

HOME WORK #1

TEx-5

Ali Zah

21K-4653

Ex-5 Q#5.4

Discrete Random Variable:- (Stochastic Variable)

A variable that is determined by the outcome of a random experiment can assume any value contained in one or more intervals. Also known as stochastic variable.

Probability Distribution of Discrete Random Variables-

List of all possible values that the random variable can assume and their corresponding probabilities.

Eg. Example of Probability Distribution Table.

Number Of Vehicles	Frequency	Relative Frequency
0	30	$30/2000 = 0.015$
1	320	$320/2000 = 0.160$
2	910	0.455
3	580	0.290
4	160	0.80
	N= 2000	Sum = 1.00

Q# 5.5

The two Characteristics of Probability distributions of a discrete random variable are:

- 1) $0 \leq P(x) \leq 1$
- 2) $\sum P(x) = 1$

⑤

⑤

Q # 5.6

- (a) Valid because $\sum P(x)$
- (b) Not Valid $\sum P(x) \neq 0.97$ it should be $\sum P(x) = 1$
- (c) $\sum P(x) = 1$ Valid but $P(7)$ has negative probability.

Q # 5.7

(a) $P(3) = \boxed{0.15}$

(b) $P(0) = 0.11$

$P(1) = 0.19$

$P(2) = 0.28$

$$P(0) + P(1) + P(2) = 0.11 + 0.19 + 0.28$$

$$= \boxed{0.58}$$

(c) $P(4) + P(5) + P(6) = 0.12 + 0.09 + 0.06 = \boxed{0.27}$

(d) $P(1) + P(2) + P(3) + P(4) = 0.19 + 0.28 + 0.15 + 0.12 = \boxed{0.74}$

(e) $x < 4$

$$P(0) + P(1) + P(2) + P(3) = 0.11 + 0.19 + 0.28 + 0.15 = \boxed{0.73}$$

(f) $x > 2$

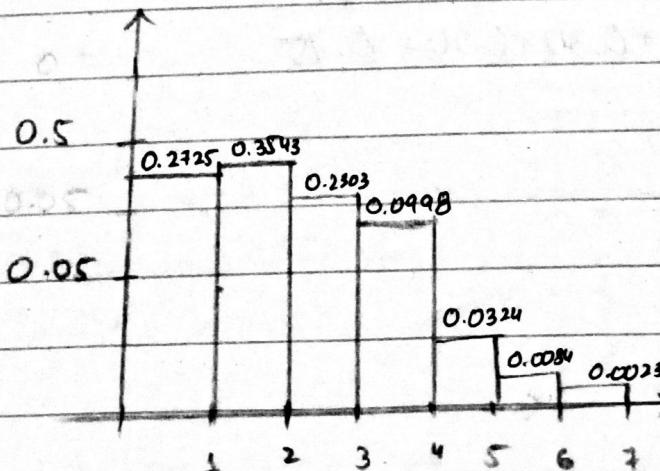
$$P(3) + P(4) + P(5) + P(6) = 0.15 + 0.12 + 0.09 + 0.06 = \boxed{0.42}$$

(g) $2 \leq x \leq 5$

$$P(2) + P(3) + P(4) + P(5) = 0.28 + 0.15 + 0.12 + 0.09 = \boxed{0.64}$$

Q # 5.8

(a)



2

(b) (P) $x \geq 2$
 $P(2) + P(3) + P(4) + P(5) + P(6)$

$$= 0.2303 + 0.0998 + 0.0324 + 0.0084 + 0.0023 \\ = 0.37320$$

ii) $x = 5$ $P(5) = 0.0084$

iii) $x \leq 3$ $P(0) + P(1) + P(2) = 0.2725 + 0.3543 + 0.2303$
 $= 0.85710$

iv) $x \geq 1$ $P(0) + P(1) = 0.2725 + 0.3543 = 0.62680$

Q# 5.9 @	x	P(x)
	1	0.1
	2	0.25
	3	0.30
	4	0.20
	5	0.15

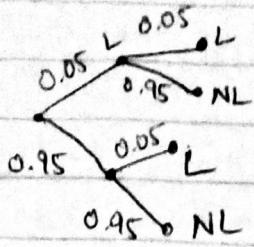
B) Approximate, because data has only been collected from the last 80 days (sample)

C) $P(3) = 0.3$

$$P(x \geq 3) = 0.15 + 0.20 + 0.30 = 0.65$$

$$P(2 \leq x \leq 4) = 0.25 + 0.30 + 0.20 = 0.75$$

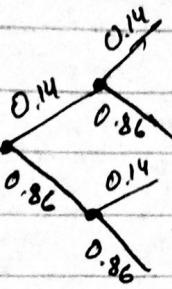
$$P(x < 4) = 0.65$$

Q5.10

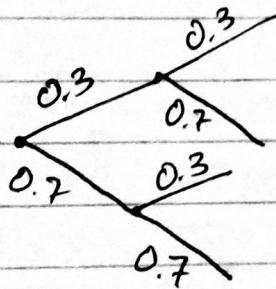
$$\text{EP}(x) =$$

L = lemon
NL = Not lemon

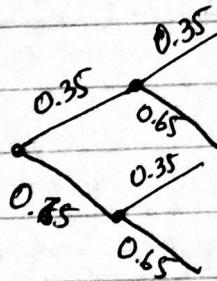
x	$P(x)$
0	0.9025
1	0.0950
2	0.0025
	$\sum P(x) = 1.0$

Q5.11

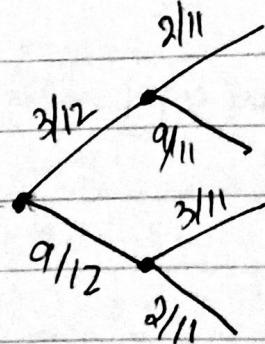
x	$P(x)$
0	0.7396
1	0.2408
2	0.0196

Q5.12

x	$P(x)$
0	0.49
1	0.42
2	0.09

Q5.13

x	$P(x)$
0	0.4225
1	0.4450
2	0.1225

Q5.14

x	$P(x)$
0	0.5454
1	0.40920
2	0.0454

Q16@ $\sum xP(x) = 1.59$

$$\sigma = \sqrt{(\sum x^2 P(x)) - \mu^2} = \sqrt{3.45 - (1.59)^2} = 0.96016$$

\therefore Mean = 1.59 Std Dev = 0.96016

(b) Mean = $\sum xP(x) = 7.07$

$$\begin{aligned}\text{Std Dev} &= \sqrt{\sum x^2 P(x) - \mu^2} \\ &= \sqrt{51.11 - (7.07)^2} \\ &= \sqrt{1.06071}\end{aligned}$$

Q17 $\sum xP(x) = 0.44$

Mean = 0.44 errors

$$\begin{aligned}\text{Std Dev} &= \sqrt{0.92 - (0.44)^2} \\ &= \sqrt{0.85229} \text{ errors.}\end{aligned}$$

Q18

$$\begin{aligned}H &= \sum xP(x) = 2.94 \\ \sigma &= \sqrt{\sum x^2 P(x) - \mu^2} \\ &= \sqrt{10.72 - (2.94)^2} = 1.44097 \text{ cans sold}\end{aligned}$$

Q19 $\mu = \sum xP(x) = 1.2997 \text{ patients}$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 1.13828 \text{ patients}$$

Q20 $H = \sum xP(x) = 2.48 \text{ houses sold}$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 1.29985 \text{ houses sold}$$

Q21

$$\mu = \sum x P(x) = 2.561$$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 1.32245$$

Q22

$$\mu = \sum x P(x) = 3.05$$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 1.20312$$

Q23

$$\mu = \sum x P(x) = 3.9 \text{ $ million}$$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 3.015 \text{ $ million}$$

Q24

x	P(x)
-2	0.8894
3	0.1
8	0.01
998	0.0005
4998	0.0001

$$\mu = \sum x P(x) = -0.4 \text{ $}$$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} \\ = \$4.78084 \text{ $}$$

Q25

$$\mu = \sum x P(x) = 0.5$$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = 0.584$$

Q26

(a) NO, number of outcomes $1=2$ since there are 6 possible outcomes

(b) Yes, it satisfies all 4 condition of binomial distribution

(c) Yes.

Q29 ~~Q29~~ =

(a) Yes

(b) No, trials are dependent

(c) Yes

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~~Q30~~

② $P(x) = {}_n C_x P^x Q^{n-x}$

$P = 0.7$

$Q = 0.3$

$n=8$

$x=5$

$n-x=3$

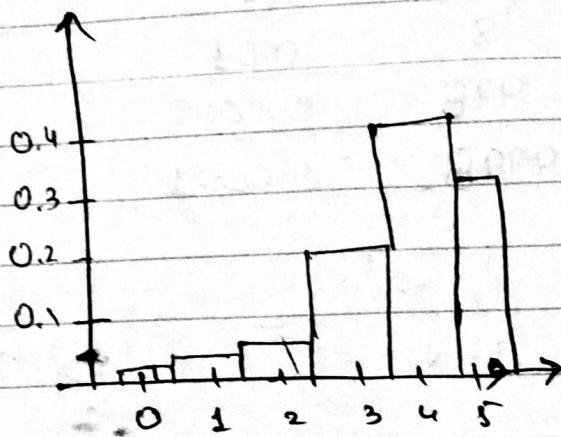
$$P(5) = {}_8 C_5 P^5 Q^3 = {}_8 C_5 (0.7)^5 (0.3)^3 \\ = 0.25412$$

③ $D(3) = {}_4 C_3 (0.4)^3 (0.6)^1 \\ = 0.1536$

④ $P(2) = {}_6 C_2 (0.3)^2 (0.7)^4 \\ = 0.32413$

~~Q31~~

x	$P(x)$
0	0.0003
1	0.0064
2	0.0512
3	0.2048
4	0.4096
5	0.3277



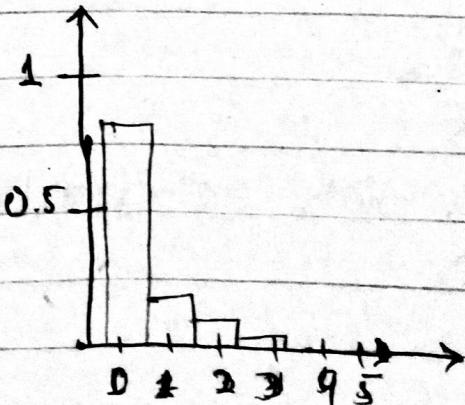
$H = np = 5(0.8) = 4$

$\sigma = \sqrt{npq} = \sqrt{5(0.8)(0.2)} = 0.89443$

~~Q32~~

for $n=5$, $P=0.05$

x	$P(x)$
0	0.7738
1	0.2036
2	0.0214
3	0.011
4	0.00
5	0.00



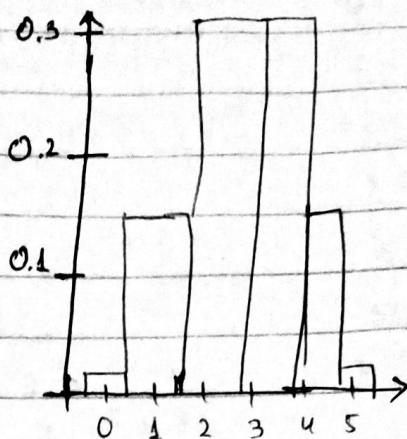
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$$n=5, P=0.5$$

x $P(x)$

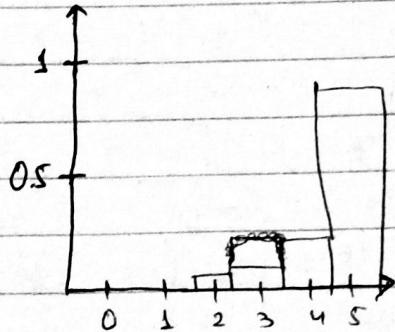
0	0.0312
1	0.1562
2	0.3125
3	0.3125
4	0.1562
5	0.0312



$$n=5, P=0.95$$

x $P(x)$

0	0.0
1	0.0
2	0.0011
3	0.0214
4	0.2036
5	0.7738



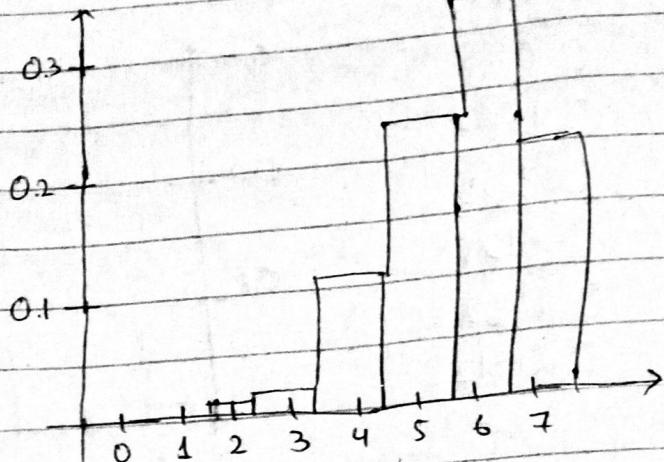
~~Q38~~ (a) $P(8) = 8C_8 (0.85)^8 (0.15)^0$
= 0.27249

(b) $P(5) = 8C_5 (0.85)^5 (0.15)^3$
= 0.08386

(8)

Q39

x	$P(x)$
0	0.0000
1	0.0064
2	0.0043
3	0.0287
4	0.1147
5	0.2753
6	0.3670
7	0.2097



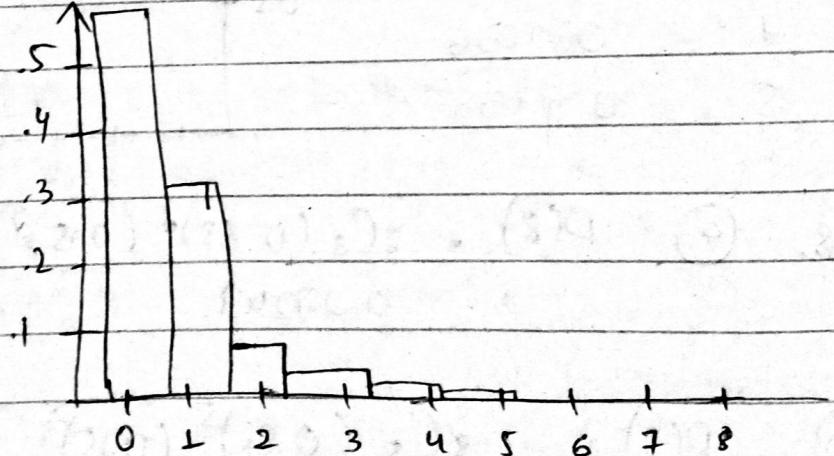
$$\mu = np = (7)(0.8) = 5.6 \text{ customers}$$

$$\sigma = \sqrt{npq} = \sqrt{(7)(0.8)(0.2)} = 1.058 \text{ customers}$$

$$P(4) = 0.1147$$

Q40

x	$P(x)$
0	0.5987
1	0.3151
2	0.0746
3	0.0105
4	0.0010
5	0.0001
6	0.0000
7	"
8	"
9	"
10	"



$$\mu = np = (10)(0.05) = 0.5$$

$$\sigma = \sqrt{npq} = \sqrt{10(0.05)(0.95)}$$

$$= 0.68920$$

(b) 0.0746

~~Q42~~

$$P(x) = \frac{rC_r \cdot n-r C_{n-r}}{n C_n}$$

$$\textcircled{a} \quad P(2) = \frac{3C_2 \cdot 5C_2}{8C_4} = 0.42857$$

$$\textcircled{b} \quad P(0) = \frac{3C_0 \cdot 5C_4}{8C_4} = 0.07143$$

$$\textcircled{c} \quad P(x \leq 1) = P(0) + P(1) \\ = 0.5 + \frac{3C_1 \cdot 5C_3}{8C_4} = 0.92857$$

~~Q43~~

$$\textcircled{a} \quad P(2) = \frac{4C_2 \cdot 7C_2}{11C_4} = 0.3818$$

$$\textcircled{b} \quad P(4) = \frac{4C_4 \cdot 7C_0}{11C_4} = 0.0030$$

$$\textcircled{c} \quad P(x \leq 1) = \frac{4C_0 + 7C_4}{11C_4} + \frac{4C_1 + 7C_3}{11C_4} = 0.5303$$

~~Q44~~ N=20 r=4, n=6

$$\textcircled{a} \quad P(1) = \frac{4C_1 \cdot 16C_5}{20C_6} = 0.45077$$

$$\textcircled{b} \quad P(0) = \frac{4C_0 \cdot 16C_6}{20C_6} = 0.20660$$

(10)

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$$(c) P(x \leq 2) = P(1) + P(0) + P(2)$$
$$= 0.45077 + 0.20660 + \frac{4 C_2 \cdot 16 C_4}{20 C_6}$$
$$= 0.93910$$

Q6 $P(4) = \frac{11 C_0 \cdot 7 C_0}{18 C_4} = 0.1078$

$$P(x \leq 2) = \frac{11 C_0 \cdot 7 C_4}{18 C_4} + \frac{11 C_1 \cdot 7 C_3}{18 C_4} + \frac{11 C_2 \cdot 7 C_2}{18 C_4}$$
$$= 0.5147$$

$$P(x > 1) = 1 - P(0) - P(1)$$
$$= 1 - 0.014 - 0.1258$$
$$= 0.86280$$

Q6 x_2 : Keyboard is defective

$$P(\text{accept}) = \frac{6 C_0 \cdot 14 C_5}{20 C_5} + \frac{6 C_1 \cdot 14 C_4}{20 C_5}$$
$$= 0.3875$$

$$P(\text{reject}) = 1 - (P(0) + P(1))$$
$$= 1 - 0.3875$$
$$= 0.6125$$

Q49 $P(x \leq 1)$ for $\lambda = 5$

$$P(1) = \frac{\lambda^x e^{-\lambda}}{x!} \cdot P(x \leq 1) = P(0) + P(1)$$

$$= \frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!}$$

$$P(x \leq 1) = 0.04043$$

 $P(2)$ for $\lambda = 2.5$

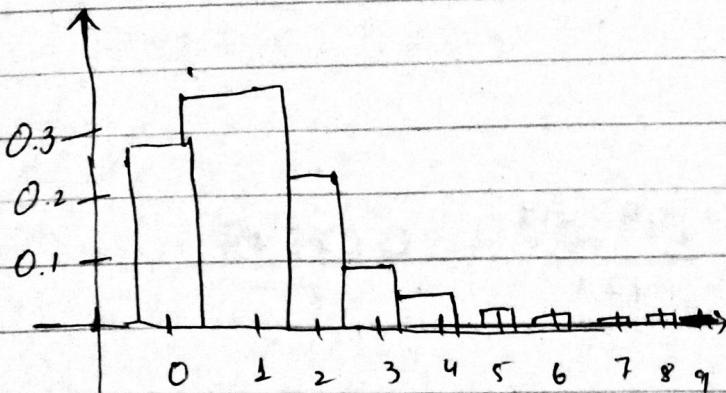
$$P(2) = \frac{2.5^2}{2!} \cdot e^{-2.5} = 0.25652$$

Q50 a) $\lambda = 1.3$

x	$P(x)$	x	$P(x)$
0	.2725	5	0.0084
1	.3543	6	0.0018
2	.2303	7	0.0003
3	.0998	8	0.0001
4	.0324	9	0.0000

$$\mu = \lambda = 1.3 = \sigma^2$$

$$\sigma = \sqrt{1.3} = 1.14018$$



Q51

$$\lambda = 5.4 \quad -5.4 \\ P(3) = \frac{12.8^3 \cdot e^{-5.4}}{3!} = \text{[redacted]} \quad 0.11853$$

Q52

$$\lambda = 12.5 \\ P(3) = \frac{12.5^3 \cdot e^{-12.5}}{3!} = 0.0019$$

Q53

$$\lambda = 3.7$$

$$\textcircled{a} \quad P(x \leq 1) = P(0) + P(1) \\ = \frac{3.7^0 \cdot e^{-3.7}}{0!} + \frac{3.7^1 \cdot e^{-3.7}}{1!} \\ = 0.1162$$

b

- i) 0.6625
- ii) 0.1699
- iii) 0.4941

Q54

$$P(3) = \frac{1.6^3 \cdot e^{-1.6}}{3!} = 0.13783$$

- b) i) 0.3962
- ii) 0.0787
- iii) 0.7833

Q55

$$P(12) = \frac{19^{12} \cdot e^{-19}}{12!} = 0.02589$$

- (b) i) 0.2314 ii) 0.0015

(m)

Q56

$$(a) P(0) = \frac{3.2^0 \cdot e^{-3.2}}{0!}$$

$$= 0.04070$$

x	$P(x)$	x	$P(x)$	x	$P(x)$
0	0.0408	5	0.1140	10	0.003
1	0.1304	6	0.0608	11	0.0004
2	0.2087	7	0.0278	12	0.0001
3	0.2226	8	0.0111	13	0.0000
4	0.1781	9	0.0040	14	0.0000

$$\lambda = 3.2$$

$$\sigma = \sqrt{3.2} = 1.78885$$

Q57

$$P(0) = \frac{0.8^0 \cdot e^{-0.8}}{0!} = 0.4493$$

x	$P(x)$	x	$P(x)$
0	0.4493	4	0.0077
1	0.3595	5	0.0012
2	0.438	6	0.0002
3	0.0383		

Q

$$\mu = 0.8$$

$$\sigma = \sqrt{0.8} = 0.8944$$

Q58

$$\lambda = 20, x = 25$$

$$P(25) = \frac{20^{25} \cdot e^{-20}}{25!} = 0.04459$$

- b) (i) 0.039 (ii) 0.182 (iii) 0.0018

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Q89

P(25)

$$\frac{20^{25} \cdot e^{-20}}{25!} = 0.04459$$

- b) (i) 0.0021 (ii) 0.4542 (iii) 0.0218

(15)