SS Lab # 9

OBJECTIVES OF THE LAB

This lab aims at the understanding of:

- Power of Continuous & Discrete time Signals
- Application of Fourier Series
- Synthesis of Square Wave
- Synthesis of Triangular Wave

9.1 SIGNAL POWER

Average power of continuous time signal can be calculated using the formula:

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$

To carry out the integral, Euler Approximation can be used. It simply tells that a definite integral can be approximated using a sum i.e.

$$\int_a^b x(t)dt \cong \sum_{n=0}^{N-1} x(a+n\Delta t)\Delta t, \quad \Delta t = \frac{b-a}{N}$$

In this method, the region over which integral is carried out is divided into N parts or intervals, each of duration Δt, such that function stays constant over those short intervals. Approximating function in this way is shown in Figure 9.1. Note that as the number of intervals N is increased, the approximation gets better.

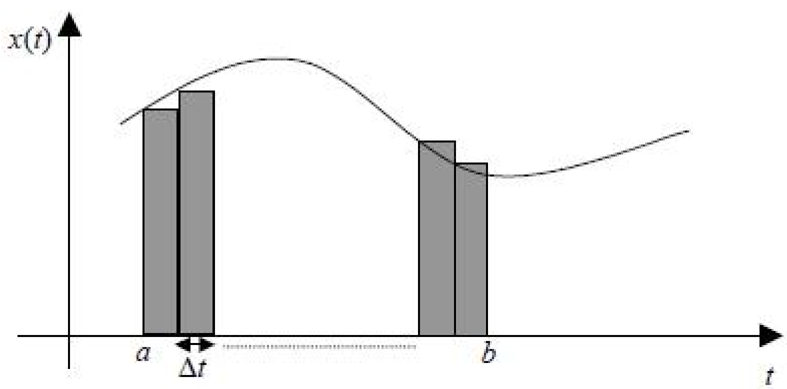

Figure 9.1 – Definite Integral Approximation using Summation

Approximating integrals using sums is a deep subject of numerical analysis by itself; therefore, its further detail is out of scope. It is enough to know that Euler's formula is easy to implement and produces good results for almost all the signals that will be studied here as long as N is selected large enough.

Example – Power of Continuous Time Cosine

```
clc; clear; close all

t = -1:0.005:0.995;        % time duration of given signal;
xt = cos(2*pi*t/2);        % generate signal
```

```
plot(t, xt);                          % plot signal
 xlabel('time, t');
ylabel('Amplitude, A');
title('Continuous Time Cosine');

abs_xt_2 = abs(xt).^2;                % take absolute square of signal

T=2;                                  % length of interval
delta_t = 0.005;                      % interval duration
pxt = sum(abs_xt_2)*delta_t/T         % power of given signal
```

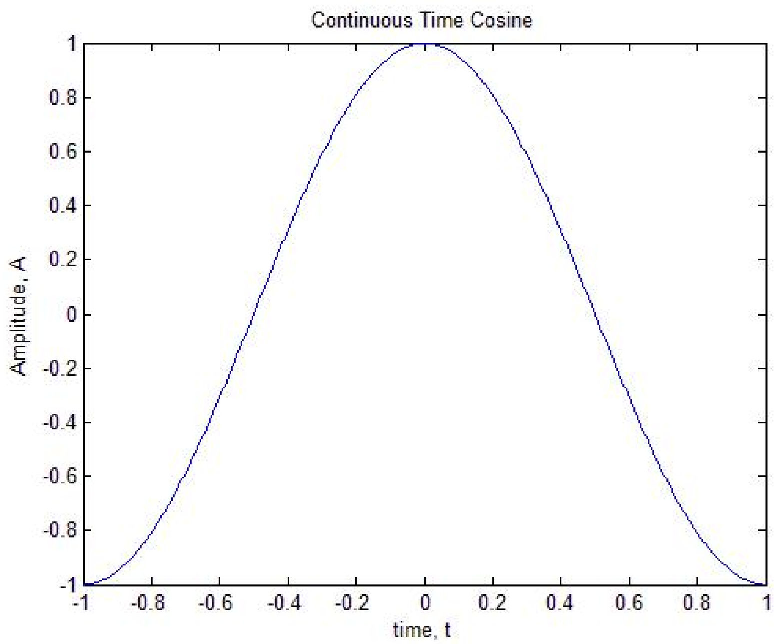

Figure 9.2 – Continuous-Time Cosine Signal

```
pxt =

   0.5000
```

---------------------------TASK 1---------------------------

Calculate the power of discrete-time cosine signal with period 20, defined over interval 0:19 using the following formula:

$$P = \frac{1}{N}\sum_{0}^{N-1} | x[n] |^2$$

9.2 FOURIER SERIES

Fourier series theory states that a periodic wave can be represented as a summation of sinusoidal waves with different frequencies, amplitudes and phase values. In this lab, we synthesize two signals i.e. square wave and triangular wave using Fourier Series. Figure 9.3, Figure 9.4, and Figure 9.5 shows this for square wave while Figure 9.6 and Figure 9.7 shows this for triangular wave.

9.2.1 Synthesis of Square wave

The square wave for one cycle can be represented mathematically as:

$$x(t) = \begin{cases} 1 & 0 <= t < T/2 \\ -1 & T/2 <= t < T \end{cases}$$

The Complex Amplitude is given by:

$$X_k = \begin{cases} (4/j*pi*k) & \text{for} \quad k=\pm1, \pm3, \pm5..... \\ 0 & \text{for} \quad k=0,\pm2, \pm4, \pm6..... \end{cases}$$

For f = 1/T = 25Hz, only the frequencies ±25, ±50, ±75 etc. are in the spectrum.

i. Effect of Adding Fundamental, third, fifth, and seventh Harmonics

Example

```
clc
clear
close all

t=0:0.0001:8;
ff=0.5;
% WE ARE USING SINE FUNCTION BECAUSE FROM EXPONENTIAL FORM OF FOURIER
% SERIES FINALLY WE ARE LEFT WITH SINE TERMS
y = (4/pi)*sin(2*pi*ff*t);
% COMPLEX AMPLITUDE = (4/(j*pi*k))
for k = 3:2:7
   fh=k*ff;
```

```
   x = (4/(k*pi))*sin(2*pi*fh*t);
   y=y+x;
end

plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st, 3rd, 5th, and 7th');
xlabel('Time');
ylabel('Amplitude');
```

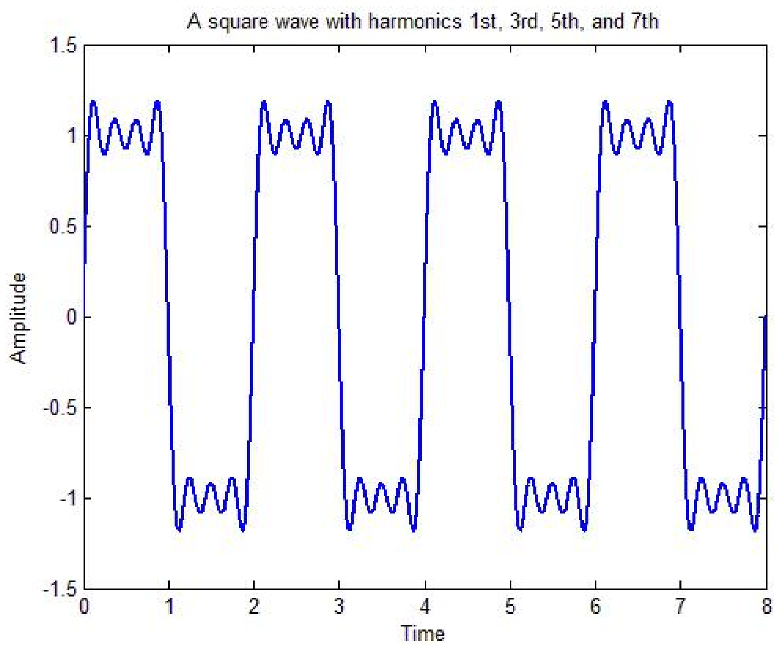

Figure 9.3 – Square Wave Synthesis using 1st, 3rd, 5th, and 7th Harmonic

ii. Effect of Adding 1st to 17th harmonics

Example

```
clc
clear all

t=0:0.0001:8;
ff=0.5;

% WE ARE USING SINE FUNCTION BECAUSE FROM EXPONENTIAL FORM OF FOURIER
```

```
% SERIES FINALLY WE ARE LEFT WITH SINE TERMS
y = (4/pi)*sin(2*pi*ff*t);
% COMPLEX AMPLITUDE = (4/(j*pi*k))
for k = 3:2:17
   fh=k*ff;
   x = (4/(k*pi))*sin(2*pi*fh*t);
   y=y+x;
end

plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st-17th');
xlabel('Time');
ylabel('Amplitude');
```

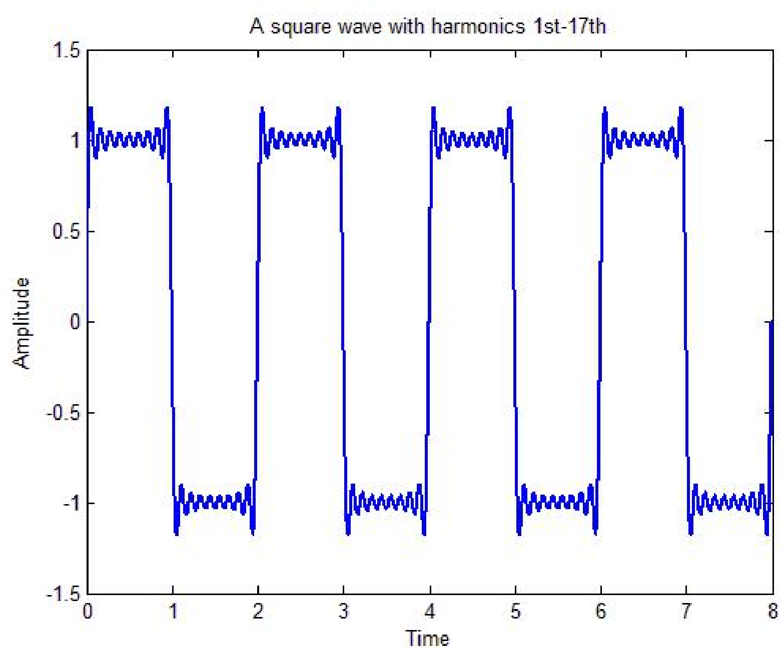

Figure 9.4 – Square Wave Synthesis using 1st to 17th Harmonic

iii. Effect of Adding 1st to 27th harmonics

Example

```
clc
 clear all
```

```
close all

t=0:0.0001:8;
ff=0.5;

% WE ARE USING SINE FUNCTION BECAUSE FROM EXPONENTIAL FORM OF FOURIER
% SERIES FINALLY WE ARE LEFT WITH SINE TERMS
y = (4/pi)*sin(2*pi*ff*t);
% COMPLEX AMPLITUDE = (4/(j*pi*k))
for k = 3:2:55
   fh=k*ff;
   x = (4/(k*pi))*sin(2*pi*fh*t);
   y=y+x;
end
plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st to 27th');
xlabel('Time');
ylabel('Amplitude');
```

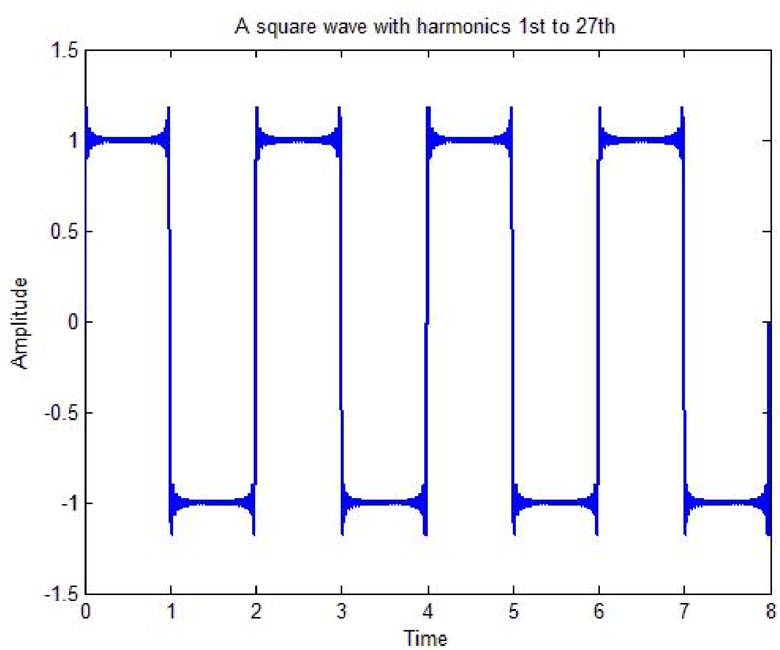

Figure 9.5 – Square Wave Synthesis using 1st to 27th Harmonic

----------------------------TASK 2----------------------------

Write a program that plots the signal s(t).

$$s(t) = \sum_{n=1}^{N} \frac{\sin(2\pi\ nt)}{n}$$ where n = 1, 3, 5, 7, 9 and N = 9 or

$$s(t) = \sin(2\pi * t) + \frac{\sin(6\pi * t)}{3} + \frac{\sin(10\pi * t)}{5} + \frac{\sin(14\pi * t)}{7} + \frac{\sin(18\pi * t)}{9}$$

----------------------------TASK 3----------------------------

Write a program that plots the signal s(t) but with N = 100.

----------------------------TASK 4----------------------------

What do you conclude from TASKS 2 & 3?

9.2.2 Synthesis of Triangular wave

The Complex Amplitude is given by:

$$Xk = \begin{cases} (-8/*pi^2*k^2) & \text{for k is an odd integer} \\ 0 & \text{for k for k is an even integer} \end{cases}$$

For f = 1/T = 25Hz

Example: Triangular wave with N=3

```
clc; clear all; close all
t=0:0.001:5;

x=(-8/(pi*pi))*exp(i*(2*pi*0.5*t));
y=(-8/(9*pi*pi))*exp(i*(2*pi*0.5*3*t));
s=x+y;

plot(t,real(s),'linewidth',3);
title('Triangular Wave with N=3');
ylabel('Amplitude');
xlabel('Time');
```

```
grid;
```

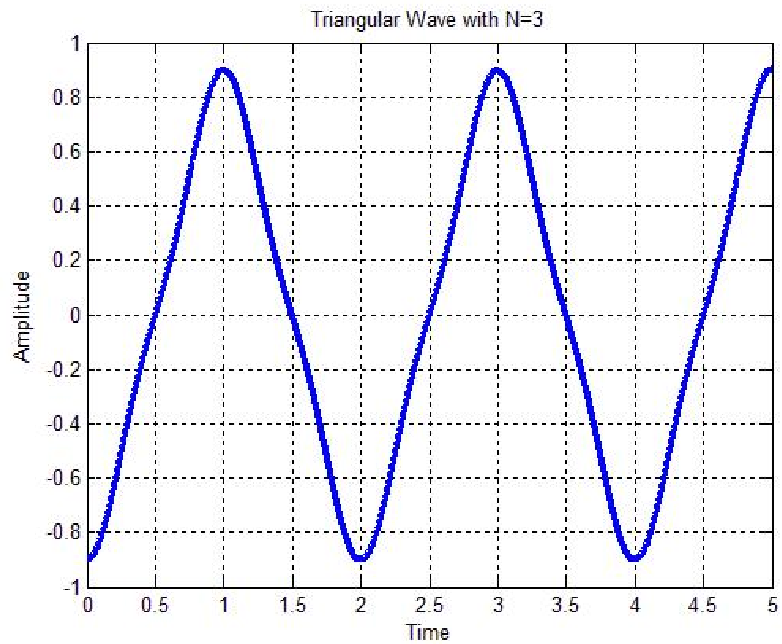

Figure 9.6 – Triangular Wave Synthesis with N = 3

Example: Triangular wave with N=11

```
clc; clear all; close all

t=0:0.01:0.25;
ff=25;

x1=(-8/(pi^2))*exp(i*(2*pi*ff*t));
for k=3:2:21,
   fh=ff*k;
   x=(-8/(pi^2*k^2))*exp(i*(2*pi*fh*t));
   y=x1+x;
end

plot(t,real(y),'linewidth',3);
title('Triangular Wave with N=11');
ylabel('Amplitude');
xlabel('Time');
grid;
```

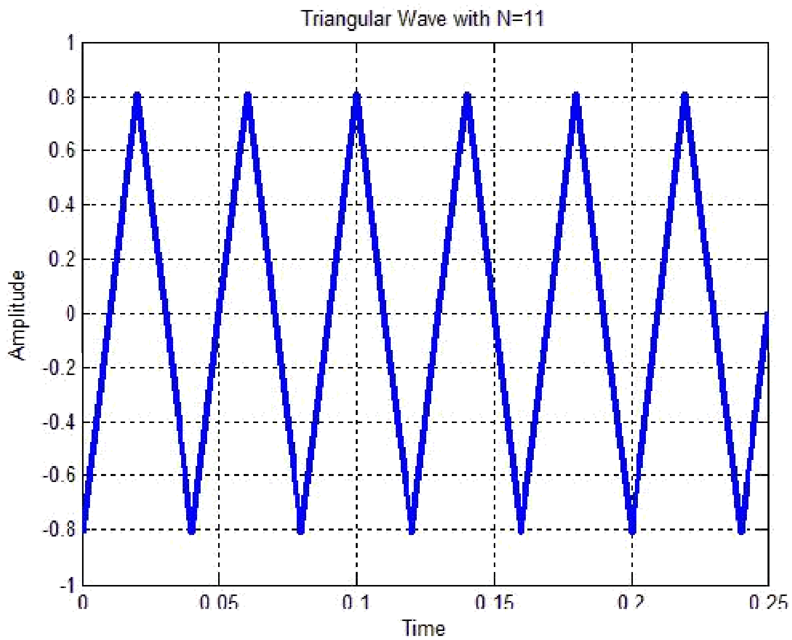

Figure 9.7 – Triangular Wave Synthesis with N = 11