Concordia University Department of Computer Science and Software Engineering

SOEN 331 Section S: Formal Methods

for Software Engineering

Assignment 4

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Contents

1 Our assignment

1. (10 pts) Find a logically equivalent formula for $\phi\ W\ \psi$ and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

   Solution:

   The logically equivalent formula for this would be:

   $(\phi\ \mathbf{W}\ \psi) \equiv (\phi\ \mathbf{U}\ \psi) \lor \Box(\psi)$

   These are equivalent, because the principle of the strong until operator **U**. Since the ψ is never guaranteed to be true, we would need to add an extra or statement because if the statement ψ becomes true it would mean that ϕ can't be true. This means that ϕ would be true until a certain condition (ψ is true) is met.

2. (10 pts) Find a logically equivalent formula for $\phi\ U\ \psi$, and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

   Solution:

   The logically equivalent formula for this would be:

   $(\phi\ \mathbf{U}\ \psi) \equiv (\phi\ \mathbf{W}\ \psi) \land \bigcirc\Diamond(\psi)$

   These are equivalent, because the principle of the strong until operator U. We know that this ψ will eventually become true. We then now that we can use the weak until clause with an eventually operator, because the only way that ϕ is not true is when the ψ is not true. We add the and operator, because we want to insure that the next one we actually get ψ as being true, because this may never happen.

3. (10 pts) Find a logically equivalent formula for $\phi\ R\ \psi$ in terms of W , and provide a short reasoning to support your answer. Represent this equivalence between the two expressions with the appropriate logical connective, and support your reasoning.

Solution:

The logically equivalent formula for this would be:

$(\phi\ \mathbf{R}\ \psi) \equiv (\phi\ \mathbf{W}\ \psi) \wedge \Diamond(\psi)$

These are equivalent as the weak until will make the operation hold until something is triggered. We need the second part of the equation to guarantee that ψ will eventually has to be true, because if it is not then, this will never hold. Hence the and statement.

This paragraph refers to Questions 4 - 5: Consider a railroad with a single rail and a road level-crossing. We introduce the following propositions that represent events:

a : A train is approaching.

b : The barrier is down

c : A train is crossing

l: A light is blinking

4. (15 pts) Express each of the following requirements formally. For each one, proceed to find a logically equivalent formula that captures the safety property of the system (i.e. in terms of "something bad never happens"):

   (a) (5 pts) When a train is crossing, the barrier must be down.

   Solution:

   $\Box(\mathrm{c} \rightarrow \Box b)$

   (b) (5 pts) If a train is approaching or crossing, then the light must be blinking.

   Solution:

   $\Box(\mathrm{a} \vee c \rightarrow \Box l)$

   (c) (5 pts) If the barrier is up and the light is off, then no train is coming or crossing.

   Solution:

   $\Box(\neg\ (b \wedge l) \rightarrow \neg\ \Box(a \vee c))$

5. (10 pts) Express each of the following requirements formally in terms of the liveness property (i.e. in terms of "something good eventually happens"):

(a) (5 pts) When a train is approaching, it will eventually cross..

Solution:

$\Box(\mathrm{a} \rightarrow \Diamond c)$

(b) (5 pts) When a train is approaching and no train is crossing, then the barrier will eventually go down before the train crosses.

Solution:

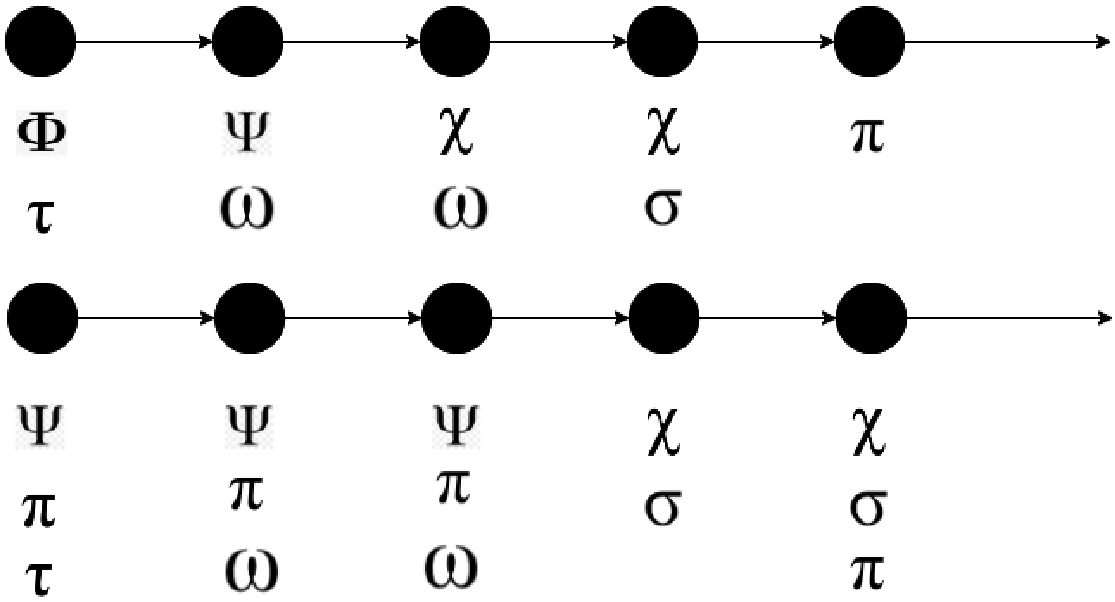
$\Box((\mathrm{a} \wedge \neg\, c) \rightarrow \Diamond(\neg\, b \ \mathbf{U}\ c))$

6. (45 pts) The behavior of a program is expressed by the following temporal formula:

$$\Box \begin{bmatrix} \mathbf{start} \rightarrow (\phi \oplus \psi) \\ \mathbf{start} \rightarrow \tau \\ \phi \rightarrow \bigcirc(\psi\ \mathcal{U}\ \chi) \\ \psi \wedge \tau \rightarrow \bigcirc(\psi\ \mathcal{W}\ \chi) \\ \tau \wedge \bigcirc\psi \rightarrow \bigcirc\omega \\ \psi \wedge \omega \rightarrow \bigcirc^2\chi \\ \omega \wedge \bigcirc^2\chi \rightarrow \bigcirc^2\sigma \\ \psi \wedge \bigcirc\sigma \rightarrow \bigcirc^2\pi \\ \psi \wedge \tau \rightarrow \sigma\ \mathcal{R}\ \pi \\ \phi \wedge \bigcirc\psi \rightarrow \bigcirc^2\chi \end{bmatrix}$$

(a) (20 pts) Visualize all models of behavior.

Solution:



(b) (10 pts) Is the set of requirements satisfiable in all models of behavior? Explain why or why not.

Solution:

(c) (10 pts) In the case where the set of requirements is not satisfiable, what modification(s) to the requirements would you make (you may temporarily assume the role of a stakeholder) in order to achieve satisfiability.

Solution:

(d) (5 pts) Having resolved any possible conflicts in requirements, specify conditions (models of behavior), if any exist, under which the program can terminate. If none exist, please indicate so.

Solution: