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400102233 - سیستم های مخابراتی - تمرین سری سوم - علی بداللهی -

سوال یک)

1. $E = \lim_{T\to\infty} \int_{-T}^{T} e^{-2t^2} \cos^2(2\pi f_0 t)\, dt$

$= \lim_{T\to\infty} \frac{1}{2} \int_{-T}^{T} e^{-2t^2} \left(1 + \cos(4\pi f_0 t)\right) dt$

$= \lim_{T\to\infty} \left[\frac{1}{2} \int_{-T}^{T} e^{-2t^2} + \frac{1}{2} \int_{-T}^{T} e^{-2t^2} \cos(4\pi f_0 t)\, dt \right.$

$= \frac{1}{2} \left[\sqrt{\frac{\pi}{2}} + \sqrt{\frac{\pi}{2}}\, e^{-2\pi^2 f^2} \right] < \infty \rightarrow$ سیگنال انرژی است

2.

$g(t) = e^{-t^2} \rightarrow \frac{dg(t)}{dt} = -2t\, e^{-t^2}$

$\rightarrow j2\pi f\, G(f) = -\frac{j}{\pi} \frac{d}{df} G(f)$

$\frac{j}{\pi} \frac{d}{df} G(f) + j2\pi f\, G(f) = 0$

$\rightarrow \frac{d}{df} G(f) = -2\pi^2 f\, G(f)$

$\rightarrow \frac{dG(f)}{G(f)} = -2\pi^2 f\, df$

$\rightarrow \ln(G(f)) = -\pi^2 f^2 + c_1 \rightarrow G(f) = e^{c_1} e^{-\pi^2 f^2} = k\, e^{-\pi^2 f^2}$

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$G(0) = \int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi} \rightarrow G(f) = \sqrt{\pi}\, e^{-\pi^2 f^2}$

$x(t) = \frac{1}{2} g(t) \left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \right.$

$= \frac{1}{2} \left(G(f-f_0) + G(f+f_0) \right)$

$= \frac{\sqrt{\pi}}{2} \left(e^{-\pi^2 (f-f_0)^2} + e^{-\pi^2 (f+f_0)^2} \right)$

3) $S_x(f) = |X(f)|^2 = \frac{\pi}{4} \left(e^{-\pi^2(f-f_0)^2} + e^{-\pi^2(f+f_0)^2} \right)^2$

$= \frac{\pi}{4} \left(e^{-2\pi^2(f-f_0)^2} + e^{-2\pi^2(f+f_0)^2} + 2 \underbrace{e^{-\pi^2 [(f-f_0)^2 + (f+f_0)^2]}}_{*} \right)$

$*$: $e^{-\pi^2 [f^2 + f_0^2 - 2ff_0 + f_0^2 + f^2 + 2ff_0]}$

$= e^{-2\pi^2 (f^2 + f_0^2)} = e^{-2\pi^2 f_0^2} e^{-2\pi^2 f^2}$

$\rightarrow S_x(f) = \left(e^{-2\pi^2(f-f_0)^2} + e^{-2\pi^2(f+f_0)^2} + 2 e^{-2\pi^2 f_0^2} e^{-2\pi^2 f^2} \right) \frac{\pi}{4}$

4) $R_x(t) = F^{-1}\{ S_x(f) \}$

$*$ $e^{-t^2} \xrightarrow{F} \sqrt{\pi}\, e^{-\pi^2 f^2}$, $e^{-\frac{t^2}{2}} \xrightarrow{F} \sqrt{2\pi}\, e^{-2\pi^2 f^2}$

$\rightarrow R_x(t) = \frac{\pi}{4} \left(\frac{e^{-t^2/2}}{\sqrt{2\pi}} e^{j2\pi f_0 t} + \frac{e^{-t^2/2}}{\sqrt{2\pi}} e^{-j2\pi f_0 t} + 2 e^{-2\pi^2 f_0^2} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} \right)$

$= \frac{\pi}{2} \frac{1}{\sqrt{2\pi}} \left(e^{-t^2/2} \cos(2\pi f_0 t) + e^{-2\pi^2 f_0^2} e^{-t^2/2} \right)$

$\rightarrow R_x(\tau) = \frac{\sqrt{\pi}}{2\sqrt{2}} e^{-\frac{t^2}{2}} \left(e^{-2\pi^2 f_0^2} + \cos(2\pi f_0 t) \right)$

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سؤال 2)

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_T(t)(t+nT) = x_T(t) * \underbrace{\sum_{n=-\infty}^{\infty} \delta(t+nT)}_{\delta_T(t)}$$

$$* \lim_{T\to\infty} R_{x_p}(\tau) = R_x(\tau) \rightarrow \lim_{T\to\infty} S_{x_p}(f) = S_x(f)$$

$$R_{x_p}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} x_p(t)\, x_p^*(t-\tau)\,dt = \frac{1}{T}\int_{-T/2}^{T/2} x_T(t)\, x_p^*(t-\tau)\,dt$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} x_T(t)\, x_p^*(t-\tau)\,dt$$

$$\rightarrow x_p^*(t-\tau) = \int_{-\infty}^{\infty} x_T^*(t-\tau-\tau')\,\delta_T(\tau')\,d\tau'$$

$$\rightarrow R_{x_p}(\tau) = \frac{1}{T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_T(t)\, x_T^*(t-\tau-\tau')\,\delta_T(\tau')\,d\tau'\,dt$$

$$= \frac{1}{T}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_T(t)\, x_T^*(t-\tau-\tau')\,dt\right]\delta_T(\tau')\,d\tau'$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} R_{x_T}(\tau+\tau')\,\delta_T(\tau')\,d\tau' = \frac{1}{T}\int_{-\infty}^{\infty} R_{x_T}(\tau-\tau')\,\delta_T(-\tau')\,d\tau'$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} R_{x_T}(\tau-\tau')\,\delta(\tau')\,d\tau' = \frac{1}{T} R_{x_T}(\tau) * \delta_T(\tau)$$

$$S_{x_p}(f) = F\{R_{x_p}(\tau)\} = \frac{1}{T} S_{x_T}(f)\, \frac{1}{T}\,\delta_{\frac{1}{T}}(f)$$

$$S_x(f) = \lim_{T\to\infty} \frac{1}{T} S_{x_T}(f)\,\frac{1}{T}\,\delta_{\frac{1}{T}}(f) \quad , * \lim_{T\to\infty}\frac{1}{T}\,\delta_{\frac{1}{T}}(f) = 1$$

$$\rightarrow S_x(f) = \lim_{T\to\infty}\frac{1}{T} S_{x_T}(f)$$

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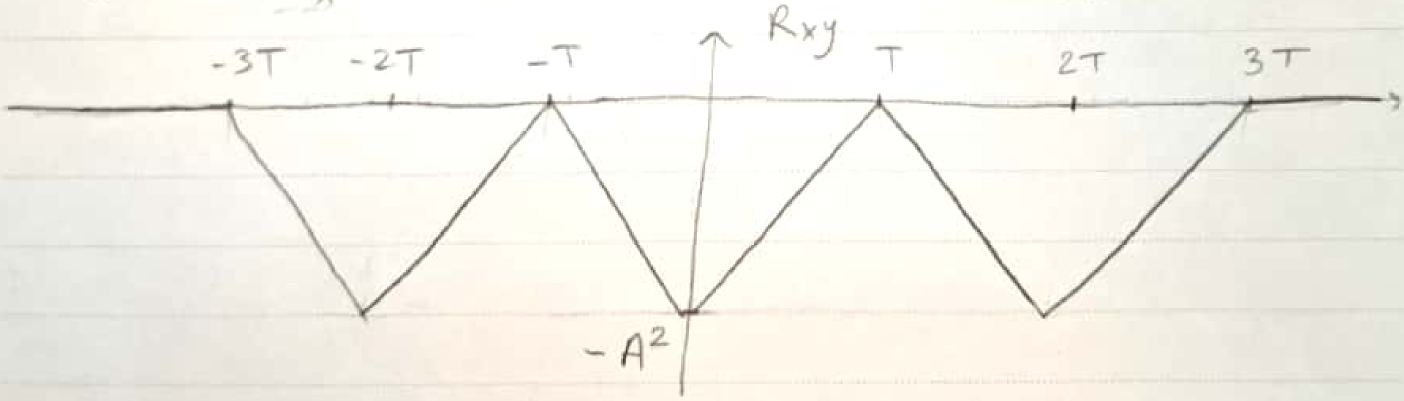
سؤال 3)

$x(t)$: Rectangular pulse , $y(t)$: Triplet pulse

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y^*(t-\tau)\, dt \xrightarrow{\text{y حقیقی}} \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt$$



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4)

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x_T(t)\, x_T^*(t-\tau)\, dt$$

$$S_x(f) = \lim_{T\to\infty} \frac{|x_T(f)|^2}{T}$$

$$R_x(f) = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x_T(t)\, x_T^*(t-\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x_T(t) \int_{-\infty}^{\infty} x_T(t-\tau)\, e^{-j2\pi f\tau}\, d\tau\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x_T(t) \int_{-\infty}^{\infty} x_T^*(\tau)\, e^{-j2\pi ft + j2\pi f\tau}\, d\tau\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x_T(t)\, e^{-j2\pi ft} \int_{-\infty}^{\infty} x_T^*(\tau)\, e^{j2\pi f\tau}\, d\tau\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} X_T(f)\, X_T(-f) \quad = \lim_{T\to\infty} \frac{1}{T} |X_T(f)|^2$$

$$\longrightarrow S_x(f) = F\{R_x(\tau)\}$$

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تمرین سیستم سری 3

سوال ۹)

$$x(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI system}\\ h(t)\end{array}} \longrightarrow y(t)$$

$$y(t) = h(t) * x(t)$$

$$R_y(\tau) = y(\tau) * y^*(-\tau) = h(\tau) * \underbrace{x(\tau) * x^*(-\tau)}_{R_x(\tau)} * h^*(-\tau)$$

$$= R_x(\tau) * h(\tau) * h^*(-\tau)$$

$$\rightarrow F\{R_y(\tau)\} = F\{R_x(\tau)\}\, F\{h(\tau) * h^*(-\tau)\}$$

$$S_y(f) = S_x(f)\, \underbrace{F\{h(\tau) * h^*(-\tau)\}}_{*}$$

$$* : F\{h(\tau) * h^*(-\tau)\} = F\{h(\tau)\}\, F\{h^*(-\tau)\} = |H(f)|^2$$

$$\rightarrow S_y(f) = S_x(f)\, |H(f)|^2$$

تمرین سیستم های مخابراتی

سؤال 5)

$$x_c(t) = A\cos(8000\pi t)\left[2\cos(400t) + 4\sin\left(500t + \frac{\pi}{3}\right)\right]$$

$$= 2A\cos(8000\pi t)\cos(400t) + 4A\cos(8000\pi t)\sin(500t + \pi/3)$$

$\Rightarrow$ روابط تبدیل جمع به ضرب مثلثاتی $= A\left[\cos((8000\pi + 400)t) + \cos((8000\pi - 400)t)\right]$

$$+ 2A\left[\sin\left((8000\pi + 500)t + \pi/3\right) - \sin\left((8000\pi - 500)t - \pi/3\right)\right]$$

$$\rightarrow x_c(t) = A\cos\left((8000\pi + 400)t\right) + A\cos\left((8000\pi - 400)t\right)$$

$$+ 2A\sin\left((8000\pi + 500)t + \pi/3\right) - 2A\sin\left((8000\pi - 500)t - \pi/3\right)$$

$$X_c(f) = \frac{A}{2}\left[\delta\left(f - \left(4000 + \frac{200}{\pi}\right)\right) + \delta\left(f + \left(4000 + \frac{200}{\pi}\right)\right)\right]$$

$$+ \frac{A}{2}\left[\delta\left(f - \left(4000 - \frac{200}{\pi}\right)\right) + \delta\left(f + \left(4000 - \frac{200}{\pi}\right)\right)\right]$$

$$+ \frac{A}{j}\left[e^{j\pi/3}\delta\left(f - \left(4000 + \frac{250}{\pi}\right)\right) - e^{-j\pi/3}\delta\left(f + \left(4000 + \frac{250}{\pi}\right)\right)\right]$$

$$- \frac{A}{j}\left[e^{-j\pi/3}\delta\left(f - \left(4000 - \frac{250}{\pi}\right)\right) - e^{j\pi/3}\delta\left(f + \left(4000 - \frac{250}{\pi}\right)\right)\right]$$

$$\rightarrow X_c(f) = \frac{A}{2}\left[\delta\left(f - \left(4000 + \frac{200}{\pi}\right)\right) + \delta\left(f + \left(4000 + \frac{200}{\pi}\right)\right)\right]$$

$$+ \frac{A}{2}\left[\delta\left(f - \left(4000 - \frac{200}{\pi}\right)\right) + \delta\left(f + \left(4000 - \frac{200}{\pi}\right)\right)\right]$$

$$+ Aj\left[e^{-j\pi/3}\delta\left(f + \left(4000 + \frac{250}{\pi}\right)\right) - e^{j\pi/3}\delta\left(f - \left(4000 + \frac{250}{\pi}\right)\right)\right]$$

$$+ Aj\left[e^{-j\pi/3}\delta\left(f - \left(4000 - \frac{250}{\pi}\right)\right) - e^{j\pi/3}\delta\left(f + \left(4000 - \frac{250}{\pi}\right)\right)\right]$$

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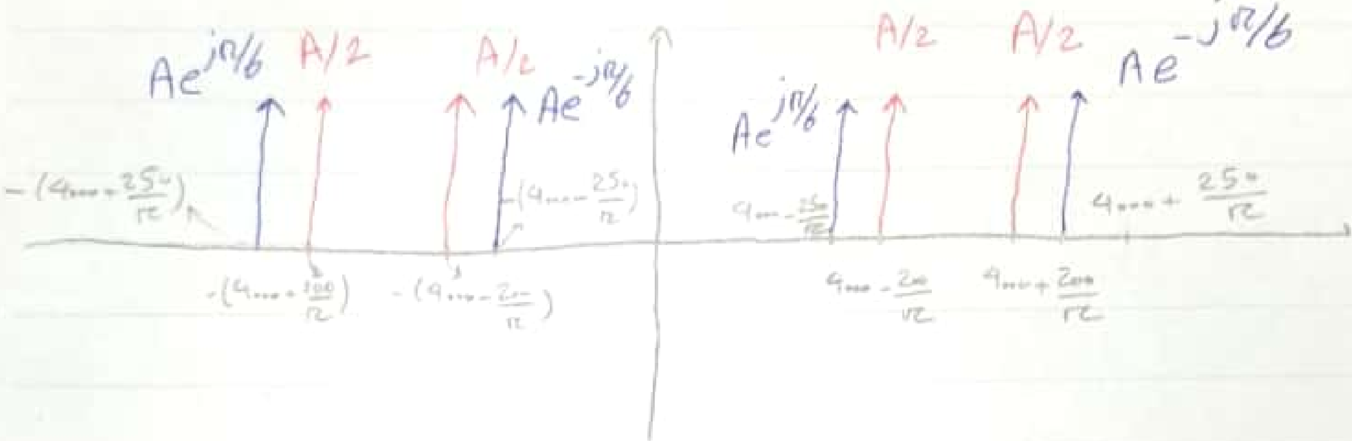
سؤال 5 _ ادامه

$$* \; Aj\,e^{-j\pi/3} = A e^{j\pi/2} e^{-j\pi/3} = A e^{j\pi/6}$$

$$* \; -Aj\,e^{j\pi/3} = A e^{-j\pi/2} e^{j\pi/3} = A e^{-j\pi/6}$$



$$P_{xc} = \frac{A_c^2}{2} P_m$$

$$P_m = \frac{1}{T} \int_{-T/2}^{T/2} \left[2\cos(400t) + 4\sin(500t + \pi/3) \right]^2 dt$$

$$= \frac{50}{\pi} \int_{-\pi/100}^{\pi/100} \left[2\cos(400t) + 4\sin(500t + \pi/3) \right]^2 dt$$

$$= 10$$

$$P_{xc} = \frac{A^2}{2} P_m = 5A^2$$

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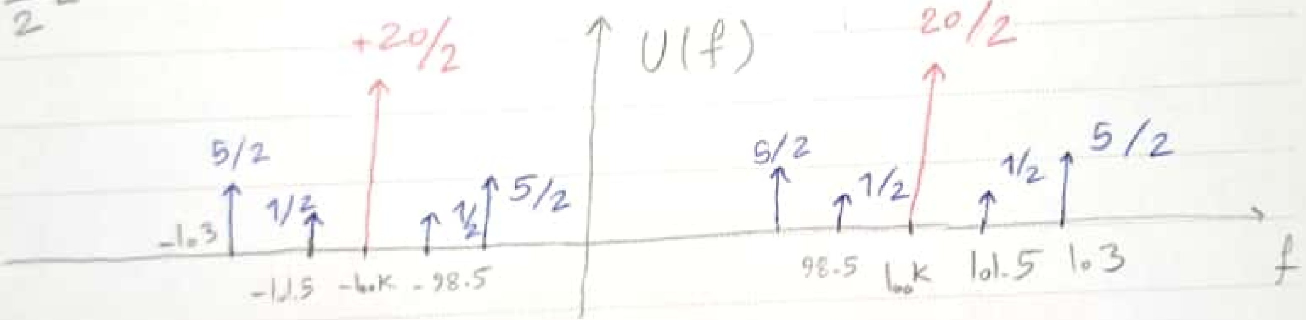
سوال 6)

1) $u(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)]\cos(200000\pi t)$

$\rightarrow u(t) = 20\cos(200000\pi t) + \cos(203000\pi t)$
$+ \cos(-197000\pi t) + 5\cos(206000\pi t) + 5\cos(-194000\pi t)$

$U(f) = 10[\delta(f-100000) + \delta(f+100000)] + \frac{1}{2}[\delta(f-101500) + \delta(f+101500)]$
$+ \frac{1}{2}[\delta(f-98500) + \delta(f+98500)] + \frac{5}{2}[\delta(f-103000) + \delta(f+103000)]$
$+ \frac{5}{2}[\delta(f-97000) + \delta(f+97000)]$



2) $R_{x_c}(\tau) = \frac{A^2}{2}(1 + \mu^2 R_m(\tau))\cos(2\pi f_c \tau)$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(\frac{0.1}{\mu}\cos(300\pi t) + \frac{0.5}{\mu}\cos(600\pi t)\right) \times$
$\left(\frac{0.1}{\mu}\cos(300\pi(t-\tau)) + \frac{0.5}{\mu}\cos(600\pi(t-\tau))\right) dt$

$= \frac{1}{200\mu^2}\cos(300\pi\tau) + \frac{1}{8\mu^2}\cos(600\pi\tau)$

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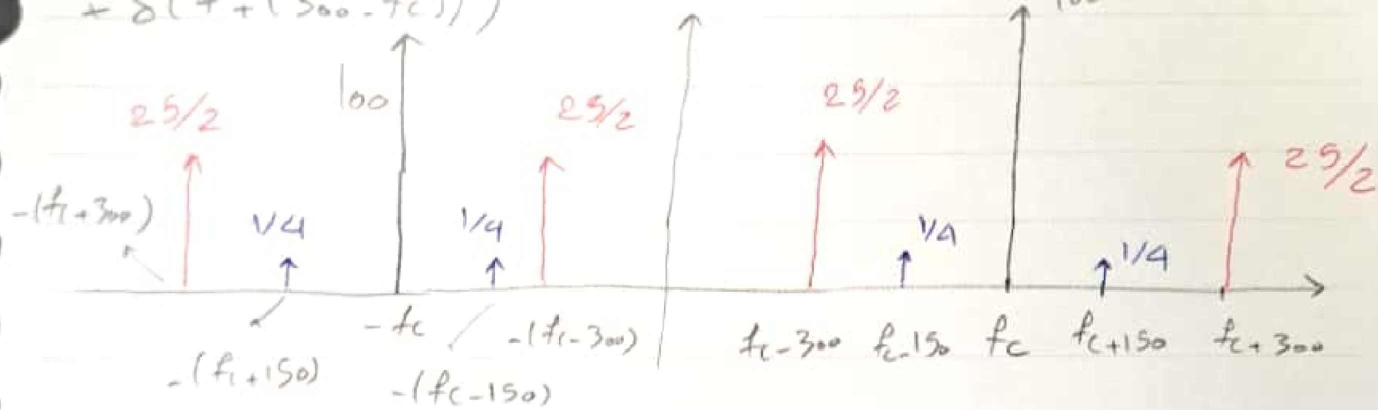
(ادامه) 2.6

$$S(f) = \frac{A^2}{4}\Big[\delta(f-f_c)+\delta(f+f_c)+\frac{1}{400}\Big(\delta(f-(150+f_c))+\delta(f-(150-f_c))+\delta(f+(150+f_c))+\delta(f+(150-f_c))\Big)$$

$$+\frac{1}{8}\Big(\delta(f-(300+f_c))+\delta(f-(300-f_c))+\delta(f+(300+f_c))+\delta(f+(300-f_c))\Big)$$



3)

$$u(t) = 20(1+\mu m(t))\cos(2\pi f_c t)$$

$$m(t) = \frac{0.1}{\mu}\cos(3000\pi t)+\frac{0.5}{\mu}\cos(600\pi t)$$

$$m(0) = \frac{0.1}{\mu}+\frac{0.5}{\mu} = \frac{0.6}{\mu} < 1 \longrightarrow \mu > 0.6$$

$$P_m = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|m(t)|^2\,dt = \frac{0.13}{\mu^2} < 1$$

$$\rightarrow \mu^2 > 0.13 \rightarrow \mu > 0.36$$

$\longrightarrow$ در نهایت : $0.6 < \mu < 1$

سوال 6)

4~~3~~) $P_u = \frac{A_c^2}{2} + \frac{A_c^2}{2}\mu^2 P_m$

$P_u = 200 + 26 = 226$

$P_{sb} = \frac{A_c^2}{2}\mu^2 P_m = 26$

$\frac{P_{sb}}{P_u} = \frac{26}{226} \simeq 0.115$

$= 11.5\%$

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