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EXERCISE 4.4

(11) $y''' - 10y'' + 25y' = 0$

Ans $r^3 - 10r^2 + 25r = 0$

$r(r^2 - 10r + 25) = 0$

$r((r-5)^2) = 0$

$r = 0, \ (r-5)^2 = 0$

$r = 0, \ r = 5$ (Multiplicity of 2)

It's general Sol. is

$y(t) = c_1 e^{0t} + c_2 e^{5t} + c_2 \cdot t e^{5t}$

$y(t) = c_1 + c_2 e^{5t} + c_2 t e^{5t}$

12) $8y''' + y'' = 0$

Ans $8r^3 + r^2 = 0$

$r^2(8r + 1) = 0$

$r^2 = 0, \quad 8r + 1 = 0$

$r = 0$ (Multiplicity of 2), $r = -1/8$

$\therefore y(t) = c_1 e^{0t} + c_2 t e^{0t} + c_3 e^{-1/8 t}$

OR

$$y(t) = c_1 e^{-1/8t} + c_2 e^{0t} + c_3 t e^{0t}$$

$$y(t) = c_1 e^{-1/8t} + c_2 + c_3 t$$

(13) $y''' + 7y'' + 17y' + 15y = 0$

$$r^3 + 7r^2 + 17r + 15 = 0 \quad \text{—(i)}$$

Put $r = -3$

$$(-3)^3 + 7(-3)^2 + 17(-3) + 15 = 0$$

$$-27 + 63 - 51 + 15 = 0$$

$$-78 + 78 = 0$$

$$0 = 0$$

$\therefore$ $r+3$ is one of the factor of eq (i)

Calculating other two factors by synthetic division.

	1	7	17	15
-3		-3	-12	-15
	1	4	5	0

$$r^2 + 4r + 5 = 0$$

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$$r = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$r = \frac{-4 \pm \sqrt{16-20}}{2(1)}$$

$$r = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2}$$

$$r = -2 \pm i$$

$$\alpha = -2, \quad \beta = 1$$

Gen. Sol. B

$$y(t) = c_1 e^{-3t} + e^{-2t}\left[\cos 1t + \sin 1t\right]$$

EXERCISE 4.6

(6) $y'' + 4y = \sec 2t$

$$y(t) = y_h(t) + y_p(t) \longleftarrow (1)$$

General Solution:-

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

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$y_h(t) = e^{0}[\cancel{c_1} \cos 2t c_1 + \sin 2t c_2]$

$y_h(t) = c_1 \cos 2t + c_2 \sin 2t$

$S = \{\cos 2t, \sin 2t\}$

Particular Solution :-

$y_p(t) = y_1(t)\, u_1(t) + y_2(t) \cdot u_2(t)$

$u_1(t) = -\int \frac{\sin 2t \cdot \sec 2t}{W(S)}$ —(ii)

$W(S) = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$

$W(S) = (\cos 2t)(2\cos 2t) - (\sin 2t)(-2\sin 2t)$

$W(S) = 2\cos^2 2t + 2\sin^2 2t$

$W(S) = 2[1] = 2 \neq 0$

Put in (ii)

$u_1(t) = -\int \frac{\sin 2t}{2} \frac{1}{\cos 2t} = \frac{-1}{2}\int \frac{\sin 2t}{\cos 2t} \times \frac{2}{2}$

$u_1(t) = \frac{1}{4}\int \frac{-2\sin 2t}{\cos 2t} = \frac{1}{4} \ln|\cos 2t|$

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$$u_2(t) = \int \frac{\cancel{\cos 2t} \cdot \cancel{\sec 2t}}{2}$$

$$u_2(t) = \int \frac{1}{2} = \frac{1}{2}t$$

$$y_p(t) = (\cos 2t) \cdot \frac{1}{4} \ln|\cos 2t| + \frac{1}{2} t \cdot \sin 2t$$

Gen. Sol:-

$$y(t) = c_1 \cos 2t + c_2 \sin 2t + \frac{1}{4} \cos 2t \cdot \ln|\cos 2t| + \frac{1}{2}$$

4. $y'' - 4y' - 5y = 8e^{2t} \cdot \cos 3t$

Ans:- General Sol.

$$y(t) = y_h(t) + y_p(t) \quad \text{—— (i)}$$

Finding $y_h(t)$:-

$$y'' - 4y' - 5y = 0$$

$$r^2 - 4r - 5 = 0$$

$$r = \frac{-(-4) \pm \sqrt{-4^2 - 4(1)(-5)}}{2(1)}$$

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$$r = \frac{4 \pm \sqrt{16+20}}{2}$$

$$r = \frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2} = \frac{\cancel{2}(2 \pm 3)}{\cancel{2}}$$

$r = 2 \pm 3$

$r_1 = 2+3, \quad r_2 = 2-3$

$r_1 = 5, \quad r_2 = -1$

$y_h(t) = c_1 e^{5t} + c_2 e^{-t}$

$S = \{e^{5t}, e^{-t}\}$

Finding $y_p(t)$:-

$y_p(t) = y_1 u_1 + y_2 u_2$

$$u_1 = -\int \frac{8e^{2t} \cdot \cos 3t \cdot e^{-t}}{W(S)} \quad \text{—(ii)}$$

$$W(S) = \begin{vmatrix} e^{5t} & e^{-t} \\ 5e^{5t} & -e^{-t} \end{vmatrix}$$

$W(S) = (e^{5t})(-e^{-t}) - (e^{-t})(5e^{5t})$

$W(S) = -e^{4t} - 5e^{4t} = -6e^{4t}$

Put in (ii)

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$$u_1 = -\int \frac{8e^{2t} \cdot \cos 3t \cdot e^{-t}}{-6e^{4t}}$$

$$u_1 = -\int \frac{8e^{t} \cos 3t}{-6e^{4t}}$$

$$u_1 = -\frac{8}{-6}\int e^{-3t} \cdot \cos 3t.$$

$$u_1 = \frac{\overset{2}{\cancel{4}}}{3}\left\{ \frac{e^{-3t}(\sin 3t - \cos 3t)}{\cancel{6}\,3} \right\}$$

$$\boxed{u_1 = \frac{2}{9}\left[e^{-3t}(\sin 3t - \cos 3t)\right]}$$

$$u_2 = \int \frac{8e^{2t} \cdot \cos 3t \cdot e^{5t}}{-6e^{4t}}$$

$$u_2 = \frac{8}{-6}\int e^{7t-4t} \cos 3t$$

$$u_2 = -\frac{4}{3}\int e^{3t} \cdot \cos 3t$$

$$u_2 = -\frac{\overset{2}{\cancel{4}}}{3}\left[\frac{e^{3t}(\sin 3t + \cos 3t)}{\cancel{6}\,3} \right]$$

$$u_2 = -\frac{2}{9}\left[e^{3t}(\sin 3t + \cos 3t)\right]$$

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$$y_p(t) = e^{5t}\left\{\frac{2}{9}\left[e^{-3t}(\sin 3t - \cos 3t)\right]\right\} + e^{-t}\left\{-\frac{2}{9}\left[e^{3t}(\sin 3t + \cos 3t)\right]\right\}$$

$$y_p(t) = \frac{2}{9} \cdot e^{2t}(\sin 3t - \cos 3t) + -\frac{2}{9} e^{2t}(\sin 3t + \cos 3t)$$

$$\text{''} = \frac{2}{9} e^{2t}\left[\cancel{\sin 3t} - \cos 3t - \cancel{\sin 3t} - \cos 3t\right]$$

$$\text{''} = \frac{2}{9} e^{2t}(-2\cos 3t)$$

$$\text{''} = -\frac{4}{9} e^{2t} \cos 3t$$

eq (5)(i) $\rightarrow$

$$\boxed{y(t) = c_1 e^{5t} + c_2 e^{-t} - \frac{4}{9} e^{2t} \cos(3t)}$$

(5) $y'' + 4y' + 20y = 2t\,e^{-2t}$

Ans:

$y_h(t) = ?$

$y'' + 4y' + 20y = 0$

$r^2 + 4r + 2\ldots$