Day: Tuesday. Date: 22. March. 2022.

stocastic, deterministic, mathematical, simulation and emulation model.

Probability and random process for electrical engineering.

yu2iscp.

Frequency: No. of occurance, How many times one thing is repeated.

Relative frequency:

$$f_R(N) = \frac{N_*(n)}{n}$$

frequency.

Total no. of possible outcomes.

Example 1:

Toss a coin. Relative F of Head.

Example 2:

Roll a dice. Relative F of number 3.

Example 3:

Number of students from 1 to 10

∴ possible outcome = sample space.

Example 1:

Sample Space $S = \{H, T\}$.

Head is one time in S.S and total 2.

So: $F_R(N) = \frac{1}{2}$

Scanned with CamScanner

Example 2:

Sample Space: $\{1, 2, 3, 4, 5, 6\} = S$.

$f_k(N) = \frac{1}{6}$

→ Any ~~Relative~~ frequency is:

$0 \leq N_k(n) \leq n$.

R.F can not be zero. It is zero meaning it cannot occur.

→ For Relative frequency:

$$\frac{0}{n} \leq \frac{N_k(n)}{n} \leq \frac{n}{n}$$

$$0 \leq \frac{N_k(n)}{n} \leq 1.$$

So in between 0 & 1.

Example 1. Slide:

$n = 40$.

No. 4 6 time aya:

Frequency $= 6$.

Relative frequency $= \frac{6}{40}$ = decimal.

Example 2

$n = 20$

heads frequency $= 15$.

Relative Frequency $= \frac{15}{20} = \boxed{0.75}$

if Tails than $\frac{5}{20} = \boxed{0.25}$

Scanned with CamScanner

Example. Travel Survey:

$n = 92$.

Cars: $R.F = \frac{35}{92} = 0.380$

Public: $R.F = F_R(M) = \frac{42}{92} = 0.456$

Bicycle: $F_R(M) = \frac{8}{92} = 0.0869$

Walk: $F_R(M) = \frac{7}{92} = 0.076$.

Sum of all Relative Frequency is 1.

Example 2:

A) = Yellow & Red so Yellow + Red.

$6+6 = 12$.

So, $F_R(M) = \frac{12}{20} = \boxed{0.6}$

B) $\frac{5}{20} = \boxed{0.25}$

Scanned with CamScanner

Day: Monday. Date: 28.March.2022.

Axioms of Property:

P = Probability.

① $0 \leq P[A] \leq 1$

② $P[S] = 1$.

③ $P(A \cup B) = P(A) + P(B)$.

→ if no common.

→ if common then.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Example:

$S = \{1, 2, 3, 4, 5, 6\}$.

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$.

$P[A] = 4/6$, $P[B] = 4/6$.

$P[A \cap B] = 2/6$.

So $P[A] + P[B] - P[A \cap B]$.

$\frac{4}{6} + \frac{4}{6} - \frac{2}{6} \Rightarrow \frac{4+4-2}{6} \Rightarrow \frac{6}{6} = 1$.

Problem 1:

part a) $S_1 = \{Head, Tails\}$.

$S_2 = \{1, 2, 3, 4, 5, 6\}$.

$S_3 = \{0 \ldots 9\}$.

part b:

(i) $f_H = 1/2$, $f_T = 1/2$.

(ii) $f_1 = 1/6$, $f_2 = 1/6 \ldots\ldots$ $f_6 = 1/6$.

(iii) $f_0 = 1/10$, $f_2 = 1/10 \ldots$ $f_9 = 1/10$.

Scanned with CamScanner

Problem 2:

(a) $S_1 = \{ (H,H), (H,T), (T,H), (T,T) \}$.

(b) $S_2 = \{ (1,1), (1,2), (1,3) (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4) (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5) (6,6) \}$.

(c) $S_3 = \{ (1,1) \ldots\ldots\ldots (1,52),$
$\vdots$
$(52,1) \ldots\ldots (52,52) \}$.

With replacement ↑

Without replacement ↓

$S_3 = \{ (1,1) \ldots\ldots (1,52)$
$(2,2) \ldots\ldots$
$(3,3)$
$\vdots$
$(52,1) \ldots\ldots (52,52) \}$

Problem 3:

$S = \{1, 2, 3, 4\}$.

$P_1 = 1/2, \quad P_2 = 1/4, \quad P_3 = 1/8, \quad P_4 = 1/8$.

a) $S = \{$

H	H	H	
H	H	T	
H	T	H	
H	T	T	
T	H	H	
T	H	T	
T	T	H	
T	T	T	

$2/8 \to 1/4$

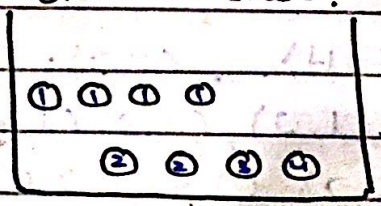
$4/8 \to 1/2$

$1/8$

$1/8$

Scanned with CamScanner

(b) Suppose a bucket:



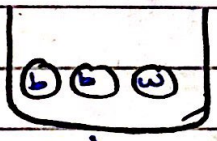
P of 1 = 4/8 = 1/2.
P of 2 = 2/8 = 1/4.
P of 3 = 1/8
P of 4 = 1/8.

(c)

Ignore 4 cards then:

$S_1 = (1-24) \rightarrow P_1 = 24/46 = 1/2.$
$S_2 = (25-36) \rightarrow P_2 = 12/48 = 1/4.$
$S_3 = (37-42) \rightarrow P_3 = 1/8.$
$S_4 = (43-46) \rightarrow P_4 = 1/8.$

Problem 4:



(a)

$S = \{(b,b), (b,w), (w,b)\}$ ∴ without replacement.

$S = \{(b,b), (b,w), (w,b), (w,w)\}$ ∴ with replacement

Suppose: b_1, b_2, w.

then: $S = \{b_1, b_1, b_2, b_2, b_1, b_2, b_2, b_1, b_1, w, b_2, w, w, b_1, w, b_2, w, w\}$ ⟷ $\{b, b, b, w, w, b, w, w\}$

(c) in a '0'
in b 1/9.

Problem 5:

As: $\frac{n_A(K)}{n} + \frac{n_B(K)}{n} = \frac{n}{n}$

$f_A(K) + f_B(K) = 1.$

So, $f_B(K) = 1 - f_A(K)$:

Problem 7:

a) $S = \{1, 2, 3, 4 \ldots\}$.

(b) $S = \{R, B, G, W\}$.

then $\{$ RB
RG
RW $\}$

Scanned with CamScanner

Day: Tuesday Date: 29.Mar.2022.

Lecture 2: Slide 9:

Problem:

Lisa: label reader works fine.
Homer: label reader stuck at most significant 1.
Bart: " " " " least " 0.

Sample Spaces:

$S_L = \{00, 01, 10\}$.
$S_H = \{10, 11\}$.
$S_B = \{00, 10\}$

Relative frequency:

① $P_{L\,00} = 1/3$, $P_{L\,01} = 1/3$, $P_{L\,10} = 1/3$.

② $P_{H\,10} = 2/3$ as if 00 it'll read 10 as 1 is stuck.
$P_{H\,11} = 1/3$. as if 01 it'll read 11.

③ $P_{B\,00} = 2/3$ as if 00 or 01, it'll read 00.
$P_{B\,10} = 1/3$

Slide 14:

Problem:

ⓐ $S = \{2, 3, 4, 5 \ldots\ldots\}$.

ⓑ RBGW, Two distinct So;

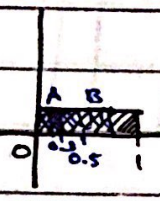
$S = \{$ RB
RG
RW
BR
BG
BW
GR
GB
GW
WR
WB
WG $\}$

∴ record order matters thats why RG & GR not same. if order didn't matter then.

(c) as 20 mins, in hours $20/60 = 1/3$.

So $S = \{0, 1/3\}$.

As



$S = \{0, 1\}$.

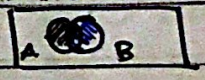
$A = \{0, 0.3\}$, $B = \{0.2, 0.7\}$

Lecture 3 Slide 5

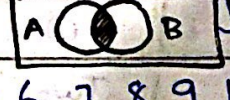
$U = \{1,2,3,4,5,6,7,8,9,10\}$

~~(a) $A \cup B$~~ $A = \{1,2,3,4,5\}$ $B = \{4,5,6,7,8\}$.

(a) $A \cup B = \{1,2,3,4,5,6,7,8\}$.

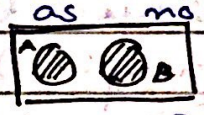


(b) $A \cap B = \{4,5\}$.



(c) $A^c = A - U = \{6,7,8,9,10\}$.

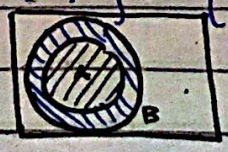
(d) $A \cap B = \emptyset$ as no intersection then of c zero/null set.



(f) $A - B = \{1,2,3,4,5\} - \{4,5,6,7,8\} = \{1,2,3\}$.

(e) Let $B = \{1,2,3,4,5\}$

$A = \{1,2,3\}$.

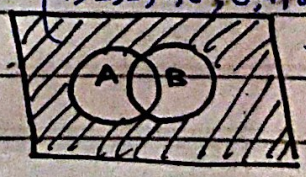


So $A \subset B$. A is a subset of B.

(g) $A \cup B = \{1,2,3,4,5,6,7,8\}$

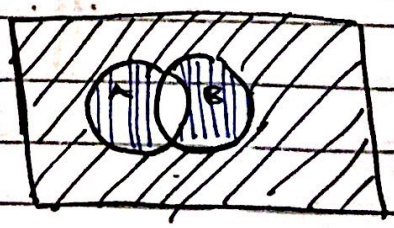
$(A \cup B)^c = U - (A \cup B) = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8\}$.

$(A \cup B)^c = \{9,10\}$.



(h) $A^c = \{6,7,8,9,10\}$, $B^c = \{1,2,3,9,10\}$.

$A^c \cap B^c = \{9,10\}$.



Scanned with CamScanner

Day: Monday Date: 4.April.2022.

Stochastic Process
Random Process.
Probability
Set theory

Set Theory:

- Universal Set (all elements)
- Subset
- Single output is Element

Probability

- Sample Space (Total outcomes)
- Event.
- Single output is Outcome.
- Mutual Exclusive:

One outcome at a time.
Two events can't occur at same time.
Example: flip a coin, it can show head or tail not both.

- Independant Event:

The occurance of one event does not effect the occurance of other.
Example: Let suppose, marker in Lab 2.

Scanned with CamScanner

- **Axioms:**

$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}^{0}$ (if Mutual Exclusive).

Example 1

A

$A = \{1, 3, 5, 7, 9\}$.

$P[A] = 5/10$.

B $B = \{3, 6, 9\}$.

$P[B] = 3/10$.

C $C = \{0, 1, 2, 3, 4\}$.

$P(C) = 5/10 = 1/2$.

D $D = A \cup B$. $D = \{1, 3, 5, 6, 7, 9\}$.

$P(D) = 5/10 + 3/10 - 2/10$.

$= 8/10 - 2/10$

$= 6/10$

E $E = A \cup B \cup C$. $E = \{0, 1, 2, 3, 4, 5, 6, 7, 9\}$.

$P(E] = 5/10 + 3/10 + 5/10 - 2/10 - 1/10 - 2/10 + 1/10$

$= 14/10 - 2/10 - 1/10 - 2/10$

$P[E] = 9/10$.

Scanned with CamScanner

Day: Tuesday Date: 5 April 2022.

Lecture 3

Slide 9

$P_1 = 1/2$. pehli dafa.

$P_2 = 1/4$. doosri dafa.

$P_3 = 1/8$ teesri dafa.

$P_4 = \frac{1}{16}$ chohti dafa.

So: $P_2 = (1/2)^2$, $P_3 = (1/2)^3$, $P_4 = (1/2)^4$.

So: $\sum_{j=1}^{\infty} P_j = \{ 1/2 + (1/2)^2 + (1/2)^3 + (1/2)^4 + \dots \}$.

if $\alpha = 1/2$.

then:

$$\sum_{j=1}^{\infty} P_j = \{ \alpha + \alpha^2 + \alpha^3 + \alpha^4 \dots \}.$$

α common.

$$\sum_{j=1}^{\infty} P_j = \alpha \{ 1 + \underbrace{\alpha + \alpha^2 + \alpha^3 \dots}_{\sum_{j=1}^{\infty} P_j} \}.$$

So:

$$\sum_{j=1}^{\infty} P_j = \alpha \{ 1 + \sum_{j=1}^{\infty} P_j \}.$$

$$\sum_{j=1}^{\infty} P_j = \alpha + \alpha \sum_{j=1}^{\infty} P_j$$

$$\sum_{j=1}^{\infty} P_j - \alpha \sum_{j=1}^{\infty} P_j = \alpha$$

$$\sum_{j=1}^{\infty} P_j (1 - \alpha) = \alpha$$

Scanned with CamScanner

$$\boxed{\sum_{j=1}^{\infty} P_j = \frac{\alpha}{1-\alpha}}$$

if $\alpha < 1$

$$\boxed{\sum_{j=1}^{\infty} P_j = 1}$$

$$\frac{1/2}{1-1/2} = \frac{1/2}{1/2} = 1$$

$$\frac{1/2}{2-1/2} = \frac{1/2}{1/2} = 1$$

$\boxed{= 1}$ Proved

Question:

A fair dice is ~~tossed~~ rolled multiple times until the first 'three' numbers shows up. The outcome of the trial is basically the number of rolling dice until we get first three. Using the propability law, Solve this experiment.

So, $P_1 = 1/6$, $P_2 = 1/12$, $P_3 = 1/18$, $P_4 = 1/24$

~~Example~~

So, $P_1 = \frac{1}{6}$, $P_2 = \frac{1}{6 \times 2^{(2)}}$, $P_3 = \frac{1}{6 \times 2^{(3)}}$

Scanned with CamScanner

$$\sum_{j=1}^{\infty} P_j(1-\alpha) = \alpha$$

$$\sum_{j=1}^{\infty} P_j = \frac{\alpha}{1-\alpha}$$

$\therefore \alpha < 1.$

$$\boxed{\sum_{j=1}^{\infty} P_j = 1}$$

Slide 10

So, $S = [0, 1]$ $\therefore$ continuous.

Part A) Another interval $A = [0, 0.5]$.

$P[A] = 1/2.$

$$P[A] = \frac{\text{Final point of interval } A - \text{Initial point of interval } A}{\text{Final point of sample space} - \text{initial point of sample space}}.$$

$$= \frac{0.5 - 0}{1 - 0}$$

$P[A] = 0.5$

part B

$A = [0.5, 1].$

$$P[A] = \frac{1 - 0.5}{1 - 0}$$

$$= \frac{0.5}{1}$$

$P[A] = 0.5$

Scanned with CamScanner

part c:

outcome is 0.5

as sample space is infinite.

So $P[A] = \frac{1}{\infty} = 0$.

also can prove from previous formula.

Scanned with CamScanner

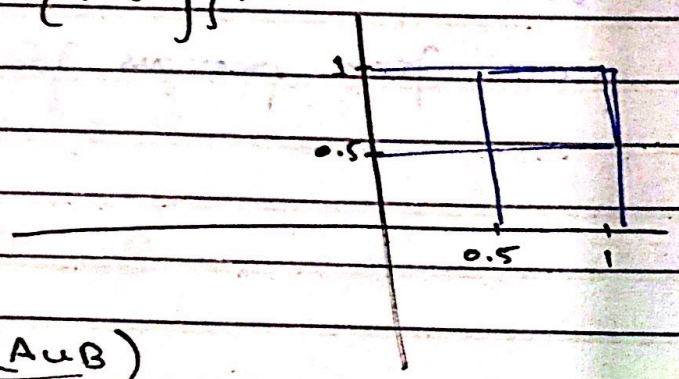
Day: Monday Date: 11 April 2022.

Quiz Question Solve:

$A = \{x \geq 0.5\}$.

$B = \{y \geq 0.5\}$.

$C = \{x > y\}$.



- Find $P(A \cup B)$

- $P(A \cup B) = P[A] + P[B] - P(A \cap B)$.

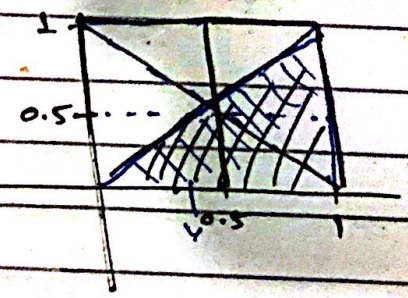
$$\frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$

$$= 1 - \frac{1}{4}$$

$$= \frac{4-1}{4}$$

$P[A \cup B] = 3/4$

- Find $P[A \cup C]$



$P[C] = 1/2$.

$P[A \cap C] = 3/8$

Scanned with CamScanner

Quiz Section A

Q) Let A, B & C be the events in Sample Space. Suppose $A \cup B \cup C = S$.
$P[A] = 1/2$, $P[B] = 2/3$, $P[A \cup B] = 5/6$.
Find $P[A \cap B] = ?$ $P[C - (A \cup B)] = ?$

$$P[A \cup B] = P[A] + P[B] - P(A \cap B).$$
$$5/6 = 1/2 + 2/3 - P(A \cap B).$$
$$5/6 = 3 + 2/6 - P[A \cap B].$$
$$\cancel{5/6}$$
$$P[A \cap B] = 1/2 + 2/3 - 5/6$$
$$P[A \cap B] = 1/3$$

② Find $P[C - (A \cup B)]$.

Assuming:

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}.$$
$$A = \{1, 2, 4\}, \quad B = \{2, 3, 4, 5\}.$$
$$A \cup B = \{1, 2, 3, 4, 5\}.$$
$$P[A] = 3/6 = 1/2, \quad P[B] = 4/6 = 2/3.$$
$$P[A \cup B] = 1/2 + 2/3 - 1/6.$$
$$C = \{6\}.$$

So $P[C] = 1/6$.

Now $P[C - (A \cup B)]$.

as $C = \{6\}$ & $A \cup B = \{1, 2, 3, 4, 5\}$.

then:

$$P[C - (A \cup B)] = [C - (A \cup B)] = \{6\} - \{1, 2, 3, 4, 5\}.$$
$$= 1/6$$

Scanned with CamScanner

Day: Tuesday. Date: 12. April. 2022.

Lecture 4

Counting Methods

① . Sampling with replacement with odering. (SWW)
② . " " " without " (SW w/o)
③ . " without " with " (S w/o w)
④ . " " " without " (S w/o w/o)

Example Dice.

① $n = 6$ (outputs) order matters.
$k = 2$ (no. of draws).

Formula: n^k

② order does not matter.
e.g (1, 2) & (2, 1) are same.

1. n^k
2. Permutation.
3. Combination

③ $^nP_k = \frac{n!}{(n-k)!}$

another formula: $n.(n-1)\ldots(n-k+1)$.

e.g $n = 6$ $k = 2$.
if $k = 2$, $n.(n-k+1)$
if $k = 3$, $n.(n-1)(n-k+1)$
if $k = 4$ $n.(n-1)(n-2)(n-k+1)$.
$6.5 = 30$ (possible combinations).

④

$^nC_k = \frac{n!}{(n-k)!\,k!}$

Scanned with CamScanner

Example 1: Sampling with R with O.

$S = \{1, 2, 3, 4, 5\}$.

$n = 5 \qquad K = 2$.

(1) $n^K \qquad 5^2 = 25$ (Sample Space).

(2) $P = \frac{5}{25}$ (Event)

Event -> (1,1) (2,2) (3,3) (4,4) (5,5)

-> (1) Elements of Sample Space = 25.

(2) $P[E] = \frac{5}{25}$ $\quad \therefore E$ = Event.

Example 2: Sampling w/o R w O.

$n = 5 \qquad K = 2$.

So $n.(n-K+1)$

$5.(5-2+1)$

$5.4 = 20$

$^5P_2 = \frac{5!}{3!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$

$= 20$.

① So Elements of S.S = 20.

② $E = \{(2,1)\ (3,1)\ (4,1)\ (5,1)\ (3,2)\ (4,2)\ (5,2)$
$(4,3)\ (5,3)\ (5,4)\}$

$P[E] = \frac{10}{20}$

Example 3: (S w w)

(1) $S.S = n^K \qquad \therefore n = 5 \quad K = 3$.

$= 5^3 \Rightarrow 125$

(2) Now event is converted to without R w O.

$E = {}^5P_3 = \frac{5!}{2!} \rightarrow \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$

$= 5 \times 4 \times 3 = 60$

$P[E] = 60/125$.

Scanned with CamScanner

Permutations:

Combinations:

Example 4:

(Q1) $n = 3$

(1) $K = 3$

(2) $K = 2$.

(1) $\frac{n}{(n-K)!} = \frac{3!}{3-3} = \frac{3!}{0} \quad (1) \frac{3!}{1!} = 3! = 6$

or $n.(n-1)(n-K+1) \Rightarrow 3(2)(1) \Rightarrow \boxed{6.} \ (2)$

(Q2) $n = 3$.

$K = 3$. $\quad {}^3C_3 = \frac{3!}{0!\,3!} = 1$.

$K = 2$.

${}^3C_2 = \frac{3!}{1!\,2!} = \frac{3 \times 2!}{2!} = 3$.

Example 5:

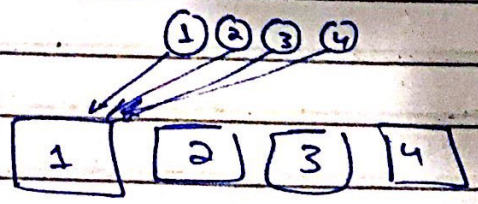
balls $= 4$ cells $= 4$

$n^K = 4^4 = 216$.

$E = {}^4P_4 = \frac{4!}{0!} = 4 \times 3 \times 2 = 24$.

$P(E) = \frac{24}{216}$

1 2 3 4
O O O O
O O O O
O O O O
O O O O



Scanned with CamScanner

Day: Monday. Date: 18.April.2022.

Lecture 5:

Slide 3:

- Sampling w/o r. w/o ordering

(1) $n=6$, $k=2$.

$$^nC_k \Rightarrow {}^6C_2 = \frac{n!}{(n-k)!\,k!}$$

$$= \frac{6!}{4!\,2!}$$

$$= \frac{6\times5\times\cancel{4!}}{\cancel{4!}\,2!}$$

$$= \frac{6\times5}{2}$$

$$= 30/2$$

$$\boxed{{}^6C_2 = 15}$$

(2) Permutation.

$n=6$, $k=2$.

$$^6P_2 = \frac{6!}{4!}$$

$$= \frac{6\times5\times\cancel{4!}}{\cancel{4!}}$$

$$\boxed{{}^6P_2 = 30}$$

Scanned with CamScanner

Question :

In how many ways can 7 different books be arranged on a shelf?

$n = 7$, $K = 7$.

$$^7P_7 = \frac{7!}{(7-7)!}$$

$$= \frac{7!}{0!}$$

$$\boxed{^7P_7 = 7!}$$

Slide 4:

① ② ③ ④

$n = 4$, $K = 4$ distinct Kana.

① $^4P_4 = \frac{4!}{(4-4)!} = 4! = 24.$

② 2 black & 2 white balls.

$b_1\ b_2\ w_1\ w_2$

bb ww

2.

it is permutation but it'll be solved using combination as

b b w w

& b b w w is same as compared to

$b_1\ b_2\ w_1\ w_2$

$b_2\ b_1\ w_1\ w_2$

So

$$^nC_K = {}^4C_2 = \frac{4!}{(4-2)!\,2!}$$

$$\frac{4!}{2!\,2!} = \frac{4!}{2!\,2!} = \frac{4\times3\times2!}{2!\,2!} = \frac{4\times3}{2} = \frac{12}{2} = 6$$

Scanned with CamScanner

③ $n = 5, \quad K = 2.$

$$^5C_2 = \frac{5!}{(5-2)!\,2!}$$

$$= \frac{5!}{3!\,2!}$$

$$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!}$$

$$= 5 \times 4$$

$$\boxed{^5C_2 = 20/2 = 10}$$

if $K = 3.$

$n = 5, \; K = 3.$

$$^5C_3 = \frac{5!}{(5-3)!\,3!}$$

$$= \frac{5!}{2!\,3!}$$

$$= \frac{5 \times 4 \times \cancel{3!}}{2!\,\cancel{3!}}$$

$$= \frac{20}{2}$$

$$\boxed{^5C_3 = 10}$$

Scanned with CamScanner

AC working.

$$\frac{\binom{3}{2}\binom{5}{2}}{\binom{8}{4}}$$

$\binom{3}{2}$ → AC working

$\binom{5}{2}$ → AC not working

$\binom{8}{4}$ ← Total

$$\frac{{}^3C_2 \cdot {}^5C_2}{{}^8C_4}$$

DCSE has 8 classrooms, in 5 AC works. If 4 C.R are selected what is the probability that 2 AC work & 2 not.

Slide 5

$$\frac{\binom{3}{2}\binom{3}{2}}{\binom{6}{4}}$$

$$\frac{{}^3C_2 \cdot {}^3C_2}{{}^6C_4}$$

Scanned with CamScanner

Slide 6 :

$$\frac{\binom{6}{3}\binom{4}{2}}{\binom{10}{5}}$$

$$\frac{{}^6C_3 \times {}^4C_2}{{}^{10}C_5}$$

$$\frac{\frac{6!}{3!\,3!} \times \frac{4!}{2!\,2!}}{\frac{10!}{5!\,5!}}$$

$$\frac{\frac{6\times5\times4\times3!}{3!\,3!} \times \frac{4\times3\times2!}{2!\,2!}}{\frac{10\times9\times8\times7\times6\times5!}{5!\,5!}}$$

$$\frac{120}{6} \cdot \frac{12}{2}$$

$$\frac{30240}{120}$$

$$\frac{20.6}{252}$$

$$\boxed{= 0.46875}$$

Scanned with CamScanner

Day: Tuesday. Date: 19.April.2022.

Lecture 5: Slide 7:

$$C_k^{n-1+k}$$ $\therefore k$ is greater than n

if k or $n-1$, Answer will be same.

Slide 8:

Example:

$n = 3.$ $K = 5.$

$$C_5^7 = \frac{7!}{(7-5!)\,5!}$$

$$= \frac{7!}{2!\;5!}$$

$$= \frac{7 \times 6 \times \cancel{5!}}{2!\;\cancel{5!}}$$

$$= \frac{42}{2}$$

$$\boxed{C_5^7 = 21}$$

$n-1+k.$

$3-1+5.$

$= 7$

if $n-1.$

$$C_2^{7!} = \frac{7!}{(7-2)!\,2!}$$

$$= \frac{7!}{5!\;\cancel{2}\times 2!}$$

$$= \frac{7 \times 6 \times \cancel{5!}}{\cancel{2!}\;\cancel{5!}}$$

$$= 7 \times 6/2!$$

$$\boxed{= 21}$$

$E = \{(1,1,1,1,1), (2,2,2,2,2)$
$(3,3,3,3,3)\}.$

So

$$\boxed{P[E] = 3/21}$$

probability of getting same ball in all draw.

Scanned with CamScanner

Lecture 6:

Conditional Probability:

When an event depend ~~occur~~ on another event.

Example: P of dice for 3 is 1/6.
but if dice can only take num less than 5 then P changes for 3. and is New 1/5.

B = occured. , A = wants to occur.

P[A|B] event depend on B.

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

∴ intersection of both ÷ by event occured.

& Slide 4

$S = \{(1,b), (2,b), (3,w), (4,w)\}$.

$A = \{(1,b), (2,b)\}$.

$B = \{(2,b), (4,w)\}$.

$C = \{(3.w), (4,w)\}$.

$P[A|B] = ?$ $\quad P[A|B] = \frac{P[A \cap B]}{P[B]}$.

$(A \cap B) = (2,b)$ only 1.

then $P[A \cap B] = 1/4$ as $S.S = 4$.

$P[B] = 2/4 = 1/2$

Scanned with CamScanner

put in formula:

$$\boxed{P[A \cap B] = \frac{1/4}{1/2} = 0.5}$$

Now $P[A|C] = ?$

$$P[A|C] = \frac{P[A \cap C]}{P[C]}$$

As $(A \cap C) = \{\ \}$ nothing.

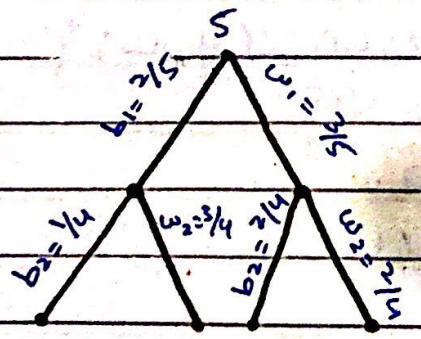
So $P(A \cap C) = 0$.

So $$\boxed{P[A|C] = \frac{0}{P(C)} = 0}$$

Slide 5:

$n = 5$, black $= 2$, white $= 3$.

So $P[b] = 2/5$ $P[w] = 3/5$.



$\therefore P[A \cap B] = P[A] \cdot P[B]$.
if both are independent.

So, $P[b_1] = 2/5$.

$P[b_2 | b_1] = 1/4$.

$P[b_1 \cap b_2] = ?$

Scanned with CamScanner

As we know that

$$P[b_2|b_1] = \frac{P(b_2 \cap b_1)}{P[b_2]}$$

$$P[b_2 \cap b_1] = P[b_2/b_1] \cdot P[b_1]$$

$$= \frac{1}{4} \cdot \frac{2}{5}$$

$$= \frac{2}{20}$$

$$P[b_2 \cap b_1] = \frac{1}{10}$$

Slide 6:

$P[0] = 1-p$

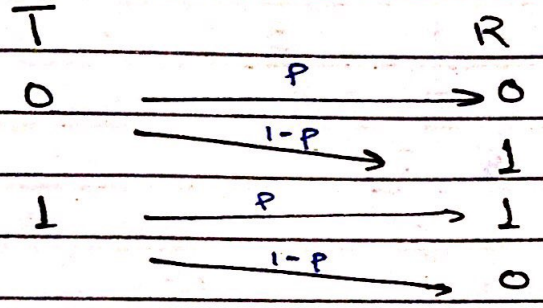
$P[1] = p$

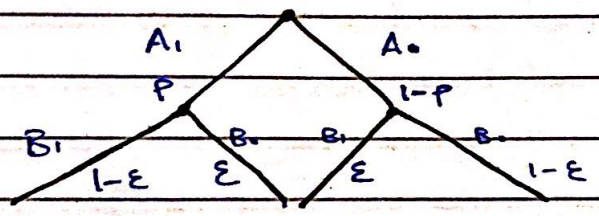
Scanned with CamScanner

Day: Monday Date: 25.April.2022.

Lecture 7:

Lecture 6 Slide 6:





Transmitter: $A \begin{cases} 0 \\ 1 \end{cases}$

Reciever: $B \begin{cases} 0 \\ 1 \end{cases}$.

$P[A_0 \cap B_0] \longrightarrow (1-\varepsilon)(1-p)$

$P[A_1 \cap B_1] \longrightarrow p(1-\varepsilon)$

$P[A_0 \cap B_1] \longrightarrow \varepsilon(1-p)$

$P[A_1 \cap B_0] \longrightarrow p\varepsilon$.

Formula:

$$P[B|A] = \frac{P[A \cap B]}{P[A]}$$

$P[A_0 \cap B_0] = P[B_0|A_0] \cdot P[A_0]$.

$= (1-\varepsilon).(1-p)$

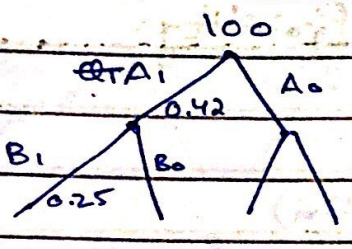
$P[A_1 \cap B_1] = P[B_1|A_1] \cdot P[A_1] \longrightarrow \boxed{p(1-\varepsilon)}$

$P[A_0 \cap B_1] = P[B_1|A_0] \cdot P[A_0] \longrightarrow \boxed{(1-p)\varepsilon}$

$P[A_1 \cap B_0] = P[B_0|A_1] \cdot P[A_1] \longrightarrow \boxed{\varepsilon p}$

Scanned with CamScanner

Lecture 6 Slide 7:



$A_1 = \text{Pass} = B_1$

$A_0 = \text{Fail} = B_0$

$$P[B_1 | A_1] = \frac{P[A_1 \cap B_1]}{P[A_1]}$$

$$= \frac{0.25}{0.42}$$

$$\boxed{P[B_1 | A_1] = 0.595}$$

Scanned with CamScanner

Day: Tuesday Date: 26.April.2022.

Lecture 7:

Theorem on Total Probility:

S → Sample Space.

$P(A \cap S) = P[A]$

$P[A] = A \cap (B_1 \cup B_2 \cup B_3 \cup \ldots B_n)$

$\Rightarrow P(A \cap B_1) \cup P(A \cap B_2) \cup P(A \cap B_2) \cup \ldots P(A \cap B_n)$.

$P[A] =$ So,

$$P(A \cap B_1) = P(A|B_1)\ P(B_1).$$

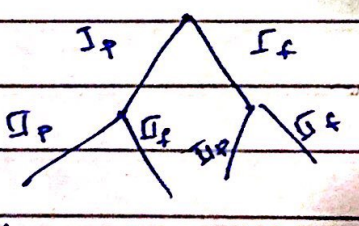
put this in (A)

$$P[A] = P(A|B_1)P(B_1) + P(A|B_2)\ P(B_2) \ldots\ldots P(A|B_n)\ P(B_n).$$

So

$$P[A] = \sum_{i=1}^{n} P(A|B_i)P(B_i).$$

Example:



I = first Year

II = Second Year

p = pass, f = fail.

We had ~~use~~ found $P(II_p|I_p)$

but if we want $P(II_p)$ only,

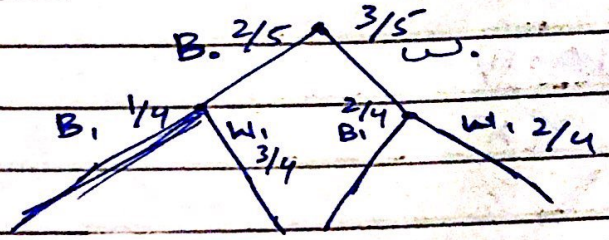
then this would formula.

$$P(II_p) = P(II_p|I_p).P(I_p) + P(II_p|I_f).P(I_f)$$

Scanned with CamScanner

Slide 4:

$P[W_1] = ?$



$$P[W_1] = \sum_{i=1} P(W$$

$$P[W_1] = P[W_1 | B_0].P[B_0] + P[W_1 | W_0].P[W_0].$$

$$= 3/4 . 2/5 + 3/5 . \cancel{3/5}\ 2/4.$$

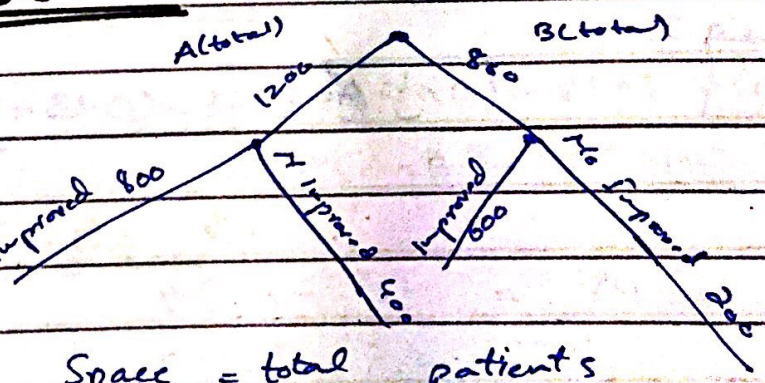
$$= 6/20 + 6/20$$

$$= \cancel{3/10} + 9/25 \qquad 12/20$$

$$= \cancel{6/20 + 9/25}$$

$\boxed{\cancel{P[W_1] = 0.60}}$

Slide 5:



Sample Space = total patients

= total patients of A + total patient of B

= 1200 + 800

Sam Space = 2000

$P[H|A] = ?$

$$P[H|A] = \frac{P[A \cap H]}{P[A]}$$

OR $(H \cap A)$

$[H|A] \cap [A] = 800$

So

$$\boxed{P[H|A] = \frac{800/2000}{1200/2000}}$$

$\therefore P[A \cap H] = 800/2000$

$\because P[A] = 1200/2000$

(3) $P[H] = ?$

$P[H] = P[H|A].P[A] + P[H|B].P[B].$

$= \frac{0.4}{0.6}(0.6)$

$= 0.4 + \frac{0.3}{0.4}(0.4$

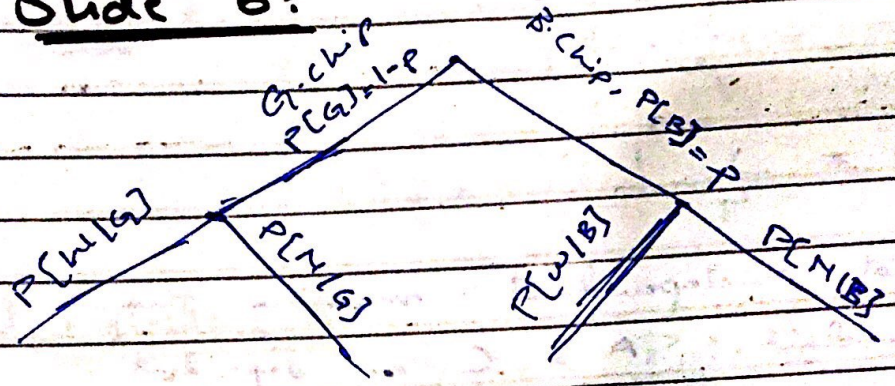
$P[H|B] = \frac{600/2000}{800/2000}$

$= 0.4 + 0.3$

$\boxed{P[H] = 0.7}$

$\boxed{\text{Ans is } 0.37}$

Slide 6:



$P[G] = 1-p$, $P[B] = p$.

$P[W|G] = e^{-\alpha t}$

$P[W|B] = e^{-1000\alpha t}$

$P[N] = ?$.

So $P[W] = P[W|G].P[G] + P[W|B] \cdot P[B]$.

$= e^{-\alpha t} \cdot 1-p + e^{-1000\alpha t} \cdot p$.

$= (1-p)e^{-\alpha t} + (p)e^{-1000\alpha t}$.

Lecture 9:

independent $\rightarrow$ $P[A \cap B] = P[A].P[B]$.

dependent $\rightarrow$ $P[A \cap B] = P[A|B].P[B]$

Slide 2:

$P[A] = 2/4 = 1/4 \cdot 2$.

$P[B] = 2/4 = 1/2$ $\quad \therefore A \cap B = 1$

$P[C] = 2/4 = 1/2$.

$P[A \cap B] = P[A].P[B]$. $\rightarrow$ Prove this.

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$ So A & B are independent.

Scanned with CamScanner

$P[A \cap C] = P[A] \cdot P[C]$.

$0 = \frac{1}{2} \cdot \frac{1}{2}$

$0 = \frac{1}{4}$

So not independant. meaning they are dependant. A & C are dependant.

Slide 4:

$P(A) = 1/2$, $P(\cancel{A}\,B) = 1/2$, $P(A \cap B) = 1/4$.

So $P[A \cap B] = P(A) \cdot P(B)$.

$1/4 = 1/2 \cdot 1/2$.

$1/4 = 1/4$ So independant.

$P(C) = 1/2$, $P[A \cap C] = 3/8$.

independant se ek question aur ga.

Scanned with CamScanner

Slide 5:

$$P[H \cdot HH] = P(H) \cdot P(H) \cdot P(H) \cdot$$

$$1/8 = 1/2 \cdot 1/2 \cdot 1/2 \cdot$$

$$\boxed{1/8 = 1/8 \cdot}$$

Lecture 8:

'Baye's Rule'

We know that:

$$P[A|B] = \frac{P(A \cap B)}{P[B]} \quad \text{or} \quad P[B|A] = \frac{P(A \cap B)}{P[A]}$$

$$P(A \cap B) = P[A|B]\,P[B] \qquad P(A \cap B) = P[B|A] \cdot P[A] \cdot$$

$$\boxed{P[A|B] \cdot P[B] = P[B|A]\,P[A] \cdot}$$

as $P(A \cap B) = P(A \cap B)$

So,

$$\boxed{P[B|A] = \frac{P[A|B] \cdot P[B]}{P[A]}} \quad \text{— (i)}$$

While using total theorem:

$$P[A] = \sum_{i=1}^{n} P[A|B_i]\,P[B_i] \cdot$$

put in eq (i)

So eq (i) becomes:

$$P[B|A] = \frac{P[A|B] \cdot P[B]}{\sum_{i=1}^{n} P[A|B_i]\,P[B_i]}$$

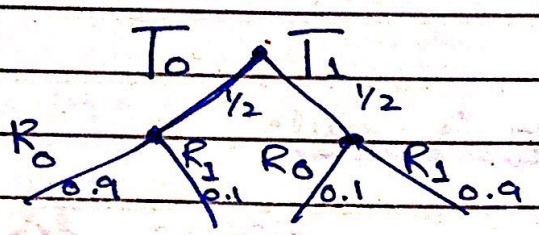
Scanned with CamScanner

Slide 3:

output = 1 (Given)

input = ?

Assume input $\rightarrow \begin{cases} 0 \\ 1 \end{cases}$



0.1 = error.

no error = 1 − 0.1

= 0.9.

(1) $P[T_0 | R_1] = \dfrac{P[R_1 | T_0] \cdot P[T_0]}{P[R_1]}$ —(A)

$\cancel{P[T_1 | R_1]}$

$= \cancel{0.1 \cdot 1/2}$

$\cancel{P[R_1 | T_0] = \dfrac{P(R_1 \cap T_0)}{P(T_0)}}$

$P[R_1] = ?$ use total Pro... wala form

$P[R_1] = P[R_1 | T_0]\, P[T_0] + P[R_1 | T_1] \cdot P[T_1]$.

$= 0.1 \cdot 0.5 + 0.9 \cdot 0.5$

$= 0.05 + 0.45$

$\boxed{P[R_1] = 0.5}$ put in (A)

$P[T_0 | R_1] = \dfrac{0.1 \cdot 0.5}{0.5}$

$\boxed{P[T_0 | R_1] = 0.1}$

Scanned with CamScanner

(2) $P[T_1|R_1] = \dfrac{P[R_1|T_1] \cdot P[T_1]}{P[R_1]}$

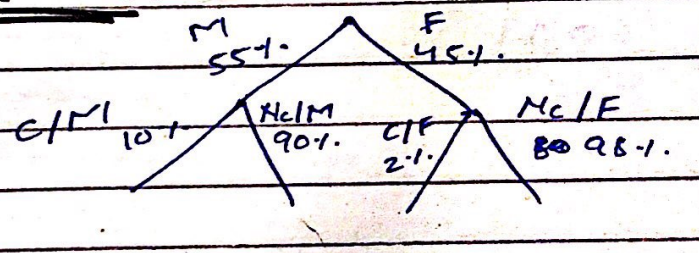
$= \dfrac{0.9 \cdot (0.5)}{0.5}$

$\boxed{P[T_1|R_1] = 0.9}$

So $P[T_0|R_1]$ is less while $P[T_1|R_1]$ is greater.

So $[T_1|R_1]$ is more probable.

Slide 4:



$\therefore 55\% = 0.55$

$45\% = 0.45$

$10\% = 0.10$

$90\% = 0.90$

$2\% = 0.02$

$98\% = 0.98$

$P[C/M] = ?$

~~P[C/M]~~ =

$P[\cancel{C/M}\ M|C/M] = \dfrac{P[C/M|M] \cdot P[M]}{P[C/M]}$

$= \dfrac{0.1 \cdot 0.55}{0.1}$

$P[C/M] = \cancel{P[C/M|M]\ P[M]}$

$= \cancel{0.1(0.55) + 0.2(0.45)}$

$\cancel{P[M|C/M] = 0.55}$

$= 0.145$

$= 0.1 \quad \dfrac{0.1 \cdot 0.55}{0.145}$

$\cancel{0.1 \times 0.55 + 0.02(0.45) = 0.054}$

$P[F|Nc/F] = ?$

$0.1 \times 0.55 + 0.9 \times 0.55$

$= 0.55$

is continued on next page:

Scanned with CamScanner

$P[M|C/M] = ?$

$$P[M|C/M] = \frac{P[C/M|M] \cdot P[M]}{P[C/M]}$$

$P[C/M] = ?$

Total probability theorem.

$$P[C/M] = P[C/M|M] \cdot P[M] + P[No/M|M] \cdot$$

$$= 0.1(0.55) + 0.9(0.55)$$

$$\boxed{P[C/M] = 0.55}$$

put in above formula.

$$P[M|C/M] = \frac{0.1\cancel{(0.55)}}{\cancel{0.55}}$$

$$\boxed{P[M|C/M] = 0.1}$$

Scanned with CamScanner