

SS Lab # 6

OBJECTIVES OF THE LAB

In this lab, we will cover the following topics:

- *Generating Sinusoids*
 - *Addition of Sinusoids with Variation in Parameters and their Plots*
 - *Linear Phase Shift Concept When Dealing with Sum of Sinusoids*
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6.1 GENERATING SINUSOIDS

Sinusoidal sequences are implemented using `sin()` & `cos()` functions. An example continuous-time sinusoidal signal is shown in Figure 6.1, while an example discrete-time sinusoidal signal is shown in Figure 6.2.

Example: Continuous-Time Sinusoid

```
clc;  
clear all;  
close all;  
  
f0 = 3;  
A = 5;  
t = -1:0.005:1;  
y = A*cos(2*pi*f0*t);  
  
figure, plot(t, y, '*:');  
xlabel('Time, sec'), ylabel('Amplitude');  
title('Graph of sinusoid');
```

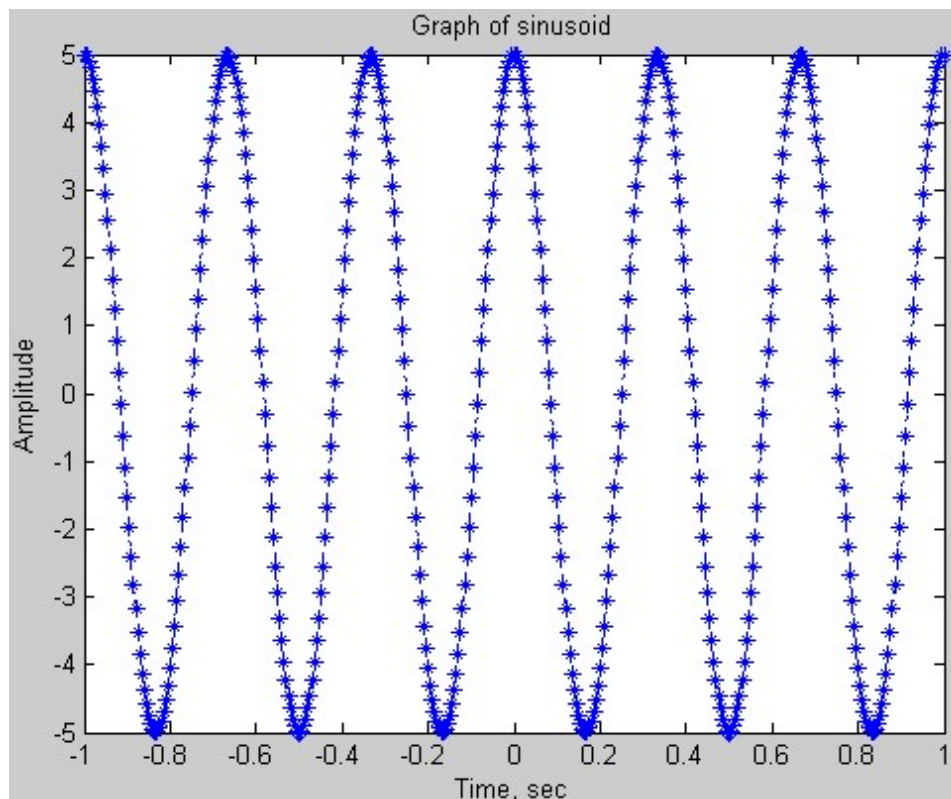


Figure 6.1 – Example Continuous-time Sinusoidal Signal

Program: Discrete-Time Sinusoid

```
clc;
clear all;

close all;

M=10;           %samples/sec
n=-3:1/M:3;
A=2;
phase=0;
f=1;
x=A * sin(2*pi*f*n + phase);

stem(n,x,'linewidth', 2)
title('Discrete-Time Sine Wave:  $A \sin(2\pi f n + \phi)$ ')
xlabel('Time Index')
ylabel('Signal Amplitude')
axis([n(1) n(end) -A A])
grid
```

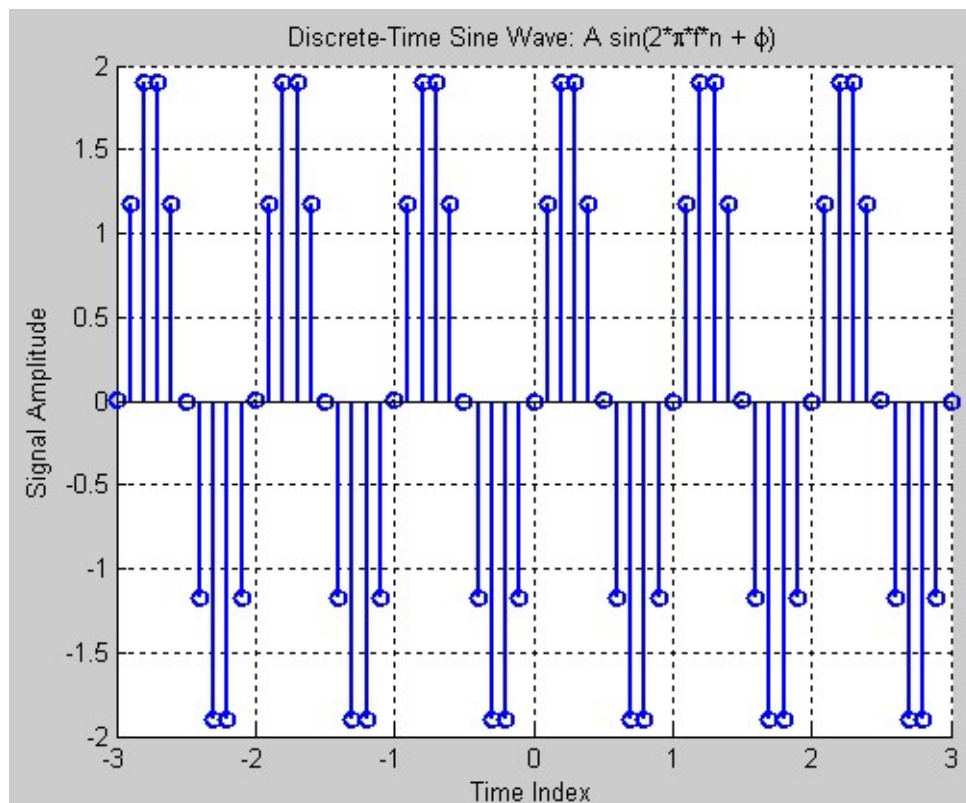


Figure 6.2 – Example Discrete-time Sinusoidal Signal

6.2 CREATING PHASE SHIFT

Phase shift can be created by adding an angle to $2\pi ft$ for a sinusoid. This is shown in Figure 6.3.

Example

```
clc;
clear all;
close all;

fs=1000;
t=-3:1/fs:3;
A=2;
phase=0;
f=1;
x=A * sin(2*pi*f*t + phase);

plot(t,x, 'linewidth', 2)
title('Continuous-Time Sine Wave: A sin(2*\pi*f*t + \phi)')
xlabel('Time Index')
ylabel('Signal Amplitude')
axis([t(1) t(end) -A A])
grid
```

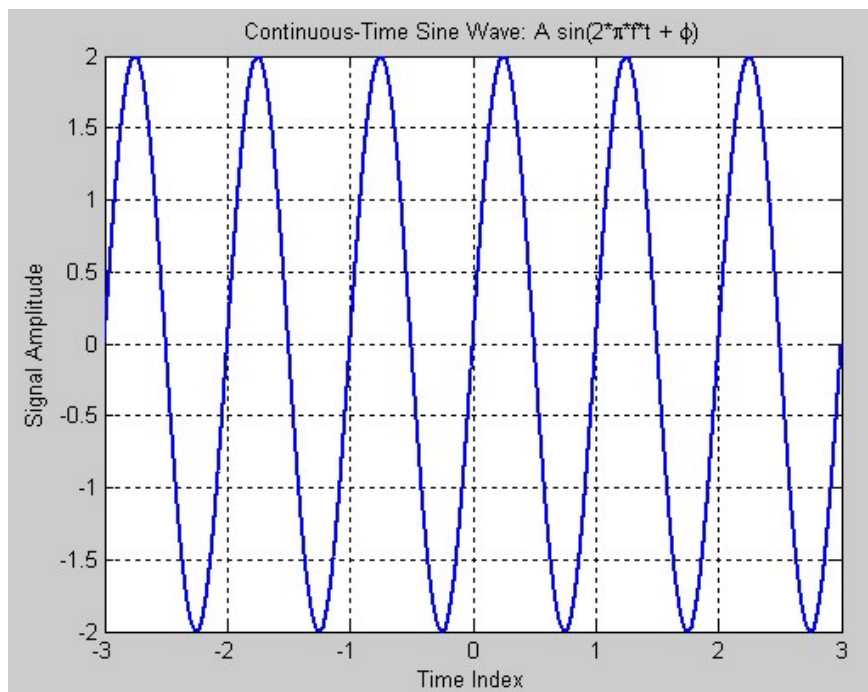


Figure 6.3 – Phase Shift Addition in Sinusoidal Signal

-----TASK 01-----

Generate the 1x10 row vector \mathbf{v} , whose i -th component is $\cos(i\pi/4)$.

-----TASK 02-----

Write Matlab code that draw graphs of $\sin(n\pi x)$ on the interval $-1 \leq x \leq 1$ for $n = 1, 2, 3, \dots, 8$.
(Hint: Use for loop)

-----TASK 03-----

Given the signal $\exp(-x)\sin(8x)$ for $0 \leq x \leq 2\pi$, plot its continuous-time and discrete-time representations. Use subplot and label properly.

-----TASK 04-----

Modify the example given in topic 6.2 to generate a sine wave with phase shift of $+\pi/2$. Then plot a cosine wave of same frequency, amplitude, and phase shift of 0 in another subplot. Compare both the signals and determine the relationship between the two.

-----TASK 05-----

Write a program to generate a continuous-time sine wave of frequency 3 Hz, positive phase shift of $\pi/2$, and amplitude of 5. Also generate a continuous-time cosine wave of frequency 3 Hz, amplitude of 5, and phase shift of 0. Plot the two signals on separate subplots and properly label them. Determine the relationship between the two signals.

6.3 ADDITION OF SINUSOIDS

Sinusoidal signals can be added in four different ways. These are shown in Figure 6.4, Figure 6.5, Figure 6.6 and Figure 6.7 respectively.

6.3.1 CASE 1: When Frequency, Phases, and amplitude of the sinusoids are same

```
clc;
clear all;
close all;
t=-2:0.01:2;
x1=cos(2*pi*0.5*t);
x2=cos(2*pi*0.5*t);
x3=x1+x2;

subplot(3,1,1);
plot(t,x1,'linewidth',3); grid;
ylabel('Amplitude'); xlabel('Time');
```

```

title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase = 0RADIAN');
subplot(3,1,2);
plot(t,x2,'linewidth',3);
grid;
ylabel('Amplitude');
xlabel('Time');
title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase= 0RADIAN');
subplot(3,1,3);
plot(t,x3,'linewidth',3);
grid;
ylabel('Amplitude');
xlabel('Time');
title('SUM OF THE ABOVE TWO COSINE SIGNALS');

```

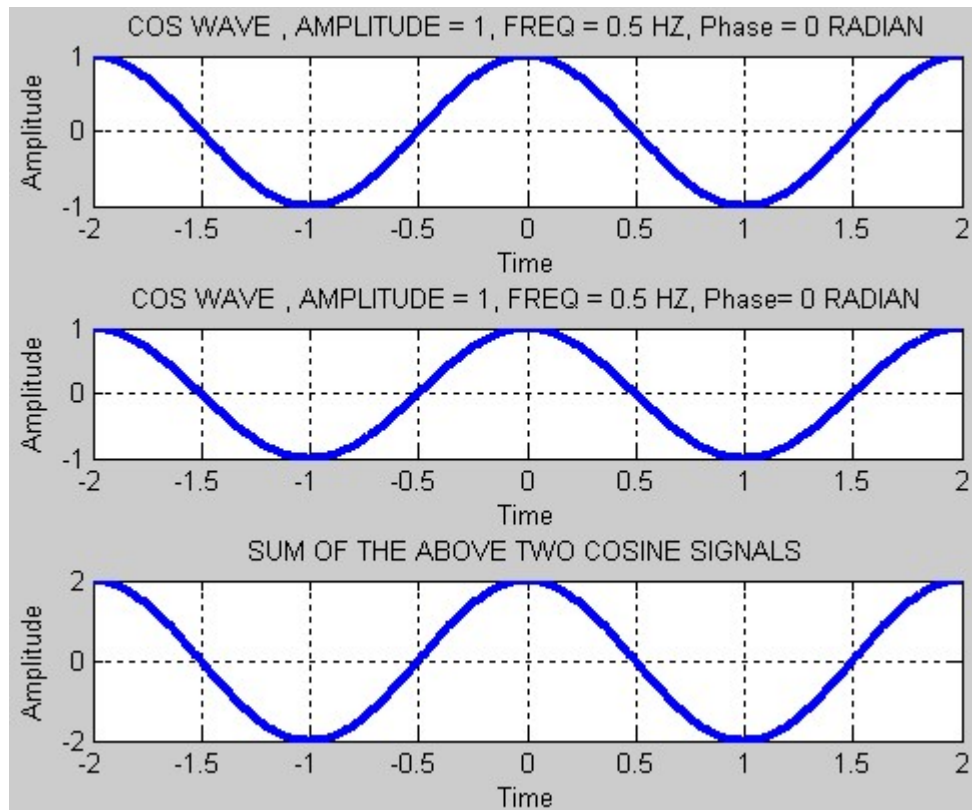


Figure 6.4 – Result of Case 1

6.3.2 CASE 2: When Frequencies and Phases of the sinusoids are same but Amplitudes are different.

```

t=-2:0.01:2;

```

```

x1=2*cos(2*pi*0.5*t);
x2=cos(2*pi*0.5*t);
x3=x1+x2;

subplot(3,1,1);
plot(t,x1,'linewidth',3); grid;
ylabel('Amplitude'); xlabel('Time');
title('COS WAVE , AMPLITUDE = 2, FREQ = 0.5 HZ, Phase = 0RADIAN');
subplot(3,1,2);
plot(t,x2,'linewidth',3); grid;
ylabel('Amplitude'); xlabel('Time');
title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase= 0RADIAN');
subplot(3,1,3);
plot(t,x3,'linewidth',3); grid;
ylabel('Amplitude'); xlabel('Time');
title('SUM OF THE ABOVE TWO COSINE SIGNALS');

```

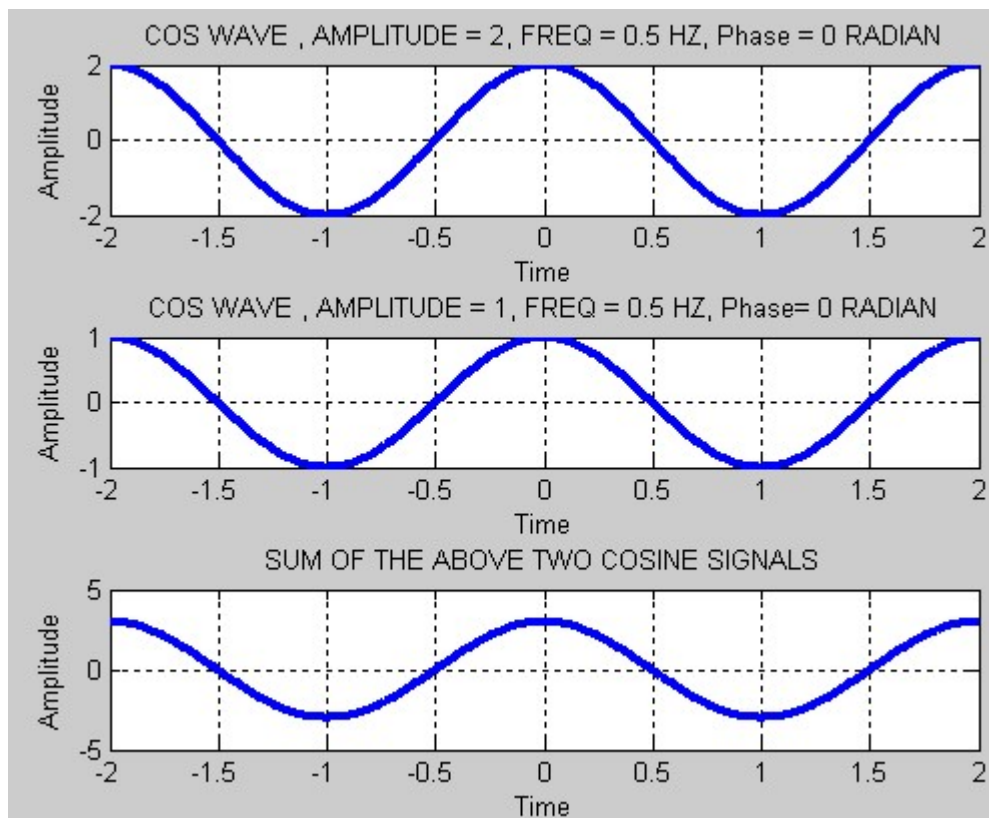


Figure 6.5 – Result of Case 2

6.3.3 **CASE 3:** When Amplitudes and Phases of the sinusoids are the same but Frequencies are different.

```
t=-2:0.01:2;  
x1=cos(2*pi*0.5*t);  
x2=cos(2*pi*1*t);  
x3=x1+x2;  
  
subplot(3,1,1);  
plot(t,x1,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase = 0RADIAN');  
subplot(3,1,2);  
plot(t,x2,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title('COS WAVE , AMPLITUDE = 1, FREQ = 1 HZ, Phase = 0RADIAN');  
subplot(3,1,3);  
plot(t,x3,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title('SUM OF THE ABOVE TWO COSINE SIGNALS');
```

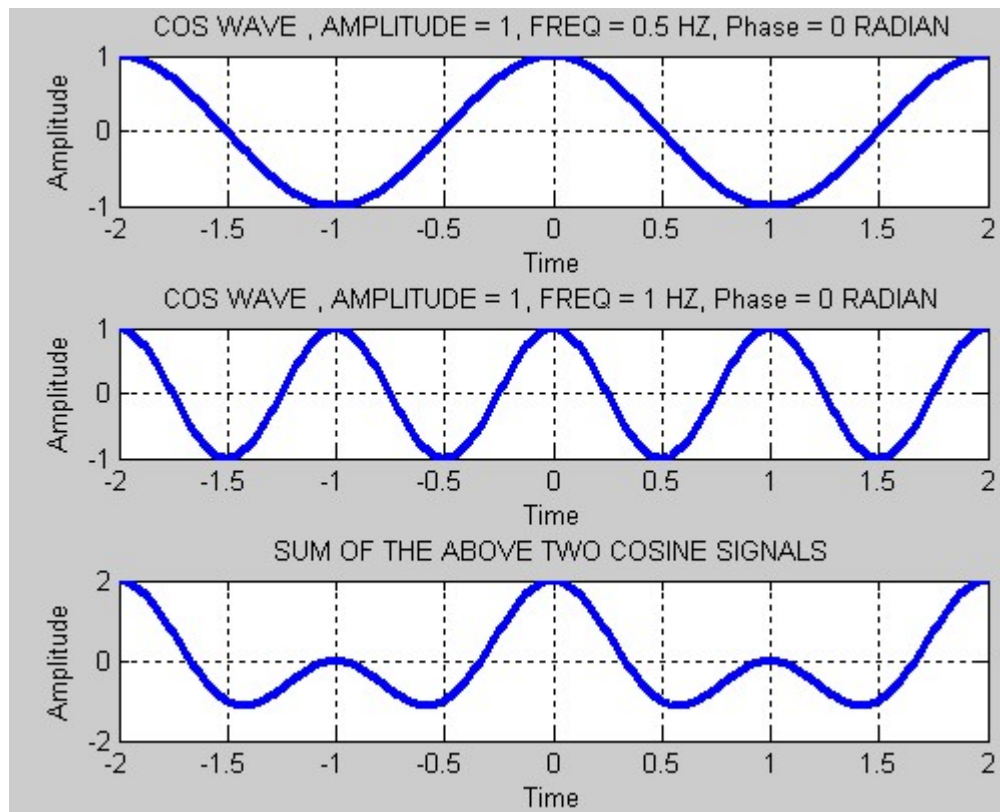


Figure 6.6 – Result of Case 3

6.3.4 **CASE 4: When Amplitudes and Frequencies of the sinusoids are the same but Phases are different**

```
t=-2:0.01:2;  
x1=cos(2*pi*0.5*t);  
x2=cos((2*pi*0.5*t)+1);  
x3=x1+x2; subplot(3,1,1);  
plot(t,x1,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase = 0RADIAN');  
subplot(3,1,2);  
plot(t,x2,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title('COS WAVE , AMPLITUDE = 1, FREQ = 0.5 HZ, Phase = 1RADIAN');  
subplot(3,1,3);  
plot(t,x3,'linewidth',3); grid;  
ylabel('Amplitude'); xlabel('Time');  
title(' SUM OF THE ABOVE TWO COSINE SIGNALS ');
```

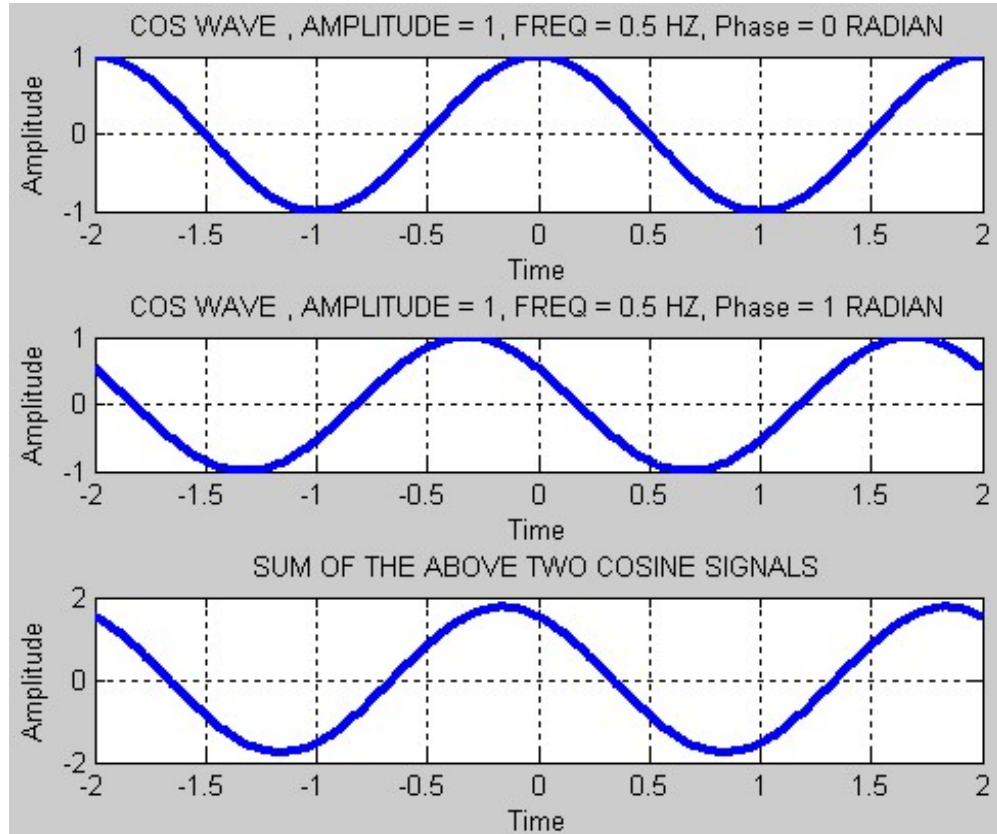


Figure 6.7 – Result of Case 4

Important note about Case 1:

When two sinusoids with the same frequency, phase, and amplitude are added together, the resulting waveform is a sinusoid with the same frequency and phase as the original waveforms, and with an amplitude that is twice that of the original waveforms. This can be seen in Figure 6.4 where the amplitude of resultant sinusoid is varying between -2 and 2.

Mathematically, if we have two sinusoids with the same frequency, phase, and amplitude given by:

$$y_1 = A \sin(\omega t + \theta)$$

$$y_2 = A \sin(\omega t + \theta)$$

where A is the amplitude, ω is the angular frequency, t is the time, and θ is the phase angle, then the sum of these two waveforms is given by:

$$y = y_1 + y_2 = A \sin(\omega t + \theta) + A \sin(\omega t + \theta)$$

Using the trigonometric identity for the sum of two sines, we can simplify this expression to:

$$y = 2A \sin(\omega t + \theta)$$

This shows that the resulting waveform has the same frequency and phase as the original waveforms, but the amplitude is doubled.

Important note about Case 2:

When two sinusoids with the same frequency and phase but different amplitudes are added together, the resulting waveform will have the same frequency and phase as the original waveforms, but its amplitude will be the sum of the amplitudes of the two original waveforms. This is known as constructive interference, where the waves reinforce each other and add up to create a larger amplitude wave. This can be seen in Figure 6.5, where the amplitude of resultant sinusoid is varying between -3 and 3, whereas the amplitude of first sinusoid is varying between -2 and 2 and the second sinusoid is varying between -1 and 1.

Important note about Case 3:

When two sinusoids with the same amplitude and phase but different frequencies are added together, the resulting waveform is a "beat" waveform. This can be seen in Figure 6.6. The frequency of the beat waveform is equal to the absolute value of the difference between the frequencies of the two original sinusoids. The amplitude of the beat waveform varies with time, oscillating between a maximum and minimum value. This effect is known as beats and is used in tuning musical instruments. For example, when a tuning fork of a specific frequency is struck, the sound waves it generates can be compared to a sound wave generated by an instrument to determine whether the instrument is in tune or not.

Mathematically, if we have two sinusoids with same amplitude A, phase phi, and frequencies f1 and f2, the resulting waveform can be expressed as:

$$y(t) = A\cos(2\pi f_1 t + \phi) + A\cos(2\pi f_2 t + \phi)$$

Using the trigonometric identity: $\cos(a) + \cos(b) = 2\cos((a+b)/2)\cos((a-b)/2)$, $y(t)$ can be simplified into following expression:

$$y(t) = 2A\cos(2\pi((f_1+f_2)/2)t + \phi)\cos(2\pi((f_1-f_2)/2)t)$$

The first term in this expression represents a sinusoid with frequency $(f_1+f_2)/2$ and amplitude $2A\cos(\phi)$, which is the average frequency of the two original signals. The second term represents a sinusoid with frequency $(f_1-f_2)/2$ and amplitude $A\cos(\phi)$, which is the beating pattern with a frequency equal to the difference between the frequencies of the two original signals. The beating pattern is a cosine wave that oscillates between positive and negative values at a rate determined by the frequency difference.

Important note about Case 4:

When two sinusoids with the same amplitude and frequency but different phases are added together, the result is a new sinusoid with the same frequency and amplitude but with a phase that is the average of the two original phases. This can be seen in Figure 6.7.

Mathematically, if we have two sinusoids with the same amplitude A and frequency f, but with different initial phases ϕ_1 and ϕ_2 , then the sum of the two sinusoids is:

$$y(t) = A\cos(2\pi f t + \phi_1) + A\cos(2\pi f t + \phi_2)$$

Using the trigonometric identity for the sum of two cosines, we get:

$$y(t) = 2A\cos((2\pi f t + \phi_1 + 2\pi f t + \phi_2)/2)\cos((2\pi f t + \phi_1 - 2\pi f t - \phi_2)/2)$$

Simplifying the argument of the first cosine and the difference of the arguments of the two cosines, we get:

$$y(t) = 2A\cos(2\pi f t + (\phi_1 + \phi_2)/2)*\cos((\phi_1 - \phi_2)/2)$$

Thus, the resulting waveform is a sinusoid with the same frequency f and amplitude $2A*\cos((\phi_1 - \phi_2)/2)$, but with a phase that is the average of the two original phases (i.e., $(\phi_1 + \phi_2)/2$).

-----TASK 06-----

Write a general program that takes 'n' sinusoids from user of same frequency, amplitude, and phase. Plot the individual sinusoids & the resultant using subplot function on same figure. Do perform proper labeling. Note: Take the amplitude, frequency, and phase given in example of case 1. Run the code for different values of n and state the result on paper.

-----TASK 07-----

Write a general program that takes 'n' sinusoids from user of same frequency and phase with varying amplitudes. Take amplitude from user on run time. Plot the individual sinusoids & the resultant using subplot function on same figure. Do perform proper labeling. Note: Take the amplitude and frequency given in example of case 2. Run the code for different values of n and state the result on paper.

-----TASK 08-----

Write a general program that takes 'n' sinusoids from user of same amplitude and phase with varying frequencies. Take each frequency from user on run time. Plot the individual sinusoids & the resultant using subplot function on same figure. Do perform proper labeling. Note: Take the amplitude and phase given in example of case 3. Run the code for different values of n and state the result on paper.

-----TASK 09-----

Write a general program that takes 'n' sinusoids from user of same amplitude and frequency with varying phases. Take each phase from user on run time. Plot the individual sinusoids & the resultant using subplot function on same figure. Do perform proper labeling. Note: Take the amplitude and frequency given in example of case 4. Run the code for different values of n and state the result on paper.