.

4 Attenuation and Amplification (2)

Considering the output power of a system is 6.25 watts, the output signal is to be transmitted via a 500 km repeater system consisting of n identical fiber optic cable sections with an attenuation rate of $2\frac{\text{dB}}{\text{km}}$ and n identical amplifiers. Find the minimum required number of sections and gain per amplifier so that P_{out} is at least 100 μW and the input power to each amplifier is at least 20 μW.

5 (∗) RMS Bandwidth and RMS Duration

The root mean-square (RMS) bandwidth of a low-pass signal of finite energy is defined by:

$$W_{\text{RMS}} = \left[\frac{\int_{-\infty}^{\infty} f^2 \left|G(f)\right|^2 df}{\int_{-\infty}^{\infty} \left|G(f)\right|^2 df} \right]^{\frac{1}{2}}$$

where $\left|G(f)\right|^2$is the energy spectral density of the signal. Correspondingly, the root mean-square (RMS) duration of the signal is defined by:

$$T_{\text{RMS}} = \left[\frac{\int_{-\infty}^{\infty} t^2 \left|g(t)\right|^2 dt}{\int_{-\infty}^{\infty} \left|g(t)\right|^2 dt} \right]^{\frac{1}{2}}$$

1. Show that:

$$T_{\text{RMS}} W_{\text{RMS}} \geq \frac{1}{4\pi}.$$

   Assume that $|g(t)| \to 0$ faster than $\frac{1}{\sqrt{|t|}}$ as $|t| \to \infty$.

2. Consider a Gaussian pulse defined by:

$$g(t) = e^{-\pi t^2}.$$

   Show that, for this signal, the above inequality holds with equality:

$$T_{\text{RMS}} W_{\text{RMS}} = \frac{1}{4\pi}.$$

   Hint: Use Cauchy–Schwarz's inequality and the fact that for any complex number c, $c + c^* = 2\mathbf{Re}\{c\} \leq 2|c|$ to show that for any two complex functions $g_1(t), g_2(t)$, we have:

$$\left\{ \int_{-\infty}^{\infty} [g_1^*(t) g_2(t) + g_1(t) g_2^*(t)] dt \right\}^2 \leq 4 \int_{-\infty}^{\infty} |g_1(t)|^2 \, dt \int_{-\infty}^{\infty} |g_2(t)|^2 \, dt.$$

   and then set $g_1(t) = t g(t)$ and $g_2(t) = \frac{dg(t)}{dt}$.