

Advance Neuroscience Simulation 01

Based on Softky & Koch, 1993

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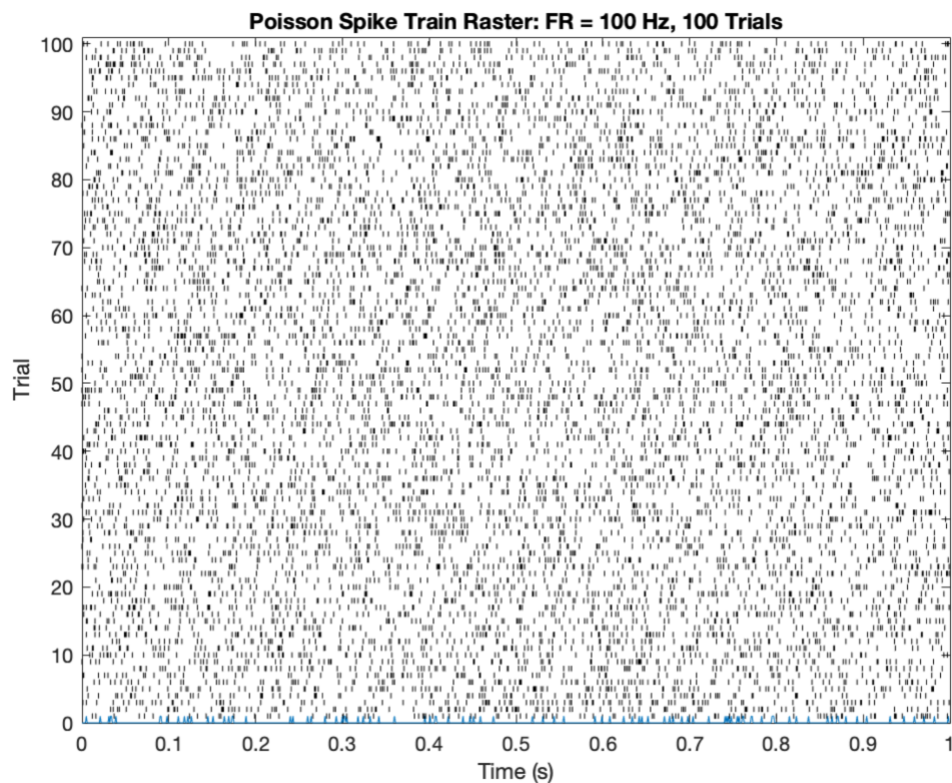
Note: all the plot are .svg so you can zoom in also I bring them in plot folder

1 Integrate and Fire Neuron

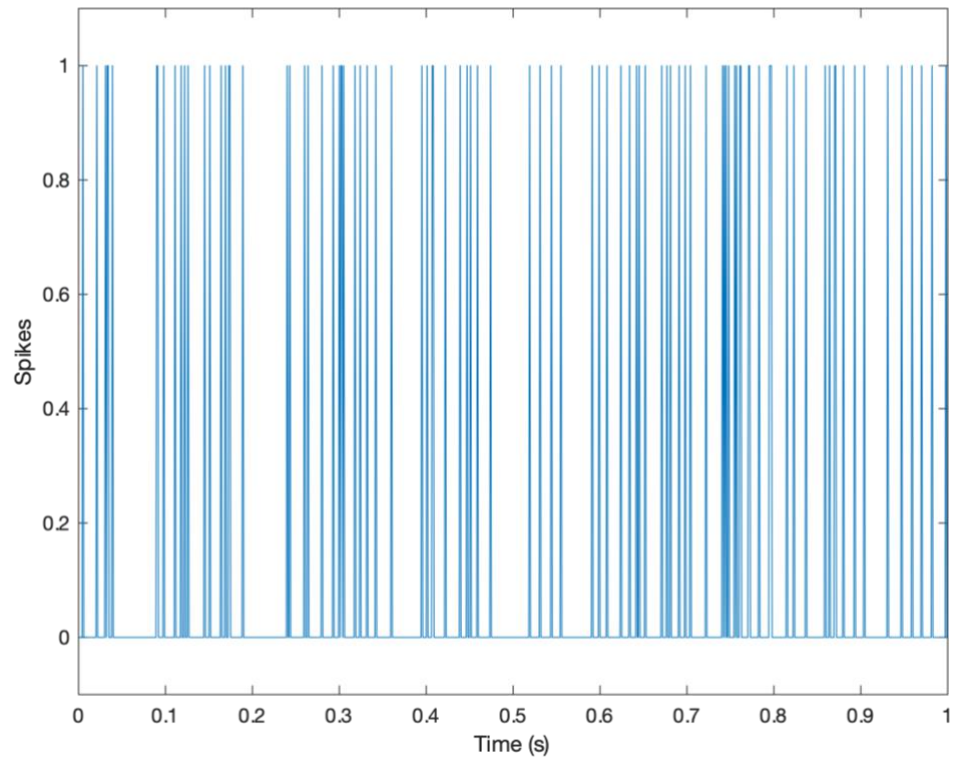
1.1 a

Generate a spike train using a Poisson process with $r = 100$ and $\Delta\tau = 1ms$.

Answer: with two methods, it is plotted; first, I use poissonSpikeGen, which prepares the 100 trials Poisson Spike train with $r = 100$ and $\Delta\tau = 1ms$. And the result brings in figure 1:



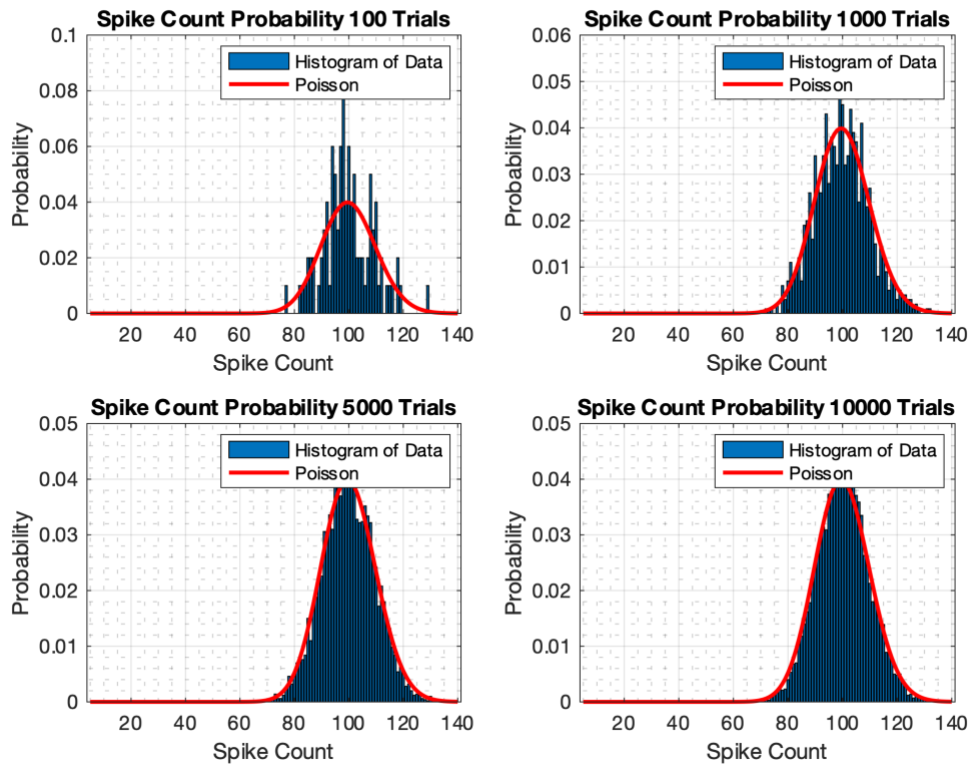
In addition, I calculate inter-spike intervals, then plot each spike and repeat it for 100 trials and plot it with the Stem function in Matlab, figure 2:



1.2 b

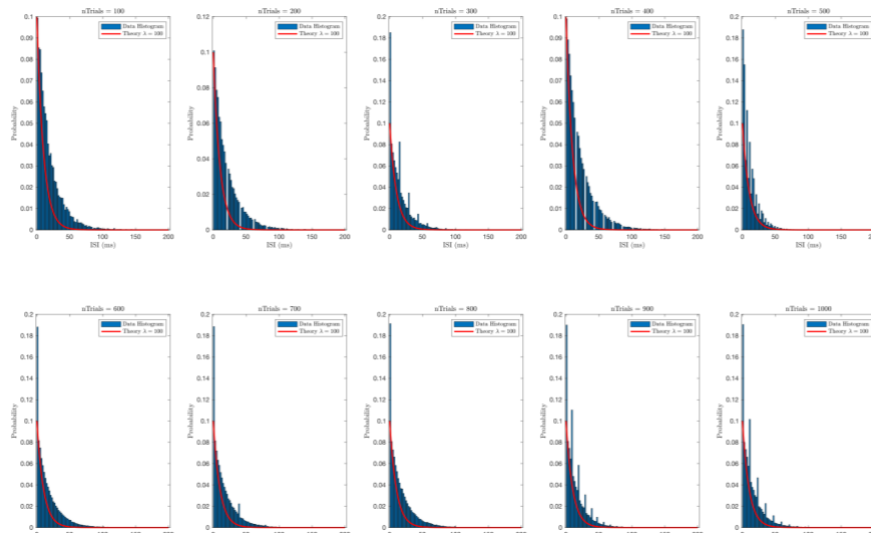
is calculated on 1000 trials, each 1-second

Spike Count Probability



1.3 c

Inter Spike Interval(ISI) is calculated by function. ISI histogram is shown in plot. The probability distribution function of ISI values of Poisson distribution is $f_{\tau}(\tau) = \lambda e^{-\lambda\tau}$. This theoretical distribution has a remarkable similarity with simulation results. (plots save as .svg so you can zoom and observe)



1.4 A-2

ANSWER TO QUESTION :

QUESTION:

A way to generate a renewal process spike train is to start with a Poisson spike train and delete all but every kth spike! This procedure is similar to integration over postsynaptic input with Poisson ISI distribution (Why?)

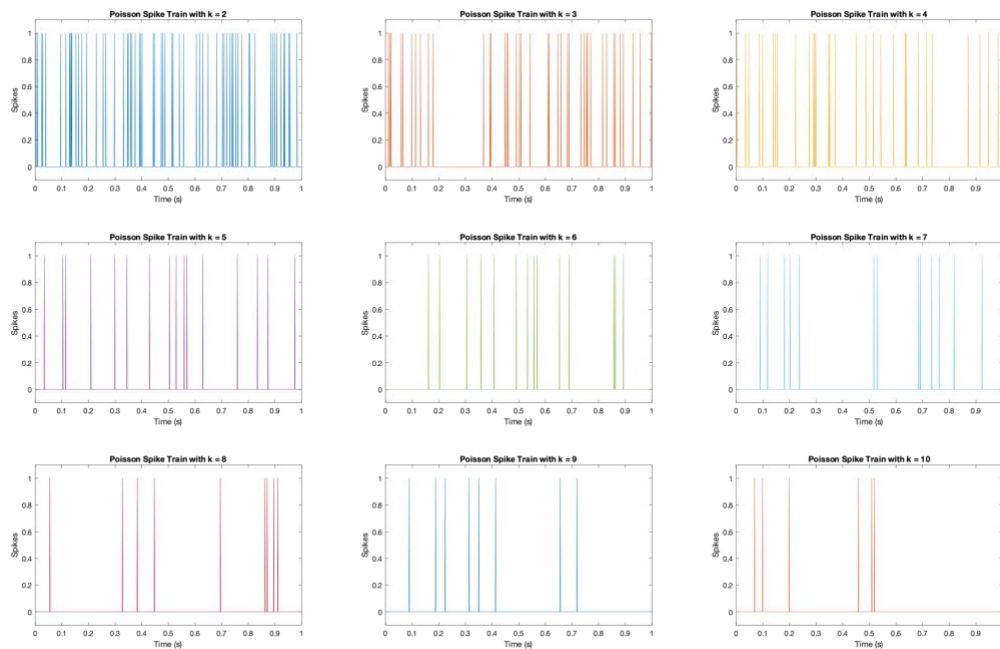
ANSWER:

This procedure is similar to integration over postsynaptic input with Poisson ISI distribution because it is based on the concept of sub-sampling or sub-sampling of the original Poisson process. In the case of the Poisson process, the probability of an event occurring at any given time is constant and independent of

past events. This means the probability distribution of inter-spike intervals (ISIs) for the Poisson process is exponential.

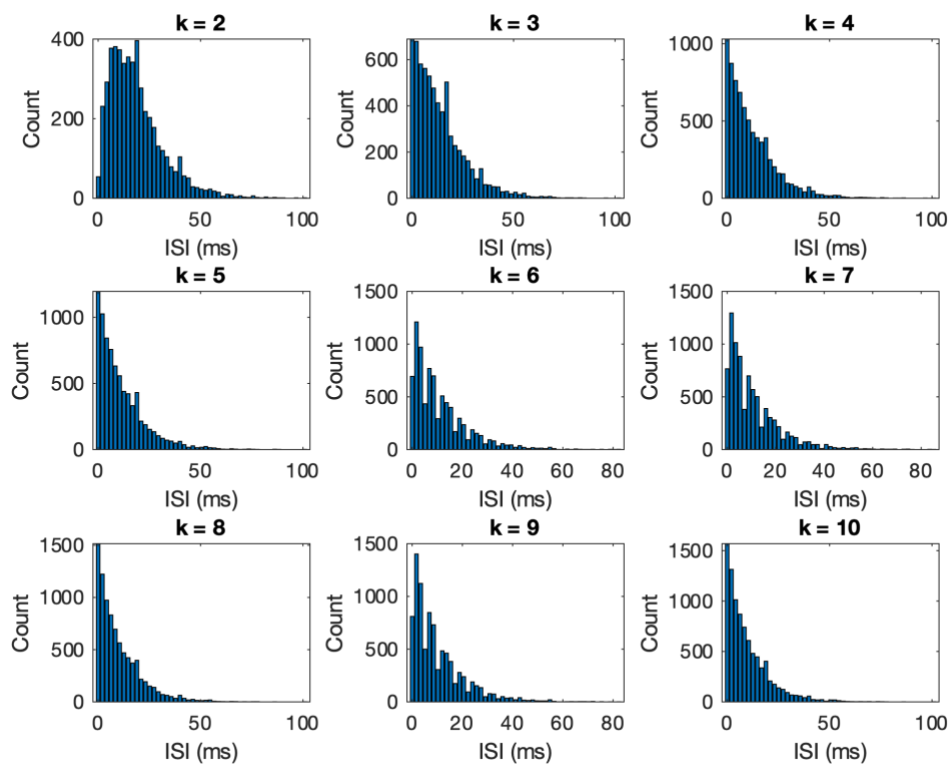
If we subsample the original Poisson process by keeping every k th spike and discarding the others, we end up with a new process with a modified ISI distribution. The modified ISI distribution will remain exponential but with an adjusted rate parameter. The adjusted rate parameter will be k times the original rate parameter, as we effectively speed up the actual process by a factor of k . This means that the modified process will have a higher frequency of events, which can be interpreted as an increase in the input to the postsynaptic neuron.

Plot the Spike Train (Renewal Poisson Process) for $K = 2$ to 10:



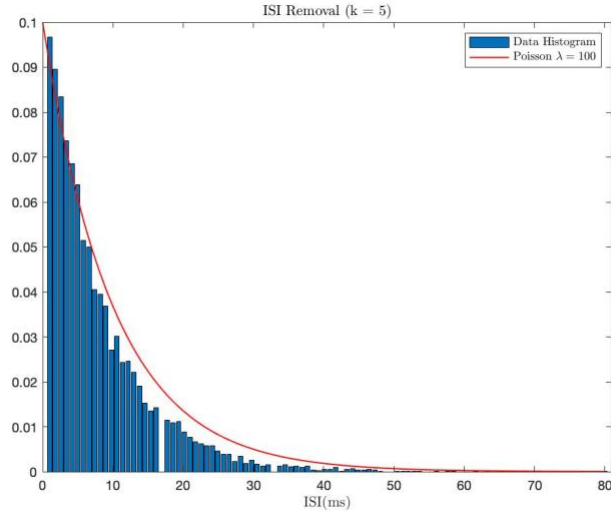
1.5 B-2

ISI distribution histogram for spike train for $K = 2$ to 10:



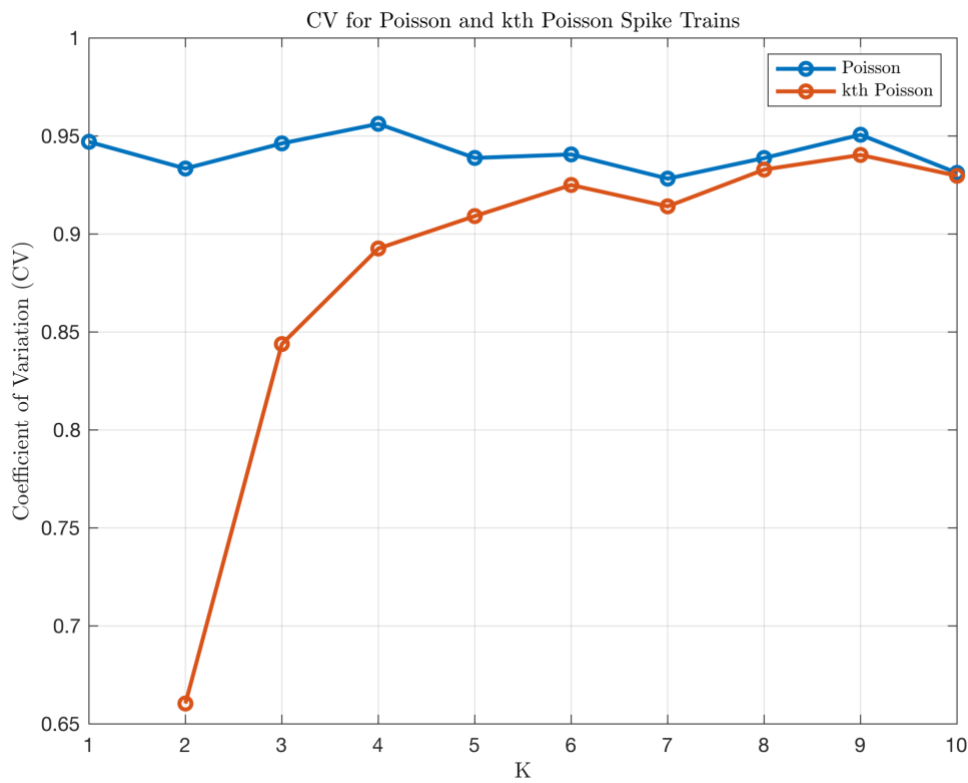
1.6 C-2

Spike count histogram for a renewal process sampled from a Poisson process with $K=5$:



1.7 d

The coefficient of variation value equals $CV = \frac{std(ISI)}{mean(ISI)}$. The CV value for Poisson distributed spike train from simulation is equal to 0.947057, which is close to 1, but for the renewal Poisson process, the CV value equals 0.6, which is less than the Poisson spikes train.



1.8 e

The probability of n renewal process spikes is equal to the probability of kn Poisson process spikes.

$$P_r(N(T) = n) = P_p(N(T) = kn) = \frac{(\lambda T)^{kn} e^{-\lambda T}}{(kn)!}$$

ISI pdf function can be reached by taking the below limit:

$$f_\tau(\tau_0) = \lim_{\Delta t \rightarrow 0} \frac{P_r(\tau_0 < \tau < \tau_0 + \Delta t)}{\Delta t} = \frac{\lambda^k \Delta t^{k-1}}{k!} e^{-\lambda \Delta t}$$

The mean and std values of the above distribution can be calculated as:

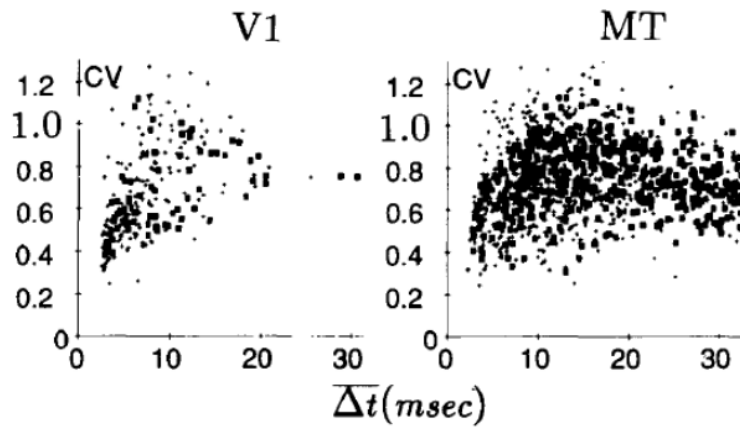
$$E[\Delta t] = \frac{\int_0^\infty \Delta t \frac{\lambda^k \Delta t^{k-1}}{k!} e^{-\lambda \Delta t} d\Delta t}{\int_0^\infty \frac{\lambda^k \Delta t^{k-1}}{k!} e^{-\lambda \Delta t} d\Delta t} = \frac{k}{\lambda}$$

$$\sigma_{\Delta t}^2 = \frac{\int_0^\infty (\Delta t - E[\Delta t])^2 \frac{\lambda^k \Delta t^{k-1}}{k!} e^{-\lambda \Delta t} d\Delta t}{\int_0^\infty \frac{\lambda^k \Delta t^{k-1}}{k!} e^{-\lambda \Delta t} d\Delta t} = \frac{k}{\lambda^2}$$

$$CV = \frac{\sigma_{\Delta t}}{E[\Delta t]} = \frac{1}{\sqrt{k}}$$

1.9 f

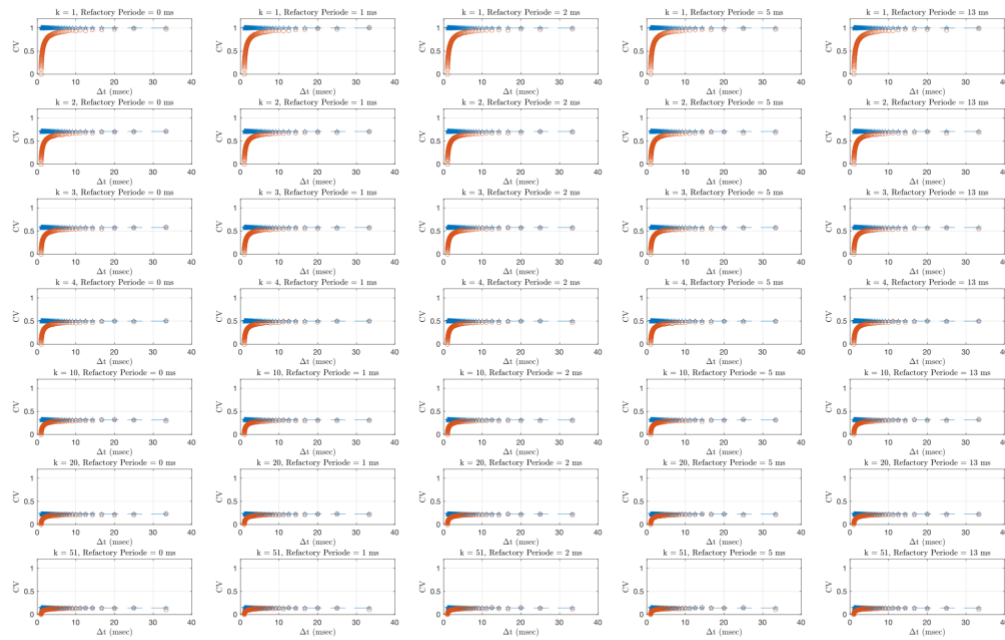
Softky & Koch's CV for neurons in V1 & MT areas are like this:



The CV values for simulated spike trains are constant for a constant firing rate. This means that the variety of variations for simulated data is constant. However, CV values for actual data need to be more consistent. The reason is that the neuron's firing rate is not constant due to adaptation which causes decrements in the neuron's firing. And this plot that from Softky & Koch is saying neurons with more firing rates have less variability (CV)

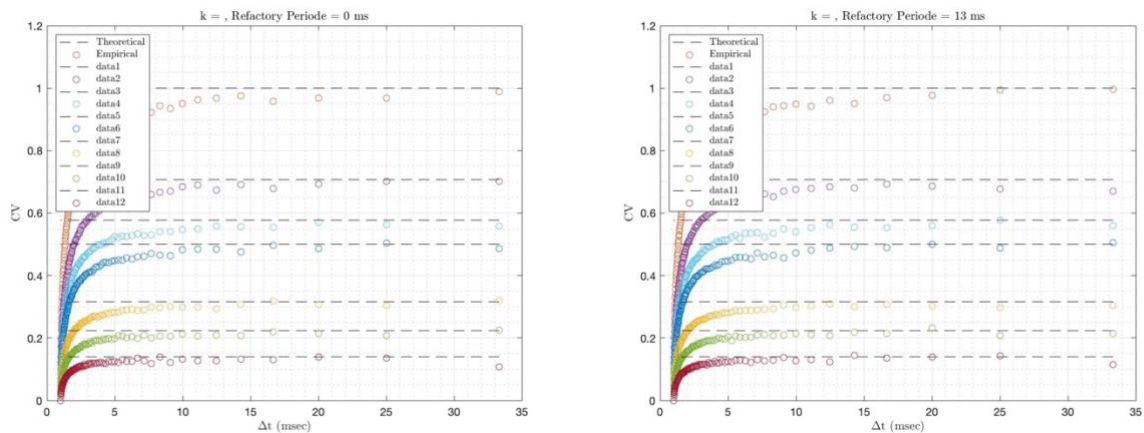
1.10 g

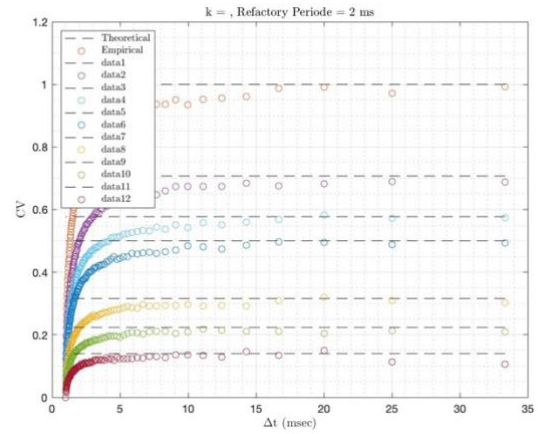
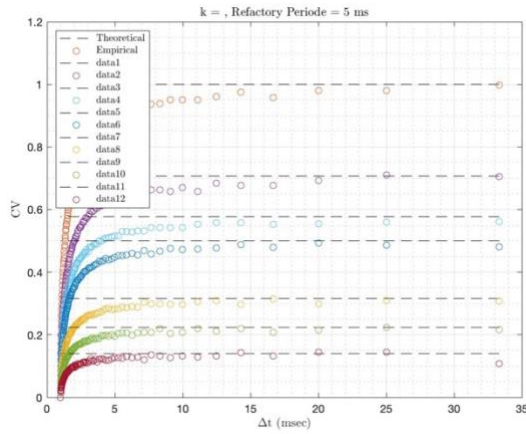
it is plotted for $k = 1, 2, 3, 4, 10, 20, 51$ and with Refractory Period $T = 1\text{ms}, 2\text{ms}, 5\text{ms}, 13\text{ms}$



Also I plot it in one plot for many k and data is k from $k_vec = [1, 2, 3, 4, 10, 20, 51]$

Title: Comparison of CV from integrator model. The horizontal lines correspond to the model without consideration of refractory period.

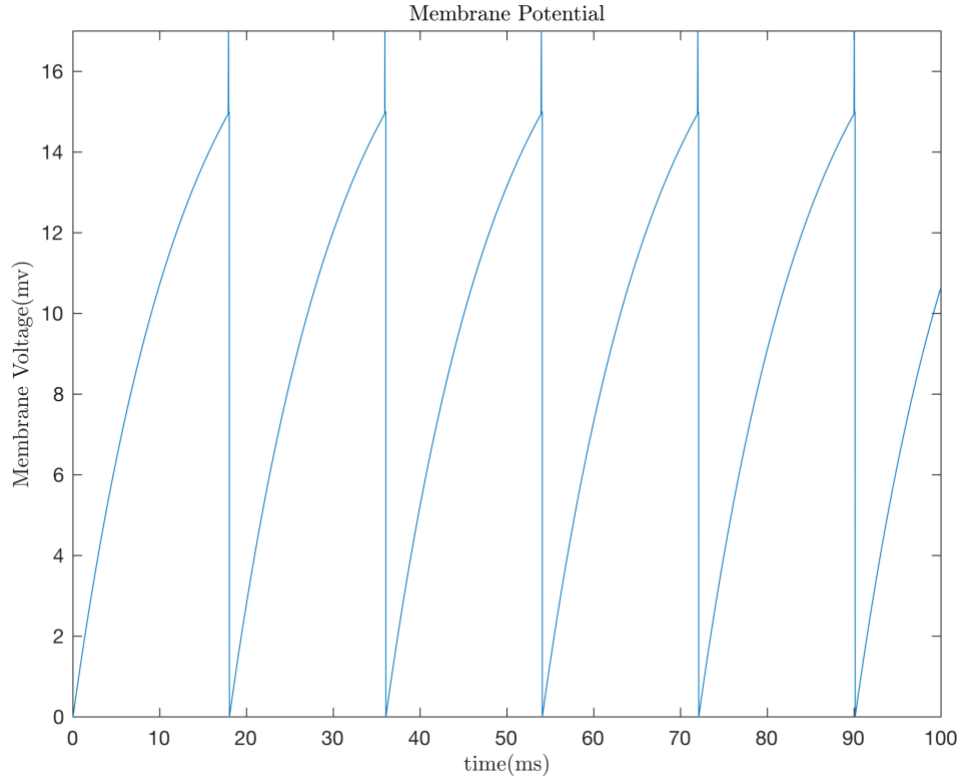




2 Leaky Integrate and Fire Neuron

2.1 a

equation is solved by Euler method with $\tau_m = 13\text{msec}$



2.2 b

The membrane potential differential equation is:

$$\tau_m \frac{dv}{dt} = -v(t) + RI(t)$$

Solving equation (6):

$$v(t) = RI(t) \left[1 + e^{\frac{-t}{\tau_m}} \right]$$

Setting $v(t) = v_{th}$:

$$t_{v_{th}} = \tau_m \ln \left(\frac{RI(t)}{RI(t) - v_{th}} \right)$$

Let the refractory period time be t_r . Then the total time required for one action potential is:

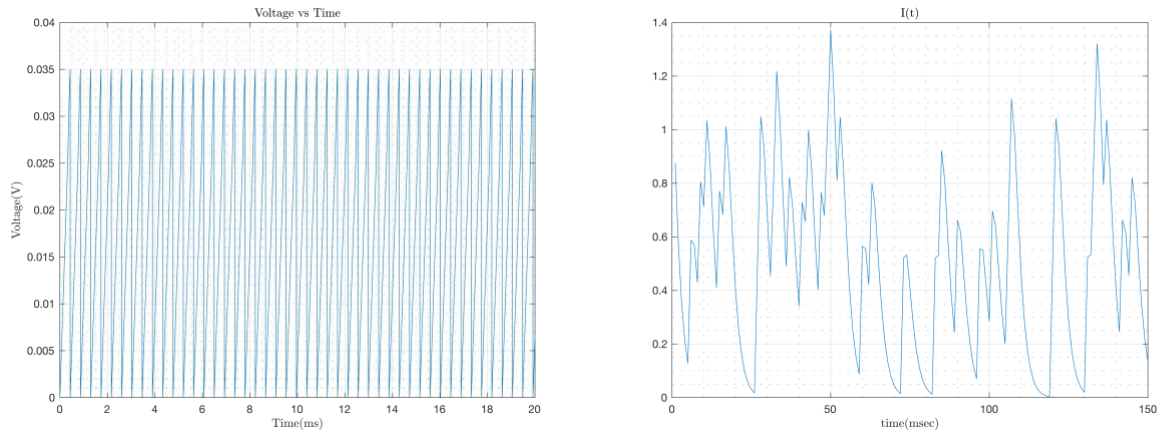
$$T = t_{v_{th}} + t_r = \tau_m \ln \left(\frac{RI(t)}{RI(t) - v_{th}} \right) + t_r \rightarrow \text{rate} = \frac{1}{\tau_m \ln \left(\frac{RI(t)}{RI(t) - v_{th}} \right) + t_r}$$

2.3 c

I set this parameter as said in the document

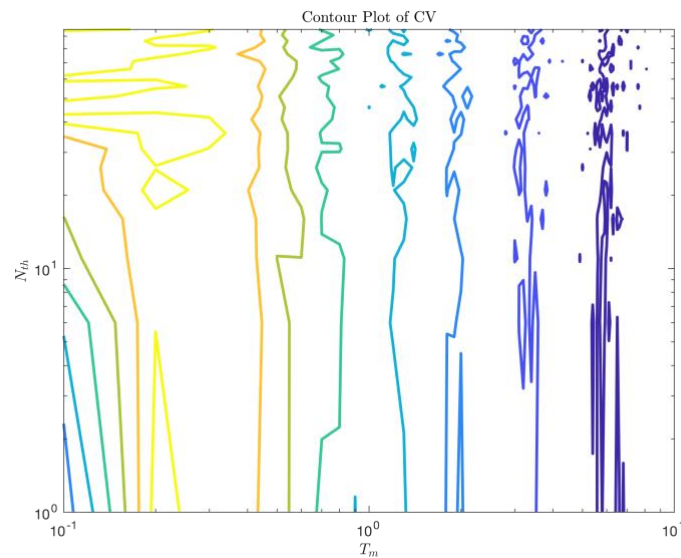
$\tau_m = 13\text{ms}$, $R = 1\text{m}\Omega$, $v_{th} = 15\text{mV}$, $v_r = 0$, $\tau_r = 1\text{ms}$, $\tau_{peak} = 1.5\text{ms}$, $fr = 200$

and this is the result for $V(t)$ and $I(t)$:

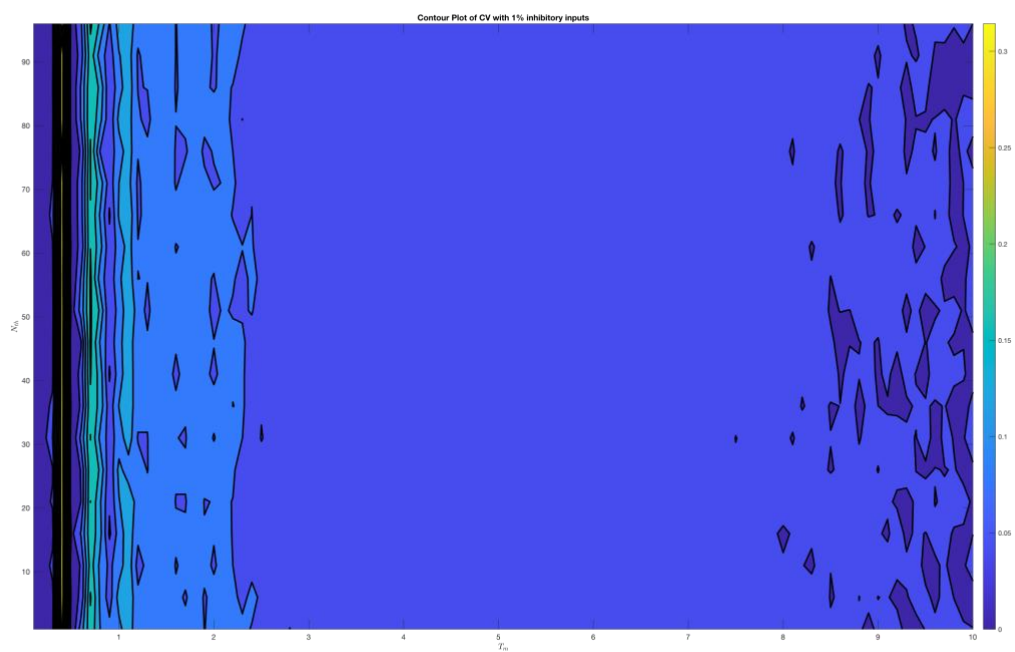


To reconstruct figure [8] of the paper, first, we have to ensure the output mean firing rate stays around 200Hz. A set of several frs is found so that the mean firing rate remains at 200Hz for each τ_m . Then for every τ_m , we simulate the model for 10 seconds.

To generate a realistic $I(t)$, ten spike trains of 1-second duration with Poisson distribution are generated and convoluted in the EPSC kernel. In two "for" loops, τ_m , and N_{th} values are changed. In each simulation, the $I(t)$ is generated. Then, using the Euler method, action potential of the neuron is determined. The action potential is then turned into a spike train.

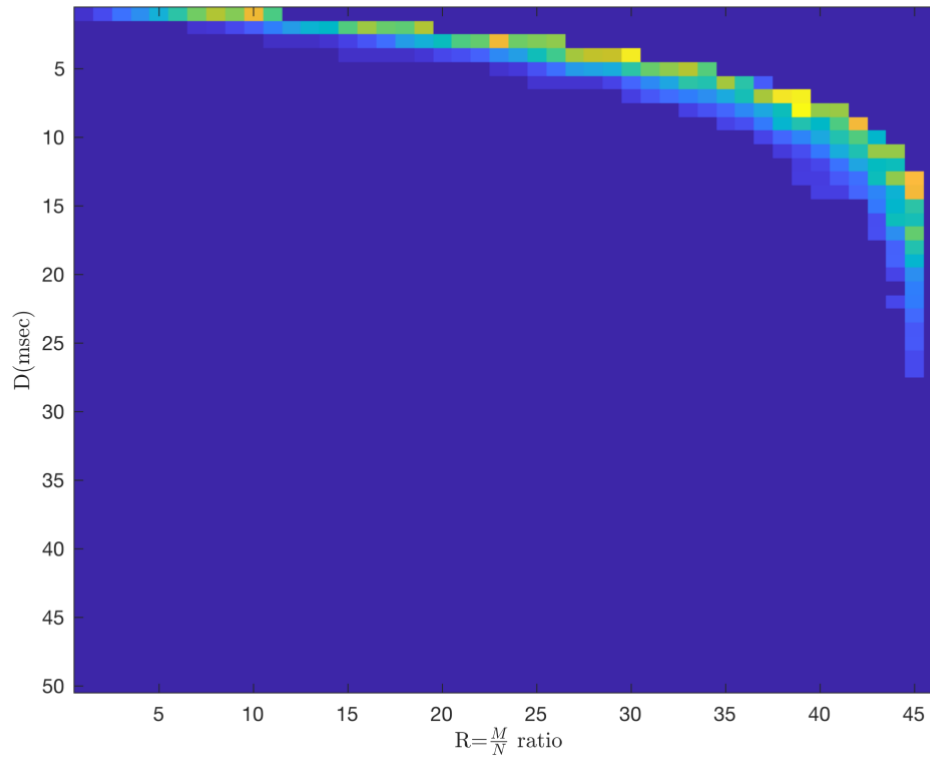


2.4 d



2.5 e

However, for very small values of D , the probability of the output neuron firing is low. Therefore, the output neuron does not fire at all. Then, the mean values of ISI would be zero, and CV values would be NaN. By increasing D , at some threshold, the output neuron starts firing. This value of D in which the output neuron fires increases as the R value increases. The reason is larger



2.6 f

the number of the inhibitory neurons be small, they won't have such a big effect on the Cv and if their number is so big, the post synaptic neurons wouldn't be able to fire an action potential.