Ex: 1.1 8th Edition

Linear Algebra Bernard Kolman

Q1 $x+2y=8$ —— (1)

$3x-4y=4$ —— (2)

Xing Equation (1) by 3 and subtring from (2)

$$\begin{array}{r} \cancel{3x}+6y=24 \\ \underline{\cancel{3x}\mp4y=\underline{-}4} \\ 10y=20 \end{array} \Rightarrow \boxed{y=2}$$

eq (1) $\Rightarrow x+2(2)=8 \Rightarrow x=8-4=4 \Rightarrow \boxed{x=4}$

Hence $(x,y)=(4,2)$ which is unique solution.

Q3 $3x+2y+z=2$ —— (i)

$4x+2y+2z=8$ —— (ii)

$x-y+z=4$ —— (iii)

Solving eq (i) & (iii)

Xing eq (iii) by 3 then subtrcig from (i)

$$\begin{array}{r} \cancel{3x}-3y+3z=12 \\ \underline{\cancel{3x}\pm2y\pm z=\underline{-}2} \\ -5y+2z=10 \end{array}$$ —— (iv).

Now Solving Equation (iii) & (ii)

xing Equation (iii) by 4 then Subtracting from (ii)

$$\cancel{4x} - 4y + 4z = 16$$
$$\underline{-\cancel{4x} \mp 2y \mp 2z = -8}$$
$$-6y + 2z = 8 \text{ —— (iv)}$$

Now Solving eq (iv) & (v).

(iv) – (v)

$$-5y + 2\cancel{z} = 10$$
$$\underline{-6y + 2\cancel{z} = 8}$$
$$\boxed{y = 2}$$

eq (iv) $\Rightarrow -5(2) + 2z = 10$

$$-10 + 2z = 10$$
$$2z = 10 + 10 \Rightarrow \cancel{2}z = \cancel{20}^{10} = \boxed{z = 10}$$

Putting the value of $y + z$ in eq (i)

$$3x + 2(2) + 10 = 2$$
$$3x = 2 - 14 = -12$$
$$\frac{\cancel{3}x}{\cancel{3}} = \frac{-\cancel{12}^{4}}{\cancel{3}} = \boxed{x = -4}$$

Hence $(x, y, z) = (-4, 2, 10)$ which is unique Solution.

② Ex: 1.1

Q7 $x + 4y - z = 12$ —— (i)

$3x + 8y - 2z = 4$ —— (ii)

xing Equ: (i) by 3 then Sub: from (ii)

$$\cancel{3x} + 12y - 3z = 36$$
$$\underline{-\cancel{3x} \mp 8y \pm 2z = -4}$$
$$4y - z = 32$$

$\Rightarrow 4y = 32 + z \Rightarrow y = 8 + z/4$ —— (iii)

where z is any real no then $z = r$.

eq (iii) $\Rightarrow y = 8 + r/4$.

eq (i) $\Rightarrow x = 12 - 4y + z$

$$x = 12 - 4(8 + z/4) + z$$
$$x = 12 - 32 - \cancel{4}\frac{z}{\cancel{4}} + z$$
$$x = 12 - 32 - \cancel{z} + \cancel{z}$$
$$x = -20$$

Hence $x = -20$, $y = \frac{1}{4}r + 8$, $z = r$, where r is any real number.

Q9 $x+y+3z=12$ —— (i)

$2x+2y+6z=6$ —— (ii)

X'ing Eq (i) by (2) then subt: from eq: (ii)

$$\cancel{2x}+\cancel{2y}+\cancel{6z}=24$$
$$\underline{\cancel{2x}+\cancel{2y}+\cancel{6z}=6}$$
$$0 = 18$$

No Solution.

Q11 $2x+3y=13$ —— (i)

$x-2y=3$ —— (ii)

$5x+2y=27$ —— (iii)

X'ing eq (ii) by (2) and Substr: from (i)

$$\cancel{2x}+3y=13$$
$$\underline{\cancel{2x}-4y=6}$$
$$7y=7 \Rightarrow \boxed{y=1}$$

X'ing eq (ii) by 5 then subtr: from (iii)

$$\cancel{5x}-10y=15$$
$$\underline{\cancel{5x}+2y=27}$$
$$-12y=-12 \Rightarrow \boxed{y=1}$$

X'ing eq (i) by 5 & Eq (iii) by 2 then subtr:

$$\cancel{10x}+15y=65$$
$$\underline{\cancel{10x}+2y=54}$$
$$11y=11 \Rightarrow \boxed{y=1}$$

Put $y=$ in (1) then

$2x+3(1)=13 \Rightarrow 2x=13-3=10 \Rightarrow \boxed{x=5}$

Hence $y=1$ & $x=5$ which is unique solution.

Q17 without using the method of elimination solve the linear system

$2x+y-2z=-5$ —— (i)

$3y+z=7$ —— (ii)

$z=4$ —— (iii)

Sol: $z=4$ then eq (ii) $\Rightarrow 3y+4=7 \Rightarrow 3y=7-4=3$

$3y=3 \Rightarrow \boxed{y=1}$

If $z=4$ & $y=1$ eq (i) $\Rightarrow$

$2x+1-2(4)=-5$

$2x=-5+7=2 \Rightarrow \cancel{2}x=\cancel{2}$ $\boxed{x=1}$

Hence $x=1$, $y=1$ & $z=4$ Ans

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Q19 Is there a value of r so that $x=1, y=2, z=r$ is a solution to the following linear system? If there is, find it.

$$2x+3y-z=11 \quad \text{---(i)}$$
$$x-y+2z=-7 \quad \text{---(ii)}$$
$$4x+y-2z=12 \quad \text{---(iii)}$$

Given that $x=1, y=2, z=r$

eq (i) $\Rightarrow 2(1)+3(2)-r=11$

$2+6-r=11 \Rightarrow -r=11-8=3 \Rightarrow \boxed{r=-3}$

eq (ii) $\Rightarrow 1-2+2r=-7$

$-1+2r=-7 \Rightarrow 2r=-7+1=-6$

$\Rightarrow \boxed{r=-3}$

eq (iii) $\Rightarrow 4(1)+2-2r=12$

$-2r=12-6$

$-2r=6$

$\boxed{r=-3}$

Hence $\boxed{r=-3}$ Ans.

Q23 Let x_1 denotes lower-sulfur & x_2 denotes high-sulfur for each ton

$5x_1+4x_2=3\times 60=180$

$4x_1+2x_2=2\times 60=120$

or

$5x_1+4x_2=180$ ---(i)

$4x_1+2x_2=120$ ---(ii)

Xing eq (ii) by 2 then subtr: from (i)

$5x_1+\cancel{4x_2}=180$

$\underline{-8x_1 \pm \cancel{4x_2} = \mp 240}$

$-3x_1=-60 \Rightarrow x_1=\frac{\cancel{60}^{20}}{\cancel{3}}=20$

$\boxed{x_1=20}$

Put $x_1=20$ in eq (i)

$5\times 20+4x_2=180$

$4x_2=180-100=80$

$4x_2=80$

$x_2=80/4=20$

$\boxed{x_1=x_2=20 \text{ tons}}$

unique solution.

(24) Let x_1 denotes regular plastic ton
& x_2 denotes special plastic ton

Now $2x_1 + 2x_2 = 8$ — (i)
$5x_1 + 3x_2 = 15$ — (ii)

Solving (i) & (ii)

$x_1 = 1.5$ tons
& $x_2 = 2.5$ tons

—— X ——

Q25 let x_1 denotes of ounce A
x_2 denotes of ounce B
& x_3 denotes of ounce C.

$$\begin{array}{lll} & A \quad B \quad C & \\ \text{Now} & 2x_1 + 3x_2 + 3x_3 = 25 & \text{— (i)} \\ & 3x_1 + 2x_2 + 3x_3 = 24 & \text{— (ii)} \\ & 4x_1 + x_2 + 2x_3 = 21 & \text{— (iii)} \end{array}$$

Xing eq (iii) by 2 then Subtr: from (ii)

$3x_1 + \cancel{2x_2} + 3x_3 = 24$
$\underline{8x_1 + \cancel{2x_2} + 6x_3 = 42}$
$-5x_1 - 3x_3 = -18$
$\Rightarrow 5x_1 + 3x_3 = 18$ —— (iv).

Xing eq (iii) by 3 then Subtr: from (i)

$2x_1 + \cancel{3x_2} + 3x_3 = 25$
$\underline{12x_1 + \cancel{3x_2} + 6x_3 = 63}$
$-10x_1 - 3x_3 = -38$
$\Rightarrow 10x_1 + 3x_3 = 38$ —— (v)

$2 \times\ 5x_1 + x_3 = 18$
$10x_1 + 3x_3 = 38$
$-x_3 = -2$

Xing eq (iv) by 2 then Subtr: from (v)

$5x_1 + x_3 = 18$

$5x_1 + 2 = 18$

$\cancel{10x_1} + 2x_3 = 36$
$\underline{\cancel{10x_1} + 3x_3 = 38}$
$-x_3 = -2 \Rightarrow \boxed{x_3 = 2}$

Xing eq (iv) by 3 then Subtr: from (v)

$15x_1 + \cancel{3x_3} = 54$
$\underline{10x_1 + \cancel{3x_3} = 38}$
$+5x_1 = 16 \Rightarrow x_1 = 16/5 = 3.2$

$\boxed{x_1 = 3.2}$

Put $x_1 = 3.2$ & $x_3 = 2$ in eq (i)

eq (i) $\Rightarrow 2(3.2) + 3x_2 + 3(2) = 25$
$3x_2 = 25 - 12.5 = 12.6$
$3x_2 = 12.6 \Rightarrow x_2 = 12.6/3 = 4.2.$

$\boxed{x_2 = 4.2}$

Ex: 1.1

Q26 Let x_1 denotes tons of 2 minute
x_2 denotes tons of 6-minute
& x_3 denotes tons of 9-minute

Now $6x_1 + 12x_2 + 12x_3 = 10 \times 60 = 600$
$24x_1 + 12x_2 + 12x_3 = 16 \times 60 = 960$

or

$6x_1 + 12x_2 + 12x_3 = 600$
$24x_1 + 12x_2 + 12x_3 = 960$

or

$x_1 + 2x_2 + 2x_3 = 100$ —— (i)
$2x_1 + x_2 + x_3 = 80$ —— (ii)

X'ing eq (ii) by 2 then subtr: from (i)

$x_1 + \cancel{2x_2} + \cancel{2x_3} = 100$
$\underline{4x_1 + 2x_2 + 2x_3 = 160}$
$-3x_1 = -60 \Rightarrow x_1 = \frac{-60}{-3} = 20$

$\boxed{x_1 = 20}$

X'ing eq (i) by 2 then subtr: from (ii)

$2x_1 + 4x_2 + 4x_3 = 200$
$\underline{2x_1 + x_2 + x_3 = 80}$
$3x_2 + 3x_3 = 120$

$\Rightarrow x_2 + x_3 = 40$ —— (iii)

Let $x_3 = r$ any real no: & r.

then eq (iii) $\Rightarrow x_2 + r = 40 \Rightarrow \boxed{x_2 = 40 - r}$

Hence $\boxed{x_1 = 20}$, $\boxed{x_2 = 40 - r}$, $\boxed{x_3 = r}$ Ans.

Q27 (a) Given that $P_1(x) = (1, -5)$
$P_2(x) = (-1, 1)$
$P_3(x) = (2, 7)$

& Parabola $P(x) = ax^2 + bx + c$.

3000
130
100
200000.
11/2

(a) $P_1(x) = y = ax^2 + bx + c$

$-5 = a(1) + b(1) + c$

$-5 = a + b + c$ —— (i)

$P_2(x) = y = ax^2 + bx + c$

$1 = a(-1)^2 + b(-1) + c$

& $1 = a - b + c$ —— (ii)

$P_3(x) = y = a(2)^2 + b(2) + c$

$7 = 4a + 2b + c$ —— (iii)

eq (1) $\Rightarrow Z_1 = 24000 - 3x_1$

$Z_1 = 24000 - 3(7000)$

$Z_1 = 24000 - 21000$

$\boxed{Z_1 = 3000\$}$

Mr. Jamal Nasir

Lecturer U.E.T Peshawar.

Spring-2012 Elect Deptt.

Q1 Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$ and $C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}$.

a) $a_{12} = -3,\ a_{22} = -5,\ a_{23} = 4$

b) $b_{11} = 4,\ b_{31} = 5$

c) $c_{13} = 2,\ c_{31} = 6,\ c_{33} = -1$

Q2 If $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$ find a, b, c & d.

Sol:

$a+b = 4$ —— (i)

$c+d = 6$ —— (ii)

$c-d = 10$ —— (iii)

$a-b = 2$ —— (iv)

(i) + (iv)

$a + \cancel{b} = 4$

$a - \cancel{b} = 2$

$2a = 6 \Rightarrow \boxed{a = 3}$

eq (i) $\Rightarrow 3 + b = 4 \Rightarrow b = 4 - 3 = 1 \Rightarrow \boxed{b = 1}$

(ii) + (iii)

$c + \cancel{d} = 6$

$c - \cancel{d} = 10$

$2c = 16 \Rightarrow \boxed{c = 8}$

eq (ii) $\Rightarrow 8 + d = 6 \Rightarrow d = 6 - 8 = -2 \Rightarrow \boxed{d = -2}$

Q3 is similarly to Q2.

In Exercises 4 through 7, let

$A=\begin{bmatrix}1&2&3\\2&1&4\end{bmatrix}$, $B=\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix}$, $C=\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}$

$D=\begin{bmatrix}3&-2\\2&4\end{bmatrix}$, $E=\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}$, $F=\begin{bmatrix}-4&5\\2&3\end{bmatrix}$

and $O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$.

Q4 a) C+E & E+C

$$C+E=\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}+\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}=\begin{bmatrix}5&-5&8\\4&2&9\\5&3&4\end{bmatrix}$$

$$\& \ E+C=\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}+\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}=\begin{bmatrix}5&-5&8\\4&2&9\\5&3&4\end{bmatrix}$$

commutative addition.

(b) $A+B=\begin{bmatrix}1&2&3\\2&1&4\end{bmatrix}+\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix}$ addition is not

Possible because the order of matrices is not same.

(c) $D-F=\begin{bmatrix}3&-2\\2&4\end{bmatrix}-\begin{bmatrix}-4&5\\2&3\end{bmatrix}=\begin{bmatrix}3+4&-2-5\\2-2&4-3\end{bmatrix}=\begin{bmatrix}7&-7\\0&1\end{bmatrix}$

(d) $-3C+5O = -3\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}+5\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$$=\begin{bmatrix}-9&3&-9\\-12&-3&-15\\-6&-3&-9\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}-9&3&-9\\-12&-3&-15\\-6&-3&-9\end{bmatrix} \text{ Ans.}$$

(f) $2B+F=2\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix}+\begin{bmatrix}-4&5\\2&3\end{bmatrix}$

$=\begin{bmatrix}2&0\\4&2\\6&4\end{bmatrix}+\begin{bmatrix}-4&5\\2&3\end{bmatrix}$ addition is not Possible

because the order of matrices is not same.

Q5 If Possible compute the indicated linear combination

(a) $3D+2F=3\begin{bmatrix}3&-2\\2&4\end{bmatrix}+2\begin{bmatrix}-4&5\\2&3\end{bmatrix}=\begin{bmatrix}9&-6\\6&12\end{bmatrix}+\begin{bmatrix}-8&10\\4&6\end{bmatrix}$

$$=\begin{bmatrix}1&4\\10&18\end{bmatrix}.$$

(b) $3(2A)$ and $6A$

$$3(2A) = 6A = 6\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}.$$

(f) $3(B+D) = 3\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}\right)$ addition is not Possible because the order of matrices is not same.

Q6 If Possible, compute

(a) A^T and $(A^t)^t$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}^t = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}.$$

$$\& \ (A^t)^t = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

So $A^t = (A^t)^t$.

(b) $(C+E)^T$ and $C^T + E^T$.

$$C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}.$$

$$(C+E)^t = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}^t = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}.$$

$$C^t = \begin{bmatrix} 3 & 4 & 2 \\ -1 & 1 & 1 \\ 3 & 5 & 3 \end{bmatrix} \ \& \ E^t = \begin{bmatrix} 2 & 0 & 3 \\ -4 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix}.$$

$$C^t + E^t = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}.$$

So $(C+E)^t = C^t + E^t$ Ans..

Q7 is Similarly to Q 6.

Q9 Is the matrix $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

Sol $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix} = c_1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_1 \end{bmatrix} + \begin{bmatrix} c_2 & 0 \\ 0 & 0 \end{bmatrix}.$

$$\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} c_1+c_2 & 0 \\ 0 & c_1 \end{bmatrix}.$$

$c_1 + c_2 = 4$ & $c_1 = -3$ $\quad c_2 = 7$

So $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix} \neq \begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}$

So it's not a linear combination.

Q13 Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ bit matrices.

(a) Find B so that $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ bit.}$$

(b) Find C so that $A + C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - A$$

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Q15 Given that $U = [0\ 1\ 0\ 1]$ & $U + V = [1\ 1\ 1\ 1]$

Let $V = [a, b, c, d]$

then $[0\ 1\ 0\ 1] + [a, b, c\ d] = [1\ 1\ 1\ 1]$

$[a\ \ b+1\ \ c\ \ d+1] = [1\ 1\ 1\ 1]$

$\boxed{a = 1}$ $\quad b + 1 = 1 \Rightarrow \boxed{b = 0}$ $\quad \boxed{c = 1}$ $\quad d + 1 = 1 \Rightarrow \boxed{d = 0}$

So $V = [a\ b\ c\ d] = [1\ 0\ 1\ 0]$.

Q1 (a) $a = [1, 2]$, $b = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. Find $a \cdot b$

Sol: $a \cdot b = [1\ 2] \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$= 1 \times 4 + 2(-1)$

$= 4 - 2 = 2$ Ans.

Q2 is similarly to Q1

Q3 Let $a = [-3, 2\ x]$ and $b = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$ if $a \cdot b = 17$ find x.

Sol: Given that $a \cdot b = 17$ — (i)

$a \cdot b = [-3\ 2\ x] \cdot \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$

$= -3 \times -3 + 2(2) + x \cdot x$

$= 9 + 4 + x^2$

$a \cdot b = 13 + x^2$ — (ii)

comparing eq (i) & eq (ii)

$17 = 13 + x^2$

$\Rightarrow x^2 = 17 - 13 = 4$

$x^2 = 4$

$\boxed{x = \pm 2}$ A.

Q4 Let $W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$. compute $W \cdot W$

Sol: $W \cdot W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$

$= \sin^2\theta + \cos^2\theta$

$\boxed{W \cdot W = 1}$ Ans.

Q5 Find all values of x so that $V \cdot V = 1$ where $V = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$.

Sol: $V \cdot V = 1$

$\begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} = 1$

$(\frac{1}{2})(1/2) + (-1/2)(-1/2) + x \cdot x = 1$

$1/4 + 1/4 + x^2 = 1$

$\frac{1+1}{4} - 1 = -x^2$

$\frac{\cancel{2}}{\cancel{4}_2} - 1 = -x^2$

$\frac{1-2}{2} = -x^2$

$\cancel{-}1/2 = \cancel{-}x^2$

$x^2 = 1/2 = \frac{2}{4}$

$\boxed{x = \pm\frac{\sqrt{2}}{2}}$ Ans.

Q6 Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ if $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ find x & y.

Sol: Given that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ — (i)

Now $A \cdot B = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$= \begin{bmatrix} y+2x+x \\ 3y-x+2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix}$ — (ii)

comparing eq (i) & eq (ii)

$\begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$y+3x = 6$ — (i) $\quad 3y-x+2 = 8$ — (ii)

Xing eq (i) by 3 then subtr. from (ii)

$3y + 9x = 18$

$\underline{\overset{-}{3y} \overset{+}{-x} = \overset{-}{6}}$

$10x = 12 \Rightarrow \boxed{x = 6/5}$

eq (i) $\Rightarrow y + 3(6/5) = 6$

$y = 6 - 18/5 = \frac{30-18}{5} = 12/5$

$\boxed{y = 12/5}$

In Exercises 7 and 8 let

$A=\begin{bmatrix}1 & 2 & -3\\ 4 & 0 & -2\end{bmatrix}$ $B=\begin{bmatrix}3 & 1\\ 2 & 4\\ -1 & 5\end{bmatrix}$ $C=\begin{bmatrix}2 & 3 & 1\\ 3 & -4 & 5\\ 1 & -1 & -2\end{bmatrix}$.

$D=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$ $E=\begin{bmatrix}1 & 0 & -3\\ -2 & 1 & 5\\ 3 & 4 & 2\end{bmatrix}$ and $F=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}$.

Q7 If Possible compute

a) AB

$\because$ if $C_1 = R_1$ then xing the

$$A.B=\begin{bmatrix}1 & 2 & -3\\ 4 & 0 & -2\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 4\\ -1 & 5\end{bmatrix}$$

$$=\begin{bmatrix}\cancel{3}1\times3+2\times2+(-3)(-3) & 1\times1+2\times4+(-3)(5)\\ 4\times3+0\times2+(-2)(-1) & 4\times1+0\times4+(-2)(5)\end{bmatrix}$$

$$=\begin{bmatrix}3+4+3 & 1+8-15\\ 12+0+2 & 4+0-10\end{bmatrix}$$

$$=\begin{bmatrix}10 & -6\\ 14 & -6\end{bmatrix} \text{ A.}$$

Q9 Let $A=\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1 & 3\\ 1 & 2 & 4\end{bmatrix}$.

(a) The (1,2) entry

Sol: (1,2) means = 1st row & 2nd column of AB matrix.

$[2,3]\begin{bmatrix}-1\\ 2\end{bmatrix} = [-2+3\times2] = [-2+6] = 4$. A.

(b) (2,3) entry.

$[-1\ 4]\begin{bmatrix}3\\ 4\end{bmatrix} = [-3+16] = 13$ A

Here was my notes
My notes were placed here

(c) (3,1) entry.

$[0\ 3]\begin{bmatrix}3\\ 1\end{bmatrix} = 0+3 = 3$ A

(d) (3,3) entry

$[0\ 3]\begin{bmatrix}3\\ 4\end{bmatrix} = 12$ A.

Q10 If $I_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $D=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$ compute DI_2 & I_2D.

Sol: $I_2D = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix} = \begin{bmatrix}2+0 & 3-0\\ 0-1 & 0-2\end{bmatrix} = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$.

& $DI_2 = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$.

So $I_2D = DI_2 = D$ A

Q11 Let $A=\begin{bmatrix}1&2\\3&2\end{bmatrix}$ & $B=\begin{bmatrix}2&-1\\-3&4\end{bmatrix}$

Show that $AB\neq BA$.

Sol $A.B=\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}2&-1\\-3&4\end{bmatrix}=\begin{bmatrix}2-6&-1+8\\6-6&-3+8\end{bmatrix}=\begin{bmatrix}-4&7\\0&5\end{bmatrix}$.

$B.A=\begin{bmatrix}2&-1\\-3&4\end{bmatrix}\begin{bmatrix}1&2\\3&2\end{bmatrix}=\begin{bmatrix}2-3&4-2\\-3+12&-6+8\end{bmatrix}=\begin{bmatrix}-1&2\\9&2\end{bmatrix}$.

So $AB\neq BA$.

Q12 $A=\begin{bmatrix}1&2&-1\\3&1&4\end{bmatrix}_{2\times3}$ $\quad O=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}_{3\times2}$

$AO=\begin{bmatrix}0&0\\0&0\end{bmatrix}_{2\times2}$.

Q13 a) The 1st column AB $A=\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}$ $\quad B=\begin{bmatrix}1&0&-1&2\\3&3&-3&4\\4&2&5&1\end{bmatrix}$.

Sol
$$\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}\begin{bmatrix}1\\3\\4\end{bmatrix}=\begin{bmatrix}1-3+8\\3+6+16\\4-6+12\\2+3+20\end{bmatrix}=\begin{bmatrix}6\\25\\10\\25\end{bmatrix}$$

b) The 3rd colum.

$$\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}\begin{bmatrix}-1\\-3\\5\end{bmatrix}=\begin{bmatrix}\\ \\ \\ \end{bmatrix}$$

Q14 is Similarly Q13.

Q15 Let $A=\begin{bmatrix}2&-3&4\\-1&2&3\\5&-1&-2\end{bmatrix}$ and $C=\begin{bmatrix}2\\1\\4\end{bmatrix}$.

$c_1\,col_1A+c_2\,col_2A+c_3\,col_3A$

$=2\begin{bmatrix}2\\-1\\5\end{bmatrix}+1\begin{bmatrix}-3\\2\\-1\end{bmatrix}+4\begin{bmatrix}4\\3\\-2\end{bmatrix}$.

Q16 $A=\begin{bmatrix}1&-2&-1\\2&4&3\\3&0&-2\end{bmatrix}$ and $B=\begin{bmatrix}1&-1\\3&2\\2&4\end{bmatrix}$.

$col_1(AB)=A\,col_1(B)$

$=1\begin{bmatrix}1\\2\\3\end{bmatrix}+3\begin{bmatrix}-2\\4\\0\end{bmatrix}+2\begin{bmatrix}-1\\3\\-2\end{bmatrix}$.

$col_2(AB)=A\,col_2(B)$

$=-1\begin{bmatrix}1\\2\\3\end{bmatrix}+2\begin{bmatrix}-2\\4\\0\end{bmatrix}+4\begin{bmatrix}-1\\3\\-2\end{bmatrix}$.

Q17 $A=\begin{bmatrix}2&-3&1\\1&2&4\end{bmatrix}$ and $B=\begin{bmatrix}3\\5\\2\end{bmatrix}$.

a) Verify $AB = 3a_1 + 5a_2 + 2a_3$.

$$AB = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-15+2 \\ 3+10+8 \end{bmatrix} = \begin{bmatrix} -7 \\ 21 \end{bmatrix} \text{ — (1)}$$

Now $3a_1 + 5a_2 + 2a_3 = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5\begin{bmatrix} -3 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

$$= \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -15 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 6-15+2 \\ 3+10+8 \end{bmatrix} = $$

from eq (1) & eq (2) we have

$AB = 3a_1 + 5a_2 + 2a_3$ Hence Verified.

(b) Verify that $AB = \begin{bmatrix} (row_1(A))B \\ (row_2(A))B \end{bmatrix}$.

from Part a $AB = \begin{bmatrix} -7 \\ 21 \end{bmatrix}$ — (1)

$[row_1(A)]B = [2\ -3\ 1]\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = [6-15+2] = -7$.

$[row_2(A)]B = [1\ 2\ 4]\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = [3+10+8] = 21$

Now $\begin{bmatrix} (row_1(A))B \\ (row_2(A))B \end{bmatrix} = \begin{bmatrix} -7 \\ 21 \end{bmatrix}$ — (2)

from (1) & (2) Hence Verified.

Q18 Sol:- $\begin{bmatrix} -2 & 2 & 3 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$.

Q19 a) $\begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 2 & 3 & 0 \\ 2 & 3 & -4 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix}$ which is coefficient matrix

b) $\begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 2 & 3 & 0 \\ 2 & 3 & -4 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 5 \end{bmatrix}$ which is the linear system in matrix form.

c) $\left[\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 7 \\ 3 & 2 & 3 & 0 & -2 \\ 2 & 3 & -4 & 0 & 3 \\ 1 & 0 & 3 & 0 & 5 \end{array}\right]$ which is augmented matrix.

Q20 a) Sol:-

$$-2x - y + 0z + 4w = 5$$
$$-3x + 2y + 7z + 8w = 3$$
$$x + 0y + 0z + 2w = 4$$
$$3x + 0y + z + 3w = 6$$

Q21 similarly Q20

Q22 similarly Q19.

Q23 Sol: ~~These~~ They are equivalent as third row have zero entry.

Q24. a) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Q25 a) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

b) $\begin{bmatrix} 2 & -3 & 5 \\ 1 & 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

$x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} -3 \\ 4 \end{bmatrix} + z\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

Q26 a) ~~can~~ Say nothing.

b) can say nothing.

Q27 a) $AB^t = 0$

$\begin{bmatrix} x & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 0$

$1(x) + 3(1) - 1(-2) = 0$

$x + 3 + 2 = 0$

$\boxed{x = -5}$

Q28 Similarly Q27.

Q31 $A = \begin{array}{cc} \begin{array}{cc} A.P & F.P \end{array} & \\ \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} & \begin{array}{l} \text{Chair} \\ \text{Table} \end{array} \end{array}$ $B = \begin{array}{cc} \begin{array}{cc} S.L.C & \text{Chicago} \end{array} & \\ \begin{bmatrix} 9 & 10 \\ 10 & 12 \end{bmatrix} & \begin{array}{l} A.P \\ F.P \end{array} \end{array}$

$AB = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 & 10 \\ 10 & 12 \end{bmatrix} = \begin{array}{cc} \begin{array}{cc} S.L.C & \text{Chicago} \end{array} & \\ \begin{bmatrix} 38 & 44 \\ 67 & 78 \end{bmatrix} & \begin{array}{l} \text{Chair} \\ \text{table} \end{array} \end{array}$

where AB gives total cost of Producing each kind of Product in each city.

Q32 $A = \begin{array}{cc} \begin{array}{ccc} S.D & N.O & P.M \end{array} & \\ \begin{bmatrix} 300 & 100 & 150 \\ 200 & 250 & 400 \end{bmatrix} & \begin{array}{l} \text{Product P} \\ \text{Product Q} \end{array} \end{array}$

$B = \begin{array}{cc} \begin{array}{cc} \text{Plant X} & \text{Plant Y} \end{array} & \\ \begin{bmatrix} 8 & 12 \\ 7 & 9 \\ 15 & 10 \end{bmatrix} & \begin{array}{l} S.D \\ N.O \\ P.M \end{array} \end{array}$

$A \cdot B = \begin{bmatrix} 300 & 100 & 150 \\ 200 & 250 & 400 \end{bmatrix}\begin{bmatrix} 8 & 12 \\ 7 & 9 \\ 15 & 10 \end{bmatrix}$

$= \begin{array}{cc} \begin{array}{cc} \text{Plant X} & \text{Plant Y} \end{array} & \\ \begin{bmatrix} 5350 & 6000 \\ 9350 & 8650 \end{bmatrix} & \begin{array}{l} \text{Product P} \\ \text{" Q.} \end{array} \end{array}$

where AB gives each Product quantity in each Plant.

Q33 $A = \begin{array}{cc|c} \text{Adults} & \text{children} & \\ \end{array} \begin{bmatrix} 80 & 120 \\ 100 & 200 \end{bmatrix} \begin{array}{l} \text{Male} \\ \text{Female} \end{array}$ $B = \begin{array}{ccc} P & F & C \end{array} \begin{bmatrix} 20 & 20 & 20 \\ 10 & 20 & 30 \end{bmatrix} \begin{array}{l} \text{Adult} \\ \text{child} \end{array}$

a) $\begin{bmatrix} 20 \\ 10 \end{bmatrix} \begin{bmatrix} 80 & 120 \end{bmatrix} = 80 \times 20 + 10 \times 120 = 2800g.$

b) $\begin{bmatrix} 20 \\ 20 \end{bmatrix} \begin{bmatrix} 100 & 200 \end{bmatrix} = 20 \times 100 + 20 \times 200 = 6000g.$

Q34 (a) $\begin{bmatrix} 220 \\ 330 \\ 120 \end{bmatrix} \begin{bmatrix} 200 & 150 & 120 \end{bmatrix} = 103400$

(b) $\begin{bmatrix} 50 & 40 & 25 \end{bmatrix} \begin{bmatrix} 100 \\ 120 \\ 250 \end{bmatrix} = 16050.$

Q35 a) $S_1 = \begin{bmatrix} 18.95 & 14.75 & 8.98 \end{bmatrix}$

$S_2 = \begin{bmatrix} 17.80 & 13.50 & 10.79 \end{bmatrix}$

$\varphi = \begin{bmatrix} 18.95 & 14.75 & 8.98 \\ 17.80 & 13.50 & 10.79 \end{bmatrix}$

(b) 20% Reduced.

$\varphi = \begin{bmatrix} 18.95 \times 80\% & 14.75 \times 80\% & 8.98 \times 80\% \\ 17.80 \times 80\% & 13.50 \times 80\% & 10.79 \times 80\% \end{bmatrix}.$

$= \begin{bmatrix} 15.16 & 11.8 & 7.18 \\ 14.24 & 10.8 & 8.63 \end{bmatrix}.$

Q36 (b) $a \cdot b = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = 0$

(a) $a \cdot b = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 1$

Q37 Similarly to Q36.

Q38 $a = \begin{bmatrix} 1 & x & 0 \end{bmatrix}$ & $b = \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix}$ find $x = ?$ & $a \cdot b = 0$

$a \cdot b = \begin{bmatrix} 1 & x & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix} = 0$

$= 1[x] + 1[x] + 0[1] = 0$

$= x + x = 0$

$2x = 0 \Rightarrow \boxed{x = 0}$

Q39 $AB = \begin{bmatrix} 1 & 1 & x \\ 0 & y & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$

$= \begin{bmatrix} x + 0 \\ y + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{x = 0,\ y = -1 = 1}$ for Bit M

Q41 $AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{c = 0}, \boxed{d = 1}$

$a + c = 1 \Rightarrow a + 0 = 1 \Rightarrow \boxed{a = 1}$

$b+d=0 \Rightarrow$ ~~1+d~~. $b+1=0 \Rightarrow b=-1=1$ (fr

$B=\begin{bmatrix} 1 & +1 \\ 0 & 1 \end{bmatrix}$

———X———

Mr. Jamal Nasir
Lecturer U.E.T Peshawer.

Q1 Verify theorem 1.1

(a) $A+B=B+A$

$A+B=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix}$ and $B+A=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix}$.

(b) $A+(B+C)=(A+B)+C$.

$B+C=\begin{bmatrix} -2 & -6 & 2 \\ 5 & 1 & 5 \end{bmatrix}$.

$A+(B+C)=\begin{bmatrix} -1 & -4 & 0 \\ 8 & 5 & 10 \end{bmatrix}$.

Similarly

$A+B=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix}$

$(A+B)+C=\begin{bmatrix} -1 & -4 & 0 \\ 8 & 5 & 10 \end{bmatrix}$.

Here $\boxed{A+(B+C)=(A+B)+C}$

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Q2 Verify (a) of theorem 1.2 for
$A(BC) = (AB)C$.

Sol:
$$BC = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1+9+2 & -3+4 \\ 1-9+4 & 3+8 \end{bmatrix} = \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix} = \begin{bmatrix} 10-12 & 1+33 \\ 20+4 & 2-11 \end{bmatrix} = \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}.$$

$$= \begin{bmatrix} -1+3 & 3-9 & 2+12 \\ -2-1 & 6+3 & 4-4 \end{bmatrix} = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}.$$

$$(AB)C = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}.$$

Hence $\boxed{A(BC) = (AB)C.}$

Q3 Verify (b) of th. 1.2

$A(B+C) = AB + AC$

S.Y.S.

Q4 Verify (a), (b) and (c) of theorem 1.3 for $\gamma = 6$, $S = -2$.

(a) $\gamma(SA) = (\gamma S)A$

(b) $(\gamma + S)A = \gamma A + SA$

(c) $\gamma(A+B) = \gamma A + \gamma B$

~~(d) A+B~~

(a) $\gamma(SA) = (\gamma S)A$

$$SA = -2\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix}.$$

$$\gamma(SA) = 6\begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}.$$

$(\gamma S) = (-2)(6) = -12$

$$(\gamma S)A = -12\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}.$$

Hence $\boxed{\gamma(SA) = (\gamma S)A}$ verified.

$\boxed{\text{(b) \& (c) Similarly to a.}}$

Q5 verify (d) of thm: 1.3 for $r = -3$.

(d) $A(rB) = r(AB) = (rA)B$.

S.Y.S.

Q6 verify (b) & (d) of thm: 1.4 for $r = -4$

(b) $(A+B)^T = A^T + B^T$

(d) $(rA)^T = rA^T$

S.Y.S.

Q7 verify (c) of Thm: 1.4 for

(c) $(AB)^T = B^T A^T$

S.Y.S.

Q10, Q11 S.Y.S

Q12 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ show $A^2 = I_2$

$A.A = A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

Hence $A^2 = I_2$

Q13 & Q14 S.Y.S.

Q15 $Ax = rx$.

$Ax = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

$rx = r\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$

given $Ax = rx$

$\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix} \Rightarrow \boxed{r = 3}$.

Q16 $(kA)^T(kA) = 1$

$kA = k\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2k \\ k \\ -k \end{bmatrix}$.

$(kA)^T = [-2k \; k \; -k]$.

$(kA)^T(kA) = [-2k \; k \; -k]\begin{bmatrix} -2k \\ k \\ -k \end{bmatrix} = 4k^2 + k^2 + k^2 = 1$

$6k^2 = 1$

$k^2 = 1/6$

$\boxed{k = \pm 1/\sqrt{6}}$ A.

Q18 a) $Ax_0 = \begin{bmatrix} 1/3 & 2/5 \\ 2/3 & 3/5 \end{bmatrix} \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$.

$Ax_0 \begin{bmatrix} 2/9 + 2/15 \\ 4/9 + 3/15 \end{bmatrix}$.

b) For stable distribution

$Ax_0 = x_0$

$\begin{bmatrix} 1/3 & 2/5 \\ 2/3 & 3/5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$= \begin{bmatrix} 1/3 a + 2/5 b \\ 2/3 a + 3/5 b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$1/3 a + 2/5 b = a$ — (i).

$2/3 a + 3/5 b = b$ — (ii).

eq (i) $\Rightarrow \frac{1}{3}a - a + 2/5 b = 0 \Rightarrow$ $\frac{3}{3}$

$-\frac{2}{3}a + 2/5 b = 0$

$-10a + 6b = 0$ ——— (iii)

eq (ii) $\Rightarrow \frac{2}{3}a + 3/5 b - b = 0$

$\Rightarrow 10a - 6b = 0$ ——— (iv)

Solving eq (iii) & (iv)

$a = 3/8$ & $b = 5/8$ $\quad x_0 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3/8 \\ 5/8 \end{bmatrix}$.

Q19

$$A = \begin{array}{cc} \begin{array}{cc} M & N \end{array} & \\ \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} & \begin{array}{c} M \\ N \end{array} \end{array} \qquad x_0 = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}.$$

(a) $Ax_0 = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 1/9 + 2/6 \\ 2/9 + 2/6 \end{bmatrix} = \begin{bmatrix} 4/9 \\ 5/9 \end{bmatrix}$.

(b) $Ax_0 = x_0$

$\begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$\frac{1}{3}a + 1/2 b = a$ —— (i)

$2/3 a + 1/2 b = b$ —— (ii)

S.Y.S

$a = 3/7$ & $b = 4/7$

$x_0 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3/7 \\ 4/7 \end{bmatrix}$.

Q20 (a) $x_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$

$$A = \begin{array}{cc} \begin{array}{ccc} R & S & T \end{array} & \\ \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} & \begin{array}{c} R \\ S \\ T \end{array} \end{array}.$$

$Ax_0 = \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 13/36 \\ 17/36 \\ 1/6 \end{bmatrix}$.

after 2 years

$x_2 = Ax_1 = A(\cancel{x}Ax_0)$

$$= \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} 13/36 \\ 17/36 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 43/108 \\ 191/432 \\ 23/144 \end{bmatrix}.$$

(b) $Ax_0 = x_0$, $x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$a+b+c=1$ —— (1)

$$\begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$\frac{1}{3}a + 1/2 b + 1/4 c = a$ —— (2)

$2/3 a + 1/4 b + 1/2 c = b$ —— (3)

$1/4 b + 1/4 c = c$ —— (4)

$eq (2) \Rightarrow \frac{4a+6b+3c}{12} = a$

$4a+6b+3c = 12a$

$\Rightarrow -8a+6b+3c$ —— (5)

$eq (3) \Rightarrow \frac{8a+3b+6c}{12} = b$

$8a+3b+6c = 12b$

$\Rightarrow 8a-9b+6c=0$ —— (6)

$eq (4) \Rightarrow b+c=4c$

$\Rightarrow b-3c=0$ —— (7).

$b=3c$

$eq (5) \Rightarrow -8a+6(3c)+3c=0$.

$\Rightarrow \boxed{a = 21/8 c}$ —— (8)

$eq (1) \Rightarrow 21/8 c + 3c + c = 1$

$\Rightarrow \boxed{c = 8/53}$

$eq (8) \Rightarrow a = \frac{21}{\cancel{8}} \times \frac{\cancel{8}}{53} = \boxed{21/53 = a}$

$eq (7) \Rightarrow b = 3 \cdot 8/53 = \boxed{24/53 = b}$

So $x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 21/53 \\ 24/53 \\ 8/53 \end{bmatrix}$ An.

12 (a) $x_1 = Ax_0$

$$x_1 = \begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$

$$x_1 = \begin{bmatrix} 4/15 \\ 3/10 \\ 13/30 \end{bmatrix}.$$

After 2 years.

$Ax_1 = A(Ax_0)$

$$= \begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 4/15 \\ 3/10 \\ 13/30 \end{bmatrix}$$

$$= \begin{bmatrix} 84/300 \\ 97/300 \\ 119/300 \end{bmatrix}.$$

(b) $Ax_0 = x_0$ $\quad x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$a+b+c = 1$ ——— (1)

Now

$$\begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

S.Y.S as Q 20.

$$x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10/37 \\ 12/37 \\ 15/37 \end{bmatrix}.$$

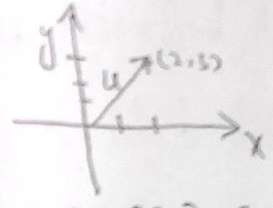
In Exercises 1 through 8, sketch u and its image under the given matrix transformation f.

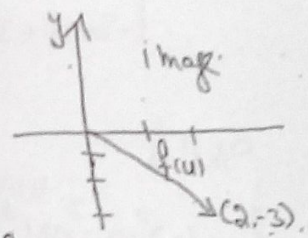
Q1 $f: R^2 \to R^2$ defined by.

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Given $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

sketch





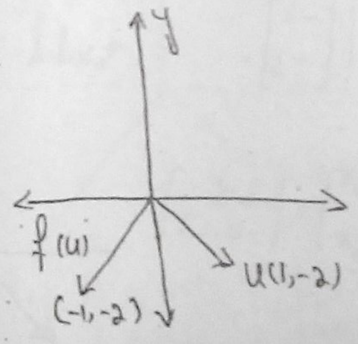
Now $f(u) = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

2 $f: R^2 \to R^2$ (reflection w.r.t y-axis) defined by

$$f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$.

Sketch



Now $f(u) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$

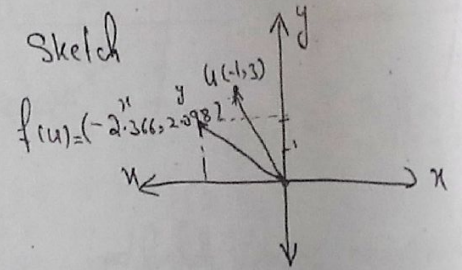
Q3 $f: R^2 \to R^2$ is a counter-clockwise rotation through $30°$, u

$$f(u) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \quad \text{Exp-9}$$

$$= \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$$

$$= \begin{bmatrix} -\cos 30° - 3\sin 30° \\ -\sin 30° + 3\cos 30° \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} - 3/2 \\ -1/2 + \frac{3\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} -2.366 \\ 2.098 \end{bmatrix}.$$

Sketch

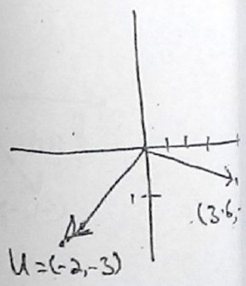


Q4 $f: R^2 \to R^2$ is a counter clock-wise rotation through $2/3\,\pi$ radians $u = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$$f(u) = \begin{bmatrix} \cos 2/3\pi & -\sin 2/3\pi \\ \sin 2/3\pi & \cos 2/3\pi \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}.$$

$$= \begin{bmatrix} -2\cos 2/3\pi + 3\sin 2/3\pi \\ -2\sin 2/3\pi - 3\cos 2/3\pi \end{bmatrix} = \begin{bmatrix} 3.6 \\ -0.23 \end{bmatrix}$$

Sketch

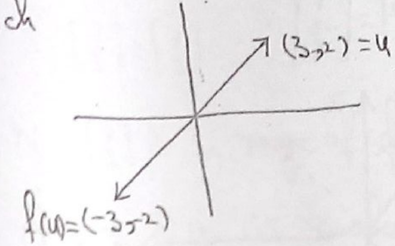


Q5 $f: R^2 \to R^2$ defined by

$$f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}.$$

Sketch



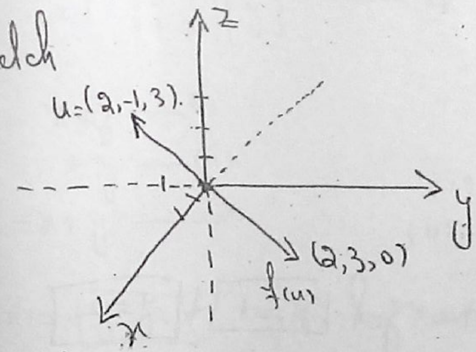
Q6 is similarly to Q2.

Q7 $f: R^3 \to R^3$ defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

Given $u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Sketch



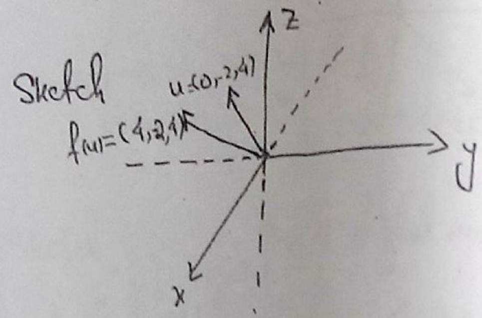
Now $f(u)$ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2+1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}.$

Q8 $f: R^3 \to R^3$ defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad u = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

$$f(u) = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}.$$

Sketch



Q9 Given that $f(x) = Ax$ & $A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$. & $W = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$.

Now $Ax = f(x) = W$

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \text{ --- } (*)$$

$$\begin{bmatrix} x+3y \\ -x+2y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$x + 3y = 7$ — (i)

$-x + 2y = 3$ — (ii)

Solving eq (ii) & (i) we get $\boxed{x = 1}$ & $\boxed{y = 2}$

eq (*) ⇒

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1+6 \\ -1+4 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Hence $W = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ is in range of f ✓

Q10 & Q11 is similarly do Q9

Q11 $f: R^2 \to R^3$.

Given that $f(x) = Ax$ & $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ & $W = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

Now $Ax = f(x) = W$, where $x = \begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow (*)$$

$1 = x + 2y$ — (i)

$-1 = \quad + y$ — (ii)

$2 = x + y$ — (iii)

eq (ii) ⇒ $\boxed{y = -1}$ eq (iii) ⇒ $x = 2 + 1 = 3 \Rightarrow \boxed{x = 3}$

$\therefore (*) \Rightarrow$

$$\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$\begin{bmatrix}3-2\\0-1\\3-1\end{bmatrix}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$\begin{bmatrix}1\\-1\\2\end{bmatrix}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

Hence $W=\begin{bmatrix}1\\-1\\2\end{bmatrix}$ is in the range of f.

Q13 Given $w=\begin{bmatrix}1\\1\\1\end{bmatrix}$ & $x=\begin{bmatrix}x\\y\end{bmatrix}$

Now $Ax=f(x)=w$

$$\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \text{——} (*)$$

$$\begin{bmatrix}x+2y\\0+y\\x+y\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$x+2y=1$ —— (i)

$+y=1$ —— (ii)

$x+y=1$ —— (iii)

$\therefore (ii) \Rightarrow \boxed{y=1}$ $\therefore (iii) \Rightarrow \boxed{x=0}$ & $\therefore (i) \Rightarrow \boxed{x=-}$

x has two values.

$\therefore (*) \Rightarrow$

$$\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}2\\1\\1\end{bmatrix}\neq\begin{bmatrix}1\\1\\1\end{bmatrix}$$

w is not in range of f

Again $\therefore (*) \Rightarrow$

$$\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}-1+2\\0+1\\-1+1\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\1\\0\end{bmatrix}\neq\begin{bmatrix}1\\1\\1\end{bmatrix}$$

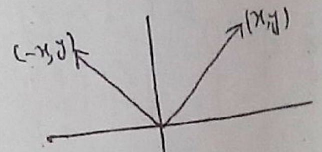
W is not in range of f.

Mr. Jamal Nasir

Lecturer U.E.T Peshawar.

Q15 (a) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ let $u = \begin{bmatrix} x \\ y \end{bmatrix}$

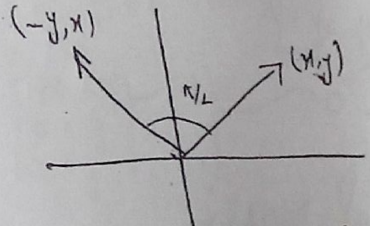
$$Au = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x+0y \\ 0x+y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}.$$



Reflection about y-axis.

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $u = \begin{bmatrix} x \\ y \end{bmatrix}$

$$Au = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0-y \\ x+0 \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$



Rotate counter clockwise through $\pi/2$.

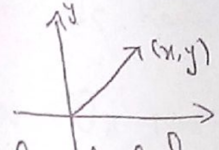
$x = -y =$

$y = x \Rightarrow$

$= (x, y) = ($

16(a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, let $u = \begin{bmatrix} x \\ y \end{bmatrix}$.

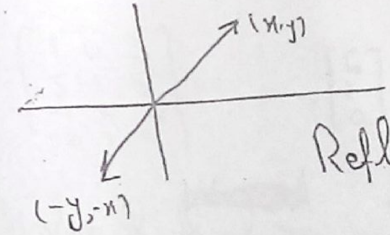
$$Au = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}.$$



Reflection about line $y = x$.

(b) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ let $u = \begin{bmatrix} x \\ y \end{bmatrix}$.

$$Au = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}.$$



Reflection about line $y = -x$.

Q17 (a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, let $u = \begin{bmatrix} x \\ y \end{bmatrix}$ $Au = \begin{bmatrix} x \\ 0 \end{bmatrix}$.

Projection onto x-axis.

(b) $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $u = \begin{bmatrix} x \\ y \end{bmatrix}$ then $Au = \begin{bmatrix} 0 \\ y \end{bmatrix}$.

Projection onto y-axis.

Q18(a) $A=\begin{bmatrix}1&2&0\\0&1&-1\end{bmatrix}$, $W=\begin{bmatrix}0\\-1\end{bmatrix}$.

$Ax=w$

$$\begin{bmatrix}1&2&0\\0&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\-1\end{bmatrix}$$

$x+2y=0$

$y-z=-1$

$x=-2y$ — (i) & $y+1=z$ — (2)

Let $z=r$ be real no.

Let $z=0$ then $\boxed{y=-1}$ & $\boxed{x=2}$

Let $z=1$, $y=0$, $x=0$

So $u=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $v=\begin{bmatrix}2\\-1\\0\end{bmatrix}$.

(b) $A=\begin{bmatrix}2&1&0\\0&2&-1\end{bmatrix}$, $W=\begin{bmatrix}4\\4\end{bmatrix}$

$Ax=w$

$$\begin{bmatrix}2&1&0\\0&2&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\4\end{bmatrix} \Rightarrow 2x+y=4 \Rightarrow \boxed{x=\frac{4-y}{2}}$$

$$2y-z=4 \Rightarrow \boxed{y=\frac{4+z}{2}}$$

Let $z=4$, $y=4$, $x=0$

Let $z=0$ $y=2$, $x=1$

So $u=\begin{bmatrix}0\\4\\4\end{bmatrix}$, $v=\begin{bmatrix}1\\2\\0\end{bmatrix}$

Ex: 1.6

Q1 Reduced row echelon form,

Q2 Neither Q3 Reduced row echelon form.

Q4 Neither Q5 Row echelon form Q6 Neither.

Q7 Neither Q8. Neither.

Q9 $A=\begin{bmatrix}1&0&3\\-3&1&4\\4&2&2\\5&-1&5\end{bmatrix}$.

a) Interchanging the second and fourth rows.

$$A=\begin{bmatrix}1&0&3\\5&-1&5\\4&2&2\\-3&1&4\end{bmatrix}.$$

b) Multiplying the third row by 3.

$$A=\begin{bmatrix}1&0&3\\-3&1&4\\12&6&6\\5&-1&5\end{bmatrix}$$

(c) Adding (-3) times the 1st row to the fourth row

$$A=\begin{bmatrix}1 & 0 & 3\\ -3 & 1 & 4\\ 4 & 2 & 2\\ 2 & -1 & -4\end{bmatrix}.$$

R.w

-3 0 -9 (-3

+with R_4

$$\begin{array}{ccc}-3 & 0 & -9\\ 5 & -1 & 5\\ \hline \end{array}$$
$$\boxed{2 \quad -1 \quad -4}$$

Q_{10} is similarly to Q_9.

Q_{11} Find three matrices that are row equivalent to

$$A=\begin{bmatrix}2 & -1 & 3 & 4\\ 0 & 1 & 2 & -1\\ 5 & 2 & -3 & 4\end{bmatrix}.$$

1st Possibility

(a) $\begin{bmatrix}2 & -1 & 3 & 4\\ 5 & 2 & -3 & 4\\ 0 & 1 & 2 & -1\end{bmatrix} \underset{\sim}{R_{23}}$

2nd Possibility

(b) $\begin{bmatrix}4 & -2 & 6 & 8\\ 0 & 1 & 2 & -1\\ 5 & 2 & -3 & 4\end{bmatrix} \underset{\sim}{2R_1}$

3rd Possibility

(c) $\begin{bmatrix}2 & -1 & 3 & 4\\ 0 & 1 & 2 & -1\\ 7 & 1 & 0 & 8\end{bmatrix} \underset{\sim}{R_1 + R}$

Q_{13} In Exercise 13 through 16, find a row echelon form of the given matrix.

Q_{13} $\begin{bmatrix}0 & -1 & 2 & 3\\ 2 & 3 & 4 & 5\\ 1 & 3 & -1 & 2\\ 3 & 2 & 4 & 1\end{bmatrix}$

$\begin{bmatrix}1 & 3 & -1 & 2\\ 2 & 3 & 4 & 5\\ 0 & -1 & 2 & 3\\ 3 & 2 & 4 & 1\end{bmatrix} \underset{\sim}{R_{13}}$ row operation

$\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & -3 & 6 & 1\\ 0 & -1 & 2 & 3\\ 0 & -7 & 7 & -5\end{bmatrix} \begin{array}{l}R_2 - 2R_1\\ R_4 - 3R_1\end{array}$

$R_4 - 3R_1$

$$\begin{array}{cccc}3 & 2 & 4 & 1\\ -3 & -9 & +3 & -6\\ \hline 0 & -7 & 7 & -5\end{array}$$

$\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & -1 & 2 & 3\\ 0 & -3 & 6 & 1\\ 0 & -7 & 7 & -5\end{bmatrix} \underset{\sim}{R_{23}}$

$\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & 1 & -2 & -3\\ 0 & -3 & 6 & 1\\ 0 & -7 & 7 & -5\end{bmatrix} (-1)R_2$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & -7 & -26 \end{bmatrix} \underset{R_4+7R_2}{\overset{R_3+3R_2}{\sim}}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -7 & -26 \\ 0 & 0 & 0 & -8 \end{bmatrix} \underset{\sim}{R_{34}}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & \phi & -2 & -3 \\ 0 & 0 & 1 & 26/7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underset{R_4/-8}{\overset{R_3/-7}{\sim}}$$

A.

Q4

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & -3 & -1 & 5 \\ 1 & 3 & 2 & 5 \\ 1 & 1 & 0 & 2 \\ 2 & -6 & -2 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & 2 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & -2 & -2 & -3 \end{bmatrix} \begin{matrix} \underset{\sim}{R_2-2R_1} \\ \underset{\sim}{R_3-R_1} \\ \underset{\sim}{R_4-R_1} \\ \underset{\sim}{R_5-R_1}. \end{matrix}$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 7 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 4 & 1 \end{bmatrix} \begin{matrix} \underset{\sim}{R_3-5R_2} \\ \underset{\sim}{R_4-3R_2} \\ \underset{\sim}{R_5+2R_2}. \end{matrix}$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 4 & 1 \end{bmatrix} \underset{\sim}{\tfrac{1}{7}R_3}.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & -15/7 \\ 0 & 0 & 0 & 15/7 \end{bmatrix} \begin{matrix} \underset{\sim}{R_4-3R_3} \\ \underset{\sim}{R_5-4R_3}. \end{matrix}$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15/7 \end{bmatrix} \left(-\tfrac{7}{15}\right)R_4.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \; R_5-\frac{15}{7}R_4$$

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Q18 Let $A=\begin{bmatrix}1&2&1\\-1&1&2\\2&1&-2\end{bmatrix}$ & $Ax=b$.

(a) $X=\begin{bmatrix}1\\2\\3\end{bmatrix}$; $b=0$

Now $Ax=b$

$$\begin{bmatrix}1&2&1\\-1&1&2\\2&1&-2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1+4+3\\-1+2+6\\2\ \ 2\ \ -6\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}8\\7\\-2\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$$ No Solution.

(b) $X=\begin{bmatrix}0\\0\\0\end{bmatrix}$; $b=0$

$Ax=b$

$$\begin{bmatrix}1&2&1\\-1&1&2\\2&1&-2\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0+0+0\\0+0+0\\0+0+0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Solution Exist. / Qc & Qd Similarly.

In exercises 20 through 22, find all solutions to the given linear system

Q20(a) $x+y+2z=-1$
$x-2y+z=-5$
$3x+y+z=3$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & -5 & 6 \end{array}\right] \begin{array}{l} \underset{\sim}{R_2-R_1} \\ \underset{\sim}{R_3-3R_1} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 3 & 1 & 4 \\ 0 & -2 & -5 & 6 \end{array}\right] (-1)R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & -2 & -5 & 6 \end{array}\right] R_2+R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & -13 & 26 \end{array}\right] R_3+2R_2.$$

$x+y+2z=-1$ —— (i)
$y-4z=10$ —— (ii)
$-13z=26$ —— (iii)

by (iii) $\Rightarrow z=\frac{26}{-13} \Rightarrow \boxed{z=-2}$

by (ii) $\Rightarrow y-4(-2)=10$
$y=10-8$
$\boxed{y=2}$

by (i) $\Rightarrow x+(2)+2(-2)=-1$
$x+2-4=-1 \Rightarrow \boxed{x=1}$.

Qb, Qc, Qd similarly

Q21 (a) $x+y+2z+3w=13$
$x-2y+z+w=8$
$3x+y+z-w=1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \begin{array}{l} R_2-R_3 \\ R_3-3R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 3 & 1 & 2 & 5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] (-1)R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] R_2+R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & -13 & -26 & -104 \end{array}\right] R_3+2R_2$$

$x+y+2z+3w=13$ — (i)

$y-4z-8w=-33$ — (ii)

$-13z-26w=-104$ — (iii)

Let us $\boxed{w=r}$

by (iii) $\Rightarrow$ $-13z-26r=-104$.

$-13(z+2r)=-104$

$z+2r=8$

$\boxed{z=8-2r}$

by (ii) $\Rightarrow$

$y-4(8-2r)-8r=-33$

$y-32+\cancel{8r}-\cancel{8r}=-33$

$y=-33+32=-1 \Rightarrow \boxed{y=-1}$

by (i) $\Rightarrow$ $x+(-1)+2(8-2r)+3r=13$

$x-1+16-4r+3r=13$

$x+15-r=13$

$x=13-15+r$

$\boxed{x=r-2}$

$\boxed{\text{Qb, Qd, Similarly}}$

Q21(c)
$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] R_{13}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & -1 & | & -1 \\ 0 & -5 & 4 & -3 & | & 1 \\ 0 & -1 & 3 & 0 & | & 3 \\ 0 & -6 & 7 & -3 & | & 4 \\ 0 & -6 & 7 & -3 & | & 8 \end{bmatrix} \begin{matrix} R_2-3R_1 \\ R_3-2R_1 \\ R_4-6R_1 \\ R_5-5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & -1 & | & -1 \\ 0 & 1 & -3 & 0 & | & -3 \\ 0 & -5 & 4 & -3 & | & 1 \\ 0 & -6 & 7 & -3 & | & 4 \\ 0 & -6 & 7 & -3 & | & 8 \end{bmatrix} \begin{matrix} -R_3 \\ R_{32}. \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & -1 & | & -1 \\ 0 & 1 & -3 & 0 & | & -3 \\ 0 & 0 & -11 & -3 & | & 1 \\ 0 & 0 & -11 & -3 & | & -14 \\ 0 & 0 & -11 & -3 & | & -10 \end{bmatrix} \begin{matrix} R_3+5R_2 \\ R_4+6R_2 \\ R_5+6R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & -1 & | & -1 \\ 0 & 1 & -3 & 0 & | & -3 \\ 0 & 0 & 1 & 3/11 & | & -1/11 \\ 0 & 0 & -11 & -3 & | & -14 \\ 0 & 0 & -11 & -3 & | & -10 \end{bmatrix} -\frac{1}{11}(R_3)$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & -1 & | & -1 \\ 0 & 1 & -3 & 0 & | & -3 \\ 0 & 0 & 1 & 3/11 & | & -1/11 \\ 0 & 0 & 0 & 0 & | & -15 \\ 0 & 0 & 0 & 0 & | & -11 \end{bmatrix} \begin{matrix} R_4+11R_3 \\ R_5+11R_3 \end{matrix}$$

No Solution.

23
$$x + y - z = 2$$
$$x + 2y \quad z = 3$$
$$x + y + (a^2-5)z = a$$

$$\begin{bmatrix} 1 & 1 & -1 & | & 2 \\ 1 & 2 & 1 & | & 3 \\ 1 & 1 & a^2-5 & | & a \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & a^2-4 & | & a-2 \end{bmatrix} \begin{matrix} R_2-R_1 \\ R_3-R_1 \end{matrix}$$

a) If $a=-2$ no Solution

(b) If $a \neq \pm 2$ then unique Solution

(c) If $a=2$ Infinite many solution

Need $a^2-4=0 \Rightarrow a^2=4 \Rightarrow a=\pm 2$

If $a=-2$: $a^2-4=a-2$

$4-4=-2-2 \Rightarrow 0=-4$ not soln

If $a=2$ $4-4=2-2 \Rightarrow 0=0$ Infinite Sol:

If $a \neq \pm 2$ unique Sol.

Q24. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & (a^2-1) & a+1 \end{array}\right]$.

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-2R_1 \end{array}$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right]$

a) $a=\pm\sqrt{3}$ (b) $a \neq \pm\sqrt{3}$, (c) None.

Q25. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right] \begin{array}{l} R_2-R_1 \\ R_3-R_1 \end{array}$

a) $a=\pm\sqrt{6}$, (b) $a \neq \pm\sqrt{6}$ (c) None.

Q26 $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right] R_2-R_1$

(a) $a=-3$, (b) $a \neq \pm 3$, (c) $a=3$.

Q 27 — to — Q 30 Similarly to Q20

———X———.

Q31 Sol:- $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ —— (i)

& $f\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right] = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$ —— (ii)

comparing eq (i) & (ii) we get

$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$.

$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$.

By augmented matrix

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 1 & 1 & 0 & -1/2 \end{array}\right] \frac{R_3}{2}$.

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 2 & -1 & 3 & 5 \\ 4 & 1 & 3 & 4 \end{array}\right] R_{13}$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{1}{2} \\ 0 & -3 & 3 & 6 \\ 0 & -3 & 3 & 6 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 3 & 6 \end{array}\right] -\frac{R_2}{3}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3+3R_2.$$

Now $x+y=-\frac{1}{2}$ —— (i)

$y-z=-2$ —— (ii)

Let $z=t$

By (ii) $\Rightarrow$ $\boxed{y=-2+t}$

By (i) $\Rightarrow$ $x+(-2+t)=-\frac{1}{2}$

$x=-\frac{1}{2}+2-t$

$\boxed{x=\frac{3}{2}-t}$

~o~

Mr. Jamal Nasir

Lecturer U.E.T Peshawer

33 Sol:-

$$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

A.M

$$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix}=\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right]$$

Augmented matrix

$$\sim \left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right] R_{13}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right] (\tfrac{1}{2})R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & -3 & 3 & a-2c \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & 1 & -1 & (\frac{-b+c}{3}) \\ 0 & -3 & 3 & a-2c \end{array}\right] -R_2/3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & 1 & -1 & -b+c/3 \\ 0 & 0 & 0 & a-c-b \end{array}\right] R_3+3R_2$$

Solution is possible if

$a-c-b=0$

$\boxed{-a+c+b=0}$

Q34 is similarly to Q33

Q35 $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$, $b_1=\begin{bmatrix}1\\ -8\end{bmatrix}$, $b_2=\begin{bmatrix}5\\ -5\end{bmatrix}$.

$Ax=b_1$

$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ -8\end{bmatrix}$.

$\begin{bmatrix}x-y\\ 2x+3y\end{bmatrix}=\begin{bmatrix}1\\ -8\end{bmatrix}$.

New Augmented matrix

$\left[\begin{array}{cc|c}1 & -1 & 1\\ 2 & 3 & -8\end{array}\right] \Rightarrow \sim \left[\begin{array}{cc|c}1 & -1 & 1\\ 0 & 5 & -10\end{array}\right]$ R_2-2R_1

$\Rightarrow 5y=-10 \Rightarrow \boxed{y=-2}$

$x-y=1 \Rightarrow x=-2+1=\boxed{x=-1}$

$\boxed{x=-1,\ y=-2}$

2nd Part $Ax=b_2$

$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}$. $\Rightarrow \begin{bmatrix}x-y\\ 2x+3y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}$.

A.M

$\left[\begin{array}{cc|c}1 & -1 & 5\\ 2 & 3 & -5\end{array}\right]$

$\sim\left[\begin{array}{cc|c}1 & -1 & 5\\ 0 & 5 & -15\end{array}\right]$ R_2-2R_1

$5y=-15 \Rightarrow \boxed{y=-3}$

$x-y=5 \Rightarrow x=5-3 \Rightarrow \boxed{x=2}$

$\boxed{x=2,\ y=-3}$

3rd Part $[A \vdots b_1\ b_2]$

$\left[\begin{array}{cc|cc}1 & -1 & 1 & 5\\ 2 & 3 & -8 & -5\end{array}\right]$

$\sim\left[\begin{array}{cc|cc}1 & -1 & 1 & 5\\ 0 & 5 & -10 & -15\end{array}\right]$ R_2-2R_1

Same results

o

Q36 is similarly to Q35.

Q37 $A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix}$ & $(-4I_3 - A)X = 0$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$-4I_3 = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}.$

$-4I_3 - A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -5 \\ -1 & -5 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$

Now $(-4I_3 - A)X = 0$

$\Rightarrow \begin{bmatrix} -5 & 0 & -5 \\ -1 & -5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -5x + 0y - 5z \\ -1x - 5y - 1z \\ 0x - 1y + 0z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

A.M.

$\left[\begin{array}{ccc|c} -5 & 0 & -5 & 0 \\ -1 & -5 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} -1 & -5 & -1 & 0 \\ -5 & 0 & -5 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_{21}$

$\sim \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ -5 & 0 & -5 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] -R_1$

$\sim \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_2 + 5R_1$

$\sim \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_2/25$

$\sim \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 + R_2$

$x + 5y + z = 0$ — (i)

$y = 0$ — (2)

Let $z = r$ (real no:)

eq (i) $\Rightarrow x + 0 + r = 0 \Rightarrow \boxed{x = -r}$

$\boxed{x = -r,\ y = 0,\ z = r}$

Q38 is similarly to Q37

Q39 Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & -9 & 2a-b \\ 0 & 0 & 0 & c-3a-b \end{array}\right] R_2+R_3,\ -R_2$$

$$c-3a-b=0$$

$$\Rightarrow \boxed{-3a-b+c=0}\ \text{Ans.}$$

Q40 is similarly to Q39

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Q41 $A=\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \end{bmatrix}$

$$AX=\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=4\begin{bmatrix} x \\ y \end{bmatrix}=4X.$$

Now $4X-AX=0$

$$(4I_2-A)X=0$$

$$\left(\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}-\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 4-4 & 0-1 \\ 0-0 & 4-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0x-y \\ 0x+2y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A.M

$$\left[\begin{array}{cc|c} 0 & -1 & 0 \\ 0 & 2 & 0 \end{array}\right].$$

$$\sim\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 2 & 0 \end{array}\right] -R_1$$

$$\sim\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2-2R_1$$

$y=0$, let $x=\gamma$ (real no.)

$$X=\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \gamma \\ 0 \end{bmatrix}.$$

Q42 $A=\begin{bmatrix}2&1\\1&2\end{bmatrix}$ & $(3I_2-A)X=0$

Now $3X-AX=0$

$(3I_2-A)X=0$

$\left(3\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}2&1\\1&2\end{bmatrix}\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\left(\begin{bmatrix}3&0\\0&3\end{bmatrix}-\begin{bmatrix}2&1\\1&2\end{bmatrix}\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}3-2&0-1\\0-1&3-2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}x-y\\-x+y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

A.M

$\left[\begin{array}{cc|c}1&-1&0\\-1&1&0\end{array}\right]$.

$\sim\left[\begin{array}{cc|c}1&-1&0\\0&0&0\end{array}\right] R_2+R_1$

$x-y=0$ let $x=\gamma$.

$y=\gamma$

$X=\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\gamma\\\gamma\end{bmatrix}$ A

Q43. $(3I_3-A)X=0$

$\left(\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}-\begin{bmatrix}1&2&-1\\1&0&1\\4&-4&5\end{bmatrix}\right)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

$\begin{bmatrix}2&-2&1\\-1&3&-1\\-4&4&-2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}2x-2y+z\\-1x+3y-z\\-4x+4y-2z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

A. M

$\left[\begin{array}{ccc|c}2&-2&1&0\\-1&3&-1&0\\-4&4&-2&0\end{array}\right]$

$\sim\left[\begin{array}{ccc|c}1&-3&1&0\\2&-2&1&0\\-4&4&-2&0\end{array}\right] \begin{matrix}-R_2\\R_{21}\end{matrix}$

$\sim\left[\begin{array}{ccc|c}1&-3&1&0\\0&4&-1&0\\0&-8&2&0\end{array}\right] \begin{matrix}R_2-2R_1\\R_4+4R_3\end{matrix}$

$\sim\left[\begin{array}{ccc|c}1&-3&1&0\\0&1&-1/4&0\\0&-8&2&0\end{array}\right] R_4/4$

$$\sim\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3+8R_2$$

$x-3y+z=0$ — (i)

$y-\frac{1}{4}z=0$ — (2)

let $\boxed{z=r}$ (real no any)

$(2) \Rightarrow \boxed{y=\frac{1}{4}r}$

eq (i) $\Rightarrow x-\frac{3}{4}r+r=0$

$x - \frac{-3r+4r}{4}=0 \Rightarrow x+\frac{r}{4}=0 \Rightarrow \boxed{x=-}$

Hence $\boxed{x=-\frac{r}{4},\ y=\frac{1}{4}r,\ z=r}$

Q44 is similarly to Q43

Mr. Jamal Nasir

Lecturer U.E.T Peshawar.

Q45 Given $X = X_p + X_h$,

when X_p is particular solution

& X_h is a solution to the associated homg: system.

A.M

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 2 & 1 & -2 & 3 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & -4 & -1 & 6 \end{array}\right]$$

$$\sim\left[\begin{array}{cccc|c} 1 & +2 & -1 & -2 & 2 \\ 0 & -3 & 0 & 7 & -2 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & -3 & 0 & 7 & -2 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-R_1 \\ R_4-4R_1 \end{array}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 0 & 1 & 0 & -7/3 & 2/3 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & -3 & 0 & 7 & -2 \end{array}\right] -R_2/3$$

$$\sim\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 0 & 1 & 0 & -7/3 & 2/3 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_4+3R_2$$

$x+2y-z-2w=2$ — (i)

$y-\frac{7}{3}w=\frac{2}{3}$ — (ii)

$4z+6w=3$ — (iii)

let $w=r$ (any real no.)

$eq(iii) \Rightarrow 4z + 6r = 3$

$4z = 3 - 6r$

$z = 3/4 - \frac{6}{4}r \Rightarrow \boxed{z = 3/4 - 3/2\, r}$

$eq(ii) \Rightarrow \boxed{y = 3/2 + 7/3\, r}$

$eq(i) \Rightarrow x + 2(3/2 + 7/3\, r) - (3/4 - 3/2\, r) - 2r = 2$

$x = 17/12 - \frac{50}{12} r$

$x = 17/12 - 25/6\, r$

$X = X_p + X_h$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 17/12 \\ 3/2 \\ 3/4 \\ 0 \end{bmatrix} + \begin{bmatrix} -25/6\, r \\ 7/3\, r \\ -3/2\, r \\ r \end{bmatrix}$$ Ans:

Q46 is similarly to Q45

Q47 (1,2) (3,3) (5,8).

Since quadratic polynomials $a_2x^2 + a_1x + a_0 = y$ ~~fun~~ ⊛

(1,2) Point

$eq ⊛ \Rightarrow$ ~~y~~ $2 = a_2 + a_1 + a_0$ —— (i)

(3,3)

$eq ⊛ \Rightarrow 3 = 9a_2 + 3a_2 + a_0$ —— (ii)

For Point (5,8)

$eq ⊛ \Rightarrow 8 = 25a_2 + 5a_1 + a_0$ —— (iii)

in ~~xx~~ A.M

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 9 & 3 & 1 & 3 \\ 25 & 5 & 1 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -6 & -8 & -15 \\ 0 & -20 & -24 & -42 \end{array}\right].$$

$$\sim\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4/3 & 5/2 \\ 0 & -20 & -24 & -42 \end{array}\right] -\tfrac{1}{6}R_2$$

$$\sim\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4/3 & 5/2 \\ 0 & 0 & 8/3 & 8 \end{array}\right] R_3+20R_2$$

$a_2+a_1+a_0=2$ —— (a)

$a_1+\frac{4}{3}a_0=\frac{5}{2}$ —— (b)

$\frac{8}{3}a_0=8$ —— (c)

$eq(c) \Rightarrow a_0=\cancel{8}\times\frac{3}{\cancel{8}} \Rightarrow \boxed{a_0=3}$

$eq(b) \Rightarrow a_1+\frac{4}{\cancel{3}}(\cancel{3})=\frac{5}{2}$

$a_1=\frac{5}{2}-4=\frac{5-8}{2}=-\frac{3}{2}$

$\boxed{a_1=-\frac{3}{2}}$

$eq(a) \Rightarrow a_2+(-\frac{3}{2})+(3)=2$

$a_2=2-3+\frac{3}{2}=\frac{4-6+3}{2}=\frac{1}{2}$

$\boxed{a_2=\frac{1}{2}}$

$eq(*) \Rightarrow \boxed{y=\frac{1}{2}x^2-\frac{3}{2}x+3}$ Ans.

Q48, Q49, Q50 similarly to Q47

Q51 $10x_1+12x_2+15x_3=16\times60=960$

$6x_1+8x_2+12x_3=11\times60=660$

$12x_1+12x_2+18x_3=18\times60=1080$

Aug: M

$$\left[\begin{array}{ccc:c} 10 & 12 & 15 & 960 \\ 6 & 8 & 12 & 660 \\ 12 & 12 & 18 & 1080 \end{array}\right]$$ S.Y.S.

Q52 $x_1 + 2x_2 + 3x_3 = 6 \times 60 = 360$

$2x_1 + 4x_2 + 5x_3 = 11 \times 60 = 660$

Aug: Matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 2 & 4 & 5 & 660 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 0 & 0 & -1 & -60 \end{array}\right] \quad R_2 - 2R_1$$

$\boxed{x_3 = 60}$ (deluxe binding)

$x_1 + 2x_2 + 3x_3 = 360$ — (i)

let $\boxed{x_2 = r}$ (any real no.)

eq (i) $\Rightarrow x_1 + 2r + 3(60) = 360$

$x_1 = 360 - 180 - 2r$

$\boxed{x_1 = 180 - 2r}$

$\boxed{x_1 = 180 - 2r, \quad x_2 = r, \quad x_3 = 60}$

Q53 Given $P(x) = ax^2 + bx + c$ — (*)

$P(0) = f(0)$, $P'(0) = f'(0)$, & $P''(0) = f''(0)$ & $f(x) = e^{2x}$.

$f(x) = e^{2x}$ — (i)

Put $x = 0$

$f(0) = e^0 = 1 \Rightarrow \boxed{f(0) = 1}$

eq (*) $\Rightarrow P(0) = a0 + b0 + c$

$\boxed{P(0) = c}$

But Given $P(0) = f(0)$

$\boxed{c = 1.}$

eq (*) $\Rightarrow P'(x) = 2ax + b$

$P'(0) = 2a(0) + b \Rightarrow P'(0) = b.$

eq (i) $\Rightarrow f'(x) = 2e^{2x}$

$f'(0) = 2e^0 = 2$

But Given $f'(0) = P'(0)$

$\boxed{2 = b}$

Similarly $\boxed{a = 2}$

eq (*) $\Rightarrow \boxed{P(x) = 2x^2+2x+1}$

~~~ o ~~~ o ~~~

Q54 is similarly to Q53

~~~ o ~~~ o ~~~

Ex 1.7

Q55

$$T_1 = \frac{30^\circ + 50^\circ + T_2 + T_3}{4}$$

$$4T_1 = 80^\circ + T_2 + T_3$$

$$4T_1 - T_2 - T_3 = 80^\circ \text{ ——— (i)}$$

$$T_2 = \frac{30 + 50^\circ + T_1 + T_4}{4}$$

$$4T_2 - T_1 - T_4 = 80^\circ \text{ ——— (ii)}$$

$$T_3 = \frac{T_1 + T_4 + 50^\circ + 0^\circ}{4}$$

$$4T_3 - T_1 - T_4 = 50^\circ \text{ ——— (iii)}$$

$$T_4 = \frac{T_3 + T_2 + 50^\circ + 0}{4}$$

$$4T_4 - T_3 - T_2 = 50^\circ \text{ ——— (iv)}$$

Now Aug: Matrix

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 80 \\ -1 & 4 & 0 & -1 & 80 \\ -1 & 0 & 4 & -1 & 50 \\ 0 & -1 & -1 & 4 & 50 \end{array}\right] \text{ S.Y.S}$$

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

BIT Matrices (optional) P-17

Table ①

+	0	1
0	0	1
1	1	0

Table

×	0	1
0	0	0
1	0	1

Exp) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1+1 & 0+1 \\ 1+0 & 1+1 \\ 0+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Note:- We have $0+0=0$ and $1+1=0$. Thus the additive inverse of 0 is 0 and the additive inverse of 1 is 1. Hence to compute the difference of bit matrices A & B we proceed as follows

$A-B = A+(\text{inverse of } 1)B = A+1B = A+B$

$A-B = A+B$

Note:- A bit is a binary digit i.e either 0 or 1. A bit matrix is also called a Boolean matrix.

Ans: M

Q57(a) $\begin{bmatrix} 1 & 1 & 0 & 1 & \vdots & 0 \\ 1 & 0 & 1 & 1 & \vdots & 1 \\ 0 & 1 & 1 & 1 & \vdots & 1 \end{bmatrix}.$

$\sim \begin{bmatrix} 1 & 1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & 1 & 0 & \vdots & 1 \\ 0 & 1 & 1 & 1 & \vdots & 1 \end{bmatrix} R_2+R_1$

$\sim \begin{bmatrix} 1 & 1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & 1 & \vdots & 0 \end{bmatrix} R_3+R_2$

$x+y+w=0$ —— (i)

$y+z=1$ —— (ii)

$\boxed{w=0}$

eq (ii) $\Rightarrow y = 1-z$

let $z=0$

$y = 1-0 = 1$

$y = 1.$

eq (i) $x = -y$

$x = -(1-z)$

$x = -(1-0)$

$x = 1$

let $z = 1$

$y = 1-1 = 0$

$x = -(1-1)$

$x = 0$

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ 4.

(b)
$$\begin{bmatrix} 1 & 1 & 0 & 0 & \vdots & 0 \\ 1 & 1 & 1 & 0 & \vdots & 1 \\ 1 & 1 & 1 & 1 & \vdots & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 1 & \vdots & 0 \end{bmatrix} \begin{matrix} R_2+R_1 \\ R_3+R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & 1 & \vdots & 1 \end{bmatrix} R_3+R_2$$

$x+y=0 \Rightarrow x=-y$

$z=1$ let $y=0$

$w=1$ $x=0$

let $y=1$ $x=1$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ Ans}$$

Mr. Jamal Nasir

Lecturer U.E.T Peshawar

Q5 (a) $Ax = C$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x+y+0z \\ 0x+y+0z \\ x+y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Aug: M

$$\begin{bmatrix} 1 & 1 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 1 \\ 1 & 1 & 1 & \vdots & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix} R_3+R$$

$x+y=0 \Rightarrow x=-y \Rightarrow x=1$

$y=1$

$z=0$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Q1 Show that $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ is nonsingular.

Sol- Let $A = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ To find A^{-1}, we let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. — (*)

Then we must have

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So that

$$\begin{bmatrix} 2a+c & 2b+d \\ -2a+3c & -2b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$2a+c = 1$ — (1) $-2a+3c = 0$ — (2)

$2b+d = 0$ — (3) $-2b+3d = 1$ — (4)

(1) + (2)

$4c = 1 \Rightarrow \boxed{c = 1/4}$

eq (1) $\Rightarrow 2a = 1 - 1/4 = 3/4 \Rightarrow \boxed{a = 3/8}$

(3) + (4)

$4d = 1 \Rightarrow \boxed{d = 1/4}$

eq (3) $\Rightarrow 2b = -1/4 \Rightarrow \boxed{b = -1/8}$

eq (*) $\Rightarrow A^{-1} = \begin{bmatrix} 3/8 & -1/8 \\ 1/4 & 1/4 \end{bmatrix}$ we conclude A^{-1} is exist

So A is nonsingular.

Q2 let $A=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$, let $A^{-1}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$.

Now $AA^{-1}=I_2$

$$\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

$$=\begin{bmatrix}2a+c & 2b+d\\ -4a-2c & -4b-2d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

$2a+c=1$ — (1)

$-4a-2c=0$ — (2)

$2b+d=0$ — (3)

$-4b-2d=1$ — (4)

x'ing eq (1) by (2) & adding eq (2)

$$\begin{array}{r} 4a+2c=2\\ -4a-2c=0\\ \hline 0=2 \end{array}$$ impossible

A^{-1} doesn't exist so it is singular

~o~

Q_3 is similarly to Q_2 & Q_1

Q4 $A=\begin{bmatrix}1 & 2 & -1\\ 3 & 2 & 3\\ 2 & 2 & 1\end{bmatrix}$ let $A^{-1}=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

Now $AA^{-1}=I_3$

$$\begin{bmatrix}1 & 2 & -1\\ 3 & 2 & 3\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

$$=\begin{bmatrix}a+2d-g & b+2e-h & c+2f-i\\ 3a+2d+3g & 3b+2e+3h & 3c+2f+3i\\ 2a+2d+g & 2b+2e+h & 2c+2f+i\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$a+2d-g=1$ — (1) $\quad b+2e-h=0$ — (4) $\quad c-2f-i=0$ — (7)

$3a+2d+3g=0$ — (2) $\quad 3b+2e+3h=1$ — (5) $\quad 3c+2f+3i=0$ — (8)

$2a+2d+g=0$ — (3) $\quad 2b+2e+h=0$ — (6) $\quad 2c+2f+i=1$ — (9)

(1) − (2)

$$\begin{array}{r} a+\cancel{2d}-g=1\\ \underset{-}{3a}\ \underset{-}{+}\cancel{2d}\ \underset{-}{+3g}=\underset{-}{0}\\ \hline -2a-4g=1 \end{array}$$ — (10)

(1) − (3)

$$\begin{array}{r} a+\cancel{2d}-g=1\\ \underset{-}{2a}\ \underset{-}{+}\cancel{2d}\ \underset{-}{+g}=\underset{-}{0}\\ \hline -a-2g=1 \end{array}$$ — (11)

x'ing 2 by eq (11) then sub: from 10

$$\begin{array}{r} -\cancel{2a}-\cancel{4g}=2\\ \underset{+}{-}\cancel{2a}\ \underset{+}{-}\cancel{4g}=\underset{-}{1}\\ \hline 0=1 \end{array}$$ impossible. A^{-1} doesn't exists so it is singular.

Q5 (a) $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$. let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. — (*)

Now $AA^{-1} = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} a-2b & 3a+6b \\ c-2d & 3c+6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$a-2b=1$ — (1) $\quad 3a+6b=0$ — (3)

$c-2d=0$ — (2) $\quad 3c+6d=1$ — (4)

Solving the above equation we get

$\boxed{a=1/2}$ $\boxed{b=-1/4}$ $\boxed{c=1/6}$ $\boxed{d=1/12}$

by (*) $\Rightarrow A^{-1} = \begin{bmatrix} 1/2 & 1/4 \\ 1/6 & 1/12 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

Now $[A \vdots I_3]$ is

$$[A \vdots I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

We now compute the reduced row echelon form of the matrix. To find A^{-1}, we proceed as follows.

$$\begin{array}{c} \quad A \qquad\qquad I_3 \\ \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] R_2-R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] -R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} R_3-R_2 \\ R_1-2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} R_1-R_3 \\ R_2-R_3 \end{array}$$

So $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

Q6 — to — Q10 is similarly to Q5

Q11 Which of the following linear system have a non-trivial solution?

(b) $2x+y-z=0$

$x-2y-3z=0$

$-3x-y+2z=0$

Method 1 A.M

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & -2 & -3 & 0 \\ -3 & -1 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 2 & 1 & -1 & 0 \\ -3 & -1 & 2 & 0 \end{array}\right] R_{12}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & -7 & -7 & 0 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3+3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -7 & -7 & 0 \end{array}\right] \tfrac{1}{5}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3+7R_2$$

As infinite solution exists so it is non-trivial.

Method 2 $[A ; I_3]$

$$A : I_3 \Rightarrow \begin{array}{c} \quad A \qquad\qquad I_3 \\ \left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 1 & -2 & -3 & 0 & 1 & 0 \\ -3 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 1 & -2 & -3 & 0 & 1 & 0 \\ -3 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \tfrac{1}{2}R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.5 & -0.5 & 0.5 & 0 & 0 \\ 0 & -2.5 & -2.5 & -0.5 & 1 & 0 \\ 0 & 0.5 & 0.5 & 1.5 & 0 & 1 \end{array}\right] \begin{array}{l} R_2-R_1 \\ R_3+3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.5 & -0.5 & 0.5 & 0 & 0 \\ 1 & 1 & 1 & 0.2 & 0.4 & 0 \\ 0 & 0.5 & 0.5 & 1.5 & 0 & 1 \end{array}\right] \frac{-R_2}{2.5}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.5 & -0.5 & 0.5 & 0 & 0 \\ 1 & 1 & 1 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 1.5 & 0 & 1 \end{array}\right] R_3 - 0.5R_2$$

So no inverse exist hence it's non-trivial.

Q12 is similarly to Q11.

Q13 $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ find A. let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ —— (*)

Since $A^{-1}A = I_2$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 2a+3c & 2b+3d \\ a+4c & b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$2a+3c=1$ —— (1) $\quad 2b+3d=0$ —— (3)

$a+4c=0$ —— (2) $\quad b+4d=1$ —— (4)

Solving the above equation we get

$\boxed{a = 4/5}$, $\boxed{b = -3/5}$ $\boxed{c = -1/5}$ $\boxed{d = 2/5}$

eq (*) $\Rightarrow A = \begin{bmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{bmatrix}.$

Q14 is similarly to Q13.

Q16 Given $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ $A^{-1} = ?$ let $A^{-1} = \begin{bmatrix} b & c & d \\ e & f & g \\ h & i & j \end{bmatrix}$ —— (*)

Since $AA^{-1} = I_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}\begin{bmatrix} b & c & d \\ e & f & g \\ h & i & j \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} b+e & c+f & d+g \\ b & c & d \\ b+2e+ha & c+2f+ai & d+2g+ja \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$b+e=1$ —— (1) $\quad b=0$ —— (4) $\quad b+2e+ha=0$ —— (7)

$c+f=0$ —— (2) $\quad c=1$ —— (5) $\quad c+2f+ai=0$ —— (8)

$d+g=0$ —— (3) $\quad d=0$ —— (6) $\quad d+2g+ja=1$ —— (9)

eq (1) $\Rightarrow 0+e=1 \Rightarrow \boxed{e=1}$

eq (2) $\Rightarrow 1+f=0 \Rightarrow \boxed{f=-1}$

eq (3) $\Rightarrow 0+g=0 \Rightarrow \boxed{g=0}$

eq (7) $\Rightarrow 0+2+ha=0 \Rightarrow \boxed{h=-2/a}$

eq (8) $\Rightarrow 1+2(-1)+ai \Rightarrow \boxed{i=1/a}$

eq (9) $\Rightarrow 0+2(0)+ja=1 \Rightarrow \boxed{j=1/a}$

eq (*) $\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -2/a & 1/a & 1/a \end{bmatrix}$ when $a \neq 0$.

Q17 Let x be input matrix i.e. $x = A^{-1}b$ given output matrix

first we find A^{-1} let $A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (4) & $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

Now $AA^{-1} = I_3$

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 2a+d+3g & 2b+1e+3h & 2c+f+3i \\ 3a+2d-1g & 3b+2e-h & 3c+2f-i \\ 2a+d+g & 2b+e+h & 2c+1f+i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2a+d+3g=1$ —(1) $\quad 2b+e+3h=0$ —(4) $\quad 2c+h+3i=0$ —(7)

$3a+2d-g=0$ —(2) $\quad 3b+2e-h=1$ —(5) $\quad 3c+2f-i=0$ —(8)

$2a+d+g=0$ —(3) $\quad 2b+e+h=0$ —(6) $\quad 2c+f+i=1$ —(9)

(3) + (2) are added

$3a+2d-\cancel{g}=0$
$2a+d+\cancel{g}=0$

$5a+3d=0$ —— (2)'

add 3 times (2) to (1)

$2a+d+3g=1$
$9a+6d-3g=0$

$11a+7d=1$ —— (3)'

Solving (2)' & (3)'

$5a=-3d \Rightarrow a = -\frac{3}{5}d$

$11(-\frac{3}{5}d)+7d=1$

$-33d+35d=5 \Rightarrow \boxed{d=5/2}$

$\boxed{a=-3/2}$

Put values in (1)

$2a+d+3g=1$

$2(-3/2)+(5/2)+3g=1$

$-3+5/2+3g=1 \Rightarrow \boxed{g=1/2}$

(5)+(6)

$3b+2e-\cancel{h}=1$
$2b+e+\cancel{h}=0$

$5b+3e=1$ —— (5)'

Adding 3 times (5) to (4)

$2b+e+\cancel{3h}=0$
$9b+6e-\cancel{3h}=3$

$11b+7e=3$ —— (6)'

Solving (5)' & (6)'

$5b = 1 - 3e \Rightarrow b = \frac{1-3e}{5}$

$11\left(\frac{1-3e}{5}\right) + 7e = 3$

$\Rightarrow \boxed{e = 2}$

$b = \frac{1-3(2)}{5} = -5/5 = -1$

$\boxed{b = -1}$

eq (4) $\Rightarrow 2b + e + 3h = 0$

$2(-1) + 2 + 3h = 0$

$\Rightarrow \boxed{h = 0}$

(8) + (9)

$3c + 2f - \cancel{i} = 0$

$2c + f + \cancel{i} = 1$

$5c + 3f = 1$ —— (8)'

Add (3) times (8) to (7)

$2c + f + 3\cancel{i} = 0$

$9c + 6f - \cancel{3i} = 0$

$11c + 7f = 0$ —— (9)'

Solving (8)' & (9)'

$11c = -7f \Rightarrow c = -7/11\, f$

$5(-7/11)f + 3f = 1 \Rightarrow \boxed{f = -11/2}$

$c = -7/11 \times -11/2 = 7/2$

$\boxed{c = 7/2}$

$3c + 2f - i = 0$

$3(7/2) + 2(-11/2) - i = 0$

$\Rightarrow \boxed{i = -1/2}$

eq (8) $\Rightarrow A = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$

$X = A^{-1}b, \quad b = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$X = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$X = \begin{bmatrix} -30 \\ 60 \\ 10 \end{bmatrix}$ A

17(b) $x = A^{-1}b, \quad b = \begin{bmatrix} 12 \\ 8 \\ 14 \end{bmatrix}$

$X = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \\ 14 \end{bmatrix} = \begin{bmatrix} 33 \\ -31 \\ -1 \end{bmatrix}$ A

Q18 $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

(a) A^{-1}

New $AA^{-1} = I_2$ let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ —— (*)

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a+3c & b+3d \\ 2a+7c & 2b+7d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a+3c = 1$ —— (1) $\quad b+3d = 0$ —— (3)

$2a+7c = 0$ —— (2) $\quad 2b+7d = 1$ —— (4)

Solving the above equation we get

$\boxed{a=7}$ $\boxed{b=-3}$ $\boxed{c=-2}$ $\boxed{d=1}$

eq (*) $\Rightarrow A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ —— (a)

(b) $(A^T)^{-1} = ?$

$A^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$. let $(A^T)^{-1} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ —— (*)

$(A^T)(A^T)^{-1} = I_2$

$= \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$= \begin{bmatrix} x+2z & y+2w \\ 3x+7z & 3y+7w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$x+2z = 1$ —— (1) $\quad y+2w = 0$ —— (3)

$3x+7z = 0$ —— (2) $\quad 3y+7w = 1$ —— (4)

Solving the above equations we get

$\boxed{x=7}$ $\boxed{y=-2}$ $\boxed{z=-3}$ $\boxed{w=1}$

eq (*) $\Rightarrow (A^T)^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$. Ans.

eq (a) $\Rightarrow (A^{-1})^T = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

So $(A^{-1})^T = (A^T)^{-1}$.

—— o ——

Q19 (a) let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Then $(A+B)^{-1}$ exists but A^{-1} & B^{-1} dont.

(b) Yes for A non-singular & $c \neq 0$.

$$(cA)^{-1} = \frac{1}{c} A^{-1}$$

$$I_n = cA\left(\frac{1}{c}A\right)^{-1}$$

$$I_n = \cancel{c}(1/\cancel{c}) \cdot AA^{-1}$$

$$I_n = I_n$$

Q21

$$(\lambda-1)x + 2y = 0 \quad —(1)$$
$$2x + (\lambda-1)y = 0 \quad —(2)$$

from (2) $x = \frac{(1-\lambda)y}{2}$

eq (1) $\Rightarrow \frac{(\lambda-1)(1-\lambda)y}{2} + 2y = 0$

$$\Rightarrow -\frac{(1-\lambda)(1-\lambda)y}{2} + 2y = 0$$

$$= \frac{-(1-\lambda)^2 y + 4y}{2} = 0$$

$$= (-(1-\lambda)^2 + 4)y = 0$$

$$= -(1+\lambda^2-2\lambda) + 4 = 0$$

$$= +\lambda^2 - 2\lambda - 3 = 0 \quad \lambda = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$$

$$\boxed{\lambda = 3, -1}$$

Q23 Similarly to Q5

Q24 $A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$ & $B^{-1} = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}$

find $(AB)^{-1}$

Since $(AB)^{-1} = (B^{-1})(A^{-1}) = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6+5 & 4+15 \\ 9-2 & 6-6 \end{bmatrix} = \begin{bmatrix} 11 & 19 \\ 7 & 0 \end{bmatrix}.$$

Q25 $X = A^{-1}b$

$$X = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 10+9 \\ 20+3 \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}.$$

Q27

$$\begin{array}{c} \quad A \qquad\quad I_3 \\ \left[\begin{array}{ccc|ccc} 5 & 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_1/5$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 0 & -0.2 & 0 & -0.6 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_2-3R_1$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] -\frac{R_2}{0.2}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_1-0.4R_2$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/4 \end{array}\right] -\frac{R_3}{4}$$

So $A^{-1}=\begin{bmatrix} -1 & 2 & 0 \\ 3 & -5 & 0 \\ 0 & 0 & -1/4 \end{bmatrix}$.

Mr. Jamal Nasir

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Q29(c)

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_2+R_1$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_{32}$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_4+R_3$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_2+R_4$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_2+R_3$$

$$\sim\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_1+R_2$$

So $A^{-1}=\begin{bmatrix}1&0&1&1\\0&0&1&1\\1&1&0&0\\1&1&0&1\end{bmatrix}$.

Q30 is similarly to Q29.

Q32 is similarly to Q31

Q31 (a) $\left[\begin{array}{ccc:ccc}1&1&1&1&0&0\\1&0&1&0&1&0\\0&1&0&0&0&1\end{array}\right]$

$\sim\left[\begin{array}{ccc:ccc}1&1&1&1&0&0\\0&1&0&1&1&0\\0&1&0&0&0&1\end{array}\right]$ R_2+R_3

$\sim\left[\begin{array}{ccc:ccc}1&1&1&1&0&0\\0&1&0&1&1&0\\0&0&0&1&1&1\end{array}\right]$ R_3+R_2

it's nontrivial so matrix is singular.

(b) $\left[\begin{array}{ccc:ccc}1&0&0&1&0&0\\1&1&1&0&1&0\\1&0&1&0&0&1\end{array}\right]$.

$\sim\left[\begin{array}{ccc:ccc}1&0&0&1&0&0\\0&1&1&1&1&0\\0&0&1&1&0&1\end{array}\right]$ R_2+R_1, R_3+R_1

$\sim\left[\begin{array}{ccc:ccc}1&0&0&1&0&0\\0&1&0&0&1&1\\0&0&1&1&0&1\end{array}\right]$ R_2+R_3 As inverse exist so it's trivial

Q1 $A=\begin{bmatrix}2&8&0\\2&2&-3\\1&2&7\end{bmatrix}$; $b=\begin{bmatrix}18\\3\\12\end{bmatrix}$, $L=\begin{bmatrix}2&0&0\\2&-3&0\\1&-1&4\end{bmatrix}$, $U=\begin{bmatrix}1&4&0\\0&2&1\\0&0&2\end{bmatrix}$.

Now $UX=Z$ — (i) $LZ=b$ — (2)

eq (ii) => $LZ=b$

$$\begin{bmatrix}2&0&0\\2&-3&0\\1&-1&4\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}=\begin{bmatrix}18\\3\\12\end{bmatrix}$$

$2z_1=18$ — (i) $2z_1-3z_2=3$ — (2)

$z_1-z_2+4z_3=12$ — (3)

Solving the above equation we get

$\boxed{z_1=9}$ $\boxed{z_2=5}$ $\boxed{z_3=2}$

eq (i) => $UX=Z$

$$\begin{bmatrix}1&4&0\\0&2&1\\0&0&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}9\\5\\2\end{bmatrix}=\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}$$

$x_1+4x_2=9$ — (1)

$2x_2+x_3=5$ — (2)

$2x_3=2$ — (3)

From above equation we get

$\boxed{x_1 = 1}$ $\boxed{x_2 = 2}$ $\boxed{x_3 = 1}$

& $X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ Ans.

Q2 Q4 is similarly to Q1.

Q5 $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 10 \\ 4 & 8 & 2 \end{bmatrix}$; $b = \begin{bmatrix} 6 \\ 16 \\ 2 \end{bmatrix}$.

For matrix U (upper triangular)

$\sim \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 2 & -6 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 2R_1 \end{matrix}$

$\sim \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix} R_3 + 2R_2$

For matrix L (lower triangular)

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$

Now $LZ = b$

$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 16 \\ 2 \end{bmatrix}$

$z_1 = 6$ —— (1)

$2z_1 + z_2 = 16$ —— (2)

$2z_1 - 2z_2 + z_3 = 2$ —— (3)

Solving we get

$\boxed{z_1 = 6}$ $\boxed{z_2 = 4}$ $\boxed{z_3 = -2}$

Now solving

$UX = Z$

$\begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix}$

$2x_1 + 3x_2 + 4x_3 = 6$ —— (1)

$-x_2 + 2x_3 = 4$ —— (2)

$-2x_3 = -2$ —— (3)

Now we get

$\boxed{x_1 = 4}$ $\boxed{x_2 = -2}$ $\boxed{x_3 = 1}$

$$X = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

Q6 — to Q10 similarly to Q5

Mr. Jamal Nasir

Lecturer U.E.T Peshawar.

Spring Semester-2012

Electrical Engg:

Q1 $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ & $V = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix}$. (1,1) (2,1) (1,3) (2,3)

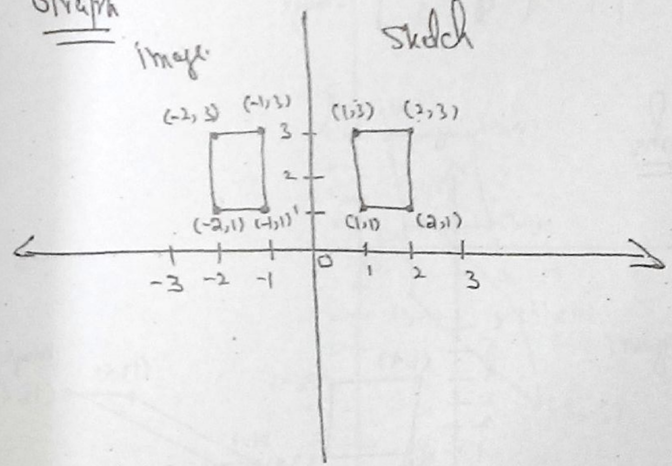
Given $f(v) = AV$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix}.$$

$$= \begin{bmatrix} -1\times1+0\times1 & -1\times2+0\times1 & -1\times1+0\times3 & -1\times2+0\times3 \\ 0\times1+1\times1 & 0\times2+1\times1 & 0\times1+1\times3 & 0\times2+1\times3 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} -1 & -2 & -1 & -2 \\ 1 & 1 & 3 & 3 \end{bmatrix}$$ image.

Graph



Mr. Jamal Nasir

Lecturer. U.E.T Peshawar

Spring Semester-2012 Electrical Engg:

Q2 Let f be the shear in the x-direction

$$f(v) = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} V$$

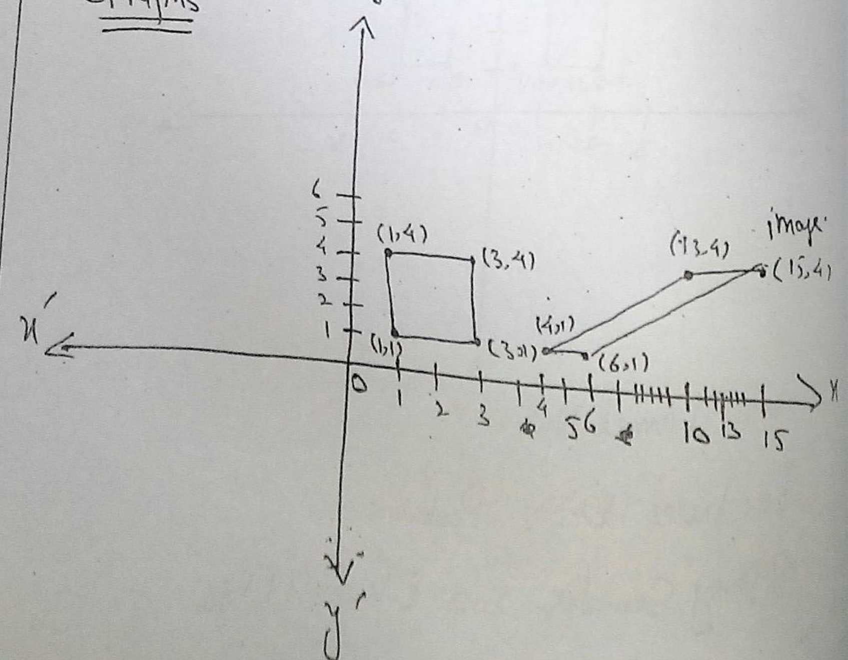
where $k=3$ & $V = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

$$f(v) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} V$$

$$\Rightarrow f(v) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 & 6 & 15 \\ 1 & 4 & \cancel{1} & 4 \end{bmatrix} \text{ Image}$$

Graphs



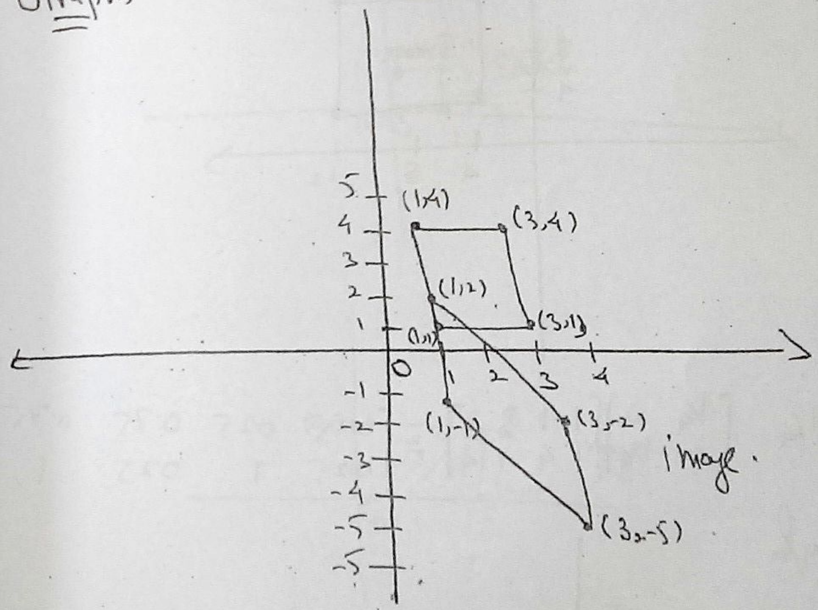
Q3 $A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ & $k=-2$ then $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$.

& $V = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

Now $f(v) = AV = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

$$= \begin{bmatrix} 1 & 1 & 3 & 3 \\ -1 & 2 & -5 & -2 \end{bmatrix} \text{ Image.}$$

Graphs



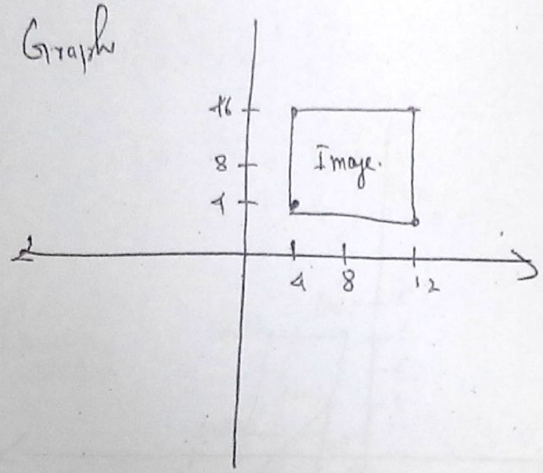
Q4(a) $A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$. $k=4$

$V = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

$f(v) = AV = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

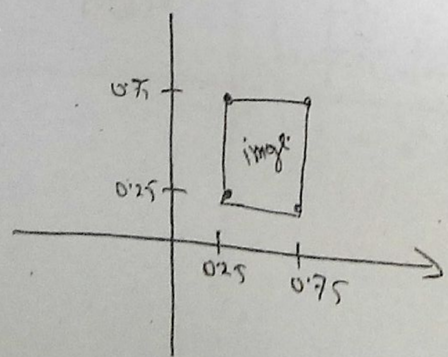
$= \begin{bmatrix} 4 & 4 & 12 & 12 \\ 4 & 16 & 4 & 16 \end{bmatrix}$ image.

Graph



(b) $k = 1/4$ $\begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.75 & 0.75 \\ 0.25 & 1 & 0.25 & 1 \end{bmatrix}$.

Graph



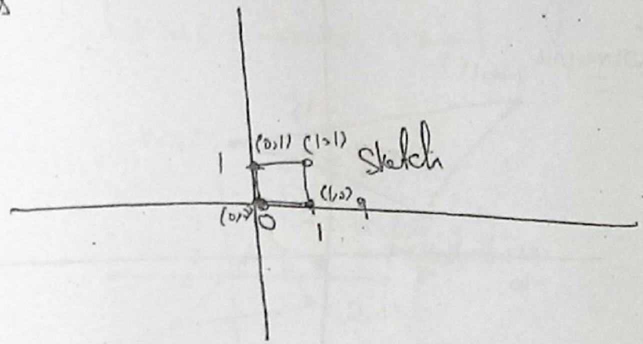
Q5 $k=2$. $A = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

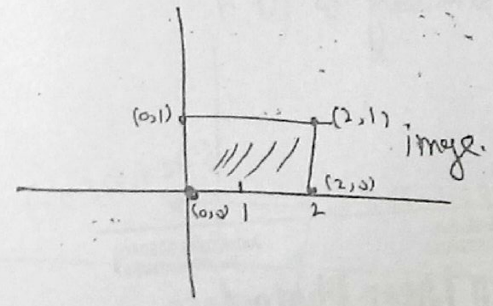
$V = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$f(v) = AV = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. image

Graphs





Q6 is Similarly to Q5

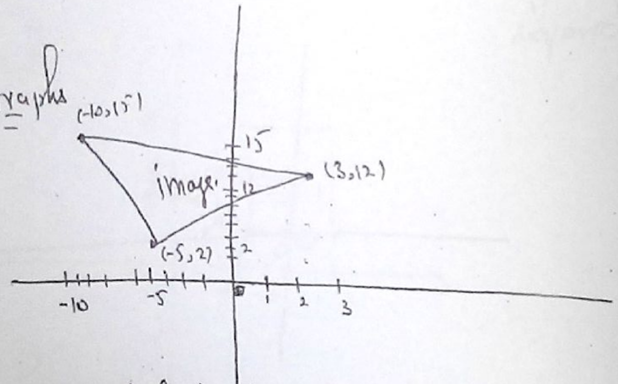
Q7 $f(v)=\begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}V$

$T=V=\begin{bmatrix}5 & 0 & 2\\ 0 & 3 & -1\end{bmatrix}$.

$f(v)=\begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 0 & 2\\ 0 & 3 & -1\end{bmatrix}$

$=\begin{bmatrix}-10 & 3 & -5\\ 15 & 12 & 2\end{bmatrix}$.

Graphs



Q8 is similarly to Q7.

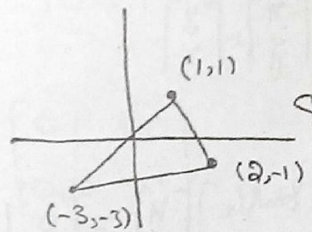
Q9 $T=\begin{bmatrix}1 & -3 & 2\\ 1 & -3 & -1\end{bmatrix}$ & $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ & $\theta=60°$

$A=\begin{bmatrix}\cos 60 & -\sin 60\\ \sin 60 & \cos 60\end{bmatrix}=\begin{bmatrix}0.5 & -0.866\\ 0.866 & 0.5\end{bmatrix}$ (∵ Exp 9 Ex 1.5)

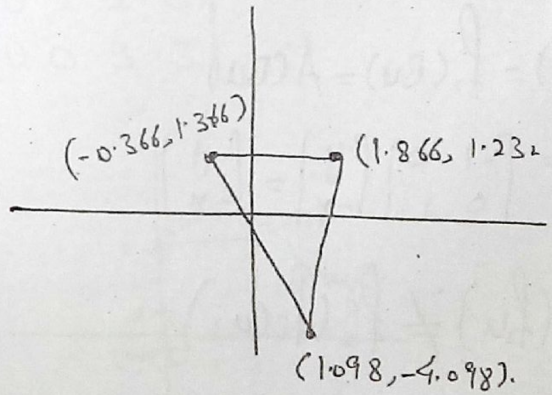
$f(T)=AT=\begin{bmatrix}0.5 & -0.866\\ 0.8666 & 0.5\end{bmatrix}\begin{bmatrix}1 & -3 & 2\\ 1 & -3 & -1\end{bmatrix}$

$=\begin{bmatrix}-0.366 & 1.098 & 1.866\\ 1.366 & -4.098 & 1.232\end{bmatrix}$

Graph



Sketch



Q10 (Assg:)

$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (exps ex1.5) & $u = \begin{bmatrix} x \\ y \end{bmatrix}$.

$f_2(u) = Au = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$.

$f_1(f_2(u)) = f_1(Au) = B(Au)$

$B = \begin{bmatrix} \cos\pi/2 & \sin\pi/2 \\ -\sin\pi/2 & \cos\pi/2 \end{bmatrix}$ (for y-axis).

$B(Au) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$.

Now

$f_1(u) = \begin{bmatrix} \cos\pi/2 & \sin\pi/2 \\ -\sin\pi/2 & \cos\pi/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$

$f_2(f_1(u)) = f_2(Bu) = A(Bu)$

$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ -x \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$.

Here $f_1(f_2u) \neq f_2(f_1(u))$.

Q11 $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $T = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$.

$f(v) = Av = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$.

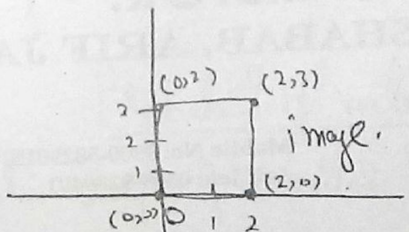
$= \begin{bmatrix} 3 & -9 & 0 \\ 6 & -18 & 0 \end{bmatrix}$.

Q12 $A = \begin{bmatrix} h & 0 \\ 0 & k \end{bmatrix}$ & $h = 2$, $k = 3$ (Given)

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. $V = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

$AV = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

$= \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$.



Q13 is similarly to Q12.

The transition Probabilities arranged in $n \times n$ matrix $T = [t_{ij}]$ is called transition matrix of markov chain where

$$t_{1j} + t_{2j} + t_{3j} + \cdots + t_{nj} = 1 \quad \text{—— (1)}$$

entries in each column of T are non-negative & add upto 1 as in (1)

(a) not transition b/c $a_{11} + a_{21} = 0.3 + 0.4 \neq 1$

(b) transition as
$0.2 + 0.8 + 0.0 = 1$
$0.3 + 0.5 + 0.2 = 1$
$0.1 + 0.7 + 0.2 = 1.$

(c) transition because
$0.55 + 0.45 = 1$
$0.33 + 0.67 = 1.$

(d) Not transition because All each column are not equal to 1.

Probability vector

The vector $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is called Probability vector if

$$u_1 + u_2 + \cdots u_n = 1$$

Q2 (a) $\frac{1}{2}+\frac{1}{3}+\frac{2}{3} = \frac{3+2+4}{6} = 9/6 \neq$ not P.

(b) $0+1+0=1$ Prob: vector

(c) $\frac{1}{4}+\frac{1}{6}+\frac{1}{3}+1/4 = \frac{3+2+4+3}{12} = \cancel{12}/\cancel{12} = 1$ Prob.

(d) $\frac{1}{5}+\frac{2}{5}+\frac{1}{10}+\frac{2}{10} = \frac{2+4+1+2}{10} = \frac{9}{10} \neq 1$ not P.V

Q3 $\begin{bmatrix} \square & 0.4 & 0.3 \\ 0.3 & \square & 0.5 \\ \square & 0.2 & \square \end{bmatrix}$

For $j=1$

$a_{11}+a_{21}+a_{31}=1$

$\square+0.3+\square=1$

$a_{11}+a_{31} = 1-0.3 = 0.7$

$a_{11}+a_{31}=0.7$ So there are more than possible values. ($(0.5+0.2)$ etc).

For $j=2$

$a_{12}+a_{22}+a_{32}=1$

$0.4+\boxed{a_{22}}+0.2=1$

$a_{22}=1-0.2-0.4=0.4 \quad = \boxed{a_{22}=0.4}$

For $j=3$

$a_{13}+a_{23}+a_{33}=1$

$0.3+0.5+a_{33}=1$

$a_{33}=1-0.3-0.5=0.2$

$\boxed{a_{33}=0.2}$

Q4 $j=1$

$0.2+0.3+a_{13}=1$

$a_{13}=1-0.2-0.3=0.5$

$\boxed{a_{13}=0.5}$

$j=2$ $\quad 0.1+a_{22}+a_{32}=1$

$a_{22}+a_{32}=1-0.1=0.9$ (So there are more than possible values $\boxed{a_{22}+a_{32}=0.9}$

$j=3$

$0.3+0.5+a_{33}=1$

$a_{33}=1-0.3-0.5=0.2$

$\boxed{a_{33}=0.2}$

Q5 $T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}$.

(a) If $x^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x^{(1)} = Tx^{(0)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$.

$x^{(2)} = Tx^{(1)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.61 \\ 0.39 \end{bmatrix}$.

$x^{(3)} = Tx^{(2)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.61 \\ 0.39 \end{bmatrix} = \begin{bmatrix} 0.583 \\ 0.417 \end{bmatrix}$.

(b) Show that T is regular.

Now $T^2 = T.T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.61 & 0.52 \\ 0.39 & 0.48 \end{bmatrix}$.

As all enteries are non-zero so it's regular

T is regular if all enteries in same Power of T are Positive

$0.575 + 0.425 = 1$

Steady state vector

$x^{(4)} = Tx^{(3)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.583 \\ 0.417 \end{bmatrix} = \begin{bmatrix} 0.575 \\ 0.425 \end{bmatrix}$

$x^{(5)} = Tx^{(4)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.575 \\ 0.425 \end{bmatrix} = \begin{bmatrix} 0.573 \\ 0.428 \end{bmatrix}$.

$x^{(6)} = Tx^{(5)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.573 \\ 0.428 \end{bmatrix} = \begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix}$.

$x^{(7)} = Tx^{(6)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix} = \begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix}$ Same.

It's steady state vector.

Q6 is similarly to Q5

Q7 (a) $T^2 = \begin{bmatrix} 0 & 1/2 \\ 1 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ 1 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$.

So no entry is 0 in T^2 so it's regular.

(b) $T^2 = T.T = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$= \begin{bmatrix} 1/4 & 0 & 0 \\ 1/4 & 1 & 3/4 \\ 1/2 & 0 & 1/4 \end{bmatrix}$

So all entries in T^2 are not non-zero so its not regular.

* Q (c) & d Part are similarly to a & b Part *

Q9 (a)

$$T^2 = T.T = \begin{bmatrix} 1/2 & 0 \\ 1/2 & \phi \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 \\ 3/4 & 1 \end{bmatrix}.$$

it's not regular because $t_{22} = 0$.

(b) let $x^{(0)} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$. (standard)

$$x^{(1)} = Tx^{(0)} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}.$$

$$x^{(2)} = Tx^{(1)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}$$

$$x^{(3)} = Tx^{(2)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 0.025 \\ 0.975 \end{bmatrix}$$

$$x^{(4)} = Tx^{(3)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.025 \\ 0.975 \end{bmatrix} = \begin{bmatrix} 0.0125 \\ 0.9875 \end{bmatrix}$$

$$x^{(5)} = Tx^{(4)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.013 \\ 0.99 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.00 \\ 1 \end{bmatrix}$$

Q10 (a)

$$T = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0.333 & 0.5 \\ 0.6667 & 0.5 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 0.444 & 0.4166 \\ 0.5555 & 0.5833 \end{bmatrix}$$

$$T^4 = \begin{bmatrix} 0.4259 & 0.4305 \\ 0.574 & 0.5694 \end{bmatrix}.$$

$$T^5 = \begin{bmatrix} 0.429 & 0.4282 \\ 0.5709 & 0.5717 \end{bmatrix}.$$

$$T^6 = \begin{bmatrix} 0.4284 & 0.4286 \\ 0.5715 & 0.5719 \end{bmatrix}$$

$$T^7 = \begin{bmatrix} 0.4285 & 0.4285 \\ 0.5714 & 0.5714 \end{bmatrix}$$

$$T^8 = \begin{bmatrix} 0.4285 & 0.4285 \\ 0.5714 & 0.5714 \end{bmatrix}$$

Same.

Steady State.

Q3 $\vec{PQ} = \begin{bmatrix}-2\\5\end{bmatrix}$ where $P = (3,2)$ $Q = ?$ let $Q(a,b)$

then $\vec{PQ} = (a-3, b-2) = (-2,5)$

$a-3 = -2 \Rightarrow \boxed{a=1}$ & $b-2 = 5 \Rightarrow \boxed{b=7}$

Q4 is similarly to Q3.

Q5 Find u+v, u-v, 2u, & 3u-3v if

(a) $u = (2,3)$, $v = (-2,5)$.

$u+v = (2-2,\ 3+5) = (0,8)$

$u-v = (2+2,\ 3+5) = (4,-2)$

$2u = 2[2,3] = [4,6]$.

$3u-2v = 3[2,3] - 2[-2,5] = (6,9) - (-4,10)$

$= (6+4,\ 9-10)$

$= (10,-1)$.

(b) & (c) Part similarly to Part(a).

Q6 is similarly to Q5.

Q7 $u = (1,2)$, $v = (-3,4)$, $w = (w_1,4)$ & $x = (-2, x_2)$.

(a) $w = 2u$

$(w_1,4) = 2(1,2)$

$(w_1, 4) = (2, 4)$.

$\Rightarrow w_1 = 2$

(b) $\frac{3}{2}x = v$

$\frac{3}{2}(-2, x_2) = (-3, 4)$

$(-3, \frac{3}{2}x_2) = (-3, 4)$

$\Rightarrow \frac{3}{2}x_2 = 4 \quad \Rightarrow \boxed{x_2 = 8/3}$

Q8 is similarly to Q7.

Q9 (a) $(1, 2)$

$\|u\| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$.

b, c, d is similarly to a

Q10 is similarly to Q9.

Q11 (a) $\|u\| = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{1+1} = \sqrt{2}$.

Q12 is similarly to Q11.

Q13. $(-5, 6) = c_1(1, 2) + c_2(3, 4)$ — (*)

$= (c_1 + 2c_1) + (c_2 3 + c_2 4)$.

$(-5, 6) = (c_1 + 3c_2, 2c_1 + 4c_2)$

$c_1 + 3c_2 = -5$ — (1)

$2c_1 + 4c_2 = 6$

$\div 2$

$\Rightarrow c_1 + 2c_2 = 3$ — (2)

(1) − (2)

$\boxed{c_2 = -8}$

eq (1) $\Rightarrow c_1 - 24 = -5 \quad \Rightarrow \boxed{c_1 = 19}$

eq (*) $\Rightarrow (-5, 6) = 19(1, 2) - 8(3, 4)$

Q14 $c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1\\2c_1\end{bmatrix} + \begin{bmatrix}3c_2\\4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1 + 3c_2\\2c_1 + 4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$.

Augmented form

$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 4 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -2 & 0 \end{array}\right] R_2 - 2R_1$$

$-2c_2 = 0 \Rightarrow \boxed{c_2 = 0}$

So it is impossible.

Q15

$$= \frac{1}{2}\left| \det\left(\begin{bmatrix} 3 & 3 & 1 \\ -1 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix}\right)\right|$$

$$= \frac{1}{2}\left| 3\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left| 3(-1-1) + 1(3-1) + 4(3+1) \right|$$

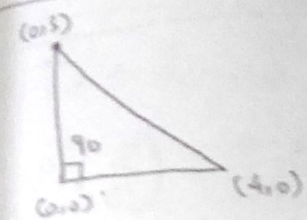
$$= \frac{1}{2}\left| 3(-2) + 2 + 4(4) \right|$$

$$= \frac{1}{2}\left| -6 + 2 + 16 \right|$$

$$= \frac{1}{2}\left| 18 - 6 \right|$$

$$= \frac{1}{2}(12) = 6$$

Q16



$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(4)(3) = 6.$$

or

$$= \frac{1}{2}\left| \det\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 1 \\ 4 & 0 & 1 \end{bmatrix}\right)\right|$$

$$= \frac{1}{2}\left| 4\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|(4(0-3)\right| = \frac{1}{2}\left|-12\right| = \frac{1}{2}(12) = 6$$

Q17 $\text{Area} = \left| \det\left(\begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}\right)\right|$ (P-221).

$$= \left| 2\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix}\right|$$

$$= \left| 2(3-5) - 5(3-5) + 4(3-3) \right|$$

$$= \left| -4 + 10 \right| = |6|$$

$$= 6.$$

Q19 (a) $X=(3,4)$

$$= \frac{3,4}{\sqrt{3^2+4^2}} = \frac{3,4}{\sqrt{25}}, \quad \frac{3,4}{5} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

b & c Part similarly to Part "a"

Q20 is similarly to Q19.

Q21 (a) $\cos\theta = \frac{U \cdot V}{\|U\| \|V\|}$ — ① $\quad U=(1,2), V=(2,-3)$

where $\|U\| = \sqrt{1^2+2^2} = \sqrt{5}$

& $\|V\| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$.

$U \cdot V = (1,2)(2,-3) = (1)(2)+(2)(-3) = 2-6=-4$

eq ① => $\cos\theta = \frac{-4}{\sqrt{5}\cdot\sqrt{13}}$.

Q (b) (c) & (c) is similarly to Part (a).

Q22 is similarly to Q21.

Q23 is S.Y.S

Q24 (a) For orthogonal? $U \cdot V = 0$

$U_1 \cdot U_6 = (i+2j)\cdot(-6i+3j) = (-\cancel{6}+\cancel{6}) = 0$

So $U_1 \cdot U_4 = (i+2j)\cdot(2i+j) = (-\cancel{2}+\cancel{2}) = 0$.

$U_3 \cdot U_4 = (-2i-4j)\cdot(-2i+j) = (-\cancel{4}+\cancel{4}) = 0$

$U_3 \cdot U_6 = (-2i-4j)\cdot(-6i+3j) = (\cancel{12}-\cancel{12}) = 0$

$U_4 \cdot U_5 = (-2i+j)\cdot(2i+4j) = (-4+4) = 0$

$U_5 \cdot U_6 = (2i+4j)\cdot(-6i+3j) = (-12+12) = 0$.

(b) U_1 & U_5 because $U_5 = 2U_1$

U_4 & U_6 because $U_6 = 3U_4$.

(c) U_1 & U_3 because $U_3 = -2U_1$

U_3 & U_5 because $U_3 = -U_5$.

Q25 ~~Find~~ Slope m_1 = Slope m_2 (vector are parallel if slopes are equal).

$m_1 = 4/a, \quad m_2 = 5/2$

i.e. $4/a = 5/2$

$5a = 8$

$\boxed{a = 8/5}$

Scanned by CamScanner

Q26 Vector are orthogonal if $\cos\theta = 0$.

$(a,2).(a,-2) = 0$

$a^2 + (-4) = 0$

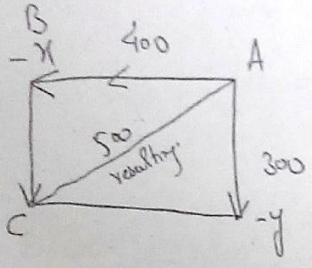
$a^2 = 4$

$\boxed{a = \pm 2}$

Q27 (a) $i + 3j$, (b) $-2i - 3j$ (c) $-2i$ (d) $3j$

Q28 (a) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}_{2\times1}$, (b) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$.

Q29



$u = 0i - 300j$ & $v = -400i + 0j$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \vec{V} + \vec{U}$

$= (-400i + 0j) + (0i - 300j) = -400i - 300j$

$|\overrightarrow{AC}| = \sqrt{(400)^2 + (300)^2} = 500$

Q1 (a) $u = (1,2,-3)$, $v = (0,1,-2)$

$u + v = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 2+1 \\ -3-2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$

$u - v = \begin{bmatrix} 1-0 \\ 2-1 \\ -3+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

$2u = 2\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$

$3u - 2v = 3\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix} - \begin{bmatrix} 0 \\ +2 \\ -4 \end{bmatrix} = \begin{bmatrix} 3-0 \\ 6-2 \\ -9+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$.

Part b is same as "a"

Q2 as same as Q1

Q3 $u = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} a \\ -1 \\ b \end{bmatrix}$, $x = \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix}$.

Find a, b & c so that.

(a) $w = \frac{1}{2}u \Rightarrow \begin{bmatrix} a \\ -1 \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1 \\ 3/2 \end{bmatrix}$

$\Rightarrow a = 1/2$
$b = 3/2$.

(b) $W+V=U$

$$\Rightarrow \begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} a-3 \\ -2 \\ b+3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$a-3=1 \Rightarrow a=1+3=4 \Rightarrow \boxed{a=4}$

$b+3=3 \Rightarrow \boxed{b=0}$

(c) $W+X=V$

$$\begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+3 \\ c-1 \\ b+2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} \Rightarrow a+3=-3 = -3-3 = -6 \Rightarrow \boxed{a=-6}$$

$c-1=-1 \Rightarrow \boxed{c=0}$ $b+2=3 \Rightarrow \boxed{b=1}$.

Q4 as same as Q3.

Q5 $u=(4,5,-2,3)$ & $v=(3,-2,0,1)$; $c=2$, $d=3$

(a) $u+v=v+u$

$u+v=(4+3, 5-2, -2+0, 3+1)=(7,3,-2,4)$

$v+u=(3+4, -2+5, 0-2, 1+3)=(7,3,-2,4)$.

(b) $u+(v+w)=(u+v)+w$.

L.H.S $u+(v+w)$

$(4,5,-2,3)+[(3,-2,0,1)+(-3,2,-5,3)]$.

$(4,5,-2,3)+(0,0,-5,4)$

$(4,5,-7,7)$.

R.H.S $(u+v)+w$

$[(4,5,-2,3)+(3,-2,0,1)]+(-3,2,-5,3)$.

$(7,3,-2,4)+(-3,2,-5,3)$

$(4,5,-7,7)$.

Here $u+(v+w)=(u+v)+w$

(c) $u+0=0+u$.

L.H.S $=(4,5,-2,3)+(0,0,0,0)=(4,5,-2,3)$

R.H.S $(0,0,0,0)+(4,5,-2,3)=(4,5,-2,3)$.

(d) $U + (-U) = 0$

$(4,5,-2,3) + (-(4,5,-2,3))$

$(4,5,-2,3) + (-4,-5,+2,-3)$

$(4-4, 5-5, -2+2, 3-3)$

$(0,0,0,0)$.

(e) $c(U+V) = cU + cV$

$L.H.S = 2[(4,5,-2,3) + (3,-2,0,1)]$

$= 2(7,3,-2,4)$

$= (14,6,-4,8)$.

$R.H.S \quad 2(4,5,-2,3) + 2(3,-2,0,1)$

$= (8,10,-4,6) + (6,-4,0,2)$

$= (14,6,-4,8)$.

(f) $(c+d)u = cu + du$

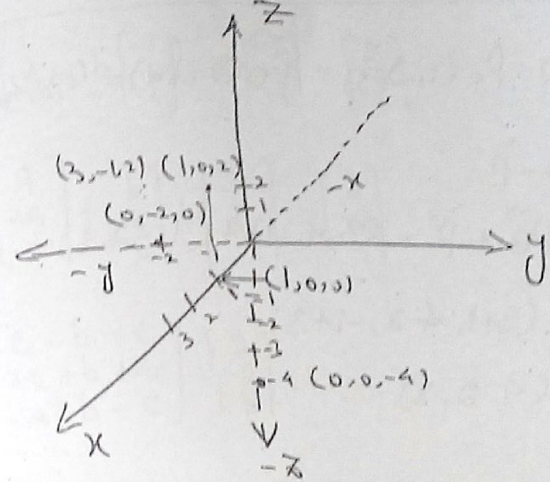
$L.H.S \quad (2+3)(4,5,-2,3) = 5(4,5,-2,3) = (20,25,-10,15)$

$R.H.S \quad 2(4,5,-2,3) + 3(4,5,-2,3)$

$(8,10,-4,6) + (12,15,-6,9)$

$= (20,25,-10,15)$

Q6



Q7 Same as Q6

Q8 (a) P(1st Point), Q(Second Point).

$\vec{PQ} = \vec{Q} - \vec{P} = (0,0,2) - (2,3,-1)$

$= (-2,-3,3)$

$\vec{PQ} = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix}$.

Part b, c, d Same as Part a.

Q9 head $Q = ?$ $P = (1,-2,3)$ $\vec{PQ}$ (vector) $= (3,4,-1)$.

$\vec{PQ} = \vec{Q} - \vec{P}$

$\Rightarrow Q = \vec{PQ} + \vec{P} = (3,4,-1) + (1,-2,3)$

$\vec{Q} = (3+1, 4-2, -1+3)$

$\vec{Q} = (4,2,2)$.

Q10 (a) $\|U\| = \sqrt{(1)^2+(2)^2+(-3)^2} = \sqrt{1+4+9} = \sqrt{14} = \sqrt{14}$.

Part b, c, d Same as part a.

Q11 Same Q10.

Q12 (a) $(1,-1,2), (3,0,2)$.

$\|U-V\| = \sqrt{(1-3)^2+(-1+0)^2+(2-2)^2}$.

$= \sqrt{4+1+0}$

$= \sqrt{5}$.

Part b, c, d Same as part a.

Q13 Same as Q12.

Q4 $a\begin{bmatrix}1\\2\\-3\end{bmatrix} + b\begin{bmatrix}-1\\1\\1\end{bmatrix} + c\begin{bmatrix}-1\\4\\-1\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$\begin{bmatrix}a\\2a\\-3a\end{bmatrix} + \begin{bmatrix}-b\\b\\b\end{bmatrix} + \begin{bmatrix}-c\\4c\\-c\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$\begin{bmatrix}a-b-c\\2a+b+4c\\-3a+b-c\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$a-b-c = 2$ —— (1)

$2a+b+4c = -2$ —— (2)

$-3a+b-c = 3$ —— (3).

add (-2) time first to (2)

$-2\cancel{a}+2b+2c = -4$

$\cancel{2a}+b+4c = -2$

$3b+6c = -6$

$b+2c = -2$ —— (2)'

add (3) times (1) to (3).

$3\cancel{a}-3b-3c = 6$

$-\cancel{3}a+b-c = 3$

$-2b-4c = 9$

$2b-4c = -9$ —— (3)'

add (-2) times (2)' to (3)'

$$-2\cancel{b}-4\cancel{c}=4$$
$$\cancel{2b}+\cancel{4c}=-9$$
$$0=-5$$

which is impossible.

Q15 $c_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-2\end{bmatrix}+c_3\begin{bmatrix}3\\7\\-4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

$$\begin{bmatrix}c_1\\2c_1\\-c_1\end{bmatrix}+\begin{bmatrix}c_2\\3c_2\\-2c_2\end{bmatrix}+\begin{bmatrix}3c_3\\7c_3\\-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$\begin{bmatrix}c_1+c_2+3c_3\\2c_1+3c_2+7c_3\\-c_1-2c_2-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$c_1+c_2+3c_3=0$ —— ①

$2c_1+3c_2+7c_3=0$ —— ②

$-c_1-2c_2-4c_3=0$ —— ③

add (-2) times ① to ②.

$$-2\cancel{c_1}-2c_2-6c_3=0$$
$$\cancel{2c_1}+3c_2+7c_3=0$$
$$c_2+c_3=0$$
$$\boxed{c_2=-c_3}$$

add (+2) times ① to ③.

$$2c_1+\cancel{2c_2}+6c_3=0$$
$$-c_1-\cancel{2c_2}-4c_3=0$$
$$c_1+2c_3=0$$
$$\boxed{c_1=-2c_3}$$

let $c_3=r$

then $c_2=-r$ & $c_1=-2r$.

For possible solution put $r=1$ (unit value)

$c_3=1$, $c_2=-1$, $c_3=-2(1)=-2$

then $\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}-2\\-1\\1\end{bmatrix}$.

Q16 $\|(1,a,-3,2)\|=5$

$$\sqrt{(1)^2+(a)^2+(-3)^2+(2)^2}=5$$

$$\sqrt{14+a^2}=5$$

Sq: B.S

$$14+a^2=25$$
$$a^2=25-14=11 \Rightarrow \boxed{a=\pm\sqrt{11}}$$

Q17 $U=(a,2,1,a)$ & $V=(a,-1,-2,-3)$.

& $U\cdot V=0$.

$(a,2,1,a)\cdot(a,-1,-2,-3)=0$

$(a^2-2,-2-3a)=0$

$a^2-3a-4=0$

$a=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}$

$a=\dfrac{3\pm\sqrt{9+16}}{2}$

$a=\dfrac{3\pm\sqrt{25}}{2}=\dfrac{3\pm5}{2}\cdot\dfrac{8}{2},-2/2$

$\boxed{a=4,-1}$

Q18 $C=3$, $U=(1,2,3)$, $V=(1,2,-4)$, $W=(1,0,2)$.

(a) $U\cdot U\geq 0$.

$(1,2,3)\cdot(1,2,3)\geq 0$.

$1+4+9\geq 0$

$14\geq 0$.

(b) $U\cdot V=-7=V\cdot U$.

(c) $(U+V)\cdot W=U\cdot W+V\cdot W$.

L.H.S $[(1,2,3)+(1,2,-4)]\cdot(1,0,2)$

$=(2,4,-1)\cdot(1,0,2)$

$=\cancel{(2+1\,4)}$

$=(2-2)=0$.

R.H.S $(1,2,3)\cdot(1,0,2)+(1,2,-4)\,(1,0,2)$

$(1+6)+(1-8)$

$(\cancel{2}-\cancel{2})=0$.

Q19 $|U\cdot V|\leq \|U\|\,\|V\|$

$|U\cdot V|=|(1,2,3)\cdot(1,2,-4)|$

$=|1^2+2^2+3(-4)|=|1+4-12|=|-7|=7$.

$\|U\|=\sqrt{U\cdot U}=\sqrt{(1,2,3)\cdot(1,2,3)}=\sqrt{1^2+4+9}=\sqrt{14}$

$\|V\|=\sqrt{V\cdot V}=\sqrt{(1,2,-4)(1,2,-4)}=\sqrt{1+4+16}=\sqrt{21}$

$\|U\|\,\|V\|=\sqrt{14}\sqrt{21}=\sqrt{14\times21}=7\sqrt{6}$.

So $7\leq 7\sqrt{6}$ verified.

Q20 (a) $u=(1,2,3)$, $v=(-4,4,5)$.

$\cos\theta = \dfrac{u\cdot v}{\|u\|\|v\|}$ — ①

$u\cdot v = (1,2,3)\cdot(-4,4,5) = (-4+8+15) = 19$.

$\|u\| = \sqrt{1+2^2+3^2} = \sqrt{14}$

$\|v\| = \sqrt{(-4)^2+(4)^2+5^2} = \sqrt{57}$.

eq ① $\Rightarrow \cos\theta = \dfrac{19}{\sqrt{14}\sqrt{57}} = 0.673$.

(b) $u=(0,2,3,1)$ $v=(-3,1,-2,0)$

$\cos\theta = \dfrac{u\cdot v}{\|u\|\cdot\|v\|}$ — ①.

$u\cdot v = (0,2,3,1)\cdot(-3,1,-2,0) = (0+2-6+0) = -4$.

$\|u\| = \sqrt{4+9+1} = \sqrt{14}$ $\|v\| = \sqrt{9+4+1} = \sqrt{14}$.

eq ① $\Rightarrow \cos\theta = \dfrac{-4}{\sqrt{14}\sqrt{14}} = \dfrac{-4}{14} = \dfrac{-2}{7}$

$\boxed{\cos\theta = -2/7}$.

c & d part similarly.

Q21 is similarly to Q20.

Q22 S.Y.S

Q23 is similarly Q24 (Ex 4.1).

Q24 For orthogonal $v\cdot w = 0$.

$(2,c,3)\cdot(1,-2,1) = 0$

$2-2c+3 = 0$

$-2c = -5$

$\boxed{c = 5/2}$

Q25 $v=(a,b,c)$, $w=(1,2,1)$ $x=(1,-1,1)$

$v\cdot w = (a,b,c)\cdot(1,2,1) = (a+2b+c) = 0$ — ①

$v\cdot x = (a,b,c)\cdot(1,-1,1) = (a-b+c) = 0$ — ②

① − ②

$\cancel{a}+2b+\cancel{c} = 0$

$\underline{-\cancel{a} \mp b -\cancel{c} = 0}$

$3b = 0 \Rightarrow \boxed{b=0}$.

eq(1) $\Rightarrow a+c=0 \Rightarrow a=-c$

eq(2) $\Rightarrow a+c=0 \Rightarrow a=-c$.

Let $c=r$ then $a=-r$, $b=0$

For possible solution put $r=1$

$c=1$, $a=-1$, $b=0$.

$\boxed{a=-1, b=0, c=1}$

Q26 triangle inequality $\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

$u=(1,2,3,-1)$ & $v=(1,0,-2,3)$

$u+v=(2,2,1,2)$

$\|u+v\| = \sqrt{(2)^2+(2)^2+(1)^2+(2)^2} = \sqrt{4+4+1+4} = \sqrt{13}$.

$\|u+v\|^2 = (\sqrt{13})^2 = 13$ — (1)

$\|u\| = \sqrt{1+4+9+1} = \sqrt{15}$

$\|v\| = \sqrt{(1)^2+(0)^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$

Now $(\|u\|+\|v\|)^2 = (\sqrt{15}+\sqrt{14})^2 = 57.98 = 58$ — (2)

From (1) & (2) we

$\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

Q27 $X=(2,-1,3)$

unit vector in the direction of X

then $u = \dfrac{X}{\|X\|}$ — (1)

$\|X\| = \sqrt{4+1+9} = \sqrt{14}$

eq(1) $\Rightarrow u = \dfrac{(2,-1,3)}{\sqrt{14}} = \left(\dfrac{2}{\sqrt{14}}, \dfrac{-1}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$.

Part b, c, d are same as part (a).

Q28 is same as Q27.

Q29 (a) $(1,2,-3) = i+2j-3k$.

Part b, c, d are same as part (a).

Q30 (a) $2i+3j-4k$

$2i = 2I_1 = 2\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\0\\0\end{bmatrix}$

$3j = 3I_2 = 3\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\3\\0\end{bmatrix}$

$-4k = -4I_3 = -4\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\-4\end{bmatrix}$.

3x1 matrix $\begin{bmatrix}2\\0\\0\end{bmatrix}+\begin{bmatrix}0\\3\\0\end{bmatrix}+\begin{bmatrix}0\\0\\-4\end{bmatrix} = \begin{bmatrix}2\\3\\-4\end{bmatrix}_{3\times1}$.

(Part b, c, d same as part a)

Q31 For isosceles triangle two sides are equal.

$\|P_1-P_2\| = \sqrt{(2-3)^2+(3-1)^2+(4-2)^2} = \sqrt{41}$

$\|P_1-P_3\| = \sqrt{(2-3)^2+(3)^2+(4-4)^2} = \sqrt{100} = 10$

$\|P_2-P_3\| = \sqrt{(3-3)^2+(1)^2+(2-4)^2} = \sqrt{41}.$

From above we have.

$\|P_1-P_2\| = \|P_2-P_3\|$ verified.

——— x ———

Q32 For right triangle angle b/w two sides are 90° &

$\cos\theta = 0$

$\vec{u} = \overrightarrow{P_1P_2} = (3-2, 1-3, 2+4) = (1,-2,6)$

$\vec{v} = \overrightarrow{P_1P_2} = (7-2, 0-3, 1+4) = (5,-3,5)$

$\vec{w} = \overrightarrow{P_2P_3} = (7-3, 0-1, 1-2) = (4,-1,-1)$

$\vec{u}\cdot\vec{v} = (5+6+30) = 41$

$\vec{v}\cdot\vec{w} = (5+6+30) = 41$

& $\vec{u}\cdot\vec{w} = (4+2-6) = 0.$

Q33 $u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{2000} \end{bmatrix}$

8 % increase

$u = \begin{bmatrix} u_1 + 0.08u_1 \\ u_2 + 0.08u_2 \\ \vdots \\ u_{2000} + 0.08u_{2000} \end{bmatrix} = \begin{bmatrix} (1+0.08)u_1 \\ (1+0.08)u_2 \\ \vdots \\ (1+0.08)u_{2000} \end{bmatrix}$

$u = \begin{bmatrix} 1.08u_1 \\ 1.08u_2 \\ \vdots \\ 1.08u_{2000} \end{bmatrix} = 1.08u.$

——— x ———

Q34 $u \cdot v$ will show price of total CD players speakers and cassette recorders.

——— x ———

Q35 $\frac{1}{2}(t+b)$ (average).

Q36 $U=(1,1,0,0)$ $U+V=0$ Let $V=\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

Now $U+V=0$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a+1 \\ b+1 \\ c \\ d \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a=1,\ b=1,\ c=0,\ d=0$

$V=(1,1,0,0)$.

Q37 Same as Q36.

Q39 $U=(1,0,1)$ & Let $V=(a,b,c)$.

$U \cdot V=0$

$(1,0,1).(a,b,c)=0$

$a+0+c=0$

$a+c=0$. — (1)

For bit matrix $b=0,\ b=1$.

If (1) $\Rightarrow b=0$. then $a=c=0$

if $b=1$ then $a=c=1$.

So $V=(0,0,0),\ (0,1,0),\ (1,0,1),\ (1,1,1)$.

Q38 Same as Q39.

Linear transformation:- A linear transformation L of R^n into R^m is a function assigning a unique vector $L(u)$ in R^m to each u in R^n such that

(i) $L(u+v)=L(u)+L(v)$, for every u and v in R^n

(ii) $L(ku)=kL(u)$, for every u in R^n and every scalar k.

Q1 Which of the following are linear transformation?

(a) $L(x,y)=(x+1,y,x+y)$.

Let $\vec{u}=(x_1,y_1)$ and $\vec{V}=(x_2,y_2)$ be the two vectors in R^2

Then $L(\vec{u})=L(x_1,y_1)=(x_1+1,y_1,x_1+y_1)$.

& $L(\vec{V})=L(x_2,y_2)=(x_2+1,y_2,x_2+y_2)$

$L(\vec{u})+L(\vec{V})=(x_1+1,y_1,x_1+y_1)+(x_2+1,y_2+x_2+y_2)$.

$=(x_1+x_2+2,\ y_1+y_2,\ x_1+y_1+x_2+y_2)$ — (1)

Now $\vec{u}+\vec{V}=(x_1,y_1)+(x_2,y_2)$

$=(x_1+x_2,\ y_1+y_2)$.

$L(\vec{u}+\vec{v}) = (x_1+x_2+1,\ y_1+y_2,\ x_1+x_2+y_1+y_2)$ —— ②

From eq ① & eq ② we have.

$L(\vec{u}+\vec{v}) \neq L(\vec{u})+L(\vec{v})$.

it's not a linear transformation.

(b) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y \\ x-z \end{bmatrix}$.

Sol: $L: R^3 \to R^3$

Let $\vec{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ be the two vectors in R^3.

then

$L(\vec{u}) = L\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix}$

and

$L(\vec{v}) = L\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$.

Now $L(\vec{u})+L(\vec{v}) = \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$

$= \begin{bmatrix} (x_1+y_1)+(x_2+y_2) \\ y_1+y_1 \\ (x_1+x_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ①$

Now

$(\vec{u}+\vec{v}) = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}$.

$L(\vec{u}+\vec{v}) = L\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix} = \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ y_1+y_2 \\ (x_1+x_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ②$

From eq ① & eq ② we have.

$L(\vec{u}+\vec{v}) = L(\vec{u})+L(\vec{v})$ —— (A)

checking 2nd Property

$L(k\vec{u}) = k(L\vec{u})$.

$k\vec{u} = k\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$.

$L(k\vec{u}) = L\begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix} = \begin{bmatrix} kx_1+ky_1 \\ ky_1 \\ kx_1-kz_1 \end{bmatrix}$.

$= k\begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix}$

$\Rightarrow L(k\vec{u}) = k(L\vec{u})$ —— (B)

From eq (A) & eq (B) it's clear that it's a Linear transformation from R^3 to R^3.

(c) $L(x,y) = (x^2+x,\ y-y^2)$.

Sol:- Let $\vec{u} = (x_1, y_1)$ & $\vec{V} = (x_2, y_2)$ be the two vector in R^2

Then $L(\vec{U}) = L(x_1, y_1) = (x_1^2+x_1,\ y_1-y_1^2)$

& $L(\vec{V}) = L(x_2, y_2) = (x_2^2+x_2,\ y_2-y_2^2)$.

$$L(\vec{u})+L(\vec{v}) = (x_1^2+x_1, y_1-y_1^2)+(x_2^2+x_2, y_2-y_2^2)$$

$$= [(x_1^2+x_1)+(x_2^2+x_2),\ (y_1-y_1^2)+(y_2-y_2^2)] \quad -(1)$$

Now $\vec{U}+\vec{V} = (x_1, y_1)+(x_2, y_2)$

$$= (x_1+x_2,\ y_1+y_2)$$

$$L(\vec{U}+\vec{V}) = L(x_1+x_2,\ y_1+y_2)$$

$$= (x_1+x_2)^2+(x_1+x_2),\ (y_1+y_2)-(y_1+y_2)^2 \quad -(2)$$

From eq (1) & eq (2) we have.

$$L(\vec{U})+L(\vec{V}) \neq L(\vec{U}+\vec{V})$$

it's not a linear transformation.

Q_2 & Q_3 is similarly to Q_1

Q4 (a)
$$L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}\right) = \begin{bmatrix} u_1 \\ u_1^2+u_2 \\ u_1-u_3 \end{bmatrix}.$$

Let $\vec{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ be the two vector in R^4 then.

$$L(\vec{U}) = L\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_1^2+u_2 \\ u_1-u_3 \end{bmatrix}.$$

$$L(\vec{V}) = L\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1^2+v_2 \\ v_1-v_3 \end{bmatrix}.$$

Now $L(\vec{U})+L(\vec{V}) = \begin{bmatrix} u_1 \\ u_1^2+u_2 \\ u_1-u_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_1^2+v_2 \\ v_1-v_3 \end{bmatrix}$

$$= \begin{bmatrix} u_1+v_1 \\ (u_1^2+u_2)+(v_1^2+v_2) \\ (u_1-u_3)+(v_1-v_3) \end{bmatrix} \longrightarrow (1)$$

Again Now

$$\vec{U}+\vec{V} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \\ u_4+v_3 \end{bmatrix}.$$

$$L(\vec{U}+\vec{V}) = L\left(\begin{bmatrix} U_1+V_1 \\ U_2+V_2 \\ U_3+V_3 \\ U_4+V_4 \end{bmatrix}\right) = \begin{bmatrix} U_1+V_1 \\ (U_1+V_1)^2 + (U_2+V_2) \\ (U_1+V_1)-(U_3+V_3) \end{bmatrix} \longrightarrow ②$$

From eq ① & eq ② we have

$$L(\vec{U}+\vec{V}) \neq L(\vec{U})+L(\vec{V})$$

it's not a linear transformation.

(b) $$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} x+y+0z \\ 0x-y+2z \\ x+y-z \end{bmatrix} = \begin{bmatrix} x+y \\ -y+2z \\ x+y-z \end{bmatrix}.$$

Let $\vec{U} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ and $\vec{V} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ be the two vector in R^3, then

$$L(\vec{U}) = L\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}\right) = \begin{bmatrix} x_1+y_1 \\ -y_1+2z_1 \\ x_1+y_1-z_1 \end{bmatrix}.$$

$$\& L(\vec{V}) = L\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2+y_2 \\ -y_2+2z_2 \\ x_2+y_2-z_2 \end{bmatrix}.$$

$$L(\vec{U})+L(\vec{V}) = \begin{bmatrix} x_1+y_1 \\ -y_1+2z_1 \\ x_1+y_1-z_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ -y_2+2z_2 \\ x_2+y_2-z_2 \end{bmatrix}$$

$$= \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ -(y_1+y_2)+2(z_1+z_2) \\ (x_1+x_2)+(y_1+y_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ①.$$

Now $$\vec{U}+\vec{V} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}.$$

$$L(\vec{U}+\vec{V}) = L\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ -(y_1+y_2)+2(z_1+z_2) \\ (x_1+x_2)+(y_1+y_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ②$$

From eq ① & eq ② we have.

$$L(\vec{U})+L(\vec{V}) = L(\vec{U}+\vec{V}). \text{———} Ⓐ$$

checking 2nd Property.

$L(k\vec{U}) = k(L\vec{U})$ where $k \in R$ then.

$$k\vec{U} = k\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$$

$$L(k\vec{u}) = L\begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$$

$$= \begin{bmatrix} kx_1 + ky_1 \\ -ky_1 + 2kz_1 \\ kx_1 + ky_1 - kz_1 \end{bmatrix} = k\begin{bmatrix} x_1 + y_1 \\ -y_1 + 2z_1 \\ x_1 + y_1 - z_1 \end{bmatrix}.$$

$\Rightarrow L(k\vec{u}) = k(L\vec{u}) \longrightarrow$ (B)

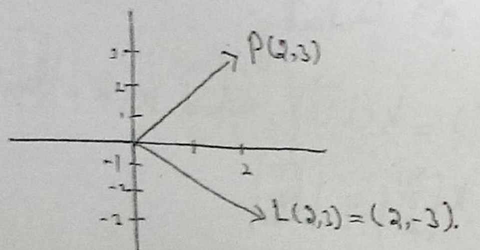
From eq (A) & eq (B) it's clear that it's a linear transformation from R^3 to R^3.

(c) Same as

In Exercises 5–12, sketch the image of the given point P or vector u under the given linear transformation L.

Q5 $L(x,y) = (x-y) : P(2,3)$

$L(2,3) = (2,-3)$.

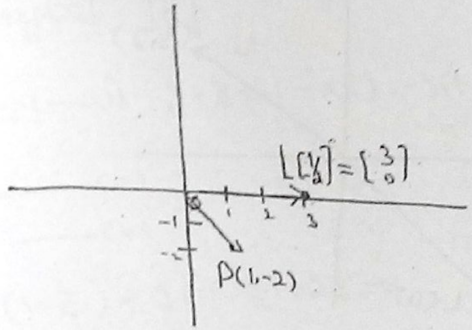


Q6 $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} ; u = (1,-2)$

$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ 2x+y \end{bmatrix}$

$L\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 1-(-2) \\ 2(1)+(-2) \end{bmatrix} = \begin{bmatrix} 1+2 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$

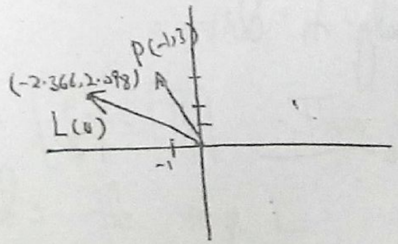
$L\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$



Q7 $L = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \theta = 30^\circ \quad P = (-1,3) = u$

$LU = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}.$

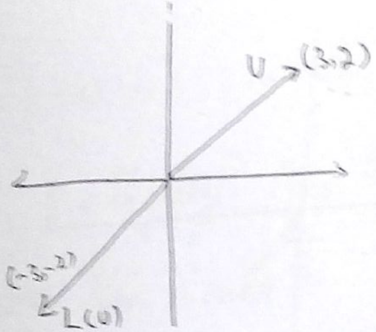
$= \begin{bmatrix} -\cos 30^\circ - 3\sin 30^\circ \\ -\sin 30 + 3\cos 30^\circ \end{bmatrix} = \begin{bmatrix} -2.366, 2.098 \end{bmatrix}.$



Q8 is similarly to Q7.

Q9 $L(U) = -U$, $U = (3,2)$

$L(3,2) = -(3,2) = (-3,-2)$



Q10 is similarly to Q9.

Q11 $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x-y \end{bmatrix}$ $\quad U = \overset{x\ \ y\ \ z}{(2,-1,3)}$

$L(u) = L\left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2-(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Q12 is similarly to Q11

~o~

Q13 (a) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix}$ $\quad W = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

Since $L(\vec{U}) = \vec{W}$

$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\Rightarrow x+z = 1$ — (i) $\quad y+z = -1$ — (ii), $\quad x+2y+2z = 0$ — (iii)

$x = 1-z$ ——— (iv)

& $y = -1-z$ ——— (v)

eq (3) $\Rightarrow (1-z) + 2(-1-z) + 2z = 0$

$1 - z - 2 - \cancel{2z} + \cancel{2z} = 0$

$-z - 1 = 0 \Rightarrow \boxed{z = -1}$

eq (iv) $\Rightarrow x = 1-(-1) = \boxed{x = 2}$

eq (v) $\Rightarrow y = -1-(-1) = -1+1 = 0 = \boxed{y = 0}$.

eq (iii) $\Rightarrow 2 + 2(0) + 2(-1) = 0$

$\cancel{2} + 0 - \cancel{2} = 0$

$0 = 0$.

$\because \vec{U} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ This shows the image and so it's the range L.

(b) $L(\vec{u}) = w$

$$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

$x+z = 2$ —— (i)

$y+z = -1$ —— (ii)

$x+2y+2z = 3$ —— (iii)

eq (i) $\Rightarrow x = 2-z$ —— (iv)

eq (ii) $\Rightarrow y = -1-z$ —— (v)

eq (iii) $\Rightarrow (2-z)+2(-1-z)+2z = 3$

$\cancel{2}-z+(\cancel{-2}-\cancel{2z})+\cancel{2z} = 3$

$-z = 3 \Rightarrow \boxed{z=-3}$

eq (iv) $\Rightarrow x = 2+3 = 5 \Rightarrow \boxed{x=5}$

eq (v) $\Rightarrow y = -1+3 \Rightarrow \boxed{y=2}$.

eq (iii) $\Rightarrow 5+2(2)+2(-3) = 3$

$5+4-6 = 3$

$9-6 = 3$

$3 = 3$

$\therefore \vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$ This show the image so it's the range L.

Q14 is Similarly to Q13.

Q15 Let $\vec{U} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ~~the~~ such that

$L\vec{U} = \vec{w}$

$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$

$$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$\Rightarrow 4x+y+3z = a$ —— (i)

$2x-y+3z = b$ —— (ii)

$2x+2y = c$ —— (iii).

× eq (ii) by (2) then subtr: from (i)

$\cancel{4x}+y+3z = a$

$\underline{\cancel{4x}-2y+6z = 2b}$

$3y-3z = a-2b$ —— (4).

eq ② − eq ③

$$\cancel{2x} - y + 3z = b$$
$$\cancel{2x} \pm 2y = c$$
$$-3y + 3z = b - c \quad ⑤$$

eq ④ + eq ⑤

$$\cancel{3y} - \cancel{3z} = a - 2b$$
$$\cancel{-3y} + \cancel{3z} = b - c$$
$$0 = a - 2b + b - c$$

$$a - b - c = 0$$

$$\Rightarrow c - a + b = 0 \quad \text{Ans.}$$

~~~~0~~~~0~~~~.

Q16 is similarly to Q15.

Q17 As $\begin{bmatrix} 4 \\ -3 \end{bmatrix} = 4i - 3j$ where $L(i) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ & $L(j) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

$$\Rightarrow L\begin{bmatrix} 4 \\ -3 \end{bmatrix} = L(4i - 3j)$$
$$= 4L(i) - 3L(j)$$
$$= 4\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$
$$= \begin{bmatrix} 8+3 \\ 12-6 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \end{bmatrix}.$$

$$\Rightarrow L\left(\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 11 \\ 6 \end{bmatrix}.$$

Q18 As $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = 2i - j + 3k$

$$\Rightarrow L\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = L(2i - j + 3k) = 2L(i) - L(j) + 3L(k)$$
$$= 2\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 2-1+3 \\ 4-0+3 \\ -2-2+9 \end{bmatrix}$$

$$L\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix} \quad \text{Ans.}$$
~~~~

Q: let $\vec{U}=(x_1,y_1)$ & $\vec{V}=(x_2,y_2)$. be the two vector in R^2, then

$L(\vec{U})= L(x_1,y_1) = (x_1+y_1+1, x_1-y_1)$

$L(\vec{V})= L(x_2,y_2)= (x_2+y_2+1, x_2-y_2)$.

$L(\vec{U})+L(\vec{V})=(x_1+y_1+1, x_1-y_1)+(x_2+y_2+1, x_2-y_2)$

$=((x_1+y_1)+(x_2+y_2)+2, (x_1+x_2)-(y_1+y_2)) \longrightarrow (1)$

Now $\vec{U}+\vec{V}= (x_1,y_1)+(x_2+y_2)$

$= (x_1+x_2, y_1+y_2)$

$L(\vec{U}+\vec{V})= L(x_1+x_2, y_1+y_2)$

$= (x_1+x_2+y_1+y_2+1, x_1+x_2-(y_1-y_2))$ —— (2)

From eq (1) & eq (2) it's clearly that

$L(\vec{U}+\vec{V}) \neq L(\vec{U})+L(\vec{V})$.

$\therefore$ it's not a linear transformation.

Q24 Let $\vec{U} = (x_1, y_1)$ and $\vec{V} = (x_2, y_2)$ be the two vectors in R_2, then

$\vec{U} + \vec{V} = (x_1 + y_1) + (x_2 + y_2) = (x_1 + x_2, y_1 + y_2)$.

$L(\vec{U} + \vec{V}) = L(x_1 + x_2, y_1 + y_2)$.

$L(\vec{U} + \vec{V}) = (\sin(x_1 + x_2), \sin(y_1 + y_2))$ —— (1)

Now $L(\vec{U}) = L(x_1, y_1)$

$= \sin x_1 + \sin y_1$

& $L(\vec{V}) = L(x_2, y_2)$

$= \sin x_2 + \sin y_2$.

$\Rightarrow L(\vec{U}) + L(\vec{V}) = (\sin x_1 + \sin y_1) + (\sin x_2 + \sin y_2)$

$= (\sin(x_1 + x_2), \sin(y_1 + y_2))$ —— (2).

From eq (1) & eq (2) we have

$L(\vec{U} + \vec{V}) = L(\vec{U}) + L(\vec{V})$. —— (A)

Now let $k \in R$ then $k\vec{U} = k(x_1, y_1) = (kx_1, ky_1)$

$L(k\vec{U}) = L(kx_1, ky_1) = (k\sin x_1 + k\sin y_1) = k(\sin x_1 + \sin y_1)$

$\Rightarrow L(k\vec{U}) = k(L\vec{U})$ —— (B)

From eq (A) & eq (B), we can say that the given function is a linear transformation.

Theorem If $L: R^n \to R^m$ is a L.T. then

$L(c_1\vec{U_1} + c_2\vec{U_2} + \cdots + c_k\vec{U_k})$

$= c_1L(\vec{U_1}) + c_2L(\vec{U_2}) + \cdots + c_kL(\vec{U_k})$.

for any vector $\vec{U_1}, \vec{U_2}, \cdots \vec{U_k}$ in R^n and any scalar $c_1, c_2 \cdots c_k$.

For R^2 $\vec{i} = \vec{e_1} = (1, 0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

$\vec{j} = \vec{e_2} = (0, 1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

For R^3 $\vec{i} = \vec{e_1} = (1, 0, 0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\vec{j} = \vec{e_2} = (0, 1, 0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{k} = \vec{e_3} = (0, 0, 1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

For R^n $\vec{i} = \vec{e_1} = (1, 0, 0, \cdots 0)$

$\vdots$

$\vec{e_n} = (0, 0, 0 \cdots 1)$.

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Q26 $L: R^2 \to R^2$ $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$

For R^2 $e_1 = (1,0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. & $e_2 = (0,1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$L(e_1) = L\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

& $L(e_2) = L\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-1 \\ 0+1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

then $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Ans

Q28 $\theta = 60°$

$L(u) = \begin{bmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{x}{2} & -\frac{y\sqrt{3}}{2} \\ \frac{x\sqrt{3}}{2} & y/2 \end{bmatrix}$.

For R^2 $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$L(e_1) = L\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ \sqrt{3}/2 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$.

$L(e_2) = L\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{3}/2 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -\sqrt{3}/2 \\ 1/2 \end{bmatrix}$.

$A = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$.

Q27 is same as Q28

Q29 $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-y \\ x+z \\ y-z \end{bmatrix}$.

For R^3 $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ & $e_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

$L(e_1) = L\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 1+0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

$L(e_2) = L\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-1 \\ 0+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

$L(e_3) = L\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0+1 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Thus $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

Q30 $L(u) = -2u = -2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2x \\ -2y \\ -2z \end{bmatrix}$.

$L(e_1) = L\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$

$L(e_2) = L\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

$L(e_3) = L\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$.

Then $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

Q4 (a)

S E N D H I M M O N E Y
19 5 14 4 8 9 13 13 15 14 5 25.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

Breaking message into 4 vector

$\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix}$.

Now $L(x) = Ax$.

$Ax = A\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$

$A\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$.

$Ax = A\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}$

$A\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}$.

$Ax = A\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 84 \\ 56 \\ 43 \end{bmatrix}$

$A\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 84 \\ 56 \\ 43 \end{bmatrix}$.

$Ax = A\begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} = \begin{bmatrix} 99 \\ 69 \\ 56 \end{bmatrix}$.

So the message is 71 52 33 47 30 26 84 56 43 99 69 56.

(b) 67 44 41 49 39 19 113 76 62 104 69 55

Breaking into vector in R^3

$\underset{L(x_1)}{\begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix}} \underset{L(x_2)}{\begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix}} \underset{L(x_3)}{\begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix}} \underset{L(x_4)}{\begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix}}$.

find $A^{-1} = ?$ $L(x) = Ax \Rightarrow x = A^{-1}L(x)$

$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

$x_1 = A^{-1}L(x_1) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}$.

$x_2 = A^{-1}L(x_2) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} = \begin{bmatrix} 20 \\ 1 \\ 9 \end{bmatrix}$

$x_3 = A^{-1}L(x_3) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 25 \end{bmatrix}$.

$x_4 = A^{-1}L(x_4) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix} = \begin{bmatrix} 14 \\ 15 \\ 20 \end{bmatrix}$

So the message is

3 5 18 20 1 9 14 12 25 14 15 20
C E R T A I N L Y N O T

certainly not.

Q32 is same as Q31.

Jamal Nasir

Lecturer U.E.T Peshawar. ~~[illegible]~~

Qaiser Shahab
0300 5829015

Aarif: 0345 9289401
Ph 091-5610987

Jan Laser Photostate

Matric to M.A.
Notes are available

Shop#58 coffee shop bazar Islamia college Peshawar university

(1)
EX 5.1

In Exercises 1 and 2 compute $U \times V$.

Q1(a) $U = 2i + 3j + 4k$, $V = -i + 3j - k$

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$i \begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix}$$

$i(-3-12) - j(-2+4) + k(6+3)$

$i(-15) - j(2) + k(9)$

$= -15i - 2j + 9k.$

(b) $U = (1, 0, 1)$, $V = (2, 3, -1)$

$\vec{U} = i + 0j + k$ $\vec{V} = 2i + 3j - k$

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 3 & -1 \end{vmatrix} = i(0-3) - j(-1-2) + k(3)$$
$$= -3i + 3j + 3k.$$

(c) Same as Part (a)

(d) $U = (2, -1, 1)$, $V = -2U = -2(2, -1, 1) = (-4, 2, -2)$

$\vec{U} = 2i - j + k$ & $\vec{V} = -4i + 2j - 2k$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ -4 & 2 & -2 \end{vmatrix} = i(2-2) - j(-4+4) + k(4-4)$$
$$= 0\hat{i} + 0j + 0\vec{k}.$$

Q2 is same as Q1.

Q3 Let $U = i+2j-3k$, $V = 2i+3j-k$, $W = 2i-j+2k$, & $c = -3$

(i) $U \times V = -(V \times U)$.

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= 11i - 7j - k \quad \text{—— (1)}$$

$$-(V \times U) = -\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= -(-11i + 7j + k) = 11i - 7j - k \quad \text{—— (2)}$$

From eq (1) & eq (2) we have.

$U \times V = -(V \times U)$ Proved.

Part b, c, d same as Part (a)

Q4 $U = 2i - j + 3k$

$V = 3i + j - k$

$W = 3i + j + 2k$.

(a) verify Equation (3) $(U \times V) \cdot W = U \cdot (V \times W)$.

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix}$$

$$= -2i + 11j + 5k$$

$$(U \times V) \cdot W = (-2i + 11j + 5k) \cdot (3i + j + 2k)$$

$$= -6 + 11 + 10 = 15 \quad \text{—— (1)}$$

Now

$$V \times W = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 3 & 1 & +2 \end{vmatrix} = 3i - 9j + 0k$$

$$U \cdot (V \times W) = (2i - j + 3k) \cdot (3i - 9j + 0k)$$

$$= 6 + 9 + 0 = 15 \quad \text{—— (2)}$$

From eq (1) & eq (2) we have

$(U \times V) \cdot W = U \cdot (V \times W)$ Proved.

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Part (b) is same as Part (a).

Q5 is same as Q4.

<u>orthogonal</u> :. If $u \times v$ is orthogonal to both u & v i.e

$(u \times v) \cdot u = 0.$

& $(u \times v) \cdot v = 0.$

Q6 Exercis a — to — (d)

(a) $\vec{U} = 2i + 3j + 4k$ & $\vec{V} = -i + 3j - k$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$= -15i - 2j + 9k.$

Now $(u \times v) \cdot u = (-15i - 2j + 9k) \cdot (2i + 3j + 4k)$

$= -30 - 6 + 36$

$= -\cancel{36} + \cancel{36} = 0.$

$\therefore (u \times v) \cdot u = 0.$

Also $(u \times v) \cdot v = (-15i - 2j + 9k) \cdot (-i + 3j - k)$

$= 15 - 6 - 9 \Rightarrow \cancel{15} - \cancel{15} = 0$

$(u \times v) \cdot v = 0.$

which is orthogonal.

Q7 is same as Q6

Q9 Area of triangle $= \frac{1}{2}|\vec{U} \times \vec{V}|$ — (1)

let $\vec{U} = P_1P_2 = (-3-1, 1-(-2), 4-3)$

$= (-4, 3, 1)$

& $\vec{V} = P_1P_3 = (0-1, 4-(-2), 3-3)$

$= (-1, 6, 0)$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ -4 & 3 & 1 \\ -1 & 6 & 0 \end{vmatrix}$$

$\vec{U} \times \vec{V} = -6i - j - 21k$

Now $|\vec{U} \times \vec{V}| = \sqrt{(-6)^2 + (-1)^2 + (-21)^2} = \sqrt{478}$

eq (1) => Area of triangle $= \frac{1}{2}|\vec{U} \times \vec{V}|$

$= \frac{1}{2}\sqrt{478}$ (units)^2.

Q10 is same as Q9.

Q11 Area of //gm $= |\vec{U}\times\vec{V}|$. —①

Now $\vec{U}\times\vec{V} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 3 & -1 & -1 \end{vmatrix}$

$= -5i - 5j - 10k$.

Now $|\vec{U}\times\vec{V}| = \sqrt{(-5)^2+(-5)^2+(-10)^2} = \sqrt{150}$

eq① ⟹ Area of //gm $= \sqrt{150}$ (unit)^2.

Q12 $U = 2i - j$, $V = i - 2j - 2k$ & $W = 3i - j + k$.

Volume of //Pied $= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$= \begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & -2 \\ 3 & -1 & 1 \end{vmatrix}$

$= 2(-2-2) + 1(1+6) + 0$

$= 2(-4) + 7$

$= -8 + 1 = |-1|$

Vol //P $= 1$ (unit)^3

Q13 is same as Q12

Ex 5.2 ①

Q1 (a) Let $P_1(x_1, y_1) = (-2, -3)$ & $P_2(x_2, y_2) = (3, 4)$.

Now $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & y & 1 \\ -2 & -3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$

$x(-3-4) - y(-2-3) + 1(-8+9) = 0$

$-7x + 5y + 1 = 0$

This is the required equation of line for the given Points.

Part (b), (c), (d) same as Part (a)

Q2 is same as Q1.

Q3 $x = 3 + 2t \Rightarrow t = \frac{x-3}{2}$.

$y = -2 + 3t \Rightarrow t = \frac{y+2}{3}$

$z = 4 - 3t \Rightarrow t = \frac{z-4}{-3}$.

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Now $\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{-3}$ — ①

(a) $(1,1,1)$

eq ① $\Rightarrow \frac{1-3}{2} = \frac{1+2}{3} = \frac{1-4}{-3}$

$\frac{-2}{2} = \frac{3}{3} = \frac{-3}{-3}$

$-1 \neq 1 \neq 1$

Hence all the value of t are not same, Therefore pts are not lies on the lines.

Part (b) & (c) Same as Part (a)

(d) $(4, -1/2, 5/2)$

eq ① $\Rightarrow \frac{4-3}{2} = \frac{-1/2+2}{3} = \frac{5/2-4}{-3}$

$1/2 = 1/2 = 1/2$

Hence all the value of t are same so it's lie on line

Q4 is same as Q3

Q5 (a) $P_0 = (3, 4, -2)$, $u = (4, -5, 2)$. $-\infty < t < \infty$

$x = x_0 + at \Rightarrow x = 3 + 4t$

$y = y_0 + bt \Rightarrow y = 4 - 5t$

$z = z_0 + ct \Rightarrow z = -2 + 2t$.

Part b, c, d Same as Part (a)

Q6 (a) $P_0 = (2, -3, 1)$ & $P_1 = (4, 2, 5)$

$\vec{U} = \overrightarrow{P_0P_1} = (4-2, 2+3, 5-1) = (2, 5, 4)$

Now $P_0 = (2, -3, 1)$ & $U = (2, 5, 4)$

$x = x_0 + at \Rightarrow x = 2 + 2t$

$y = y_0 + bt \Rightarrow y = -3 + 5t$

$z = z_0 + ct \Rightarrow z = 1 + 4t$

Part (b), (c) & (d) Same as Part (a).

Q7(a) $P_0 = \overset{x_0\ y_0\ z_0}{(2,-3,1)}$ & $u = \overset{a\ b\ c}{(2,5,4)}$

For symmetric form

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-1}{4}$$

(b), (c), & (d) Part as same as Part (a)

Jamal Nasir
Lecturer U.E.T Peshawar

Q8 $3(x-2)+2(y+3)-4(z-4)=0$ — ①

(a) $(0,-2,3)$

eq ① $\Rightarrow 3(0-2)+2(-2+3)-4(3-4)=0$

$0=0$

So it's lie on plane.

(b), (c), & (d) Same as Part (a).

Q9 (d) $P_0 = (x_0, y_0, z_0) = (5, 2, \ )$

$n = (a, b, c) = (-1, -2, 4)$

Equation of plane is

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$-1(x-5) - 2(y-2) + 4(z-3) = 0$

$-x - 2y + 4z - 3 = 0$.

Part (a), (b) & (c) Same as Part (d)

Q10 (b) $P_1(2,3,4)$, $P_2(-1,-2,3)$ $P_3(-5,-4,2)$

Let $U = \overrightarrow{P_1P_2} = (-3,-5,-1)$

$V = \overrightarrow{P_1P_3} = (-7,-7,-2)$

Now $\vec{U} \times \vec{V} = \begin{vmatrix} x & y & z \\ -3 & -5 & -1 \\ -7 & -7 & -2 \end{vmatrix}$

$\Rightarrow x(10-7) - y(6-7) + z(21-35) = 0$

$\Rightarrow 3x + y - 14z = 0$

Which is the required equation of Plane.

others Parts are Same as.

Q11 (b) $3x-2y-5z+4=0$ —— ①
$2x+3y+4z+8=0$ —— ②

Xing eq (1) by "3" & eq (2) by "2"

$9x-\cancel{6y}-15z+12=0$
$4x+\cancel{6y}+8z+16=0$

$13x-7z+28=0$

$\frac{13x+28}{7}=z$ —— ③

Xing eq ① by "2" & eq ② by "3" & subtracting

$\cancel{6x}-4y-10z+8=0$
$\cancel{6x}+9y+12z+24=0$
$-\quad-\quad-\quad-$

$-13y-22z-16=0$

$\frac{-13y-16}{22}=z$ —— (4)

comparing eq (3) & eq (4)

$$\frac{13x+28}{7}=\frac{-13y-16}{22}=z=t \text{ (say)}.$$

Q14 let $P_1(x_1,y_1,z_1)=(-2,4,2)$
$P_2(x_2,y_2,z_2)=(3,5,1)$
$P_3(x_3,y_3,z_3)=(4,2,-1)$.

Now $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}=0$

$\begin{vmatrix} -2 & 4 & 2 \\ 3 & 5 & 1 \\ 4 & 2 & -1 \end{vmatrix}=0$

$-2(-5-2)-4(-3-4)+2(6-20)=0$

$14\neq 0$

These Points are not on the same plane.

Q13 is same as Q14.

Q5

$x = 2-3s \qquad x = 5+2t$

$y = 3+2s$ and $y = 1-3t$

$z = 4+2s \qquad z = 2+t$

$2-3s = 5+2t$ —— (1)

$3+2s = 1-3t$ —— (2)

$4+2s = 2+t$ —— (3)

Subtracting (3) from eq (2), we get

$$\begin{array}{r} 3+2s = 1-3t \\ \underline{\overset{-}{4} \overset{-}{+} 2s = \overset{-}{2} \overset{-}{+} t} \\ -1 = -1-4t \end{array}$$

$-4t = 0$

$\boxed{t=0}$

eq (1) $\Rightarrow 2-3s = 5+2(0) \Rightarrow \boxed{s=-1}$

eq (2) $\Rightarrow 3+2s = 1-3(0)$

$3+2s = 1 \Rightarrow 2s = -2 \Rightarrow \boxed{s=-1}$

$\because t=0,\ s=1$

Now

$x = 2-3s$	$x = 5+2t$
$x = 2-3(-1)$	$x = 5+2(0)$
$\boxed{x=5}$	$\boxed{x=5}$

Now $\dfrac{13x+28}{7} = t$

$\Rightarrow x = \dfrac{7t-28}{13} \longrightarrow$ (a)

& $\dfrac{-13y-16}{22} = t$

$\Rightarrow y = \dfrac{22t+16}{-13} \longrightarrow$ (b)

Also $z = t \longrightarrow$ (c)

Equation (a), (b) & (c) are the required parametric equations of the line of intersection of the given planes.

Q12 (b) $\dfrac{x-2}{-2} = \dfrac{y-3}{4} = \dfrac{z+4}{3}$

Now $\dfrac{x-2}{-2} = \dfrac{y-3}{4}$

$\Rightarrow 4x+2y-14=0$ —— (a) which is 1st equation of plane

And $\dfrac{x-2}{-2} = \dfrac{z+4}{3}$

$3x+2z+2=0$ which is 2nd equation of plane.

(c) $x = 4t \longrightarrow (a)$

$y = 1+5t \longrightarrow (b)$

$z = 2-t \longrightarrow (c)$

eq (a) $\Rightarrow \frac{x}{4} = t$ ——— (i)

eq (b) $\Rightarrow \frac{y-1}{5} = t$ ——— (ii)

eq (c) $\Rightarrow 2-z = t$ ——— (iii)

From eq (i), (ii) & eq (iii)

$$\frac{x}{4} = \frac{y-1}{5} = 2-z = t$$

Now $x/4 = y-1/5$

$\Rightarrow 5x - 4y + 4 = 0$ ——— ① which is 1st eq. of Plane.

& $y-1/5 = 2-z$

$y + 5z - 11 = 0$ ——— ② Which is 2nd eq: of Plane.

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Jamal Nasir

Lecturer UET Peshawar

$y = 3+2s$

$y = 3+2(-1)$

$y = 3-2 = 1$

$\boxed{y = 1}$

$z = 4+2s$

$z = 4+2(-1)$

$z = 4-2$

$\boxed{z = 2}$

$y = 1-3t$

$y = 1-3(0)$

$\boxed{y = 1}$

$z = 2+t$

$z = 2+0$

$\boxed{z = 2}$

Therefore the point of intersection of the given line are $P(x,y,z) = P(5,1,2)$.

~o~o~o

Q16 (a) $x = 2+2t$ and $x = 2+t$

$y = -3-3t$ and $y = 4-t$

$z = 4+4t$ and $z = 5-t$

Let $\vec{u} = (2,-3,4)$

& $\vec{v} = (1,-1,-1)$

Now $\vec{U}\cdot\vec{V} = (2,-3,4)\cdot(1,-1,-1)$.

$$= 2+3-4 = 1$$

$$\vec{U}\cdot\vec{V} \neq 0$$

The given lines are not $\perp$.

$\boxed{\text{Note } \vec{U}\cdot\vec{V} = 0 \text{ then } \perp}$

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Real Vector Spaces

Definition :- A real vector space is a set of elements V together with two operation $\oplus$ and $\odot$ satisfying the following Properties

(α) If U and V are any elements of V then $U \oplus V$ is in V (ie V is closed under the operation $\oplus$).

(a) $U \oplus V = V \oplus U$ for u and v in V

(b) $U \oplus (V \oplus W) = (U \oplus V) \oplus W$ for U, V and W in V.

(c) There is an element 0 in V such that

$$U \oplus 0 = 0 \oplus u = u \text{ for all } u \text{ in } V$$

(d) For each u in V there is an element $-u$ in V such that $u \oplus -u = 0$.

(β) If u is any element of V and c is any real number then $c \odot u$ is in V (ie V is closed under the operation $\odot$).

(e) $c \odot (U \oplus V) = c \odot u \oplus c \odot V$, for all real number c and all u and V in V

(f) $(c+d) \odot u = c \odot u \oplus d \odot u$ for all real number c and d and all u in V

(g) $c\odot(d\odot u) = (cd)\odot u$ for all real number c and d and all u in V.

(h) $1\odot u = u$ for all u in V.

The elements of V are called vector; the real number are called scalars The operation $\oplus$ is called vector addition the operation $\odot$ is called scalar.

Example 2 consider the set V of all ordered triple of real number of the form $(x,y,0)$ and define the operations $\oplus$ and $\odot$ by.

$$(x,y,0)\oplus(x',y',0) = (x+x', y+y', 0)$$

$$c\odot(x,y,0) = (cx,cy,0)$$

Sol. checking the Properties to show that it is a vector space or not

let $u = (x,y,0)$ & $V = (x',y',0)$.

(a) $u\oplus V = V\oplus u$

$\Rightarrow u\oplus V = (x,y,0)\oplus(x',y',0)$.

$= (x+x', y+y', 0) \longrightarrow$ (i)

& $V\oplus u = (x',y',0)\oplus(x,y,0)$

$= (x'+x, y'+y, 0)$ — (ii)

From (i) & (ii) we have

Hence $u\oplus V = V\oplus u$.

(b) $u\oplus(V\oplus W) = (u\oplus V)\oplus W$. let $W = (a,b,c)$.

$U \oplus (V \oplus W) = U \oplus [(x', y', 0) \oplus (a, b, c)]$

$= U \oplus [x'+a, y'+b, 0+c]$

$= (x, y, 0) \oplus (x'+a, y'+b+c)$

$= (x+x'+a, y+y'+b, c)$ — (i)

& $(U \oplus V) \oplus W = [(x, y, 0) \oplus (x', y', 0)] \oplus (a, b, c)$

$= (x+x', y+y', 0) \oplus (a, b, c)$

$= (x+x'+a, y+y'+b, c)$ — (ii)

From eq (i) & eq (ii) we have

$U \oplus (V \oplus W) = (U \oplus V) \oplus W$.

(c) $U \oplus 0 = 0 \oplus U = U$.

$U \oplus 0 = (x, y, 0) \oplus 0 = (x+0, y+0, 0+0) = (x, y, 0) = U$

& $0 \oplus U = (0, 0, 0) \oplus (x, y, 0) = (0+x, 0+y, 0+0) = (x, y, 0) = U$

Hence $U \oplus 0 = 0 \oplus U = U$.

(d) $U \oplus -U = 0$.

$(x, y, 0) \oplus -(x, y, 0) = (x, y, 0) \oplus (-x, -y, 0)$

$(x-x, y-y, 0) = (0, 0, 0) = 0$

Hence $U \oplus -U = 0$.

Now checking the properties for scalar multiplication

(e) $c \odot (U \oplus V) = c \odot ((x, y, 0) \oplus (x', y', 0))$

$= c \odot (x+x', y+y', 0)$

$= (c(x+x'), c(y+y'), 0)$ — (i)

& $c \odot U \oplus c \odot V = c \odot (x, y, 0) \oplus c \odot (x', y', 0)$

$= (cx, cy, 0) \oplus (cx', cy', 0)$

$= (cx+cx', cy+cy', 0)$

$= (c(x+x'), c(y+y'), 0)$ — (ii)

From eq (i) & eq (ii) we have.

$c \odot (U \oplus V) = c \odot U \oplus c \odot V$.

(f) $(c+d) \odot U = c \odot U \oplus d \odot U$

$(c+d) \odot U = (c+d) \odot (x, y, 0)$

$= (cx, cy, 0 + dx, dy, 0)$

$= (cx+dx, cy+dy, 0)$ — (1)

& $c \odot U \oplus d \odot U = c \odot (x, y, 0) \oplus d \odot (x, y, 0)$

$= (cx, cy, 0) \oplus (dx, dy, 0)$

$= (cx + dx, cy + dy, 0)$ — (ii)

From eq (i) & eq (ii) we have

$(c+d) \odot u = c \odot u \oplus d \odot u.$

(g) $c \odot (d \odot u) = (cd) \odot u.$

$c \odot (d \odot u) = c \odot (d \odot (x, y, 0))$

$= c \odot (dx, dy, 0)$

$= (cdx, cdy, 0)$ — (i)

& $(cd) \odot u = (cd) \odot (x, y, 0)$

$= (cdx, cdy, 0)$ — (ii)

From eq (i) & eq (ii) we have

$c \odot (d \odot u) = (cd) \odot u.$

(h) $1 \odot u = u$

$1 \odot (x, y, 0) = (1 \cdot x, 1 \cdot y, 1 \cdot 0)$

$= (x, y, 0)$

$1 \odot (x, y, 0) = u$

Hence it satisfies all the Properties Therefore V is a vector space.

Q1 V is the set of all ordered Pairs of real number (x, y) where $x > 0$ & $y > 0$

$(x, y) \oplus (x', y') = (x + x', y + y').$

& $c \odot (x, y) = (cx, cy).$

sol. Checking the Properties of vector addition $\oplus$

let $u = (x, y)$ & $v = (x', y')$

(a) $u \oplus v = v \oplus u$

$u \oplus v = (x, y) \oplus (x', y')$

$= (x + x', y + y')$ — (i)

& $v \oplus u = (x', y') \oplus (x, y)$ —

$= (x' + x, y' + y)$ — (ii)

From eq (i) & eq (ii) we have

$u \oplus v = v \oplus u.$

(b) $u \oplus (v \oplus w) = (u \oplus v) \oplus w$ let $w = (a, b)$

$U \oplus (V \oplus W) = U \oplus ((x', y') \oplus (a, b))$

$= (x, y) \oplus (x'+a, y'+b)$

$= (x + x' + a, y + y' + b)$ — (i)

& $(U \oplus V) \oplus W = ((x, y) \oplus (x', y')) \oplus (a, b)$

$= (x + x', y + y') \oplus (a, b)$

$= (x + x' + a, y + y' + b)$ — (ii)

From eq (i) & eq (ii) we have

$U \oplus (V \oplus W) = (U \oplus V) + W$

(c) $U \oplus 0 = 0 \oplus U = U$

$U \oplus 0 = (x, y) \oplus (0, 0) = (x+0, y+0) = (x, y) = U$

$0 \oplus U = (0, 0) \oplus (x, y) = (0 + x, 0 + y) = (x, y) = U$

Hence $U \oplus 0 = 0 \oplus U = U$.

(d) $U \oplus -U = 0$

$(x, y) \oplus -(x, y) = (x, y) \oplus (-x, y)$

$(x - x, y - y) = (0, 0) = 0$

Hence $U \oplus -U = 0$. closed under $\oplus$

Now checking the properties for scalar multiplication.

(e) $c \odot (U \oplus V) = c \odot U \oplus c \odot V$

$c \odot (U \oplus V) = c \odot ((x, y) \oplus (x', y'))$

$= c \odot (x + x', y + y')$

$= c(x + x'), c(y + y')$ — (i)

& $c \odot U \oplus c \odot V = c \odot (x, y) \oplus c \odot (x', y')$

$= (cx, cy) \oplus (cx', cy')$

$= (cx + cx', cy + cy')$

$= c(x + x'), c(y + y')$ — (ii)

From eq (i) & eq (ii) we have

$c \odot (U \oplus V) = c \odot U \oplus c \odot V$

But if $c < 0$ then $(x + x') + (y + y') < 0$ so it not closed under $\odot$.

Q2 V is the set of all ordered triples of real numbers of the form $(0,y,z)$.

$$(0,y,z) \oplus (0,y',z') = (0, y+y', z+z')$$

& $c \odot (0,y,z) = (0,0,cz)$.

Sol: let $U = (0,y,z)$ & $V = (0,y',z')$

Checking the Properties for vector addition (+)

(a) $U \oplus V = V \oplus U$

$\Rightarrow U \oplus V = (0,y,z) \oplus (0,y',z')$

$= (0+0, y+y', z+z')$

$= (0, y', z+z')$. — (i)

& $V \oplus U = (0,y',z') \oplus (0,y,z)$

$= (0+0, y'+y, z'+z)$.

$= (0, y'+y, z'+z)$. — (ii)

From eq (i) & eq (ii) we have.

$U \oplus V = V \oplus U$.

(b) $U \oplus (V \oplus W) = (U \oplus V) \oplus W$ & let $W = (a,b,c)$

$\Rightarrow U \oplus (V \oplus W) = U \oplus [(0,y',z') \oplus (a,b,c)]$

$= U \oplus (0+a, y'+b, z'+c)$

$= (0,y,z) \oplus (a, y'+b, z'+c)$

$= (a, y+y'+b, z+z'+c)$ — (i)

& $(U \oplus V) \oplus W = [(0,y,z) \oplus (0,y',z')] \oplus W$

$= (0, y+y', z+z') \oplus W$

$= (0, y+y', z+z') \oplus (a,b,c)$

$= (a, y+y'+b, z+z'+c)$ — (ii)

From eq (i) & eq (ii) we have.

$U \oplus (V \oplus W) = (U \oplus V) \oplus W$

(c) $U \oplus 0 = 0 \oplus U = U$.

$U \oplus 0 = (0,y,z) \oplus (0,0,0) = (0+0, y+0, z+0) = (0,y,z) = U$

& $0 \oplus U = (0,0,0) \oplus (0,y,z) = (0+0, 0+y, 0+z) = (0,y,z) = U$

Hence $U \oplus 0 = 0 \oplus U = U$.

(d) $U \oplus -U = 0$

$(0, y, z) \oplus -(0, y, z)$

$(0, y, z) \oplus (0, -y, -z) = (0, y-y, z-z) = (0,0,0) = 0$

Hence $U \oplus -U = 0$

closed under $\oplus$.

Now checking the properties for scalar multiplication

(e) $c \odot (U+V) = (c \odot U) \oplus (c \odot V)$

$c \odot (U+V) = c \odot [(0, y, z) \oplus (0, y', z')]$

$= c \odot [0+0, y+y', z+z']$

$= c \odot (0, y+y' + z+z')$

$= (0, c(y+y'), c(z+z'))$ —— (i)

$(c \odot U) \oplus (c \odot V) = [c \odot (0, y, z)] \oplus [c \odot (0, y', z')]$

$= (0, cy, cz) \oplus (0, cy', cz')$

$= (0+0, cy+cy', cz+cz')$

$= (0, c(y+y') + c(z+z'))$ —— (ii)

From eq (i) & eq (ii) we have.

$c \odot (U+V) = (c \odot U) \oplus (c \odot V)$.

(f) $(c+d) \odot U = c \odot U \oplus d \odot U$.

$(c+d) \odot U = (c+d) \odot (0, y, z)$

$= (0, cy, cz + 0, dy, dz)$

$= (0, cy + dy, cz + dz)$ — ①.

$c \odot U \oplus d \odot U = c \odot (0, y, z) \oplus d \odot (0, y, z)$

$= (0, cy, cz) \oplus (0, dy, dz)$

$= (0, cy + dy, cz + dz)$ —— (ii)

From eq ① & eq (ii) we have.

$(c+d) \odot U = c \odot U \oplus d \odot U$.

(g) $c \odot (d \odot U) = (cd) \odot U$

$\Rightarrow c \odot (d \odot U) = c \odot (d \odot (0, y, z))$

$= c \odot (0, dy, dz)$

$= (0, cdy, cdz)$ — (i)

& $(cd) \odot U = (cd) \odot (0, y, z)$

$= (0, cdy, cdz)$ —— (ii)

From eq (i) & eq (ii) we have.

$c \odot (d \odot U) = (cd) \odot U$.

(h) $1 \odot U = U$

$1 \odot (0, y, z) = (0, y, z) = U$

Hence it satisfies all the Properties Therefore V is a vector space.

Q3 V is the set of all polynomials of the form at^2+bt+c where a, b and c are real numbers with $b = a+1$

$(a_1t^2+b_1t+c_1) \oplus (a_2t^2+b_2t+c_2)$.

$= (a_1+a_2)t^2+(b_1+b_2)t+(c_1+c_2)$

& $r \odot (at^2+bt+c) = (ra)t^2+(rb)t+rc$.

Sol: Let $U = a_1t^2+b_1t+c_1$ & $V = a_2t^2+b_2t+c_2$.

(a) $U \oplus V = V \oplus U$

$\Rightarrow U \oplus V = (a_1t^2+b_1t+c_1) \oplus (a_2t^2+b_2t+c_2)$

$U \oplus V = (a_1+a_2)t^2+(b_1+b_2)t+(c_1+c_2)$

Put $b_1 = a_1+1$, $b_2 = a_2+1$

$U \oplus V = (a_1+a_2)t^2+(a_1+1+a_2+1)t+(c_1+c_2)$

$= (a_1+a_2)t^2+(a_1+a_2+2)t+(c_1+c_2)$

Not closed under $\oplus$ here we get 2 instead of 1. as given (at^2+bt+c) & $b = a+1$.

Now checking the Properties of scalar multiplication

$r \odot (a_1t^2+b_1t+c_1) = ra_1t^2+rb_1t+rc_1$

$= ra_1t^2+r(a_1+1)t+rc_1$

$= ra_1t^2+(ra_1+r)t+rc_1$

Not closed under $\odot$ b/c we get r instead of 1.

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Hint

Q4 $U = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ & $V = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$.

Q5 Let $U = (x, y)$, $V = (x', y')$; $w = (a, b)$

Q6 $U = (x, y, z)$, $V = (x', y', z')$, $w = (a, b, c)$.

Q11, Q12 + Q13 same as Q2.

Q16 same as Q1

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Subspaces: Let V be a vector space. then W will be subspace of V if

(a) $U \oplus V$ is in W

(b) $k \in \mathbb{R}$, $k \odot U$ is in W.

Q5 Which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$? the set of all vectors of the form

(a) $(a, b, 2)$

Let $\vec{U} = (a, b, 2)$ & $\vec{V} = (a', b', 2)$ are two vector in W then

$\vec{U} \oplus \vec{V} = (a, b, 2) \oplus (a', b', 2)$

$= (a+a', b+b', 4) \notin W$

Which is not in W Hence W is not a subspace.

(b) Let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are two vector in W then $a+b=c$, $a'+b'=c'$

(a) $\vec{U} \oplus \vec{V} = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c') \in W$

or

$\vec{U} \oplus \vec{V} = (a+a', b+b', a+b+a'+b')$

$= (a+a', b+b', (a+a')+(b+b')) \in W.$

(b) $k \odot U = k \odot (a, b, c)$

$= k \odot (a, b, a+b)$

$= (k \odot a, k \odot b, k \odot (a+b)) \in W$

$= (k \odot a, k \odot b, k \odot c) \in W$

Hence W is the Subspace of R^3

16 (a) let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are the two vector in W then $a = c = 0$, $a' = c' = 0$

(a) $U \oplus V = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c')$

$= (0, b+b', +0) \in W$

(b) $k \odot U = k \odot (a, b, c)$

$= (k \odot a, k \odot b, k \odot c)$

$= (k \odot 0, k \odot b, k \odot 0)$

$= (0, k \odot b, 0) \in W$

$= (0, kb, 0) \in W$

Hence W is the Sub space of R^3.

(b) (a, b, c) where $a = -c$

let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are two vector in W then $a = -c$, $a' = -c$

(a) $U \oplus V = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c')$

$= (-c - c' + b+b', c+c')$

$\vec{U} \oplus \vec{V} = (-(c+c') + (b+b') + (c+c')) \in W$

(b) $k \odot \vec{U} = k \odot (a, b, c)$

$= k \odot a, k \odot b, k \odot c$

$= k \odot (-c), k \odot b, k \odot c$

$= (-kc, kb, kc) \notin W$

if $k < 0$ & $k(c) > 0$

(c) Let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ be two vector in W

then $b = 2a+1$, $b' = 2a'+1$

$\vec{U} \oplus \vec{V} = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c')$

$= (a+a', (2a+1)(2a'+1), c+c')$

$= (a+a', 2(a+a')+2, c+c') \notin W$

So W is not a subspace of R^3.

Ex 6.2

Q7 (a) Let $\vec{U} = (a, b, c, d)$ & $\vec{V} = (a', b', c', d')$ be two vector in R^4 then $a-b=2$ & $a'-b'=2$

$a = 2+b$ & $a' = 2+b'$

(a) $\vec{U} \oplus \vec{V} = (a, b, c, d) \oplus (a', b', c', d')$

$= (a+a', b+b', c+c', d+d')$

$= (b+2+b'+2, b+b', c+c', d+d')$

$= (b+b'+4, b+b', c+c', d+d') \notin W$

So W is not a subspace of R^4.

Part (b), (c) & Q8 is similarly to Q7

Q10 (a) $a_2t^2 + a_1t + a_0$ where $a_1 = 0$ & $a_0 = 0$

Let $\vec{U} = a_2t^2 + a_1t + a_0$ & $\vec{V} = a_2't^2 + a_1't + a_0$

(a) $\vec{U} \oplus \vec{V} = (a_2t^2 + a_1t + a_0) \oplus (a_2't^2 + a_1't + a_0)$

$= (a_2+a_2')t^2 + (a_1+a_1')t + (a_0+a_0') \in W$

(b) $k \odot U = k \odot (a_2t^2 + a_1t + a_0)$

$= (ka_2t^2 + ka_1t + ka_0) \in W$ Subspace.

(b) $a_2t^2+a_1t+a_0$ where $a_1=2a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ & $\vec{V}=a_2't^2+a_1't+a_0'$

(a) $\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(a_2+a_2')t^2+2(a_0+a_0')t+(a_0+a_0')\in W$.

(b) $k\odot U=k\odot(a_2t^2+a_1t+a_0)$

$=ka_2t^2+ka_1t+ka_0$

$=ka_2t^2+2ka_0t+ka_0\in W$ subspace.

(c) $a_2t^2+a_1t+a_0$, where $a_2+a_1+a_0=2. \Rightarrow a_2=2-a_1-a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ + $\vec{V}=a_2't^2+a_1't+a_0'$

$\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(2-a_1-a_0+2-a_1'-a_0')t^2+(a_1+a_1')t+(a_0+a_0')\notin W$

not a subspace.

Q9 is similarly to Q10

Q14 let $w_1=a_1U+b_1V$

& $w_2=a_2U+b_2V$.

(a) $w_1\oplus w_2=(a_1U+b_1V)\oplus(a_2U+b_2V)$

$=a_1U+a_2U+b_1V+b_2V$

$=(a_1+a_2)U+(b_1+b_2)V\in W$

(b) $k\odot w_1=k\odot(a_1U+b_1V)$

$=(ka_1)U+(kb_1)V\in W$

So w is a subspace of R^3.

Q15 is as same as Q14.

Q16 let $\vec{U}=\begin{bmatrix}a & b & c\\ d & 0 & 0\end{bmatrix}$ & $\vec{V}=\begin{bmatrix}a_1 & b_1 & c_1\\ d_1 & 0 & 0\end{bmatrix}$, $b=a+k$, $b_1=a_1+k_1$

(a) $\vec{U}\oplus\vec{V}=\begin{bmatrix}a & b & c\\ d & 0 & 0\end{bmatrix}\oplus\begin{bmatrix}a_1 & b_1 & c_1\\ d_1 & 0 & 0\end{bmatrix}$.

$=\begin{bmatrix}a+a_1 & b+b_1 & c+c_1\\ d+d_1 & 0 & 0\end{bmatrix}$.

$$U \oplus V = \begin{bmatrix} a+a_1 & (a+c)+(a_1+c_1) & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$$

(b) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$

$= k \odot \begin{bmatrix} a & a+c & c \\ d & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} ka & k(a+b) & kc \\ kd & 0 & 0 \end{bmatrix} \in W$

Subspace.

(h) $\vec{U} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$ & $c > 0$.

(a) $\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$

(b) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & 0 & 0 \end{bmatrix}$

if $k < 0$ $kc < 0$ so $\notin W$ not subspace.

(c) let $\vec{U} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$ & $a = -2c$. $f = 2e+d$.

(a) $\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \oplus \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$

$= \begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & f+f_1 \end{bmatrix}$

$= \begin{bmatrix} -2(c+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & (2e+d)+(2e_1+d_1) \end{bmatrix}$

$= \begin{bmatrix} -2(c+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & 2(e+e_1)+(d+d_1) \end{bmatrix} \in W$.

(b) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$= \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

$= \begin{bmatrix} -2ck & kb & kc \\ kd & ke & k(2e+d) \end{bmatrix} \in W$.

Which is subspace.

Q17 is similarly Q16.

Definition:- If $S=\{v_1, v_2, \dots v_k\}$ is a set of vector in a vector space V. then the set of all vectors in V that are linear combinations of the vectors in S is denoted by spanS or span$\{v_1, v_2 \dots v_k\}$.

Q25 $v_1=(1,0,0,1)$ $v_2=(1,-1,0,0)$, $v_3=(0,1,2,1)$

(a) $V=(-1,4,2,2)$.

let $c_1v_1+c_2v_2+c_3v_3=V$ then $v_1,v_2,v_3 \in V$.

$c_1(1,0,0,1)+c_2(1,-1,0,0)+c_3(0,1,2,1)=(-1,4,2,2)$

$(c_1,0,0,c_1)+(c_2,-c_2,0,0)+(0,c_3,2c_3,c_3)=(-1,4,2,2)$

$c_1+c_2=-1$ —— (i)

$-c_2+c_3=4$ —— (ii)

$2c_3=2$ —— (iii)

$c_1+c_3=2$ —— (iv)

eq (iii) $\Rightarrow 2c_3=2 \Rightarrow \boxed{c_3=1}$

eq (ii) $\Rightarrow -c_2+1=4 \Rightarrow -c_2=4-1=3 \Rightarrow \boxed{c_2=-3}$

eq (iv) $\Rightarrow c_1+1=2 \Rightarrow c_1=2-1=1 \Rightarrow \boxed{c_1=1}$

eq (i) $\Rightarrow c_1+(-3)=-1 \Rightarrow c_1=-1+3=2 \Rightarrow \boxed{c_1=2}$

So solution is not possible. So V is not span of (v_1,v_2,v_3).

Part (b), (c), (d) is same as part (a).

$A_1=\begin{bmatrix}1 & -1\\0 & 3\end{bmatrix}$, $A_2=\begin{bmatrix}1 & 1\\0 & 2\end{bmatrix}$, $A_3=\begin{bmatrix}2 & 2\\-1 & 1\end{bmatrix}$.

$\begin{bmatrix}5 & 1\\-1 & 9\end{bmatrix}$.

$\begin{bmatrix}5 & 1\\-1 & 9\end{bmatrix}=c_1\begin{bmatrix}1 & -1\\0 & 3\end{bmatrix}+c_2\begin{bmatrix}1 & 1\\0 & 2\end{bmatrix}+c_3\begin{bmatrix}2 & 2\\-1 & 1\end{bmatrix}$.

$=\begin{bmatrix}c_1 & -c_1\\0 & 3c_1\end{bmatrix}+\begin{bmatrix}c_2 & c_2\\0 & 2c_2\end{bmatrix}+\begin{bmatrix}2c_3 & 2c_3\\-c_3 & c_3\end{bmatrix}$.

$\begin{bmatrix}5 & 1\\-1 & 9\end{bmatrix}=\begin{bmatrix}c_1+c_2+2c_3 & -c_1+c_2+2c_3\\-c_3 & 3c_1+2c_2+c_3\end{bmatrix}$

$c_1+c_2+2c_3=5$ —— (1)

$-c_1+c_2+2c_3=1$ —— (2)

$-c_3=-1$ —— (3)

$3c_1+2c_2+c_3=9$ —— (4)

eq (3) $\Rightarrow -c_3=-1 \Rightarrow \boxed{c_3=1}$

Put c_3 in eq (1) & eq (2)

eq (1) $\Rightarrow c_1+c_2+2=5 \Rightarrow c_1+c_2=3$ —— (5)

eq (2) $\Rightarrow -c_1+c_2+2=1 \Rightarrow -c_1+c_2=-1$ —— (6)

Put $c_3 = 1$ in eq (4)

$3c_1 + 2c_2 + 1 = 9$

$3c_1 + 2c_2 = 8$ —— (7)

(5) + (6)

$$\begin{array}{r} \cancel{c_1} + c_2 = 3 \\ -\cancel{c_1} + c_2 = -1 \\ \hline 2c_2 = 2 \end{array}$$

$\boxed{c_2 = 1}$.

X'ing eq (6) by (3) & add with (7)

$$\begin{array}{r} -\cancel{3c_1} + 3c_2 = -3 \\ \cancel{3c_1} + 2c_2 = 8 \\ \hline 5c_2 = 5 \end{array}$$

$\boxed{c_2 = 1}$

eq (7) $\Rightarrow$ $3c_1 + 2(1) = 8$

$3c_1 = 8 - 2 = 6$

$\boxed{c_1 = 2}$

So $\begin{bmatrix} 5 & 1 \\ -9 & \end{bmatrix}$ belongs to space if $c_1 = 2$, $c_2 = 1$, $c_3 = 1$.

Part (b), (c) & (d) same as part (a).

Q28 $W = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. bit matrices

Let $w_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $w_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $w_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

$w_1 + w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w_3$

$w_1 + w_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = w_2$

$w_2 + w_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = w_1$

$w_1 + w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \neq W$ so not a subspace.

Q29 $W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Let $w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$w_1 + w_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = w_3$

$w_1 + w_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w_2$

$w_1 + w_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = w_1$

$w_1 + w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_2 + w_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_3 + w_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_4 + w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_2 + w_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = w_1$

$w_2 + w_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w_2$

$w_3 + w_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = w_3$

it's subspace.

Q30 let $V = B^4$. Determine If W the set of all Vector in V with first entry zero is a Subspace of V.

$$W = \left\{ \begin{array}{cccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\ \begin{bmatrix}0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \end{array} \right\}$$

$$w_1 + w_2 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = w_2$$

$$w_1 + w_3 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix} = w_3$$

$$w_1 + w_4 = \begin{bmatrix}0\\1\\1\\1\end{bmatrix} = w_4$$

$$w_1 + w_5 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = w_5$$

$w_1 + w_6 = w_6$

$w_1 + w_7 = w_7$

$w_1 + w_8 = w_8$

$w_1 + w_1 = w_1$

$$w_2 + w_2 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = w_1$$

Similarly other also hold.

Q31 let $V = B^4$. Determine If W, the set of all vector in V with second entry one is a Subspace of V

$$W = \left\{ \begin{array}{cccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\ \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\1\end{bmatrix} & \begin{bmatrix}1\\1\\1\\1\end{bmatrix} & \begin{bmatrix}1\\1\\1\\0\end{bmatrix} & \begin{bmatrix}1\\1\\0\\1\end{bmatrix} & \begin{bmatrix}1\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \end{array} \right\}$$

Not Vector Space as $w_1 + w_8 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \notin W$.

Q32 $u \in S$ if

$u = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1\\0\\0\end{bmatrix} + \begin{bmatrix}c_2\\c_2\\0\end{bmatrix} + \begin{bmatrix}c_3\\c_3\\c_3\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1+c_2+c_3\\c_2+c_3\\c_3\end{bmatrix}.$$

$c_1 + c_2 + c_3 = 1$ —— (1)

$c_2 + c_3 = 0$ —— (2)

$c_3 = 1$ —— (3)

$\boxed{c_3 = 1}$

eq (2) $\Rightarrow c_2 + 1 = 0$ bit max

$\boxed{c_2 = -1}$ or $\boxed{c_2 = 1}$

eq (1) $\Rightarrow c_1 + 1 + 1 = 1$

$c_1 = 1 - 2 = -1 = 1$ (for b.m)

$\boxed{c_1 = 1}$

It belong to when $c_1 = c_2 = c_3 = 1$.

Q33 is Same as Q 32.

Jamal Nasir

Lecturer U.E.T Peshawar.

Spring Semester Fall 2013

Electrical Engg:

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Which of the following vectors span R^2?

a) $(1,2), (-1,1)$

Sol. Let $V = (a,b)$ & $V_1 = (1,2)$, $V_2 = (-1,1)$

Now $V = c_1V_1 + c_2V_2$

$(a,b) = c_1(1,2) + c_2(-1,1)$

$(a,b) = (c_1, 2c_1) + (-c_2, c_2)$

$(a,b) = (c_1 - c_2, 2c_1 + c_2)$

$c_1 - c_2 = a$ — (i)

$2c_1 + c_2 = b$ — (ii)

(i)+(ii)

$$\begin{aligned} c_1 - \cancel{c_2} &= a \\ 2c_1 + \cancel{c_2} &= b \\ \hline 3c_1 &= a+b \end{aligned} \Rightarrow \boxed{c_1 = \frac{a+b}{3}}$$

eq (i) $\Rightarrow \frac{a+b}{3} - c_2 = a$

$$c_2 = \frac{a+b}{3} - a = \frac{a+b-3a}{3} = \frac{-2a+b}{3}$$

$$\boxed{c_2 = \frac{-2a+b}{3}}$$

So solution exist so span R^2

~~so span R^3~~

(b) $(0,0), (1,1), (-2,-2)$

let $V=(a,b)$, $V_1=(0,0)$, $V_2=(1,1)$ $V_3=(-2,-2)$

Now $V = c_1V_1 + c_2V_2 + c_3V_3$

$(a,b) = c_1(0,0) + c_2(1,1) + c_3(-2-2)$

$(a,b) = (0,0) + (c_2,c_2) + (-2c_3, -2c_3)$

$(a,b) = (0+c_2-2c_3, 0+c_2-2c_3)$

$0+c_2-2c_3 = a$ —— ①

$0+c_2-2c_3 = b$ —— ②

① - ②

$0+\cancel{c_2}-\cancel{2c_3}=a$
$0+\cancel{c_2}-\cancel{2c_3}=b$
$\underline{\ \ -\ \ \ \ +\ \ \ \ \ -\ }$

$0 \neq a-b$

Solution doesn't exist so V_1, V_2, V_3 don't span R^2

Part c same Part b & (d) same (a)

which of the following sets of vector span R^3?

(a) $(1,-2,2), (0,1,1)$

let $V=(a,b,c)$, $V_1=(1,-1,2)$ $V_2=(0,1,1)$.

Now $V = c_1V_1 + c_2V_2$

$(a,b,c) = c_1(1,-1,2) + c_2(0,1,1)$

$(a,b,c) = (c_1, -c_1, 2c_1) + (0, c_2, c_2)$

$(a,b,c) = (c_1, -c_1+c_2, 2c_1+c_2)$

$c_1 = a$ —— ①

$-c_1+c_2 = b$ —— ②

$2c_1+c_2 = c$ —— ③

② - ③

$-c_1+\cancel{c_2} = b$
$\underline{2c_1+\cancel{c_2} = c}$

$-3c_1 = b-c$

$c_1 = -\left(\frac{b-c}{3}\right) = \boxed{\frac{c-b}{3} = c_1}$

There are different values of c_1 so solution does not exist Not a span.

Ex 6.3

Q3 Which of the following vectors span R^4?

(a) $(1,0,0,1)$
$(0,1,0,0)$
$(1,1,1,1)$
$(1,1,1,0)$

let $V=(a,b,c,d)$, $V_1=(1,0,0,1)$, $V_2=(0,1,0,0)$, $V_3=(1,1,1,1)$
$V_4=(1,1,1,0)$

then $V = C_1V_1 + C_2V_2 + C_3V_3 + C_4V_4$

$(a,b,c,d) = C_1(1,0,0,1) + C_2(0,1,0,0) + C_3(1,1,1,1) + C_4(1,1,1,0)$

$(a,b,c,d) = (C_1,0,0,C_1) + (0,C_2,0,0) + (C_3,C_3,C_3,C_3) + (C_4,C_4,C_4,0)$

$(a,b,c,d) = (C_1+C_3+C_4,\ C_2+C_3+C_4,\ C_3+C_4,\ C_1+C_3)$

$C_1+C_3+C_4 = a$ — (1)
$C_2+C_3+C_4 = b$ — (2)
$C_3+C_4 = c$ — (3)
$C_1+C_3 = d$ — (4)

Put $C_1+C_3=d$ in eq (1)

eq (1) $\Rightarrow C_4+d=a \Rightarrow \boxed{C_4=a-d}$

eq (3) $\Rightarrow C_3+(a-d)=c \Rightarrow \boxed{C_3=c-a+d}$

eq (4) $\Rightarrow C_1+(c-a+d)=d \Rightarrow C_1 = d-(c-a+d)$
$C_1 = \cancel{d}-c+a-\cancel{d} = -c+a$

$\boxed{c_1 = a - c}$

eq② $\Rightarrow$ $c_2 + (c - a + d) + (a - d) = b$

$c_2 + c - \cancel{a} + \cancel{d} + \cancel{a} - \cancel{d} = b$

$\boxed{c_2 = b - c}$

Hence solution exist so span R^4

other Parts are same as Part a.

Q4 (a) $\{t^2+1, t^2+t, t+1\}$

Let $P = (a, b, c)$, $P_1 = (t^2+1)$, $P_2 = (t^2+t)$, $P_3 = (t+1)$

$P = c_1P_1 + c_2P_2 + c_3P_3$

$(a,b,c) = c_1(t^2+1) + c_2(t^2+t) + c_3(t+1)$

$= (c_1t^2 + c_1) + (c_2t^2 + c_2t) + (c_3t + c_3)$

$=$ ~~[illegible]~~

$(a,b,c) = (c_1+c_2)t^2 + (c_2+c_3)t + (c_1+c_3)$

$c_1 + c_2 = a$ — ①

$c_3 + c_2 = b$ — ②

$c_1 + c_3 = c$ — ③

eq① $\Rightarrow$ $c_1 = a - c_2$

eq③ $\Rightarrow$ $a - c_2 + c_3 = c$

$-c_2 + c_3 = c - a$ — ④

② + ④

$c_3 + \cancel{c_2} = b$

$c_3 - \cancel{c_2} = c - a$

$2c_3 = c - a + b$

$\boxed{c_3 = \frac{c-a+b}{2}}$

eq② $c_2 = b - c_3$

$c_2 = b - \left(\frac{c-a+b}{2}\right)$

$= \frac{2b-c+a-b}{2} = \frac{b-c+a}{2}$

$\boxed{c_2 = \frac{b-c+a}{2}}$

eq① $\Rightarrow$ $c_1 = a - c_2$

$= a - \left(\frac{b+a-c}{2}\right) = \frac{2a-b-a+c}{2}$

$\boxed{c_1 = \frac{a-b+c}{2}}$

So the solution exist hence a Polyno: Span P_2

Q5 is same as Q4.

Q6 Augmented form

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 \\ 2 & 1 & 3 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_2-R_1 \\ -2R_3-2R_1 \\ R_3-R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \end{array}\right] \begin{array}{l} R_3-\frac{1}{2}R_2 \\ R_4-\frac{1}{2}R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_4-R_3$$

$\frac{1}{2}C_4=0 \Rightarrow C_4=0$

$2C_2+2C_3+C_4=0$

$2C_2+2C_3+0=0$

$2C_2+2C_3=0$

$2(C_2+C_3)=0$

$C_2+C_3=0$

$C_2=-C_3$ let $C_3=r$ (any real no.)

$C_2=-r$

$C_1+C_3=0$

$C_1=-C_3$

$C_1=-r$

So span exist has infinite many solution

Q7 is same Q6

— o —

Q9 $X_1=\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$, $X_2=\begin{bmatrix}1\\0\\-1\\1\end{bmatrix}$, $X_3=\begin{bmatrix}1\\6\\2\\0\end{bmatrix}$, $X=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

Now

$X=C_1X_1+C_2X_2+C_3X_3$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=C_1\begin{bmatrix}1\\2\\0\\1\end{bmatrix}+C_2\begin{bmatrix}1\\0\\-1\\1\end{bmatrix}+C_3\begin{bmatrix}1\\6\\2\\0\end{bmatrix}.$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=\begin{bmatrix}C_1+C_2+C_3\\2C_1+0+6C_3\\0-C_2+2C_3\\C_1+C_2+0\end{bmatrix}$$

$c_1+c_2+c_3=0$ — (1)

$2c_1 + 6c_3=0$ — (2)

$-c_2+2c_3=0$ — (3)

$c_1+c_2=0$ — (4)

Put c_1+c_2 values in eq (1)

eq (1) $\Rightarrow 0+c_3=0 \Rightarrow \boxed{c_3=0}$

eq (3) $\Rightarrow -c_2+2(0)=0 \Rightarrow \boxed{c_2=0}$

eq (4) $\Rightarrow c_1+0=0 \Rightarrow \boxed{c_1=0}$

Hence solution exist so $[x_1, x_2, x_3]$ is L.I.

—— X ——

Q8 is same as Q9.

—— X ——

Q10 $V_1=(1,2,-1)$ & $V_2=(3,2,5)$

Now $c_1V_1+c_2V_2=0$

$c_1(1,2,-1)+c_2(3,2,5)=0$.

$c_1+3c_2=0$ — (1)

$2c_1+2c_2=0$ — (2)

$-c_1+5c_2=0$ — (3)

(1) − (3)

$\cancel{c_1}+3c_2=0$

$-\cancel{c_1}+5c_2=0$

$8c_2=0$

$\boxed{c_2=0}$

eq (1) $\Rightarrow c_1+3(0)=0 \Rightarrow \boxed{c_1=0}$

Hence linearly independent.

(b), (c), (d) is similarly.

Q11 is similarly to Q10

~ o ~ o ~

Q12 (a) $P_1=t^2+1$, $P_2=t-2$, $P_3=t+3$.

Now $c_1P_1+c_2P_2+c_3P_3=0$

$c_1(t^2+1)+c_2(t-2)+c_3(t+3)=0$

$(c_1t^2+c_1)+(c_2t-c_2)+(c_3t+c_3)=0$

$(c_1)t^2+(c_2+c_3)t+(c_1-c_2+c_3)=0$.

$c_1=0$ — (1) $\Rightarrow \boxed{c_1=0}$

$c_2+c_3=0$ — (2)

$c_1-c_2+c_3=0$ — (3)

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Put $c_1=0$ in eq ③

eq ③ $\Rightarrow 0 - c_2 + c_3 = 0$

$-c_2 + c_3 = 0$ —— ④

② + ④

$c_2 + c_3 = 0$

$-c_2 + c_3 = 0$

$2c_3 = 0$

$\boxed{c_3 = 0}$

eq ② $\Rightarrow c_2 + 0 = 0$

$\boxed{c_2 = 0}$

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$ so linearly independent.

Part b, c, d is similarly

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Q13 Consider the vector space M_{22}. Follow the direction of Exercise 10

(a) $\left\{\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}, \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, \begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}, \begin{bmatrix}2 & 2\\ 1 & 1\end{bmatrix}\right\}$

Now $c_1V_1 + c_2V_2 + c_3V_3 + c_4V_4 = 0$

$c_1\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} + c_2\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} + c_3\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix} + c_4\begin{bmatrix}2 & 2\\ 1 & 1\end{bmatrix} = 0$

$c_1 + 2c_2 + 3c_3 + 2c_4 = 0$ —— ①

$c_1 + 3c_2 + c_3 + 2c_4 = 0$ —— ②

$c_1 + c_2 + 2c_3 + c_4 = 0$ —— ③

$c_1 + 2c_2 + c_3 + c_4 = 0$ —— ④

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0\\ 1 & 3 & 1 & 2 & 0\\ 1 & 1 & 2 & 1 & 0\\ 1 & 2 & 1 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0\\ 0 & 1 & -2 & 0 & 0\\ 0 & -1 & -1 & -1 & 0\\ 0 & 0 & -2 & -1 & 0 \end{array}\right] \begin{array}{l} R_2-R_1\\ R_3-R_1\\ R_4-R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0\\ 0 & 1 & -2 & 0 & 0\\ 0 & 0 & -3 & -1 & 0\\ 0 & 0 & -2 & -1 & 0 \end{array}\right] R_2+R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] -\tfrac{1}{3}R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & -1/3 & 0 \end{array}\right] 2R_3+R_4$$

$-\frac{1}{3}c_4 = \boxed{c_4 = 0}$

$c_3 + \frac{1}{3}c_4 = 0$

$c_3 + \frac{1}{3}(0) = 0$

$\boxed{c_3 = 0}$

$c_2 - 2c_3 + 0c_4 = 0$

$c_2 - 2(0) + 0 = 0$

$\boxed{c_2 = 0}$

$c_1 + 2c_2 + 3c_3 + 2c_4 = 0$

$c_1 + 2(0) + 3(0) + 2(0) = 0$

$\boxed{c_1 = 0}$

$c_1 = c_2 = c_3 = c_4 = 0$

So it's linearly Independent.

Q4 (a) $\{\cos t, \sin t, e^t\}$.

$V_1 = \cos t$, $V_2 = \sin t$, $V_3 = e^t$

Now $c_1V_1 + c_2V_2 + c_3V_3 = 0$

$c_1\cos t + c_2\sin t + c_3 e^t = 0$

$c_1 = c_2 = c_3 = 0$ linearly Independent.

(b) $\{t, e^t, \sin t\}$

$V_1 = t$, $V_2 = e^t$, $V_3 = \sin t$

Now $c_1V_1 + c_2V_2 + c_3V_3 = 0$

$c_1 t + c_2 e^t + c_3 \sin t = 0$

$c_1 = c_2 = c_3 = 0$ L. Independent.

(d) $\{\cos^2 t, \sin^2 t, \cos 2t\}$

$V_1 = \cos^2 t$, $V_2 = \sin^2 t$, $V_3 = \cos 2t = \cos^2 t - \sin^2 t$

Now $c_1V_1 + c_2V_2 + c_3V_3 = 0$

$c_1\cos^2 t + c_2\sin^2 t + c_3(\cos^2 t - \sin^2 t) = 0$

$(c_1 + c_3)\cos^2 t + (c_2 - c_3)\sin^2 t = 0$

$c_1 + c_3 = 0$ — (1)

$c_2 - c_3 = 0$ — (2)

let $\boxed{c_3 = \gamma}$

eq ② $\Rightarrow c_2 - \gamma = 0 \Rightarrow \boxed{c_2 = +\gamma}$

eq ① $\Rightarrow c_1 + \gamma = 0 \Rightarrow \boxed{c_1 = -\gamma}$.

Which is linearly dependent as non-trivial.

Q15 let

$V_1 = (-1, 0, -1)$

$V_2 = (2, 1, 2)$

$V_3 = (1, 1, c)$

Now $c_1 V_1 + c_2 V_2 + c_3 V_3 = 0$

$c_1(-1, 0, -1) + c_2(2, 1, 2) + c_3(1, 1, c) = 0$

$-c_1 + 2c_2 + c_3 = 0$ —— ①

$+c_2 + c_3 = 0$ —— ②

$-c_1 + 2c_2 + c c_3 = 0$ —— ③

given L. Dependent so the solution is non-trivial then det $= 0$

$$\begin{vmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & c \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 1 & 1 \\ 2 & c \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ -1 & c \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 0$$

$-1(c-2) - 2(0+1) + 1(0+1) = 0$

$-c + \cancel{2} - \cancel{2} + 1 = 0$

$-c + 1 = 0$

$\Rightarrow \boxed{c = 1}$

Q16 let $V_1 = t + 3$

$V_2 = 2t + \lambda^2 + 2$

Now $c_1 V_1 + c_2 V_2 = 0$.

$c_1(t+3) + c_2(2t + \lambda^2 + 2) = 0$

$c_1 t + 3c_1 + 2c_2 t + c_2(\lambda^2 + 2) = 0$

$c_1 t + 2c_2 t = 0$ —— ①

$3c_1 + c_2(\lambda^2 + 2) = 0$ —— ②

Det $= 0$ ∵ for non-trivial solution

$$\begin{vmatrix} 1 & 2 \\ 3 & \lambda^2 + 2 \end{vmatrix} = 0 \qquad \lambda^2 + 2 - 6 = 0$$

$\lambda^2 - 4 = 0 \Rightarrow \lambda^2 = 4 \Rightarrow \boxed{\lambda = \pm 2}$

Q17 $V_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}, V_2 \begin{bmatrix}1\\0\\1\end{bmatrix}, V_3 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ & $V = \begin{bmatrix}a\\b\\c\end{bmatrix}$.

$c_1V_1 + c_2V_2 + c_3V_3 = V$

$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\begin{bmatrix}c_1\\c_1\\0\end{bmatrix} + \begin{bmatrix}c_2\\0\\c_2\end{bmatrix} + \begin{bmatrix}c_3\\c_3\\c_3\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\begin{bmatrix}c_1 + c_2 + c_3\\c_1 + 0 + c_3\\0 + c_2 + c_3\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$c_1 + c_2 + c_3 = a$ —— ①
$c_1 + 0 + c_3 = b$ —— ②
$0 + c_2 + c_3 = c$ —— ③

① − ②

$\cancel{c_1} + c_2 + \cancel{c_3} = a$
$\cancel{c_1} + 0 + \cancel{c_3} = b$

$c_2 = a - b \Rightarrow \boxed{c_2 = a - b} = \boxed{c_2 = a + b}$ For B.M

① − ③

$c_1 + \cancel{c_2} + \cancel{c_3} = a$
$0 + \cancel{c_2} + \cancel{c_3} = c$

$c_1 = a - c \Rightarrow \boxed{c_1 = a - c} \Rightarrow \boxed{c_1 = a + c}$ For B.M

eq ① $\Rightarrow a + c + a + b + c_3 = a$

$(1+1)a + c_3 = a + c + b = 0(a) + c_3 = a + c + b$ For B.M
$c_3 = a + c + b - 0a = \boxed{c + b + a = c_3}$

$c_3 = a + c + b$
$c_3 = c_1 + b$
$c_3 - c_1 \pm b$
$c_3 + c_1 = b$ (For B.M).
$c + b + a + a + c = b$
$c(1+1) + (1+1)a + b = b$
$c(0) + (0)a + b = b$
$0 + 0 + b = b$
$b = b$ verified.

So it span R^3

Q18 is same as Q17.
Q19 is same.
Q20 is same as Q3
Q21 is similarly
Q22 " "

Q1 let $S = \{V_1, V_2, V_3, V_4, V_5\}$ where.

$V_1 = (1,2,3)$,
$V_2 = (2,1,4)$,
$V_3 = (-1,-1,2)$.
$V_4 = (0,1,2)$. & $V_5 = (1,1,1)$

Not that V is the row space of matrix A whose rows are the given vectors

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ -1 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \begin{matrix} \\ -2R_1+R_2 \\ R_1+R_3 \\ -R_1+R_5 \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & -3 & -2 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} R_{23}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \begin{matrix} -2R_2+R_1 \\ 3R_4+R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} -R_2+R_4 \\ R_4+R_5 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} R_3/4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} -5R_3+R_2 \\ 7R_3+R_1 \\ 3R_3+R_4 \end{matrix}$$

Which is reduced row echelon form. ~~the vectors~~

~~Note~~ Here $w_1 = (1,0,0)$
$w_2 = (0,1,0)$
$w_3 = (0,0,1)$ from a basis for V.

Q_2 is Similarly to Q_1

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} \quad V_3 = \begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix} \quad V_5 = \begin{bmatrix} 5 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \quad V_4 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

Now

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 3 & 2 & 3 & 2 \\ 3 & 3 & 3 & 3 \\ 5 & 3 & 5 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -3 & 0 & -3 \\ 0 & -4 & 0 & -4 \\ 0 & -3 & 0 & -3 \\ 0 & -7 & 0 & -7 \end{bmatrix} \begin{matrix} -2R_1+R_2 \\ -3R_1+R_3 \\ -3R_1+R_4 \\ -5R_1+R_5 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -4 & 0 & -4 \\ 0 & -3 & 0 & -3 \\ 0 & -7 & 0 & -7 \end{bmatrix} -\tfrac{1}{3}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -2R_2+R_1 \\ 4R_2+R_3 \\ 3R_2+R_4 \\ 7R_2+R_5 \end{matrix}$$

Hence Possible solution $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

Q4 $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 2 \\ 0 & 2 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 5 & 0 & 0 & -1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & -4 & -2 & 1 \\ 0 & -10 & -5 & -6 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_4 - 3R_1 \\ R_5 - 5R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 2 & 1 & 2 \\ 0 & -4 & -2 & 1 \\ 0 & -10 & -5 & -6 \end{bmatrix} -(R_2 + R_3)$

$\sim \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & -10 & -7 \\ 0 & 0 & -25 & -26 \end{bmatrix} \begin{matrix} R_1 - 2R_2 \\ R_3 - 2R_2 \\ R_4 + 4R_2 \\ R_5 + 10R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & -10 & -7 \\ 0 & 0 & -25 & -26 \end{bmatrix} R_3/5$

$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{matrix} R_4 + 10R_3 \\ R_5 + 25R_3 \\ R_2 + 2R_3 \\ R_1 - 5R_3 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \frac{1}{5}R_4$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_5 - 4R_4 \\ R_3 - 6/5 R_4 \\ R_2 - 2/5 R_4 \\ R_1 + R_4 \end{matrix}$

So $w_1 = (1,0,0,0)$

$w_2 = (0,1,0,0)$

$w_3 = (0,0,1,0)$

$w_4 = (0,0,0,1)$ form a basis for V

Q5 (a) consisting of vector that are not row vectors of A

$$A=\begin{bmatrix}1 & 2 & -1\\ 1 & 9 & -1\\ -3 & 8 & 3\\ -2 & 3 & 2\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 2 & -1\\ 0 & 7 & 0\\ 0 & 14 & 0\\ 0 & 7 & 0\end{bmatrix}\begin{matrix}-R_1+R_2\\ 3R_1+R_3\\ 2R_1+R_4\end{matrix}$$

$$\sim\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 0\\ 0 & 14 & 0\\ 0 & 7 & 0\end{bmatrix}\frac{1}{7}R_2$$

$$\sim\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{matrix}-2R_2+R_1\\ -14R_2+R_3\\ -7R_2+R_1\end{matrix}$$

Here $w_1=(1,0,-1)$

$w_2=(0,1,0)$ is basis for A.

~~$w_3=$~~

(b) consisting of vector that are row vector of A

whose augmented matrix is

$$\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 2 & 9 & 8 & 3 & 0\\ -1 & -1 & 3 & 2 & 0\end{array}\right]=[A^T \,\vdots\, 0].$$

i.e. the coefficient matrix is A^T. Transforming the augmented matrix $[A^T\vdots 0]$ is to reduced row echelon form we obtain

$$\sim\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 0 & 7 & 14 & 7 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]\begin{matrix}-2R_1+R_3\\ R_1+R_3\end{matrix}$$

$$\sim\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]R_2/7$$

$$\sim\left[\begin{array}{cccc|c}1 & 0 & -5 & -3 & 0\\ 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]-R_2+R_1$$

Since the leading 1s in column 1, 2, we conclude that the 1st two rows of A form a basis for the row space A that is $\{(1,2,-1),(1,9,-1)\}$.

Q6 is similarly to Q5.

Q7 $A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 3 & 2 \\ -2 & -1 & 2 & 1 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix} 2R_1+R_2$

$\sim \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 0 & -3 & -24 & -15 \\ 0 & 0 & 5 & 3 \end{bmatrix} -7R_1+R_3$

$\sim \begin{bmatrix} 1 & 0 & -5 & -3 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} \begin{matrix} -R_2+R_1 \\ 3R_2+R_3 \\ 1/5 R_4. \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & -5 & -3 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} -8R_4+R_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} 5R_4+R_1$

Hence $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3/5 \end{bmatrix} \right\}$ Ans.

is the basis for the column space A.

(b) when augmented matrix is

$\left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 1 & -1 & 4 & 0 & 0 \\ 3 & 2 & -3 & 5 & 0 \\ 2 & 1 & -1 & 3 & 0 \end{array}\right] = [\bar{A}^T \,\vdots\, 0]$

i.e the coefficient matrix is $\bar{A}^T$. Transforming the augmented matrix $[\bar{A}^T \,\vdots\, 0]$ is to reduced row echelon from we obtain

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 8 & -24 & 5 & 0 \\ 0 & 5 & -15 & 3 & 0 \end{array}\right] \begin{matrix} -R_1+R_2 \\ -3R_1+R_3 \\ -2R_1+R_4 \end{matrix}$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right] \begin{matrix} -8R_2+R_3 \\ -5R_2+R_4 \end{matrix}$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \tfrac{1}{3}R_4$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 1 & & 0 \end{array}\right] -5R_4+R_3$$

Since the leading 1's in colum 1, 2 & 4 we conclude that the 1st two rows of A form a basis for the row space A that is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \\ 3 \end{bmatrix} \right\}$$

Q8 is similarly to Q7

— x —

Jamal Nasir

Lecturer U.E.T Peshawar.

Q9 $A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & 3 & 2 \\ 0 & -7 & 8 \end{bmatrix}$

(a) basis for row space of A

$$\sim \begin{bmatrix} 1 & -2 & 5 \\ 0 & 7 & -8 \\ 0 & -7 & 8 \end{bmatrix} R_2-2R_1$$

$$\sim \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -8/7 \\ 0 & -7 & 8 \end{bmatrix} \tfrac{1}{7}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 19/7 \\ 0 & 1 & -8/7 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1+2R_2 \\ R_3+7R_2 \end{matrix}$$

Basis for row space of A

$$A = \{[1, 0, 19/7], [0, 1, -8/7]\}.$$

(b) Basis for colum space of A

$$A^T = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & -7 \\ 5 & 2 & 8 \end{bmatrix}.$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & -7 & 8 & 0 \end{array}\right] \quad [A:0]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 19/7 & 0 \\ 0 & 1 & -8/7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad (\text{From } 1^{st} \text{ part (a)})$$

As leading 1s occur in col 1 & 2 so 1^{st} & 2nd col of A^T form basis

$$\begin{bmatrix} 1 \\ 0 \\ 19/7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 8/7 \end{bmatrix}$$

(c) Basis for row space of A^T

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ -2 & 3 & -7 & 0 \\ 5 & 2 & 8 & 0 \end{array}\right] \quad [A^T : 0]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right] \begin{array}{l} R_2+2R_1 \\ R_3-5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right] \tfrac{1}{7}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ R_3+8R_2 \end{array}$$

$\{[1\ 0\ 2], [0\ 1\ -1]\}$ form Basis

(d) column space of A

$$\left[\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right] : [A : 0]$$

Basis for col space of A

$\left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$ form Basis

———— x ————

Q10 is similarly to Q9.

———— x ————

Q11

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}$$

augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 2 & 1 & 8 & 0 \\ 3 & -5 & -1 & 0 \\ 2 & -2 & 2 & 0 \\ 1 & 1 & 5 & 0 \end{array}\right] = [A^T \mid 0]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 0 & -5 & -6 & 0 \\ 0 & -14 & -22 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ -3R_1+R_3 \\ -2R_1+R_4 \\ -R_1+R_5 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & -14 & -22 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] -\tfrac{1}{5}R_2$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \begin{array}{l} -3R_2+R_1 \\ -7R_5+R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] -4R_5+R_4$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 2/5 & 0 \end{array}\right] 2R_2+R_5$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \tfrac{5}{2}R_5$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} 4R_5+R_4 \\ 8R_5+R_3 \\ -\tfrac{6}{5}R_5+R_2 \\ -\tfrac{17}{5}R_5+R_1 \end{array}$$

Since the non-zero row is 3 so rank = 3.

(b) where augmented matrix is $[A \,|\, 0]$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 3 & 1 & -5 & -2 & 1 & 0 \\ 7 & 8 & -1 & 2 & 5 & 0 \end{array}\right] = [A\,|\,0]$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 0 & -5 & -11 & -8 & -2 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] \begin{array}{l} -3R_1+R_2 \\ -7R_1+R_3 \end{array}$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] -\tfrac{1}{5}R_2$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] -2R_2+R_1$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & 0 & -48/5 & -12/5 & 2/5 & 0 \end{array}\right] \quad 6R_2+R_3$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & 0 & 1 & 12/45 & -2/45 & 0 \end{array}\right] \quad -\tfrac{5}{45}R_3$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -62/75 & 31/225 & 0 \\ 0 & 1 & 0 & 76/75 & 68/225 & 0 \\ 0 & 0 & 1 & 12/45 & -2/45 & 0 \end{array}\right] \quad \begin{array}{l} \frac{-11}{5}R_3+R_2 \\ \frac{7}{5}R_3+R_1 \end{array}$$

So non-zero row = 3 rank = 3

Row rank = column rank = 3.

—— x ——

Q12 is similarly to Q11

—— y ——

Jamal Nasir

Lecturer U.E.T Peshawar

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 4 \\ 0 & -5 & -7 \end{bmatrix} \quad \begin{array}{l} R_2+R_1 \\ R_3-3R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -7 \end{bmatrix} \quad \tfrac{1}{4}R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad R_3+5R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad R_1-2R_2$$

Rank of A = No: of non-row = 3

So rank = 3.

For nullity $Ax = 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$-2x_3 = 0 \Rightarrow \boxed{x_3 = 0}$

$x_2 + x_3 = 0 \Rightarrow \boxed{x_2 = 0}$

$x_1 + 3x_3 = 0 \Rightarrow \boxed{x_1 = 0}$

Q13, Q14, Q16, Q_{17} is similarly to Q_{15}

Q26 If Rank $A = n = 3$
then there is unique solution

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 8 & -7 \\ 3 & -2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 8 & -7 \\ 0 & -8 & 7 \end{bmatrix} R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7/8 \\ 0 & -8 & 7 \end{bmatrix} R_2/8$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7/8 \\ 0 & 0 & 0 \end{bmatrix} R_3 + 8R_2$$

Non-zero row is 2

so rank = 2

No unique solution

Q27 is same as Q26

Q28 Linearly independent holds when det $\neq 0$

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 3 & 2 & 3 \end{vmatrix}$$

$2(0-2) - 1(6-3) + 0(4-0)$

$-4 - (3) + 0$

$-4 - 3$

$-7 \neq 0$ so linearly independent

Q29 is same as Q28

Q30 Non-trivial solution occurs when rank $< n$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} 2R_2 + R_3$$

Rank $A < 3$ so non-trivial.

Q 30 & Q 32 is same as Q 31

~~~~ o ~~~~ 6 ~~~~

Solution exist when

Rank A = Rank of $[A:b]$.

$$\left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 2 & 3 & -2 & 4 & 0 \\ 5 & 1 & 0 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 0 & -1 & -12 & 8 & 0 \\ 0 & -9 & -25 & 12 & 0 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 5R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 0 & 1 & 12 & -8 & 0 \\ 0 & -9 & -25 & 12 & 0 \end{array}\right] - R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -19 & 14 & 0 \\ 0 & 1 & 12 & -8 & 0 \\ 0 & 0 & 83 & -60 & 0 \end{array}\right] \begin{array}{l} R_3 + 9R_2 \\ R_1 - 2R_2 \end{array}$$

Rank of A = Rank of $[A:o]$

$\Rightarrow$ so it has Solution

~~~~ 8 ~~~~

Solution exist when

Rank of A = Rank of $[A:b]$

$$\left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 4 & -1 & -5 & 6 & 2 \\ 2 & 3 & 1 & -2 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 0 & 7 & 7 & -10 & -2 \\ 0 & 7 & 7 & -10 & 0 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 0 & 7 & 7 & -10 & -2 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right] R_3 - R_1$$

Rank of A = 3 & Rank of $[A:b] = 2$

Rank of A $\neq$ Rank of $[A:b]$

$\Rightarrow$ so it's has no solution

Q 34, Q 36 is similarly

~~~~ 6 ~~~~
~~~~

bit matrix

Q37 $\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\end{bmatrix} \sim \begin{bmatrix}1&0&1\\0&1&1\\0&1&1\end{bmatrix} R_3+R_1$

$\sim \begin{bmatrix}1&0&1\\0&1&1\\0&0&0\end{bmatrix} R_3+R_2$

Non-zero row $=2$

So rank $=2$.

Q38 $\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&1&0\\1&1&0&0\end{bmatrix}$.

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&1&0\\0&1&1&0\end{bmatrix} R_4+R_1$

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&1&1\\0&0&1&1\end{bmatrix} \begin{matrix}R_3+R_2\\R_4+R_2\end{matrix}$

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&1&1\\0&0&0&0\end{bmatrix} R_4+R_3$

non-zero row $=3 \Rightarrow$ rank $=3$

Q39 + Q40 similarly.

let $A=\begin{bmatrix}3&-1\\-2&2\end{bmatrix}$.

For eigenvalues

$\det(\lambda I_n - A)=0$

$\det\left(\lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\right)=0$.

$\det\left(\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\right)=0$.

$\det\left(\begin{bmatrix}\lambda-3&1\\2&\lambda-2\end{bmatrix}\right)=0$.

$(\lambda-3)(\lambda-2)-2=0$.

$\lambda^2-2\lambda-3\lambda+6-2=0$

$\lambda^2-5\lambda+4=0$.

$\boxed{\lambda=1}$ $\boxed{\lambda=4}$

For eigenvector

$(\lambda I_n - A)X=0$.

For $\lambda=1$

$1\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=0$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

$$\begin{bmatrix}-2 & 1\\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

$-2x_1 + x_2 = 0$

$2x_1 - x_2 = 0$

$x_1 = \frac{1}{2}x_2$

$x_2 = r$ (Any real no's) $X = \begin{bmatrix}\frac{1}{2}r\\ r\end{bmatrix}.$

$\boxed{\text{For } \lambda = 4}$

$$4\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

$$\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} - \begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

$x_1 + x_2 = 0$

$2x_1 + 2x_2 = 0.$

$x_1 = -x_2$

$x_2 = r$ (any real no's)

$X = \begin{bmatrix}-r\\ r\end{bmatrix}$

Ex 8.1

Let $A = \begin{bmatrix}2 & 2 & 3\\ 1 & 2 & 1\\ 2 & -2 & 1\end{bmatrix}.$

For eigenvalues.

$\det(\lambda I_n - A) = 0$

$$\det\left(\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}2 & 2 & 3\\ 1 & 2 & 1\\ 2 & -2 & 1\end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix} - \begin{bmatrix}2 & 2 & 3\\ 1 & 2 & 1\\ 2 & -2 & 1\end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix}\lambda-2 & -2 & -3\\ -1 & \lambda-2 & -1\\ -2 & 2 & \lambda-1\end{bmatrix}\right) = 0$$

$\lambda-2[(\lambda-1)(\lambda-2)+2] + 2[-(\lambda-1)-2] - 3[-2+2(\lambda-2)] = 0$

$\Rightarrow (\lambda-2)(\lambda^2-3\lambda+4) + (-2\lambda-2) + (-6\lambda+18) = 0$

$\lambda^3 - 5\lambda^2 + 2\lambda + 8 = 0$

$\lambda = -1$

S.D

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8\\ & & -1 & 6 & -8\\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$(\lambda+1)(\lambda^2 - 6\lambda + 8) = 0$

$\lambda+1=0$

$\lambda=-1$

$\lambda^2-6\lambda+8=0$

$\lambda^2-4\lambda-2\lambda+8=0$

$\lambda(\lambda-4)-2(\lambda-4)=0$

$(\lambda-2)(\lambda-4)=0$

$\lambda-2=0,\ \lambda-4=0$

$\lambda=2,\ \lambda=4.$

Now $\lambda=-1,\ \lambda=2,\ \lambda=4.$

For eigenvector

$(\lambda I_n-A)X=0$

$\lambda=-1$

$$=\left(-1\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}2&2&3\\1&2&1\\2&-2&1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$=\left(\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}-\begin{bmatrix}2&2&3\\1&2&1\\2&-2&1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$=\begin{bmatrix}-3&-2&-3\\-1&-3&-1\\-2&2&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$(A/b)=\left[\begin{array}{ccc|c}-3&-2&-3&0\\-1&-3&-1&0\\-2&2&-2&0\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&3&1&0\\0&1&0&0\\0&0&-1&0\end{array}\right]$$

$x_3=-1$

$x_2=0$

$x_1=-3x_2-x_3$

$x_3=-3(0)-(-1)=+1$

$$X=\begin{bmatrix}1\\0\\-1\end{bmatrix}.$$

$\lambda=2,\ \lambda=4$ S.Y.S

Qaisr Shabab
0300 5829015
Aarif: 0345 9289401
Ph :091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Q3 Let $A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ -1 & 3 & 2\end{bmatrix}$

The characteristic Poly: is

$P(\lambda)= \det(\lambda I_n - A)$

$= \det\left(\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix} - \begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ -1 & 3 & 2\end{bmatrix}\right)$

$= \det\left(\begin{bmatrix}\lambda-1 & -2 & -1\\ 0 & \lambda-1 & -2\\ 1 & -3 & \lambda-3\end{bmatrix}\right)$

$= (\lambda-1)\left[(\lambda-1)(\lambda-2)-6\right] + 2(0+2) - 1(0+1-\lambda)$

$= (\lambda-1)(\lambda^2-3\lambda-4) + 4 + \lambda - 1$

$P(\lambda) = \lambda^3 - 4\lambda^2 + 7$ which is required ch: Poly:

Q4, Q5, Q6, Q7 is Similarly to Q3

Qaisar Shahab
0300 5829015
Aarif: 0345 9289401
Ph: 091-5610987
Jan Laser Photostate
Matric to M.A.
Notes are available
Shop#58 coffee shop bazar islamia college Peshawar university

Q8 $A=\begin{bmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{bmatrix}$

The characteristic Poly: of matrix A is given by:

$P(\lambda) = \det(\lambda I_3 - A)$

$= \det\left(\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{bmatrix}\right)$

$= \det\left(\begin{bmatrix}\lambda & -1 & -2\\ 0 & \lambda & -3\\ 0 & 0 & \lambda\end{bmatrix}\right)$

$= \lambda(\lambda^2+0)+1(0+0)-2(0+0)$

$P(\lambda) = \lambda^3$

Now $P(\lambda) = \lambda^3 = 0$

$\lambda = 0$ Eigen value.

For Eigen vector

$(\lambda I_3 - A)x = 0$

$\left(\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

$\left(0\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

$$\left(0 - \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & 0 & -3 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(A/h) $\left[\begin{array}{ccc|c} 0 & -1 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$

$\boxed{x_3 = 0}$

$-x_2 - 2x_3 = 0$

$-x_2 = 2x_3$

$-x_2 = 2(0)$

$-x_2 = 0$

$\boxed{x_2 = 0}$

let $x_1 = r$

$X = \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}.$

Q9, Q10, Q11, Q12, Q13, Q14, Q15

Similarly: