DSP

Date 27/11/2023

After Mids :-

Systems

1. Linear / Non-linear System
2. Time Invariant / Time Variant "
3. Memory / Memoryless "
4. Stable / Unstable "
5. Static / Dynamic "

Type ① → follows BIBO
Bounded i/p / Bounded o/p

Type ②

$$x[n] \longrightarrow \boxed{\text{Sys}} \longrightarrow y[n]$$

e.g

$y = x(\sin t)$ / $y = \sin t$

$y = \cos t$ (stable)

$-1 \leq y \leq 1$

$y = \tan(\theta)$ (unstable)

Type ②

Impulse Respont

$\int$ $h(t) = ?$

Σ $h(n) = ?$

(i) Check whether it's is cont/discrete.

For Cont. $\int_{-\infty}^{\infty} |h(t)| dt$

$\Downarrow$

if

Some finite value → stable

$\int_{-\infty}^{\infty} |h(t)| dt < \infty$

∞ → Unstable.

$\int_{-\infty}^{\infty} |h(t)| dt = \infty$

For discrete,

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \rightarrow \text{stable}$$

otherwise unstable.

Question :-

$$h(t) = e^{-t} \cdot \sin t \cdot u(t)$$

$$h(t) = e^{-at} \cdot \sin bt \cdot u(t).$$

$$h(t) = \underset{v}{\underline{e^{-t}}} \cdot \underset{u}{\underline{\sin t \cdot u(t)}}$$

$$= \int_{-\infty}^{\infty} e^{-t} \cdot \sin t \cdot u(t)$$

$$h(t) = \int_{0}^{\infty} e^{-t} \cdot \sin t \cdot dt$$

$$-e^{-t} \cdot \sin t + \int e^{-t} \cdot \cos t \, dt$$

ILATE

$$-e^{-t} \cdot \sin t \Big|_0^{\infty} + \int -e^{-t} \cdot \cos t$$

$$= (0 - 0) + \frac{\cos t \cdot e^{-t}}{-1} \Big|_0^{\infty} - \int_0^{\infty} (-\sin t) \frac{e^{-t}}{-1} dt$$

$$I = -(\cos t \cdot e^{-t}) \Big|_0^{\infty} - \int_0^{\infty} e^{-t} \cdot \sin t$$

$$2I = -(0 - 1)$$

$$\boxed{I = 1/2}$$

$$I = \int_{-\infty}^{\infty} |h(t)| = \frac{1}{2} < \infty$$

Stable

If the sys is stable then find the range of a and

if $h(t) = e^{-\alpha t} \cdot \sin(\beta t) \cdot u(t)$

$$I = \int_0^{\infty} e^{-\alpha t} \sin(\beta t)$$

$$I = -\frac{e^{-at} \cdot \sin bt}{a} \Big|_0^{\infty} - \int_0^{\infty} \frac{e^{-at}}{-a} \cdot \frac{\cos bt}{b}$$

(2) $h(t) = e^{at} u(t) + e^{-bt} u(t)$

(3) $h[n] = \begin{cases} b^{n} & n < 0 \\ a^{n} & n \geq 0 \end{cases}$ piece-wise

if system is stable determine the range of a and b for which sys will be stable,

$= \sum_{-\infty}^{-1}$

$$\sum_{-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{-\infty}^{-1} b^{n} + \sum_{0}^{\infty} a^{n} < \infty$$

$$\sum_{1}^{\infty} b^{n} + \sum_{0}^{\infty} a^{n} < \infty$$

$$\sum_{1}^{\infty} b^{n} + \sum_{0}^{\infty} a^{n} + 1 - 1 < \infty$$

$\sum^{\infty} b^{n} + 1$

$$\left(\sum_{1}^{\infty} b^{n} + 1\right) + \sum_{0}^{\infty} a^{n} - 1 < \infty$$

$\sum_{0}^{\infty} b^{n} - 1 + 1$

$$\sum_{0} |b^{n}| + \sum_{0}^{\infty} |a^{n}| + 1$$

$$\underbrace{\frac{1}{1-|b|} + \frac{1}{1-|a|}}_{\text{Issue here}} + 1 < \infty$$

$b \neq \pm 1 \quad , \quad a \neq \pm 1$

STATIC AND DYNAMIC SYSTEMS :-

STATIC $\Downarrow$ memoryless

DYNAMIC $\Downarrow$ Memory

(a) $\int$ (b) Σ (c) $\frac{d}{dt}$ (d) $t - t_d$

(i) $y(t) = a \cdot x(t) \Rightarrow$ Static

for $t = 0 \quad y(0) = a \cdot x(0)$

$t = 1 \quad y(1) = a \cdot x(1)$

(ii) $y(t) = t \cdot x(t^2) \rightarrow$ Dynamic

$t = 0 \quad y(0) = 0$

$t = 1 \quad y(1) = 1 \cdot x(1)$

$t = 2 \quad y(2) = 2 \cdot x(4)$

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LTI Systems

→ Mostly used in books

→ Reason; Convolution

$$x(t) \longrightarrow \boxed{\text{Sys}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

1. Sys Must be linear
2. Sys Must be Time Invariant

LTV → Linear Time Variant System

Invertible System

Time Check

1. Delay (k units)
2. Replace
   $n \rightarrow n-k$
   $t \rightarrow t-k$

Prob 1 $y(t) = t \cdot x(t)$

i) $x_1(t) \Rightarrow y_1(t) = t \cdot x_1(t)$

ii) $x_2(t) \rightarrow y_2(t) = t \cdot x_2(t)$

$y_1(t) + y_2(t) = t \cdot x_1(t) + t \cdot x_2(t)$

$y'(t) = t\left(x_1(t) + x_2(t)\right)$ — (i)

(iii) $x_1(t) + x_2(t) \Rightarrow y'' = t\{x_1(t) + x_2(t)\}$ — (ii)

Prob $y(t) = 10x(t^2) + 5$

(i) $x_1(t) \rightarrow y_1(t) = 10x_1(t^2) + 5$

(ii) $x_2(t) \rightarrow y_2(t) = 10\,x_2(t^2) + 5$

$y_1(t) + y_2(t) = 10x_1(t^2) + 10\,x_2(t^2) + 10$

$y' = 10\left(x_1(t^2) + x_2(t^2) + 1\right)$

(iii) $x_1(t) + x_2(t) \Rightarrow y'' = 10\left[x_1(t^2) + x_2(t^2)\right] + 5$

$\Rightarrow y'' = 10\,x_1(t^2) + 10\,x_2(t^2) + 5$

LTV/LTI

i) Delay $x(t)$ by k units
ii) Replace t with $t-k$

$$y(t) = x(t) - 3u(t)$$

(i) $x(t-td) \rightarrow y(t) = x(t-td) - 3.u(t)$

(ii) $y(t-td) = x(t-td) - 3u(t-td)$

TV

Invertible & Non-Invertible Systems.

$$x[n] \longrightarrow \underset{\text{Source}}{\boxed{h_1(n)}} \xrightarrow[y_1[n]]{} \underset{h_2(n)}{\boxed{\text{Encrypt}}} \xrightarrow[y_2[n]]{}$$

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Convolution:-

$\int$ = cont.

Σ = discrette

For converting to freq. domain.

i) F.T

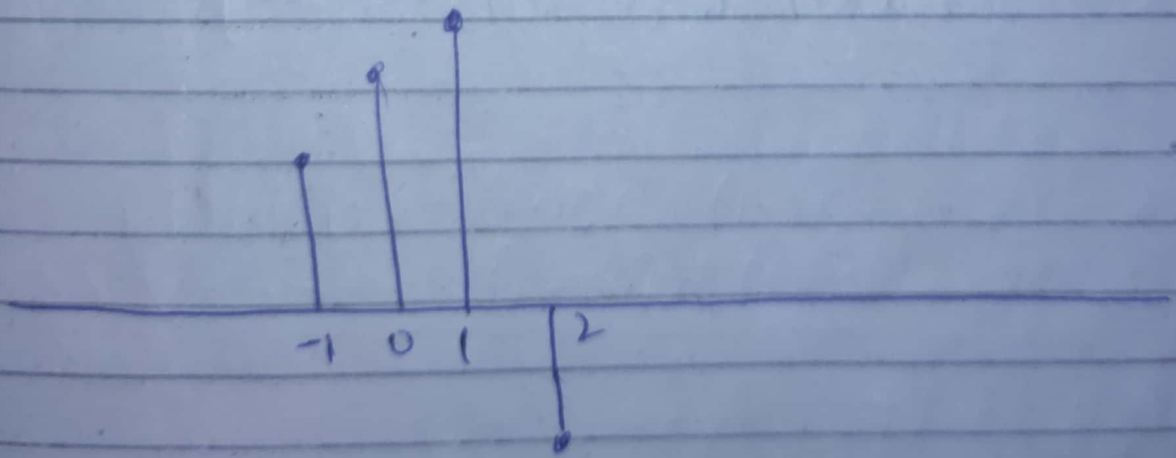
ii) L.T

$$\underbrace{y[n] = \sum_{n=-\infty}^{\infty} x[m] \cdot h[n-m]}$$

Total o/p = ZIR + ZSR

Any discrete signal is represented as:

$$x[n] = \sum_{k=\infty}^{\infty} x[k] \cdot \delta(n-k)$$



$$x[n] = x(-1) \cdot \delta(n+1) + x(0) \cdot \delta(n) + x(1) \cdot \delta(n-1)$$
$$+ x(2) \cdot \delta(n-2)$$

$$x(n) \longrightarrow \boxed{T} \xrightarrow{y(n)}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta(n-k)$$

$$y[n] = T[x[n]] = T\left[\sum_{k=-\infty}^{\infty} x[k] \cdot \delta(n-k)\right]$$

$$〃 = T\{ \ldots x(-1) \cdot \delta(n+1) + x(0) \cdot \delta(n) + x(p) \cdot \delta(n+1) \ldots \}$$

$$〃 = \{ \ldots x(-1) \cdot T[\delta(n+1)] + x(0) \cdot T[\delta(n)] + x(1) \cdot T[\delta(n-1)] \ldots \}$$

$$〃 = \ldots x(-1) \cdot h(n+1) + x(0) \cdot h[n] + x(1) \cdot h[n-1] \ldots$$

$$〃 = \sum_{k=-\infty}^{\infty} x(k) \cdot h[n-k]$$

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Important

Properties of Convolution :-

i) Commutative Prop.

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Let $\left.\begin{array}{l} n-k = m \\ n-m = k \end{array}\right\}$ change of var

If $k \to -\infty$ then $m \to \infty$

$k \to \infty$ then $m \to -\infty$

$$y[n] = \sum_{m=\infty}^{-\infty} h[m]\, x[n-m]$$

$$y[n] = \sum_{m=-\infty}^{\infty} h[m].\, x[n-m] = h[n] * x[n]$$

~~Prop~~ (ii) Dist. Property.

$$x[n] \to \boxed{h_1[n]} \xrightarrow{y_1[n]} \boxed{h_2[n]} \to y[n]$$

$$x[n] \to \boxed{h_2[n]} \xrightarrow{y_2[n]} \boxed{h_1[n]} \to y'[n]$$

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$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n]$$

LTI

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

① Causal System (realistic)

present + past values
No future values

③ Non Causal System (unrea

past + futur
values

② Anti Causal

only future

So care must be taken for
$h[-1]$, $h[-2]$, - - - - - -

Distr. Prop.

$$y[n] = \sum_{r=-\infty}^{\infty} h_2[r] \cdot y_1[n-r]$$

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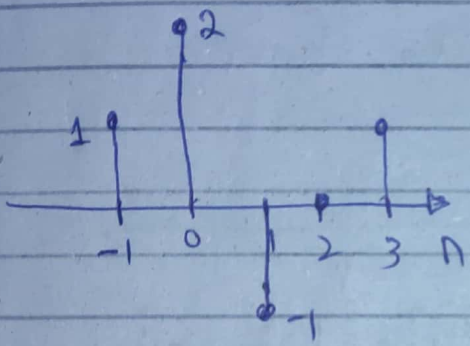
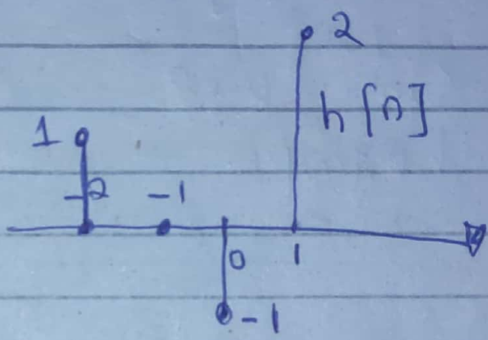
Convolution (Graphically):

1. $x[n] \rightarrow \{1, \underset{\uparrow}{2}, -1, 0, 1\}$

2. $h[n] \rightarrow \{1, 0, \underset{\uparrow}{-1}, 2\}$





3. $y(n) = ?$

starting index $= -1 - 2 = -3$

Ending " $= 4$

Length $=$ length$(x(n))$ + length$(h(n)) - 1$

$= 5 + 4 - 1 = 8$

4. $x(n) \rightarrow x(k)$

$h(n) \rightarrow h(k)$

5. $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ or

$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

6. $h(-k) \rightarrow$ folding

Mujhe shifting nazar nahi arahi.

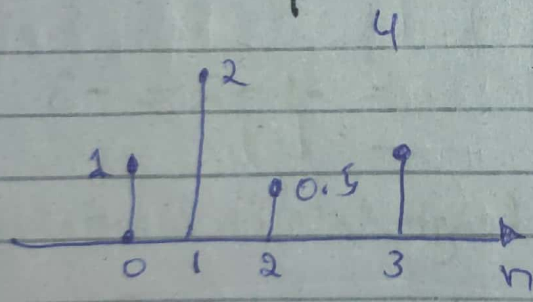
7. $h(n-k) \rightarrow$ shifting

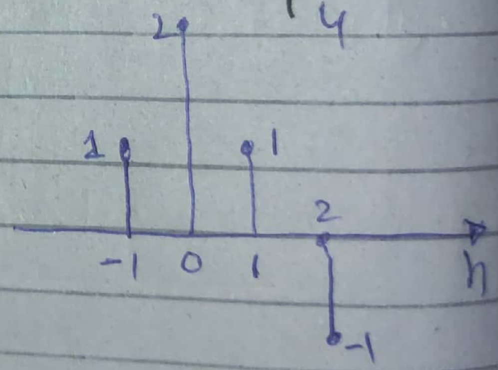
8. Multiply $x(k) \cdot h(n-k)$

9. Sum krlo, Aap ka Kam hogaya.

$x[n] = \{\underset{\uparrow}{1}, 2, 0.5, 1\}$ $h[n] = \{1, \underset{\uparrow}{2}, 1, -1\}$

4   4

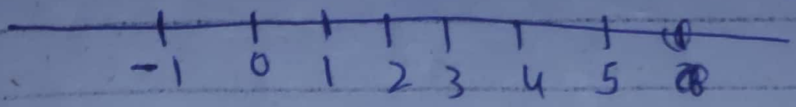




Step (1)

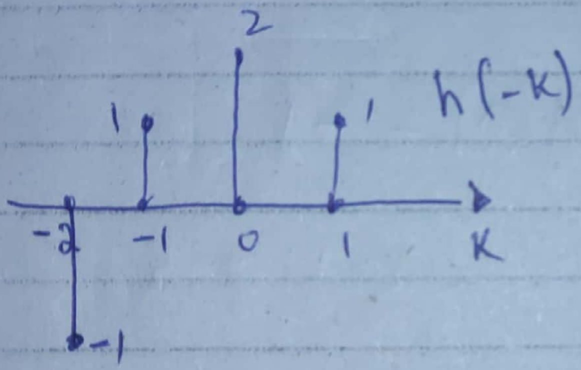
Starting index of $x[n] = 0$
" " " $h[n] = -1$
" " " $y[n] = 0 + (-1) = -1$

Len of $x(n) = 4$
Len of $h[n] = 4$
" " $y(n) = 7$

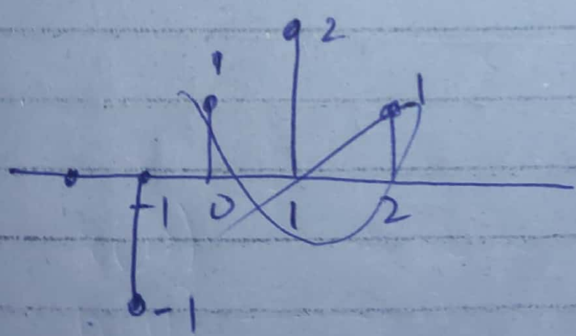


Hence, Ending index of $y[n] = 5$

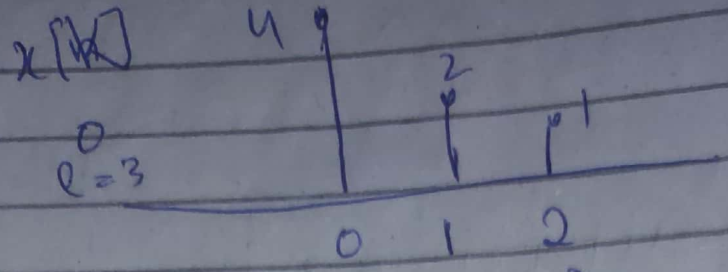
Replace n by k,

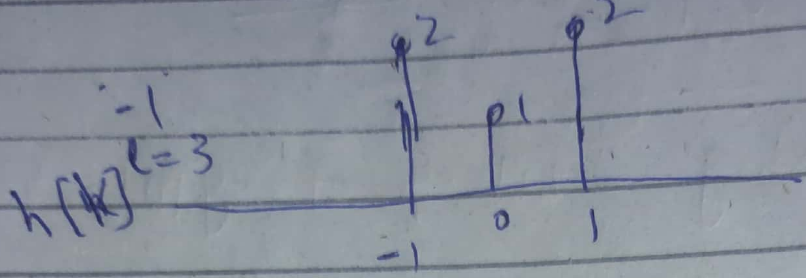


h(+1-k)



Output sequence $= \{1, \underset{\uparrow}{4}, 5.5, 3, 0.5, 0.5, -1\}$



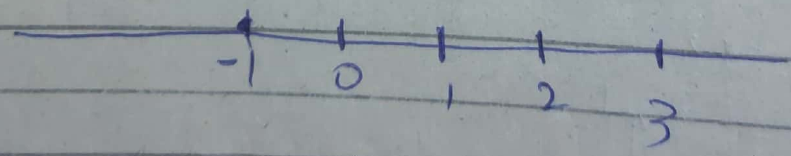


h[-k]

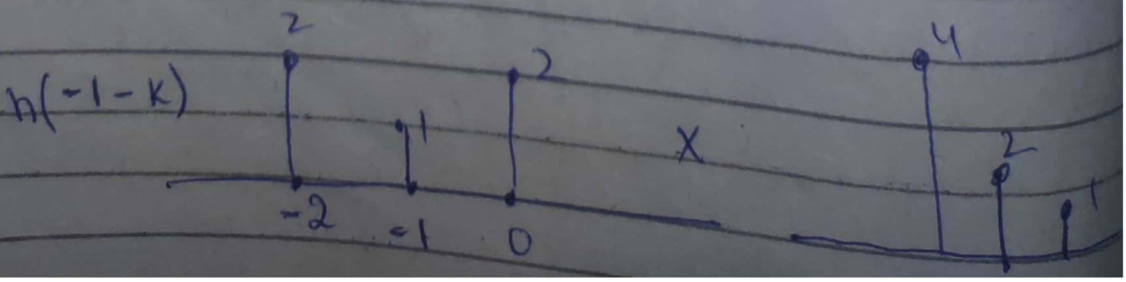
St. ind of y[n] = -1

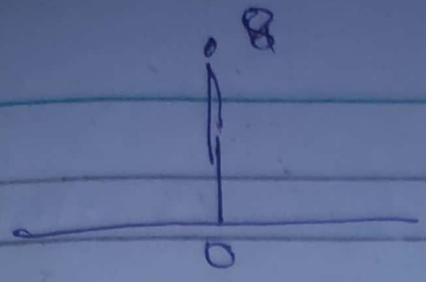
len of y(n) = 3+3-1 = 5

y(n)



For y n = -1 :

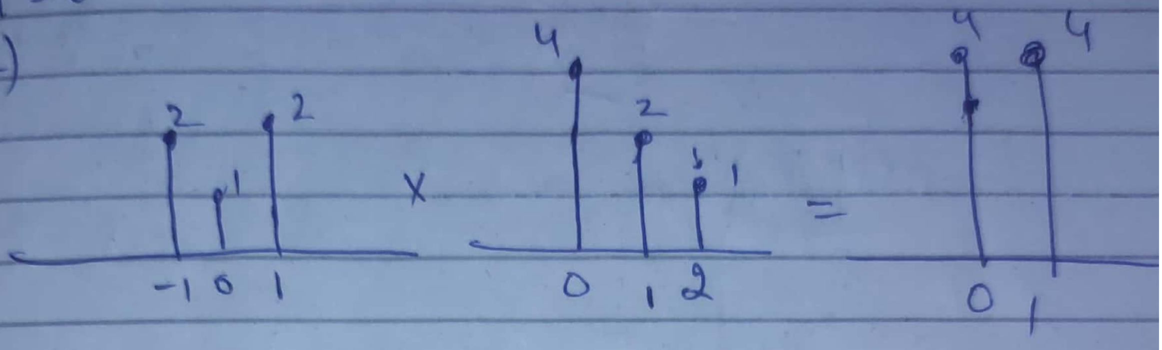




$\boxed{y(-1) = 8}$

For $n = 0$

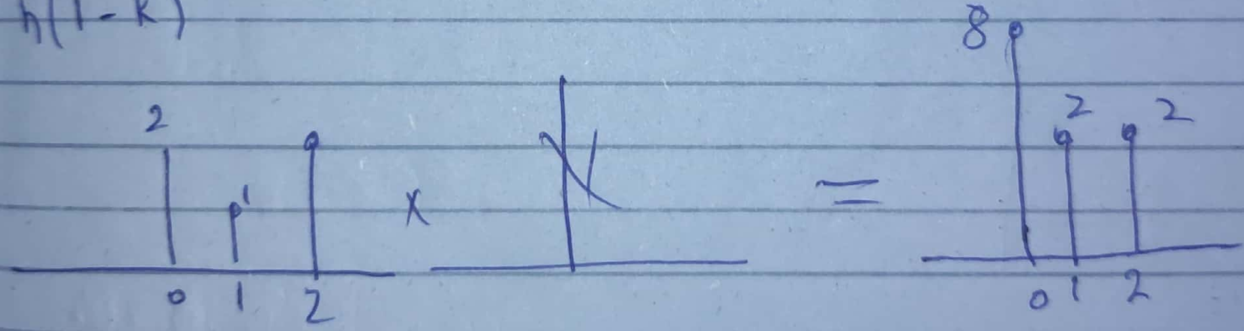
$h(-k)$



$y(0) = 8$

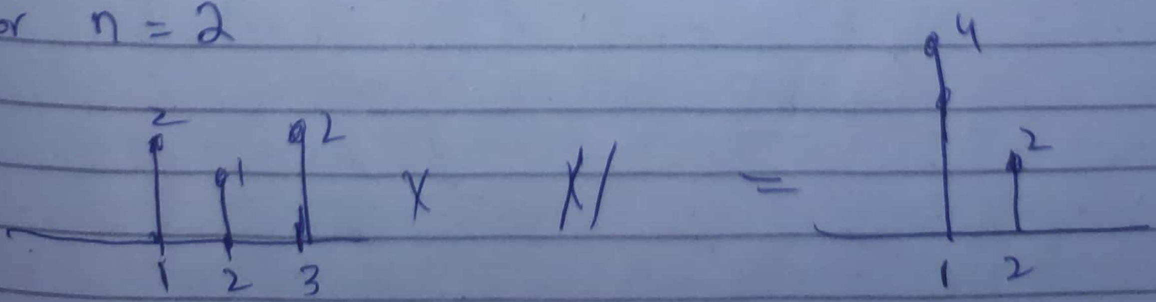
For $n = 1$;

$h(1-k)$



$y(1) = 12$

For $n = 2$



$y(2) = 6$

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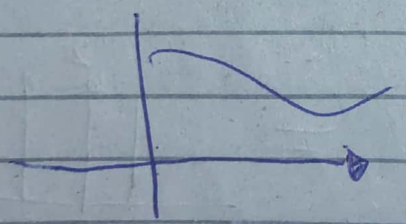
Time Domain	Frequency Domain	Time-Frequency Domain
	DTFS	Higher from Curriculum
	DFS	3D Graph



DTFT$(X(e^{j\omega}))$

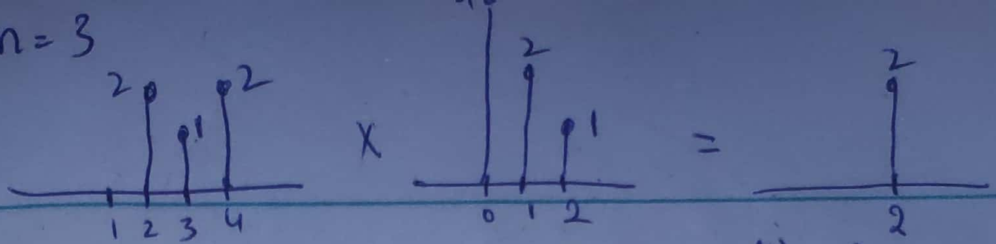
DFT$(X(k))$

$\rightarrow$ FFT

$h(-n)$

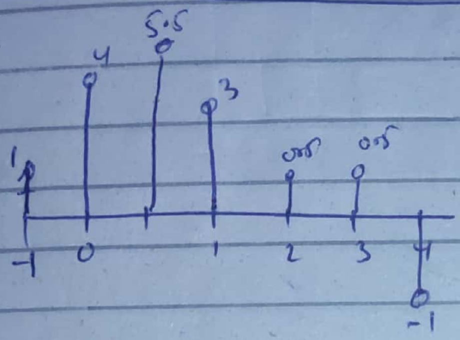
$h(-n-1)$

For $n = 3$



$y(3) = 2$

$y(n) = 8, 8, 12$



$y(n)$	$h(n)$: 1	2	1	-1
$x(n)$: 1	1	2	1	-1
2	2	4	2	-2
0.5	0.5	1	0.5	-0.5
1	1	2	1	-1

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$X(e^{j\omega})$ → Complex
↳ continous

$x(t)$
↓
$x(n)$ → discrete
↓
DTFT $(x(n))$ → $X(\omega)$ → digital freq.

DFT → output is discrete
↳ $x(k)$
↳ discrete.

$$x(n) \xrightarrow[\sum_{-\infty}^{\infty}]{F} X(e^{j\omega})$$

$$x(n) \xleftarrow{\int} X(e^{j\omega})$$

$$\begin{tikzcd}
X(n) \arrow[r, "DTFT"] & X(e^{j\omega}) \arrow[r, "DFT"] \arrow[l, bend left, "IDTFT"] & X(k) \arrow[ll, bend left=60, "IDFT"]
\end{tikzcd}$$

$$DTFT\{x(n)\} = \sum_{n=0}^{\infty} x(n).e^{-j\omega n}$$

$$X(\omega) = \sum_{n=0}^{\infty} x(n).e^{-j\omega n}$$

$$x(n) \longleftrightarrow X(e^{j\omega})$$

$N \rightarrow$ No. of samples

$$\omega = \pi/4 = \underset{k}{1} \cdot \frac{2\pi}{8} = \frac{\pi}{4}$$

$$= 2 \cdot 2\pi/8 = \frac{\pi}{2}$$

$$3 \cdot 2\pi/8 = 3\pi/4$$

$$\vdots$$

$$7 \cdot 2\pi/8 = 7\pi/4$$

generally we can write as:

$$\boxed{\omega = k \cdot \frac{2\pi}{N}}$$

we sampled over frequency.

Now DFT is:

$$X(k) = \sum_{n=0}^{N-1} x(k) \cdot e^{-j\frac{2\pi k}{N} n}$$

$$x(n) \xrightarrow{DTFT} X(\omega) \qquad \sum_{n=0}^{\infty} x(n) \cdot e^{-j\omega n}$$

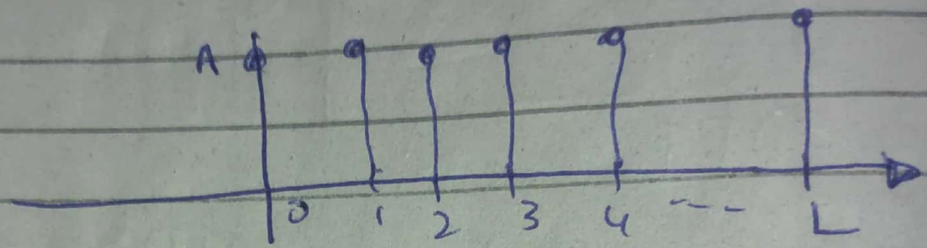
$$x(n) \xleftarrow{IDTFT} X(\omega) \qquad \frac{1}{2\pi} \int_{0}^{\infty} X(\omega) \cdot e^{j\omega n}$$

$$x(n) \xrightarrow{DFT} X(k) \qquad \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi nk}{N}}$$

$$x(n) \xleftarrow{IDFT} X(k) \qquad \sum_{k=0}^{N-1} X(k) \cdot e^{j\frac{2\pi nk}{N}}$$

Ex 2

$$x(n) = \begin{cases} A & 0 \le n \le L \\ 0 & \text{otherwise} \end{cases}$$



DTFT $\longrightarrow$ $X(\omega) = ?$

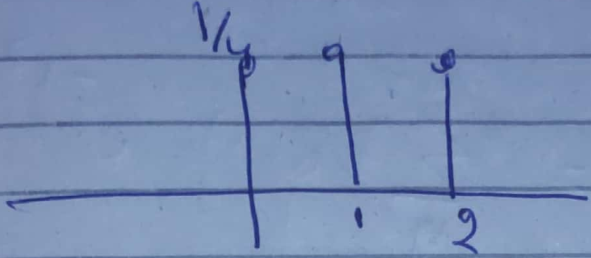
$$X(\omega) = \sum_{n=0}^{L} A \cdot e^{-j\omega n}$$

$$= A \cdot \sum_{n=0}^{L} e^{-j\omega n}$$

$$=$$

Ex ③

$$x(n) = \begin{cases} 1/4 & 0 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$$



$N = 3$

$k = 0, 1, 2$

DFT $X(k) = ?$

$x(n) = a^n \qquad 0 \le n \le N-1$

$$X(k) = \sum_{n=0}^{N-1} a^n \cdot e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \left(a \cdot e^{-j\frac{2\pi k}{N}}\right)^n$$

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Compute the N-Points DFT

$$x(n) = \delta(n)$$

$$X(k) = \sum_{n=0}^{N-1} \delta(n) \cdot e^{-j\frac{2\pi k}{N}n}$$

$$X(k) = \delta(0) \cdot e^{\frac{-j2\pi}{N}k(0)}$$

$$X(k) = 1 + 0j$$

Mag

phase

$$mag = \sqrt{1^2+0^2} = 1$$

$$phase = \tan^{-1}(b/a) = \tan^{-1}\left(\frac{0}{1}\right) = 0$$

$$x(n) = \delta(n-n_0) \quad 0 < n_0 < N$$

$$X(k) = \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j\frac{2\pi k}{N}n}$$

$$X(k) = 1 \; e^{-\frac{j2\pi k n_0}{N}}$$

at

$$\delta(n-n_0) = \begin{cases} 1 ; & n = n_0 \\ 0 ; & n \neq n_0 \end{cases}$$

$$x(k) = e^{-j\frac{2\pi k}{N} n_0}$$

delay in time, produces phase shift in frequency domain.

(iii) $x(n) = a^n \qquad 0 \leq n \leq N-1$

$$x(k) = \sum_{n=0}^{N-1} a^n . e^{-j\frac{2\pi k}{N} n}$$

$$x(k) = \sum_{n=0}^{N-1} \left(a . e^{-j\frac{2\pi k}{N}}\right)^n$$

$$x(k) = \frac{(1) \left(a.e^{-j\frac{2\pi k n}{N}} - 1\right)}{a . e^{-j\frac{2\pi k}{N}} - 1}$$

$$x(K) = \frac{ae^{-j\frac{2\pi k n}{N}} - 1}{a . e^{-j\frac{2\pi k}{N}} - 1}$$

$$\sum_{N1}^{N2} a^n = \frac{a^{N_1} - a^{N_2-1}}{1-a}$$

$$e^{-j2k\pi} = 1$$

$$x(n) = \begin{Bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 0 \end{Bmatrix}$$ For $N=4$

$$x(k) = \sum_{n=0}^{N-1} x(n) . e^{-j\frac{2\pi kn}{N}} ; \quad k=$$

$$x(k) = x(0) + x(1).e \quad +$$

Inverse DFT:-

$$X(k) = \{ \underset{0}{6}, \underset{1}{-2+2j}, \underset{2}{-2}, \underset{3}{-2}, \underset{4}{-2j} \}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{\frac{-j2\pi kn}{N}}$$

Properties of DFT:-

Linearity:-

$$DFT\{a_1 x_1(t) + a_2 x_2(t)\} =$$

$$= \sum_{n=0}^{N-1} a_1 x_1(n) . e^{\frac{-j2\pi K}{N}} + \sum_{n=0}^{N-1} a_2 x_2(n) e^{\frac{-j2\pi K}{N}}$$

$$= a_1 \sum_{n=0}^{N-1} x_1(n) . e^{\frac{-j2\pi K}{N}} + a_2 \sum_{n=0}^{N-1} x_2(n) . e^{\frac{-j2\pi K}{N}}$$

$$= a_1 x_1(K) + a_2 x_2(K)$$

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Periodicity:-

$$x(n+N) = x(n)$$
$$X(K+N) = X(K)$$

Proof:-

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) \cdot e^{\frac{j2\pi kn}{N}}$$

$$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(k)\, e^{\frac{j2\pi k(n+N)}{N}}$$

$$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(k) \cdot e^{\frac{j2\pi kn}{N}} \cdot e^{j2\pi K}$$

$$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(k) \cdot e^{\frac{j2\pi kn}{N}}$$