In the Name of God

Communication Systems (25751-4)

Python Assignment 01

Fall Semester 1402-03

Department of Electrical Engineering

Sharif University of Technology

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Due on Azar 3, 1402 at 23:59

($*$) starred problems are optional and have a bonus mark!

1 Channel Disortion

Consider the signal $x(t)$ as follows:

$$x(t) = (t - T_0)^4 e^{-(t-T_0)} \cos^2(2\pi f_0(t - T_0))u(t - T_0) \tag{1}$$

1.1

Create a time range from 0 to 100 seconds in 0.01 second increments. Sample $x(t)$ in this interval and plot it in the time domain. Consider the parameters of the signal as $T_0 = 10$, $f_0 = 10$.

1.2

Consider the following channel:

$$H(f) = \begin{cases} (1 + k\cos(2\pi f T_0))e^{-j2\pi f t_d} & |f| \leq B \\ 0 & |f| > B \end{cases} \tag{2}$$

Consider $t_d = \frac{T_0}{3}$ and $k = 0.4$. Plot the channel output for the $x(t)$ input.
Based on what you have learned, explain what type of distortion this channel implies.

1.3

Consider the following channel:

$$y(t) - \alpha y(t - T_0) - \underline{\beta} y(t - 2T_0) = x(t) \tag{3}$$

Calculate the frequency response of the channel.

1.4

Let $\alpha = 0.3$ and $\underline{\beta}$ a random variable with probability density function $f_{\underline{\beta}}(\beta) = \frac{\beta}{\sigma}\exp(\frac{-\beta^2}{2\sigma^2})u(\beta)$. Also suppose $\sigma = 1$. For $N = 3$ occurrences of $\underline{\beta}$, plot the output of the channel for the input, $x(t)$. Plot all three outputs on the same graph.

1.5

We want to estimate $\mathbb{E}[Y(t)]$. For this, we calculate the output of the channel for N random occurrences of $\underline{\beta}$ and show them as $Y_1(t), Y_2(t), ..., Y_N(t)$, then we take the average of these N output signals and use the following signal as an estimate of $\mathbb{E}[Y(t)]$:

$$\overline{Y}(t) = \sum_{i=1}^{N} Y_i(t) \tag{4}$$

Plot $Y(t)$ for $N = 10,\ 50,\ 100,\ 200$.

1.6

Explain the effect of this channel on the input signal in time domain.

2 Signal Recovery

As you saw in the previous section, the random channel damages the received signal. In this section, we try to recover the original signal. You have seen in the lesson that the frequency response of the ideal channel is as follows:

$$H(f) = ke^{-2j\pi f t_0} \tag{5}$$

But due to the arrival of the wave from several paths, the channel is in the form of equation (2). In order to recover the exact input signal in the receiver, a equalizer is used in such a way that the whole system operation can be modeled as an ideal channel.
If we call the frequency response of the equalizer $H_{Eq}(f)$, we have:

$$H_{Eq}(f)H_C(f) = ke^{-2j\pi f t_0} \tag{6}$$

So, if we have $H_C(f)$, we can design $H_{E_q}(f)$:

$$H_{Eq}(f) = \frac{k\ \exp(-2j\pi f t_0)}{H_C(f)} \tag{7}$$

2.1

Suppose we replace the value of the random coefficient of $\underline{\beta}$ with $\mathbb{E}[\underline{\beta}]$. In this case, calculate the frequency response of the equalizer for the channel of equation (3) and show that the structure of the equalizer will be as figure 1.
This method is called **m-Tapped-Delay Line Equalizer** (The parameter m indicates the number of delay blocks and is a measure of the level of complexity, and the capabality of the equalizer).

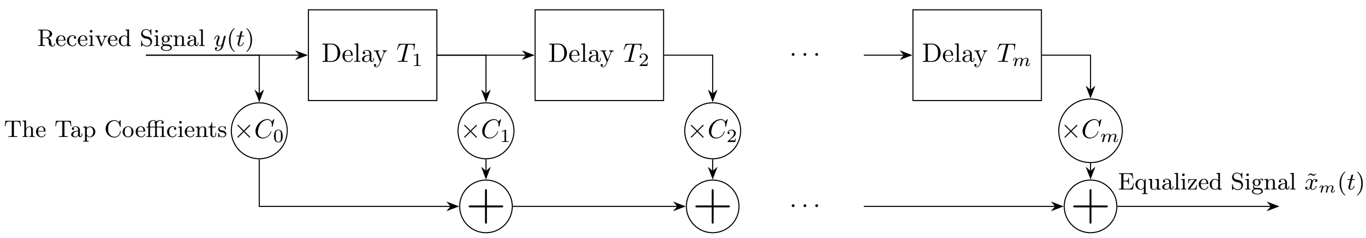

Figure 1: m–Tapped-Delay Line Equalizer

2.2

Now suppose that $x(t)$ which was mentioned in equation (1) passes through the following channel:

$$y(t) = x(t) + \gamma x(t - T_0) \tag{8}$$

$\gamma = 0.3$.
In this case, try to neutralize the effect of the channel with the help of a m-Tapped-Delay Line Equalizer. You may need to use Taylor series. For $m = 3,\ 4,\ , ..., 10$ calculate the coefficients and the amount of delays.

2.3

Suppose we denote the recovered signal by using m-Tapped-Delay Line Equalizer as $\tilde{x}_m(t)$. Also, we pass the input signal through the ideal channel of equation (5) and get $\hat{y}(t)$ as the output.
We define the error criterion as follows:

$$\text{RMS Error}_m = \int_{-\infty}^{+\infty} |\tilde{x}_m(t) - \hat{y}(t)|^2 \, dt \tag{9}$$

Calculate the error rate for $m = 3,\ 4,\ ...,\ 10$ and plot its graph. Select the vertical axis once as linear and once as logarithmic.

2.4

plot $x(t)$, $y(t)$, $\hat{y}(t)$ and $\tilde{x}_5(t)$ in a graph and compare them.