$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 3(3)e^{5t} - (-5)e^{-2t} \\ 4(3)e^{5t} + (-5)e^{-2t} \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 9e^{5t} + 5e^{-2t} \\ 12e^{5t} - 5e^{-2t} \end{pmatrix}$$

CHAPTER 8

EXAMPLE 1:

$$\mathcal{L}\{1\} = ?$$

Sol:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\, dt$$

$$\mathcal{L}\{1\} = \frac{e^{-st}}{-s}\Big|_0^\infty$$

$$\mathcal{L}\{1\} = \left(\frac{e^{-s(\infty)}}{-s} - \frac{e^0}{-s}\right)$$

$$y = e$$

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$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\cdot(1)\,dt = \lim_{M\to\infty}\int_0^M e^{-st}\,dt$$

$$\text{"} = \lim_{M\to\infty}\left[\frac{e^{-st}}{-s}\right]_0^M$$

$$\text{"} = \lim_{M\to\infty}\left[\frac{e^{-sM}}{-s} - \frac{e^{0}}{-s}\right]$$

$$\text{"} = 0 - \frac{1}{-s} = \boxed{\frac{1}{s}} \text{ For } s>0$$

EXAMPLE ②:-

$$\mathcal{L}\{e^{at}\} = ?$$

Sol:

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st}\cdot e^{at}\,dt$$

$$\text{"} = \int_0^\infty e^{(-s+a)t}\,dt$$

$$\text{"} = \lim_{M\to\infty}\int_0^M e^{(-s+a)t}\,dt$$

$$\lim_{M\to\infty}\left[\frac{e^{(-s+a)t}}{-s+a}\right]_0^M$$

$$\text{"} = \lim_{M\to\infty}\left[\frac{e^{(-s+a)M}}{-s+a} - \frac{e^{(-s+a)0}}{-s+a}\right]$$

$$\text{"} = \lim_{M\to\infty}\left[\frac{e^{(-s+a)M}}{-s+a} - \frac{1}{-s+a}\right]$$

$\int e^{-st} \sin t = \cancel{e^{-st} \cdot -\cos t} = \int \cancel{\quad} + \cos t \, s \, e^{-st}$

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~~$u\,dv = uv - \int v\,du$~~

$$\text{''} = \cancel{-e^{-st}\cos t - s\int \cos t \, e}$$

$$\text{''} = \frac{e^{(-s+a)\infty}}{-s+a} - \frac{1}{-s+a}$$

$$\text{''} = 0 - \frac{1}{-s+a}$$

$$\text{''} = -\frac{1}{-(s-a)} = \boxed{\frac{1}{s-a}}$$

For s

EXAMPLE 5

$$\mathcal{L}\{\sin t\} = ?$$

$$\mathcal{L}\{\sin t\} = \int_0^\infty (e^{-st} \cdot \sin t)\,dt$$

$$\mathcal{L}\{\sin t\} = \lim_{M\to\infty} \int_0^M (e^{-st} \cdot \sin t)\,dt \quad (i)$$

Let $I = \int e^{-st} \cdot \sin t$

$I = e^{-st} \cdot -\cos t - \int -\cos t \cdot -s e^{-st}$

$I = -e^{-st} \cdot \cos t - s\int \cos t \cdot e^{-st}$

$I = -e^{-st} \cdot \cos t - s\left[e^{-st} \cdot \sin t - \int \sin t \cdot -s e^{-st}\right]$

$I = -e^{-st} \cdot \cos t - s\left[e^{-st} \cdot \sin t + s\int \sin t \, e^{-st}\right]$

$I = -e^{-st} \cdot \cos t - s\left[e^{-st} \cdot \sin t + s I\right]$

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$$I = -e^{-st}.\cos t - s.e^{-st}.\sin t - s^2 I$$

$$I + s^2 I = -e^{-st}.\cos t - s.e^{-st}.\sin t$$

$$I(1+s^2) = -e^{-st}(\cos t + \sin t)$$

$$I = \frac{-e^{-st}(\cos t + \sin t)}{1+s^2}$$

(i) $\Rightarrow$ $\mathcal{L}\{\sin t\} = \lim_{M\to\infty} \left[\frac{-e^{-st}(\cos t + \sin t)}{1+s^2} \right]_0^M$

$$" = \lim_{M\to\infty} \left[\left(\frac{-e^{-sM}(\cos M + \sin M)}{1+s^2} \right) - \left(\frac{e^{0}(\cos 0 + \sin 0)}{1+s^2} \right) \right]$$

$$" = \lim_{M\to\infty} \left[\frac{-e^{-sM}(\cos M + \sin M)}{1+s^2} - \frac{-1(1)}{1+s^2} \right]$$

$$" = \frac{0(\cos\infty + \sin\infty)}{1+s^2} + \frac{1}{1+s^2}$$

$$" = 0 + \frac{1}{s^2+1} = \boxed{\frac{1}{s^2+1}},\ s > 0$$

EXAMPLE 5 :-

$$\mathcal{L}\{t\} = ?$$

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$$\mathcal{L}\{t\} = \int_0^\infty e^{-st} \cdot t \, dt$$

$$\| = \lim_{M \to \infty} \int_0^M e^{-st} \cdot t \, dt \quad \text{---(i)}$$

Let $I = \int e^{-st} \cdot t$

$$I = t \cdot \frac{e^{-st}}{-s} - \int (1) \frac{e^{-st}}{-s}$$

$$I = \frac{te^{-st}}{-s} + \frac{1}{s}\int e^{-st}$$

$$\boxed{\begin{aligned} I &= -ste^{-st} + s\int e^{-st} \\ I &= \frac{-ste^{-st}}{1} + \frac{se^{-st}}{-s} \\ I &= \frac{-ts \cdot e^{-st} - e^{-st}}{s} \\ I &= \frac{e^{-st}(-ts-1)}{s} \end{aligned}}$$

$$I = \frac{te^{-st}}{-s} + \frac{1}{s} \cdot \frac{e^{-st}}{-s}$$

$$I = \frac{e^{-st}}{-s}\left[t + \frac{1}{s}\right]$$

$$I = \frac{e^{-st}}{-s}\left[\frac{st+1}{s}\right]$$

OR

$$= \frac{1}{s^2}\left[-e^{-st} \cdot st - e^{-st}\right]$$

(i) $\Rightarrow$ $$\mathcal{L}\{t\} = \lim_{M \to \infty} \left[\frac{e^{-st}}{-s}\left[\frac{st+1}{s}\right]\right]_0^M$$

$$\| = \lim_{M \to \infty} \left[\left(\frac{e^{-sM}}{-s}\left(\frac{sM+1}{s}\right)\right) - \left(\frac{(1)(1)}{-s^2}\right)\right]$$

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$$// = \lim_{M\to\infty}\left[\left(\frac{e^{-SM}}{-s}\left(\frac{SM+1}{s}\right)\right)+\frac{1}{s^2}\right]$$

$$// = \frac{0}{-s}\left(\frac{s(\infty)+1}{s}\right)+\frac{1}{s^2}$$

$$// = 0 + 1/s^2 = \boxed{1/s^2}, \; s>0$$

EXERCISE 8.1

(7) $f(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ 1, & t>1 \end{cases}$

Ans:

$$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(0) + \int_1^\infty e^{-st}\cdot 1\,dt$$

$$// = \int_0^1 0\,dt + \lim_{M\to\infty}\int_1^M e^{-st}\,dt$$

$$// = 0 + \lim_{M\to\infty}\left[\frac{e^{-st}}{-s}\right]_1^M$$

$$// = \lim_{M\to\infty}\left[\frac{e^{-SM}}{-s} - \frac{e^{-s}}{-s}\right]$$

$$// = 0 - \frac{e^{-s}}{-s} = \boxed{\frac{e^{-s}}{s}}$$

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Date:

⑧ $f(t) = \begin{cases} 1-t , & 0 \le t \le 1 \\ 0 , & t > 1 \end{cases}$

Ans:-

$$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(1-t)\,dt + \int_1^\infty e^{-st}.(0)\,dt$$

$$\mathcal{L}\{f(t)\} = \int_0^1 (e^{-st} - e^{-st}t) + 0$$

$$'' = \int_0^1 e^{-st} - \int_0^1 e^{-st}.t$$

$$'' = \left[\frac{e^{-st}}{-s}\right]_0^1 - \left[\frac{e^{-st}(-ts-1)}{s^2}\right]_0^1$$

$$'' = \left\{\frac{e^{-s}}{-s} - \frac{e^{-0}}{-s}\right\} - \left\{\frac{e^{-s}(-s-1)}{s^2} - \frac{(1)(0-1)}{s^2}\right\}$$

$$'' = \left\{\frac{e^{-s}}{-s} - \frac{1}{-s}\right\} - \left\{\frac{e^{-s}(-s-1)}{s^2} + \frac{1}{s^2}\right\}$$

$$'' = \left\{\frac{-e^{-s}}{s} + \frac{1}{s}\right\} - \left\{\frac{e^{-s}(-s-1)+1}{s^2}\right\}$$

$$'' = \frac{-e^{-s}+1}{s} - \frac{-e^{-s}(+s+1)+1}{s^2}$$

$$'' = \frac{(-e^{-s}+1)s - (-e^{-s}(s+1)+1)}{}$$

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$$// = \frac{-\cancel{e^{-s}.s} + 8 + \cancel{e^{-s}.s} + e^{-s} - 1}{s^2}$$

$$// = \boxed{\frac{s - 1 + e^{-s}}{s^2}}$$

(3) $f(t) = 2e^{t}$

Ans:- By using def. of Laplace Transform.

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} . 2e^{t} \, dt$$

$$// = \lim_{M \to \infty} \int_0^{M} 2e^{-st+t} \, dt$$

$$// = 2\lim_{M \to \infty} \int_0^{M} e^{+t(-s+1)} \, dt$$

$$// = 2\lim_{M \to \infty} \left[\frac{e^{+t(-s+1)}}{-s+1} \right]_0^M$$

$$// = 2\lim_{M \to \infty} \left[\left(\frac{e^{+M(-s+1)}}{-s+1} \right) - \left(\frac{e^{+0(-s+1)}}{-s+1} \right) \right]$$

$$// = 2\left[0 - \frac{1}{-s+1} \right] = 2 . \frac{\cancel{-}1}{\cancel{-}(s-1)}$$

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$$= \boxed{\frac{2}{s-1}}$$

USE THE PROPERTIES OF LAPLACE TRANSFOM AND TABLE 8.1. COMPUTE THE LAPLACE TRANSFORM OF EACH FUNCTION.

30. $f(t) = 1 + \cos 2t$

Ans: $\mathcal{L}\{f(t)\} = ?$

$$f(t) = 1 + \cos 2t$$

Taking Laplace Transform ~~on~~ of both sides

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{1 + \cos 2t\}$$

$$'' = \mathcal{L}\{1\} + \mathcal{L}\{\cos 2t\}$$

$$'' = \frac{1}{s} + \frac{s}{s^2+4}$$

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(28) $f(t) = 28e^t$

Ans:- $\mathcal{L}\{f(t)\} = ?$

$f(t) = 28e^t$

Taking Laplace Transform of each side.

$\mathcal{L}\{f(t)\} = \mathcal{L}\{28e^t\}$

$" = 28\,\mathcal{L}\{e^t\}$

$" = 28 \cdot \frac{1}{s-1}$

$" = \frac{28}{s-1}$

(49) $f(t) = e^{-2t} \cdot \cos 4t$

Ans:-

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos 4t\}\Big|_{s \to s+2}$

$" = \frac{s}{s^2 + (4)^2}\Big|_{s \to s+2}$

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$$\text{"} = \frac{s+2}{(s+2)^2 + 16}$$

INVERSE LAPLACE TRANSFORM

(66) $1/s^8$

Ans:- Let $F(s) = 1/s^8$

Taking Inverse Laplace Transform.

$$f(t) = \mathcal{L}^{-1}\left\{ 1/s^8 \right\}$$

$$\text{"} = \mathcal{L}^{-1}\left\{ \frac{7!}{7!} \cdot \frac{1}{s^8} \right\}$$

$$\text{"} = \frac{1}{7!} \mathcal{L}^{-1}\left\{ \frac{7!}{s^8} \right\}$$

$$\text{"} = \frac{1}{7!} \cdot t^{\cancel{8}\,7}$$

$$\text{"} = \boxed{\frac{1}{5040} \cdot t^7}$$

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(68) $\dfrac{1}{(s-5)^6}$

Ans:

Let $F(s) = \dfrac{1}{(s-5)^6}$

Taking Inverse Laplace Transform

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s-5)^6}\right\}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{5!}{5!}\cdot\frac{1}{(s-5)^6}\right\}$$

$$f(t) = \frac{1}{5!}\cdot\mathcal{L}^{-1}\left\{\frac{5!}{(s-5)^6}\right\}$$

$$\boxed{f(t) = \frac{1}{120}\cdot t^5\cdot e^{5t}}$$

(72) $\dfrac{1}{s^2+9}$

Ans

Let $F(s) = \dfrac{1}{s^2+9}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\}$$

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$$f(t) = \mathcal{L}^{-1}\left\{ \frac{3}{3} \cdot \frac{1}{s^2+3^2} \right\}$$

$$f(t) = \frac{1}{3}\mathcal{L}^{-1}\left\{ \frac{3}{s^2+3^2} \right\}$$

$$\boxed{f(t) = \frac{1}{3}\sin 3t}$$

74) $\frac{1}{s^2+15s+56}$

Ans:- Let $F(s) = \frac{1}{s^2+15s+56}$ —(1)

$$\frac{1}{s^2+15s+56} = \frac{1}{s^2+8s+7s+56} = \frac{1}{s(s+8)+7(s+8)}$$

$$\frac{1}{(s+8)(s+7)} = \frac{A}{s+8} + \frac{B}{s+7}$$

$$\frac{1}{\cancel{(s+8)(s+7)}} = \frac{A(s+7)+B(s+8)}{\cancel{(s+8)(s+7)}}$$

$$1 = A(s+7) + B(s+8)$$

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For $S = -7$

$1 = 0 + B(-7+8)$

$\boxed{1 = B}$

For $S = -8$

$1 = A(-8+7) + 0$

$1 = A(-1)$

$\boxed{A = -1}$

$$\frac{1}{(S+8)(S+7)} = \frac{-1}{S+8} + \frac{1}{S+7}$$

put in (i)

$$F(S) = \frac{1}{S+7} - \frac{1}{S+8}$$

Now taking inverse laplace transform.

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{S+7}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{S+8}\right\}$$

$$f(t) = e^{-7t} - e^{-8t}$$

(77) $\dfrac{S}{S^2 - 5S - 14}$

Ans Let $F(S) = \dfrac{S}{S^2 - 5S - 14}$ —— (i)

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$$\frac{s}{s^2-5s-14} = \frac{s}{(s-7)(s+2)} = \frac{A}{s-7} + \frac{B}{s+2} \text{ --(ii)}$$

$$\Rightarrow \frac{s}{\cancel{(s-7)(s+2)}} = \frac{A(s+2)+B(s-7)}{\cancel{(s-7)(s+2)}}$$

$$s = A(s+2) + B(s-7) \text{ --(iii)}$$

For $s = -2$ in (iii)

$$-2 = A(0) + B(-9)$$

$$-2 = -9B$$

$$\boxed{B = 2/9}$$

For $s = 7$ in (iii)

$$7 = A(9) + B(0)$$

$$7 = 9A$$

$$\boxed{A = 7/9}$$

Put in (ii)

$$\frac{s}{(s-7)(s+2)} = \frac{7}{9(s-7)} + \frac{2}{9(s+2)}$$

Put in (i)

$$F(s) = \frac{7}{9(s-7)} + \frac{2}{9(s+2)}$$

$$F(s) = \frac{1}{9}\left[\frac{7}{s-7} + \frac{2}{s+2}\right]$$

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$$I = \frac{e^{-st}}{-s} \cdot t + \int \frac{e^{-st}}{-s} \Rightarrow I = -\frac{e^{-st} \cdot t}{s} + \frac{1}{s} \cdot \frac{e^{-st}}{-s}$$

$$\Rightarrow I = -\frac{e^{-st} \cdot t}{s} - \frac{e^{-st}}{s^2}$$

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Taking Inverse Laplace Transform

$$f(t) = \frac{1}{9} \mathcal{L}^{-1}\left\{ \frac{7}{s-7} + \frac{2}{s+2} \right\}$$

$$f(t) = \frac{1}{9}\left[7 \cdot e^{7t} + 2e^{-2t} \right]$$

Remaining Questions.

Q1 $f(t) = 21t$

Ans.

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$\mathcal{L}\{21t\} = \int_0^\infty e^{-st} \cdot 21t\, dt$$

$$\mathcal{L}\{21t\} = 21 \lim_{M \to \infty} \int_0^M e^{-st} \cdot t\, dt$$

$$\text{"} = 21 \lim_{M \to \infty} \left[-e^{-st}\left(\frac{t}{s} - \frac{1}{s^2} \right) \right]_0^M$$

$$\text{"} = 21 \cdot \lim \left\{ 0 - (1)\left(0 - \frac{1}{s^2}\right) \right\}$$

$$\text{"} = 21 \left\{ \frac{1}{s^2} \right\} = \boxed{\frac{21}{s^2}}$$

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② $f(t) = 7e^{-t}$

Ans:

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} \cdot f(t)\, dt$$

$$\mathcal{L}\{7e^{-t}\} = \int_0^{\infty} e^{-st} \cdot 7e^{-t}\, dt$$

$$'' = 7 \int_0^{\infty} e^{-st-t}\, dt$$

$$'' = 7 \cdot \int_0^{\infty} e^{-t(s+1)}\, dt$$

$$'' = 7 \lim_{M \to \infty} \int_0^{M} e^{-t(s+1)}\, dt$$

$$'' = 7 \lim_{M \to \infty} \left[\frac{e^{-t(s+1)}}{-(s+1)} \right]_0^{M}$$

$$'' = 7 \cdot \lim_{M \to \infty} \left[\frac{e^{-M(s+1)}}{-(s+1)} - \frac{1}{-(s+1)} \right]$$

$$'' = 7 \cdot \left[0 + \frac{1}{s+1} \right]$$

$$'' = \boxed{\frac{7}{s+1}}$$

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EXERCISE 8.2

Solve the IVP

(i) $y'' - y' - 2y = 0$, $y(0) = 8$, $y'(0) = 7$

Ans: Taking Laplace Transform of both sides

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) - s\mathcal{L}\{y\} + y(0) - 2\mathcal{L}\{y\} = 0$$

$$s^2 Y(s) - s(8) - 7 - sY(s) + 8 - 2Y(s) = 0$$

$$Y(s)\left[s^2 - s - 2\right] - 8s + 1 = 0$$

$$Y(s) = \frac{8s - 1}{s^2 - s - 2}$$

$$Y(s) = \frac{8s - 1}{s^2 - 2s + s - 2} = \frac{8s - 1}{s(s-2) + 1(s-2)}$$

$$Y(s) = \frac{8s - 1}{(s+1)(s-2)} \quad \text{---(i)}$$

Let $\frac{8s - 1}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$ ---(ii)

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$$\cancel{y(t) = \frac{1}{3}\left[\mathcal{L}^{-1}\left\{\frac{-7}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{31}{s-2}\right\}\right]}$$

$$\cancel{y(t) = \frac{1}{3}\left[-7e^{-t} + 31e^{2t}\right]}$$

$$\cancel{\frac{8s+1}{(s+1)(s-2)} = \frac{A(s-2)+B(s+1)}{(s+1)(s-2)}}$$

$$\cancel{8s+1 = A(s-2) + B(s+1)} \quad \text{— (iii)}$$

~~For $s = -1$ put in (iii)~~ | ~~For $s = +2$ put in (iii)~~

$$\cancel{8(-1)+1 = A(-1-2) + 0}$$

$$\cancel{-7 = -3A}$$

$$\cancel{\boxed{A = 7/3}}$$

$$\cancel{17 = A(0) + B(3)}$$

$$\cancel{\boxed{B = 17/3}}$$

$$\frac{8s-1}{(s+1)(s-2)} = \frac{A(s-2)+B(s+1)}{(s+1)(s-2)}$$

$$8s-1 = A(s-2) + B(s+1) \quad \text{— (iii)}$$

For $s = -1$, put in (iii)

$$-9 = A(-3) + 0$$

$$\boxed{A = 3}$$

For $s = 2$ put in (iii)

$$15 = 3B$$

$$\boxed{B = 5}$$

Put in (ii)

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$$\frac{8s-1}{(s+1)(s-2)} = \frac{3}{s+1} + \frac{5}{s-2}$$

Put in (1)

$$Y(s) = \frac{3}{s+1} + \frac{5}{s-2}$$

Taking Inverse Laplace Transform

$$\boxed{y(t) = 3 \cdot e^{-t} + 5e^{-2}}$$

(6) $y'' + y = \cos t$, $y(0) = 3$, $y'(0) = 4$

Ans: Taking laplace transform on both sides

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\cos t\}$$

$$s^2 \mathcal{L}\{y\} - s'y(0) - y'(0) + s'\mathcal{L}\{y\} - y(0) = \frac{s}{s^2+1}$$

$$s^2 Y(s) - s.(3) - 4 + s \cdot Y(s) - 3 = \frac{s}{s^2+1}$$

$$Y(s)\left[s^2 + s\right] - 3s - 7 = \frac{s}{s^2+1}$$

$$\cancel{Y(s)[s^2+s] = \frac{s}{s^2+1} + \frac{3s0-1}{1}}$$

$$\cancel{\frac{Y(s)[s^2+s]}{s^2+s} = \frac{s+3s(s^2+1)-(s^2+1)}{(s^2+1)(s^2+s)}}$$

$$\cancel{Y(s) = \frac{s+3s^3+3s-s^2-1}{s^4+s^3+s^2+s}}$$

$$s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0) + \mathcal{L}\{y\} = \frac{s}{s^2+1}$$

$$s^2 . Y(s) - s.(3) - (4) + Y(s) = \frac{s}{s^2+1}$$

$$Y(s)(s^2+1) - 3s - 4 = \frac{s}{s^2+1}$$

$$Y(s)(s^2+1) = \frac{s}{s^2+1} + \frac{3s+4}{1}$$

$$Y(s) = \frac{s}{(s^2+1)^2} + \frac{3s+4}{s^2+1}$$

$$Y(s) = \frac{s+(s^2+1)(3s+4)}{(s^2+1)^2} = \frac{s+3s^3+4s^2+3s+4}{(s^2+1)^2}$$

$$Y(s) = \frac{3s^3+4s^2+4s+4}{(s^2+1)^2} \quad -(1)$$

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Let $$\frac{3s^3+4s^2+5s+4}{(s^2+1)^2} = \frac{As+B}{s^2+1} + \frac{Cs+D}{(s^2+1)^2} \quad \text{(ii)}$$

$$\frac{3s^3+4s^2+5s+4}{\cancel{(s^2+1)^2}} = \frac{(As+B)(s^2+1)+Cs+D}{\cancel{(s^2+1)^2}}$$

$$3s^3+4s^2+5s+4 = (As+B)(s^2+1)+Cs+D$$

$$3s^3+4s^2+5s+4 = As^3+As+Bs^2+B+Cs+D$$

$$3s^3+4s^2+5s+4 = As^3+As+Cs+Bs^2+B+D$$

Comparing Co-efficients of s^3, s^2, s and Constants

$\boxed{A=3}$ $\boxed{B=4}$, $A+C=5$

$B+D=4$

$\therefore 3+C=5 \Rightarrow \boxed{C=2}$

$4+D=4 \Rightarrow \boxed{D=0}$

Put A, B, C and D values in (ii)

$$\frac{3s^3+4s^2+5s+4}{(s^2+1)^2} = \frac{3s+4}{s^2+1} + \frac{2s}{(s^2+1)^2}$$

Put in (i)

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$$Y(s) = \frac{3s+4}{s^2+1} + \frac{2s}{(s^2+1)^2}$$

$$Y(s) = \frac{3s}{s^2+1} + \frac{4}{s^2+1} + \frac{2s}{(s^2+1)^2}$$

Taking inverse laplace transform,

$$f(t) = \mathcal{L}^{-1}\left\{\frac{3s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{4}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2s}{(s^2+1)^2}\right\}$$

$$f(t) = 3.\cos t + 4\sin t + \cancel{2}\frac{1}{\cancel{2}}t^1.\sin t$$

$$f(t) = 3\cos t + 4\sin t + t\sin t.$$

② $y'' + 8y' + 7y = 0, \quad y(0) = 0, \; y'(0) = 1$

Ans:

$$\mathcal{L}\{y'' + 8y' + 7y\} = 0$$

$$\mathcal{L}\{y''\} + 8\mathcal{L}\{y'\} + 7\mathcal{L}\{y\} = 0$$

$$s^2.\mathcal{L}\{y\} - s\,y(0) - y'(0) + 8[s\mathcal{L}\{y\} - y(0)] + 7\mathcal{L}\{y\} = 0$$

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$s^2 Y(s) - 1 + 8s \cdot Y(s) + 7 \cdot Y(s) = 0$

$Y(s)[s^2 + 8s + 7] = 1$

$$Y(s) = \frac{1}{s^2+8s+7} = \frac{1}{s^2+1s+7s+7} = \frac{1}{s(s+1)+7(s+1)}$$

$$Y(s) = \frac{1}{(s+1)(s+7)} = \frac{1}{6(s+1)} - \frac{1}{6(s+7)}$$

Now taking inverse laplace transform

$$f(t) = \frac{1}{6}\left[\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+7}\right\}\right]$$

$$f(t) = \frac{1}{6}\left[e^{-t} - e^{-7t}\right]$$

⑧ $y''' + 6y'' + 9y' + 4y = 0, \quad y(0) = 0, \; y'(0) = -3$

$y''(0) = 2$

Ans: $\mathcal{L}\{y'''\} + 6\mathcal{L}\{y''\} + 9\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 0$

$[s^3 \mathcal{L}\{y\} - s^2 y''(0) - s y'(0) - y(0)] + 6[s^2 \mathcal{L}\{y\} - s y'(0)$

$- y(0)] + 9[s \mathcal{L}\{y\} - y(0)] + 4\mathcal{L}\{y\} = 0$

$-1+6-$

$(s+1)$

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$$\left[s^3 . Y(s) - s^2(2) - s(-3)\right] + 6\left[s^2 . Y(s) - s(-3)\right]$$
$$+9\left[s . Y(s)\right] + 4 . Y(s) = 0$$

$$s^3 . Y(s) - 2s^2 + 3s + 6s^2 . Y(s) + 18s + 9s . Y(s)$$
$$+4 . Y(s) = 0$$

$$s^3 . Y(s) + 6s^2 . Y(s) + 9s . Y(s) + 4 . Y(s) = 2s^2 - 3s - 18s$$

$$\frac{Y(s)\left[\cancel{s^3 + 6s^2 + 9s + 4}\right]}{\cancel{s^3 + 6s^2 + 9s + 4}} = \frac{2s^2 - 21s}{s^3 + 6s^2 + 9s + 4}$$

$$Y(s) = \frac{2s^2 - 21s}{s^3 + 6s^2 + 9s + 4} \quad \text{—— (1)}$$

Taking factors of denominator. By inspection $(s+1)$ is a factor, now finding other factors by using synthetic division.

	1	6	9	4
-1		-1	-5	-4
	1	5	4	0

$$s^2 + 5s + 4 = 0$$
$$s^2 + 4s + s + 4 = 0$$
$$s(s+4) + 1(s+4) = 0$$
$$(s+4)(s+1) = 0$$

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(i) $\Rightarrow$ $Y(s) = \dfrac{2s^2 - 21s}{(s+1)^2(s+4)}$

By partial fractions

$$Y(s) = \frac{-98}{9(x+1)} + \frac{23}{3(x+1)^2} + \frac{116}{9(x+4)}$$

Taking inverse Laplace Transform

$$f(t) = -\frac{98}{9}\mathcal{L}^{-1}\left\{\frac{1}{x+1}\right\} + \frac{116}{9}\mathcal{L}^{-1}\left\{\frac{1}{x+4}\right\} + \mathcal{L}^{-1}\left\{\frac{23}{3(x+1)^2}\right\}$$

$$f(t) = -\frac{98}{9}\cdot e^{-t} + \frac{116}{9}e^{-4t} + \frac{23}{3}\cdot e^{-t}\cdot t$$

EXERCISE 8.3

① $-28\,u(t-3)$

Ans: $f(t) = -28$

& $a = 3$

$\therefore \mathcal{L}^{a}\{28\,u(t-3)\} = e^{3s}\cdot\dfrac{-28}{s}$

③ $3\,u(t-8) - u(t-4)$

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Sol:

$$G(s) = \mathcal{L}\{3u(t-8)\} - \mathcal{L}\{u(t-4)\}$$

$$G(s) = \frac{3}{s} \cdot e^{-8s} - \frac{1}{s} e^{-4s}$$

23 $\frac{-3}{se^{\pi s}}$

Ans:

$$G(s) = -\frac{3}{s} \cdot e^{-\pi s}$$

Taking inverse laplace transform

$$f(t) = -3u(t-\pi)$$

25 $\cancel{2e^{3s} - 4e^{3s} - 3}$

$$\frac{2e^{3s} - 3}{se^{4s}}$$

Ans: let $G(s) = \frac{2e^{3s} - 3}{se^{4s}}$

$$G(s) = \frac{2e^{3s}}{se^{4s}} - \frac{3}{se^{4s}}$$

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$$G(s) = \frac{2}{s} e^{-s} - \frac{3}{s} \cdot e^{-4s}$$

Taking inverse laplace transform

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s} \cdot e^{-s}\right\} - \mathcal{L}\left\{\frac{3}{s} e^{-4s}\right\}$$

$$f(t) = 2u(t-1) - 3u(t-4)$$