SS Lab # 5

OBJECTIVES OF THE LAB

In this lab, we will cover the following topics:

- *Gain familiarity with Complex Numbers and plot them*
- *Complex exponential signals*
- *Real exponential signals*

5.1 COMPLEX NUMBERS

A complex number *z* is an ordered pair *(x, y)* of real numbers. Complex numbers can be represented in rectangular form as *z = x + iy,* which is the vector in two-dimensional plane. The horizontal coordinate *x* is called the *real part* of z and can be represented as *x = Re {z},* while the vertical coordinate *y* is called the *imaginary part* of z and represented as *y = Imag {z}*. That is:

z = (x, y)

= x + iy

= Re {x} + i Imag {x}

Another way to represent a complex number is in polar form. In polar form, the vector is defined by its length (r) or magnitude (|z|) and its direction (θ). A rectangular form can be converted into polar form using formulas:

$|z| = r = (x^2 + y^2)^{½}$

θ = arctan (y/x)

$z = r\ e^{j\theta}$

where $e^{j\theta} = \cos\theta + i \sin\theta$, and known as the Euler's formula.

5.2 BUILT-IN MATRIX FUNCTIONS

```
Function Description
====================================================
real         returns the real part x of z
imag         returns the imaginary part y of z
abs          returns the length r of z
angle        returns the direction θ of z
conj         returns the complex conjugate z̰ of z
====================================================
```

Here are some examples:

Example

To define the complex number, for instance, z = (3, 4) in Matlab, write in Matlab editor:

```
>> z = 3 + 4i

z =

    3.0000 + 4.0000i
```

Example

To find the real and imaginary parts of the complex number, write

```
>> x = real(z)
x =
    3
>> y = imag(z)
y =
    4
```

Example

To find the length and direction of z, write

```
>> r = abs(z)
r =
    5
>> θ = angle(z)
θ =
    0.9273
```

Example

To find the conjugate of z, write

```
>> zx = conj(z)
zx =
    3.0000 – 4.0000i
```

--------------------------TASK 01--------------------------

Write Matlab function **zprint,** which takes a complex number and returns it real part, imaginary part, magnitude, phase in radians, and phase in degrees.

A sample run of program is:

```
>> zprint(z)
Z =   X  +  jY   Magnitude  Phase  Ph(deg)
      3     4      5         0.927   53.13
```

--------------------------TASK 02--------------------------

Compute the conjugate ź (i.e. z_conj [give variable name]) and the inverse 1/z (i.e. z_inv [give variable name]) for any complex number z. Display the results numerically with zprint.

--------------------------TASK 03--------------------------

Take two complex number and compute $z_1 + z_2$ and display the results numerically using zprint.

--------------------------TASK 04--------------------------

Take two complex numbers and compute $z_1 z_2$ and z_1/z_2. Use zprint to display the results numerically.

5.3 COMPLEX EXPONENTIAL SIGNALS

The complex exponential signal is defined as

$$x'(t) = A\, e^{j(w_0 t + \phi)}$$

which is a complex-valued function of t, where the magnitude of x'(t) is

$|x'(t)| = A$ → magnitude or length of x'(t)

$\arg x'(t) = (w_0 t + \phi)$ → angle or direction of x'(t)

Using Euler's formula, it can be expressed in rectangular or Cartesian form, i.e.

$$x'(t) = A\, e^{j(w_0 t + \phi)} = A \cos(w_0 t + \phi) + j\, A \sin(w_0 t + \phi)$$

where

A = amplitude,

ø=phase shift

w_0 = frequency in rad/sec

Figure 5.1 shows an example complex exponential signal with both real and imaginary parts.

Example

```
clear, close all, clc
n=0:1/10:10;
k=5;
a=pi/2;
x=k * exp(a*n*i);
% plot the real part
subplot(2,1,1)
```

```
stem(n, real(x), 'filled')
title('Real part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid
% plot the imaginary part
subplot(2,1,2)
stem(n, imag(x), 'filled')
title('Imaginary part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid
```

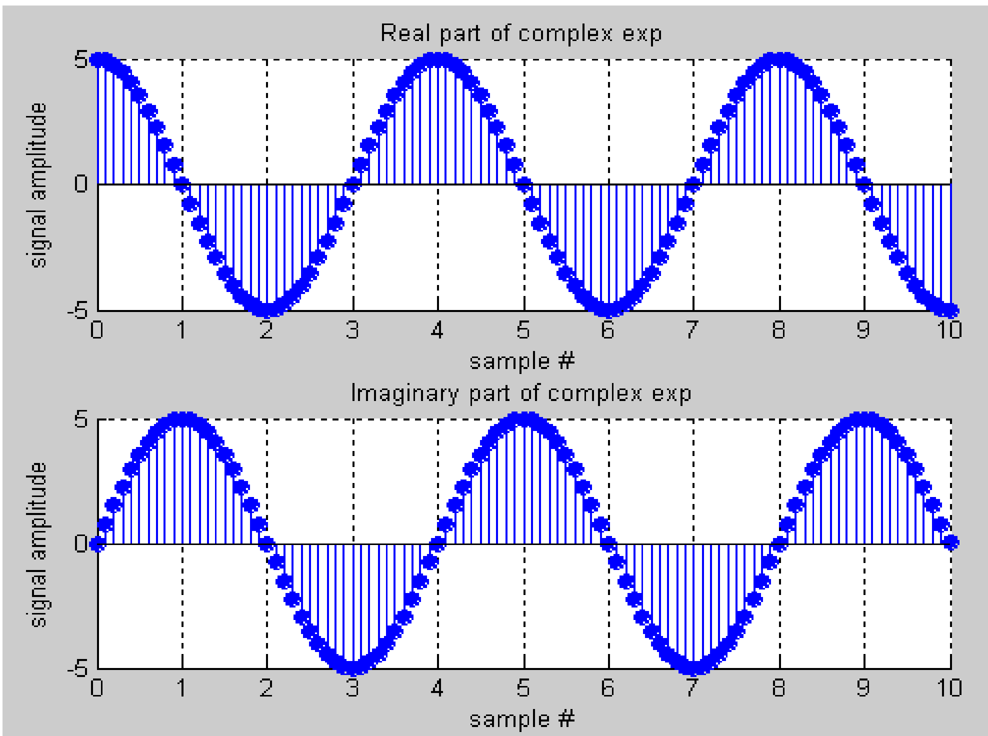

Figure 5.1 – Example Complex Exponential Signal with Real and Imaginary Parts

-------------------------TASK 05-------------------------

Determine the complex conjugate of the exponential signal given in above example and plot its real and imaginary portions.

--------------------------TASK 06--------------------------

Generate the complex valued signal

$$y(n) = \exp^{(-0.2 + j0.5)n}, \quad -10 \le n \le 10$$

and plot its magnitude, phase, the real part, and the imaginary part in separate subplots.

--------------------------TASK 07--------------------------

a) Generate a real-exponential $x=a^n$ for a=0.7 and n ranging from 0-10. Find the discrete time as well as the continuous time version of this signal. Plot the two signals on same graph (holding both the graphs).

b) Repeat the same program with value of a=1.3.

--------------------------TASK 08--------------------------

Multiply the two discrete signals $x_1=5\exp^{(i*n*pi/4)}$ and $x2= a^n$ (use point-by-point multiplication of the two signals). Plot the real as well as the exponential parts for 0<a<1 and a>1.

--------------------------TASK 09--------------------------

Plot the discrete signal x=a^|n| for n ranging from -10 to 10. Draw two subplots for 0<a<1 and a>1.

--------------------------TASK 10--------------------------

a) Generate the signal $x(t) = Ae^{j(\omega t + \pi)}$ for A = 3, π= -0.4, and ω = 2π(1250). Take a range for t that will cover 2 or 3 periods.

b) Plot the real part versus t and the imaginary part versus t. Use subplot(2,1,i) to put both plots in the same window.

c) Verify that the real and imaginary parts are sinusoids and that they have the correct frequency, phase, and amplitude.