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Exercise 4.7

(22) $x^2y'' - 5xy' + 9y = x^3$

$$x^2\left[\frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right] - 5x\left[\frac{1}{x}\frac{dy}{dt}\right] + 9y = e^{3t}$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 5\frac{dy}{dt} + 9y = e^{3t}$$

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = e^{3t}$$

$y_h(t) = ?$

$$r^2 - 6r + 9 = 0$$

$$r^2 - 3r - 3r + 9 = 0$$

$$r(r-3) - 3(r-3) = 0$$

$$(r-3)(r-3) = 0$$

$$r = r_1 = r_2 = 3$$

$$\therefore \quad y_h(t) = c_1e^{3t} + c_2te^{3t}$$

$$S = \{e^{3t}\}$$

$$S' = \{te^{3t}\}$$

$$S'' = \{t^2e^{3t}\}$$

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$e^{mt}(t^n . m + n t^{n-1})$ $t^n e^{mt}$

$t^n . m e^{mt} + n t^{n-1} . e^{mt}$

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$\therefore y_p(t) = At^2 e^{3t}$

$y_p'(t) = A[t^2 . 3e^{3t} + e^{3t} . 2t]$

$y_p'(t) = A[3t^2 e^{3t} + 2t e^{3t}]$

$y_p''(t) = A[3\{3t^2 e^{3t} + e^{3t} . 2t\} + 2\{t . 3e^{3t} + e^{3t} . 2\}]$

$y_p''(t) = A[9t^2 e^{3t} + 2t . e^{3t} + 2t . e^{3t} + 2e^{3t}]$

$y_p''(t) = A[9t^2 e^{3t} + 4t e^{3t} + 2e^{3t}]$

$\cancel{e^{3t}} A[9t^2 + 4t + 2] - 6[\cancel{e^{3t}} A[3t^2 \cancel{e^{3t}} + 2t \cancel{e^{3t}}]] +$

$9[At^2 \cancel{e^{3t}}] = \cancel{e^{3t}}$

e^{3t} $\cancel{9At^2} + 4At + 2A - \cancel{18At^2} - 12At + \cancel{9At^2} = 0$

$-8At + 2A = 1$

$2A = 1$

$A = 1/2$

$\therefore y_p(t) = \frac{1}{2} t^2 e^{3t}$

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$$y(t) = c_1 e^{3t} + c_2 e^{3t} + \frac{1}{2} t^2 e^{3t}$$

Now put $t = \ln x$

$$y(t) = c_1 e^{3\ln x} + c_2 e^{3\ln x} + \frac{1}{2} (\ln x)^2 e^{3\ln x}$$

$$y(t) = c_1 x^3 + c_2 x^3 + \frac{1}{2} (\ln x)^2 x^3$$

(25) $x^2 y'' + 2xy' - 6y = 2x$

$x = e^t$

$$\cancel{x^2}\left[\frac{1}{\cancel{x^2}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right] + 2\cancel{x}\left[\frac{1}{\cancel{x}} \cdot \frac{dy}{dt}\right] - 6y = 2e^t$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 2\frac{dy}{dt} - 6y = 2e^t$$

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 2e^t$$

$$y'' + y' - 6y = 2e^t$$

$y_h(t) = ?$

$$y'' + y' - 6y = 0$$

$$r^2 + r - 6 = 0$$

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$$r^2 - 2r + 3r - 6 = 0$$

$$r(r-2) + 3(r-2) = 0$$

$$(r-2)(r+3) = 0$$

$$r-2 = 0, \quad r+3 = 0$$

$$r = +2, \quad r = -3$$

$$y_h(t) = c_1 e^{+2t} + c_2 e^{-3t}$$

$y_p(t) = ?$

Suppose $y_p(t) = A e^t$

$$\therefore (Ae^t)'' + (Ae^t)' - 6(Ae^t) = 2e^t$$

$$Ae^t + Ae^t - 6Ae^t = 2e^t$$

$$-4Ae^t = 2e^t$$

$$-4A = 2$$

$$A = -\frac{1}{2}$$

$$\therefore y_p(t) = -\frac{1}{2} e^t$$

$$\therefore y(t) = c_1 e^{+2t} + c_2 e^{-3t} - \frac{1}{2} e^t$$

$$t = \ln x$$

$$y(x) = c_1 e^{+2\ln x} + c_2 e^{-3\ln x} - \frac{1}{2} e^{\ln}$$

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$y(x) = c_1 x^{+2} + c_2 x^{-3} - \frac{1}{2} x$

(26) $x^2 y'' + \cancel{2}xy' \cancel{+}\cancel{y} - 16y = \ln x$

Ans:=

$\cancel{x^2}\left[\frac{1}{\cancel{x^2}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right] + x\left[\frac{1}{x}\frac{dy}{dt}\right] - 16y =$

$\cancel{\ln}(\cancel{e}^t)$

$\frac{d^2y}{dt^2} - \cancel{\frac{dy}{dt}} + \cancel{\frac{dy}{dt}} - 16y = t$

$y'' - 16y = t$

$y_h(t) = ?$

$r^2 - 16 = 0$

$r^2 = 16$

$r = \pm 4$

$\therefore y_h(t) = c_1 e^{-4t} + c_2 e^{4t}$

$y_p(t) = ?$

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$$y_p(t) = At^2 + Bt + C$$

$$y_p'(t) = 2At + B$$

$$y_p''(t) = 2A$$

$$(2A) - 16(At^2 + Bt + C) = t$$

$$2A - 16At^2 - 16Bt - 16C = t$$

$$-16At^2 - 16Bt - 16C + 2A = t$$

$$-16A = 0, \quad -16B = 1, \quad -16C + 2A = 0$$

$$\boxed{A = 0}, \quad \boxed{B = -\tfrac{1}{16}}, \quad \boxed{C = 0}$$

$$\therefore \quad y_p(t) = -\tfrac{1}{16}t$$

Now $y(t) = c_1 e^{-4t} + c_2 e^{4t} - \frac{1}{16}t$

$$y(x) = c_1 e^{-4\ln x} + c_2 e^{4\ln x} - \tfrac{1}{16}\ln(x)$$

$$y(x) = c_1 x^{-4} + c_2 x^4 - \frac{\ln(x)}{16}$$

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(28) $x^2y'' + xy' + 36y = x^2$

$$x^2\left[\frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right] + x\left[\frac{1}{x}\frac{dy}{dt}\right] + 36y = e^{2t}$$

$$\frac{d^2y}{dt^2} - \cancel{\frac{dy}{dt}} + \cancel{\frac{dy}{dt}} + 36y = e^{2t}$$

$$y'' + 36y = e^{2t}$$

$y_h(t) = ?$

$$y'' + 36y = 0$$

$$r^2 + 36 = 0$$

$$r^2 = -36$$

$$r = \pm\sqrt{-36}$$

$$r = \pm 6i$$

$$\alpha = 0, \quad \beta = 6$$

$\therefore$ $y_h(t) = e^{0t}\left[c_1 \cos 6t + c_2 \sin 6t\right]$

$$y_h(t) = c_1 \cos 6t + c_2 \sin 6t$$

$y_p(t) = ?$

$$y_p(t) = Ae^{2t}$$

$$y_p'(t) = 2Ae^{2t}$$

$$y_p''(t) = 4Ae^{2t}$$

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$$\therefore \left(4Ae^{2t}\right) + 36\left(Ae^{2t}\right) = e^{2t}$$

$$40A\, e^{2t} = e^{2t}$$

$$40A = 1$$

$$\boxed{A = 1/40}$$

$$y_p(t) = \frac{1}{40} e^{2t}$$

$$\therefore y(t) = c_1 \cos 6t + c_2 \sin 6t + \frac{1}{40} e^{2t}$$

$$y(x) = c_1 \cos(6(\ln x)) + c_2 \sin(6(\ln x)) + \frac{1}{40} x^2$$

41 $\quad 4x^2 y'' + y = x^3 \quad , y(1) = 1, y'(1) = 1$

$$4\cancel{x^2}\left[\frac{1}{\cancel{x^2}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right] + y = e^{3t}$$

$$4\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + y = e^{3t}$$

$$4y'' - 4y' + y = e^{3t}$$

$y_h(t) = ?$

$$4y'' - 4y' + y = 0$$

$$4r^2 - 4r + 1 = 0$$

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$$r = r_1 = r_2 = \frac{1}{2}$$

$$\therefore \; y_h(t) = c_1 e^{\frac{1}{2}t} + t c_2 e^{\frac{1}{2}t}$$

$\underline{y_p(t) = ?}$

$$y_p(t) = A e^{3t}$$

$$4(Ae^{3t})'' - 4(Ae^{3t})' + Ae^{3t} = e^{3t}$$

$$4(3Ae^{3t})' - 4(3Ae^{3t}) + Ae^{3t} = e^{3t}$$

$$4(9Ae^{3t}) - 12Ae^{3t} + Ae^{3t} = e^{3t}$$

$$36Ae^{3t} - 11Ae^{3t} = e^{3t}$$

$$25A\cancel{e^{3t}} = \cancel{e^{3t}}$$

$$25A = 1$$

$$A = \frac{1}{25}$$

$$\therefore \; y_p(t) = \frac{1}{25} e^{3t}$$

So $y(t) = c_1 e^{\frac{1}{2}t} + t c_2 e^{\frac{1}{2}t} + \frac{1}{25} e^{3t}$

Put $t = \ln x$

$$y(x) = c_1 e^{\frac{1}{2}\ln x} + (\ln x) c_2 e^{\frac{1}{2}\ln x} + \frac{1}{25} e^{3\ln x}$$

$$y(x) = c_1 e^{\ln x^{1/2}} + (\ln x) c_2 e^{\ln x^{1/2}} + \frac{1}{25} e^{\ln x^3}$$

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$$y(x) = c_1 x^{1/2} + (\ln x) c_2 x^{1/2} + \frac{1}{25} x^3$$

$$\cancel{y'(x) = c_1 \frac{1}{2} x^{1/2 - 1} + c_2 \left[\frac{1}{2} x^{1/2 - 1} + x^{1/2} \right] +}$$

$$\cancel{\frac{1}{25} 3x^2}$$

$$\cancel{y'(x) = \frac{1}{2} c_1 \cdot x^{-1/2} + c_2 \left[\frac{1}{2} x^{-1/2} + \right.}$$

$$y'(x) = \frac{1}{2} c_1 \cdot \frac{1}{\sqrt{x}} + c_2 \left[\ln x \cdot \frac{1}{2} \frac{1}{\sqrt{x}} + x^{1/2} \cdot \frac{1}{x} \right]$$

$$+ \frac{1}{25} 3x^2$$

$$y'(x) = \frac{c_1}{2\sqrt{x}} + c_2 \left[\frac{\ln x}{2\sqrt{x}} + x^{1/2 - 1} \right] + \frac{3x^2}{25}$$

$$y'(x) = \frac{c_1}{2\sqrt{x}} + c_2 \left[\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right] + \frac{3x^2}{25}$$

$$y'(x) = \frac{c_1}{2\sqrt{x}} + c_2 \left[\frac{\ln x + 2}{2\sqrt{x}} \right] + \frac{3x^2}{25}$$

Now $y(1) = c_1 (1)^{1/2} + \ln 1 . c_2 (1)^{1/2} + \frac{1}{25} (1)^3$

$$1 = c_1 + (0) c_2 (1) + 1/25$$

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$$1 = C_1 + 1/25$$

$$C_1 = \frac{1}{1} - 1/25$$

$$C_1 = \frac{25-1}{25} = \boxed{24/25}$$

$$y'(1) = \frac{C_1}{2\sqrt{1}} + C_2\left[\frac{\ln 1 + 2}{2(\sqrt{1})}\right] + \frac{3(1)^2}{25}$$

$$1 = \frac{C_1}{2} + C_2\left(\frac{0+2}{2}\right) + \frac{3}{25}$$

$$1 = \frac{C_1}{2} + C_2\left(\frac{\cancel{2}^1}{\cancel{2}1}\right) + \frac{3}{25}$$

$$1 = \frac{\cancel{24}^{12}}{25} \times \frac{1}{\cancel{2}_1} + C_2 + \frac{3}{25}$$

$$1 = \frac{\cancel{15}^3}{\cancel{25}_5} + C_2$$

$$C_2 = \frac{1}{1} - 3/5$$

$$C_2 = \frac{5-3}{5} = \boxed{2/5}$$

$$y(t) = \frac{24}{25}x^{1/2} + \frac{2}{5}\ln x \cdot x^{1/2} + 1/25\, x^3$$

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