

Concordia University Department of Computer Science and Software Engineering

SOEN 331 Section S: Formal Methods
for Software Engineering

Assignment 4

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Contents

1 Our assignment

1. (10 pts) Find a logically equivalent formula for $\phi \mathbf{W} \psi$ and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

Solution:

The logically equivalent formula for this would be:

$$(\phi \mathbf{W} \psi) \equiv (\phi \mathbf{U} \psi) \vee \Box(\psi)$$

These are equivalent, because the principle of the strong until operator \mathbf{U} . Since the ψ is never guaranteed to be true, we would need to add an extra or statement because if the statement ψ becomes true it would mean that ϕ can't be true. This means that ϕ would be true until a certain condition (ψ is true) is met.

2. (10 pts) Find a logically equivalent formula for $\phi \mathbf{U} \psi$, and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

Solution:

The logically equivalent formula for this would be:

$$(\phi \mathbf{U} \psi) \equiv (\phi \mathbf{W} \psi) \wedge \bigcirc\Diamond(\psi)$$

These are equivalent, because the principle of the strong until operator \mathbf{U} . We know that this ψ will eventually become true. We then now that we can use the weak until clause with an eventually operator, because the only way that ϕ is not true is when the ψ is not true. We add the and operator, because we want to insure that the next one we actually get ψ as being true, because this may never happen.

3. (10 pts) Find a logically equivalent formula for $\phi \mathbf{R} \psi$ in terms of \mathbf{W} , and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

Solution:

The logically equivalent formula for this would be:

$$(\phi \mathbf{R} \psi) \equiv (\phi \mathbf{W} \psi) \wedge \Diamond(\psi)$$

These are equivalent as the weak until will make the operation hold until something is triggered. We need the second part of the equation to guarantee that ψ will eventually has to be true, because if it is not then, this will never hold. Hence the and statement.

This paragraph refers to Questions 4 - 5: Consider a railroad with a single rail and a road level-crossing. We introduce the following propositions that represent events:

a : A train is approaching.

b : The barrier is down

c : A train is crossing

l: A light is blinking

4. (15 pts) Express each of the following requirements formally. For each one, proceed to find a logically equivalent formula that captures the safety property of the system (i.e. in terms of “something bad never happens”):

- (a) (5 pts) When a train is crossing, the barrier must be down.

Solution:

$$\Box(c \rightarrow \Box b)$$

- (b) (5 pts) If a train is approaching or crossing, then the light must be blinking.

Solution:

$$\Box(a \vee c \rightarrow \Box l)$$

- (c) (5 pts) If the barrier is up and the light is off, then no train is coming or crossing.

Solution:

$$\Box(\neg(b \wedge l) \rightarrow \neg \Box(a \vee c))$$

5. (10 pts) Express each of the following requirements formally in terms of the liveness property (i.e. in terms of “something good eventually happens”):

(a) (5 pts) When a train is approaching, it will eventually cross..

Solution:

$$\Box(a \rightarrow \Diamond c)$$

(b) (5 pts) When a train is approaching and no train is crossing, then the barrier will eventually go down before the train crosses.

Solution:

$$\Box((a \wedge \neg c) \rightarrow \Diamond(\neg b \text{ U } c))$$

6. (45 pts) The behavior of a program is expressed by the following temporal formula:

$$\Box \left[\begin{array}{l} \text{start} \rightarrow (\phi \oplus \psi) \\ \\ \text{start} \rightarrow \tau \\ \\ \phi \rightarrow \bigcirc(\psi \text{ U } \chi) \\ \\ \psi \wedge \tau \rightarrow \bigcirc(\psi \text{ W } \chi) \\ \\ \tau \wedge \bigcirc\psi \rightarrow \bigcirc\omega \\ \\ \psi \wedge \omega \rightarrow \bigcirc^2\chi \\ \\ \omega \wedge \bigcirc^2\chi \rightarrow \bigcirc^2\sigma \\ \\ \psi \wedge \bigcirc\sigma \rightarrow \bigcirc^2\pi \\ \\ \psi \wedge \tau \rightarrow \sigma \text{ R } \pi \\ \\ \phi \wedge \bigcirc\psi \rightarrow \bigcirc^2\chi \end{array} \right]$$

- (a) (20 pts) Visualize all models of behavior.

Solution:

- (b) (10 pts) Is the set of requirements satisfiable in all models of behavior? Explain why or why not.

Solution:

- (c) (10 pts) In the case where the set of requirements is not satisfiable, what modification(s) to the requirements would you make (you may temporarily assume the role of a stakeholder) in order to achieve satisfiability.

Solution:

- (d) (5 pts) Having resolved any possible conflicts in requirements, specify conditions (models of behavior), if any exist, under which the program can terminate. If none exist, please indicate so.

Solution: