Date: 17.May.2022. Day: Tuesday.

Lecture 10:

Discrete Distribution:

Diff b/w continuous, discrete & digital. (Do yourself).

- The data which is frequently transmitted is represented in fewer bits.
- The data occuring sometimes is represented in larger bits is no issue.

Bernoulli Distribution:

has only two possible outcomes:

① Success ② failure.

interest area is always success.

→ Repitition of this Bernoulli bhai is Binomial Process

Slide 5:

Total = 20.

With Replacement: (Pw= White)

$P_w = 10/20 = 1/2$.

again $P_w = 10/20 = 1/2$, third time its same: $1/2$.

Without Replacement:

First time $P_w = 1/2$.

Second time $P_w = 9/19$.

Third time $P_w = 8/18$

- In Bernoulli, Sample Space/Prob remains same, but in 'Without Replacement' it ~~is not~~ changes so not Bernoulli trial.
- Probability should be constant in Bernoulli trials.

Binomial Process:

$$P_K = C^n_K \, p^k \, q^{n-k}$$

$$\therefore q = 1 - p.$$

Slide 9:

$n = 3 \quad p = 0.4 \quad q = 1 - 0.4 = 0.6.$

(0 Head) $P = P[T].P[T].P[T].$

$= (0.6).(0.6).(0.6)$

$\boxed{P[\text{no head}] = (0.6)^3}.$

(1 Head) $P = P[HTT]\,P[THT].P[TTH].$

$= 3[.P[T].P[T].P[H]].$

$= 3(0.6 \quad 0.6 \quad 0.4).$

$\boxed{P[1\text{ Head}] = 3(0.6)^2(0.4)}$

(2 Head) $P = P[HHT] \,\&\, P[HTH]\,P[THH].$

$= 3(P[H]\,P[H]\,P[T]).$

$\boxed{P[2\text{ Head}] = 3(0.6)(0.4)^2}.$

(3 Head) $P = P[HHH].$

$= P[H].P[H].P[H].$

$= 0.4 \;.\; 0.4 \quad 0.4$

$\boxed{P[3\text{ Head}] = (0.4)^3}$

~~Slide~~ Slide 10:

$n=3$ $\quad p=0.4$, $q=0.6$.

$P_0 = C^3_0 (0.4)^0 (0.6)^3$

$= \frac{3!}{3!\,0!} 1 (0.6)^3$

$\boxed{P_0 = (0.6)^3}$

$P_1 = \frac{3!}{2!\,1!} (0.4)^1 (0.6)^2$.

$\boxed{P_1 = 3(0.4)(0.6)^2}$

$P_2 = \frac{3!}{1!\,2!} (0.4)^2 (0.6)^1$.

$\boxed{P_2 = 3(0.4)^2(0.6)}$

$P_3 = \frac{3!}{0!\,3!} (0.4)^3 (0.6)^0$

$\boxed{P_3 = (0.4)^3}$

Which are same as previous method.

Slide 11

$n=8$, $\quad K =$ active speakers >6.

$p = 1/3$.

→ if $K=6$.

$P_6 = \frac{8!}{2!\,6!} (0.33)^6 (0.67)^2$

$\boxed{P_6 = 0.016}$

→ if $K=7$

$P_7 = \frac{8!}{1!\,7!} (0.33)^7 (0.67)^1$.

$= 2.28 \times 10^{-3}$

$\boxed{P_7 = 0.002}$

$P_8 = \frac{8!}{0!\,8!} (0.33)^8 (0.67)^0$

$= (0.33)^8$.

$= 1.406 \times 10^{-4}$.

$\boxed{P_8 = 0.00014}$

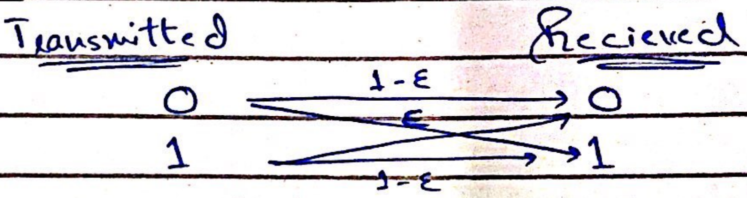
Now for Probability of active Speakers greater than 6, add Probability of $K=7$ & $K=8$.

$P[K>6] = P_7 + P_8.$

$= 0.002 + 0.000015$

$\boxed{P[K>6] = 0.00215}$

Slide 12:



$p = \varepsilon = 0.1 \qquad q = 1 - 0.1 = 0.9.$

$n = 3 \qquad K = 0, 1, 2, 3.$

$P[\text{error}] = P_0 + P_1.$

$= \binom{3}{0}(0.9)^0(0.1)^3 + \binom{3}{1}(0.9)^1(0.1)^2.$

$= 0.001 + 0.027.$

$\boxed{P[\text{error}] = 0.028.}$

$P[\text{no-error}] = 1 - 0.028.$

$\boxed{P[\text{no-error}] = 0.972}$

Slide 13:

$n=6, \quad K=2,$

$p=0.5 \quad q=0.5.$

$\cancel{P_2 = \left(\frac{6!}{2!4!}\right.}$

$$P_2 = \frac{6!}{4!2!}(0.5)^2(0.5)^4.$$

$$= 15(0.5)^6$$

$$\boxed{P_2 = 0.234}$$

Slide 14:

$n=4 \qquad K=5,6$

$p=0.6 \qquad q=0.4$

$$P[K>4] = \binom{6}{5}(0.6)^5(0.4)^1 + \binom{6}{6}(0.6)^6(0.4)^0.$$

$$= \frac{6!}{1!5!}(0.6)^5(0.4) + \frac{6!}{0!6!}(0.6)^6$$

$$= 0.19 + 0.05$$

$$\cancel{= 0.25}$$

$$\boxed{P[K>4] = 0.24}$$

Slide 15:

$p = 10\% = 0.1$

$n = 8 \qquad K = 1,$

$$P_1 = \frac{8!}{7!\,1!}(0.1)^1(0.9)^7.$$

$$= 8\,(0.1)(0.9)^7.$$

$$\boxed{P_1 = 0.383}$$

Date: 23 · May · 2022. Day: Monday.

Lecture 11:

I reject.
BB.
Brilliant Brains.
Huh!

Geometric Discrete Distribution:

e.g like you get success at 4^{th} trial.

$$P(m) = (1-p)^{m-1} \cdot p.$$

failure $= 1-p = q$

Success $= p$.

m = pehli dafa success.

if like 4 se ziada pe ata hai.

$$P(m > k) = (1-p)^{\cancel{m}\,k} \cdot p + (1-p)^{\cancel{m}\,k+1} \cdot p + (1-p)^{\cancel{m}\,k+2} \cdot p + \cdots$$

$$= (1-p)^{\cancel{m}\,k} \cdot p \left(1 + (1-p) + (1-p)^2 + \cdots\right)$$

$$= (1-p)^{k} \cdot p \left[\frac{1}{1-(1-p)} \right]$$

So 'k' would be equal to 4.

$$= (1-p)^{k} \cdot p \left[\frac{1}{\cancel{1}-(\cancel{1}-p)} \right]$$

$$= (1-p)^{k} \cdot \cancel{p} \cdot \frac{1}{\cancel{p}}$$

$$= (1-p)^{k}.$$

as $1-p = q$

so

when k is such that success comes after it then:

$$P(m > k) = q^{k}$$

Slide 4:

$P[H] = 0.6$ $P[T] = 0.4$.

(i) $m = 4$ times (ii) $m > 4$ times.

$P(m) = (1-p)^{m-1} \cdot p$ $P(m > K) = q^{K}$.

$P(4) = (0.4)^3 \cdot 0.6$ $P(m > 4) = (0.4)^4$.

$=$

Sequence of Dependent Experiments:

As $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ $\therefore P(A \cap B) = P(A|B)\, P(B)$.

Three experiments & they depend on each other.

$\underbrace{P\{S_2\}}_{A} \cap \underbrace{P\{S_1\} \cap P\{S_0\}}_{B}$.

So, '$P(A \cap B)$'

'A' represents single experiment

'B' " two "

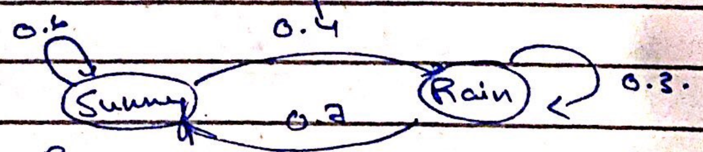
$= P\{S_2 | S_1 \cap S_0\} \cdot P\{S_1 \cap S_0\}$

$\because B = P(S_1 \cap S_0)$

$P(A \cap B) = P\{S_2 | S_1 \cap S_0\}\, P\{S_1 | S_0\}\, P\{S_0\}$.

'Markov chain process'

Eg.

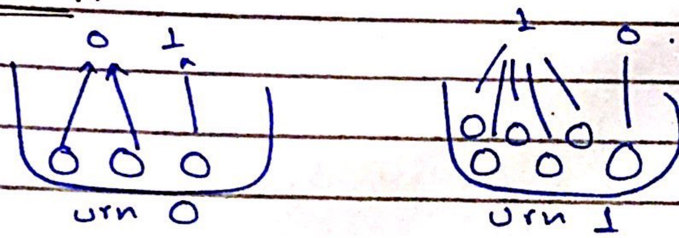


$$\begin{array}{c|cc} & S & R \\ \hline S & 0.6 & 0.4 \\ R & 0.7 & 0.3 \end{array} \quad \begin{array}{l} = 1 \\ = 1 \end{array}$$

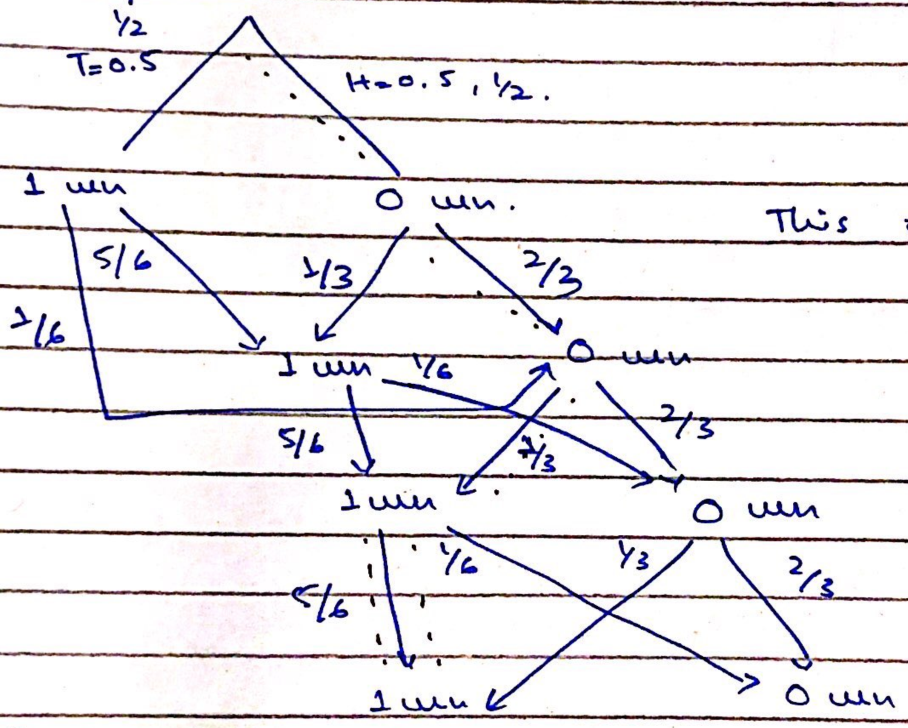
$\therefore$ Kal ki prediction aaj per.

memory less.

Slide 7:



Flip a coin.



This is thellis diagram.

Slide 8

'0011'

$= P[0] \cdot P[0|0] \cdot P[1|0] \cdot P[1|1]$.

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{5}{6}$

$\boxed{P[0011] = 0.09}$

→ 11001101

P[1] . P[1|1] . P[0|1] P[0|0] P[1|0] P[1|1] P[0|1] P[1|0]

$\frac{1}{2} \cdot \frac{5}{6} \cdot \frac{1}{3} \frac{2}{3} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{3} \cdot \frac{1}{6}$

$\boxed{0.0007}$

→ 10001110

P[1] P[0|1] P[0|0] P[0|0] P[1|0] . P[1|1] P[1|1] P[0|1].

$\frac{1}{2} \cdot \frac{1}{6} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$

→ 1111001

P[1] P[1|1] P[1|1] P[1|1] P[0|1] P[0|0] P(1|0].

$\frac{1}{2} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{2}{3} \cdot \frac{1}{3}$.

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Day: Monday. Date: 30 May 2022.

Lecture 12:

Random Variables: RV.

A function of assigning a number to each outcome of random experiment.

Example: Toss a coin pair.

Sample Space = {HH, HT, TT, TH} = 4.

X = no. of Heads.

$S_x = \{0, 1, 2\}$.

$P_x = (0) = 1/4$.

$P_x = (1) = 1/2 . (2/4)$.

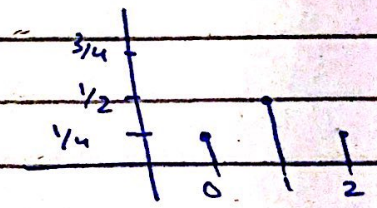
$P_x = (2) = 1/4$.

also represented as:

$P_x(0) = P[x = 0] = 1/4$.

$P_x(1) = P[x=1] = 1/2$.

$P_x(2) = P[x = 2] = 1/4$.



- Types of random Variables:

Discrete (Probability Mass function) PMF

- samples

e.g: coin toss twice.

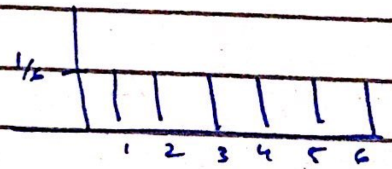
$S_x = \{0, 1, 2\}$.

coin toss thrice.

$S_y = \{0, 1, 2, 3\}$.

dice rolled

$S_z = \{1, 2, 3, 4, 5, 6\}$.



Slide 5:

x = no. of heads.

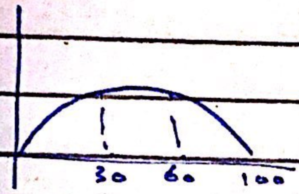
S.S: {HHH, HHT .... TTT} = 8.

$$\boxed{S_x = \{0, 1, 2, 3\}}$$

∴ 0 = No head.
∴ 1 = 1 head.
∴ 2 = 2 heads.
∴ 3 = 3 heads

Continuous (Probability Density function) PDF

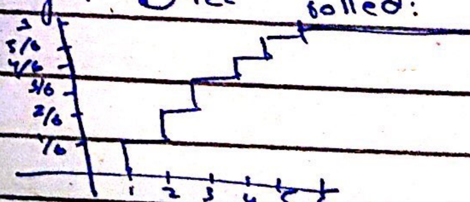
- intervals.



Commulative Density function

- in CDF, the probability before first event is zero, and after last event, probability is 1. and in b/w samples, it is always increasing. You add p of first current & previous.

e.g: Dice rolled:



Slide 7:

(random Variable, y).

- y = no. of goals
  $S_y = \{0, 1, 2, 3, 4 \ldots\ldots\}$.
- z = average no. of goals scored by players during the season.
  $S_z = \{0, 1, 2, 3, 4 \ldots\ldots\}$.
- x = sum of numbers facing up.
  $S_x = \{\cancel{0, 1, 2}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.
- F = no. of throws until six.
  $S_F = \{1, 2, 3, 4, 5 \ldots\ldots\}$.

$S_z = \{0/10, 1/10, 2/10, 3/10 \ldots\}$.
$S_z = \{0, 0.1, 0.2, 0.3 \ldots\}$.

Slide 9

- Discrete.
- Discrete.
- Continuous.

Slide 10:

- Discrete.
- Discrete.
- Discrete.
- Continuous.

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Lecture 13:

Properties:

$P_x(0) \geq 0$.

$\sum P_x = 1$.

Slide 3:

X = no. of heads.

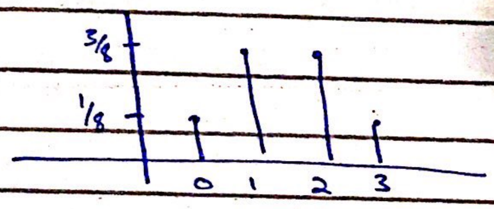
$S_x = \{0, 1, 2, 3\}$.

$P_x(0) = \cancel{1/8}$. $P[x=0] = 1/8$.

$P_x(1) = P[x=1] = 3/8$.

$P_x(2) = P[x=2] = 3/8$.

$P_x(3) = P[x=3] = 1/8$.



Slide 4:

y = points.

$S_y = \{0, 1, 8\}$

1 point when 2 heads.
8 point when 3 heads.

So using above wala problem.

when 0, $S_y(0) = P_x(0) + P_x(1)$

$= \frac{1}{8} + \frac{3}{8}$

$= \frac{4}{8} = 1/2$.

when 1 point, $S_y(1) = P_x(2) = P[x=2] = 3/8$

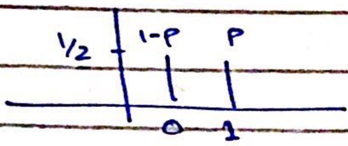
when 8 point, $S_y(8) = P_x(3) = P[x=3] = 1/8$

Slide 5:

$\cancel{x=0}$

$P_x(0) = P[x=0] = 1-p$ (failure.)

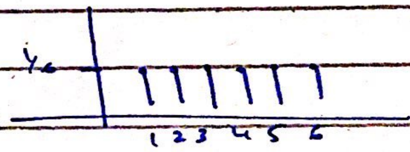
$P_x(1) = P[x=1] = p$ (Success.)



Slide 6:

$x =$ outcome.

$S_x = \{1, 2, 3, 4, 5, 6\}$.



~~Set~~

$P_x(1) = P[x=1] = 1/6$. $P_x(4) = P[x=4] = 1/6$.

$P_x(2) = P[x=2] = 1/6$. $P_x(5) = P(x=5] = 1/6$.

$P_x(3) = P(x=3] = 1/6$ $P_x(6) = P(x=6] = 1/6$.

Slide 7: $x =$ no. of times msg is transmitted.

$S_x = \{1, 2, 3, 4, 5, \ldots \ldots\}$.

$P_x(1) = p$ (Success)

$P_x(2) = pq$ (Success, failure).

$P_x(3) = pq^2$ (Success, failure^2).

$P_x(4) = pq^3$

$\vdots$

$P_x(x = \text{even number}] = P_x(2) + P_x(4) + P_x(6) + P_x(8) \ldots$

$= pq + pq^3 + pq^5 + pq^7 \ldots$

$= pq(1 + q^2 + q^4 + q^6 \ldots)$

$=$

$$= Pq\left(\frac{1}{1-q^2}\right)$$

$$= P.q\left(\frac{1}{(1+q)(1-q)}\right)$$

$$\therefore P = (1-q).$$

$$= \cancel{(1-q)}\, q\left(\frac{1}{(1+q)\cancel{(1-q)}}\right)$$

$$\boxed{P[x = \text{even No.}] = \frac{q}{1+q}}$$

Quiz Question:

x	0	1	2	3	4	5	6	7.
P_x	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$2K^2+K$

a) Find K.

$\sum P_x = 1.$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 2K^2 + K = 1.$

$9K + 5K^2 = 1.$

$K(9+5K) = 1.$

$K = 1 \quad , 9 + 5K = 1.$

$\cancel{9K = 0.5} \,/\, \cancel{5K^2 = 0.5}$

$\cancel{K = 0.5}$

Lets say:

$10K + 10K^2 = 1.$

$10K(1+K) = 1$

$K = 1/10 \quad , K = 0.$

Day: Tuesday. Date: 31·May·2022.

Lecture 14

Slide 13 :

x = no. of errors in four independent transmissions.

maybe error would be:

$S_x = \{0, 1, 2, 3, 4\}$.

i] p = success (no error)

$q = 0.1$ (error).

$\boxed{\text{so } p = 0.9}$

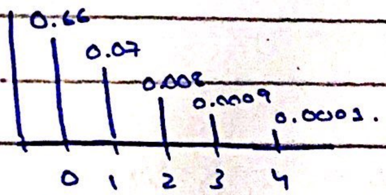
$P_x(0) = \cancel{PPPP}\ p \cdot p \cdot p \cdot p = p^4 = (0.9)^4 = 0.656$ (no error, means only success).

$P_x(1) = p^3 q = (0.9)^3 (0.1) = 0.073.$ (1 error)

$P_x(2) = p^2 q^2 = (0.9)^2 (0.1)^2 = 0.008$ (2 errors).

$P_x(3) = p q^3 = (0.9)(0.1)^3 = 0.0009$ (3 errors).

$P_x(4) = q^4 = \cancel{0.9^4}\ (0.1)^4 = 0.0001$ (4 errors).



Second part:

$P_x(x \leq 1) = P_x(0) + P_x(1)$

$= 0.656 + 0.073$

$\boxed{P_x(x \leq 1) = 0.729 \approx 0.73}$

Expected Value:

$$E[x] = x \cdot P(x=x)$$

Solving ~~same q~~ question with this:

flipping 2 coins.

x = no. of Heads, $S_x = \{0, 1, 2\}$.

$P_x(0) = 1/4$. (0 Head).

$P_x(1) = 2/4 = 1/2$. (1 head)

$P_x(2) = 1/4$ (2 head

Expected value:

~~E[x] = E[0] = 0~~

$$E[x] = 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$$

$$\boxed{E[x] = 1.}$$

Slide 7:

x = no. of heads in 3 tosses.

$S_x = \{0, 1, 2, 3\}$.

$P_x(0) = 1/8$

$P_x(1) = 3/8$.

$P_x(2) = 3/8$

$P_x(3) = 1/8$

$$E_x = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{6}{8} + \frac{6}{8}$$

$\boxed{E[x] = 1.5}$ it is average value.

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Slide 5:

success $= p$, failure $= 1-p$.

Sample Space $= 1$ $= 0$.

$S_x = \{0, 1\}$.

$P_x(0) = 1-p$. $P_x(1) = p$.

So $E[x] = 0(1-p) + 1p$.

$\boxed{E(x) = p}$

Success' probability is p, so failure's will be 1-p. but 0 & 1 is value for success & failure meaning sample space

Slide 6:

$x =$ no. of heads in 5 tosses.

$S_x = \{0, 1, 2, 3, 4, 5\}$.

$P_x(0) = 1/32$.

$P_x(1) = \cancel{1/32} =$

${}^nC_r = \frac{32!}{(32-1)!\,1!}$

$= 32$

$P_x(1) = \frac{5}{32}$

$P_x(2) = 10/32$.

$P_x(3) = 10/32$.

$P_x(4) = 5/32$.

$P_x(5) = 1/32$

${}^5C_0 \cdot p^0 q^5 = \frac{5!}{5!} 1 \cdot \left(\frac{1}{2}\right)^5 = 1/32$.

${}^5C_1\, p^1 q^4 = \frac{5!}{4!} \frac{1}{2} \left(\frac{1}{2}\right)^4 = 5/32$.

$E[x] = 0 \times 1/32 + 1 \times 5/32 + 2 \times 10/32 + 3 \times 10/32 + 4 \times 5/32 + 5 \times 1/32$.

$\boxed{E[x] = 2.5}$

Slide 8:

x = no. of dots for dice.

$S_x = \{1, 2, 3, 4, 5, 6\}$.

$P_x(0) = 1/6$, $P_x(1) = 1/6$, $P_x(2) = 1/6$, $P_x(3) = 1/6$, $P_x(4) = 1/6$

$P_x(5) = 1/6$, $P_x(6) = 1/6$.

$E[x] = 1 \times \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$

$\boxed{E[x] = 3.5}$

Lecture 15:

Slide 3

Sample mean = 140

median = 79 (mid).

mode = 68 (repeated)

Slide 4:

y = points.

$S_y = \{0, 1, 8\}$.

$P_y(0) = 4/8 = 1/2 \; (3/8 + 1/8 = 4/8)$

$P_y(1) = 3/8$

$P_y(8) = 1/8$.

$E[y] = 0 \times \frac{4}{8} + \frac{3}{8} + \frac{8}{8}$

$\boxed{E(y) = 1.37}$

Slide 5: $S_u = \{0, 1, 2, 3\}$

$S_y = \{0, 1, 8\}$.

$S_x(0) = 30/200$.

$S_x(1) = 75/200$

$S_x(2) = 80/200$

$S_x(3) = 15/200$.

Now S_y.

$$S_y(0) = \frac{30}{200} + \frac{75}{200} = \frac{105}{200}$$

$$S_y(1) = \frac{80}{200}$$

$$S_y(8) = 15/200.$$

$$E[Y] = \frac{80}{200} + \frac{8 \times 15}{200}$$

$$\boxed{E(Y) = 1}$$

Slide 6:

n = no. of times a message is transmitted.

success' probability = p, failure's probability = $q(1-p)$

$S_x = \{1, 2, 3, 4, 5, 6 \ldots\ldots\}$.

$P_x = \{p, qp, q^2p, q^3p, q^4p \ldots\ldots\}$.

$$E[x] = 1 \times p + 2 \times qp + 3 \times q^2 p + 4 \times q^3 p + 5 \times q^4 p \ldots$$

$$= p + 2pq + 3q^2 p + 4q^3 p \ldots$$

$$= p[1 + 2q + 3q^2 + 4q^3 + 5q^4 \ldots].$$

take integration on b/s w.r.to q.

$$\int E[x]\, dq = p\left[q + \frac{2q^2}{2} + \frac{3q^3}{3} + \frac{4q^4}{4} + \frac{5q^5}{5} \ldots\right]$$

$= pq\,[1+q+q^2+q^3+q^4+q^5 \ldots]$

$= pq\left[\frac{1}{1-q}\right].$

$\int E[x]\,dq = \frac{pq}{1-q} \begin{matrix}\rightarrow a \\ \rightarrow b\end{matrix}$

let $\frac{pq}{1-q} = \frac{a}{b}$ & take derivative on b/s.

$\frac{d}{dq}\int E[x]\,dq = \frac{d}{dq}\frac{a}{b}$

$\therefore \quad \frac{d}{dq}\frac{a}{b} = \frac{a'b - ab'}{b^2}$

so,

$E[x] = p\frac{(1-q) - pq(-1)}{(1-q)^2}$

$= \frac{p - \cancel{pq} + \cancel{pq}}{(1-q)^2}$

$= \frac{p}{(1-q)^2}$

$= \frac{p}{(p)^2}$

$$\boxed{E[x] = \frac{1}{p}}$$

Date: 6.June.2022. Day: Monday.

Lecture 16:

in discrete : $E[g(x)] = \sum_{x \in x} g(x) f(x)$

in cont: $E[g(x)] = \int g(x) f(x) dx$.

Slide 4:

find $E[z]$ where $z = x^2$.

and $S_x = \{-3, -1, 1, 3\}$. , $P_x(k) = 1/4$.

$E(z) = (-3)^2 (1/4) + (-1)^2 (1/4) + (1)^2 (1/4) + (3)^2 (1/4)$

$= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4}$

$= \frac{20}{4}$

$\boxed{E[z] = 5}$ "This was method 1".

Second method:

after x^2, S_x will be $S_x = \{9, 1, 1, 9\}$ with probabilities $1/4$, So S_x becomes $S_x = \{1, 9\}$. Then probabilities become $1/2$ for each Now:

$E[x] = \frac{1}{2}(1) + \frac{1}{2}(9)$

$= \frac{1}{2} + \frac{9}{2}$

$= \frac{10}{2}$

$\boxed{E[x] = 5}$ same as above.

Slide 5:

$S_x = \{-3, -1, +1, 3\}$. $P_x(k) = 1/4$.

$Z = (2x+10)^2$.

~~for -3, $z = (16)^2 = 256$.~~

~~for -1, $z = (12)^2 = 144$.~~

~~for 1, $z =$~~

$z = (2x+10)^2 \longrightarrow (a+b)^2 = a^2+b^2+2ab$.

So $E[z] = E[4x^2 + 40x + 100]$.

$= 4E[x^2] + 40E[x] + 100$.

put values:

$= 4\left[\frac{9\times1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4}\right] + 40\left[\frac{-3\times\cancel{1}}{\cancel{4}} - \frac{\cancel{1}}{\cancel{4}} + \frac{\cancel{1}}{\cancel{4}} + \frac{\cancel{3}}{\cancel{4}}\right] + 100.$

$= 4(5) + 0 + 100$

$= 20 + 100$

$\boxed{E[z] = 120}$

my method:

~~for $3 = 256$~~, for $-3 = 16$, for $-1 = 64$.

for $3 = 256$ for $1 = 144$.

Now:

$E[z] = \frac{16}{4} + \frac{64}{4} + \frac{256}{4} + \frac{144}{4}$

$= \frac{480}{4}$

$\boxed{E[z] = 120}$

Slide 6:

$$Z = X^2.$$

$$S_x = [0, 1, 2, 3].$$

$$P_x = \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}.$$

$$E[z] = z.$$

$$E[z] = E[x^2] = g(x^2)\, p(x)$$

$$= 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}.$$

$$= \frac{3}{8} + \frac{12}{8} + \frac{9}{8}.$$

$$= \frac{24}{8}$$

$$\boxed{E[z] = 3}$$

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Date: 7.June.2022. **Day:** Tuesday.

Assignment 6: (CLO 3).

write python codes for:

- Conditional Property.
- Bias Theorem.
- Bernouli Distribution.
- Binomial Distribution.
- Geometric Distribution.

Take a problem, Solve it yousely & byusing code then compare the results.

Deadline: ~~15~~ 20 June.

Lecture 16:

Variance:

It is used to measure the spread or vaiability of distribution of a random variable. In short, Basically it measures the ~~distance~~ difference from the expected value.

Formula:

$$Var[x] = E[x - E[x]]^2.$$

$$= E[x^2 + E[x]^2 - 2xE[x]].$$

$$= E[x^2] - 2E[x]E[x] + E[x]^2.$$

$$= E[x^2] - 2E^2[x] + E^2[x]$$

$$\sigma = \boxed{Var[x] = E[x^2] - E^2[x].}$$

Standard Deviation:

under root of variance

$\sqrt{\sigma} = \sqrt{var[x]} = \sqrt{E[x^2] - E^2[x]}$

Slide 10:

X = no. of heads.

Find $var[x] = ?$

$S_x = \{0, 1, 2, 3\}$. $P_x = \{1/8, 3/8, 3/8, 1/8\}$.

$E[x] = 0 \times \frac{1}{8} + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$

$= \frac{12}{8}$

$= \frac{3}{2}$

$\boxed{E[x] = 1.5}$

Find $E[x^2]$

$E[x^2] = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$

$= \frac{24}{8}$

$\boxed{E[x^2] = 3}$

put in formula.

$var = E[x^2] - E^2[x] \Rightarrow 3 - (1.5)^2$.

$= 3 - 2.25$

$\boxed{var[x] = 0.75}$

Question 1:

x	0	1	2	3	4
P_x	1/5	1/5	1/5	1/5	1/5

Find $var[x] = ?$

$E[x] = \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}$

$\boxed{E[x] = 2}$ so $E^2[x] = 2^2 = 4.$

find $E[x^2] = \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5}$

$\boxed{E[x^2] = 6}$

put in formula $var[x] = E[x^2] - E^2[x].$

$\sigma = var[x] = 6 - 4$

$\boxed{var[x] = 2}$

Question 2:

$$M = \begin{cases} 1 & P(1) = 0.2 \\ 2 & P(2) = 0.4 \\ 3 & P(3) = 0.3 \\ 4 & P(4) = 0.1 \\ & \text{otherwise } 0 \end{cases}$$

Find variance.

$E[x] = 0.2 + 2(0.4) + 3(0.3) + 4(0.1)$

$E[x] = 2.3$, $\boxed{E^2[x] = 5.29}$

$E[x^2] = 0.2 + 4(0.4) + 9(0.3) + 16(0.1)$

$\boxed{E[x^2] = 6.1}$

put in formula.

$\sigma = var[x] = 6.1 - 2.3$

$\boxed{var[x] = \cancel{3.8} \; 0.81}$

Lecture 17:

pehla sawal:

Find the variance of geometric distribution.

$S_x = \{1, 2, 3, 4, 5 \ldots\ldots\}$.

$P_x = \{p, pq, pq^2, pq^3, pq^4 \ldots\ldots\}$.

$E[x] = p + 2pq + 3pq^2 + 4pq^3 + 5pq^4 \ldots\ldots$

$= \sum_{K=1}^{\infty} K . pq^{K-1}$ $\quad \therefore$ put $K = 1, 2 \ldots$

$E[x^2] = \sum_{K=1}^{\infty} K^2 . pq^{K-1}$

By adding & subtracting 'K'.

$E[x^2] = \sum_{K=1}^{\infty} (K^2 + K - K) pq^{K-1}$

$\therefore$ separating.

$E[x^2] = \underbrace{\sum_{K=1}^{\infty} (K^2+K) pq^{K-1}}_{a} - \underbrace{\sum_{K=1}^{\infty} K pq^{K-1}}_{E[x]}$

$a = p \sum_{K=1}^{\infty} K(K+1) q^{K-1}$

integrate w.r.to q.

$\int a \, dq = p \int \sum_{K=1}^{\infty} K(K+1) q^{K-1}$

$= p \sum_{K=1}^{\infty} \cancel{K}(K+1) \frac{q^K}{\cancel{K}}$

$$= P\sum_{k=1}^{\infty}(k+1)\,q^{k}.$$

$$\int a\,dq = P\left[2q + 3q^2 + 4q^3 + 5q^4 + \dots\dots\right].$$

~~A~~ Again integrate.

$$\iint a\,dq = P\left[\frac{\cancel{2}q^2}{\cancel{2}} + \frac{\cancel{3}q^3}{\cancel{3}} + \frac{\cancel{4}q^4}{\cancel{4}} + \frac{\cancel{5}q^5}{\cancel{5}} \dots\right]$$

$$= P\left[q^2 + q^3 + q^4 + q^5 \dots\right].$$

$$= pq^2\left[1 + q + q^2 + q^3 + q^4 \dots\right].$$

$$= pq^2\,\frac{1}{(1-q)^{4}}$$

$$= \frac{pq^2}{1-q} \quad \begin{matrix}\to a \\ \to b\end{matrix}$$

$$\iint a\,dq = \frac{a}{b}$$

Taking derivative.

$$\int a\,dq = \frac{2pq(1-q) - (-1)(pq^2)}{(1-q)^2}$$

$$= \frac{2pq - 2pq^2 + pq^2}{(1-q)^2}$$

$$= \frac{2pq - pq^2}{(1-q)^2} \quad \begin{matrix}— a \\ — b\end{matrix}$$

Take second derivative.

$$a = \frac{2p - 4pq + 2pq^2 - 2pq + 4pq^2 - 2pq^3 - [-4pq^2 + 4pq^2 + 2pq^2 - 2pq^3]}{(1-q)^4}$$

$$\boxed{a = \frac{2-2q}{p^3}}$$

$$= \frac{2(1-q)}{p^3} = \frac{2p}{p^3}$$

$\boxed{a = \frac{2}{p^2}}$ put there.

as you rember $E[x] = \frac{1}{p}$

var[x]

So $\cancel{E[x^2]} = \frac{2}{p^2} - \frac{1}{p^{\cancel{4}}} - \frac{1}{p^2}$

$$= \frac{1-p}{p^2}$$

$$\boxed{\text{var}[x] = \frac{q}{p^2}}$$

Lecture 17:

Slide(5)

$E[x] = np$ [Prove this].

Binomial Random Variable

$S_x = \{0, 1, 2, 3, 4, \ldots\ldots n\}$.

$P_x = {}^nC_k\, p^k q^{n-k}$

$E[x] = k \cdot P_x$.

$= \sum_{k=0}^{n} k \cdot {}^nC_k\, p^k q^{n-k}$.

as if $k=0$, everything will be zero, So $k=1$.

$= \sum_{k=1}^{n} k \cdot \frac{n!}{(n-k)!\,k!} p^k q^{n-k}$.

$= \sum_{k=1}^{n} \cancel{k} \cdot \frac{n(n-1)!}{(n-k)!\,\cancel{k}(k-1)!} p \cdot p^{k-1} \cdot q^{n-k}$.

$= \sum_{k=1}^{n} \frac{n(n-1)!}{(n-k)!\,(k-1)!} p \cdot p^{k-1} q^{n-k}$

np is constant.

$= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!\,(k-1)!} p^{k-1} q^{n-k}$.

Suppose $n-1=m$ then $n=m+1$

" $k-1=j$ then $k=j+1$

$= np \sum_{j=0}^{m} \frac{(m+\cancel{1}-\cancel{1})!}{(m+1-k)!\,(j+\cancel{1}-\cancel{1})!} p^{\cancel{k-1}\; j+\cancel{1}-\cancel{1}} q^{\cancel{n-k}\; m+1-k}$

$= np \sum_{j=0}^{m} \frac{m!}{(m+\cancel{1}-(j+\cancel{1}))!\; j!} p^j q^{m+\cancel{1}-(j+\cancel{1})}$

$= np \sum_{j=0}^{m} \frac{m!}{(m-j)!\; j!} p^j q^{m-j}$

$$E[X] = np \cdot \sum_{j=0}^{m+1} \frac{m!}{(m-j)!\; j!} \, p^{j} \, q^{m-j}$$

Formula of probabability mass function.

→ submission of probabilities from 0 to n is equal to 1. (Rule of PMF).

So,

$$E[X] = np\,(1)$$

$$\boxed{E[X] = np}$$ Proved

Uniform Random Variable

$$S_x = \{0, 1, 2, 3 \ldots\ldots L\}.$$

$$P_x = 1/L.$$

$$E[X] = \sum_{k=0}^{L} k \cdot \frac{1}{L}$$

$$\therefore 1+2+3+4+5 \ldots = \frac{n(n+1)}{2}$$

$$E[X] = \sum_{k=1} k \cdot \frac{1}{L}$$

$$= \sum_{k=1} \frac{k(L+1)}{2} \cdot \frac{1}{k}$$

$$\boxed{E[X] = \frac{L+1}{2}}$$

Lecture 18:

Poisson Random Variable:

where probability of success is very less. Like finding happiness in life.

e.g: Arrival Rate $= \lambda$

Time of Interest $= t$.

$\alpha = \lambda t$.

So formula is:

$$P_K = \frac{\alpha^K e^{-\alpha}}{K!}$$

$\therefore e =$ Euler constant $= 2.718$.

Slide: 4:

Arrival Rate $= 4/\text{min} = \lambda$

part a

$t = 10\text{sec}$.

first convert 4/min to sec

So $\lambda = 4/60 = 1/15$ sec.

So, $\alpha = 10 \cdot 1/15$

$= 10/15 = \boxed{2/3 = \alpha}$

put values:

$$P_K = \frac{\alpha^K e^{-\alpha}}{K!}$$

Laos.

$$P_4 = \frac{2/3^4 \; 2.718^{-2/3}}{4!}$$

$= 4.22 \times 10^{-3}$

$\boxed{P_4 = 0.004225}$

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part 2: more than 4.

$$P[n>4] = 1 - \sum_{k=0}^{4} \frac{\alpha^k e^{-\alpha}}{k!}$$

So for $n>4$ we need to find

$k=0$, $k=1$, $k=2$, $k=3$.

i) $k=0$.

$\boxed{\cancel{P_0 = 1}}$ $\boxed{P_0 = 0.513}$

i) $k=1$.

$$P_1 = \frac{2/3^1}{1!} 2.718^{-1}$$

$\boxed{P_1 = \cancel{0.4245}}$ $\boxed{\cancel{0.426}}$ $\boxed{0.342.}$

i) $k=2$

$$P_2 = \frac{2/3^2}{2!} 2.718^{-2/3}.$$

$\boxed{P_2 = 0.114}$

$P_3 = 0.025$

$P[n>4] = 1 - 0.994$

$\boxed{P[n>4] = 0.006}$

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$\boxed{\alpha = \text{column wise} \quad \& \quad K = \text{horizontal}}$

part 3: less than or equal to 5 in 2 minutes.

$\lambda = 4/\text{min}.$

$t = 2\text{min}$

$\alpha = \lambda t.$

$= 4 \times 2 = \boxed{8 = \alpha}$

P[

Slide 5: $\boxed{e^{-\lambda t}}$

$\alpha = \lambda t$

$K = 0$

$$P_0 = \frac{\lambda t^0 \, e^{-\lambda t}}{0!}$$

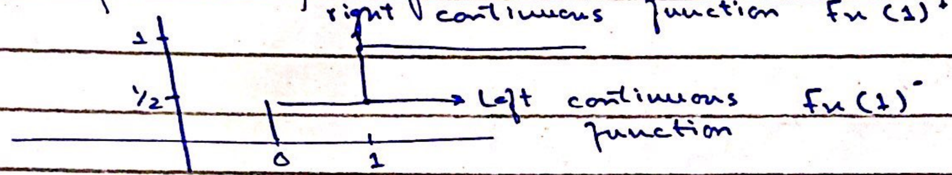
$$\boxed{P_0 = e^{-\lambda t}}$$

Date: 20.June.2022. Day: Monday.

CDF Commulative Density Function

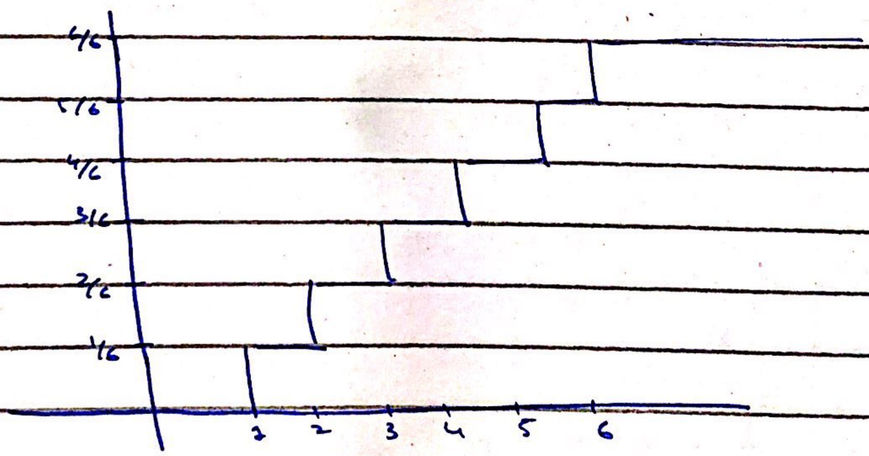
adds previous probabilities:

e.g probability of head (0) tail (1)



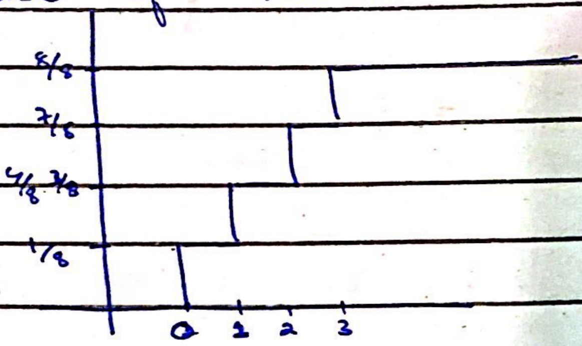
- $F_X(x) = P[a < x \leq b]$.
- $0 \leq F_X(x) \leq 1$.
- $\lim_{x \to -\infty} F_X(x) = 0$.
- $\lim_{x \to \infty} F_X(x) = 1$.

Lecture 18 slide 7

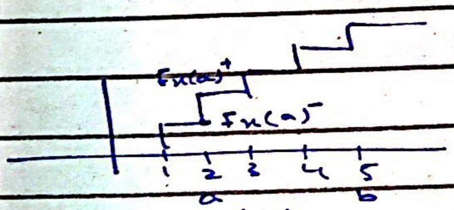


cdf of RV X where x is the number of outcomes of rolling a dice.

Slide 8: cdf of RV X, where x is the number of heads in three tosses.



Lecture 19:



Properties of CDF: Read from slides.

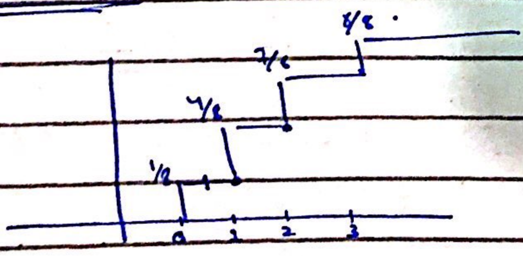
if two points a & b.

the ~~Eule~~ $P(a < X \leq b) = F_x(b) - F_x(a)$.

if one point $\rightarrow P(x = a) = F_x(a)^+ - F_x(a)^-$

if greater than $\rightarrow P[x > X] = 1 - F_x(X)$.

Slide 5:



part 1: $P[1 < x \leq 2)$

using formula: $P(a < x \leq b) = F_x(b) - F_x(a)$

$\Rightarrow P(1 < x \leq 2) = F_x(2) - F_x(1)$.

$= 7/8 - 4/8$

$= 3/8$

part 2: $P[0.5 \leq x \leq 2.5]$.   ∴ as $0.5 \leq x$, equal comes here but in formula it doesn't.

↓ change it to.

$P(0.5 < x \leq 2.5) + P(x = 0.5)$

So using formulae.

$= F_x(2.5) - \cancel{F_x(0.5)} + \cancel{F_x(0.5)} - F_x(0.5)^-$

$= 7/8 - 1/8$ ∴ this is '+1' in graph.

$= 6/8$.

part 3: $P[1 \leq x < 2]$.

↓ change it to.

$P(1 < x \leq 2) + P(x = 1) - P(x = 2)$.

using formulae.

$= \cancel{F_x(2)} - \cancel{F_x(1)} + \cancel{F_x(1)} - F_x(1)^- - \cancel{F_x(2)} + F_x(2)^-$

$= - F_x(1)^- + F_x(2)^-$

$= - 1/8 + 4/8$ ∴ this is '•' in graph.

$= 3/8$.

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Date: 21.June.2022. Day: Tuesday.

Slide 6:

CDF of continuous RV:

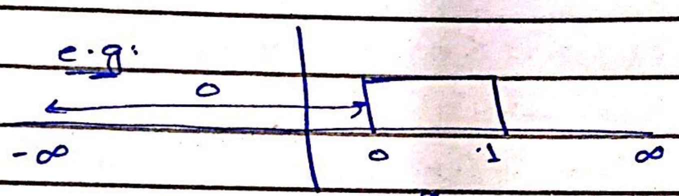
we've been studying cdf of discrete RV, So now we gonna study continuous.

→ in cont, there are infinite points b/w two points.

→ Stair case nahi bane ga. Instead smooth straight line bane gi.

→ Integration.

e.g:



Formula: $F_x(x) = \int_{-\infty}^{\infty} f(t)\, dt.$

$F_x(x) = \int_{0}^{\infty} f(t)\, dt.$

Slide 9:

$f(x) = 3x^2, \quad \therefore \quad 0<x<1.$

$F_x(x) = \int_{-\infty}^{x} 3x^2$

$= \int_{0}^{x} 3x^2$

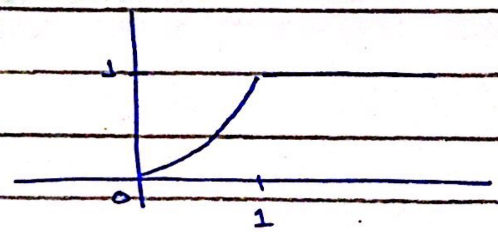
$= \frac{\cancel{3}x^3}{\cancel{3}} \Big|_0^x$

$F_x(x) = x^3$

Now range is from 0 to 1.

$x \to 0$,	$F_x(x) = 0.$
$x \to 0.2$	0.008
$x \to 0.4$	0.064
$x \to 0.6$	0.216
$x \to 0.8$	0.512
$x \to 1.$	1.

Graph



Slide 10:

$$f(x) = \frac{x^3}{4} \qquad 0 < x < 2.$$

$$F_x(x) = \int_{-\infty}^{x} f(x).$$

$$= \int_{0}^{x} \frac{x^3}{4}$$

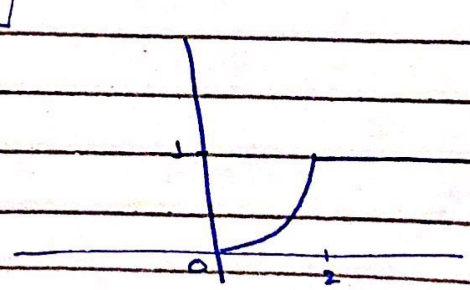
$$= \frac{1}{4} \left(\frac{x^4}{4} \right|_0^2 \qquad \frac{x^3}{4}$$

$$F_x(x) = \frac{1}{16} x^4 \Big|_0^2. \qquad \frac{1}{4} \int x^3$$

$$\frac{x^{3+1}}{3+1}$$

$x \to 0$	$F_x(0) = 0.$
$x \to 0.4$	$F_x(0) =$ ~~0.0064~~ 0.0016
$x \to 0.8$	~~0.10~~ 0.0256
$x \to 1$	~~0.25~~ 0.0625
$x \to 1.2$	~~0.548~~ 0.1296
$x \to 1.6$	~~1.638~~ 0.4096
$x \to 2$	~~4~~ 1

Graph



Slide 11:

$$f(x) = \begin{cases} x+1 & , -1 < x < 0. \\ 1-x & , 0 \leq x < 1. \end{cases}$$

$$F_X(x) = \int_{-\infty}^{x} f(x)\, dx.$$

For $-1 < x < 0$

$$F_X(x) = \int_{-1}^{0} x+1 \; dx.$$

$$= \left(\frac{x^2}{2} + x\right) \Big|_{-1}^{x}$$

$$= \left(\frac{x^2}{2} + x\right) - \left(\frac{+1}{2} - 1\right)$$

$$= \frac{x^2}{2} + x - \frac{1}{2} + 1.$$

$$= \left(\frac{x^2}{2} + x + \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(x^2 + 2x + 1\right)$$

$$\boxed{F_X(x) = 1/2\,(x+1)^2}$$

$x = -1$, $F_X(x) = 0$.

~~$x = -1.5$, $= 0.125$~~

~~$x = -1.6$,~~

$x = -0.8$, 0.02.

$x = -0.6$, 0.08.

$x = -0.2$, 0.32.

$x = 0$, 0.5

Slide 17:

Extra Question:

$$f(x) = \begin{cases} 0 & \text{for } x \le -1. \\ \frac{1}{2}(x-1) & \text{for } -1 < x \le 0. \\ 1-(1-x)/2 & \text{for } 0 < x < 1. \\ 0 & \text{for } x \geq 1. \end{cases}$$

this type ko question paper me asakta hai.

$$1 - e^{-\lambda x}$$

$$\frac{d}{dx}\left(-e^{-\lambda x}\right)$$

$$-e^{-\lambda x}\frac{d}{dx}(-\lambda x)$$

$$+\lambda e^{-\lambda x}.1.$$

$$+\lambda e^{-\lambda x}.$$

Slide 12:

$$f_X(x) =$$

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{b-a}$$

$$= \int_{a}^{x} \frac{1}{b-a}$$

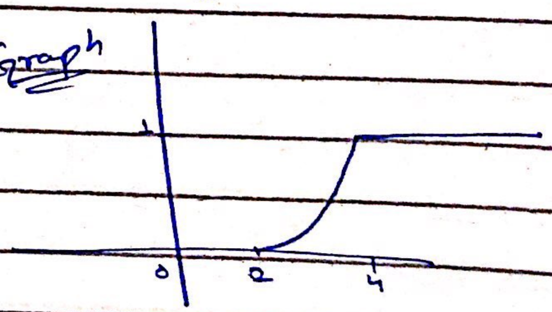
$$= \frac{x}{b-a}\Big|_{a}^{x}$$

$$F_X(x) = \frac{x-a}{b-a}$$

Let $a = 2$, $b = 4$

$x = 2.2$	0.1.
$x = 2.6$	0.3
$x = 2.8$	0.4
$x = 3.2$	0.6
$x = 3.6$	0.8
$x = 4.$	1

Graph



Date: 23·June·2022. **Day:** Thursday.

Lecture 20:

Slide 3:

$f(x) = 1/b-a. \qquad a \le x \le b$

formula:

$$E(x) = \int_{-\infty}^{\infty} t f_x(t)\, dt.$$

$$E(x) = \int_a^b t \cdot \frac{1}{b-a}\, dt.$$

$$= \frac{1}{b-a} \int_a^b t\, dt.$$

$$= \frac{1}{b-a} \left[\frac{t^2}{2} \Big|_a^b \right]$$

$$= \frac{b^2 - a^2}{2(b-a)}$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$\boxed{E(x) = \frac{b+a}{2}}$$

Slide 4:

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

Formula: $E(x) = \int_{-\infty}^{\infty} t \, f_x(t)\, dt.$

$$E(x) = \int_0^{\infty} t \cdot \lambda e^{-\lambda t}\, dt.$$

$$= \cancel{-\lambda t e^{-\lambda t}} \cancel{\int_0^{\infty} t} \cdot \cancel{\int_0 -\lambda t \, dt}.$$

$$= -\cancel{\lambda} t . e^{-\lambda t} \cancel{\left[\frac{t^2}{2}\Big|_0^{\infty}\right]} \cdot \cancel{\left[\frac{-\lambda t^2}{2}\Big|_0^{\infty}\right]}.$$

formula for integration by parts.

$$\int uv \, dx.$$
$$= u\int v\, dx - \int u'\left(\int v\, dx\right) dx.$$

$$= t\,\frac{\lambda e^{-\lambda t}}{-\lambda}\Big|_0^{\infty} - \left[\int 1 \frac{\lambda e^{-\lambda t}}{-\lambda}\, dt.\right.$$

$$= -t e^{-\lambda t}\Big|_0^{\infty} - \frac{1}{\lambda}\left[e^{-\lambda t}\right]\Big|_0^{\infty}$$

$$= (0-0) - \frac{1}{\lambda}(0-1)$$

$$\boxed{E(x) = \frac{1}{\lambda}}$$

Slide 6:

$$f_x(x) = \begin{cases} 1/b-a & a \leq x \leq b. \\ 0 & x < a \ \& \ x > b. \end{cases}$$

$$VAR = E\left[(x - E[x])^2\right].$$

as from Slide 4. $E(x) = \frac{b+a}{2}$.

put here.

$$= E\left[\left(x - \frac{a+b}{2}\right)^2\right].$$

Let $y = x - \frac{a+b}{2}$

→ if $x = a$, then

$$y = a - a+b/2$$

$$= \frac{2a - a - b}{2} \qquad = \frac{+a-b}{2}$$

$$y = \frac{-(b-a)}{2}$$

→ if $x = b$, then.

$$y = b - a+b/2.$$

$$= \frac{2b - a - b}{2}.$$

$$y = \frac{\cancel{2b} + b - a}{2}$$

So, ~~VAR[X] = 1/2~~

$$VAR = E(y^2)$$

using Expected value formula.

$$\text{VAR} = \int y^3 \cdot \frac{1}{b-a}\, dy.$$

$$= \frac{1}{b-a}\left[\frac{y^3}{3}\right]\Bigg|_{-\frac{(b-a)}{2}}^{\frac{b-a}{2}}$$

$$= \frac{1}{3(b-a)}\left[\frac{(b-a)^3}{8} + \frac{(b-a)^3}{8}\right].$$

$$= \frac{\cancel{2}(b-a)^{\cancel{3}}}{\cancel{24}(\cancel{b-a})}$$

$$\boxed{\text{VAR} = \frac{(b-a)^2}{12}}$$

Date: 27.June.2022. Day: Monday.

Lecture 20

Slide 7:

Formula is:

$$VAR = E[(x - E[x])^2].$$

OR. $VAR = E[x^2] - E^2[x].$

Pdf: $\boxed{\therefore E[x] = 1/\lambda.}$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0. \\ 0 & \text{otherwise.} \end{cases}$$

$$E[x^2] = \int_0^\infty x^2 . \lambda e^{-\lambda x} \, dx.$$

integration by parts:

$$= \left[x^2 \frac{\lambda e^{-\lambda x}}{-\lambda} \right]\Big|_0^\infty - \int_0^\infty 2x \frac{\lambda e^{-\lambda x}}{-\lambda}$$

$$= \cancel{\left(\frac{\infty^2 . 0 - 0 . 1}{-\lambda}\right)}^{0} + \frac{2\lambda}{\lambda} \int_0^\infty x \, e^{-\lambda x} \, dx.$$

$$= \frac{2\lambda}{\lambda} \left[\frac{x . e^{-\lambda x}}{-\lambda} \Big|_0^\infty - \int_0^\infty \frac{1 . e^{-\lambda x}}{-\lambda} \, dx \right]$$

$$= 2 \left[0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda x} \, dx \right].$$

$$= \frac{2}{\lambda} \int_0^\infty e^{-\lambda x} \, dx.$$

$$= \frac{2}{\lambda} \left\{ \frac{e^{-\lambda x}}{-\lambda} \Big|_0^\infty \right.$$

$$= \frac{-2}{\lambda^2} \left[0 - 1 \right]$$

$$\boxed{E[x^2] = \frac{2}{\lambda^2}}$$

$$VAR = E[x^2] - E^2[x].$$
$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$
$$\boxed{VAR = \frac{1}{\lambda^2}}$$

Markov Chain:

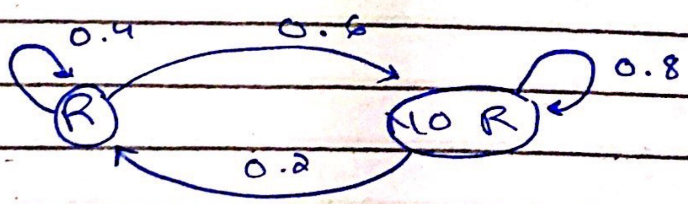
future probabilities predicted on current data.

Example:

if rain today (R), 40% chance rain tomorrow 60% chance no rain. if no rain today. 20% chance rain tomorrow. 80% chance no rain.

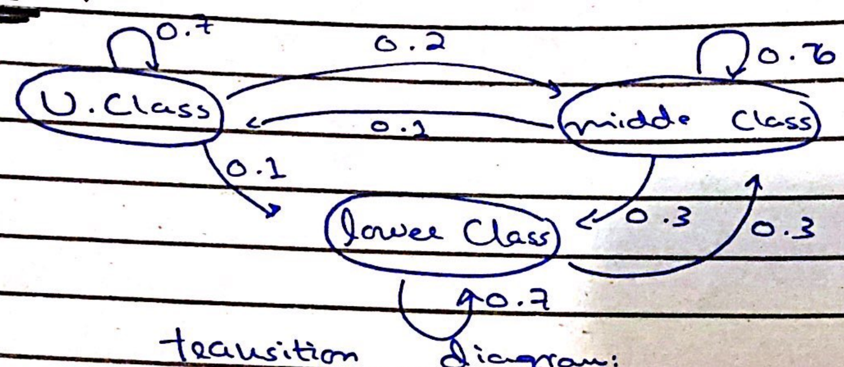
Current situation	Future event R	NoR
R	0.4	0.6
No R	0.2	0.8

Transition Diagram:



Scanned with CamScanner

Problem:



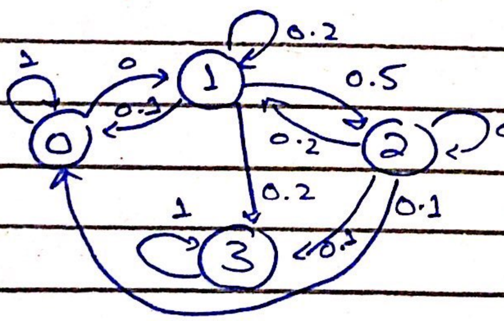
transition diagram:

probability matrix:

	U	M	L
U	0.7	0.2	0.1.
M	0.1	0.6	0.3
L	0	0.3	0.7

Problem 3:

	0	1	2	3
0	1	0	0	0
1	0.1	0.2	0.5	0.2
2	0.1	0.2	0.6	0.1
3	0	0	0	1.



Date: 28·June·2022. Day: Tuesday.

Lecture 21:

To easy calculations, we introduce two more methods.

Characteristic function:

$$\Phi_x(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} f_x(x).e^{j\omega x}\,dx.$$

$$\therefore \quad E[x] = \int_{-\infty}^{\infty} f_x(x).\cancel{e^{j\omega x}}\,x\,dx.$$

Discrete:

$$\Phi_x(\omega) = \sum_k p_x(x_k)\, e^{j\omega x_k}.$$

Slide 6:

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} f_x(x)\, e^{j\omega x}\,dx.$$

$$= \int_{-\infty}^{\infty} \lambda e^{-\lambda x}.e^{j\omega x}\,dx.$$

$$= \lambda \int_{0}^{\infty} e^{-(\lambda - j\omega)x}\,dx.$$

$$= \lambda \left(\frac{e^{-(\lambda - j\omega)x}}{-(\lambda - j\omega)} \right) \Bigg|_0^{\infty}$$

$$= \frac{-\lambda}{(\lambda - j\omega)} \left(e^{-\infty} - e^{0} \right)$$

$$= \frac{+\lambda}{\lambda - j\omega}.$$

Question: find for $E[x^2]$.

$$E[x] = \int_{-\infty}^{\infty} f_x(x) . x \, dx.$$

$$E[x^3] = \int_{0}^{\infty} x^3 . \lambda e^{-\lambda x} \, dx.$$

$$= u \int v \, dx - \int u' \left(\int v \, dx \right) dx.$$

$$= x^3 \left(\frac{\lambda e^{-\lambda x}}{-\lambda} \right) \Big|_0^{\infty} - \int 3x^2 \left(\frac{\lambda e^{-\lambda x}}{-\lambda} \right) dx.$$

$$= \cancel{\frac{\lambda x^2}{\lambda}}$$

$$= \left(x^3 . \frac{\lambda e^{-\lambda x}}{\lambda} \right) \Big|_0^{\infty} - \int \left(3x^2 \left(\frac{\lambda e^{-\lambda x}}{-\lambda} \right) \Big|_0^{\infty} - \int 6x \left(\frac{\lambda e^{-\lambda x}}{-\lambda^2} \right) \right.$$

$$= \frac{\infty^3 (0) - \cancel{0^3 . 1}}{\cancel{\lambda}} -$$