(39) $y'' + y = \tan t$ $y(0) = 2$, $y'(0) = 1$

Ans: $y_h(t) = ?$

$$y'' + y = 0$$

$$r^2 + 1 = 0$$

$$r^2 = -1$$

$$r = \pm\sqrt{-1}$$

$$r = \pm i$$

$$\alpha = 0, \quad \beta = 1$$

$$\therefore y_h(t) = e^{\alpha t}\left[c_1 \cos\beta t + c_2 \sin\beta t\right]$$

$$y_h(t) = e^{0t}\left[c_1 \cos t + c_2 \sin t\right]$$

$$y_h(t) = c_1 \cos t + c_2 \sin t$$

$y_p(t) = ?$

$$y_p(t) = u_1(t) \cdot y_1(t) + u_2(t) \cdot y_2(t)$$

$$u_1(t) = -\int \frac{\tan t \cdot \sin t}{W(s)}$$

$$\boxed{W(s) = 1}$$

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$$u_1(t) = -\int \frac{\frac{\sin t}{\cos t} \cdot \sin t}{(1)}$$

$$u_1(t) = -\int \frac{\sin^2 t}{\cos t}$$

$$u_1(t) = -\int \frac{1-\cos^2 t}{\cos t}$$

$$u_1(t) = -\int \frac{1}{\cos t} - \cos t$$

$$u_1(t) = -\left[\int \frac{1}{\cos t} - \int \cos t\right]$$

$$u_1(t) = -\left[\ln(\tan t + \sec t) - \sin t\right]$$

$$u_2 = \int \frac{\tan t \cdot \cos t}{(1)} = \int \frac{\sin t}{\cancel{\cos t}} \cdot \cancel{\cos t}$$

$$u_2 = -\cos t$$

$$\therefore y_p(t) = -\left[\ln|\tan t + \sec t| - \sin t\right]\cos t + \left[-\cos t\right] \cdot \sin t$$

$$y_p(t) = -\ln(\tan t + \sec t)\cos t + \cancel{\sin t \cos t} - \cancel{\cos t \cdot \sin t}$$

$$y_p(t) = \boxed{-\ln(\tan t + \sec t)\cos t}$$

$$y(t) = c_1 \cos t + c_2 \sin t - \cos t\left(\ln(\tan t + \sec t)\right)$$

$$- \cancel{\ldots}\, \cos t$$

$$y'(t) = -c_1 \sin t + c_2 \cos t - \Big[\cos t \frac{d}{dt}\{\ln(\tan t + \sec$$

$$+ \ln(\tan t + \sec t) \cdot -\sin t\Big]$$

$$y'(t) = -c_1 \sin t + c_2 \cos t - \Big[\cos t \cdot \frac{1}{\tan t + \sec t} \cdot \sec^2 t + \tan$$

$$- \sin t \cdot \ln(\tan t + \sec t)\Big]$$

$$y'(t) = -c_1 \sin t + c_2 \cos t - \Big[\frac{\sec t + \tan t}{\sec t + \tan t} \cancel{+ \tan t \ldots}$$

$$- \sin t \cdot \ln(\tan t + \sec t)\Big]$$

$$y'(t) = -c_1 \sin t + c_2 \cos t - \Big\{1 \cancel{\ldots}\ \sin t \cdot \ln(\tan$$

Now

$$\ldots - \cos(0)\Big[\ln(\tan 0 + \sec$$

$y(0) = C_1 + 0 - 1\left[\ln(0+1)\right]$

$2 = C_1 - 1\left[\ln(1)\right]$

$2 = C_1 - 0$

$\boxed{C_1 = 2}$

$\cancel{y'(0) = -C_1 \sin(0) + C_2 \cos 0 - \left[1 - \sin(0) \ln(\tan 0 + \sec 0)\right] + 2}$

$\cancel{1 = 0 + C_2(1) - \left[1 - 0\right] + 2}$

$\cancel{1 = C_2 - 1 + 2}$

$\cancel{1 = C_2 + 1}$

$\cancel{\boxed{C_2 = 0}}$

Now $y'(0) = -C_1 \sin(0) + C_2 \cos 0 - \left[1 - \sin 0 \left(\ln(\tan 0 + \sec 0)\right)\right]$

$1 = -0 + C_2 - \left[1 - 0(0+1)\right]$

$1 = C_2 - \left[1 - 0\right]$

$1 = C_2 - 1$

$C_2 = 1 + 1$

$\boxed{C_2 = 2}$

$y(t) = 2\cos t + 2\sin t - \cos t\left(\ln(\tan t + \sec t\right)$

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