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EXERCISE 4.1

Q1: $S = \{t, 4t-1\}$

$$W(S) = \begin{vmatrix} t & 4t-1 \\ 1 & 4 \end{vmatrix} = 4t-(4t-1)$$

$$W(S) = \cancel{4t} - \cancel{4t} + 1 = \boxed{+1}$$

Linearly Independent

Q2:- $S = \{t, e^t\}$

$$W(S) = \begin{vmatrix} t & e^t \\ 1 & e^t \end{vmatrix} = (t)(e^t) - (e^t)$$

$$W(S) = e^t(t-1)$$

Linearly independent for $\mathbb{R} - \{1\}$

Q3:- $S = \{e^{-6t}, e^{-4t}\}$

$$W(S) = \begin{vmatrix} e^{-6t} & e^{-4t} \\ -6e^{-6t} & -4e^{-4t} \end{vmatrix} = (e^{-6t})(-4e^{-4t}) - (e^{-4t})(-6e^{-6t})$$

$$W(S) = -4e^{-10t} + 6e^{-10t}$$

$$W(S) = 2e^{-10t}$$

Linearly independent

Q4: $S = \{\cos 2t, \sin 2t\}$

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$$W(s) = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$$

$W(s) = [2\cos 2t . \cos 2t] - [(\sin 2t)(-2\sin 2t)]$

$W(s) = 2\cos^2 2t + 2\sin^2 2t$

$W(s) = 2[\cos^2 2t + \sin^2 2t]$

$W(s) = 2[1] = 2$

Linearly Independent.

(5) $S = \{ e^{-3t}\cos 3t, e^{-3t}\sin 3t \}$

$$W(s) = \begin{vmatrix} e^{-3t} . \cos 3t & e^{-3t} \sin 3t \\ -3e^{-3t}(\sin 3t + \cos 3t) & 3e^{-3t}(\cos 3t - \sin 3t) \end{vmatrix}$$

$W(s) = [(e^{-3t} . \cos 3t)(3e^{-3t}(\cos 3t - \sin 3t))] - [(e^{-3t} . \sin 3t)(-3e^{-3t}(\sin 3t + \cos 3t))]$

$W(s) = [3e^{-6t}\cos 3t(\cos 3t - \sin 3t)] - [-3e^{-6t}\sin 3t(\sin 3t + \cos$

$W(s) = 3e^{-6t}.[\cos^2 3t - \sin 3t \cos 3t] + 3e^{-6t}[\sin^2 3t + \sin 3t \cos 3t]$

$W(s) = 3e^{-6t}[\cos^2 3t - \cancel{\sin 3t \cos 3t} + \sin^2 3t + \sin 3t \cos$

$3e^{-3t}$ ~~$\cos 3t - 3\cos 3t e$~~

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$$W(S) = 3e^{-6t}\left[\cos^2 3t + \sin^2 3t\right]$$

$$W(S) = 3e^{-6t}\left[1\right]$$

$$W(S) = 3e^{-6t}$$

Linearly independant for all t.

(6) $S = \{ e^{5t} \cdot \sin 4t, \; e^{5t} \sin 4t \}$

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(34) $t^2y'' - ty' + y = 0$, $y_1(t) = t$

$y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0$

$p(t) = -1/t$; $q(t) = 1/t^2$

$y_2(t) = f_1(t)\int \frac{e^{-\int p(t)\,dt}}{[f_1(t)]^2}$

$y_2(t) = t\int \frac{e^{-\int -1/t}}{(t)^2}$

$y_2(t) = t\int \frac{1}{t^2 \cdot e^{\int 1/t}}\,dt$

$y_2(t) = t\int \frac{1}{e^{\ln(t)} \cdot t^2} = t\int \frac{1}{t^3}$

$y_2(t) = t\int t^{-3}\,dt = t \cdot \frac{t^{-3+1}}{-3+1} = t \cdot \frac{t^{-2}}{-2}$

$y_2(t) = t \cdot \frac{1}{-2t^2} = -\frac{1}{2t}$

$y_2(t) = t\int \frac{e^{\int 1/t}}{(t)^2} = t\int \frac{e^{\ln t}}{t^2}$

$-\frac{1}{2} - \frac{1}{2}$

$-\frac{2}{2}$

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$$y_2(t) = t \int \frac{t}{t^2} = t \int \frac{1}{t} = t \ln t$$

$$W(s) = \begin{vmatrix} t & t \ln t \\ 1 & 1 + \ln t \end{vmatrix} = (t)(1+\ln t) - (t \ln t)(1)$$

$$= t + \cancel{t \ln t} - \cancel{t \ln t}$$

$$= t$$

Linearly independant for all t except at $t = 0$

Example

$$\frac{4t^2 y'' + 8t y' + y = 0}{4t^2} \quad , \quad y_1(t) = t^{-1/2}$$

$$y'' + \frac{2}{t} y' + \frac{1}{4t^2} y = 0$$

$$y_2(t) = t^{-1/2} \int \frac{e^{-\int 2/t \, dt}}{(t^{-1/2})^2}$$

$$y_2(t) = t^{-1/2} \int \frac{e^{-2\ln(t)}}{t^{-1}} = t^{-1/2} \int \frac{t}{e^{\ln(t)^2}}$$

$$y_2(t) = t^{-1/2} \int \frac{t}{t^2} = \frac{1}{\,} \int \frac{1}{\,} = \frac{1}{\,} \ln(t)$$

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(2) $y'' - 4y' - 12y = 0$

$$r^2 - 4r - 12 = 0$$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$

$$r = \frac{4 \pm \sqrt{16+48}}{2} = \frac{4 \pm \sqrt{64}}{2}$$

$$r = \frac{4 \pm 8}{2}$$

$$r_1 = \frac{4+8}{2}, \quad r_2 = \frac{4-8}{2}$$

$$r_1 = 6, \quad r_2 = -2$$

$$y(t) = c_1 e^{6t} + c_2 e^{-2t}$$

$$\cancel{S = \{c_1 e^{6t}, c_2 e^{-2t}\}}$$

(6) $4y'' + y' - 72y = 0$

$$4r^2 + r - 72 = 0$$

$$r = \frac{-1 \pm \sqrt{1^2 - 4(4)(-72)}}{2(4)} = -1 \pm \sqrt{1}$$

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$$r = \frac{-1 \pm \sqrt{1+1152}}{8} = \frac{-1 \pm \sqrt{1153}}{8}$$

$$r_1 = \frac{-1 + \sqrt{1153}}{8}, \quad r_2 = \frac{-1 - \sqrt{1153}}{8}$$

$$y(t) = c_1 e^{\frac{-1+\sqrt{1153}}{8}t} + c_2 e^{\frac{-1-\sqrt{1153}}{8}}$$

(10) $y'' - 8y' + 20y = 0$

$$r^2 - 8r + 20 = 0$$

$$r = \frac{-(-8) \pm \sqrt{-8^2 - 4(1)(20)}}{2(1)}$$

$$r = \frac{8 \pm \sqrt{64 - 80}}{2} = \frac{8 \pm \sqrt{-16}}{2}$$

$$r = \frac{8 \pm 4i}{2} = \frac{\cancel{2}(4 \pm 2i)}{\cancel{2}}$$

$$r = 4 \pm 2i$$

$$\alpha = 4, \ \beta = 2$$

$$y(t) = e^{4t}\left(c_1 \cos 2t + c_2 \sin 2t\right)$$