

University of Engineering and Technology Peshawar
Department of Electrical Engineering

Course: Linear Algebra (BSI-111)

Semester: Spring (Final)

Time allowed: 180minutes

Total Marks: 60

Q1(a) (i) Code the message **WORK HARD** using the matrix

$$\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

(ii) Decode the message **67 44 41 49 39 19 113 76 62 104 69 55**.

(b) Let $L : R^3 \rightarrow R^3$ be linear transformation for which we have $L(\hat{i}) = (1, 2, -1)$,

$$L(\hat{j}) = (1, 0, 2), L(\hat{k}) = (1, 1, 3). \text{ Find } L(2, -1, 3).$$

Q2(a) Find a parametric equation of the line passing through the point $(3, -1, -3)$ and

perpendicular to the line joining $(3, -2, 4)$ and $(0, 3, 5)$.

(b) Prove the volume of a parallelepiped with a vertex at the origin and edges \vec{u}, \vec{v} , and \vec{w}

$$\text{is given by } V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$$

Q3(a) Define vector space and determine whether the set of all positive real numbers \mathbf{u} with the

operations $\mathbf{u} \oplus \mathbf{v} = 2\mathbf{u} - \mathbf{v}$ and $c \circ \mathbf{u} = c\mathbf{u}$ is a vector space?

(b) Express $\vec{v} = (3, 7, -4)$ as a linear combination of $\vec{x} = (1, 2, 3)$, $\vec{y} = (2, 3, 7)$,

$\vec{z} = (3, 5, 6)$, also check whether the vectors $\vec{x}, \vec{y}, \vec{z}$ are linearly dependent or

independent?

Q4(a) Define the null space of an $m \times n$ matrix A and prove that it is a subspace of R^n

(b) Find a basis for the eigenspace associated with λ .

$$\begin{bmatrix} 4 & 2 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \lambda = 2.$$