

Assignment 2, PME, 4th Semester, Spring 2023

Deadline: Before the start of PME mid-term exam paper

Assignment should be hand written.

Write your name, registration No. and section; else your assignment may not be marked.

Copying is not allowed.

Write in your own words.

Properly staple your pages (binding is not required).

1. You toss a fair coin three times: What is the probability of three heads, HHH? What is the probability that you observe exactly one heads? Given that you have observed at least one heads, what is the probability that you observe at least two heads?
2. Ten numbered cards are there from 1 to 15, and two cards are chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered.
3. There are 12 balls in the pool. 6 balls are blue and the rest are red. Two of the balls are picked up from the pool without replacement. What is the probability of selecting two red balls from the pool?
4. In a class, 35% of the students study science and history. 65% of the students study science. What is the probability of a student studying history given he/she is already studying science?
5. Suppose you roll two dice. What is the probability the sum is 8?
6. A bag contains red and blue marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and then a blue marble is 0.28. The probability of selecting a red marble on the first draw is 0.5. What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was red?
7. The probability of a student passing in science is $\frac{4}{5}$ and the of the student passing in both science and maths is $\frac{1}{2}$. What is the probability of that student passing in maths knowing that he passed in science?
8. A group of 200 Students were asked whether they played football or basketball. Among the group, 120 said they played football, 50 said they played basketball and 20 said they played both football and basketball.
 - a) What is the probability that a students selected at random from the group plays football given that he plays basketball?
 - b) What is the probability that a students selected at random from the group plays basketball given that he plays football?
 - c) What is the probability that a students selected at random from the group plays football given that he plays one game only.
9. Susan took two tests. The probability of her passing both tests is 0.6. The probability of her passing the first test is 0.8. What is the probability of her passing the second test given that she has passed the first test?
10. In a survey among few people, 60% read Hindi newspaper, 40% read English newspaper and 20% read both. If a person is chosen at random and if he already reads English newspaper find the probability that he also reads Hindi newspaper.

11. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?
12. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15.
 - (i) If the oil had to be changed, what is the probability that a new oil filter is needed?
 - (ii) If a new oil filter is needed, what is the probability that the oil has to be changed?
13. Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve the profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.
14. A bag contains 4 balls. Two balls are drawn at random without replacement and are found to be blue. What is the probability that all balls in the bag are blue?
15. In a neighbourhood, 90% children were falling sick due flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child develops rashes, find the child's probability of having flu.
16. There are three identical cards except that both the sides of the first card is coloured red, both sides of the second card is coloured blue and for the third card one side is coloured red and the other side is blue. One card is randomly selected among these three cards and put down, visible side of the card is red. What is the probability that the other side is blue?
17. In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.) 95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree?
18. Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is colored black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?
19. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?
20. Let's work on a simple NLP problem with Bayes Theorem. By using NLP, I can detect spam e-mails in my inbox. Assume that the word 'offer' occurs in 80% of the spam messages in my account. Also, let's assume 'offer' occurs in 10% of my desired e-

mails. If 30% of the received e-mails are considered as a scam, and I will receive a new message which contains 'offer', what is the probability that it is spam?

21. I want to solve one more example from a popular topic as Covid-19. As you know, Covid-19 tests are common nowadays, but some results of tests are not true. Let's assume; a diagnostic test has 99% accuracy and 60% of all people have Covid-19. If a patient tests positive, what is the probability that they actually have the disease?
22. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and a ball is drawn from it. If the ball that is drawn out is red, what will be the probability that the second box is chosen?
23. Two boxes containing candies are placed on a table. The boxes are labelled B1 and B2. Box B1 contains 7 cinnamon candies and 4 ginger candies. Box B2 contains 3 cinnamon candies and 10 pepper candies. The boxes are arranged so that the probability of selecting box B1 is $\frac{1}{3}$ and the probability of selecting box B2 is $\frac{2}{3}$. Suresh is blindfolded and asked to select a candy. He will win a colour TV if he selects a cinnamon candy. What is the probability that Suresh will win the TV (that is, she will select a cinnamon candy)?
24. An urn B1 contains 2 white and 3 black chips and another urn B2 contains 3 white and 4 black chips. One urn is selected at random and a chip is drawn from it. If the chip drawn is found black, find the probability that the urn chosen was B1.
25. Consider a test to detect a disease that 0.1 % of the population have. The test is 99 % effective in detecting an infected person. However, the test gives a false positive result in 0.5 % of cases. If a person tests positive for the disease what is the probability that they actually have it?
26. Two production lines produce the same part. Line 1 produces 1,000 parts per week of which 100 are defective. Line 2 produces 2,000 parts per week of which 150 are defective. If you choose a part randomly from the stock what is the probability it is defective? If it is defective what is the probability it was produced by line 1?
27. Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90 % of the time. When it doesn't rain, he incorrectly forecasts rain 10 % of the time. What is the probability that it will rain on the day of Marie's wedding? (Assume that there are no leap years)
28. You are selling a product in an area where 30 % of the people live in the city and the rest live in the suburbs. Currently 20 % of the city dwellers use your product and 10 % of the suburbanites use your product. You are presented with two new sales strategies the first will increase your market share in the suburbs to 15 %. The second will increase your market share in the city to 25 %. Which strategy should you adopt? What percentage of the people who own your product are city dwellers before your new sales drive?

29. In a casino in Blackpool there are two slot machines: one that pays out 10 % of the time, and one that pays out 20 % of the time. Obviously, you would like to play on the machine that pays out 20 % of the time but you do not know which of the two machines is the more generous. You thus adopt the following strategy: you assume initially that the two machines are equally likely to be the generous machine. You then select one of the two machines at random and put a coin into it. Given that you loose that first bet estimate the probability that the machine you selected is the more generous of the two machines
30. A different number of light bulbs are present in three boxes. The first box has 12 bulbs, out of which 5 are dead. The second box has eight bulbs, out of which the dead ones are 3. The third box has nine bulbs, out of which two are dead. Find the probability of selecting a dead bulb randomly from one of the three boxes.
31. If 40% of boys opted for maths and 60% of girls opted for maths, then what is the probability that maths is chosen if half of the class's population is girls?
32. A compound event can occur in 3 ways, each of which is equally likely. The probability in the first and the second event is observed to be $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Then the following will be true for the probability of the third event:
- A) The probability of the third event is greater than the second event.
 - B) P (the third event) is greater than the first event.
 - C) The probability of the third event is $\frac{1}{6}$.
 - D) Both the options A) and C) are exactly correct.
33. Company A produces 10% defective products, Company B produces 20% defective products and C produces 5% defective products. If choosing a company is an equally likely event, then find the probability that the product chosen is defective.
34. A person has undertaken a mining job. The probabilities of completion of the job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.
35. Three factories produce the same tool and supply it to the market. Factory A produces 30% of the tools for the market and the remaining 70% of the tools are produced in factories B and C. 98% of the tools produced in factory A, 95% of the tools produced in factory B and 97% of the tools produced in factory C are not defective.
36. What percent of tools should be produced by factories B and C so that a tool picked at random in the market will have a probability of being non defective equal to 96%?
37. Suppose 5 men out of 100 men and 10 women out of 250 women are colour blind, then find the total probability of colour blind people. (Assume that both men and women are in equal numbers.)

38. Students in a math class where 40% are males and 60% are females took a test. 50% of the males and 70% of the females passed the test. What percent of students passed the test?
39. There are four bags. Each bag has 50 balls in it. There are 30 blue balls in the first bag, 45 blue balls in the second bag, 35 blue balls in the third bag, and 15 blue balls in the fourth bag. Find the probability of selecting a blue ball from one of the four bags at random.