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COMP 352: Data Structures and Algorithms

18th April 2020

Assignment 4

1. i)

1) 32, 147, 265, 195, 207, 180, 21, 16, 189, 202, 91, 94, 162

2) 75, 37, 77, 81, 48

Hash function: $h(k) = k \bmod 13$

$$k=32 \rightarrow 32 \bmod 13 \rightarrow 6$$

$$k=147 \rightarrow 147 \bmod 13 \rightarrow 4$$

$$k=265 \rightarrow 265 \bmod 13 \rightarrow 5$$

$$k=195 \rightarrow 195 \bmod 13 \rightarrow 0$$

$$k=207 \rightarrow 207 \bmod 13 \rightarrow 12$$

$$k=180 \rightarrow 180 \bmod 13 \rightarrow 11$$

$$k=21 \rightarrow 21 \bmod 13 \rightarrow 8$$

$$k=16 \rightarrow 16 \bmod 13 \rightarrow 3$$

$$k=189 \rightarrow 189 \bmod 13 \rightarrow 7$$

$$k=202 \rightarrow 202 \bmod 13 \rightarrow 7$$

$$k=91 \rightarrow 91 \bmod 13 \rightarrow 0$$

$$k=94 \rightarrow 94 \bmod 13 \rightarrow 3$$

$$k=162 \rightarrow 162 \bmod 13 \rightarrow 6$$

$$k=75 \rightarrow 75 \bmod 13 \rightarrow 10$$

$$k=37 \rightarrow 37 \bmod 13 \rightarrow 11$$

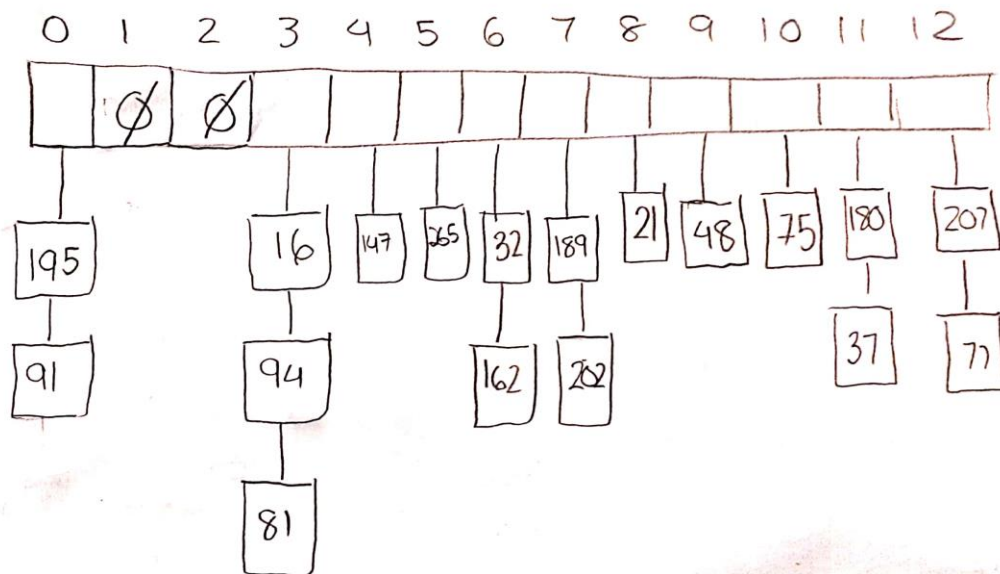
$$k=77 \rightarrow 77 \bmod 13 \rightarrow 12$$

$$k=81 \rightarrow 81 \bmod 13 \rightarrow 3$$

$$k=48 \rightarrow 48 \bmod 13 \rightarrow 9$$

Table is on the other side.

The table hence looks like this



2) 0 - 195 → 180 → 75

ii) At one index, there are at most 2 collisions, but in total there are 7 collisions.

2. This proposal would not be valid here since we would have more collisions than question 1 (see picture below). However, the idea of this practice makes sense since if we increase the table size, the better chance we have to cause less collisions due to the table having more cells to insert elements in. We can just say that the data was not optimal here.

81

2) 0 - 195 \rightarrow 180 \rightarrow 75 (2 collisions)

1 - 16 \rightarrow 91 (1 collision)

2 - 32 \rightarrow 77 (1 collision)

3 - 48

4 - 94

5 - \emptyset

6 - 21 \rightarrow 81 (1 collision)

7 - 202 \rightarrow 37 (1 collision)

8 - \emptyset

9 - 189

10 - 265

11 - \emptyset

12 - 147 \rightarrow 207 \rightarrow 162 (2 collisions)

13 - \emptyset

14 - \emptyset

We would have a total of 8 collisions

3. i)

3) 0 - 12
 1 - 39
 2 -
 3 - 29
 4 - 42
 5 -
 6 - ~~25~~ 35
 7 -
 8 -
 9 -
 10 - 48
 11 - 35
 12 - 12
 13 -
 14 -
 15 -

16 - ~~31~~ 29
 17 -
 18 - 18

/ = remove

Rest of
 the operations
 on the
 other side.

$$\text{put}(25) = 25 \bmod 19 = 6$$

$$\text{put}(12) = 12 \cdot 19 = 12$$

$$\text{put}(42) = 42 \cdot 19 = 4$$

$$\text{put}(31) = 31 \cdot 19 = 12$$

There is a collision

$$7 - 31 \bmod 7 = 4$$

$$12 + 1(4) = 16$$

put(35)

$$35 \bmod 19 = 16$$

There is a collision

$$7 - 35 \bmod 7 = 7$$

$$16 + 2(7) \bmod 19 = 11$$

$$\text{put}(39) \rightarrow 39 \bmod 19 = 1$$

$$\text{remove}(31) \rightarrow 31 \bmod 19 = 12, \text{ it's not there}$$

$$7 - 31 \bmod 7 = 9$$

$$12 + (1)(9) = 16 \rightarrow \text{This is true, remove at 16}$$

$$\text{put}(48) = 48 \bmod 19 = 10$$

$$\text{remove}(25) = 25 \bmod 19 = 6 \rightarrow \text{This is true, remove at 6}$$

$$\text{put}(18) = 18 \bmod 19 = 18$$

$$\text{put}(29) = 29 \bmod 19 = 10 \text{ There is a collision}$$

$$7 - 29 \bmod 7 = 6$$

$$10 + 1(6) = 16$$

$$\text{put}(29) \rightarrow \text{collision at 10}$$

$$10 + 1(6) = 16$$

$$10 + 2(6) = 22 \rightarrow 22 \bmod 19 = 3$$

$$\text{put}(35) \rightarrow \text{Collision at 16} \quad 7 - 35 \bmod 7 =$$

$$16 + 1(7) = 23 \bmod 19 = 4 \times$$

$$16 + 2(7) = 30 \bmod 19 = 11 \times$$

$$16 + 3(7) = 37 \bmod 19 = 18 \times$$

$$16 + 4(7) = 44 \bmod 19 = 6$$

Hence, the updated table would look like

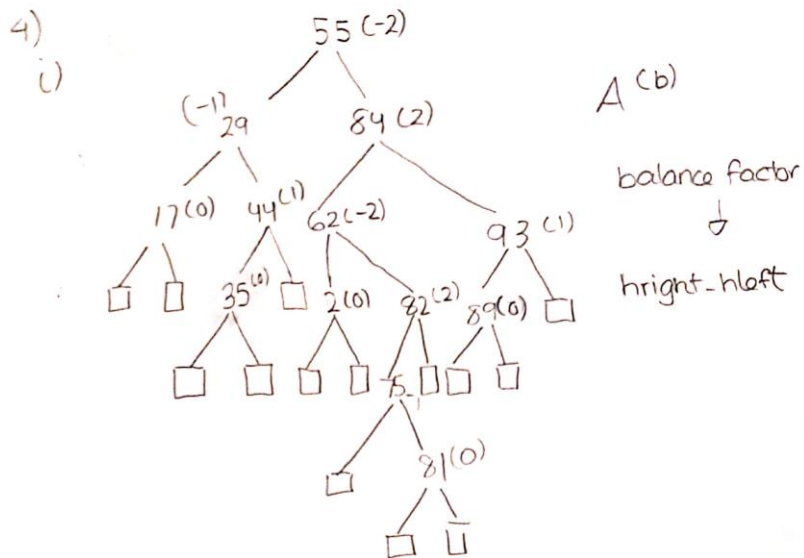
0	- \emptyset
1	- 39
2	- \emptyset
3	- 29
4	- 42
5	- \emptyset
6	- 35
7	- \emptyset
8	- \emptyset
9	- \emptyset
10	- 48
11	- 35
12	- 12
13	- \emptyset
14	- \emptyset
15	- \emptyset
16	- 29
17	- \emptyset
18	- 18

ii) The size of the longest cluster would be 3 at the indexes 10-11-12

iii) There was a total of 10 collisions

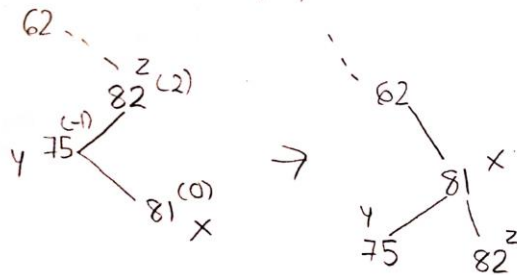
iv) The load factor is 9/19 or 47.37%

4.

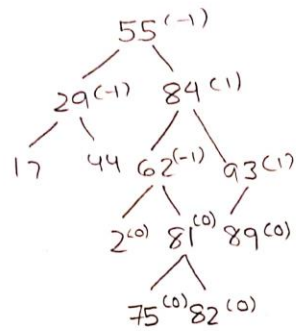


Hence, here there are errors with 55, 84, 62, 82

To correct this we use 82 (the farthest root)
and work on our way up.

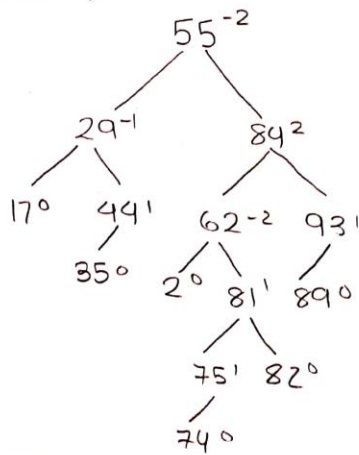


The whole tree would then look like



→ This is a balanced tree.

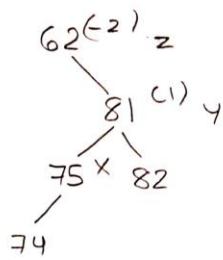
ii) put(74)



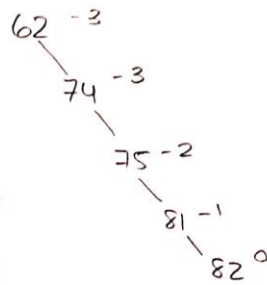
Errors at 55, 84, 62

iii

We take 62 since it is farthest from the right.

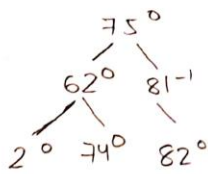


map to

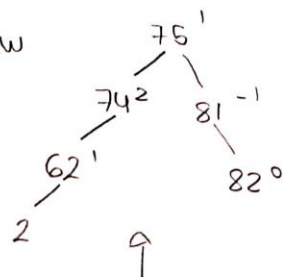


Still error
in 62, 74, 75

rotate ccw

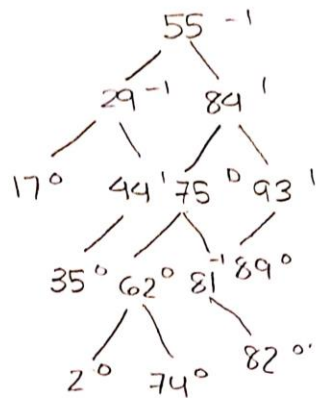


rotate cw



Still an error in 74

Hence the entire tree would look like

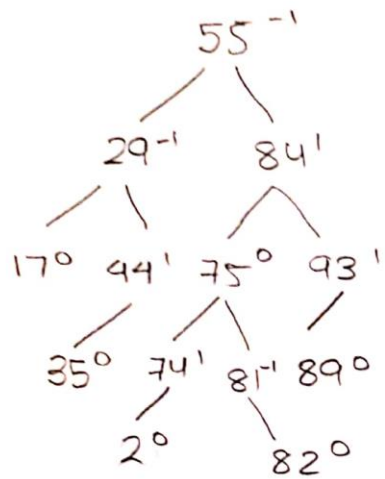


Since the tree is now balanced, we know that the complexity is $O(h)$, which in this case is $O(\log n)$

iii) remove(62) , This has two children, hence we replace the node with the predecessor (parent)

Hence, we replace 62 with 75

Hence, tree would look like



→ This is a balanced tree, no error

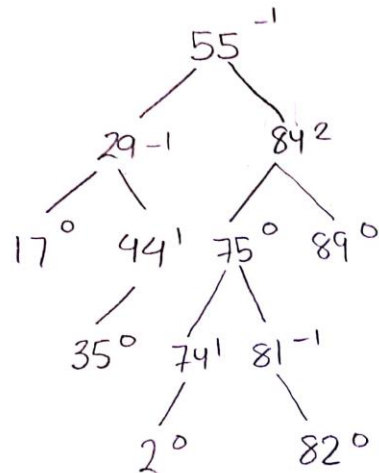
Hence, complexity is $O(\log n)$.

iv) remove (93) . Only has one child.

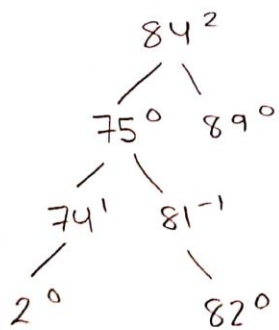
Hence we have to connect it's parent and child.

Correct 84 to 89

Hence the tree would look like,

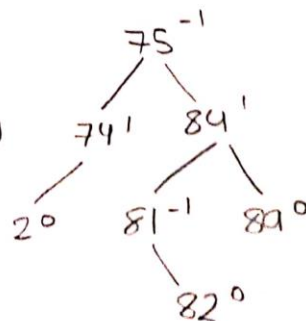


Error in 84

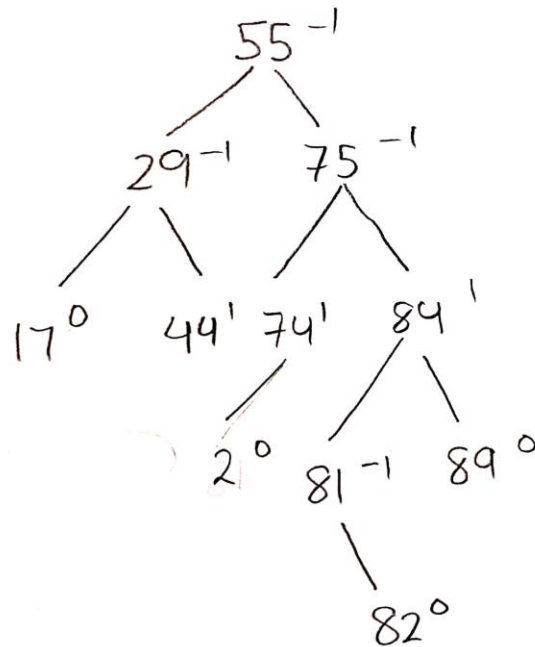


left-side imbalance.

→ rotate CW



Hence, the final tree looks like



→ Balanced tree

Hence, complexity
is $O(\log n)$.

5)

5) 832, 91, 411, 172, 243, 573, 326, 292, 682, 489
96

At most, there are 3 digits in one number

If we sort by the first digit,

91 411 832 172 292 682 243 573 326
96 489

If we sort by the second digit,

411 326 832 243 172 573 682 489 91
292 96

If we sort by the third digit,

91 96 172 243 292 326 411 489 573
682 832

↑
This is the array sorted by the radix sort