QUESTION No. IV:

A 16 lb weight stretches ------. ---------.---- return to the equilibrium position?

Part a (object released at zero initial velocity.)

Sol:- GIVEN DATA:-

$F = \boxed{16\ lb}$, $S = \frac{8}{12}\ ft = \boxed{2/3\ ft}$

$x(0) = 4^{in} \Rightarrow x(0) = \frac{4}{12}\ ft = \boxed{1/3}$

$x'(0) = 0,$

REQUIRED DATA:-

$t = ?$ (When object is at equilibrium)
i-e $x(t) = 0$

$x(4) = ?$

SOLUTION:-

We know that

$F = KS$

OR $\quad K = F/S = \frac{16}{2/3} = \frac{16 \times 3}{2} = \frac{48}{2} = \boxed{24\ lb/ft}$

Now also $F = mg$

$16 = m(32)$

$m = \frac{16}{32} = \boxed{\frac{1}{2}\ Slugs}$

The differential equation used to find the displacement of the object at time t is given by:

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \text{—(i)}$$

Putting values in (i)

$$\frac{1}{2}\cdot\frac{d^2x}{dt^2} + 24x = 0$$

Multiplying '2' both sides

$$\frac{d^2x}{dt^2} + 48x = 0$$

Now solving this homogenous equation by writing it's corresponding characteristic equation.

$$r^2 + 48 = 0$$

$$r^2 = -48$$

$$r^{\cancel{2}} = \pm\sqrt{-48}$$

$$r^{\cancel{2}} = \pm\sqrt{-1\times16\times3}$$

$$r = \pm\sqrt{-1}\cdot\sqrt{16}\cdot\sqrt{3}$$

$$r = \pm 4i\sqrt{3}$$

$$\boxed{r = \pm i\,4\sqrt{3}}$$

$\alpha = 0$ $\beta = 4\sqrt{3}$

$\therefore$ It's general Solution is given by:

$$x(t) = e^{0t}\left[c_1 \cos 4\sqrt{3}t + c_2 \sin 4\sqrt{3}t\right]$$

$$\boxed{x(t) = c_1 \cos 4\sqrt{3}t + c_2 \sin 4\sqrt{3}t} \quad \text{—(ii)}$$

Differentiating (ii) with respect to 't'.

$$x'(t) = -4\sqrt{3}.c_1.\sin 4\sqrt{3}t + c_2 \cos 4\sqrt{3}t .4\sqrt{3}$$

$$\boxed{x'(t) = 4\sqrt{3}\left(-c_1 \sin 4\sqrt{3}t + c_2 \cos 4\sqrt{3}t\right)} \quad \text{—(iii)}$$

Now applying initial conditions on $x(t)$ and $x'(t)$

$$x(0) = c_1 \cos 0 + c_2 \sin 0$$

$$\frac{1}{3} = c_1(1) + c_2(0)$$

$$\boxed{c_1 = 1/3}$$

$$x'(0) = 4\sqrt{3}\left(-c_1 \sin 0 + c_2 \cos 0\right)$$

$$0 = 4\sqrt{3}\left(-c_1(0) + c_2(1)\right)$$

$$0 = 4\sqrt{3}\left(0 + c_2\right)$$

$$0 = 4\sqrt{3}.c_2$$

$$\boxed{c_2 = 0}$$

$$\therefore x(t) = \frac{1}{3}\cos 4\sqrt{3}t$$

Now for t at equilibrium position.

$$x(t) = 0$$

$$\frac{1}{3}\cos 4\sqrt{3}t = 0$$

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$$\cos 4\sqrt{3}t = 0$$

$$4\sqrt{3}t = \cos^{-1}(0)$$

$$4\sqrt{3}t = \frac{\pi}{2}$$

$$t = \frac{\pi}{8\sqrt{3}} \approx \boxed{0.2267\,s}$$

Now put $t = 4\,s$ in $x(t)$

$$x(4) = \frac{1}{3}\cos(4\sqrt{3}.4) = \frac{1}{3}\cos 16\sqrt{3} \approx \frac{1}{3}(-0.846)$$

$$\boxed{x(4) \approx -0.282}$$

Part b

Initial condition changed i-e

$x(0) = 0$

$x'(0) = -2$

$t = ?$

eq (ii) $\Rightarrow$ $x(t) = c_1 \cos 4\sqrt{3}t + c_2 \sin 4\sqrt{3}t$

eq (iii) $\Rightarrow$ $x'(t) = 4\sqrt{3}(-c_1 \sin 4\sqrt{3} + c_2 \cos 4\sqrt{3}t)$

$$x(0) = c_1 \cos 0 + c_2 \sin 0$$

$$0 = c_1(1) + 0$$

$$\boxed{c_1 = 0}$$

$$x'(0) = 4\sqrt{3}(-c_1 \sin 0 + c_2 \cos 0)$$

$$-2 = 4\sqrt{3}(0 + c_2(1))$$

$$-2 = 4\sqrt{3}(c_2)$$

$$c_2 = \frac{-\cancel{2}^{1}}{{}_{2}\cancel{4}\sqrt{3}} = \boxed{-\frac{1}{2\sqrt{3}}}$$

$$\therefore \quad x(t) = 0 \cos 4\sqrt{3}t + \left(\frac{-1}{2\sqrt{3}}\right) \sin 4\sqrt{3}t$$

$$x(t) = -\frac{1}{2\sqrt{3}} \cdot \sin 4\sqrt{3}t$$

Now for t. at equilibrium position.

$$x(t) = 0$$

$$-\frac{1}{2\sqrt{3}} \sin 4\sqrt{3}t = 0$$

$$\sin 4\sqrt{3}t = 0$$

$$4\sqrt{3}t = \sin^{-1}(0)$$

$$4\sqrt{3}t = \pi$$

$$\boxed{t = \frac{\pi}{4\sqrt{3}}}$$

Ans

$$\therefore \quad x(t) = 0 \cos 4\sqrt{3}t + \left(\frac{-1}{2\sqrt{3}}\right) \sin 4\sqrt{3}t$$

$$x(t) = -\frac{1}{2\sqrt{3}} \cdot \sin 4\sqrt{3}t$$

Now for t. at equilibrium Position.

$$x(t) = 0$$

$$-\frac{1}{2\sqrt{3}} \sin 4\sqrt{3}t = 0$$

$$\sin 4\sqrt{3}t = 0$$

$$4\sqrt{3}t = \sin^{-1}(0)$$

$$4\sqrt{3}t = \pi$$

$$\boxed{t = \frac{\pi}{4\sqrt{3}}}$$

Ans

QUESTION No. V: Suppose that an object ....

.... in the downward

direction.

Sol:. GIVEN DATA:-

$$m = 1$$

$$K = 5/4$$

$$F_R = 2\frac{dx}{dt}$$

$$x(0) = 0$$

$$x'(0) = 3$$

REQUIRED DATA:-

$$x(t) = ?$$

SOLUTION :-

The differential equation used to find the displacement of the object at time t is given by:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + Kx = 0 \quad \text{—(i)}$$

here $c\frac{dx}{dt} = F_r$

Putting values in (i), we get

$$(1)\frac{d^2x}{dt^2} + (2)\frac{dx}{dt} + \left(\frac{5}{4}\right)x = 0$$

Multiplying '4' both sides.

$$4\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 5x = 0$$

The characteristic equation of above 2nd order differential equation is given by:

$$4r^2 + 8r + 5 = 0$$

$$r = \frac{-(8) \pm \sqrt{8^2 - 4(4)(5)}}{2(4)} = \frac{-8 \pm \sqrt{64 - 80}}{8}$$

$$r = \frac{-8 \pm \sqrt{-16}}{8} = \frac{-8 \pm 4i}{8} = -1 \pm \frac{1}{2}i$$

$$\alpha = -1, \quad \beta = 1/2$$

$$x(t) = e^{-1t}\left[c_1 \cos(\tfrac{1}{2}t) + c_2 \sin(\tfrac{1}{2}t)\right]$$

Now $x'(t) = e^{-t}\left[-\frac{1}{2}c_1 \sin(\frac{1}{2}t) + \frac{1}{2}c_2 \cos(\frac{1}{2}t)\right] +$

$$\left[c_1 \cos(\tfrac{1}{2}t) + c_2 \sin(\tfrac{1}{2}t)\right] \cdot -e^{-t}$$

$$x'(t) = e^{-t}\left[\left\{-\tfrac{1}{2}c_1 \sin(\tfrac{1}{2}t) + \tfrac{1}{2}c_2 \cos(\tfrac{1}{2}t)\right\} - \left\{c_1 \cos(\tfrac{1}{2}t) + c_2 \sin(\tfrac{1}{2}t)\right\}\right]$$

Now applying intial conditions.

$$x(0) = e^{-(0)}\left[c_1 \cos(0) + c_2 \sin(0)\right]$$

$$0 = 1\left[c_1(1) + c_2(0)\right]$$

$$\boxed{0 = c_1}$$

$$x'(0) = e^{-0}\left[\left\{-\tfrac{1}{2}c_1 \sin(0) + \tfrac{1}{2}c_2 \cos(0)\right\} - \left\{c_1 \cos(0) + c_2 \sin(0)\right\}\right]$$

$$3 = 1\left[\left\{-\tfrac{1}{2}c_1(0) + \tfrac{1}{2}c_2(1)\right\} - \left\{c_1(1) + c_2(0)\right\}\right]$$

$$3 = 0 + \tfrac{1}{2}c_2 - c_1$$

$$3 = \tfrac{1}{2}c_2 - c_1$$

Put $c_1 = 0$

$$3 = \tfrac{1}{2}c_2 - 0$$

$$\frac{1}{2} c_2 = 3$$

$$\boxed{c_2 = 6}$$

So $x(t) = e^{-t}\left[(0)\cos\left(\frac{1}{2}t\right) + (6)\sin\left(\frac{1}{2}t\right)\right]$

$$x(t) = e^{-t}\left[0 + 6\sin\left(\tfrac{1}{2}t\right)\right]$$

$$\boxed{x(t) = 6 . e^{-t} . \sin\left(\tfrac{1}{2}t\right)}$$

QUESTION VII :

DETERMINE THE LINEARLY INDEPENDANT SOLUTIONS OF EACH SYSTEM (BY USING EIGEN VALUES AND EIGENVECTORS OF MATRIX)

a) $x' = \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix} x$

Ans:- Sol:-

Let $A = \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix}$

Firstly, we find the eigen values of A and then corresponding eigenvectors of eigenvalues of A. Then the Solution of above system of differential equations is given by:

$$\boxed{X(t) = c_1 V_1 e^{\lambda_1 t} + c_2 V_2 e^{\lambda_2 t}} \quad \text{—(i)}$$

(For distinct λ values)

FINDING EIGENVALUES :-

We can find eigenvalues by the formula.

$$\det(A - \lambda I) = 0 \quad \text{—(ii)}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\therefore \quad A - \lambda I = \begin{bmatrix} 0 & 8 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -\lambda & 8 \\ 2 & -\lambda \end{bmatrix}$$

Putting this in eq (ii)

$$\begin{vmatrix} -\lambda & 8 \\ 2 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 16 = 0$$

$$\lambda^2 = 16$$

$$\lambda = \pm 4$$

So, $\lambda_1 = 4$, $\lambda_2 = -4$ are two distinct eigenvalues of A.

For $\lambda_1 = 4$

The eigenvector V_1 is given by: $V_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

For $\lambda_2 = -4$

The eigenvector V_2 is given by: $V_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

P.T.O

The eigenvector V_1 must satisfy the system,

$$\begin{bmatrix} -4 & 8 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

The augmented matrix is given by:

$$\left[\begin{array}{cc|c} -4 & 8 & 0 \\ 2 & -4 & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 2 & -4 & 0 \end{array}\right] -\frac{1}{4}R_1$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2 - 2R_1$$

$$\therefore \quad x_1 - 2y_1 = 0$$
$$0x_1 - 0y_1 = 0$$

So $x_1 = 2y_1$

The eigenvector V_2 must satisfy the system,

$$\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

The augmented matrix is given by:

$$\left[\begin{array}{cc|c} 4 & 8 & 0 \\ 2 & 4 & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right] \frac{1}{4}R_1$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2 - 2R_1$$

$$\therefore \quad x_2 + 2y_2 = 0$$
$$0x_2 + 0y_2 = 0$$

So $x_2 = -2y_2$

P.T.O

The corresponding eigenvectors V_1 and V_2 are:

$$V_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} , \quad V_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Putting values in (i)

$$X(t) = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-4t}$$

$$X(t) = \begin{pmatrix} 2c_1 e^{4t} \\ c_1 e^{4t} \end{pmatrix} + \begin{pmatrix} -2c_2 e^{-4t} \\ c_2 e^{-4t} \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 2c_1 e^{4t} - 2c_2 e^{-4t} \\ c_1 e^{4t} + c_2 e^{-4t} \end{pmatrix}$$

$$\begin{cases} x(t) = 2c_1 e^{4t} - 2c_2 e^{-4t} \\ y(t) = c_1 e^{4t} + c_2 e^{-4t} \end{cases}$$

$$W(x_1(t), x_2(t)) = \begin{vmatrix} 2e^{4t} & -2e^{-4t} \\ e^{4t} & e^{-4t} \end{vmatrix}$$

$$\quad '' \quad = (2\cancel{e^{4t}})(\cancel{e^{-4t}}) - (-2\cancel{e^{-4t}})(\cancel{e^{4t}})$$

$$\quad '' \quad = 2 + 2 = 4$$

Linearly Independant for all real number t.

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(b) $x' = -6x + 2y$
$y' = -2x - 4y$

Sol:- Writing in matrices form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{— (i)}$$

Let $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $A = \begin{bmatrix} -6 & 2 \\ -2 & -4 \end{bmatrix}$

eq (i) $\Rightarrow$ $X' = AX$

FINDING EIGENVALUES :-

We can find eigenvalues by using the formula.

$$\det(A - \lambda I) = 0 \quad \text{—(ii)}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Now $A - \lambda I = \begin{bmatrix} -6 & 2 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -6-\lambda & 2 \\ -2 & 4-\lambda \end{bmatrix}$$

$$(-6-\lambda)(-4-\lambda)-(-4)=0$$

$$-6(-4-\lambda)-\lambda(-4-\lambda)+4=0$$

$$24+6\lambda+4\lambda+\lambda^2+4=0$$

$$\lambda^2+10\lambda+28=0$$

$$\lambda=\frac{-10\pm\sqrt{10^2-4(1)(28)}}{2(1)}$$

$$\lambda=\frac{-10\pm\sqrt{100-112}}{2}=\frac{-10\pm\sqrt{-12}}{2}$$

$$\lambda=\frac{-10\pm\sqrt{-1\times4\times3}}{2}$$

$$\lambda=\frac{-10\pm i\cdot2\sqrt{3}}{2}$$

$$\lambda=\frac{\cancel{2}(-5\pm i\sqrt{3})}{\cancel{2}}$$

$$\lambda=\lambda_1=\lambda_2=-5\pm i\sqrt{3}\quad,\ \alpha=-5,\ \beta=\sqrt{3}$$

As the eigen values are complex in nature, the general solution of the given system is given by:

$$x(t)=c_1e^{\alpha t}\left[a\cos\beta t-b\sin\beta t\right]+c_2e^{\alpha t}\left[a\sin\beta t+b\cos\beta t\right]$$

(iii)

$$\begin{bmatrix} -6-(-5+i\sqrt{3}) & 2 \\ -2 & -4-(-5+i\sqrt{3}) \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -6+5-i\sqrt{3} & 2 \\ -2 & -4+5-i\sqrt{3} \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1-i\sqrt{3} & 2 \\ -2 & 1-i\sqrt{3} \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

Writing it's augmented matrix

$$\left[\begin{array}{cc|c} -1-i\sqrt{3} & 2 & 0 \\ -2 & 1-i\sqrt{3} & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & \frac{2}{-1-i\sqrt{3}} & 0 \\ -2 & 1-i\sqrt{3} & 0 \end{array}\right] \frac{1}{-1-i\sqrt{3}}R_1$$

By calculating in rough work we get

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & \frac{-1+i\sqrt{3}}{2} & 0 \\ -2 & 1-i\sqrt{3} & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 \end{array}\right] R_2+2R_1$$

$$\begin{bmatrix} -6-(-5-i\sqrt{3}) & 2 \\ -2 & -4-(-5-i\sqrt{3}) \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -6+5+i\sqrt{3} & 2 \\ -2 & -4+5+i\sqrt{3} \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1+i\sqrt{3} & 2 \\ -2 & 1+i\sqrt{3} \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

Writing it's Augmented matrix.

$$\left[\begin{array}{cc|c} -1+i\sqrt{3} & 2 & 0 \\ -2 & 1+i\sqrt{3} & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & \frac{2}{-1+i\sqrt{3}} & 0 \\ -2 & 1+i\sqrt{3} & 0 \end{array}\right] \frac{1}{-1+i\sqrt{3}}R_1$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & -\frac{1-i\sqrt{3}}{2} & 0 \\ -2 & 1+i\sqrt{3} & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{cc|c} 1 & -\frac{1-i\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 \end{array}\right] R_2+2R_1$$

P.T.O

$x_1 + \left(\frac{-1+i\sqrt{3}}{2}\right) y_1 = 0$

Multiplying 2 both sides

$2x_1 + (-1+i\sqrt{3}) y_1 = 0$

$2x_1 = -(-1+i\sqrt{3}) y_1$

$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{(1-i\sqrt{3})}{2} y_1$

$x_1 = \left(\frac{1-i\sqrt{3}}{2}\right) y_1$

$\therefore V_1 = \begin{pmatrix} \frac{1-i\sqrt{3}}{2} \\ 1 \end{pmatrix}$ OR

$V_1 = \begin{pmatrix} 1-i\sqrt{3} \\ 2 \end{pmatrix}$

$V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix} i$

So $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, b = \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix}$

$x_2 + \left(\frac{-1-i\sqrt{3}}{2}\right) y_2 = 0$

Multiplying 2 both side

$2x_2 + (-1-i\sqrt{3}) y_2 = 0$

$2x_2 = -(-1-i\sqrt{3}) y_2$

$\frac{2x_2}{2} = \frac{(1+i\sqrt{3})}{2} y_2$

$x_2 = \left(\frac{1+i\sqrt{3}}{2}\right) y_2$

$\therefore V_2 = \begin{pmatrix} \frac{1+i\sqrt{3}}{2} \\ 1 \end{pmatrix}$

$V_2 = \begin{pmatrix} 1+i\sqrt{3} \\ 2 \end{pmatrix}$

$V_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix} i$

So $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, b = \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix}$

Putting values in (iii)

$$X(t) = c_1 e^{-5t}\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\cos(\sqrt{3}t) - \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix}\sin(\sqrt{3}t)\right] + c_2 e^{-5t}\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\sin(\sqrt{3}t) + \begin{pmatrix} -\sqrt{3} \\ 0 \end{pmatrix}\cos(\sqrt{3}\cdot t)\right]$$

$$X(t) = c_1 e^{-5t}\left[\begin{pmatrix}\cos(\sqrt{3}t)\\ 2\cos(\sqrt{3}t)\end{pmatrix} - \begin{pmatrix}-\sqrt{3}.\sin(\sqrt{3}t)\\ 0\end{pmatrix}\right] +$$

$$c_2 e^{-5t}\left[\begin{pmatrix}\sin(\sqrt{3}.t)\\ 2\sin(\sqrt{3}t)\end{pmatrix} + \begin{pmatrix}-\sqrt{3}\cos(\sqrt{3}.t)\\ 0\end{pmatrix}\right]$$

$$X(t) = c_1 e^{-5t}\left[\begin{pmatrix}\cos(\sqrt{3}t) + \sqrt{3}.\sin(\sqrt{3}.t)\\ 2\cos(\sqrt{3}t)\end{pmatrix}\right] + c_2 e^{-5t}\left[\begin{pmatrix}\sin(\sqrt{3}t) - \sqrt{3}\cos(\sqrt{3}t)\\ 2\sin(\sqrt{3}t)\end{pmatrix}\right]$$

$$X(t) = \begin{pmatrix}c_1 e^{-5t}\left[(\cos(\sqrt{3}t)) + \sqrt{3}.\sin(\sqrt{3}.t)\right]\\ 2c_1 e^{-5t}.\cos(\sqrt{3}t)\end{pmatrix} + \begin{pmatrix}c_2 e^{-5t}\left[\sin(\sqrt{3}t) - \sqrt{3}.\cos(\sqrt{3}t)\right]\\ 2c_2 e^{-5t}.\sin(\sqrt{3}t)\end{pmatrix}$$

$$\begin{pmatrix}x(t)\\ \cancel{x(t)}\\ y(t)\end{pmatrix} = \begin{pmatrix}c_1 e^{-5t}\left[\cos(\sqrt{3}t) + \sqrt{3}.\sin(\sqrt{3}.t)\right] + c_2 e^{-5t}\left[\sin(\sqrt{3}t) - \sqrt{3}.\cos(\sqrt{3}t)\right]\\ 2c_1 e^{-5t}.\cos(\sqrt{3}t) + 2c_2 e^{-5t}.\sin(\sqrt{3}t)\end{pmatrix}$$

$$W(x_1(t), Y_1(t)) = \begin{vmatrix} e^{-5t}\left[\cos(\sqrt{3}t) + \sqrt{3}.\sin(\sqrt{3}t)\right] & e^{-5t}\left[\sin(\sqrt{3}t) - \sqrt{3}.\cos(\sqrt{3}t)\right]\\ 2e^{-5t}\cos(\sqrt{3}t) & 2e^{-5t}\sin(\sqrt{3}t)\end{vmatrix}$$

$$\text{"} = \boxed{2\sqrt{3}\,e^{-10t}}$$ which is never zero

Solutions are linearly independent

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