Assignment 5, PME, 4th Semester, Spring 2023

Deadline: Before the final exam paper of PME

Assignment should be hand written.
Write your name, registration No. and section; else your assignment may not be marked.
Copying is not allowed.
Properly staple your pages (binding is not required).

1. An urn contains nine Rs. 10 notes and one Rs. 50 note. Let the random variable X be the total amount that results when two notes are drawn from the urn without replacement.
   a. Plot the cdf of X.
   b. Use properties of cdf to find $P[20 \leq X < 60]$.
2. Let X be a random variable with pmf $p_k = 0.6/k^2$ for $k = 1, 2, 3, \ldots$. Plot the cdf of X for $k = 0$ to 4. Use the properties of cdf to find
   a. $P[X > 4]$.
   b. $P[6 \leq X \leq 8]$.
3. The transmission time X of messages in a communication system has an exponential distribution. If $\lambda = 1$, find
   a. $P[X > 3]$.
   b. $P[2 \leq X \leq 4]$.
4. Let Y be the difference between the number of heads and the number of tails in the 3 tosses of a fair coin.
   a. Plot the cdf of Y.
   b. Determine the mean and variance of Y.
5. $S_C = \{1, 2, 3, 4\}$ where C is a uniform random variable having four possible values of electric current. If W is a random variable and represents the corresponding power values such that $W = 3C^2$. Plot the cdf of C and W.
6. A random variable X has pdf:

$$f_X(x) = \begin{cases} c(1-x^2) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

   a. Find c and plot the pdf.
   b. Plot the cdf of X.
   c. Find $P[X = 0]$, $P[0 < X < 0.5]$ and $P[\,|X-0.5| < 0.25]$.
7. Find the characteristic function of the continuous uniform random variable X in $[-b, b]$. Find the mean and variance of X by applying the moment theorem.
8. Find the characteristic function of the geometric random variable X. Find the mean and variance of X by applying the moment theorem.
9. Find the probability generating function of the geometric random variable X. Find the mean and variance of X using the probability generating function.
10. An urn contains 16 balls: 4 balls are labeled “1”, 4 are labeled “2”, 2 are labeled “3”, 2 are labeled “4”, and the remaining balls are labeled “5”, “6”, “7”, and “8.” One ball is drawn from the urn at random and the number is noted as random variable X. Find the entropy of X.
11. Let X be the outcome of the toss of a fair dice.

a. Find the entropy of X.

b. Suppose you are told that X is even. What would the reduction in entropy?

12. A uniform random variable V has four possible values such that the set $S_V = \{-R, e, g, \#\}$. Find the mean of $Z = V^3$.
13. A uniform random variable V has four possible values such that the set $S_V = \{-R, e, g, \#\}$. Find the mean of $Z = V^2 - 2V$.
14. A uniform random variable V has four possible values such that the set $S_V = \{-R, e, g, \#\}$. Find the mean of $Z = (1/2)V^2 - 3V + 1$.
15. A uniform random variable V has four possible values such that the set $S_V = \{-R, e, g, \#\}$. Find the variance of $Z = (1/2)V^2 - 2V + 2$.
16. Find the expected value of the last four digits of your registration number. Each digit has a probability of $\frac{1}{4}$.