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Exercise 5.1

(11) Ans:

$F = 16$ lb

$s = 6$ ft $= 1/2$ ft

$x(t) = ?$

$x(0) = 1, \quad x'(0) = 0$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$F = mg$

$$m = \frac{16}{32} = \frac{1}{2} \text{ slugs}$$

$F = ks$

$16 = k(1/2) \Rightarrow \boxed{k = 32 \text{ lb/ft}}$

$$\frac{1}{2}\frac{d^2x}{dt^2} + 32x = 0$$

$$\frac{d^2x}{dt^2} + 64x = 0$$

$$r^2 + 64 = 0$$

$$r^2 = -64$$

$$r = \pm 8i$$

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$x_g(t) = e^{0t}[C_1 \sin 8t + C_2 \cos 8t]$

$x_g(t) = C_1 \sin 8t + C_2 \cos 8t$

$x(0) = C_1(0) + C_2(1)$

$1 = 0 + C_2$

$\boxed{C_2 = 1}$

$x'(t) = 8C_1(\cos 8t) + C_2 - 8 \sin 8t$

$x'(t) = 8C_1 \cos 8t - 8C_2 \sin 8t$

$0 = 8C_1(1) - 8C_2(0)$

$0 = 8C_1$

$\boxed{C_1 = 0}$

$\therefore x(t) = \cos 8t$

* For max. displacement

$x'(t) = -8 \sin 8t = 0$

$\sin 8t = 0$

$\therefore \boxed{t = \frac{n\pi}{8}}$

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5.2 Damped Motion.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

Example 1:-

$F = 8$ lb

$S = 2$ ft

$x(t) = ?$, $F_R = 2\frac{dx}{dt}$

$F = mg$

$8 = m(32)$

$m = 1/4$ Slugs

Also $F = kS \rightarrow 8 = k(2) \rightarrow k = 4$

$$\therefore \frac{1}{4}\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 4x = 0$$

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)^2 = 0$$

$$x = -4$$

$$x(t) = c_1 e^{-4t} + c_2 t e^{-4t}$$

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(a)

$$x'(t) = c_1(-4e^{-4t}) + c_2(t.-4e^{-4t} + e^{-4t})$$

$$x'(t) = -4c_1e^{-4t} + c_2(-4te^{-4t} + e^{-4t})$$

$$x(0) = 0, \quad x'(0) = 1$$

$x(0) = c_1e^0 + c_2(0)e^0$

$x(0) = c_1 + 0$

$\boxed{0 = c_1}$

$x'(0) = -4c_1e^0 + c_2(-4(0)e^0 + e^0)$

$1 = -4c_1 + c_2(0+1)$

$1 = -4c_1 + c_2$

$1 = -4(0) + c_2$

$\boxed{c_2 = 1}$

$$x(t) = te^{-4t}$$

(b)

$x(0) = -1/2$

$x'(0) = 5$

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Exercise 5.4

(1) $L = 2H$, $C = \frac{1}{32}F$, $E(t) = 220$

$R = 0$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

$$\frac{2\frac{d^2Q}{dt^2} + 32Q = 220}{2}$$

$$\frac{d^2Q}{dt^2} + 16Q = 110$$

$y_n(t) = ?$

$$\frac{d^2Q}{dt^2} + 16Q = 0$$

$$\gamma^2 + 16 = 0$$

$$\gamma^2 = -16$$

$$\gamma = \pm 4i$$

$$\alpha = 0, \quad \beta = 4$$

$$Q_n(t) = e^{\alpha}[c_1 \sin 4t + c_2 \cos 4t]$$

$$Q_n(t) = c_1 \sin 4t + c_2 \cos 4t$$

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$Q_p(t) = ?$

Let $Q_p(t) = A$

$Q_p'(t) = 0$

$Q_p''(t) = 0$

$0 + 16(A) = 110$

$\boxed{A = 55/8}$

$\therefore Q_p(t) = 55/8$

$Q(t) = C_1 \sin 4t + C_2 \cos 4t + \frac{55}{8}$

$Q'(t) = 4C_1 \cos 4t - 4C_2 \sin 4t + 0$

$Q(0) = C_1 \sin 0 + C_2 \cos 0 + \frac{55}{8}$

$Q(0) = C_1(0) + C_2(1) + 55/8$

$\boxed{-55/8 = C_2}$

$Q'(0) = 4C_1 \cos 0 - 4C_2 \sin 0$

$0 = 4C_1(1) - 4C_2(0)$

$0 = 4C_1$

$\boxed{C_1 = 0}$

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$$Q(t) = -\frac{55}{8}\cos 4t + \frac{55}{8}$$

(4)