Engineering Probability

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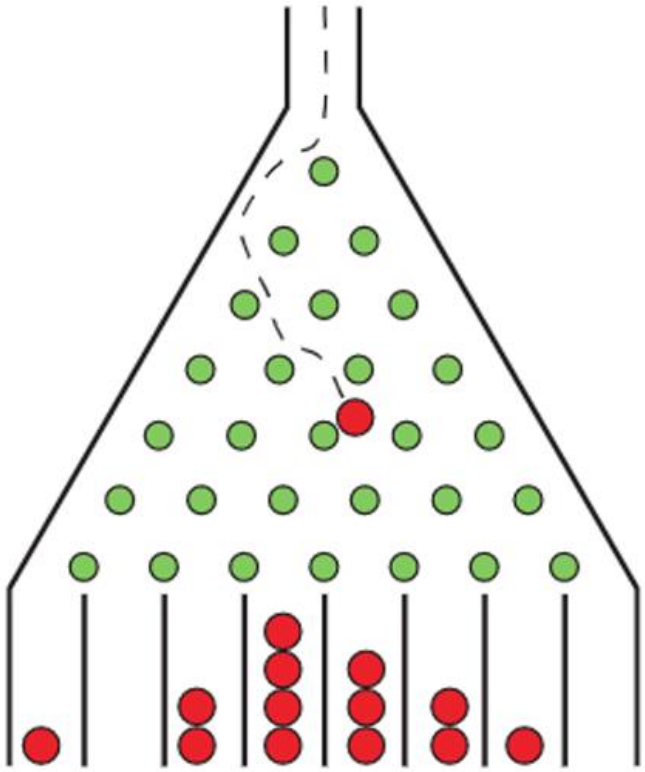
***Project (Issued:** Thursday, Dey 15, 1401 - **Due:** Monday, Dey 26, 1401)*

Problem 1 – Galton Board Simulation

Simulate a Galton board using Python or MATLAB.
In this system, a ball drops through a series of pegs, randomly bouncing left or right at each peg with equal probability. After a number of rows, the balls accumulate in bins at the bottom, forming a shape that approximates the **normal distribution**.



✓ **Tasks:**

- Implement a simulation where each ball randomly falls left or right and ends up in one of the bins.
- Repeat this process for 200 balls.
- Plot a histogram of the number of balls in each bin to show the bell-curve shape of the normal distribution.

➢ Problem 2 – Sum of Random Variables

Assume a series of independent and identically distributed (i.i.d.) random variables xix_i, each following the **exponential distribution** with probability density function:

Let Sn=x1+x2+⋯+xn. Then:

$$f_{X_i}(x) = \lambda e^{-\lambda x}$$

$$f_{S_n}(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{n-1}}{(n-1)!}$$

✓ Tasks:

- Plot fSn(x) for even values of n up to 14.
- Choose a suitable value for λ.
- Analyze and interpret the resulting graphs.

❖ Then repeat for a **uniform distribution** U(0, 1):

$$f_{X_i}(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

$$f_{S_n}(x) = \begin{cases} \frac{1}{(n-1)!}\sum_{0\le j\le x}(-1)^j \binom{n}{j}(x-j)^{n-1} & 0 < x < n \\ 0 & otherwise \end{cases}$$

Use the known result for the sum of uniform variables (Irwin-Hall distribution) and compare the shape with the exponential case.

➢ Problem 3 – Bus Arrival Problem

Assume buses are scheduled to arrive at exactly 12:00, but they experience random delays. Let xx be the random delay in minutes, modeled with a known PDF.

✓ Tasks:

- Calculate the probability that a person has to wait **more than 5 minutes** for a bus.
- Now assume the person arrives at 12:10. What is the probability that they still have to wait **more than 5 more minutes**?

Then, generate 1000 random samples from a uniform distribution over [0, 1] using Python or MATLAB.

✓ Tasks:

- Plot the histogram of the generated data.
- Choose appropriate bin width to visualize the shape of the distribution.
- Determine the type of distribution from the histogram.