In the Name of God

Communication Systems (25751-4)

Problem Set 06

Fall Semester 1402-03

Department of Electrical Engineering

Sharif University of Technology

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Due on Azar 22, 1402 at 17:00

(∗) starred problems are optional and have a bonus mark!

1 First-Order Hold Interpolation

First-order hold (FOH) performs a linear interpolation to generate the sampled signal

$$y(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\Lambda(\frac{t - kT_s}{T_s})$$

from the band-limited signal $x(t)$ with bandwidth W.

1. Find the spectrum of the sampled signal $Y(f)$. (Hint: $\delta(at) = \frac{1}{|a|}\delta(t) \quad , a \neq 0$)
2. Propose a condition on the sampling period T_s for the perfect reconstruction of the original signal $x(t)$ from the sampled signal $y(t)$.
3. Obtain an expression for the reconstruction filter $H(f)$.

2 Frequency-Domain Sampling

The frequency-domain sampling theorem says that if $x(t)$ is a time-limited signal such that $x(t) = 0$ for $|t| \geq T$, then $X(f)$ is completely determined by its sample values $X(nf_0)$ with $f_0 \leq 1/2T$. Prove that the condition $f_0 \leq 1/2T$ must be satisfied and use the samples $X(nf_0)$ to reconstruct $X(f)$.

3 Midrise Quantization

Let $X(t)$ be an analog signal that has a uniform distribution over the range $[-x_m, +x_m]$. Suppose we quantize $X(t)$ using a uniform midrise quantizer with $N = 2^\nu$ levels. Find the SQNR of the quantizer. (Hint: SQNR $= \frac{\mathbb{E}[X^2]}{\mathbb{E}[(X-Q(X))^2]}$)

4 Sampling

The lowpass signal $x(t)$ with a bandwidth of W is sampled at the Nyquist rate and the signal

$$y(t) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\delta(t - nT_s)$$

is generated.

1. Find the Fourier transform of $y(t)$.
2. How Can $x(t)$ be reconstructed from $y(t)$.

5 Quantization Erorr

A stationary source is distributed according to the triangle probability density function $f_X(x) = 0.5\Lambda(0.5x)$. This source is quantized using the four-level uniform quantizer

$$Q(x) = \begin{cases} 1.5, & 1 < x \leq 2 \\ 0.5, & 0 < x \leq 1 \\ -0.5, & -1 < x \leq 0 \\ -1.5, & -2 \leq x \leq -1 \end{cases}$$

Determine the probability density function of the random variable representing the quantizer error: $Y = X - Q(X)$.

6 Quantization and Signal Transmission

A signal $m(t)$ bandlimited to 3 kHz is sampled at a rate $\frac{1}{3}$ higher than its Nyquist rate. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak signal amplitude m_p (quantization steps are uniform). The quantized samples are binary coded. Find the minimum bandwidth of a channel required to transmit the encoded binary signal.
24 such signals are time-division-multiplexed. If 2% more bits are added to the multiplexed data for error protection and synchronization, determine the minimum transmission bandwidth required to transmit the multiplexed signal.