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$$a = \begin{pmatrix} (-5)\left(-\frac{29}{35}\right) + (6)\left(\frac{1}{7}\right) \\ 0 + (-7)\left(\frac{1}{7}\right) \end{pmatrix}$$

$$a = \begin{pmatrix} \frac{29}{7} + \frac{6}{7} \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{35}{7} \\ -1 \end{pmatrix}$$

$$a = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$a = \begin{pmatrix} -\frac{7}{35} & -\frac{6}{35} \\ 0 & -\frac{5}{35} \end{pmatrix} \begin{pmatrix} -\frac{29}{35} \\ \frac{1}{7} \end{pmatrix}$$

$$a = \begin{pmatrix} \left(-\frac{7}{35}\right)\left(\frac{-29}{35}\right) + \left(-\frac{6}{35}\right)\left(\frac{1}{7}\right) \\ 0 + \left(-\frac{5}{35}\right)\left(\frac{1}{7}\right) \end{pmatrix}$$

$$a = \begin{pmatrix} \frac{29}{175} - \frac{6}{245} \\ -\frac{1}{49} \end{pmatrix} = \begin{pmatrix} \frac{173}{1225} \\ -\frac{1}{49} \end{pmatrix}$$

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$$b = \begin{pmatrix} -1/5 & -6/35 \\ 0 & -1/7 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$b = \begin{pmatrix} 6/35 \\ 1/7 \end{pmatrix}$$

Solving (ii) for a

~~a~~

$$\cancel{\alpha} = Aa + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = b$$

$$Aa = \begin{pmatrix} 6/35 \\ 1/7 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Aa = \begin{pmatrix} \frac{6}{35} - \frac{1}{1} \\ 1/7 \end{pmatrix}$$

$$Aa = \begin{pmatrix} \frac{6-35}{35} \\ 1/7 \end{pmatrix} = \begin{pmatrix} -29/35 \\ 1/7 \end{pmatrix}$$

$$a = A^{-1} \begin{pmatrix} -29/35 \\ 1/7 \end{pmatrix}$$

$$a = \begin{pmatrix} -5 & 6 \\ 0 & -7 \end{pmatrix} \begin{pmatrix} -29/35 \\ 1/7 \end{pmatrix}$$

c, $Aa + \begin{pmatrix}1\\0\end{pmatrix} = b$ —(ii)

Solving (i) for b,

$$Ab = \begin{pmatrix}0\\-1\end{pmatrix}$$

$$b = A^{-1}\begin{pmatrix}0\\-1\end{pmatrix}$$

$$b = \begin{pmatrix}-5 & 6\\0 & -7\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}$$

$$b = \frac{1}{35-0}\begin{pmatrix}-7 & 0\\-6 & -5\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}$$

$$b = \begin{pmatrix}-7/35 & 0\\6/35 & -5/35\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}$$

$$b = \begin{pmatrix}0+0\\0+1/7\end{pmatrix} = \begin{pmatrix}0\\1/7\end{pmatrix}$$

$$b = \frac{1}{35}\begin{pmatrix}-7 & -6\\0 & -5\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}$$

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It's general solution is:

$$X_h(t) = c_1\begin{pmatrix}-3\\1\end{pmatrix}e^{-7t} + c_2\begin{pmatrix}1\\0\end{pmatrix}e^{-5t}$$

~~$X_p(t) = ?$~~

$$X_h(t) = \begin{pmatrix}-3c_1e^{-7t} + c_2e^{-5t}\\ c_1e^{-7t} + 0\end{pmatrix}$$

$$X_h(t) = \begin{pmatrix}-3c_1e^{-7t} + c_2e^{-5t}\\ c_1e^{-7t}\end{pmatrix}$$

$$X_p(t) = a + bt$$

$$X_p(t) = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}t$$

~~$X_p'(t) = b$~~

$$X_p'(t) = b$$

~~$X_p(t)$~~

$$b = (a+bt)\,\overset{A}{\cancel{X_p(t)}} + \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}$$

$$b = Aa + Abt + \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}t$$

Equating, Co-efficients,

$$Ab + \begin{pmatrix}0\\1\end{pmatrix} = 0 \quad -(1)$$

$(-5-\lambda)(-7-\lambda) = 0$

$35 + 5\lambda + 7\lambda + \lambda^2 = 0$

$\cancel{\lambda^2 + 12\lambda + 35} = 0$

$5(7+\lambda) + \lambda(7+\lambda) = 0$

$(\lambda+7)(\lambda+5) = 0$

$\lambda = -7, \quad \lambda = -5$

For $\lambda = -7$,

$$\begin{bmatrix} -5-(-7) & 6 \\ 0 & -7-(-7) \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 6 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$2x_1 + 6y_1 = 0$

$x_1 + 3y_1 = 0$

$x_1 = -3y_1$

$\therefore V_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

For $\lambda = -5$

$$\begin{bmatrix} -5+5 & 6 \\ 0 & -7+5 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 6 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$6y_2 = 0 \Rightarrow \boxed{y_2 = 0}$

$-2y_2 = 0$

$x \rightarrow$ free variable

$V_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Exercise 6.5

① $x' = \begin{pmatrix} -5 & 6 \\ 0 & -7 \end{pmatrix} x + \begin{pmatrix} 1 \\ t \end{pmatrix}$

$$x' = Ax + F(t)$$

$$F(t) = \begin{pmatrix} 1 \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} t$$

Gen. Sol. is given by:

$$X(t) = X_h(t) + X_p(t)$$

$X_h(t) = ?$:-

$$F(t) = 0$$

$$\therefore \quad x' = \begin{pmatrix} -5 & 6 \\ 0 & -7 \end{pmatrix} x$$

& $|A - \lambda I| = 0$

$$\begin{vmatrix} -5-\lambda & 6 \\ 0 & -7-\lambda \end{vmatrix} = 0$$

∴ General Solution is given by:

$$X(t) = C_1 e^{4t}\begin{pmatrix}1\\0\\0\end{pmatrix} + C_2 e^{-2t}\begin{pmatrix}0\\1\\0\end{pmatrix} + C_3 e^{-t}\begin{pmatrix}-1\\0\\5\end{pmatrix}$$

$$X(t) = \begin{pmatrix} C_1 e^{4t} + 0 - C_3 e^{-t} \\ 0 + C_2 e^{-2t} + 0 \\ 0 + 0 + 5C_3 e^{-t} \end{pmatrix}$$

$$X(t) = \begin{pmatrix} C_1 e^{4t} - C_3 e^{-t} \\ C_2 e^{-2t} \\ 5C_3 e^{-t} \end{pmatrix}$$

$$\begin{cases} x(t) = C_1 e^{4t} - C_3 e^{-t} \\ y(t) = C_2 e^{-2t} \\ z(t) = 5C_3 e^{-t} \end{cases}$$

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Now for $\lambda_3 = -1$, let $v_3 = \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$

$$\begin{bmatrix} 4+1 & 0 & 1 \\ 0 & -2+1 & 0 \\ 0 & 0 & -1+1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c} 5 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} \frac{1}{5}R_1 \\ -1R_2 \end{array}$$

$$x_3 + \frac{1}{5}z_3 = 0 \Rightarrow x_3 = -\frac{1}{5}z_3$$

$$y_3 = 0$$

$$\therefore v_3 = \begin{pmatrix} -1/5 \\ 0 \\ 1 \end{pmatrix} \text{ OR } \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$$

$$\therefore \left[\begin{array}{ccc|c} 6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\overset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \frac{1}{6}R_1$$

$$\overset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1/6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \leftrightarrow R_2$$

$$\overset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 - \frac{1}{6}R_2$$

$\therefore \; x_2 = 0,$

$z_2 = 0$

y - free variable.

$$\therefore \; v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -6 & 0 \\ 6 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = 0$$

$$\therefore \begin{aligned} 0x_1 + 0y_1 + 0z_1 &= 0 \\ 0x_1 - 6y_1 + 0z_1 &= 0 \\ 0x_1 + 0y_1 - 5z_1 &= 0 \end{aligned}$$

$$R \sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -\frac{1}{6}R_2 \\ \\ R_3 - R_1 \end{array}$$

$y_1 = 0$

$\therefore z_1 = 0$

$x_1 = 1$

$$\therefore v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Now for $\lambda_2 = -2$, let $v_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$$\begin{bmatrix} 6 & 0 & 1 \\ 6 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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(8) (27) $x' = \begin{pmatrix} 4 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix} X$

Let $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Finding eigenvalues.

$\det(A - I\lambda) = 0$

$$\begin{vmatrix} 4-\lambda & 0 & 1 \\ 0 & -2-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix} = 0$$

$(4-\lambda)\left[(-2-\lambda)(-1-\lambda) - 0\right] - 0 + 1\left[0 - 0\right] = 0$

$(4-\lambda)(-2-\lambda)(-1-\lambda) = 0$

$\therefore \quad \lambda_1 = 4, \quad \lambda_2 = -2, \quad \lambda_3 = -1$

Finding eigenvectors,

For $\lambda_1 = 4$ Let $V_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

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$$3x_2 + 0y_2 = 0$$

$$x_2 + 0y_2 = 0$$

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$$R \sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -3 & 0 \end{array}\right] \quad \frac{1}{5}R_2$$

$$x_1 - 3y_1 = 0$$

$$x_1 = 3y_1$$

$$\therefore \quad V_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

For $\lambda_2 = -8$, Let $V_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$\begin{bmatrix} 7+8 & 0 \\ 5 & -8+8 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 15 & 0 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|cc} 15 & 0 & x_2 & 0 \\ 5 & 0 & y_2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 3 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \quad \frac{1}{5}R_1, \frac{1}{5}R_2$$

$$\frac{2}{-1-i\sqrt{3}} \times \frac{-1+i\sqrt{3}}{-1+i\sqrt{3}}$$

$$\frac{-2+2i\sqrt{3}}{(-1)^2-(i\sqrt{3})^2} = \frac{-2+2i\sqrt{3}}{1+3} = \frac{2(-1+i\sqrt{3})}{4}$$

$$\frac{2}{-1+i\sqrt{3}} \times \frac{-1-i\sqrt{3}}{-1-i\sqrt{3}}$$

$$\frac{-2-2i\sqrt{3}}{(-1)^2-(i\sqrt{3})^2} = \frac{2(-1-i\sqrt{3})}{1+3}$$

finding eigenvectors

For $\lambda_1 = 7$, let $V_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$\begin{bmatrix} 7-7 & 0 \\ 5 & -8-7 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 5 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 5 & -15 & 0 \end{array}\right]$$

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$5\,|\,\cancel{2}145$ 15 15 5 15 ×

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$$\begin{vmatrix} 7-\lambda & 0 \\ 5 & -8-\lambda \end{vmatrix} = 0$$

$$(7-\lambda)(-8-\lambda) - (0)(5) = 0$$

$$-56 - 7\lambda + 8\lambda + \lambda^2 = 0$$

$$\lambda^2 + \lambda - 56 = 0$$

$$\lambda = \frac{-(1) \pm \sqrt{1^2 - 4(1)(-56)}}{2(1)}$$

$$\lambda = \frac{-1 \pm \sqrt{1 + 224}}{2}$$

$$\lambda = \frac{-1 \pm \sqrt{225}}{2}$$

$$\lambda = \frac{-1 \pm 15}{2}$$

$$\lambda_1 = \frac{-1+15}{2}, \quad \lambda_2 = \frac{-1-15}{2}$$

$$\lambda_1 = 7, \quad \lambda_2 = -8$$

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$$\begin{cases} x(t) = \frac{1}{5}c_1 e^{8t} + c_2 e^{5t} \\ y(t) = c_1 e^{8t} + c_2 e^{5t} \end{cases}$$

(17) $x' = 7x$

$y' = 5x - 8y$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 5 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$X'(t) = \begin{pmatrix} 7 & 0 \\ 5 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$X'(t) = \begin{pmatrix} 7 & 0 \\ 5 & -8 \end{pmatrix} X(t)$$

Let $A = \begin{pmatrix} 7 & 0 \\ 5 & -8 \end{pmatrix}$

Finding eigenvalues;

$$\det(A - I\lambda) = 0$$

$$\left| \begin{pmatrix} 7 & 0 \\ 5 & -8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

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$$\begin{bmatrix} 1 & -1 \\ 5 & -5 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 5 & -5 & 0 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right] \quad \frac{1}{5}R_2 - \cancel{5}R_1$$

$$x_2 - y_2 = 0$$

$$\therefore x_2 = y_2$$

$$\text{So } V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The general solution is given by:

$$X(t) = c_1 e^{1t} \begin{pmatrix} 1/5 \\ 1 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} c_1 e^t \frac{1}{5} + c_2 e^{5t} \\ c_1 e^t + c_2 e^{5t} \end{pmatrix}$$

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$$R \sim \left[\begin{array}{cc|c} 1 & -1/7 & 0 \\ 0 & 1 & 0 \end{array}\right] \frac{7}{12} R_2$$

$$x_1 - \frac{1}{7} y_1 = 0 \Rightarrow x_1 = \frac{1}{7} y_1$$

$$0x_1 + y_1 = 0$$

$$R \sim \left[\begin{array}{cc|c} 1 & -1/5 & 0 \\ 1 & -1/5 & 0 \end{array}\right]$$

$$x_1 - \frac{1}{5} y_1 = 0$$

$$x_1 = \frac{1}{5} y_1$$

$$\therefore V_1 = \begin{pmatrix} 1/5 \\ 1 \end{pmatrix}$$

For $\lambda_2 = 5$, let $V_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$\begin{bmatrix} 6-5 & -1 \\ 5 & -5 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

Finding eigenvectors

For $\lambda_1 = +1$, Let $V_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$\begin{bmatrix} 6-(+1) & -1 \\ 5 & -(+1) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & -1 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

Writing it's augmented matrix.

$$\left[\begin{array}{cc|c} 5 & -1 & 0 \\ 5 & -1 & 0 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{cc|c} 7 & -1 & 0 \\ 0 & 1+\frac{5}{7} & 0 \end{array}\right] \quad R_2 - \frac{5}{7} R_1$$

$$\underset{\sim}{R} \left[\begin{array}{cc|c} 7 & -1 & 0 \\ 0 & \frac{12}{7} & 0 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{cc|c} 1 & -\frac{1}{7} & 0 \\ 0 & \frac{12}{7} & 0 \end{array}\right] \quad \frac{1}{7} R_1$$

$$X'(t) = A\,X(t)$$

Finding eigenvalues,

$$\cancel{\det(A - \lambda I) = \begin{vmatrix} -4-\lambda & -1 \\ 5 & -\lambda \end{vmatrix} = 0}$$

$$\cancel{\begin{vmatrix} -4-\lambda & -1 \\ 5 & -\lambda \end{vmatrix} = 0}$$

$$\cancel{(-4-\lambda)(-\lambda) - (-1)(5) = 0}$$

$$\cancel{4\lambda + \lambda^2 + 5 = 0}$$

$$\cancel{\lambda^2 + 4\lambda + 5 = 0}$$

$$\det(A - \lambda I) = \begin{vmatrix} 6-\lambda & -1 \\ 5 & -\lambda \end{vmatrix} = 0$$

$$(6-\lambda)(-\lambda) - (-1)(5) = 0$$

$$-6\lambda + \lambda^2 + 5 = 0$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\lambda^2 - \lambda - 5\lambda + 5 = 0$$

$\lambda(\lambda-1) - 5(\lambda-1) = 0$ $\quad \cancel{\lambda(\lambda+1) + 5(\lambda+1) = 0}$

$(\lambda-1)(\lambda-5) = 0$ $\quad \cancel{(\lambda+1)(\lambda+5) = 0}$

$\lambda = 1, \lambda = 5$ $\quad \cancel{\lambda = -1, \lambda = -5}$

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$x(t) = c_1 e^{2t} + c_2 e^{4t}$

$y(t) = 3c_1 e^{2t} + c_2 e^{4t}$

(2) $A = \begin{pmatrix} -4 & 0 \\ -1 & 0 \end{pmatrix}$; $\lambda_1 = 0$, $\lambda_2 = -4$, $v_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

Ans: The general sol. is given by:

$$X(t) = c_1 e^{0t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$X(t) = c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

Find a general solution of the system

(15) $\begin{cases} \frac{dx}{dt} = 6x - y \\ \frac{dy}{dt} = 5x \end{cases}$

Ans: Writing in matrices form

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Let $\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = X'(t)$, $\begin{pmatrix} x \\ y \end{pmatrix} = X(t)$

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Exercise 6.4.

Use the eigenvalues and corresp. (linearly independant) eigenvectors of the matrix A to find a general solution of $x' = Ax$.

① $A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$; $\lambda_1 = 2$, $v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\lambda_2 = 4$,

$$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ans:- The general sol. is given by

$$X(t) = c_1 e^{\lambda_1 t} V_1 + c_2 e^{\lambda_2 t} V_2$$

$$X(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$X(t) = \begin{pmatrix} c_1 e^{2t} \\ 3c_1 e^{2t} \end{pmatrix} + \begin{pmatrix} c_2 e^{4t} \\ c_2 e^{4t} \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} c_1 e^{2t} + c_2 e^{4t} \\ 3c_1 e^{2t} + c_2 e^{4t} \end{pmatrix}$$