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16 Solve the IVP.

16 $y'' - y' = 0 \quad , \quad y(0) = 3, \quad y'(0) = 2$

Ans

$r^2 - r = 0$

$r(r-1) = 0$

$r = 0, \quad r = 1$

$y(t) = c_1 e^{0t} + c_2 e^{1t}$

$y(t) = c_1(1) + c_2 e^t = \boxed{c_1 + c_2 . e^t}$

$y'(t) = 0 + c_2 e^t = \boxed{c_2 e^t}$

$y(0) = c_1 + c_2 e^0$	$y'(0) = c_2 e^0$
$3 = c_1 + c_2$	$\boxed{2 = c_2}$

$3 = c_1 + 2 \Rightarrow \boxed{c_1 = 1}$

$\therefore \boxed{y(t) = 1 + 2e^t}$

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(18) $y'' + y' - 12y = 0$, $y(0) = 0$, $y'(0) = 7$

Ans:- $r^2 + r - 12 = 0$

$$r = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)}$$

$$r = \frac{-1 \pm \sqrt{1 + 48}}{2}$$

$$r = \frac{-1 \pm \sqrt{49}}{2}$$

$$r = \frac{-1 \pm 7}{2}$$

$$r_1 = \frac{-1 + 7}{2} \quad , \quad r_2 = \frac{-1 - 7}{2}$$

$$r_1 = 3 \quad , \quad r_2 = -4$$

$$y(t) = c_1 e^{3t} + c_2 e^{-4t} \quad \text{—— (i)}$$

$$y'(t) = 3c_1 e^{3t} - 4c_2 e^{-4t} \quad \text{—— (ii)}$$

$y(0) = c_1 e^{3(0)} + c_2 e^{-4(0)}$	$y'(0) = 3c_1 e^0 - 4c_2 e^0$
$y(0) = c_1 e^0 + c_2 e^0$	$y'(0) = 3c_1 - 4c_2$
$0 = c_1 + c_2$ —(iii)	$7 = 3c_1 - 4c_2$ —(iv)

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Multiplying 4 with eq

$$0 = 4c_1 + \cancel{4c_2}$$
$$7 = 3c_1 - \cancel{4c_2}$$

$$7c_1 = 7$$
$$\boxed{c_1 = 1}$$

Put in (iii)

$$1 + c_2 = 0$$
$$\boxed{c_2 = -1}$$

Putting c_1 and c_2 values in (i)

$$y(t) = 1e^{3t} + (-1)e^{-4t}$$
$$y(t) = e^{3t} - e^{-4t}$$

(20) $2y'' - 7y' - 4y = 0$, $y(0) = 0$, $y'(0) =$

$$2r^2 - 7r - 4 = 0$$

$$r = \frac{-(-7) \pm \sqrt{-7^2 - 4(2)(-4)}}{2(2)}$$

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$$r = \frac{7 \pm \sqrt{81}}{4} = \frac{7 \pm 9}{4}$$

$$r_1 = \frac{7+9}{4}, \quad r_2 = \frac{7-9}{4}$$

$$r_1 = \frac{16}{4}, \quad r_2 = \frac{-2}{4}$$

$$r_1 = 4, \quad r_2 = -1/2$$

$$y(t) = c_1 e^{4t} + c_2 e^{-1/2 t} \quad \text{—(i)}$$

$$y'(t) = 4 \cdot c_1 e^{4t} + (-1/2) c_2 e^{-1/2 t}$$

$$y'(t) = 4c_1 e^{4t} - 1/2 \cdot c_2 e^{-1/2 t} \quad \text{—(ii)}$$

$$y(0) = c_1 e^{4(0)} + c_2 e^{-1/2(0)}$$

$$y(0) = c_1 e^0 + c_2 e^0$$

$$\boxed{0 = c_1 + c_2} \quad \text{—(iii)}$$

$$y'(0) = 4c_1 e^{4(0)} - 1/2 c_2 e^{-1/2(0)}$$

$$y'(0) = 4c_1 (1) - 1/2 c_2 (1)$$

$$\boxed{1 = 4c_1 - 1/2 c_2} \quad \text{—(iv)}$$

Xing 2 with eq (iv)

$$\boxed{2 = 8c_1 - c_2} \quad \text{—(v)}$$

Adding eq (iii) and eq (v)

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$$c_1 + \cancel{c_2} = 0$$

$$8c_1 - \cancel{c_2} = 2$$

$$9c_1 = 2$$

$$\boxed{c_1 = 2/9}$$

Put in (iii)

$$\frac{2}{9} + c_2 = 0$$

$$\boxed{c_2 = -2/9}$$

$$\therefore \quad y(t) = \frac{2}{9} e^{4t} - \frac{2}{9} e^{-1/2 t}$$

$$y(t) = \frac{2}{9}\left(e^{4t} - e^{-1/2 t}\right)$$

(22) $y'' + 36y = 0 \quad , \quad y(0) = 2, \; y'(0) = -6$

$$r^2 + 36 = 0$$

$$r^2 = -36$$

$$r = \pm\sqrt{-36}$$

$$r = \pm 6i$$

$$\alpha = 0, \; \beta = 6$$

$$\therefore \quad y(t) = e^{0t}\left[c_1 \cos 6t + c_2 \sin 6t\right.$$

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$y(t) = (1)\ [c_1 \cos 6t + c_2 \sin 6t]$

$y(t) = c_1 \cos 6t + c_2 \sin 6t$ —(i)

$y'(t) = c_1 \cdot -\sin 6t \cdot 6 + c_2 \cdot \cos 6t \cdot 6$

$y'(t) = -6c_1 \sin 6t + 6 \cdot c_2 \cdot \cos 6t$ —(ii)

$y(0) = c_1 \cos 6(0) + c_2 \sin 6(0)$

$y(0) = c_1 \cos 0 + c_2 \sin(0)$

$2 = c_1(1) + c_2(0)$

$\boxed{2 = c_1}$

$y'(0) = -6c_1 \sin 6(0) + 6c_2 \cos 6(0)$

$y'(0) = -6c_1 \sin 0 + 6c_2 \cos 0$

$-6 = -6c_1(0) + 6c_2(1)$

$-6 = 0 + 6c_2$

$\boxed{c_2 = -1}$

$\therefore\ y(t) = 2\cos 6t - 1\sin 6t$

$\boxed{y(t) = 2\cos 6t - \sin 6t}$

(24) $y'' - 2y' + y = 0$, $y(0) = 4$, $y'(0) = 0$

Ans: $r^2 - 2r + r = 0$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$r = \frac{2 \pm \sqrt{4-4}}{2} = \frac{2 \pm \sqrt{0}}{2} = \frac{2}{2} = 1$$

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$r = r_1 = r_2 = 1$ (Repeated Roots)

$y(t) = e^{rt}(c_1 + c_2 t)$

$y(t) = e^{(1)t}(c_1 + c_2 t)$

$y(t) = e^{t}(c_1 + c_2 t)$

$y'(t) = e^{t}(+c_2) + (c_1 + c_2 t)e^{t}$

$y'(t) = c_2 . e^{t} + e^{t}(c_1 + c_2 t)$

$y(0) = e^{0}(c_1 + c_2(0))$

$y(0) = 1(c_1 + 0)$

$\boxed{4 = c_1}$

$y'(0) = c_2 e^{0} + e^{0}(c_1 + c_2(0$

$0 = c_2 + (1)(c_1 + 0)$

$\boxed{0 = c_2 + c_1}$

$0 = c_2 + 4$

$\boxed{c_2 = -4}$

$\therefore \boxed{y(t) = e^{t}(4 - 4t)}$

(26) $y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$

$r^2 + 2r + 5 = 0$

$r = \frac{-(2) \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$

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$$r = \frac{-2 \pm \sqrt{4-20}}{2}$$

$$r = \frac{-2 \pm \sqrt{-16}}{2}$$

$$r = \frac{-2 \pm 4i}{2}$$

$$r = \frac{2(-1 \pm 2i)}{2} = -1 \pm 2i$$

$$\alpha = -1, \quad \beta = 2$$

$$y(t) = e^{-1t}(c_1 \cos 2t + c_2 \sin 2t)$$

$$y(t) = e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$$

$$y'(t) = e^{-t}(c_1 \cdot 2 \cdot -\sin 2t + c_2 \cdot 2 \cdot \cos 2t) + (c_1 \cos 2t + c$$

$$\cancel{y'(t)} = \sin 2t) \cdot - e^{-t}$$

$$y'(t) = e^{-t}(-2c_1 \sin 2t + 2c_2 \cos 2t) - e^{-t}(c_1 \cos 2t + c$$

$$\sin 2t)$$

$$y'(t) = e^{-t}[-2c_1 \cdot \sin 2t + 2c_2 \cos 2t - c_1 \cos 2t - c_2 \sin$$

$$y'(t) = e^{-t}[-2c_1 \sin 2t - c_2 \sin 2t + 2c_2 \cos 2t - c_1 \cos 2t$$

$$y'(t) = e^{-t}[\sin 2t(-2c_1 - c_2) + \cos 2t(2c_2 - c_1)]$$

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Now

$$y(0) = e^{-0}\left[c_1 \cos 2(0) + c_2 \sin 2(0)\right]$$

$$y(0) = e^{0}\left[c_1 \cos 0 + c_2 \sin 0\right]$$

$$1 = 1\left[c_1(1) + c_2(0)\right]$$

$$1 = 1\left[c_1 + 0\right]$$

$$\boxed{c_1 = 1}$$

$$y'(0) = e^{-0}\left[\sin 2(0)(-2c_1 - c_2) + \cos 2(0)(2c_2 - c_1)\right]$$

$$0 = 1\left[0(-2c_1 - c_2) + 1(2c_2 - c_1)\right]$$

$$0 = \left[0 + 2c_2 - c_1\right] = 2c_2 - c_1$$

$$\therefore\ 0 = 2c_2 - 1$$

$$2c_2 = 1$$

$$\boxed{c_2 = 1/2}$$

$$y(t) = e^{-t}\left[(1)\cos 2t + \tfrac{1}{2}\sin 2t\right]$$

EXERCISE 4.3

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(2) $S = \{\cos 2t, \sin t, 1\}$

Ans:

$$W(S) = \begin{vmatrix} \cos 2t & \sin t & 1 \\ -2\sin 2t & \cos t & 0 \\ -4\cos 2t & -\sin t & 0 \end{vmatrix}$$

$$= \cos 2t\,[0 - 0] - \sin t\,[0 - 0] + 1\,[+2\sin 2t.\sin t + 4\cos t.\cos 2t]$$

$$= 0 - 0 + 2\sin 2t.\sin t + 4.\cos 2t.\cos t.$$

$$W(S) = 2\sin 2t.\sin t + 4\cos 2t.\cos t$$

For linear dependancy, solve for $W(S) = 0$

$$2\sin 2t.\sin t + 4\cos 2t.\cos t = 0$$

$$2[2\sin t.\cos t].\sin t + 4\cos 2t.\cos t = 0$$

$$4\sin^2 t.\cos t + 4\cos 2t.\cos t = 0$$

$$\cos t\,[4\sin^2 t + 4[\cos^2 t - \sin^2 t]] = 0$$

$$\cos t\,[\cancel{4\sin^2 t} + 4\cos^2 t - \cancel{4\sin^2 t}] = 0$$

$$4\cos^3 t = 0$$

$\therefore$ $W(S) = 0$ for $t = \pm\left(\frac{\pi}{2}(2n+1)\right)$

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⑥ $S = \{e^{3t}, e^{-2t}, e^{5t}, te^{5t}\}$

$$\begin{vmatrix} e^{3t} & e^{-2t} & e^{5t} & te^{5t} \\ 3e^{3t} & -2e^{-2t} & 5e^{5t} & \cancel{5xe^{5t}+e^{5t}} \\ 9e^{3t} & 4e^{-2t} & 25e^{5t} & \\ 27e^{3t} & -8e^{-2t} & 125e^{5t} & \end{vmatrix}$$

$$\begin{vmatrix} e^{3t} & e^{-2t} & e^{5t} & te^{5t} \\ 3e^{3t} & -2e^{-2t} & 5e^{5t} & 5t\cdot e^{5t}+e^{5t} \\ 9e^{3t} & 4e^{-2t} & 25e^{5t} & 25t\cdot e^{5t}+10e^{5t} \\ 27e^{3t} & -8e^{-2t} & 125e^{5t} & 125te^{5t}+75e^{5t} \end{vmatrix}$$

$$W(S) = e^{3t}\begin{vmatrix} -2e^{-2t} & 5e^{5t} & 5te^{5t}+e^{5t} \\ 4e^{-2t} & 25e^{5t} & 25te^{5t}+10e^{5t} \\ -8e^{-2t} & 125e^{5t} & 125te^{5t}+75e^{5t} \end{vmatrix} -$$

$$e^{-2t}\begin{vmatrix} 3e^{3t} & e^{5t} & te^{5t} \\ 9e^{3t} & 25e^{5t} & 5te^{5t}+e^{5t} \\ 27e^{3t} & 125e^{5t} & 125te^{5t}+75e^{5t} \end{vmatrix} +$$

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$\underbrace{5t \cdot e^{5t}} + e^{5t}$

$5t.5e^{5t} + e^{5t}.5$

25

$25t.5e^{5t} + e^{5t}.25 + 5e^{5t} + e^{5t}$

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$$e^{5t}\begin{vmatrix} 3e^{3t} & -2e^{-2t} & 5te^{5t}+e^{5t} \\ 9e^{3t} & 4e^{-2t} & 25te^{5t}+10e^{5t} \\ 27e^{3t} & -8e^{-2t} & 125te^{5t}+75e^{5t} \end{vmatrix} -$$

$$te^{5t}\begin{vmatrix} 3e^{3t} & -2e^{-2t} & 5e^{5t} \\ 9e^{3t} & 4e^{-2t} & 25e^{5t} \\ 27e^{3t} & -8e^{-2t} & 125e^{5t} \end{vmatrix}$$

$W(S) = -980e^{11t} \neq 0$

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Q9 $S = \{ e^{-t}, e^{3t}, te^{3t} \}$

$$y''' - 5y'' + 3y' + 9y = 0$$

For $y_1(t)$

$$y_1(t) = e^{-t}$$
$$y_1'(t) = -e^{-t}$$
$$y_1''(t) = e^{-t}$$
$$y_1'''(t) = -e^{-t}$$

$$\therefore (-e^{-t}) - 5(e^{-t}) + 3(-e^{-t}) + 9(e^{-t}) = 0$$
$$-e^{-t} - 5e^{-t} - 3e^{-t} + 9e^{-t} = 0$$
$$\cancel{-9e^{-t}} + \cancel{9e^{-t}} = 0$$
$$0 = 0$$

Verified

For $y_2(t)$

$$y_2(t) = e^{3t}$$
$$y_2'(t) = 3e^{3t}$$
$$y_2''(t) = 9e^{3t}$$
$$y_2'''(t) = 27e^{3t}$$

$$\therefore (27e^{3t}) - 5(9e^{3t}) + 3(3e^{3t}) + 9(e^{3t}) =$$
$$27e^{3t} - 45e^{3t} + 9e^{3t} + 9e^{3t} = 0$$

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$\cancel{45e^{3t}} - \cancel{45e^{3t}} = 0$

$0 = 0$

Verified

For $y_3(t)$.

$y_3(t) = te^{3t}$

$y_3'(t) = t \cdot 3e^{3t} + e^{3t} = \boxed{3te^{3t} + e^{3t}}$

$y_3''(t) = 3t \cdot 3e^{3t} + e^{3t} \cdot 3 + 3e^{3t}$

$y_3''(t) = \boxed{9t \cdot e^{3t} + 6e^{3t}}$

$y_3'''(t) = 9t \cdot 3e^{3t} + e^{3t} \cdot 9 + 18e^{3t}$

$y_3'''(t) = \boxed{27te^{3t} + 27e^{3t}}$

$\therefore (27te^{3t} + 27e^{3t}) - 5(9te^{3t} + 6e^{3t}) + 3(3te^{3t} + e^{3t})$
$+ 9(t \cdot e^{3t}) = 0$

$27te^{3t} + 27e^{3t} - 45te^{3t} - 30e^{3t} + 9te^{3t} + 3e^{3t}$
$+ 9te^{3t} = 0$

$45t\cancel{e^{3t}} - 45t\cancel{e^{3t}} + \cancel{30e^{3t}} - \cancel{30e^{3t}} = 0$

$0 = 0$

Verified