# EXERCISE 8.4

Q1:- $f(t) = 1$ , $g(t) = t^2$

Ans:- $(f * g)(t) = ?$

$$(f * g) = \int_0^t f(t-v) \cdot g(v)\, dv$$

$$'' = \int_0^t (1) \cdot v^2\, dv$$

$$'' = \int_0^t v^2\, dv$$

$$'' = \left[\frac{v^3}{3}\right]_0^t$$

$$'' = \frac{t^3}{3} - 0 = \boxed{t^3/3}$$

Scanned with CamScanner

Date:

② $f(t) = e^{-3t}, \quad g(t) = 2$ ?

Ans:- $(f * g)(t) = ?$

$$(f*g)(t) = \int_0^t f(t-v)\, g(v)\, dv$$

$$'' = \int_0^t e^{-3(t-v)} \cdot 2\, dv$$

$$'' = \int_0^t 2 \cdot e^{-3t+3v}\, dv$$

$$'' = 2 \cdot e^{-3t} \cdot \int_0^t e^{3v}\, dv$$

$$'' = 2e^{-3t} \left[ \frac{e^{3v}}{3} \right]_0^t$$

$$'' = 2e^{-3t} \left[ \frac{e^{3t}}{3} - \frac{e^0}{3} \right]$$

$$'' = 2e^{-3t} \left[ \frac{e^{3t} - 1}{3} \right]$$

$$'' = \frac{2}{3}\left(1 - e^{-3t}\right)$$

Day: M T W T F S

(13) $\dfrac{1}{s^2(s+1)}$

Ans:

Let $F(s) = \frac{1}{s^2}$, $G(s) = \frac{1}{s+1}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 1!\,\mathcal{L}^{-1}\left\{\frac{1!}{s^2}\right\} = t = f(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t} = g(t)$$

$$(f*g)(t) = \int_0^t f(t-v).g(v)\,dv$$

$$\text{"} = \int_0^t (t-v).e^{-v}\,dv$$

$$\text{"} = \int_0^t (e^{-v}.t - ve^{-v})dv$$

$$\text{"} = \int_0^t e^{-v}.t\,dv - \int_0^t ve^{-v}\,dv$$

$$\text{"} = t\left[\frac{e^{-v}}{-1}\right]_0^t - \left[e^{-v}(-v-1)\right]_0^t$$

$$\text{"} = t\left[-e^{-t} - \frac{e^0}{-1}\right] - \left[e^{-t}(-t-1) - e^0(-1)\right]$$

$$\text{"} = t\left[-e^{-t}+1\right] - \left[-e^{-t}.t - e^{-t} + 1\right]$$

$$\text{"} = -\cancel{e^{-t}.t} + t + \cancel{e^{-t}t} + e^{-t} - 1$$

$'' = t + e^{-t} - 1$

(14) $\dfrac{1}{s^3(s+3)}$

Ans:- Let $F(s) = 1/s^3$, $G(s) = \dfrac{1}{s+3}$

$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\left\{1/s^3\right\}$

$f(t) = \dfrac{1}{2!}\mathcal{L}^{-1}\left\{\dfrac{2!}{s^3}\right\}$

$f(t) = \dfrac{1}{2} . t^2$

$\mathcal{L}^{-1} G(s) = \mathcal{L}^{-1}\left\{\dfrac{1}{s+3}\right\}$

$\mathcal{L}^{-1}\ g(t) = e^{-3t}$

$(f \times g)(t) = \int_0^t f(t-v)\, g(v)\, dv$

$'' = \int_0^t \left(\dfrac{(t-v)^2}{2}\right)\left(e^{-3v}\right) dv$

$'' = \dfrac{1}{2}\int_0^t (t^2 + v^2 - 2tv)(e^{-3v})\, dv$

$'' = \dfrac{1}{2}\int_0^t (e^{-3v} . t^2 + v^2 . e^{-3v} - 2tv e^{-3v})\, dv$

$'' = \dfrac{1}{2}\left[t^2 \int_0^t e^{-3v} + \int_0^t v^2 e^{-3v} - 2t \int_0^t v e^{-3v}\right]$

Day: M T W T F S

$$I1 = \frac{1}{2}\left\{t^2\left[\frac{e^{-3v}}{-3}\right]_0^t + \left[-\frac{1}{29}\left(9e^{-3v}\cdot v^2 - 2\left(-3e^{-3v}\cdot v - e^{-3v}\right)\right)\right]_0^t - 2t\left[\frac{1}{9}\left(-3e^{-3v}v - e^{-3v}\right)\right]\right.$$

$$I1 = \frac{1}{2}\left[t^2\left(\frac{e^{-3t}}{-3} - \frac{e^0}{-3}\right)\right] + \left[-\frac{1}{29}\left(9e^{-3t}\cdot t^2 - 2\left(-3e^{-3t}\cdot t - e^{-3t}\right)\right) + \frac{1}{29}\left(9(1)(0)^2 - 2\left(-3e^0\cdot(0) - 1\right)\right)\right.$$

$$-2t\left[\frac{1}{9}\left(-3e^{-3t}\cdot t - e^{-3t}\right) - \frac{1}{9}\left(0 - e^0\right)\right]$$

$$I1 = \frac{1}{2}\left[t^2\left(-\frac{e^{-3t}}{3} + \frac{1}{3}\right)\right] + \left[-\frac{1}{29}\left(9t^2e^{-3t} + 6te^{-3t} + 2\right.\right.$$

$$+ \frac{1}{29}(2)\left.\right] - 2t\left[\frac{-3e^{-3t}\cdot t}{9} - \frac{e^{-3t}}{9} + \frac{1}{9}\right]$$

$$I1 = \frac{1}{2}\left[\frac{t(1 - e^{-3t})}{3}\right] + \left[-\frac{9t^2e^{-3t}}{29} - \frac{6}{29}te^{-3t} - \frac{2}{29}\right.$$

$$+ \frac{2}{29}\left.\right] + \left[\frac{6t^2e^{-3t}}{9} + \frac{2te^{-3t}}{9} - \frac{2t}{9}\right]$$

$$I1 = \frac{t}{6} - \frac{te^{-3t}}{6} - \frac{9t^2e^{-3t}}{29} - \frac{6}{29}te^{-3t} - \frac{2}{29}e^{-3t}$$

$$+ \frac{2}{29} + \frac{2t^2}{3}e^{-3t} + \frac{2}{9}te^{-3t} - \frac{2t}{9}$$

# EXERCISE 8.5

Day: M T W T F S

Date:

① $\begin{cases} x' - 2x + 3y = 0 & , x(0) = 0 \\ y' + 9x + 4y = 0 & , y(0) = 4 \end{cases}$

Ans:- Taking laplace transform of 1st eqtion.

$$\mathcal{L}\{x' - 2x + 3y\} = 0$$
$$\mathcal{L}\{x'\} - 2\mathcal{L}\{x\} + 3\mathcal{L}\{y\} = 0$$
$$s'\mathcal{L}\{x\} - x(0) - 2X(s) + 3Y(s) = 0$$
$$s.X(s) - 0 - 2X(s) + 3Y(s) = 0$$
$$X(s)(s-2) + 3Y(s) = 0 \quad \text{—(i)}$$

Now taking laplace transform of 2nd equation

$$\mathcal{L}\{y' + 9x + 4y\} = 0$$
$$\mathcal{L}\{y'\} + 9\mathcal{L}\{x\} + 4\mathcal{L}\{y\} = 0$$
$$s'\mathcal{L}\{y\} - y(0) + 9X(s) + 4Y(s) = 0$$
$$s.Y(s) - 4 + 9X(s) + 4Y(s) = 0$$
$$Y(s)(s+4) + 9X(s) - 4 = 0 \quad \text{—(ii)}$$

Now Multiplying (s-2) with eq(ii) and 9 with eq(i) and Subtracting eq(ii) from eq(i)

Scanned with CamScanner

Day: M T W T F S

$9(s-2)X(s) + 27Y(s) = 0$

$\pm 9(s-2)X(s) \pm (s-2)(s+4)Y(s) \mp 4(s-2) = 0$

$27Y(s) - (s-2)(s+4)Y(s) + 4(s-2) = 0$

$Y(s)[27-(s-2)(s+4)] = -4(s-2)$

$$Y(s) = \frac{-4(s-2)}{27-[s^2+2s-8]}$$

$$Y(s) = \frac{-4(s-2)}{27-s^2-2s+8} = \frac{-4(s-2)}{-s^2-2s+35}$$

$$Y(s) = \frac{-4(s-2)}{-(s^2+2s-35)} \Rightarrow \frac{4(s-2)}{s^2-5s+7s-35}$$

$$Y(s) = \frac{4(s-2)}{s(s-5)+7(s-5)} = \frac{4(s-2)}{(s+7)(s-5)}$$

By partical fraction.

$$Y(s) = \frac{3}{s+7} + \frac{1}{s-5}$$

Now taking inverse laplace transform

Day: M T W T F S

Date:

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{3}{s+7}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-5}\right\}$$

$$\boxed{y(t) = 3e^{-7t} + e^{5t}}$$

Now Xing 's+4' with eq(i) and '3' with eq(ii) and subtracting (ii) from (i)

$$X(s)(s+4)(s-2) + 3(s+4)Y(s) = 0$$

$$+27X(s) \mp 12 \pm 3(s+4)Y(s) = 0$$

$$X(s)(s+4)(s-2) - 27X(s) + 12 = 0$$

$$X(s)\left[(s+4)(s-2) - 27\right] = -12$$

$$X(s)\left[s^2+2s-8-27\right] = -12$$

$$X(s)\left[s^2+2s-35\right] = -12$$

$$X(s)\left[s^2+7s-5s-35\right] = -12$$

$$X(s)\left[s(s+7)-5(s+7)\right] = -12$$

$$X(s)(s+7)(s-5) = -12$$

$$X(s) = \frac{-12}{(s+7)(s-5)}$$

$$X(s) = \frac{-1}{s-5} + \frac{1}{s+7}$$

Now taking inverse laplace transform.

$$\boxed{X(t) = -e^{5t} + e^{-7t}}$$

$$\begin{cases} x(t) = e^{-7t} - e^{5t} \\ y(t) = 3e^{-7t} + e^{5t} \end{cases}$$

---

## EXERCISE 8.6

**20** $m = 4, \quad k = 16, \quad x(0) = 1, \quad x'(0) = 0$

Ans:
=

$$\begin{cases} m \frac{d^2x}{dt^2} + kx = 0 \\ x(0) = 1, \quad x'(0) = 0 \end{cases}$$

$$4x''(t) + 16x = 0$$

Taking Laplace transform

$$4\mathcal{L}\{x''\} + 16\mathcal{L}\{x\} = 0$$

$$4[s^2\mathcal{L}\{x\} - s^1 \cdot x(0) - x'(0)] + 16X(s) = 0$$

$$4[s^2 \cdot X(s) - s(1) - 0] + 16X(s) = 0$$

$$4\{s^2 \cdot X(s) - s\} + 16 \cdot X(s) = 0$$

$$4s^2 \cdot X(s) - 4s + 16 \cdot X(s) = 0$$

$$X(s)[4s^2 + 16] - 4s = 0$$

$$X(s) = \frac{4s}{4s^2+16} = \frac{s}{s^2+4}$$

$$\cancel{X(s) = \frac{2}{s^2+2^2} \cdot \frac{1}{2}}$$

Now taking inverse laplace transform.

$$\cancel{x(t) = \cos 2t}$$

$$\cancel{x(t) = \frac{1}{2} \sin 2t}$$

$$x(t) = \cos 2t$$

**(2) $m = 1$, $k = 9$, $x(0) = 3$, $x'(0) = -2$**

$$\begin{cases} x''(t) + 9x(t) = 0 \\ x(0) = 3, \quad x'(0) = -2 \end{cases}$$

Date:______ Day: M T W T F S

Taking Laplace Transform.

$\mathcal{L}\{x''\} + 9\,\mathcal{L}\{x\} = 0$

$s^2 \cdot \mathcal{L}\{x\} - s^1 \cdot x(0) - x'(0) + 9\,X(s) = 0$

$s^2 \cdot X(s) - s^1(3) - (-2) + 9\,X(s) = 0$

$s^2 \cdot X(s) - 3s + 2 + 9\,X(s) = 0$

$X(s)\left[s^2 + 9\right] - 3s + 2 = 0$

$X(s) = \dfrac{3s - 2}{s^2 + 9} = \dfrac{3s}{s^2+9} - \dfrac{2}{s^2+9}$

Now taking inverse laplace transform.

$x(t) = \mathcal{L}^{-1}\left\{\dfrac{3s}{s^2+9}\right\} - \mathcal{L}^{-1}\left\{\dfrac{2}{s^2+9}\right\} \times \dfrac{3}{3}$

$x(t) = 3\cos 3t - \dfrac{2}{3}\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\}$

$x(t) = 3\cos 3t - \dfrac{2}{3} \cdot \sin 3t$