

Assignment 4, PME, 4th Semester, Spring 2023

Deadline: Before the final exam paper of PME

Assignment should be hand written.

Write your name, registration No. and section; else your assignment may not be marked.

Copying is not allowed.

Properly staple your pages (binding is not required).

1. Let X be the number of heads obtained when a fair coin is flipped four times.
 - a. Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
 - b. Show the mapping from S to S_X , the range of X .
 - c. Find the probabilities for the various values of X .
2. An urn contains nine Rs. 10 notes and one Rs. 50 note. Let the random variable X be the total amount that results when two notes are drawn from the urn without replacement.
 - a. Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
 - b. Show the mapping from S to S_X , the range of X .
 - c. Find the probabilities for the various values of X .
3. Let X be a random variable with pmf $p_k = 0.6/k^2$ for $k = 1, 2, 3, \dots$.
 - a. Find $P[X > 4]$.
 - b. Find $P[6 \leq X \leq 8]$.
4. Show that the $VAR[X]$ of a binomial random variable X is npq .
5. Find 1st moment, 2nd moment and variance of X in question 1 and 2.
6. $S_C = \{1, 2, 3, 4\}$ where C is a uniform random variable having four possible values of electric current. If W is a random variable and represents the corresponding power values such that $W = 3C^2$. Find the mean and variance of C and W . Also find $E[C^3]$.
7. The number N of customers arriving in t seconds at a restaurant is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in customers/second. Assume that the arrival rate is 6 customers per hour. Find the probability of the following events:
 - a. more than 18 customers in 2 hours
 - b. less than or equal to 12 customers in 2 hours.
8. Show that the probability generating function $G_X(z)$ of a geometrically distributed random variable X with pmf $p_X(k) = pq^{k-1}$ is $pz/(1 - qz)$.
9. Find the mean of RV X if $G'_X(z)$ is $\alpha e^{\alpha(z-1)}$.
10. Find the variance of RV X if $G'_X(z)$ is $\alpha e^{\alpha(z-1)}$ and $G''_X(z)$ is $\alpha^2 e^{\alpha(z-1)}$.
11. Find the mean of a geometric RV using generating function.
12. Let X be the number of heads obtained when a fair coin is flipped four times.
 - a. Plot the cdf of X .
 - b. Use properties of cdf to find $P[2 < X \leq 3]$, $P[0.7 \leq X \leq 1.3]$ and $P[1 \leq X < 3]$.