

Concordia University Department of Computer Science and Software Engineering

SOEN 331 Section S: Formal Methods
for Software Engineering

Assignment 1

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Contents

1 Problem 1: Predicate Logic 1 (10 pts)

1.1 Description:

In the domain of all people in the room, consider the predicate $received_request(a, b)$ that is interpreted as

"[person] a has received a request from [person] b to connect on some social platform"

1. How are the following two expressions translated into plain English? Are the two expressions logically equivalent?

- $\forall a \exists b \text{ received_request}(a, b)$.
- $\exists b \forall a \text{ received_request}(a, b)$.

Solution:

The first statement $\forall a \exists b \text{ received_request}(a, b)$. reads that every person a has received a request from one person b.

The second statement $\exists b \forall a \text{ received_request}(a, b)$. reads that there exists a person b which has a received a request from all people a.

In terms of logical equivalency, we would need to determine if the truth values for both of these statements are the same. For the first statement, while it is possible that every person a has received a request from one person b, it is highly unlikely that in the second statement, that one person b has received a request from every person from a. **Hence, both of these statement are NOT logically equivalent.**

2. Discuss in detail whether we can we claim the following:

$$\forall a \exists b \text{ received_request}(a, b) \rightarrow \exists b \forall a \text{ received_request}(a, b).$$

Solution:

The statement $\forall a \exists b \text{ received_request}(a, b) \rightarrow \exists b \forall a \text{ received_request}(a, b)$. is **FALSE**.

The first predicate states that all the people from a have received a request from each

person b. While this does mean that everyone person a received a request from a person b, it does not necessarily mean that all the a people asked the same b people. Every a person could have received a request from a different b person. Predicate 2, however, states that there exists a person b who has received a request from every person a, which we have proved from the above statement is **FALSE**.

3. Discuss in detail whether we can we claim the following:

$$\exists b \forall a \text{ received_request}(a, b) \rightarrow \forall a \exists b \text{ received_request}(a, b).$$

Solution:

The statement $\exists b \forall a \text{ received_request}(a, b) \rightarrow \forall a \exists b \text{ received_request}(a, b)$. is **TRUE**. The first predicates states that there is exists one person b who has a received a request from every person a. The second predicate states that every person a has received a request from atleast one person b which we see from the first predicate to be true, hence we can claim this statement to be **TRUE**.

4. How are the following two expressions translated into plain English? Are the two expressions logically equivalent?

- $\forall b \exists a \text{ received_request}(a, b)$.
- $\exists a \forall b \text{ received_request}(a, b)$.

Solution:

The first statement $\forall b \exists a \text{ received_request}(a, b)$. reads that every person b has received a request from one such person a. The second statement $\exists a \forall b \text{ received_request}(a, b)$. states that there exists a person a which has a received from every person b. To make sure that these statements are logically equivalent, we would need to determine if the truth values for these statements are the same. While in the first statement, every person b does receive a request from one person a, we have no way to make sure that every person b get the request from the same person a. If that statement would hold, then we could say that there would exist a person a who receives a request from every person b, but since this is not the case **Both of these statements are NOT logically equivalent**.

2 Problem 2: Predicate Logic 2 (10 pts)

2.1 Description:

Given the subject "being a person" and the predicate "being bad", consider the list of propositions below:

1. "There are some nice people"
2. "There are no nice people"
3. "Everybody is bad"
4. "Some people are bad"
5. "Everybody is nice"
6. "Some people are not nice"

Associate each of the propositions below to one of the standard forms of categorical propositions.

Solution:

We can assume in this case that being a person (or people) would be the subject S, denoted by $P(x)$ and the being bad would be the predicate P, denoted by $Q(x)$. Let:

1. "There are some nice people" = This is of the form some S are not P. Translated to proposition form, this would give us $\exists x, (P(x) \wedge \neg Q(x))$, **which is O form.**
2. "There are no nice people" = This is of the form of all S are P. Translated to proposition form, this would give us $\forall x, (P(x) \rightarrow Q(x))$, **which is A form.**
3. "Everybody is bad" = This is of the form of no S are P. Translated to proposition form, this would give us $\forall x, (P(x) \rightarrow \neg Q(x))$, **which is E form.**
4. "Some people are bad" = This is of the form of some S are P. Translated to proposition form, this would give us $\exists x, (P(x) \wedge Q(x))$, **which is I form.**

5. "Everybody is nice" = This is of the form of no S are P. this would give us $\forall x, (P(x) \rightarrow \neg Q(x))$, **which is E form.**
6. "Some people are not nice" = This is of the form of some S are P. Translated to proposition form, this would give us $\exists x, (P(x) \wedge \neg Q(x))$, **which is I form.**