For $\lambda = 0$

$$y'' + (0)y = 0$$

$$y'' = 0$$

$$\therefore \quad y = 0$$

QUESTION No. II :-

Verify that all singular points of the DE are regular singular points.

a) $y'' - xy = 0$

Ans:- $x = 0$ is the singular point of given differential equation.

$$\lim_{x \to 0} (x-0)P(x) < \infty$$

$$\lim_{x \to 0} (x)(0) < \infty$$

$$0 < \infty$$

$$\lim_{x \to 0} (x-0)^2 Q(x) < \infty$$

$$\lim_{x \to 0} (x)^2(-x) < \infty$$

$$\lim_{x \to 0} -x^3 < \infty$$

$$0 < \infty$$

Since in both case we get a finite number as $x \to 0 \therefore x = 0$ is a regular singular point.

ⓑ $(1-x^2)y'' - x + p^2y = 0$

Sol:- Dividing given equation by "$1-x^2$".

$$y'' - \frac{x}{1-x^2} + \frac{p^2 y}{1-x^2} = 0$$

$$y'' + \frac{p^2 y}{1-x^2} = \frac{x}{1-x^2}$$

$$P(x) = 0 \quad , \quad Q(x) = \frac{p^2}{1-x^2}$$

$\therefore$ $x = \pm 1$ are two singular points of given differential equation.

CHECKING $x = 1$:-

$$\lim_{x \to 1} (x-1)(0) < \infty$$

$$\lim_{x \to 1} 0 < \infty$$

$$0 < \infty$$

Regular Singular Point

$$\lim_{x \to 1} (x-1)^2 \cdot \frac{p^2}{(1-x^2)} < \infty$$

$$\lim_{x \to 1} \frac{(x-1)\cancel{(x-1)}\, p^2}{(1+x)\cancel{(1-x)}} < \infty$$

$$\lim_{x \to 1} \frac{(x-1)\, p^2}{(x+1)} < \infty$$

$$0 < \infty$$

Regular Singular Point

$\cancel{\text{(c) } y'' - 2xy' + 2py = 0}$

CHECKING $x = -1$:-

$$\lim_{x \to -1} (x-(-1))(0) < \infty$$

$$\lim_{x \to -1} (0) < \infty$$

$$0 < \infty$$

$$\lim_{x \to -1} (x-(-1))^2 \left(\frac{p^2}{(1-x^2)} \right) < \infty$$

$$\lim_{x \to -1} (x+1)^{\cancel{2}} \cdot \frac{p^2}{\cancel{(1+x)}(1-x)} < \infty$$

$$\lim_{x \to -1} \frac{(x+1)p^2}{1-x} < \infty$$

$$\frac{(-1+1)p^2}{1+1} < \infty$$

$$0 < \infty$$

$\therefore$ Both $x = +1$ and $x = -1$ are regular singular points.

(c) $y'' - 2xy' + 2py = 0$

$P(x) = -2x$, $Q(x) = 2p$

$\therefore$ $x = 0$ is singular point

Now checking for regular and irregular

P-T-O

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CHECKING $x = 1$:-

$$\lim_{x \to 1} (x-1)\left(\frac{-2x}{1-x^2}\right) < \infty$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(-2x)}{\cancel{(1-x)}(1+x)} < \infty$$

$$\lim_{x \to 1} \frac{-2x}{1+x} < \infty$$

$$\frac{-2(1)}{1+1} < \infty$$

$$-\frac{2}{2} < \infty$$

$$-1 < \infty$$

$$\lim_{x \to 1} (x-1)^2\left(\frac{n(n+1)}{1-x^2}\right) < \infty$$

$$\lim_{x \to 1} \frac{(x-1)\cancel{(x-1)} \cdot n(n+1)}{-(1+x)\cancel{(-1+x)}} < \infty$$

$$\lim_{x \to 1} \frac{(x-1) \cdot n(n+1)}{-(1+x)} < \infty$$

$$\frac{(1-1)\,n(n+1)}{-(1+1)} < \infty$$

$$0 < \infty$$

Since both limits are not infinite so $x = 1$ is a regular singular point.

CHECKING $x = -1$:-

$$\lim_{x \to -1} (x-(-1))\left(\frac{-2x}{1-x^2}\right) < \infty$$

$$\lim_{x \to -1} \cancel{(x+1)}\left(\frac{-2x}{\cancel{(1+x)}(1-x)}\right) < \infty$$

$$\lim_{x \to -1} \frac{-2x}{1-x} < \infty$$

$$\lim_{x \to -1} (x+1)^2 \cdot \frac{n(n+1)}{1-x^2} < \infty$$

$$\cancel{\lim_{x \to -1} \frac{(x+1)(x-1) \cdot n(n+1)}{-(1+x)(1+x)} < \infty}$$

$$\cancel{\lim_{x \to -1} \frac{(x-1) \cdot n(n+1)}{-(1+x)} < \infty}$$

$$\lim_{x \to 0} (x-0)(-2x) < \infty$$

$$\lim_{x \to 0} -2x^2 < \infty$$

$$0 < \infty$$

$$\lim_{x \to 0} (x-0)^2 (2p) < \infty$$

$$\lim_{x \to 0} (x)^2 2p < \infty$$

$$0 < \infty$$

$\therefore$ $x = 0$ is a regular singular point.

(d) $(1-x^2)y'' - 2xy' + (n)(n+1)y = 0$

Ans:- Dividing given equation by "$1-x^2$"

$$y'' - \frac{2x}{1-x^2} y' + \frac{n(n+1)}{1-x^2} y = 0$$

$$P(x) = -\frac{2x}{1-x^2} \quad , \quad Q(x) = \frac{n(n+1)}{1-x^2}$$

$P(x)$ and $Q(x)$ are undefined at

$x = 1$ and $x = -1$

$\therefore$ $x = 1$ and $x = -1$ are singular points

for given differential equation.

$$\frac{-2(-1)}{1-(-1)} < \infty$$

$$\frac{2}{2} < \infty$$

$$1 < \infty$$

$$\lim_{x \to -1} \frac{(x+1)^2 \cdot n(n+1)}{\cancel{(1+x)}(1-x)} < \infty$$

$$\lim_{x \to -1} \frac{(x+1) \cdot n(n+1)}{1-x} < \infty$$

$$0 < \infty$$

Since both limits are finite, So $x=1$ and $x=-1$ are regular singular points.

QUESTION No. III :

Sol:-

$$y'' - 2(x-1)y' + 2y = 0$$

$$x_0 = 1$$

The series solution is given by

$$y(t) = \sum_{n=0}^{\infty} (x-1)^n a_n .$$

$$y'(t) = \sum_{n=1}^{\infty} n(x-1)^{n-1} \cdot a_n$$

$$y''(t) = \sum_{n=2}^{\infty} n(n-1)(x-1)^{n-2} \cdot a_n$$

$$\sum_{n=2}^{\infty} n(n-1)(x-1)^{n-2} a_n + \left[-2(x-1)\sum_{n=1}^{\infty} n(x-1)^n a_n\right] + 2\sum_{n=0}^{\infty}(x-1)^n a_n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)(x-1)^{n-2} \cdot a_n + \left[-2\sum_{n=1}^{\infty} n(x-1)^{n+1} a_n\right] + 2\sum_{n=0}^{\infty}(x-1)^n a_n = 0$$

$$\sum_{n=0}^{\infty}(n+2)(n+1)(x-1)^n \cdot a_{n+2} + \left[-2\sum_{n=2}^{\infty}(n-1)(x-1)^n a_{n-1}\right] + 2\sum_{n=0}^{\infty}(x-1)^n a_n = 0$$

$$(2)(1)(x-1)^0 a_2 + (3)(2)(x-1)^1 \cdot a_3 + \sum_{n=2}^{\infty}(n+2)(n+1)(x-1)^n \cdot a_{n+2} +$$

$$\left[-2\sum_{n=2}^{\infty}(n-1)(x-1)^n a_{n-1}\right] + 2(x-1)^0 a_0 + 2(x-1)^1 a_1$$

$$+ 2\sum_{n=2}^{\infty}(x-1)^n a_n = 0$$

$$2a_2 + 6a_3(x-1) + \sum_{n=2}^{\infty}(n+2)(n+1)(x-1)^n a_{n+2} + \left[-2\sum_{n=2}^{\infty}(n-1)(x-1)^n a_{n-1}\right]$$

$$+ 2a_0 + 2a_1(x-1) + 2\sum_{n=2}^{\infty}(x-1)^n a_n = 0$$

$2a_2 +$

$$2a_0 + 2a_1 x - 2a_1 + 6a_3 x - 6a_3 + \sum_{n=2}^{\infty}\Big[(n+2)(n+1)(x-1)^n a_{n+2}$$

$$-2(n-1)(x-1)^n a_{n-1} + 2(x-1)^n a_n\Big] = 0$$

$$2a_0 - 2a_1 - 6a_3 + (2a_1 + 6a_3)x + \sum_{n=2}^{\infty}\Big[(n+2)(n+1) a_{n+2}$$

$$-2(n-1)a_{n-1} + 2a_n\Big](x-1)^n = 0$$

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Comparing co-efficients of x^0, x^1 and $(x-1)^n$

$$2a_2 + 2a_0 - 2a_1 = 0 \quad \text{—(i)} \Rightarrow$$

$$2a_1 + 6a_3 = 0 \quad \text{—(ii)} \Rightarrow a_1 = -3a_3$$

$$(n+2)(n+1)a_{n+2} - 2(n-1)a_{n-1} + 2a_n = 0$$

$$\frac{(n+2)\cancel{(n+1)}a_{n+2}}{\cancel{(n+2)(n+1)}} = \frac{2(n-1)a_{n-1} - 2a_n}{(n+2)(n+1)}$$

$$a_{n+2} = \frac{2(n-1)a_{n-1} - 2a_n}{(n+1)(n+2)}$$

$\forall\ n \geq 2$

For $n=2$; $a_4 = \dfrac{2(1)a_1 - 2a_2}{(3)(4)} = \dfrac{2a_1 - 2a_2}{12}$

$$a_4 = \frac{a_1 - a_2}{6}$$

For $n=3$; $a_5 = \dfrac{2(2)a_2 - 2a_3}{(4)(5)} = \dfrac{\cancel{2}(2a_2 - a_3)}{\cancel{4}(5)} = \dfrac{2a_2 - a_3}{10}$

For $n=4$; $a_6 = \dfrac{2(3)a_3 - 2a_4}{(5)(6)} = \dfrac{\cancel{2}(3a_3 - a_4)}{5 \times \cancel{6}\,3} = \dfrac{3a_3 - a_4}{15}$

For $n=5$; $a_7 = \dfrac{2(4)a_4 - 2a_5}{(7)(6)} = \dfrac{\cancel{2}(4a_4 - a_5)}{7\times\cancel{6}3} = \dfrac{4a_4 - a_5}{21}$

$\sum_{n}$

$\sum_{n}$

(

$$y(t) = \sum_{n=0}^{\infty} a_n (x-1)^n$$

$$y(t) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + a_4(x-1)^4 + a_5(x-1)^5 + a_6(x-1)^6 + a_7(x-1)^7 \text{------}$$

$$y(t) = (a_1 - a_2) + a_1(x-1) + a_2(x-1)^2 + \left(-\tfrac{1}{3}a_1\right)(x-1)^3 + \left(\frac{a_1 - a_2}{6}\right)(x-1)^4 + \frac{2a_2 - a_3}{10}(x-1)^5 + \frac{3a_3 - a_4}{15}(x-1)^6 + \frac{4a_4 - a_5}{21}(x-1)^7 \text{-----}$$

Writing for 5 terms only. and neglecting other terms

$$y(t) = (a_1 - a_2) + a_1(x-1) + a_2(x-1)^2 - \frac{1}{3}a_1(x-1)^3 + \frac{(a_1 - a_2)}{6}(x-1)^4 \text{-------}$$