Even or odd signal

1) Even or odd signal

2) ~~Deterministic or Random~~

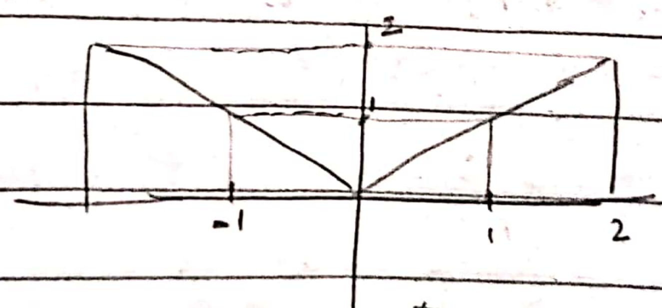
3) periodic & non-periodic signals

4) Energy & power signals

1) Even or Odd :-

A CT signal $x(t)$ is said to be even if

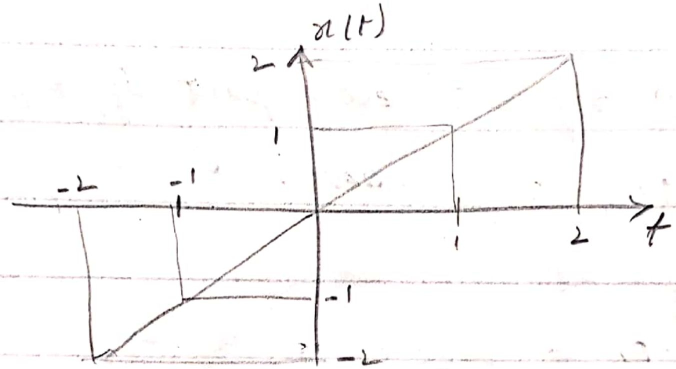
$x(t) = x(-t)$. For all values of t



An even signal are symmetric about vertical axis

Similarly, A CT signal is said to be odd if

$x(-t) = -(t)$; for all values of t



An odd signals are anti-symmetric about vertical axis

Similarly, for DT signal $x(n)$, it is even if

$$x(-n) = x(n)$$

& $x(n)$ is odd if

$x(-n) = -x(n)$ for all values of n.

* Representation of signal in its even & odd parts

Consider a signal $x(t)$ (3)

$$x(t) = x_e(t) + x_o(t) \quad \text{—— (1)}$$

where

$x_e(t)$ represent even part of $x(t)$ &

$x_o(t)$ " odd " " "

Replacing t by $-t$ in eq (1)

$$x(-t) = x_e(-t) + x_o(-t)$$

But by definition of even signal

$$x_e(-t) = x_e(t) \quad \&$$

by def of odd signals

$$x_o(-t) = -x_o(t)$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{—— (2)}$$

Add (1) + (2)

$$x(t) + x(-t) = x_e(t) + x_o(t) + x_e(t) - x_o(t)$$

$$= 2x_e(t)$$

$$\boxed{x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)}$$

① − ② ⇒

$$x(t) - x(-t) = x_e(t) + x_o(t) - x_e(t) + x_o(t)$$

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$$= 2x_o(t)$$

$$\boxed{x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)}$$

Similarly, for DT signal $x(n)$

Even part is

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

odd part is

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Properties

i) Addition properties of Odd & Even signals

a) Addition of two even signals is even

b) " , , odd " , odd

c) " of even & odd signal is neither odd nor even.

II) Multiplication properties of odd & even signals

a) Multiplication of two even signals is even

b) " , , odd " " "

c) Multiplication of even & odd signals is odd signal

* Significance of even or odd signals:

i) Even or odd symmetry of the signals have specific harmonic or frequency component

ii) Even or odd symmetry property is used in filter design

Example

Find even & odd components of $x(t)$

i) $x(t) = 5\sin(t)$ —— (1)

Replacing t by $-t$, we get

$x(-t) = 5\sin(-t)$

we know that

$\sin(-t) = -\sin(t)$ —— (2)

$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$= \frac{1}{2}\left[5\sin(t) - 5\sin(t)\right]$

$= 0$ (even component)

&

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$$= \frac{1}{2}\left[5\sin t - (-5\sin(t))\right]$$

$$= \frac{1}{2}\left[5\sin t + 5\sin(t)\right]$$

$$= 5\sin(t) \quad (\text{odd component})$$

Here we can observe that even component of sine signal is 0 (zero)

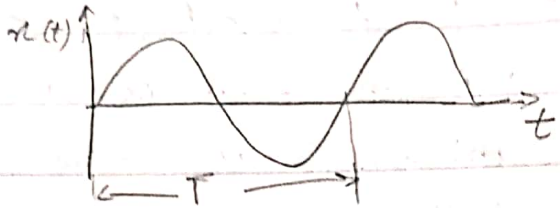
Hence sine is standard odd signal.

Similarly, cosine is standard even signal

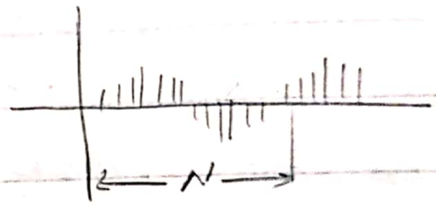
Periodic & Non-periodic signal

A signal is said to be periodic if it repeats at regular interval

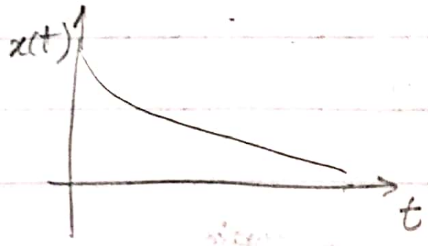
Non periodic signal do not repeat



CT periodic signal

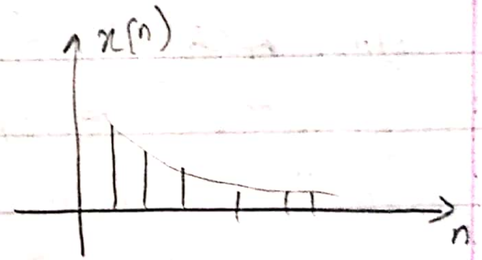


DT periodic signal



exponential decay signal

CT non-periodic signal



DT non-periodic signal

A CT signal $x(t)$ is said to be periodic if it satisfies the condition,

$$x(t) = x(t+T_0) \quad \text{for all values of } t$$

The smallest value of T_0 that satisfies above condition is called as fundamental period of $x(t)$. The fundamental period defines the time duration of one complete cycle of $x(t)$

The reciprocal of fundamental time period is called as fundamental frequency of $x(t)$

$$F_0 = \frac{1}{T_0}$$

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It describes how frequently the periodic signal $x(t)$ repeats itself.

A DT signal $x(n)$ is said to be periodic if it satisfies the condition

$$x(n) = x(n+N) \quad \text{for all values of } n$$

where N is the Number of samples/sec.

The smallest value of N that satisfies above condition is called fundamental period of $x(n)$

Condition for Periodicity of CT signal

A CT signal repeats after T_0 is periodic

$$x(t) = x(t+T_0)$$

Condition for periodicity of DT signal

Consider

$$x(n) = x(n+N)$$

Consider $x(n) = \cos(2\pi f_0 n)$ — (1)

replacing n by $n+N$

$$x(n+N) = \cos(2\pi f_0 (n+N))$$

$$= \cos(2\pi f_0 n + 2\pi f_0 N) \quad \text{--- (2)}$$

For periodicity of DT signal

$$x(n) = x(n+N)$$

$$\cos(2\pi f_0 n) = \cos(2\pi f_0 n + 2\pi f_0 N) \quad \text{--- (3)}$$

We know standard eqn

$$\cos(\theta + 2k\pi) = \cos(\theta) \quad \text{--- (4)}$$

Comparing (3) & (4), we can say that to satisfy eq (3)

$$\cancel{2\pi} f_0 N = \cancel{2} k \cancel{\pi} \qquad \text{where } k = \text{integer}$$

$$f_0 = k/N \qquad \text{where } N = \text{period of } x(n)$$

The above condition shows that DT signal is periodic only if its freq is ratio of two integer (i.e rational)

v. Periodicity of CT signal $x(t)$ if

$$x(t) = x_1(t) + x_2(t)$$

If $x_1(t)$ is periodic with period T_1 &
$x_2(t)$ " " " " T_2

Then given signal $x(t)$ is periodic if

$$\frac{T_1}{T_2} = \frac{n}{m}$$ i.e ratio of integers &

period T of $x(t)$ is

$$T = (LCM(T_1, T_2)$$

* periodicity of DT signal $x(n)$ if

$$x(n) = x_1(n) + x_2(n)$$

If $x_1(n)$ is periodic with period N_1 &
$x_2(n)$ " " " " N_2 then

$x(n)$ is periodic if

$$\frac{N_1}{N_2} = \frac{n}{m}$$ i.e ratio of two integers &

period N of $x(n)$ is

$$N = LCM(N_1, N_2)$$

For a given signals, determine whether it is periodic or non-periodic signal & if periodic find its fundamental period

i) $x(t) = 5\cos(200\pi t)$

ii) $x(n) = 12\sin(25\pi n)$

iii) $x(n) = 9\cos(25n)$

iv) $x(t) = \cos^2(2\pi t)$

v) $x(t) = 4\cos(\frac{\pi}{100} t) + 2\cos(\frac{2\pi}{150} t)$

i) $x(t) = 5\cos(200\pi t)$

Comparing given signal with standard cosine signal

$$x(t) = A\cos(2\pi f_0 t)$$

then we will get

$$2\pi f_0 = 200\pi$$

$$f_0 = \frac{200\pi}{\pi 2} = 100\ Hz$$

$$T_0 = \frac{1}{f_0} = \frac{1}{100}\ sec = 0.01 = 10\,msec$$

Given signal is periodic with period

$$T_0 = 10\ msec$$

ii) $x(n) = 12 \sin(25\pi n)$

$\Rightarrow$ Given signal is DT signal. We know that the condition for DT signal to be periodic if

$$f_0 = \frac{K}{N}$$ i.e ratio of integers

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Comparing given signal with standard sin signal

$$x(n) = A \sin(2\pi f_0 n)$$

we get,

$$2\pi f_0 = 25\pi$$

$$f_0 = \frac{25\pi}{2\pi} = \frac{25}{2}$$ i.e $\frac{K}{N}$ ratio of integers

Given signal $x(n)$ is period with period $N = 2$ samples/cycle

iii) $x(n) = 9 \cos(25n)$

Given signal is DT signal. We know that the condition for DT signal to be periodic if

$$f_0 = \frac{K}{N}$$ i.e ratio of integers

Therefore, if we compare given $x(n)$ with standard cosine

$$x(n) = A \cos(2\pi f_0 n)$$

$$2\pi f_0 = 25$$

$$f_0 = \frac{25}{2\pi} \neq \frac{k}{N}$$ i.e not ratio of two integers

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$\Rightarrow$ Given signal $x(n)$ is non periodic signal

iv)

$$x(t) = \cos^2(2\pi t)$$

$\Rightarrow$ Given signal is not in the form of standard sine/cosine form

we know standard formula

$$1 + \cos 2\theta = 2\cos^2\theta$$

i.e $$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Applying,

$$x(t) = \cos^2(2\pi t) = \frac{1 + \cos 2(2\pi t)}{2}$$

$$x(t) = \frac{1}{2} + \frac{\cos(4\pi t)}{2}$$

$$= 0.5 + 0.5\cos(4\pi t)$$

does not affect periodicity being DC term

$$\Rightarrow 2\pi f_0 t = 4\pi t$$

$$f_0 = \frac{4\pi t}{2\pi t} = \frac{2}{1} \text{ Hz}$$

Fundamental period T_0 is

$$T_0 = \frac{1}{f_0} = 1/2 = 0.5 \text{ sec}$$

Given signal $x(t)$ is periodic with period 0.5 sec

v) $$x(t) = 4\cos\left(\frac{\pi t}{100}\right) + 2\cos\left(\frac{2\pi}{180}t\right)$$

$\Rightarrow$ Given signal is of form

$$x(t) = x_1(t) + x_2(t)$$

where

$$x_1(t) = 4\cos\left(\frac{\pi t}{100}\right)$$

& $$x_2(t) = 2\cos\left(\frac{2\pi t}{180}\right)$$

$$\Rightarrow 2\pi f_1 = \frac{\pi}{100} \Rightarrow f_1 = \frac{1}{200} \Rightarrow \boxed{T_1 = 200 \text{ sec}}$$

& $$2\pi f_2 t = \frac{2\pi t}{180} \Rightarrow f_2 = \frac{2\pi}{180} \cdot \frac{1}{2\pi} = \frac{1}{180} \Rightarrow \boxed{T_2 = 180 \text{ sec}}$$

Given signal $x(t)$ is periodic if

$$\frac{T_1}{T_2} = \frac{n}{m} \quad \text{i.e ratio of integers}$$

$$\frac{T_1}{T_2} = \frac{20\cancel{0}}{18\cancel{0}} = \frac{10}{9} \quad \text{i.e } \frac{n}{m}$$

$x(t)$ is periodic with period T

$$T = \mathrm{LCM}(T_1, T_2)$$

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$$T = \lambda_1 T_1 = \lambda_2 T_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} = \frac{9}{10}$$

$$\Rightarrow \lambda_1 = 9$$
$$\lambda_2 = 10$$

$$T = \lambda_1 T_1 = 9 \times 200 = 1800 \text{ sec}$$

$\Rightarrow$ Given signal $x(t)$ is periodic with period $T = 1800$ sec

For practice

$$x(t) = 4\cos(4\pi t)$$

$$x(n) = 3\cos(0.02\pi n)$$

$$x(n) = 5\sin\left(\frac{2\pi n}{7}\right)$$

$$x(t) = 7\sin^2(4\pi t)$$

$$x(t) = \sin\left(\frac{21}{5}\pi t\right) + \cos\left(\frac{31}{7} t\right)$$