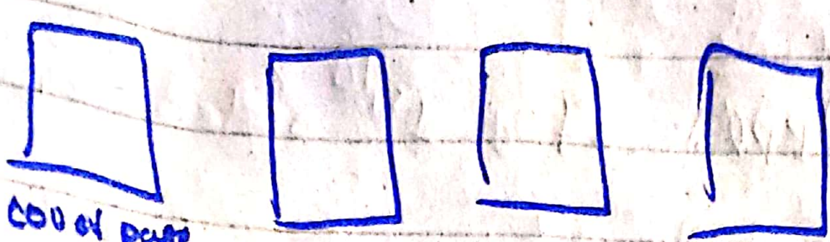

cover page

Ruberics

Mini projects/ Final Project

Differential Equations (BSI-231)

Textbook (1) Modern Differential Equations 2nd Edition by A Bell and Brosatta

(2) Mathematical Methods by Dr. S.M Yousaf Ilmi Kitab Khana

Chapter 1 Introduction to Differential Equations.

Chapter 2

First order equations.

Chap 3

Appl. of 1st order equations

Chap. 4

Higher order diff. equations

Chap 5

Appl. of higher order diff equ

Chap 6

System of diff equation

Chap 1

Differential Equation

→ An equation which involve derivative.

$$\frac{dy}{dx} + 2xy = \sin x$$

$$\left(\frac{dy}{dx}\right)^3 = 2 + x$$

CLASSIFICATION

Ordinary DE :-

An equation which involve ordinary derivative.

$$y' + y = e^{x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 6y = 2x$$

Partial DE :-

An equation which involve partial derivative

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 8z$$

ORDER OF D.E :-

Following DE's are of order 1, 2 and 3.

resp.

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 10z \quad (1^{st} \text{ order})$$

$$y'' + 2y' + 6y = \ln x \quad (2^{nd} \text{ order})$$

$$(y''')^4 + 2y'' + y' + 2y = 3 \quad (3^{rd} \text{ order})$$

DEGREE OF D.E :-

① $y''' = 2x + (y')^3$ (Degree 1)

② $\left[1 + (y')^2\right]^{3/2} = y''$

Taking square on b.s

$$\left(\left(1 + (y')^2\right)^{3/2}\right)^{\cancel{2}} = (y'')^2$$

$$\left(1 + (y')^2\right)^3 = (y'')^2 \quad (\text{Degree } 2)$$

(3) $(y''')^{3/2} = 1 + 2y'$ (Degree 3)

Classification On Linearity

(1) Linear DE :-

An ODE of order n is called linear if it is of the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots a_1(x)\frac{dy}{dx}$$

where the functions

$a_j(x)$, $j = 0, 1, 2, 3 \ldots n$

and $f(x)$

Solution Of DE :-

A ftion which satisfy the DE and does not involve any derivative.

(a) General Solution
(b) Particular "

$y'' + 9y' = 0$, $y(x) = e^{-9x}$

$y'' + 9y' = 0$; $y(x) = \underbrace{A + B}\, e^{-9x}$

Two arbitraly Constants because of order two

Exercise 1.1

Verify that each of the given ftion is a Solution to the corresponding D.E (A, B and C) are Constants.

(25) $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 0$, $y(x) = A + Be^{2x} + Ce^{-2x}$

(26) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} = 0$, $y(x) = A + Bx + Ce^{2x}$

(27) $x^2\frac{d^2y}{dx^2} - 12x\frac{dy}{dx} + 42y = 0$, $y(x) = Ax^6 + Bx^7$

IVP (Initial value Problem)

Use the indicated condition with the indicated solution to determine the solution to given problem.

$\frac{d^2y}{dx^2} + 9\frac{dy}{dx} = 0$, $y(x) = A + Be^{-9x}$

$\underbrace{y(0) = 2, \quad y'(0) = 1}$

→ Both are at same point So initial value problem. If at different points then boundary value problem.

(48) Sol:

Diff. $y(x)$ w.r.t x

Explicit f'tion → explicit Sol.
Implicit " → Implicit "

$$y'(x) = 0 + (B)-9e^{-9x}$$
$$y'(x) = -9Be^{-9x}$$

Now applying given condition

$$y(0) = A + Be^{0}$$
$$\boxed{2 = A + B} \quad (1)$$

and $y'(0) = -1$, we get

$$-1 = -9Be^{0}$$
$$\boxed{-1 = -9B} \quad (2)$$

$$A + B = 2$$
$$-9B = -1$$

The Augmented matrix is

$$[A/B] = \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -9 & -1 \end{array}\right]$$

$$\underset{R}{\sim} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & \frac{1}{9} \end{array}\right] -\tfrac{1}{9}R_2 \quad \left[\begin{array}{c}\text{Echelon} \\ \text{Form}\end{array}\right]$$

$$\underset{R}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 17/9 \\ 0 & 1 & 1/9 \end{array}\right] R_1 - R_2 \qquad \frac{2}{1} - \frac{1}{9} = \frac{17}{9}$$

$$\left[\begin{array}{c}\text{Reduced Echelon} \\ \text{Form}\end{array}\right]$$

$$Cx = d$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 17/9 \\ 1/9 \end{bmatrix}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12 = 0 \quad , \quad y(x) = C_1 e^{4x} + C_2$$

$$y(0) = -2, \quad y'(0) = 6$$

Solving D.E

14 - 04 - 2022

Quiz. No. 1

(1) $y' = f(x)$

$y = \int f(x)\, dx$

C P

Scanned with CamScanner