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Exercise 1.1

Q8.

(25) $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 0$, $y(x) = A + Be^{2x} + Ce^{-2x}$

i)

~~i) ODE~~

~~ii) 3~~

~~iii)~~

~~(26)~~

Ans: Verification :-

$\frac{dy}{dx} = \frac{d}{dx}(A + Be^{2x} + Ce^{-2x})$

$\frac{dy}{dx} = 0 + B.2.e^{2x} - 2Ce^{-2x}$

$\frac{dy}{dx} = 2Be^{2x} - 2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Be^{2x} + 4Ce^{-2x}$

$\frac{d^3y}{dx^3} = 8Be^{2x} - 8Ce^{-2x}$

Putting values in i)

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$$8Be^{2x} - 8Ce^{-2x} - 4(2Be^{2x} - 2Ce^{-2x}) = 0$$

$$\cancel{8Be^{2x}} - \cancel{8Ce^{-2x}} - \cancel{8Be^{2x}} + \cancel{8Ce^{-2x}} = 0$$

$$0 = 0$$

(26) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} = 0, \; y(x) = A + Bx + Ce^{2x}$

$$y''' - 2y'' = 0$$

$$y' = 0 + B + Ce^{2x} \cdot 2$$

$$\boxed{y' = B + 2Ce^{2x}}$$

$$\boxed{y'' = 0 + 4Ce^{2x}}$$

$$\boxed{y''' = 8Ce^{2x}}$$

$$8Ce^{2x} - 2(4Ce^{2x}) = 0$$

$$\cancel{8Ce^{2x}} - \cancel{8Ce^{2x}} = 0$$

$$0 = 0$$

(27) $x^2\frac{d^2y}{dx^2} - 12x\frac{dy}{dx} + 42y = 0, \; y(x) = Ax^6 + Bx^7$

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$$\frac{dy}{dx} = 6Ax^5 + 7Bx^6$$

$$\frac{d^2y}{dx^2} = 30Ax^4 + 42Bx^5$$

$$x^2(30Ax^4 + 42Bx^5) - 12x(6Ax^5 + 7Bx^6) + 42(Ax^6 + Bx^7) = 0$$

$$30Ax^6 + 42Bx^7 - 72Ax^6 - 84Bx^7 + 42Ax^6 + 42Bx^7 = 0$$

$$\cancel{72Ax^6} + \cancel{84Bx^7} - \cancel{84Bx^7} - \cancel{72Ax^6} = 0$$

$$0 = 0$$

(48) $\frac{d^2y}{dx^2} + 9\frac{dy}{dx} = 0, \quad y(0) = 2, \quad y'(0) = -1$

$$y(x) = A + Be^{-9x}$$

$$y(0) = A + Be^{-9(0)}$$

$$y(0) = A + Be^0 = A + B$$

$$\boxed{A + B = 2} \text{ --- (i)}$$

$$y' = A - 9Be^{-9x}$$

$$y'(0) = A - 9Be^{-9(0)} = A - 9Be^0 = \boxed{A - 9B} \text{ (ii)}$$

$= -1$

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$$A + B = 2$$

$$-A - 9B = -1$$

$$10B = 3$$

$$\boxed{B = 3/10}$$

$$A + \frac{3}{10} = 2$$

$$A = \frac{2}{1} - \frac{3}{10}$$

$$\boxed{A = \frac{17}{10}}$$

$$y(x) = \frac{17}{10} + \frac{3}{10} e^{-9x}$$

$$\boxed{y(x) = \frac{17 + 3e^{-9x}}{10}}$$

(40) $\frac{dy}{dx} = \frac{-2(x+5)}{(x+2)(x-4)}$?

$$\int \frac{dy}{dx} = \int \frac{-2(x+5)}{(x+2)(x-4)} dx$$

$$y = -2 \int \frac{x+5}{(x+2)(x-4)}$$

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$$\frac{x+5}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4}$$

$$\frac{x+5}{\cancel{(x+2)(x-4)}} = \frac{A(x-4)+B(x+2)}{\cancel{(x+2)(x-4)}}$$

$$x+5 = A(x-4)+B(x+2)$$

For $x=4$	For $x=-2$
$4+5 = A(4-4)+B(4+2)$	$-2+5 = A(-2-4)+B(-2+2)$
$9 = 0+6B$	$3 = A(-6)+0$
$B = 9/6$	$-6A = 3$
$\boxed{B = 3/2}$	$\boxed{A = -1/2}$

$$\frac{x+5}{(x+2)(x-4)} = \frac{-1/2}{x+2} + \frac{3/2}{x-4}$$

$$\text{''} = \frac{-1}{2(x+2)} + \frac{3}{2(x-4)}$$

$$y = -2\int\left(\frac{-1}{\cancel{2}(x+2)} + \frac{3}{\cancel{2}(x-4)}\right)dx$$

$$y = -1\left[\int \frac{-1}{x+2}dx + \int \frac{3}{x-4}dx\right]$$

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$$y = \int \frac{1}{x+2}dx - 3\int \frac{1}{x-4}dx$$

$$y = \ln|x+2| - 3\ln|x-4| + \ln|c|$$

$$y = \ln|x+2| - \ln|x-4|^3 + \ln|c|$$

$$y = \ln \frac{|x+2|\, c}{|x-4|^3}$$

(44) $$\frac{dy}{dx} = \frac{1}{x^2-16}$$

$$\frac{dy}{dx} = \frac{1}{(x+4)(x-4)}$$

$$\frac{1}{(x+4)(x-4)} = \frac{A}{x+4} + \frac{B}{x-4}$$

$$\frac{1}{\cancel{(x+4)(x-4)}} = \frac{A(x-4)+B(x+4)}{\cancel{(x+4)(x-4)}}$$

$$1 = A(x-4) + B(x+4) \text{ — } (A)$$

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For $x = 4$	For $x = -4$
$1 = A(4-4) + B(4+4)$	$1 = A(-4-4) + B(-4+4)$
$1 = A(0) + B(8)$	$1 = A(-8) + B(0)$
$1 = 8B$	$1 = -8A + 0$
$\boxed{B = 1/8}$	$\boxed{A = -1/8}$

$$\frac{1}{(x+4)(x-4)} = \frac{-1/8}{x+4} + \frac{1/8}{x-4}$$

$$'' = \frac{-1}{8(x+4)} + \frac{1}{8(x-4)}$$

$$'' = \frac{1}{8}\left[\frac{1}{x-4} - \frac{1}{x+4}\right]$$

$$\int \frac{dy}{dx} = \int \frac{1}{8}\left[\frac{1}{x-4} - \frac{1}{x+4}\right]$$

$$y = \frac{1}{8}\left[\int \frac{1}{x-4} - \int \frac{1}{x+4}\right]$$

$$y = \frac{1}{8}\left[\ln|x-4| - \ln|x+4| + \ln c\right]$$

$$\boxed{y = \frac{1}{8}\left[\ln\left|\frac{x-4}{x+4}\right| + \ln c\right]}$$

35 $\frac{dy}{dx} = x \sin(x^2)$

Ans:- $\int \frac{dy}{dx} = \int x \sin(x^2)\, dx$

$y = \int x \sin(x^2)\, dx \leftarrow (i)$

Let $x^2 = u$

$2x\, dx = du$

$x\, dx = \frac{du}{2}$

Putting values in (i)

$y = \int \sin(u) \frac{du}{2}$

$y = \frac{1}{2} \int \sin u\, du$

$y = \frac{1}{2}\left[-\cos u + c\right]$

$y = \frac{1}{2}\left[-\cos x^2 + c\right]$

(55) $\frac{dy}{dx} = 4x^3 - x + 2, \quad y(0) = 1$

$$\int \frac{dy}{dx} = \int (4x^3 - x + 2)\,dx$$

$$y = \frac{\cancel{4}x^4}{\cancel{4}} - \frac{x^2}{2} + 2x + c$$

$$y = x^4 - x^2/2 + 2x + c \quad \text{---(i)}$$

Put $x = 0$

$$y(0) = 0 - 0 + 2(0) + c$$

$$y(0) = c$$

$$1 = c$$

$$\boxed{c = 1}$$

Put in (i)

$$y = x^4 - \frac{x^2}{2} + 2x + 1$$

(57) $\frac{dy}{dx} = \frac{1}{x^2} \cos(1/x)$

$$y(2/\pi) = 1$$

$$\int \frac{dy}{dx} = \int \frac{1}{x^2} \cos(1/x)\,dx$$

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$$y = \int \frac{1}{x^2} \cos(1/x)\, dx$$

Let $1/x = u$

$$x^{-1} = u$$

$$-1 x^{-2}\, dx = du$$

$$-\frac{1}{x^2}\, dx = du$$

$$\frac{1}{x^2}\, dx = -du$$

$$y = \int \cos(u) - du$$

$$y = -\int \cos(u)\, du$$

$$y = -\left[\sin(u) + c\right]$$

$$\boxed{y = -\left[\sin(1/x) + c\right]} \quad \text{—(i)}$$

$$y(2/\pi) = -\left[\sin(\pi/2) + c\right]$$

$$y(2/\pi) = -\left[1 + c\right]$$

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$1 = -1 \mp C$

$1+1 = -C$

$-C = 2$

$\boxed{C = -2}$

Put in (i)

$y = -\left[\sin(1/x) - 2\right]$

$\boxed{y = -\sin(1/x) + 2}$

(58) $\frac{dy}{dx} = \frac{\ln x}{x}$, $y(1) = 0$

$\int \frac{dy}{dx} = \int \frac{\ln x}{x}$

$y = \int \frac{\ln x}{x} dx$

Let $\ln x = u$

$\frac{1}{x} dx = du$

$y = \int u \, du$

$y = \frac{u^2}{2} + C$

$y = \frac{\ln x^2}{2} + C$ —— (i)

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$$y(1) = \frac{\ln 1^2}{2} + C$$

$$y(1) = 0 + C$$

$$\boxed{0 = C}$$

$$y = \frac{\ln x^2}{2} + 0$$

$$\boxed{y = \frac{\ln x^2}{2}}$$

70) $\frac{d}{dx}(e^x y) = e^x \frac{dy}{dx} + e^x y$?

Solve $e^x \frac{dy}{dx} + e^x y = x e^x$

$$\int \frac{d}{dx}(e^x y) = \int \left(e^x \frac{dy}{dx} + e^x y\right)$$

$$e^x y = \int e^x \frac{dy}{dx} + \int e^x y$$

$\frac{1}{2} - \frac{2}{1}$

$\frac{1-4}{2} = -3/2$

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(29) Show that $u(x,y) = \tan^{-1}(y/x)$ Satisfies Laplace Equation $U_{xx} + U_{yy}$

ii) U_x :-

$$U_x = \frac{\partial}{\partial x} \tan^{-1}(y/x)$$

$$U_x = \frac{1}{1+(y/x)^2} \cdot \frac{\partial}{\partial x}\left(\frac{y}{x}\right)$$

$$U_x = \frac{1}{1+(y/x)^2} \cdot -2y \cdot \frac{1}{\sqrt{x}}$$

$$U_x = \frac{-2y}{1+(y/x)^2} \cdot \frac{1}{\sqrt{x}}$$

$$U_{xy} = \frac{-2y}{\sqrt{x} + \frac{y^2}{x^2} \cdot x^{1/2}}$$

$$U_x = \frac{-2y}{\sqrt{x} + y^2 x^{1/2 - 2}} = \frac{-2y}{\sqrt{x} + y^2 x^{-3/2}}$$

$$U_x = \frac{-2y}{x^{1/2} + y^2 x^{-3/2}}$$

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$$\cancel{U_{xx} = \frac{-2y}{\frac{1}{2}x^{-1/2} + y^2 \cdot \left(-\frac{3}{2}\right) x^{-\frac{3}{2}-\frac{1}{1}}}}$$

$$\cancel{U_{xx} = \frac{-2y}{\frac{1}{2}\frac{1}{\sqrt{x}} - \frac{3}{2} y^2 x^{-\frac{5}{2}}}}$$

$$\cancel{U_{xx} = \frac{-2y}{\frac{1}{2}\frac{1}{\sqrt{x}} - \frac{3}{2} y^2 x^{-5} \cdot x^{1/2}}}$$

$$\cancel{U_{xx} = -2y}$$

$$U_x = \frac{1}{\frac{1}{1} + y^2/x^2} \cdot \frac{-y}{x^2}$$

$$U_x = \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{-y}{x^2} \Rightarrow U_x = \frac{x^2}{x^2+y^2} \cdot \frac{-y}{x^2}$$

$$\boxed{U_x = \frac{-y}{x^2+y^2}}$$

$$U_x = -y\,(x^2+y^2)^{-1} \Rightarrow U_{xx} = -y \cdot (-1)(x^2+y^2)^{-2} \cdot (2x)$$

$$U_{xx} = 2xy\,(x^2+y^2)^{-2}$$

$$U_{xx} = \frac{2xy}{(x^2+y^2)^2} \quad \text{—(i)}$$

$$U_y = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{\partial}{\partial y}\left(\frac{y}{x}\right)$$

$$U_y = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x}$$

$$U_y = \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x}$$

$$U_y = \frac{x^2}{x^2+y^2} \cdot \frac{1}{\cancel{x}}$$

$$U_y = \frac{x}{x^2+y^2}$$

$$U_y = x.(x^2+y^2)^{-1}$$

$$U_{yy} = x \cdot -1(x^2+y^2)^{-2} . (2y)$$

$$U_{yy} = -2xy\,(x^2+y^2)^{-2}$$

$$U_{yy} = \frac{-2xy}{(x^2+y^2)^2} \quad \text{—(ii)}$$

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Adding (i) and (ii)

$$u_{xx} + u_{yy} = \cancel{\frac{2xy}{(x^2+y^2)^2}} - \cancel{\frac{2xy}{(x^2+y^2)^2}}$$

$$\text{"} \quad = \boxed{0}$$

CHAPTER 2

EXERCISE 2.1

(24) $\dfrac{dy}{dx} = \dfrac{1+2e^y}{e^y x + \ln x}$

$$\frac{\cancel{(1+2e^y)}\,dx}{(x\ln x)\cancel{(1+2e^y)}} = \frac{(e^y x \ln x)\,dy}{\cancel{(x\ln x)}(1+2e^y)}$$

$$\int \frac{1}{x\ln x}\,dx = \int \frac{e^y}{1+2e^y}\,dy$$

$$\int \frac{1/x}{\ln x}\,dx = \frac{1}{2}\int \frac{2e^y}{1+2e^y}\,dy$$

$$\ln(\ln x) = \frac{1}{2}\ln(1+2e^y) + C$$

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(5) $(6+4x^3)\,dx = (5+9/y^2)\,dy$

Integrating DE with respect to indicated variables.

$\int (6+4x^3)\,dx = \int (5+9/y^2)\,dy$

$6x + \frac{4x^4}{4} = 5x + 9\frac{y^{-2+1}}{-2+1} + c$

$x^4 + 6x = 5x + 9\frac{y^{-1}}{-1} + c$

$x^4 + 6x - 5x = -\frac{9}{y} + c$

$x^4 + x = -9/y + c$

$-9/y + c = x^4 + x$

$-\frac{1}{9} \times -\frac{9}{y} = (x^4 + x - c) \times -\frac{1}{9}$

$\frac{1}{y} = -\frac{1}{9}(x^4 + x - c)$

$$\boxed{y = -9\left(\frac{1}{x^4+x-c}\right)}$$

(6) $\frac{dy}{dx} = \frac{e^{2y} e^{10x}}{1}$?

$$\frac{\cancel{e^{2y}} e^{10x}}{\cancel{e^{2y}}} \cdot dx = \frac{dy}{e^{2y}}$$

$$e^{10x}\, dx = \frac{1}{e^{2y}} \cdot dy$$

$$\int e^{10x}\, dx = \int e^{-2y}\, dy$$

$$\frac{e^{10x}}{10} = \frac{e^{-2y}}{-2} + C$$

$$-2 \times \left(\frac{e^{10x}}{10} - C \right) = \frac{e^{-2y}}{-\cancel{2}} \times \cancel{2}$$

$$-\frac{e^{10x}}{5} - 2C = e^{-2y}$$

(25) $x \sin(x^2)\, dx = \frac{\cos\sqrt{y}}{\sqrt{y}}\, dy$

Ans: $\int x \sin(x^2)\, dx = \int \frac{1}{\sqrt{y}} \cdot \cos\sqrt{y}\, dy$ ...(1)

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$$\int x \sin(x^2)\, dx = ?$$

Let $x^2 = u$

$$2x\, dx = du$$

$$x\, dx = \frac{du}{2}$$

$$= \int \sin(u) \cdot \frac{du}{2}$$

$$= \frac{1}{2} \int \sin(u)\, du$$

$$= \frac{1}{2} \left[-\cos(u) \right]$$

$$= \boxed{-\frac{\cos(x^2)}{2}} \text{ — } A$$

$$\int \frac{\cos\sqrt{y}\, dy}{\sqrt{y}} ?$$

Let $\sqrt{y} = u$

$$\cancel{2} \times \frac{1}{\cancel{2}} \frac{1}{\sqrt{y}}\, dy = du \times 2$$

$$\frac{1}{\sqrt{y}}\, dy = 2\, du$$

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$= \int \cos u \cdot 2du$

$= 2\int \cos u\, du$

$= \boxed{2\sin u + c}$ — (B)

$\int x \sin(x^2)dx = \int \frac{1}{\sqrt{y}} \cos\sqrt{y}\; dy$

From (A) and (B)

$\frac{-\cos(x^2)}{2} = 2\sin\sqrt{y} + c$

$\frac{\frac{-\cos(x^2)}{2} - c}{2} = \frac{\cancel{2}\sin\sqrt{y}}{\cancel{2}}$

$\sin\sqrt{y} = \frac{-\cos(x^2)}{4} - \frac{c}{2}$

$\sqrt{y} = \sin^{-1}\left[\frac{-\cos(x^2)}{4} - \frac{c}{2}\right]$

$(y^{1/\cancel{2}})^{\cancel{2}} = \left[\sin^{-1}\left[\frac{-\cos(x^2)}{4} - \frac{c}{2}\right]\right]^2$

$y = \left[\sin^{-1}\left(\frac{-\cos(x^2)}{4} - \frac{c}{2}\right)\right]^2$

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(32) $\frac{dy}{dx} = \cos x, \quad y(\pi/2) = -1$

$$\int \frac{dy}{dx} = \int \cos x$$

$$y = \sin x + c \quad — (i)$$

$$y(\pi/2) = \sin(\pi/2) + c$$

$$-1 = 1 + c$$

$$-1 - 1 = c$$

$$\boxed{c = -2}$$

Put in (i)

$$\boxed{y = \sin x - 2}$$

(34) $\sin^2 y \, dy = dx, \quad y(0) = 0$

$$\int \sin^2 y \, dy = \int dx$$

$$\frac{y}{2} - \frac{1}{4}\sin 2y + c = x — (i)$$

When $x = 0, \; y = 0$

$$\frac{0}{2} - \frac{1}{4}\sin(2(0)) + c = (0)$$

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$-\frac{1}{4}(0) + C = 0$

$\boxed{C = 0}$

Put in (i)

$\frac{y}{2} - \frac{1}{4}\sin 2y + 0 = x$

$\frac{y}{2} - \frac{1}{4}\sin 2y = x$

$4\left(\frac{y}{2} - x\right) = \frac{1}{4}\sin 2y \times 4$

$\boxed{2y - 4x = \sin 2y}$ — Implicit Solution

(24) $\frac{dy}{dx} = (x+y-4)^2$?

Ans: Let $\boxed{x+y-4 = u}$

$\cancel{(1 + \frac{dy}{dx} - 0)\, dx = du}$

$\cancel{\boxed{dx = du}}$

$\int \frac{dy}{dx} = \int (x+y-4)^2 \, dx$

$y = \int (u)^2 du$

$y = \frac{u^3}{3} + C$

$y = \frac{(x+y-4)^3}{3} + C$

$x+y-4 = u$

$\frac{du}{dx} = \frac{d}{dx}(x+y-4)$

$\frac{du}{dx} = 1 + \frac{dy}{dx} - 0$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$du = dx + dy$

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EXERCISE 2.2

(8) $\frac{dy}{dx} + y \cot x = \cos x$

Comparing with general form

$\frac{dy}{dx} + P(x)y = q(x)$

$P(x) = \cot x \quad , \quad q(x) = \cos x$

Its general solution is:

$$y e^{\int \cot x \, dx} = \int \left(\cos x \cdot e^{\int \cot x \, dx} \right) dx$$

$$y e^{\ln(\sin x)} = \int \left(\cos x \cdot e^{\ln(\sin x)} \right) dx$$