



Eötvös Loránd University

Faculty of Informatics

Department of Programming Languages and Compilers

Adaptability of neural networks between games of different genres

Kitlei Róbert

Assistant lecturer

Andrey Khasanov

Software Information Technology MSc

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1. Introduction

1.1. Abstract

The goal of this thesis is to train a neural network for one task and then apply the trained neural network on the other task. This will be achieved by expressing the second task in terms of the first.

In the first part of the work, several artificial neural networks will be created and trained for the Snake game and the networks' properties, such as weights and biases, as well as the networks results and game strategies, will be compared. Some of the networks will be trained only briefly, while others will have a more extensive training. After that the it will be tested if a bigger number of layers brings better results if the total number of neurons across all layers is constant.

In the second part, the best performing Snake-trained networks will be applied to play the Maze game and the results will be compared to three other networks, all of which were exclusively trained for the Maze game and share the same structure as the best performing networks from the first part of the work. The next experiment will study the changes in the Snake-trained networks performance and hidden neurons' weight values after the networks receive an additional training in the Maze game.

Some networks will have different structures, such as different number of layers and different number of neurons in them, but there will also be similarly structured networks, which will be trained with a varying number of test games to observe if more data and training bring the best results.

1.2. Snake game history

The Snake game is the classic video game concept where the player plays for a line that represents the snake which grows in length after consuming other points. The game's main difficulty is staying alive without touching the walls or itself. This concept was originally seen in the arcade maze game Blockade developed by Gremlin and published by Sega in October 1976. [1]

There are many variations of The Snake Game, both single player and multiplayer, with the most known being Nibbler, Tron, Slither.io and Nokia Snake.

Soon after its creation in 1978, Snake, which was programmed by Peter Trefonas for TRS-80 microcomputer and Apple II, was instantly widespread on arcade machines, personal computers and other platforms due to its ease of implementation and replayability and in 1996 Next Generation magazine ranked it number 41 on their "Top Hundred Games of All Time." [2]

In 1998 Nokia resurged the Snake popularity after preloading it as an app on their monochrome mobile phones. This helped the game tremendously as it was open to a much larger audience

than before and people could play the game while spending their time in queues and at bus stops. Snake was associated with a Nokia app ever since.

1.3. Artificial neural network history

The original idea of a neural network was described in 1943 by neuropsychologist Warren McCulloch and mathematician Walter Pitts in their “A logical calculus of the ideas immanent in nervous activity”, which was on how neurons in the brain might work. The writers have modeled a simple neural network using electrical circuits. [3]

Later in 1949, Donald Hebb introduced his theory, also known as “Hebb’s Postulate”, about the neural bases of learning in his book “Organization of Behavior”, which has indicated that neural connections are enhanced every time they are being used and if two nerves fire at the same time, the connection between them becomes stronger. [4] Thus, learning is not done by brain passively, which would be an incredible feat, but rather it is a process during which the cellular structure of the brain is altered permanently. This theory has a classic status within science and is backed by recent research. [5]

In 1959, Bernard Widrow and Marcian Hoff invented the Widrow-Hoff least mean squares filter (LMS) adaptive algorithm that was the basis for the ADALINE and MADALINE (Multiple ADaptive LINear Elements) networks. MADALINE was the first neural network to ever be applied to a real-world problem. It is an adaptive filter that eliminates the echoes on phone lines, which is still in commercial use today. [6]

The success of ADALINE and MADALINE networks unfortunately did more harm than good to neural network research, particularly due to high expectations and outrageous promises on the network capabilities and severe limitations of hardware back then. Unfulfilled promises and increasing popularity of von Neumann architecture led to neural network research funding being heavily reduced.

There were a few advances in the field, however it was not until 1982 when John Hopfield presented “Neural networks and physical systems with emergent collective computational abilities” paper to the National Academy of Sciences, where he proposed using bidirectional connections between neurons, since previously the neural networks have only used single “forward” direction. [7] This has led to an invention of an associative neural network, also known as Hopfield network, which consists of long-term and short-term memories and the new training is done using the short-term memory.

In the same year, Reilly, Cooper and Elbaum have published “A neural model for category learning” that used the “hybrid” multiple layer neural network and in 1986 Rumelhart and McClelland described the use of parallel distributed processing, also known as the connectionism, in the neural network. [8] This is now known as a backpropagation network and, unlike hybrid networks, it uses more than a couple of layers, however that makes its training much slower, as it needs many more iterations over the input to produce the results.

Nowadays, networks with multiple hidden layers, also known as deep neural networks (DNN), have achieved state-of-the-art performance on computer vision problems and the current research goal is to reduce both the model size and computational cost of the networks to allow a more widespread deployment. [9]

1.4. Neural networks in games

Neural networks are quite a rare occurrence in games as those are usually fully scripted and use 30-year old artificial intelligence (AI) technology, such as A* and finite state machines instead. [10] There are exceptions to this, of course, such as the 1996 game Creatures, where the player assumes control over the small furry animals and teaches them how to properly behave. These creatures are using the neural networks to learn from the player actions to become independent later. Another example would be the 2013 Forza Motorsport 5 by Microsoft where the neural networks learn how the human players control in-game vehicles and react to the events, such as crashes or overtakes by other players and then the trained networks, also known as drivatars, play versus human players for a more realistic competition. [11]

On the other hand, the neural networks are quite commonly used for playing the games instead. One of the most famous examples of this is AlphaGo, a Google DeepMind's DNN which in the span of 2016-2017 won the official matches against two world champions of Go game (ranked 1st and 4th at the time) to become the first Go program, not only to surpass the amateur level, but also to reach the professional nine dan level in the game. Even though the nine dan is the highest possible level in the Go game, many top Go players believe that AlphaGo skills go beyond this dan and are at their own level above everyone else. [12]

1.5. Personal motivation

One of the limitations of neural networks is the fact that the networks are problem-specific, and a trained network cannot be applied to another task after it learned to solve one. [13,14]

In this work, however, I aim to create and train a neural network that can play one game and then apply it onto a different game to show that it is possible to reuse the network, instead of developing a new one altogether.

My personal goal is to familiarize myself with the neural networks playing the games and later apply that knowledge in game development field to create an alternative to the currently outdated AI technology and make the techniques of scripting and AI cheating obsolete. This will also make the AI controlled game characters actions be more human-like and give the AI an ability to learn from its mistakes and adjust its game strategy accordingly.

I hope that this will bring us one step closer to a more realistic artificial intelligence, which in turn will provide a better gaming experience for the players around the world.

2. Preparation

2.1. Abstract concepts

2.1.1. Neural network

So far there was a lot of history of neural networks in this work, but there was no explanation on what the neural network is.

A neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experiential knowledge and making it available for us. It resembles the brain in two respects: the knowledge is acquired by the network from its environment through a learning process and then stored using interneuron connection strengths, known as synaptic weights. [15]

There are two types of neural networks: biological neural networks and artificial neural networks (ANN). The biological neural networks could be any group of connected biological nerve cells, known as neurons. An example of such a network is our brain.

An artificial neural network, on the other hand, is a mathematical structure designed to simulate biological networks. ANN is formed of three different types of layers: input layer, hidden layer and output layer. Each layer consists of one or more neurons, where each neuron represents a relatively simple function that takes an input and produces an output. The neurons of an input layer are passive, meaning they do not modify the data used for training, while the neurons of hidden and output layers are active instead. A visual representation of an arbitrary ANN with one hidden layer can be seen in Figure 1.

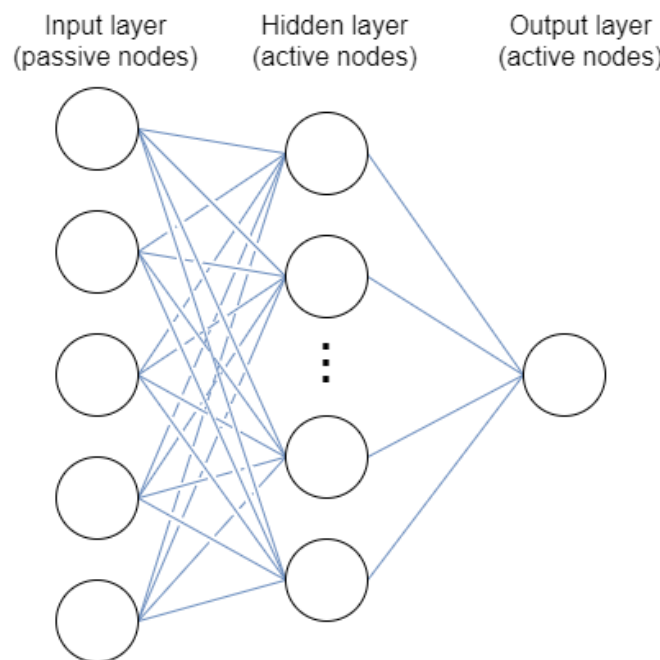


Figure 1. An arbitrary artificial neural network with one hidden layer

When the data is provided to an ANN for training, each value in an input layer is duplicated and each neuron in the hidden layers receives a copy. This is called a fully interconnected structure. After that, the values are multiplied by a set of predetermined numbers, also known as the weights, and then weighted inputs are added to produce a single number. This can be expressed as

$$y = f(x) = \sum_{i=1}^n x_i w_i + b$$

In the formula above x is the input, w is the weight and b is the bias, which is used to adjust the output. The number is passed through a non-linear function, also known as an activation function or a transfer function, which could be a sigmoid, rectifier, \tanh or any other activation function. This function takes a value between $-\infty$ and $+\infty$ and produces an output within the limits of the used function, which specifies if the neuron should fire or not. [16]

This work uses the state-of-the-art rectified linear unit (ReLU) as the activation function for a couple of reasons. First is the ease of calculation compared to other activation functions, since ReLU is a simple $f(x) = \max(0, x)$, while, for example, sigmoid uses expensive operations, such as exponentials: $f(x) = \frac{1}{1+e^{-x}}$. The other reason to use ReLU is because it returns the value of 0 more often than sigmoid or \tanh , which makes the neural network sparser, since fewer neurons fire, which more realistically simulates a biological neural network and allows for the smaller number of calculations needed.

The training of an ANN can be supervised, unsupervised or a mixture of both. Unsupervised training needs a lot more data, but the data can be “unlabeled”, which means it can lack categorization. This eliminates the need to label the data by hand, however, that also means that the network should have an effective algorithm to use the unlabeled data efficiently. The problem with this approach is that the accuracy of the said algorithm’s output cannot be evaluated. However, this way of training is closer to the true artificial intelligence and can produce the results which humans do not take into consideration due to pre-existing biases.

Supervised learning is done using “labeled” data. The input is presented to the network as the pairs of data and desired outputs. The goal of this approach is to analyze the training data and produce an accurate function, which can be used both for expected and unexpected inputs to get reasonable outputs. In this work, the neural network training will be supervised.

2.1.2. Backpropagation

During the ANN training, there will be a discrepancy between the expected and the actual outputs. This discrepancy is an error, which will be used to update the neuron weights for a better result.

It is not possible to get around this issue and choose “perfect” neuron weights during the initialization step of the neural network due to how the weights are calculated. A common

practice for choosing the initial weights since 1986, when Rumelhart and McClelland published their classic “Parallel Distributed Processing”, is to generate random non-zero numbers, which are very small. [17] As the weights are randomly generated, they will not produce the best immediate results and will need to be adjusted. Here is when backpropagation comes in.

Backpropagation, also known as backward propagation of errors or a feedback step, is a method of calculating an error made by each neuron in a network after the data is processed. Then the values are sent to an optimization method, such as gradient descent, which will adjust the neuron’s weight accordingly. This process requires multiple iterations, also known as epochs, to reach its highest potential to reduce the difference between the expected and the actual output of the network. It is most commonly used in deep neural networks.

2.1.3. Optimization algorithms

As it was previously mentioned in the Section 2.1.2, the optimization algorithm adjusts the ANN’s neuron weights to produce more accurate results. There are two types of such algorithms: first order optimization algorithms and second order optimization algorithms, where the order specifies the derivative order.

First order algorithms minimize the error using the gradient values with respect to the parameters, with an example being the gradient descent algorithm. The regular (batch) gradient descent will perform an update after calculating the gradient of the whole data, which makes it very slow and inefficient if the data is too large. There is another variant of gradient descent called stochastic gradient descent (SGD), which is more popular, as it calculates the gradient for each training input data. The SGD algorithms which are commonly used nowadays are momentum, Nesterov accelerated gradient, AdaGrad, AdaDelta and Adam, with the last being used in this work.

The second order optimization algorithms depend on the second order derivatives and, therefore, are not in demand as of now; this is due to the high cost of computation in terms of time and memory. However, they can potentially outperform the first order algorithms if their derivatives are calculated in advance. Second order algorithms include the Newton’s method, Quasi-Newton and Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithms.

2.1.4. Adam optimization algorithm

Adam, short for ADAPtive Moment estimation, is a method which computes the adaptive learning rates for different parameters. The learning rate is a measure of how fast the network arrives to the final solution, so if the learning rate is too high, the network might fluctuate around the solution without ever reaching it or miss it completely (diverge). If the learning rate is too low, however, the network will take a very long time to reach the final solution, which is also known as the converging, so the best practice is to find a good middle ground.

Adam was introduced in 2015 as an SGD method that only requires the first-order gradients with the little memory requirement and it was designed to combine the advantages of two other

popular optimization algorithms: AdaGrad, which works well with the sparse gradients and the Root Mean Square Propagation (RMSProp), which does well in realistic settings. [18]

At this moment Adam is a state-of-the-art algorithm that compares favorably to other SGD methods, as it can be seen on Figure 2.

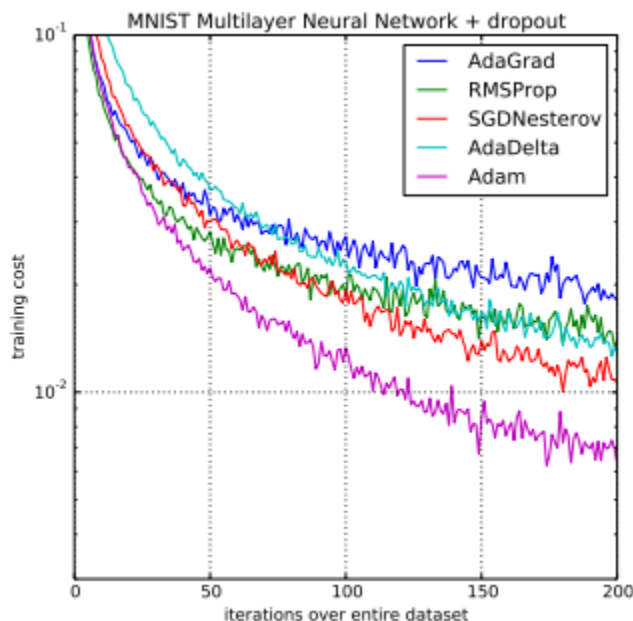


Figure 2. Comparison of Adam to other optimization algorithms training a multilayer neural network. Taken from Adam: A Method for Stochastic Optimization, 2015 [18]

2.2. Software

This work uses Python 3 programming language of version 3.6 [19] and the publicly available libraries, which can be found on its “PyPI – the Python Package Index.” [20]

2.2.1. TensorFlow

TensorFlow [21] is an open-source library for numerical computation using data flow graphs. The graph nodes represent mathematical operations, while the graph edges represent the multidimensional data arrays (tensors) that flow between them, hence the name. TensorFlow is available on all major operating systems, including Android and iOS. It was originally developed by Google Brain team for the purposes of conducting machine learning and deep neural networks research internally, but was released to public in 2015.

2.2.2. TFLearn

TFLearn [22] is a modular and transparent deep learning library for distributed machine learning built on top of TensorFlow. It features a higher-level API for implementing deep neural networks, as well as the helper functions to train the TensorFlow graphs and offers easy graph visualization. It was created by Aymeric Damien in 2016.

2.2.3. NumPy

NumPy [23] is the fundamental Python package for scientific computations which is used massively. It contains N-dimensional array objects, tools for integrating C/C++ and Fortran code, as well as the implementations of high-level mathematical functions. The ancestor of NumPy, Numeric, was created by Jim Hugunin, which was later competing with another package, Numarray. Numarray was faster while doing operations with large arrays and Numeric was faster with the smaller ones. In 2005, Travis Oliphant decided to unify the community around a single package which had the best of both worlds and thus the NumPy was created.

3. Applying a neural network to the Snake game

This part of the research concerns the Snake game and the experiments will be conducted on how modifying the settings and parameters affects the performance of the neural networks. First, the networks with a varying number of initial test cases will be compared in the Section 3.4.1, and then the changes in their neuron weight values will be discussed in the Section 3.4.3. After that the neural networks with a different number of layers, but with the same total number of neurons across them will be compared in the Section 3.5 and it will be seen if the deep neural networks perform better than the shallow ones.

The performance will be measured by the scores the neural networks achieve in their 5,000 test games and, after that, the best performing networks will be tested with the Maze game to see if the already trained networks can reach a good performance level in the task they were not trained for.

3.1. Snake game implementation

The Snake game source code was taken from an internet source [24], and extensively modified to allow extra features, such as manual play and command-line arguments. The playing field is 20x20 characters and the field borders are the impassable walls. The score is shown at the top of the playground. The characters used to print the snake are 'O' for the head and 'o' for the body, while 'x' marks an apple location.

A screenshot of the game can be seen below in the Figure 3.

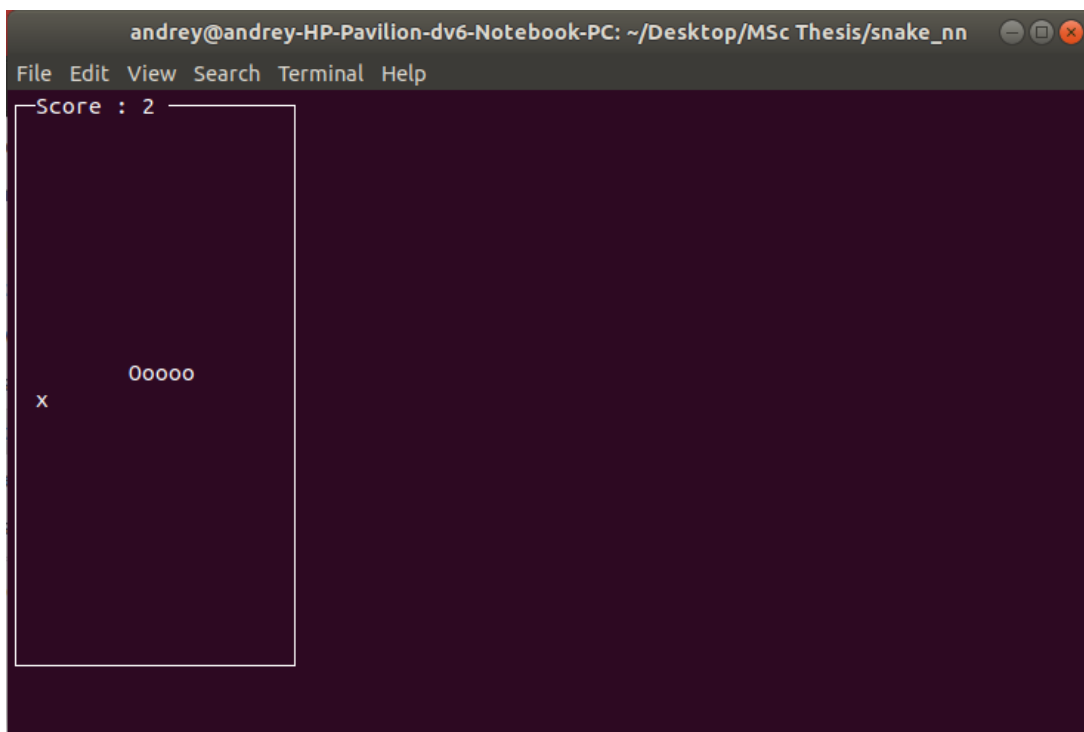


Figure 3. The Snake game written in Python using included Curses library

3.2. Neural network structure

As it was mentioned in the Section 2.1.1, the artificial neural networks consist of three types of layers: input, hidden and output layers. A more detailed information on these layers can be read below.

3.2.1. Input layer

The input layer receives the data, which it later passes on to the following layers without any changes. In this work, this data will be a list of five values, with the first three values specifying if there are any obstacles immediately to the left, front and right of the snake. The values are binary, so they can be either one (1) or zero (0), where zero means that there is no obstacle in the specified direction and one means that the obstacle is there.

The fourth value of an input is an angle between the snake's current direction vector and the direction to an apple. This angle will be normalized and divided by 180 degrees, so that the resulting value ranges from -1 to 1.

The last value will be the direction, which the snake is suggested to take, which can be one of the three predetermined values: -1 specifies a turn to the left, 0 stands for continuing the movement in the current direction and a value of 1 indicates that a turn to the right is currently suggested. This value is what makes the neural network training supervised, as the network is told what outputs are expected for the certain inputs, which should improve the network ability to generalize on the new and unexpected data once the testing begins.

3.2.2. Hidden layer

The hidden layer is the first layer where the values are expected to be modified, however the results of this layer will be sent to another hidden layer or to an output layer for additional calculations and, thus, the immediate results will remain hidden outside the network, hence the hidden layer name.

Since the neural network created for playing the Snake game does not have to be overly complicated, there was a decision to only have one single hidden layer with 25 neurons in it, with ReLU being the activation function.

There will be other networks tested, which will consist of multiple hidden layers, where the number of neurons per layer will be specified.

3.2.3. Output layer

The output layer produces the final results of the neural network and in our case this layer returns a single value, which will be one of the following three choices: -1 if the snake did not survive a turn, 0 if the snake has survived, but the direction of the snake is wrong in relation to the apple and 1 if the snake has survived and chose the correct direction during its last move.

This layer contains only one neuron and uses the default linear activation function.

3.2.4. Code implementation

Described neural network with one hidden layer of 25 neurons is trivial to implement in Python using TFLearn package. The code is as follows:

```
network = input_data(shape=[None, 5, 1], name='input')
network = fully_connected(network, 25, activation='relu')
network = fully_connected(network, 1, activation='linear')
network = regression(network, optimizer='adam',
learning_rate=0.01, loss='mean_square')
model = tflearn.DNN(network)
```

3.3. Neural network training

The training of the neural network is a time and memory consuming process depending on the amount of games to train with. In our case, we will use different networks, which will be trained with a varying number of games and then the results of these networks will be compared after 5,000 test games are played by each.

The games are generated automatically, which means that each turn of the game will be chosen randomly out of three possible directions (left, front, and right), as going backwards in the Snake game is impossible, and if the snake is alive after the previous turn, we continue to generate the new turns until the game is over.

Since the computers used for generation of these initial sets of games were not designed to contain a big amount of data in the memory at the same time (there is more than 5,800,000 randomly generated moves for 100,000 games only), the training part of the network was designed to have an ability to load an existing network and continue training it with a new set of data. At first, however, this feature was absent, and the network was trained from scratch every time the training process was activated. This quickly became a problem due to the limitations of computer memory, as the network could not be trained with more than 100,000 games, due to a memory error. Therefore, the network with 1,000,000 initial games was trained during 10 different training sessions of 100,000 games each and the data from all the 10 training sessions was recorded.

The time spent on training the networks, as well as their performance comparison over the course of 5,000 test games, is in the Table 1. The networks are single-layered and have 25 neurons in their hidden layer, except for two-layered and three-layered networks, which have 100 neurons per each hidden layer, and every network was trained with a different number of initial games. For the network with 1,000,000 training data the time was recorded over the course of 10 training sessions and summed later. Time spent on the generation of the training data set is omitted, however it was the main reason of the training slowing down tremendously. It is also important to note that the game score is the number of apples picked up by the snake and not the final length of it, which would be equal to the score plus three (as three is the length of the snake at the start of the new game).

3.4. Network comparison based on an initial number of games

3.4.1. General results

Table 1. Trained neural networks performance comparison

Number of training games	1,000	10,000	100,000	1,000,000	100,000 (two-layered)	100,000 (three-layered)
Time spent on the 1st epoch (s)	4.401	36.105	362.135	3636.172	602.469	791.910
Time spent on the 2nd epoch (s)	3.402	35.220	378.288	3843.051	614.335	820.464
Time spent on the 3rd epoch (s)	4.144	36.470	376.790	3861.501	617.867	824.834
Total training time (s)	11.947	107.795	1117.213	11340.724	1834.671	2437.208
Average steps in 5,000 games	279.8218	247.0770	426.6786	400.9064	549.5052	602.5854
Average score in 5,000 games	19.3770	16.9704	28.7722	27.3626	35.5392	38.2772
Worst score	1	1	4	2	6	5
Best score	53	67	75	64	94	89

As it can be seen from the table, the number of games to train from is an important distinction between the networks and their performances. It is expected that the less games there are, the worse the network performs, however, there are two interesting performance drops, with the first being from 1,000 games to 10,000 and the second from 100,000 games to 1,000,000. This could be attributed to the fact that the set of 5,000 games is a very small sample, however, the tests were run many times and the results were consistent across all the test runs.

The first drop in performance probably occurs since 10,000 initial games is too small of a data set and the network did not train enough to obtain good or at least better results than its predecessor. It could also mean that the training took a wrong turn during the process and the quality of the network went down, but then went up again once there was more data to train from in a 100,000 initial games network.

The reason of the second performance drop is a phenomenon known as the overtraining. [25] Overtraining happens when there are too many iterations done on the training data and the network simply memorizes the input values and their expected results, which increases the network's performance over the training set of data, but worsens its ability to generalize on the new data. In other words, the network does well when it replays the game scenarios it has trained on already, but has difficulties when present with the game scenarios it has never met during its training.

After studying the data for the single-layered networks, it was observed that the data set of 100,000 games produces the best trained neural network and thus the decision was made to train the multi-layered networks with the same amount of games in hopes to create the best networks possible.

Out of all the studied networks, the neural network of three layers produced the best results, while the two-layered network was the second best, but it was interesting to see that even though three-layered network was better on average, it still could not beat any of the high scores of the two-layered network.

The rise in performance of the deeper networks, however, is not surprising at all, because the general approach in the neural network creation is to have multiple hidden layers, with some state-of-the-art networks having over 150 of them! [26]

Since operations which are performed by the network neurons are very simple, combining them allows for much more complex calculations to be made possible. This, in turn, allows for a higher level of abstraction in deeper layers, where every next layer will recognize more and more features of the data set. An example is the visual recognition problem, where the first layer of the neural network will only recognize very simple things like the edges of the objects, the second layer will be trained to recognize more difficult shapes, such as ellipses and circles, the third layer will be able to differentiate between the ears and the eyes, while the fourth layer will already learn some of the very complex features, such as the human faces or other objects.

After considering the success of the deeper neural networks, there was a natural decision to study the networks with a constant total number of neurons across the varying number of hidden layers later in this chapter. This is done to see if the number of layers does, in fact, positively affect the performance quality of the network if the total number of neurons of the said network remains unchanged, since in the previous experiment the number of neurons was directly correlated to the number of layers, which could give the deeper networks an unfair advantage over the rest of the networks that have a fewer number of hidden layers in them.

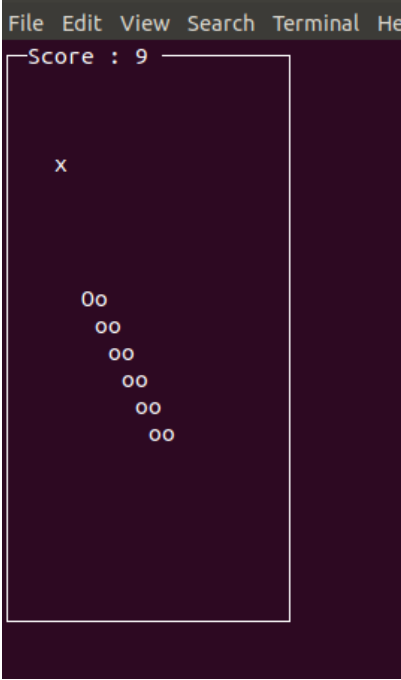


Figure 4. Diagonal turns strategy

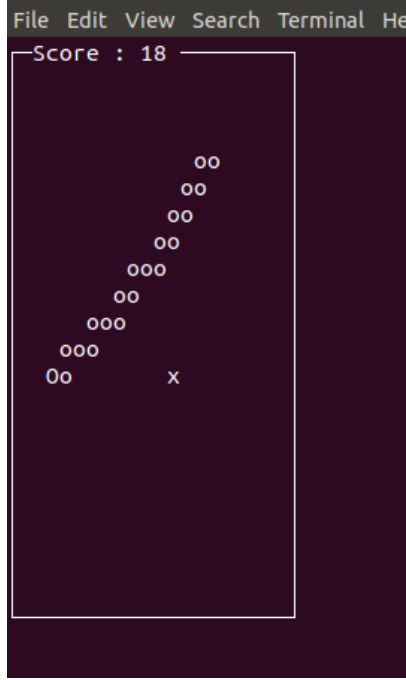


Figure 5. Diagonal turns with skips

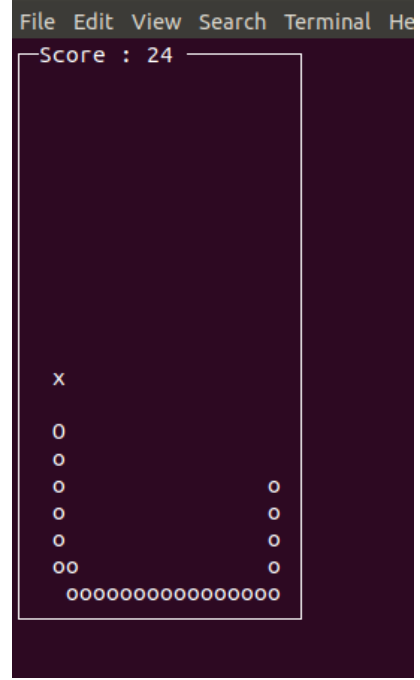


Figure 6. Transition from diagonal turns to straight turns after more training

3.4.2. Game strategies

One of the interesting things to talk about is the different strategies that some of the neural networks had. As it was expected before the training, the networks with 1,000 and 10,000 initial set of games were using a diagonal turn strategy. An example of what that means can be seen in Figure 4 and Figure 5. This strategy was always used if an angle between the snake head and an apple was 45° . When the angle was different, then instead of moving in a perfect diagonal line, the networks were skipping a turn to get to 45° sooner, but they also never skipped more than one turn at a time.

As it can be seen in the Figure 6, when the network was trained with more games, such as the 100,000 and 1,000,000 games networks, it started transitioning from the diagonal turn strategy to the straight turns, however, some of the occasional moves it made still resembled the original diagonal turn strategy.

The complete shift from the diagonal turns strategy in favor of the straight turns finally happened once the networks became deeper. Both two-layered and three-layered networks were using this strategy, which is visualized in the Figure 7.

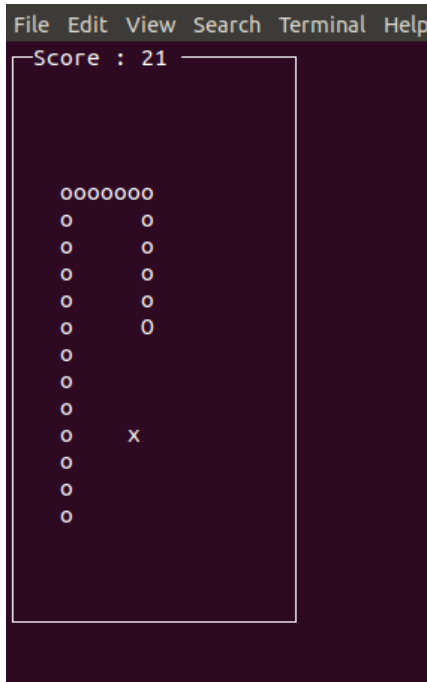


Figure 7. Straight turns strategy in deeper networks

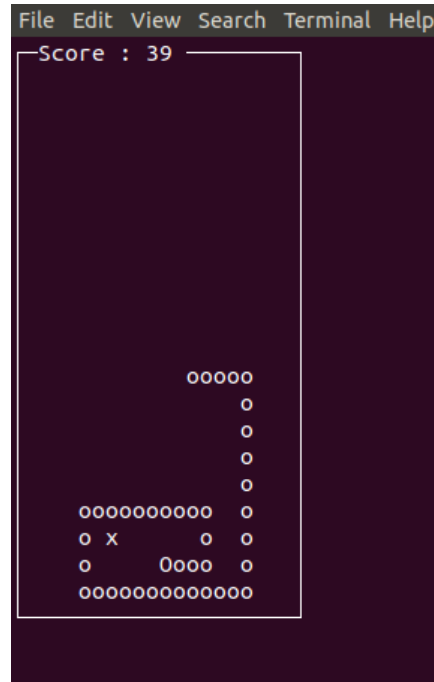


Figure 8. Unwinnable state avoidable by humans

Several people were asked to play the Snake game to compare the strategies used by the neural networks and the humans, and every human player used the straight turns strategy right from the start as well, which the deeper networks have managed to discover by themselves.

The biggest difference between the way the humans played the game compared to the neural networks was that the humans were able to tell if they were going to enter an area that was obstructed by their own body, which was not possible to get out from in a limited number of moves, such as the game state shown in Figure 8. In these situations, the humans were able to take some preliminary measures to avoid such problematic states of the game. Most players did it by going in the opposite direction from the apple first and then, when the danger of entering an unwinnable position was not present anymore, they continued playing the game normally.

This skill, however, did not help the human players obtain the higher scores than the neural networks, with the best scores being only in the range from 20 to 35 points, while the networks were consistently getting the results that were several times better than that. But, on the other hand, if the networks knew how to avoid the situations like this one, it would give them an even bigger advantage over the human players, which would lose any ability to compete with the networks at any stage of the game, like it has already happened with the Go game, as mentioned earlier in the Section 1.4 on page 6.

3.4.3. Weights and biases

Every neuron has its own weight values for every input parameter and since there are five initial parameters in our networks, every neuron has five different weights, which, in the case of the single-layered network of 25 neurons, is 125 values. The biases, however, remain constant for every input parameter, thus making them to be a total of 25 values.

Considering that the structure of the multiple layered networks and the single layered ones is different (they have different number of neurons per layer, too), the best way to compare the weights and biases is to compare them across the networks of the similar single-layered structure. Another limitation is that 125 values is too many to compare, so the weight values for one of the input parameters will be compared. There is no importance in which variable is chosen, but let us choose the one that specifies if there is an obstacle directly to the left side of the snake, which is the second parameter of the input.

Table 2. Single-layered network weights for one of the input parameters

1,000 initial games	10,000 initial games	100,000 initial games	1,000,000 initial games
0.072406173	-0.047996681	-0.012485540	-0.012485540
-0.538972318	-0.847055733	-2.822388411	-2.618926287
-0.877597332	-1.344542503	0.273763716	0.843500078
-1.143232465	-2.159810543	-3.354897499	-3.400511503
0.042361051	-0.030023403	-0.006273229	-0.006273229
-0.769199014	-0.297206699	-0.297206699	-0.297206699
-0.939412951	-0.939412951	-0.939412951	-0.939412951
0.075344317	-0.021229422	-0.001764486	-0.001764486
0.252286732	0.384311885	0.530116677	0.505924523
-0.487262487	-1.483316064	0.331819355	0.331819355
-1.041768670	-2.099506855	-2.499260902	-2.499260902
-0.188621491	-0.001163005	0.128729463	-0.046580069
-0.094241835	0.080227323	0.411132187	0.258867204
0.046578322	-0.021678241	-0.170884013	-0.071125247
-1.098333478	-1.802851558	-1.802851558	-1.802851558
-0.003732676	-0.003732676	-0.003732676	-0.003732676
-1.042869568	0.008858151	0.008858151	0.008858151
-0.299276650	0.107646152	0.086139977	0.086139977
-0.136341050	0.007103892	0.160483778	0.151804492
0.009423577	0.009423577	0.009423577	0.009423577
-0.080396853	-0.247389868	-0.437185228	-0.419417709
-0.062357765	-0.062357765	-0.062357765	-0.062357765
-0.129455134	-0.000063382	-0.000063382	-0.000063382
-0.750602961	-0.919643402	-1.939302325	-1.992216945
-0.023972072	0.034327365	0.034327365	0.034327365

The weights for four single-layered networks can be seen in the Table 2. The values that did not change throughout the additional trainings are highlighted. An interesting observation is that if the neuron weight value did not change for one of the input parameters, the weights for other parameters and the bias values of those neurons remained unchanged, too. Also, another observation is that if the value remained the same from one network to the other, it remained constant throughout all the following networks, as if the calculated value was deemed to be an “ideal” one.

When the neural networks are being trained for prolonged amounts of time, it is expected that at some point the differences between consequent networks weights start converging to zero, as the network is coming closer and closer to its final state where the additional trainings will not have much influence over the performance of the network. Since 1,000,000 initial games network was calculated across 10 different training sessions of 100,000 games each, it is possible to compare the weight value changes between the sessions, as all the intermediate network states were recorded.

The differences between the neural network weights after N initial games can be seen in Figure 9. The values do start converging to zero after a set of 200,000 initial training games, as it was already expected. There is an abnormal increase in the weights difference going from 600,000 to 700,000 initial games, which can be attributed to the network overtraining phenomenon, which was previously discussed in the Section 3.4.1, but after this irregularity the values continue converging to zero as expected.

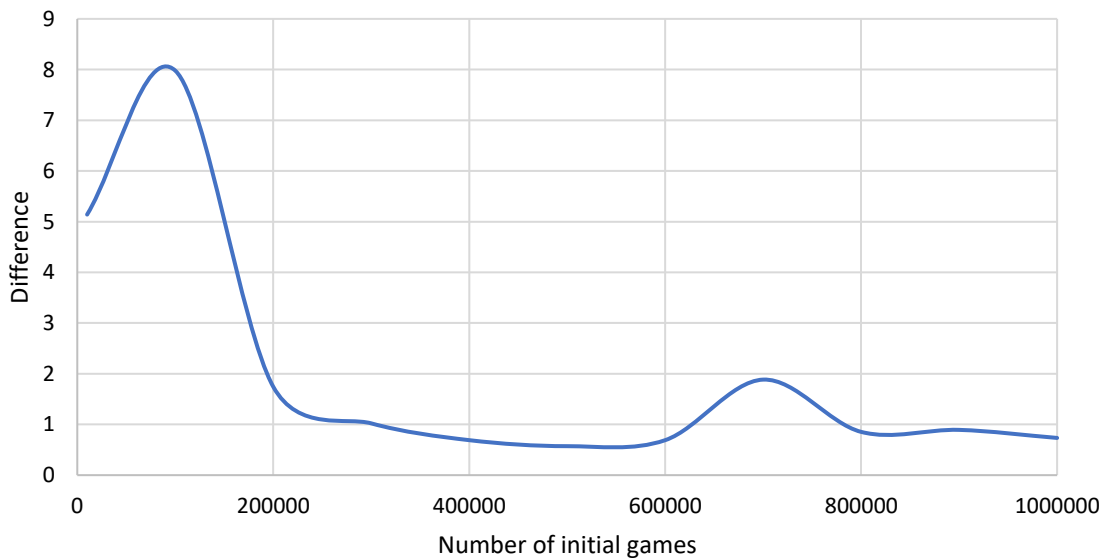


Figure 9. The weights differences converge to zero with every subsequent training

3.5. Network comparison based on layers

Previously in this work it was observed that the three-layered network produced the best results out of all the compared networks and there was a hypothesis which stated that the deeper networks should produce the better results, as opposed to the networks with a fewer number of layers.

Since the number of layers will be different across all networks, there was a decision to keep the total number of neurons the same, all split through the layers equally. The number which was chosen to be the total number of neurons was 36, since it is divisible by most of the numbers ranging from 1 to 9. Thus, the single-layered network has 36 neurons in its only hidden layer, the two-layered network has 18 neurons in each of the layers and so on. In the cases where the neurons could not be split across the layers equally, each hidden layer will contain the quotient of the total neuron number with the divisor being the total number of layers. Thus, in the five-layered network, first four layers will contain 7 neurons, while the last one will consist of 8.

The performances of every tested network are in the Table 3, which is similarly structured to the Table 1, however, the training time was omitted, as it was equal to 320 seconds per an epoch across all the neural networks, since they all contain the same total number of neurons in them.

The average scores of the networks are also present in the Figure 10 for an easier visual comparison.

As it can be seen both from the table and the chart, the single-layered and two-layered networks have a strong start, but the subsequent networks have a large performance drop compared to the first two. The last nine-layered network with four neurons per each layer was able to learn only one of the Snake game objectives, which is staying alive, but it completely missed the other important objective of picking the fruits up to gain an increase both in the snake length and the score.

There is a small improvement shown by the seven-layered network, but this improvement is so little that it can be ignored due to a rather small test sample of 5,000 games.

This shows that the previous assumption of the deeper networks being better than the shallow ones is only partially correct. Even though the two-layered network did perform better than the single-layered one, after that there was an inverse correlation between the number of layers and the scores that the networks managed to achieve on average. This observation, however, does not necessarily mean that the deeper networks are worse than the shallow ones, since we had the same total number of neurons in our networks, while in the real-life scenarios we would have additional neurons to work with, which could help us obtain the better results.

Thus, the success of the deeper neural networks in the Section 3.4.1 can be attributed to the fact that the deeper networks contained a bigger total number of neurons, which helped these networks generalize on the data they were presented with during the test games better.

Table 3. Trained multiple-layered neural networks comparison

Layers number	1	2	3	4	5	6	7	8	9
Average steps in 5,000 games	373.48	443.63	323.97	207.95	198.69	200.61	228.57	214.4	∞
Average score in 5,000 games	25.812	30.289	22.514	14.972	14.397	14.684	16.532	15.683	0
Worst score	4	2	2	1	1	1	2	1	0
Best score	63	67	67	44	43	48	47	45	0

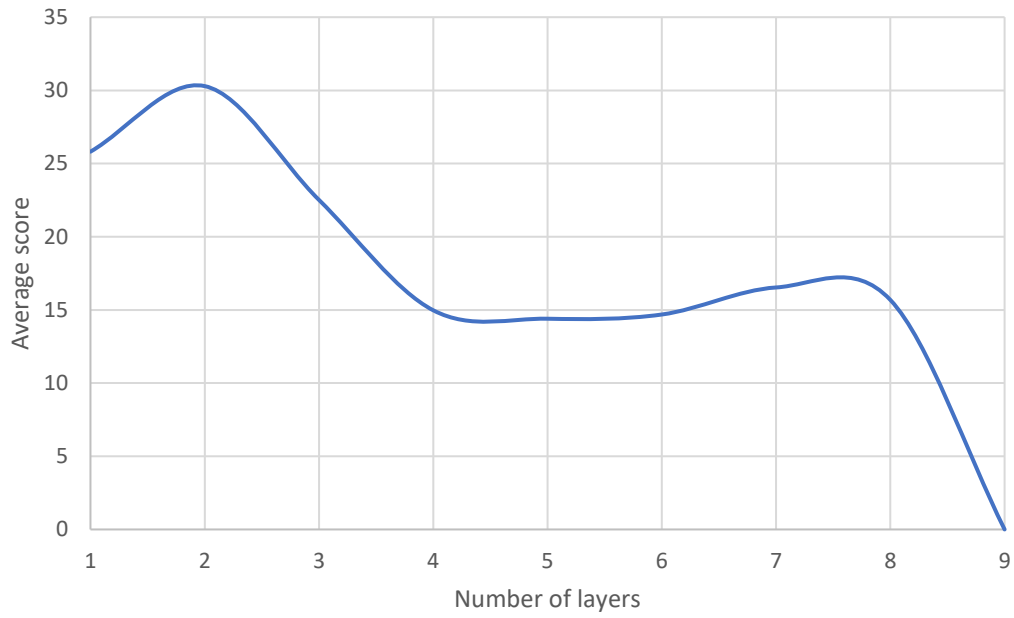


Figure 10. Average scores maintained by the networks with total same number of neurons

4. Applying a pre-trained network to a different game

This part of the research is about applying the best performing Snake-trained networks onto a Maze game. The experiments will be conducted on how these pre-trained networks perform in the new game and in the Section 4.3 the results will be compared to the networks which were exclusively trained for the Maze game, where the latter are expected to have a better performance. After that the Snake-trained networks will be trained against the Maze game and the new results will be compared in the Section 4.4. The weight changes will be discussed in the section 4.5 and both experiments will be concluded in the Section 4.6.

The training process will be the same as the one used in the previous chapter.

4.1. Maze game implementation

Just like in the Snake game, the playing field is 20x20 characters and the field borders, as well as the obstacles on the field, are the impassable walls. The character used to print the player is 'o', while an 'x' marks an exit location.

The maze itself is generated by the Kruskal algorithm and then the neighboring cells of the start and the end points of the maze are cleared of walls to have a higher chance of the maze being solvable. Since there is still a small possibility that the maze is not solvable, there is a recursive algorithm, which checks if the maze is valid or not. If the maze is invalid, the maze generation method will be activated indefinitely until the solvable maze is generated.



Figure 11. Maze game built on top of the Python Curses library

Originally, the score was incrementing by one after every successful turn that the player managed to survive, but this quickly turned out to be an abysmal design decision, as every trained network was moving in circles to obtain infinitely high scores, without even considering the main objective of reaching an exit.

Thus, there was a decision to give negative points once the network repeats the same turn more than three times during one game and if that does not help the network change its strategy, the game is declared lost once the same turn is repeated more than seven times. If the maze is solved, however, the final score of the game is automatically set to 1, which is done to simplify the solved mazes counting process.

To apply the trained neural networks from the previous chapter, the Maze game was expressed in terms of the Snake game. Thus, an apple became an exit location, while the obstacles on the field are simply the continuations of the snake body and the player represents a snake head. This made it possible for the networks to apply their knowledge of the previous game to play the new one without any additional changes.

The screenshot of the Maze game can be seen in the Figure 11.

4.2. Neural network structure

The structure of the pre-trained neural networks is the same as it was before, as the networks remained the same since they were last tested. The newly trained networks are going to have the same structure as the best performing networks from the previous chapter and will contain up to three hidden layers.

4.2.1. Input layer

The input layer is the same as it was in the previous chapter: there are five arguments passed to the network, where the first three specify if there are obstacles immediately next to the player, the fourth value is a normalized angle to the exit divided by 180 degrees and the fifth value is the direction that the player is suggested to take, which makes the neural networks training in this chapter supervised, too.

4.2.2. Hidden layer

The number of neurons per each hidden layer will be specified for every neural network in this chapter. ReLU will remain the activation function that all the neurons use.

4.2.3. Output layer

The output layer contains only one neuron, which returns one of the three values, like before: -1 if the player did not survive a turn, 0 if the player has survived, but the direction of the player is wrong in relation to the exit and 1 if the player has both survived and chosen the correct direction in their last move. Linear activation function will remain to be the one used by the output layer neuron.

4.3. Networks comparison

The pre-trained networks that were chosen to be tested with the Maze game are the single-layered network of 25 neurons that was trained with a set of 100,000 initial games from the Section 3.4, the two- and three-layered networks of 100 neurons per layer also from the Section 3.4, and the two-layered network of 18 neurons per layer from the Section 3.5.

The networks that were exclusively trained with the Maze game were chosen to be of the same structure as the pre-trained networks: the single-layered network of 25 neurons and two- and three-layered networks of 100 neurons per layer, except for the two-layered network of 18 neurons per layer, as there was already a two-layered network with a larger number of neurons.

Every new network was trained with a set of 100,000 initial games, as this value was found to produce the best performing networks in the last chapter, and the results that were achieved by both pre-trained and newly trained networks are in the Table 4. The number of the test games was increased from 5,000 to 10,000, as the Maze games take much less time to complete than the Snake ones. The scores were omitted, as it is possible to lose with a score of 0 or less and if the game is won, the score is always set to 1, thus making this statistic meaningless.

4.3.1. General results

Table 4. Pre-trained and newly-trained neural networks performance comparison

Hidden layers (neurons per layer)	1 (25)¹	2 (18)¹	2 (100)¹	3 (100)¹	1 (25)²	2 (100)²	3 (100)²
Time spent on the 1st epoch (s)	4.401	291.856	602.469	791.910	16.617	30.325	40.990
Time spent on the 2nd epoch (s)	3.402	293.645	614.335	820.464	17.969	31.594	39.716
Time spent on the 3rd epoch (s)	4.144	292.854	617.867	824.834	17.788	29.668	39.190
Total training time (s)	11.947	878.355	1834.671	2437.208	52.374	91.587	119.896
Average steps in 10,000 games	24.9078	25.6873	25.4965	19.7023	25.3470	28.0331	24.2054
Total mazes solved	932	665	895	216	3588	3712	3108

¹ Pre-trained network

² Newly-trained network

As it was said previously in the current chapter's introduction, it was already expected that the networks trained with the Maze game will perform better in the game that they were trained for compared to the other networks, which only learnt to play the Snake game, however, it was also expected that the Maze-trained networks will be able to solve more mazes, but that was not the case.

What was not expected at all was that the best performing network at the Snake game, the three-layered one, was the worst performing network out of all the previously trained networks, as if it could not adapt its previous knowledge to the new game scenarios. It could only solve 2.16% of all the mazes that were given to it, which is several times worse than a result by any other network.

While the previously best-performing network has struggled in the new game and quickly became the worst-performing one, the single-layered network, which had the weakest results of all the chosen networks, managed to obtain the best results in the Maze game with a solving rate of 9.32%. This, of course, is still much worse than what the newly-trained networks managed to get, however, considering that we were measuring the adaptability of the neural networks, the results are not completely terrible, while still being somewhat weak.

4.3.2. Game strategies

The biggest difficulty that the pre-trained networks had was their inability to avoid the dead-end game scenarios, such as the one shown in the Figure 12, which was their previous weakness in the Snake game as well. The newly-trained networks were a little better at recognizing such situations and avoiding them altogether, which gave them quite an advantage over the older networks.

In the case of the networks from the previous chapter, this difficulty is completely understandable, as it is not possible to immediately turn back in the Snake game, since that would mean that the snake would have to eat itself and instantly lose the game.

Other than that, the main difference between the playing strategies of the networks was that the networks from the previous chapter tried to turn left or right quite often when the possibility of doing so was present, while the newly-trained networks were often ignoring such distractions and continuing with their goal of reaching an exit.

There also was a different direction priority for the two types of tested networks. The older networks tried to go upwards first and then find their way to an exit from there. This resembled the straight turn strategy from the Snake game. But this strategy, however, was an issue in the Maze game, as once the network reached the top of the maze, the exit was quite often fully obstructed by the walls, thus lowering the network's total solving rate.

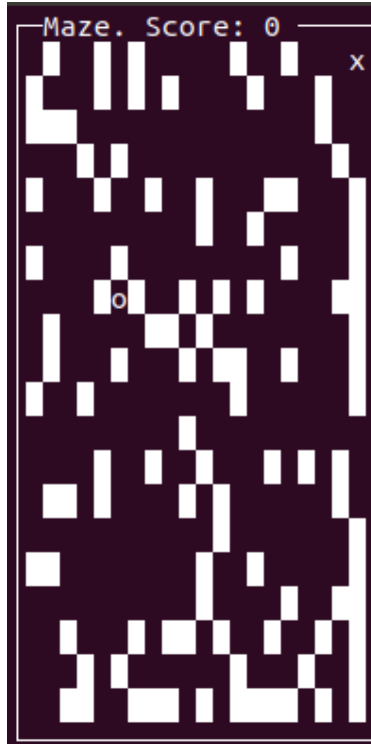


Figure 12. Pre-trained network after entering an avoidable dead-end game scenario

The newly trained networks were trying to reach an exit point evenly, in the similar style as the diagonal turn strategy, which was the inferior tactic in the Snake game. This could potentially mean that the earlier networks from the previous chapter, which use this strategy, would perform very well after being applied to a new game, but this is not the case, as the 10,000 initial games single-layered network was tested against the Maze game, too and only managed to achieve a solving rate of 4.78%, thus disproving this hypothesis.

4.4. Network comparison after an additional training

As it was said in the Section 1.5, one of the limitations of neural networks is that the networks are problem-specific, and a trained network cannot be applied to another task once it learns to solve one already. The results which were obtained during the experiment in a previous section prove that this statement is, indeed, correct.

Since applying the network directly does not yield the positive results, it would make sense to let the pre-trained networks receive an additional training against the Maze game and then measure the changes in performance. In the Table 5, the previous section results can be compared to the new ones that were obtained after an additional network training with a set of 50,000 maze games.

Originally, the training data was a set of 100,000 games, but it did not improve the networks ability to play the Maze game compared to the set of 50,000 games, as the results had virtually no difference. Meanwhile, it has lowered the networks quality of performance at the Snake game they were originally trained for, as the snakes controlled by these networks were stuck in an infinite loop around an apple.

Table 5. The Snake-trained networks performance before and after an additional training

Network	Old maze solve rate	New maze solve rate	Old average Snake score (5,000 tests)	New average Snake score (5,000 tests)
1-layered, 25 neurons per layer	9.32%	18.24%	28.7722	0
2-layered, 18 neurons per layer	6.65%	37.08%	30.2886	20.0078
2-layered, 100 neurons per layer	8.95%	16.3%	35.5392	25.4322
3-layered, 100 neurons per layer	2.16%	30.8%	38.2772	19.7258

The genetic algorithm initial population of 50,000 games was generated once, and all the networks were trained with the same batch of data to make the generation part of the training take less time. This technique sped up the training process immensely.

While the maze solving ability of the networks has improved in every single case, it would not be correct to call these results a definite success, as the original purpose of the said networks was to have a good performance in the Snake game, which, unfortunately, has deteriorated for every tested network.

What was very interesting, regardless of the result above, is that the two-layered network of 36 neurons, with a new maze solving rate of 37.08%, has managed to outperform the three-layered network of 100 neurons per layer from the previous section, which was trained with the Maze game exclusively and had the 31.08% solving rate. This is quite similar to the results we saw in the second experiment conducted in the previous chapter, where the deeper networks did not necessarily perform better than the networks with a fewer number of layers.

4.5. Weights change for additionally trained networks

The Snake-trained neural networks, which received an additional Maze training surely had their weights changed, as their performances did not stay the same for both the Snake and Maze games. Since the changes in the weight values were already looked at in the Section 3.4.3 and some of the weights remained constant throughout all networks and some were changing at first, but then stopped to, it would be interesting to see if those same weights had their values changed after an additional training with the Maze game.

Table 6. Single-layered Snake-trained network weights for one of the input parameters before and after an additional Maze training

Before	After
-0.012485540 *	0.322254717
-2.822388411	-1.844631433
0.273763716	0.437202066
-3.354897499	-3.419529915
-0.006273229 *	-0.323803365
-0.297206699 **	-0.297206699
-0.939412951 ***	-0.939412951
-0.001764486 *	-0.001764486
0.530116677	0.402083248
0.331819355 *	0.331819355
-2.499260902 *	-2.499260902
0.128729463	-0.103217460
0.411132187	0.076555967
-0.170884013	-0.922648907
-1.802851558 **	-1.802851558
-0.003732676 ***	-0.003732676
0.008858151 **	0.008858151
0.086139977 *	0.023521159
0.160483778	0.760529816
0.009423577 ***	0.009423577
-0.437185228	-0.456430763
-0.062357765 ***	-0.062357765
-0.000063382 **	-0.000063382
-1.939302325	-1.394816399
0.034327365 **	-0.320049256

Considering that we previously chose to study the weight values for the second input parameter, it would only be logical to examine the new values for that same parameter, which specifies if there is an obstacle immediately to the left of the player. The network chosen to be tested is the single-layered network of 25 neurons with the training data set of 100,000 games that was already studied in the previous chapter. The weight values for the original and for the additionally trained networks are in the Table 6, and the unchanged values are highlighted.

* Value remained the same since the 100,000 initial games network

** Value remained the same since the 10,000 initial games network

*** Value remained constant throughout all networks

To make it easier to see which weights did not change throughout the experiment conducted in the Chapter 3, these values are noted with the asterisks, which specify the network these weights have not changed since.

Same as in the previous chapter, if the neuron weight did not change for one of the input parameters, all the other weight values of that specific neuron remained the same for other parameters, too.

While most of the weights that remained constant in the Snake-trained networks have also not changed after receiving an additional training in the Maze game, there still were 4 values, which were modified. Three of these values were only considered “ideal” starting with the network of 100,000 initial games and in its successors, while the fourth value remained unchanged ever since the network of 10,000 initial games.

A very interesting observation is that none of these changed weight values were the ones that remained constant across every single Snake-trained network from the previous chapter. Those values, which were the same since the very first neural network of 1,000 initial games, continued to remain the same even after receiving an additional Maze training, which makes these weights “ideal” for both the Snake and Maze games.

4.6. Conclusion

It was expected that the networks will not have a great solving rate in the Maze game, no matter which game they were originally trained for, as they cannot compete with the A* pathfinding algorithms, which have a 100% solve rate for mazes that can be solved. In fact, the algorithm, which was checking if the maze is solvable or not in this work, was using the A* algorithm, too.

Overall, the results collected from the experiments completed in this chapter show that the networks experience difficulties after being applied to a new field of application, which was already a well-known fact in the neural networks study, as it was mentioned before in the Section 1.5. While the networks can improve their results in the new field after being trained with an additional set of data related to that task, they lose their quality and the best performing status at the task they were originally developed and trained for. And even then, the results that they manage to obtain at their new task are still not comparable to the results of the best-performing networks designed for that task specifically.

Thus, the original goal of showing that the trained neural networks can be reused for other tasks was only partially fulfilled, as these networks could not outperform the best networks at their respective tasks even after receiving an additional training. It was, however, shown that it is possible to use the same network for both tasks it was trained for, if it is not critical to consistently obtain the best possible results, as such networks will only be the so-called “jacks of all trades, masters of none”. This means that while they can have a decent performance in all tasks they were trained for, their results will always be worse than what the best performing networks specifically trained for each of these tasks, can obtain.

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