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EXERCISE 2.2

(8) $\frac{dy}{dx} + y \cot x = \cos x$

Comparing with general form

$$\frac{dy}{dx} + P(x)y = q(x)$$

$$P(x) = \cot x \quad , \quad q(x) = \cos x$$

Its general solution is:

$$y e^{\int \cot x \, dx} = \int \left(\cos x \cdot e^{\int \cot x \, dx} \right) dx$$

$$y e^{\ln(\sin x)} = \int \left(\cos x \, e^{\ln(\sin x)} \right) dx$$

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(1) $\frac{dy}{dx} + \frac{1}{x} y = x$

Here $p(x) = 1/x$, $q(x) = x$

$y e^{\int 1/x \, dx} = \int (x \cdot e^{\int 1/x}) dx$

$y e^{\ln x} = \int (x \, e^{\ln x}) dx$

(4) $\frac{dy}{dx} + \frac{1}{x} y = x e^{-x}$

$P(x) = 1/x$, $q(x) = x e^{-x}$

$y e^{\int 1/x} = \int (1/x \cdot e^{\int x e^{-x}})$

$y e^{\ln x} = \int \left(\frac{1}{x} e^{e^{-x}(-1-x)} \right) dx$

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(7) $dy = \left(2x + \frac{xy}{x^2-1}\right) dx$

Ans: $\frac{dy}{dx} = 2x + \frac{xy}{x^2-1}$

$$\frac{dy}{dx} - \frac{xy}{x^2-1} = 2x$$

$$y e^{\int -\left(\frac{x}{x^2-1}\right)} = \int (2x)\left(e^{\int -\left(\frac{x}{x^2-1}\right) dx}\right)$$

$$y e^{-\int \frac{x}{x^2-1}} = \int 2x \left(e^{-\int \frac{x}{x^2-1} dx}\right) dx \quad \text{---(1)}$$

$$\int \frac{x}{x^2-1} = \frac{x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Rightarrow \frac{x}{(x+1)(x-1)} = \frac{A(x-1) + B(x+1)}{(x+1)(x-1)}$$

$$x = A(x-1) + B(x+1)$$

For $x = 1$	For $x = -1$
$1 = A(1-1) + B(1+1)$	$-1 = A(-1-1) + B(-1+1)$
$1 = 0 + 2B$	$-1 = A(-2) + 0$

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$\boxed{B = 1/2}$ | $\boxed{A = 1/2}$

$$\frac{x}{(x+1)(x-1)} = \frac{1/2}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$\frac{x}{(x+1)(x-1)} = \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{x-1}\right] \quad \text{— (ii)}$$

Putting (ii) in (i)

$$y e^{-\int \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{x-1}\right] dx} = \int 2x \,.\, e^{-\int \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{x-1}\right] dx}$$

$$y e^{-\frac{1}{2}\left[\ln|x+1| + \ln|x-1|\right]} = \int 2x \; e^{-\frac{1}{2}\left[\ln|x+1| + \ln|x-1|\right]}$$

$$y \cdot e^{-1/2 \ln (x+1)(x-1)} = \int 2x \,.\, e^{-1/2 \ln (x+1)(x-1)}$$

$$\frac{dy}{dx} - \frac{x}{x^2-1} y = 2x$$

$$y e^{-\int + \frac{x}{x^2-1}} = \int (2x)(e^{-\int \frac{x}{x^2-1} d})$$

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$$y e^{-\frac{1}{2}\int \frac{2x}{x^2-1}} = \int (2x)\left(e^{-\frac{1}{2}\int \frac{2x}{x^2-1}dx}\right)$$

$$y e^{-\frac{1}{2}\ln(x^2-1)} = \int (2x)\left(e^{-\frac{1}{2}\ln(x^2-1)}\right) dx$$

y

(7) $dy = \left(2x + \frac{xy}{x^2-1}\right) dx$

Ans. $dy = \left(\frac{2x^3 - 2x + xy}{x^2-1}\right) dx$

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(22) $\frac{dx}{dy} = \frac{3xy^2}{1-y^3}$

Ans:- $\frac{dx}{dy} - \frac{3xy^2}{1-y^3} = 0$

$$\frac{dx}{dy} - \frac{3y^2}{1-y^3}x = 0$$

$$P(y) = \frac{-3y^2}{1-y^3}, \quad q(y) = 0$$

$$x e^{\int \frac{-3y^2}{1-y^3} dy} = \int \left(0 \cdot e^{\int \frac{-3y^2}{1-y^3} dy}\right) dy$$

$$x e^{\ln(1-y^3)} = \int 0 \, dy$$

$$\frac{x \cancel{e^{\ln(1-y^3)}}}{\cancel{e^{\ln(1-y^3)}}} = \frac{C}{e^{\ln(1-y^3)}}$$

$$\boxed{x = \frac{C}{e^{\ln(1-y^3)}}}$$

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(25) $\frac{dy}{dx} - y = 4e^x$

$p(x) = -1, \quad q(x) = 4e^x$

General Solution.

$$y e^{\int(-1)dx} = \int (4e^x \cdot e^{\int(-1)dx})\, dx$$

$$y e^{-x} = \int (4\cancel{e^x} \cdot \cancel{e^{-x}})\, dx$$

$$y e^{-x} = \int (4)\, dx$$

$$y e^{-x} = 4x + c$$

$$\boxed{y = e^x (4x + c)} \text{ --- (i)}$$

$$y(0) = e^0 (4(0) + c)$$

$$y(0) = 1(0 + c)$$

But $y(0) = 4$

$$\therefore 4 = 1(c)$$

$$\boxed{c = 4}$$

Put in (i)

$$\boxed{y = e^x (4x + 4)}$$

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(26) $\frac{dy}{dx} + y = e^{-x}$ $y(0) = -1$

$P(x) = 1$, $q(x) = e^{-x}$

General Solution is:

$$y e^{\int 1 dx} = \int (e^{-x} \cdot e^{\int 1 dx}) dx$$

$$y e^{x} = \int (\cancel{e^{-x}} \cancel{e^{x}}) dx$$

$$y e^{x} = \int 1 dx$$

$$y e^{x} = x + c$$

$$\boxed{y = \frac{x + c}{e^{x}}} \quad \text{——(i)}$$

$$y(0) = \frac{0 + c}{e^{0}} = \frac{c}{1}$$

$y(0) = 0$

But $y(0) = -1$

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$x(1-x^2)$
$x(1+x)(1-x)$

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$\therefore -1 = c$

$\boxed{c = -1}$

let $u = 1-x^2$
$du = -2x\,dx$

Put in (1)

$$\boxed{y = \frac{x-1}{e^x}}$$

(27) $\frac{dy}{dx} + 3x^2y = e^{-x^3}$, $y(0) = 2$

$P(x) = 3x^2$, $q(x) = e^{-x^3}$

$$y e^{\int 3x^2 dx} = \int (e^{-x^3} \cdot e^{\int 3x^2 dx})\,dx$$

$$y e^{x^3} = \int (e^{-x^3} \cdot e^{x^3})\,dx$$

$$y e^{x^3} = \int e^{-x^3+x^3}\,dx$$

$\int e^0\,dx$

$$y e^{x^3} = \int 1\,dx$$

$$\frac{y \cancel{e^{x^3}}}{\cancel{e^{x^3}}} = \frac{x + c}{e^{x^3}}$$

$$y = \frac{x+c}{e^{x^3}} \quad \text{—— (1)}$$

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$$y(0) = \frac{0+c}{e^{(0)^3}}$$

$$y(0) = \frac{c}{1}$$

But $y(0) = 2$

$$2 = c/1$$

$$\boxed{c = 2}$$

Put in (i)

$$\boxed{y = \frac{x+2}{e^{x^3}}}$$

(33) $\frac{dx}{dt} = x + t + 1$ $\quad x(0) = 2$

$$\frac{dx}{dt} - x = t+1$$

Here $P(t) = -1$

$q(t) = t+1$

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$= \cancel{e^{-t}\frac{t^2}{2}} \frac{t^2}{2}$ $\quad I = t.\frac{e^{-t}}{-1} - \int \frac{e^{-t}}{-1}$

$I = -te^{-t} + e^{-t}$

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It's general Solution is:

$$x e^{\int -1\, dt} = \int \left[(t+1)\, e^{\int -1\, dt}\right] dt$$

$$x e^{-t} = \int \left[(t+1)\, e^{-t}\right] dt$$

$$x e^{-t} = \int (t+1)\, e^{-t}\, dt$$

$$x e^{-t} = \int \left[(e^{-t}\, t) + e^{-t}\right] dt$$

$$x e^{-t} = \int e^{-t}.t\, dt + \int e^{-t}\, dt$$

$$x e^{-t} = t.\frac{e^{-t}}{-1} - \int \frac{e^{-t}}{-1}(1) + \frac{e^{-t}}{-1} + C_1$$

$$x e^{-t} = -t.e^{-t} + \int e^{-t} - e^{-t} + C_1$$

$$x e^{-t} = -te^{-t} - e^{-t} - e^{-t} + C_1$$

$$x e^{-t} = e^{-t}\left[-t - 1 - 1\right] + C_1$$

$$\frac{x \cancel{e^{-t}}}{\cancel{e^{-t}}} = \frac{e^{-t}\left[-t-2\right] + C_1}{e^{-t}}$$

$$x = -t - 2 + C_1/e^{-t} = -t - 2 + C \quad (1)$$

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$x(0) = -0 - 2 + C$

$2 = -2 + C$

$\boxed{C = 4}$

Put in ①

$\boxed{x = -t - 2 + 4}$

(34) $\frac{d\theta}{dt} = e^{2t} + 2\theta \qquad \theta(0) = 0$

$\frac{d\theta}{dt} - 2\theta = e^{2t}$

$p(t) = -2, \quad q(t) = e^{2t}$

General sol—

$\theta \cdot e^{\int -2\,dt} = \int (e^{2t})(e^{\int -2dt})\,dt$

$\theta \cdot e^{-2t} = \int [\cancel{e^{2t}} \cdot \cancel{e^{-2t}}]\,dt$

$\theta \cdot e^{-2t} = t + C_1$

$\theta = e^{2t}\,t + e^{2t} C_1$ — (1)

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$$\theta(0) = e^{2(0)} + e^{2(0)} C_1$$

$$0 = e^{0} + e^{0} c_1$$

$$0 = 1 + (1) C_1$$

$$\boxed{C_1 = 0}$$

Put in (i)

$$\theta = e^{2t} + e^{2t}(0)$$

$$\boxed{\theta = e^{2t}}$$

Skipping Method of Undetermined Coefficients

Substitution Methods And Special Equations.

$$\frac{dy}{dt} - ky = -ay^2 \quad \text{—(1)}$$

$$n = 2$$

Put $\omega = y^{1-2} = y^{-1}$

Now Differentiate w.r.t t.

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$$\frac{dw}{dt} = -y^{-2}\frac{dy}{dt}$$

$$\boxed{\frac{dw}{dt}\cdot -y^{2} = \frac{dy}{dt}}$$ Put in (1)

$$\frac{dw}{dt}\cdot -y^{2} - Ky = -ay^{2}$$

$$-\frac{dw}{dt}y^{2} - Ky = -ay^{2}$$

Dividing b.s by y^2

$$-\frac{dw}{dt} - \frac{K}{y} = -a$$

$$-\frac{dw}{dt} - Ky^{-1} = -a$$

Putting $y^{-1} = w$

$$-\frac{dw}{dt} - Kw = -a$$

$$\cancel{-}\left(\frac{dw}{dt} + Kw\right) = \cancel{-}a$$

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$$\frac{dw}{dt} + kw = a$$

It's general Solution is:

$$w\,e^{\int k\,dt} = \int \left(a.\,e^{\int k\,dt}\right) dt$$

$$w.\,e^{kt} = \int \left(a.\,e^{kt}\right) dt$$

$$w\,e^{kt} = a \int e^{kt}\,dt$$

$$w.e^{kt} = a.\frac{e^{kt}}{k} + c$$

Put $w = 1/y$ back

$$\frac{1}{\cancel{e^{kt}}}\left(\frac{1}{y}\,\cancel{e^{kt}}\right) = \left(\frac{a\,e^{kt}}{k} + c\right)\frac{1}{e^{kt}}$$

$$\frac{1}{y} = \frac{a}{k} + \frac{c}{e^{kt}}$$

$$\frac{1}{y} = \frac{a e^{kt} + ck}{k e^{kt}}$$

$$\boxed{y = \frac{k e^{kt}}{a e^{kt} + ck}} \quad (i)$$

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Put $t=0$, $y(0)=y_0$

$$y_0 = \frac{k e^{k(0)}}{a e^{k(0)} + c(0) k}$$

$$y_0 = \frac{k(1)}{a(1) + ck}$$

$$y_0 = \frac{k}{a+ck}$$

$$\frac{1}{y_0} = \frac{a+ck}{k}$$

$$\frac{1}{y_0} = \frac{a}{k} + \frac{c\cancel{k}}{\cancel{k}}$$

$$\frac{1}{y_0} - \frac{a}{k} = c$$

OR

$$\boxed{c = \frac{1}{y_0} - \frac{a}{k}}$$

Put c in (i)

$$y = \frac{k e^{kt}}{a e^{kt} + \left(\frac{1}{y_0} - \frac{a}{k}\right)k}$$

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$$y = \frac{k e^{kt}}{a e^{kt} + \frac{k}{y_0} - a}$$

$$y = \frac{k e^{kt}}{\frac{a e^{kt} y_0 + k - a y_0}{y_0}}$$

$$y = \frac{k e^{kt} \cdot y_0}{a e^{kt} y_0 + k - a y_0}$$

$$y = \frac{\cancel{e^{kt}} (k y_0)}{\cancel{e^{kt}} (a y_0 + k e^{-kt} - a y_0 e^{-kt})}$$

$$y = \frac{k y_0}{a y_0 + (k - a y_0) e^{-kt}}$$

Taking limit, $t \to \infty$

$$y = \frac{k y_0}{a y_0 + (k - a y_0) e^{-k(\infty)}}$$

$$y = \frac{k \cancel{y_0}}{a \cancel{y_0} + 0} = \boxed{k/a}$$

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EXERCISE 2.3

(1) $y' + y = xy^2$

Ans:- $\frac{dy}{dx} + y = xy^2$

Let $\omega = y^{1-2}$

$\omega = y^{-1}$

$\frac{d\omega}{dx} = -y^{-2} \cdot \frac{dy}{dx}$

$-y^2 \frac{d\omega}{dx} = \frac{dy}{dx}$

$-y^2 \frac{d\omega}{dx} + y = xy^2$

Dividing y^2 b/s

$-\frac{d\omega}{dx} + 1/y = x$

Put $\boxed{1/y = \omega}$

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$I = -xe^{-x} - e^{-x}$

Date: $\int \frac{d}{dx}(uv) = \int u\,dv + \int du \cdot v$ Day: M T W T F S

$uv - \int u\,dv = \int du \cdot v$ $I = e^{-x}(-x-1)$

$$-\frac{dw}{dx} + w = x$$

$$\frac{dw}{dx} - w = -x$$

$$P(x) = -1, \quad q(x) = -x$$

$$w\, e^{\int(-1)dx} = \int \left(-x \cdot e^{\int -1 dx}\right) dx$$

$$w\, e^{-x} = \int \left(-x \cdot e^{-x}\right) dx$$

$$w\, e^{-x} = -\int x\, e^{-x}\, dx$$

$$w\, e^{-x} = -\left[e^{-x}(-x-1)\right] + C$$

$$w\, e^{-x} = e^{-x}(x+1) + C$$

Put $w = 1/y$

$$\frac{1}{y} e^{-x} = e^{-x}(x+1) + C$$

$$\frac{1}{y} = e^{x}\left[e^{-x}(x+1) + C\right]$$

$$\frac{1}{y} = x + 1 + e^{x} C$$

$$\boxed{y = \frac{1}{x + 1 + e^{x} C}}$$

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(4) $y' - \frac{1}{x}y = y^3 \sin x$

Dividing y^3 both sides

$$\frac{y'}{y^3} - \frac{1}{x}\cdot\frac{1}{y^2} = \sin x \quad \text{--- (i)}$$

Let $\omega = 1/y^2$

$$\frac{d\omega}{dx} = -2y^{-3}\cdot\frac{dy}{dx}$$

$$-2y^3\frac{dw}{dx} = \frac{dy}{dx}$$

Putting values in (i)

$$\left(-2y^3\frac{dw}{dx}\right)\frac{1}{y^3} - \frac{1}{x}\omega = \sin x$$

$$-2\frac{d\omega}{dx} - \frac{1}{x}\omega = \sin x$$

$$\frac{1}{-2}\left[-2\frac{d\omega}{dx} - \frac{1}{x}\omega\right] = \frac{-1}{2}\sin x$$

$$\frac{d\omega}{dx} + \frac{1}{2x}\omega = \frac{-1}{2}\sin x$$

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$P(x) = \frac{1}{2x}$, $q(x) = -\frac{1}{2}\sin x$

$\omega\, e^{\int \frac{1}{2x}} = \int \left(-\frac{1}{2}\sin x \; e^{\int \frac{1}{2x}} \right) dx$

$\omega\, e^{\frac{1}{2}\ln(x)} = -\frac{1}{2}\int \sin . e^{\frac{1}{2}\ln x}$

?

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(5) $\frac{dy}{dx} - \frac{1}{2}y = x \cdot \frac{1}{y}$

Xing y both sides

$y \cdot \left[\frac{dy}{dx} - \frac{1}{2}y \right] = x$

$\frac{dy}{dx} \cdot y - \frac{1}{2}y^2 = x$

let $\omega = y^2$

$\frac{1}{2y}\frac{d\omega}{dx} = \frac{\cancel{2y}}{\cancel{2y}} \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2y} \cdot \frac{d\omega}{dx}$

$\frac{1}{2\cancel{y}} \cdot \frac{d\omega}{dx} \cdot \cancel{y} - \frac{1}{2}y^2 = x$

Xing 2 b. s

$\frac{d\omega}{dx} - y^2 = 2x$

$\frac{d\omega}{dx} - \omega = 2x$

$// = -2xe^{-x} - 2 \cdot - e^{-x}$

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$p(x) = -1 \quad , q(x) = 2x$

$\omega \, e^{\int -1 \, dx} = \int (2x \cdot e^{\int -dx}) \, dx$

$\omega \cdot e^{-x} = \int (2x \cdot e^{-x}) \, dx$

$\omega \cdot e^{-x} = -2xe^{-x} + 2e^{-x} + c$

$\omega = e^{x} (-2xe^{-x} + 2e^{-x} + c)$

$\omega = -2x + 2 + e^{x} c$

$\sqrt{y^2} = \sqrt{-2x + 2 + e^{x} c}$

$$\boxed{y = (-2x + 2 + e^{x} c)^{1/2}}$$

(6) $y' - 2y = \cos x \cdot \frac{1}{\sqrt{y}}$

$y' - 2y = \cos x \cdot y^{-1/2}$

Dividing $y^{-1/2}$ B. S

$$\frac{y'}{y^{-1/2}} - \frac{2y}{y^{-1/2}} = \frac{\cos x \cdot \cancel{y^{-1/2}}}{\cancel{y^{-1/2}}}$$

$$\frac{dy}{dx} \cdot y^{1/2} - 2y^{1+1/2} = \cos x$$

$$\frac{dy}{dx} \cdot y^{1/2} - 2y^{3/2} = \cos x \text{ --- } (1)$$

Let $w = y^{3/2}$

$$\frac{dw}{dx} = \frac{3}{2} y^{3/2 - 1} \cdot \frac{dy}{dx}$$

$$\frac{dw}{dx} = \frac{3}{2} y^{\frac{1}{2}} \cdot \frac{dy}{dx}$$

$$\boxed{\frac{2}{3}\frac{dw}{dx} = y^{1/2}\frac{dy}{dx}}$$

Put in 1

$$\frac{2}{3}\frac{dw}{dx} - 2y^{3/2} = \cos x$$

$$\frac{3}{2}\left[\frac{2}{3}\frac{dw}{dx} - 2y^{3/2}\right] = \frac{3}{2}\cos x$$

$$\frac{dw}{dx} - 3y^{3/2} = \frac{3}{2}\cos x$$

Put $y^{3/2} = w$

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$$\frac{dw}{dx} - 3w = \frac{3}{2} \cos x$$

$$p(x) = -3 \quad , \quad q(x) = \frac{3}{2} \cos x$$

$$w e^{\int -3\, dx} = \int \left(\frac{3}{2} \cos x . e^{\int -3 dx} \right) dx$$

$$w e^{-3x} = \frac{3}{2} \int \cos x . e^{-3x} \, dx$$

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