Solution Manual

ENGINEERING ECONOMIC ANALYSIS

Ninth Edition

  

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Chapter 1: Making Economic Decisions

1-1

A survey of students answering this question indicated that they thought about 40% of their decisions were conscious decisions.

1-2

(a)	Yes.	The choice of an engine has important money consequences so would be suitable for engineering economic analysis.
(b)	Yes.	Important economic- and social- consequences. Some might argue the social consequences are more important than the economics.
(c)	?	Probably there are a variety of considerations much more important than the economics.
(d)	No.	Picking a career on an economic basis sounds terrible.
(e)	No.	Picking a wife on an economic basis sounds even worse.

1-3

Of the three alternatives, the $150,000 investment problem is *most* suitable for economic analysis. There is not enough data to figure out how to proceed, but if the 'desirable interest rate' were 9%, then foregoing it for one week would mean a loss of:

$$^{1}/_{52}\ (0.09) = 0.0017 = 0.17\%$$

immediately. It would take over a year at 0.15% more to equal the 0.17% foregone now.

The chocolate bar problem is suitable for economic analysis. Compared to the investment problem it is, of course, trivial.

Joe's problem is a real problem with serious economic consequences. The difficulty may be in figuring out what one gains if he pays for the fender damage, instead of having the insurance company pay for it.

1-4

Gambling, the stock market, drilling for oil, hunting for buried treasure—there are sure to be a lot of interesting answers. Note that if you could double your money every day, then:

$$2^{x}\ (\$300) = \$1,000,000$$

and x is less than 12 days.

1-5

Maybe their stock market 'systems' don't work!

1-6

It may look simple to the owner because *he* is not the one losing a job. For the three machinists it represents a major event with major consequences.

1-7

For most high school seniors there probably are only a limited number of colleges and universities that are feasible alternatives. Nevertheless, it is still a complex problem.

1-8

It really is not an economic problem solely — it is a complex problem.

1-9

Since it takes time and effort to go to the bookstore, the minimum number of pads might be related to the smallest saving worth bothering about. The maximum number of pads might be the quantity needed over a reasonable period of time, like the rest of the academic year.

1-10

While there might be a lot of disagreement on the 'correct' answer, only automobile insurance represents a *substantial amount of money* and a situation where money might be the *primary* basis for choosing between alternatives.

1-11

The overall problems are all complex. The student will have a hard time coming up with examples that are truly *simple* or *intermediate* until he/she breaks them into smaller and smaller sub-problems.

1-12

These questions will create disagreement. None of the situations represents rational decision-making.

Choosing the same career as a friend might be OK, but it doesn't seem too rational.

Jill didn't consider all the alternatives.

Don thought he was minimizing cost, but it didn't work. Maybe rational decision-making says one should buy better tools that will last.

1-13

Possible objectives for NASA can be stated in general terms of space exploration or the generation of knowledge or they can be stated in very concrete terms. President Kennedy used the latter approach with a year for landing a man on the moon to inspire employees. Thus the following objectives as examples are concrete. No year is specified here, because unlike President Kennedy we do not know what dates may be achievable.

Land a man safely on Mars and return him to earth by ______.
Establish a colony on the moon by ______.
Establish a permanent space station by ______.
Support private sector tourism in space by ______.
Maximize fundamental knowledge about science through *x* probes per year or for *$y* per year.
Maximize applied knowledge about supporting man's activities in space through *x* probes per year or for *$y* per year.

Choosing among these objectives involves technical decisions (some objectives may be prerequisites for others), political decisions (balance between science and applied knowledge for man's activities), and economic decisions (how many dollars per year can be allocated to NASA).

However, our favorite is a colony on the moon, because a colony is intended to be permanent and it would represent a new frontier for human ingenuity and opportunity. Evaluation of alternatives would focus on costs, uncertainties, and schedules. Estimates of these would rely on NASA's historical experience, expert judgment, and some of the estimating tools discussed in Chapter 2.

1-14

This is a challenging question. One approach might be:

(a) Find out what percentage of the population is left-handed.
(b) What is the population of the selected hometown?
(c) Next, market research might be required. With some specific scissors (quality and price) in mind, ask a random sample of people if they would purchase the scissors. Study the responses of both left-handed and right-handed people.
(d) With only two hours available, this is probably all the information one could collect. From the data, make an estimate.

A different approach might be to assume that the people interested in left handed scissors in the future will be about the same as the number who bought them in the past.

(a) Telephone several sewing and department stores in the area. Ask two questions:
    (i) How many pairs of scissors have you sold in one year (or six months or?).
    (ii) What is the ratio of sales of left-handed scissors to regular scissor?

(b) From the data in (a), estimate the future demand for left-handed scissors.

Two items might be worth noting.

1. Lots of scissors are universal, and equally useful for left- and right-handed people.
2. Many left-handed people probably never have heard of left-handed scissors.

1-15

Possible alternatives might include:

1. Live at home.
2. A room in a private home in return for work in the garden, etc.
3. Become a Resident Assistant in a University dormitory.
4. Live in a camper-or tent- in a nearby rural area.
5. Live in a trailer on a construction site in return for 'keeping an eye on the place.'

1-16

A common situation is looking for a car where the car is purchased from either the first dealer or the most promising alternative from the newspaper's classified section. This may lead to an acceptable or even a good choice, but it is highly unlikely to lead to the best choice. A better search would begin with *Consumer Reports* or some other source that summarizes many models of vehicles. While reading about models, the car buyer can be identifying alternatives and clarifying which features are important. With this in mind, several car lots can be visited to see many of the choices. Then either a dealer or the classifieds can be used to select the best alternative.

1-17

Choose the better of the undesirable alternatives.

1-18

(a) Maximize the difference between output and input.
(b) Minimize input.
(c) Maximize the difference between output and input.
(d) Minimize input.

1-19

(a) Maximize the difference between output and input.
(b) Maximize the difference between output and input.
(c) Minimize input.
(d) Minimize input.

1-20

Some possible answers:

1. There are benefits to those who gain from the decision, but no one is harmed. (Pareto Optimum)
2. Benefits flow to those who need them most. (Welfware criterion)

3. Minimize air pollution or other specific item.
4. Maximize total employment on the project.
5. Maximize pay and benefits for some group (e.g., union members)
6. Most aesthetically pleasing result.
7. Fit into normal workweek to avoid overtime.
8. Maximize the use of the people already within the company.

1-21

Surely planners would like to use criterion (a). Unfortunately, people who are relocated often feel harmed, no matter how much money, etc., they are given. Thus planners consider criterion (a) unworkable and use criterion (b) instead.

1-22

In this kind of highway project, the benefits typically focus on better serving future demand for travel measured in vehicles per day, lower accident rates, and time lost due to congestion. In some cases, these projects are also used for urban renewal of decayed residential or industrial areas, which introduces other benefits.

The costs of these projects include the money spent on the project, the time lost by travelers due to construction caused congestion, and the lost residences and businesses of those displaced. In some cases, the loss may be intangible as a road separates a neighborhood into two pieces. In other cases, the loss may be due to living next to a source of air, noise, and visual pollution.

1-23

The remaining costs for the year are:

Alternatives:

1.	To stay in the residence the rest of the year		
	Food: 8 months at $120/month	**Total**	= $960
2.	To stay in the residence the balance of the first semester; apartment for second semester		
	Housing: 4 ½ months x $80 apartment - $190 residence		= $170
	Food: 3 ½ months x $120 + 4 ½ x $100		= $870
		Total	= $1,040
3.	Move into an apartment now		
	Housing: 8 mo x $80 apartment – 8 x $30 residence		= $400
	Food: 8 mo x $100		= $800
		Total	= $1,200

Ironically, Jay had sufficient money to live in an apartment all year. He originally had $1,770 ($1,050 + 1 mo residence food of $120 plus $600 residence contract cost). His cost for an apartment for the year would have been 9 mo x ($80 + $100) = $1,620. Alternative 3 is not possible because the cost exceeds Jay’s $1,050. Jay appears to prefer Alternative 2, and he has sufficient money to adopt it.

1-24

'In decision-making the model is mathematical.'

1-25

The situation is an example of the failure of a low-cost item that may have major consequences in a production situation. While there are alternatives available, one appears so obvious that that foreman discarded the rest and asks to proceed with the replacement.

One could argue that the foreman, or the plant manager, or both are making decisions. There is no single 'right' answer to this problem.

1-26

While everyone might not agree, the key decision seems to be in providing Bill's dad an opportunity to judge between purposely-limited alternatives. Although suggested by the clerk, it was Bill's decision.

(One of my students observed that his father would not fall for such a simple deception, and surely would insist on the weird shirt as a subtle form of punishment.)

1-27

Plan A	Profit	= Income – Cost	= \$800 - \$600	= \$200/acre
Plan B	Profit	= Income – Cost	= \$1,900 - \$1,500	= \$400/acre
Plan C	Profit	= Income – Cost	= \$2,250 - \$1,800	= \$450/acre
Plan D	Profit	= Income – Cost	= \$2,500 - \$2,100	= \$400/acre

To maximize profit, <u>choose Plan C</u>.

1-28

Each student's answer will be unique, but there are likely to be common threads. Alternatives to their current university program are likely to focus on other fields of engineering and science, but answers are likely to be distributed over most fields offered by the university. Outcomes include degree switches, courses taken, changing dates for expected graduation, and probable future job opportunities.

At best criteria will focus on joy in the subject matter and a good match for the working environment that pleases that particular student. Often economic criteria will be mentioned, but these are more telling when comparing engineering with the liberal arts than when comparing engineering fields. Other criteria may revolve around an inspirational teacher or an influential friend or family member. In some cases, simple availability is a driver. What degree programs are available at a campus or which programs will admit a student with a 2.xx GPA in first year engineering.

At best the process will follow the steps outlined in this chapter. At the other extreme, a student's major may have been selected by the parent and may be completely mismatched to the student's interests and abilities.

Students shouldn't lightly abandon a major, as changing majors represents real costs in time, money, and effort and real risks that the new choice will be no better a fit. Nevertheless, it is a large mistake to not change majors when a student now realizes the major is not for them.

1-29

The most common large problem faced by undergraduate engineering students is where to look for a job and which offer to accept. This problem seems ideal for listing student ideas on the board or overhead transparencies. It is also a good opportunity for the instructor to add more experienced comments.

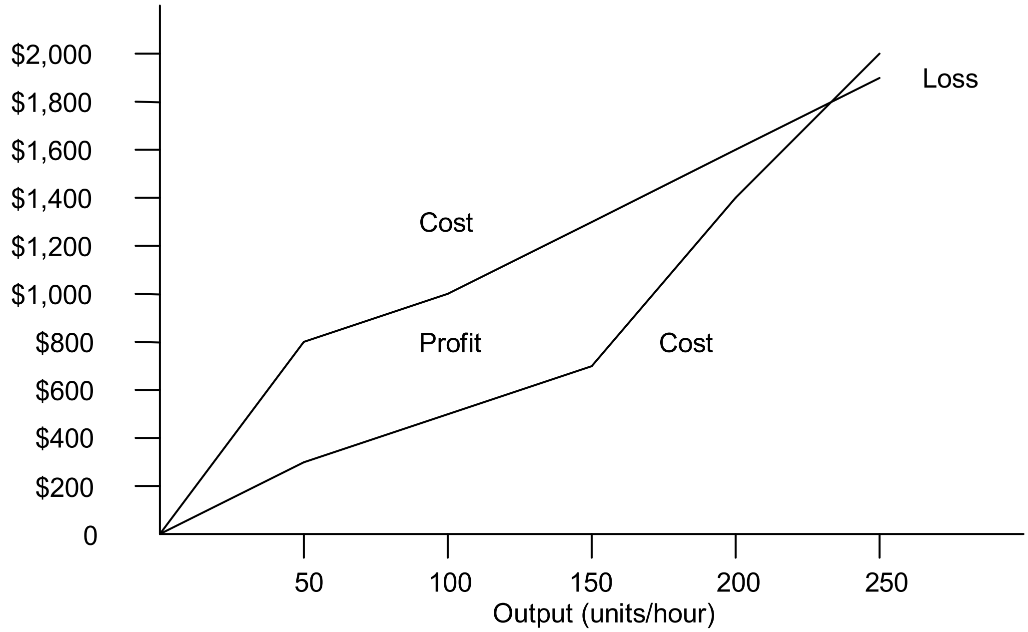
1-30

Test marketing and pilot plant operation are situations where it is hoped that solving the sub-problems gives a solution to the large overall problem. On the other hand, Example 3-1 (shipping department buying printing) is a situation where the sub-problem does not lead to a proper complex problem solution.

1-31

(a) The suitable criterion is to maximize the difference between output and input. Or simply, maximize net profit. The data from the graphs may be tabulated as follows:

Output Units/Hour	Total Cost	Total Income	Net Profit
50	\$300	\$800	\$500
100	\$500	\$1,000	\$500
150	\$700	\$1,350	\$650 □
200	\$1,400	\$1,600	\$200
250	\$2,000	\$1,750	-\$250



(b) *Minimum input* is, of course, zero, and *maximum output* is 250 units/hr (based on the graph). Since one cannot achieve maximum output with minimum input, the statement makes no sense.

1-32

Itemized expenses: $0.14 x 29,000 km + $2,000 = $6,060
Based on Standard distance Rate: $0.20 x $29,000 = $5,800

Itemizing produces a larger reimbursement.

Breakeven: Let x = distance (km) at which both methods yield the same amount.

x = $2,000/($0.20 - $0.14) = <u>33,333 km</u>

1-33

The fundamental concept here is that we will trade an hour of study in one subject for an hour of study in another subject so long as we are improving the total results. The stated criterion is to 'get as high an average grade as possible in the combined classes.' (This is the same as saying 'get the highest combined total score.')

Since the data in the problem indicate that additional study always increases the grade, the question is how to apportion the available 15 hours of study among the courses. One might begin, for example, assuming five hours of study on each course. The combined total score would be 190.

Decreasing the study of mathematics one hour reduces the math grade by 8 points (from 52 to 44). This hour could be used to increase the physics grade by 9 points (from 59 to 68). The result would be:

Math	4 hours	44
Physics	6 hours	68
Engr. Econ.	5 hours	79
Total	15 hours	191

Further study would show that the best use of the time is:

Math	4 hours	44
Physics	7 hours	77
Engr. Econ.	4 hours	71
Total	15 hours	192

1-34

Saving = 2 [$185.00 + (2 x 150 km) ($0.375/km)] = $595.00/week

1-35

Area A Preparation Cost = 2 x 10^6 x $2.35 = $4,700,000

Area B Difference in Haul

0.60 x 8 km	= 4.8 km
0.20 x -3 km	= -0.6 km
0.20 x 0	= 0 km
Total	= 4.2 km average additional haul

Cost of additonal haul/load = 4.2 km/25 km/hr x $35/hr = $5.88

Since truck capacity is 20 m^3:
Additional cost/cubic yard = $5.88/20 m^3 = $0.294/$m^3$

For 14 million cubic meters:
Total Cost = 14 x 10^6 x $0.294 = $4,116,000

Area B with its lower total cost is preferred.

1-36

12,000 litre capacity = 12 m^3 capacity
Let: L = tank length in m
d = tank diameter in m

The volume of a cylindrical tank equals the end area x length:

$$\text{Volume} = (\Pi/4)\, d^2L = 12\ m^3$$

$$L = (12 \times 4)/(\Pi\, d^2)$$

The total surface area is the two end areas + the cylinder surface area:

$$S = 2\,(\Pi/4)\,d^2 + \Pi\,dL$$

Substitute in the equation for L:

$$S = (\Pi/2)\,d^2 + \Pi d\,[(12 \times 4)/(\Pi d^2)]$$
$$= (\Pi/2)d^2 + 48d^{-1}$$

Take the first derivative and set it equal to zero:

$$dS/dd \quad = \Pi d - 48d^{-2} = 0$$

$$\Pi d = 48/d^2$$

$$d^3 = 48/\Pi \qquad = 15.28$$

$$d = 2.48 \text{ m}$$

Subsitute back to find L:

$$L = (12 \times 4)/(\Pi d^2) \qquad = 48/(\Pi(2.48)^2) \qquad = 2.48 \text{ m}$$

Tank diameter = 2.48 m (≅2.5 m)
Tank length = 2.48 m (≅2.5 m)

1-37

Quantity Sold per week	Selling Price	Income	Cost	Profit
300 packages	\$0.60	\$180	\$104	\$75
600	\$0.45	\$270	\$210	\$60
1,200	\$0.40	\$480	\$336	\$144
1,700	\$0.33	\$561	\$425*	\$136
			\$400**	\$161 □
2,500	\$0.26	\$598	\$460	\$138

* buy 1,700 packages at \$0.25 each
** buy 2,000 packages at \$0.20 each

Conclusion: Buy 2,000 packages at \$0.20 each. Sell at \$0.33 each.

1-38

Time period	Daily sales in time period	Cost of groceries	Hourly Cost	Hourly Profit
0600- 0700	\$20	\$14	\$10	-\$4
0700- 0800	\$40	\$28	\$10	+\$2
0800- 0900	\$60	\$42	\$10	+\$8
0900-1200	\$200	\$140	\$30	+\$30
1200- 1500	\$180	\$126	\$30	+\$24
1500- 1800	\$300	\$210	\$30	+\$60
1800- 2100	\$400	\$280	\$30	+\$90
2100- 2200	\$100	\$70	\$10	+\$20
2200- 2300	\$30	\$21	\$10	-\$1

2300- 2400	$60	$42	$10	+$8
2400- 0100	$20	$14	$10	-$4

The first profitable operation is in 0700- 0800 time period. In the evening the 2200- 2300 time period is unprofitable, but next hour's profit more than makes up for it.

Conclusion: Open at 0700, close at 2400.

1-39

Alternative	Price	Net Income per Room	Outcome Rate	No. Room	Net Income
1	$35	$23	100%	50	$1,150
2	$42	$30	94%	47	$1,410
3	$48	$36	80%	40	$1,440
4	$54	$42	66%	33	$1,386
5	$48	$36	70%	35	$1,260
6	$54	$42	68%	34	$1,428
7	$62	$50	66%	33	$1,650
8	$68	$56	56%	28	$1,568

To maximize net income, Jim should not advertise and charge $62 per night.

1-40

Profit = Income- Cost
= PQ- C where $PQ = 35Q - 0.02Q^2$
$C = 4Q + 8{,}000$

$d(\text{Profit})/dQ = 31 - 0.04Q = 0$
Solve for Q
$Q = 31/0.04 = 775$ units/year

$d^2(\text{Profit})/dQ^2 = -0.04$

The negative sign indicates that profit is maximum at Q equals 775 units/year.

Answer: Q = 775 units/year

1-41

Basis: 1,000 pieces

Individual Assembly:	$22.00 x 2.6 hours x 1,000 = $57,200	$57.20/unit
Team Assembly:	4 x $13.00 x 1.0 hrs x 1,000= $52,00	$52.00/unit

Team Assembly is less expensive.

1-42

Let t = time from the present (in weeks)
Volume of apples at any time = (1,000 + 120t – 20t)
Price at any time = \$3.00 - \$0.15t

Total Cash Return (TCR) = (1,000 + 120t – 20t) (\$3.00 - \$0.15t)
= \$3,000 + \$150t - \$15t^2

This is a minima-maxima problem.
Set the first derivative equal to zero and solve for t.

dTCR/dt = \$150 - \$30t = 0
t = \$150/\$30 = 5 wekes

d^2TCR/dt^2 = -10
(The negative sign indicates the function is a maximum for the critical value.)

At t = 5 weeks:

Total Cash Return (TCR) = \$3,000 + \$150 (5) - \$15 (25) = \$3,375

Chapter 2: Engineering Costs and Cost Estimating

2-1

This is an example of a 'sunk cost.' The $4,000 is a past cost and should not be allowed to alter a subsequent decision unless there is some real or perceived effect. Since either home is really an individual plan selected by the homeowner, each should be judged in terms of value to the homeowner vs. the cost. On this basis the stock plan house appears to be the preferred alternative.

2-2

Unit Manufacturing Cost
(a) Daytime Shift = ($2,000,000 + $9,109,000)/23,000 = $483/unit
(b) Two Shifts = [($2,400,000 + (1 + 1.25) ($9,109,000)]/46,000
= $497.72/unit

Second shift increases unit cost.

2-3

(a) Monthly Bill:

50 x 30	= 1,500 kWh @ $0.086	= $129.00
	= 1,300 kWh @ $0.066	= $85.80
Total	= 2,800 kWh	= $214.80

Average Cost = $214.80/2,800 = $129.00
Marginal Cost (cost for the next kWh) = $0.066 because the 2,801st kWh is in the 2nd bracket of the cost structure.

($0.066 for 1,501-to-3,000 kWh)

(b) Incremental cost of an additional 1,200 kWh/month:

200 kWh x $0.066	= $13.20
1,000 kWh x $0.040	= $40.00
1,200 kWh	$53.20

(c) New equipment:
Assuming the basic conditions are 30 HP and 2,800 kWh/month
Monthly bill with new equipment installed:

50 x 40	= 2,000 kWh at $0.086	= $172.00
	900 kWh at $0.066	= $59.40
	2,900 kWh	$231.40

Incremental cost of energy = $231.40 - $214.80 = $16.60
Incremental unit cost = $16.60/100 = $0.1660/kWh

2-4

x = no. of maps dispensed per year

(a) Fixed Cost (I) = \$1,000
(b) Fixed Cost (II) = \$5,000
(c) Variable Costs (I) = 0.800
(d) Variable Costs (II) = 0.160
(e) Set Total Cost (I) = Total Cost (II)
\$1,000 + 0.90 x = \$5,000 + 0.10 x
thus x = 5,000 maps dispensed per year.
The student can visually verify this from the figure.
(f) System I is recommended if the annual need for maps is <5,000
(g) System II is recommended if the annual need for maps is >5,000
(h) Average Cost @ 3,000 maps:
TC(I) = (0.9) (3.0) + 1.0 = 3.7/3.0 = \$1.23 per map
TC(II) = (0.1) (3.0) + 5.0 = 5.3/3.0 = \$1.77 per map
Marginal Cost is the variable cost for each alternative, thus:
Marginal Cost (I) = \$0.90 per map
Marginal Cost (II) = \$0.10 per map

2-5

$C = \$3,000,000 - \$18,000Q + \$75Q^2$

Where C = Total cost per year
Q = Number of units produced per year

Set the first derivative equal to zero and solve for Q.

$dC/dQ = -\$18,000 + \$150Q = 0$

$Q = \$18,000/\$150 = 120$

Therefore total cost is a minimum at Q equal to 120. This indicates that production below 120 units per year is most undesirable, as it costs more to produce 110 units than to produce 120 units.

Check the sign of the second derivative:

$d^2C/dQ^2 = +\$150$

The + indicates the curve is concave upward, ensuring that Q = 120 is the point of a minimum.

Average unit cost at Q = 120/year:
$= [\$3,000,000 - \$18,000\ (120) + \$75\ (120)^2]/120 \quad = \$16,000$

Average unit cost at Q = 110/year:

= [$3,000,000 - $18,000 (110) + $75 (120)^2]/110 = $17,523

One must note, of course, that 120 units per year is *not* necessarily the optimal level of production. Economists would remind us that the optimum point is where Marginal Cost = Marginal Revenue, and Marginal Cost is increasing. Since we do not know the Selling Price, we cannot know Marginal Revenue, and hence we cannot compute the optimum level of output.

We can say, however, that if the firm is profitable at the 110 units/year level, then it will be much more profitable at levels greater than 120 units.

2-6

x = number of campers

(a) Total Cost = Fixed Cost + Variable Cost
= $48,000 + $80 (12) x
Total Revenue= $120 (12) x

(b) Break-even when Total Cost = Total Revenue
$48,000 + $960 x = $1,440 x
$4,800 = $480 x
x = 100 campers to break-even

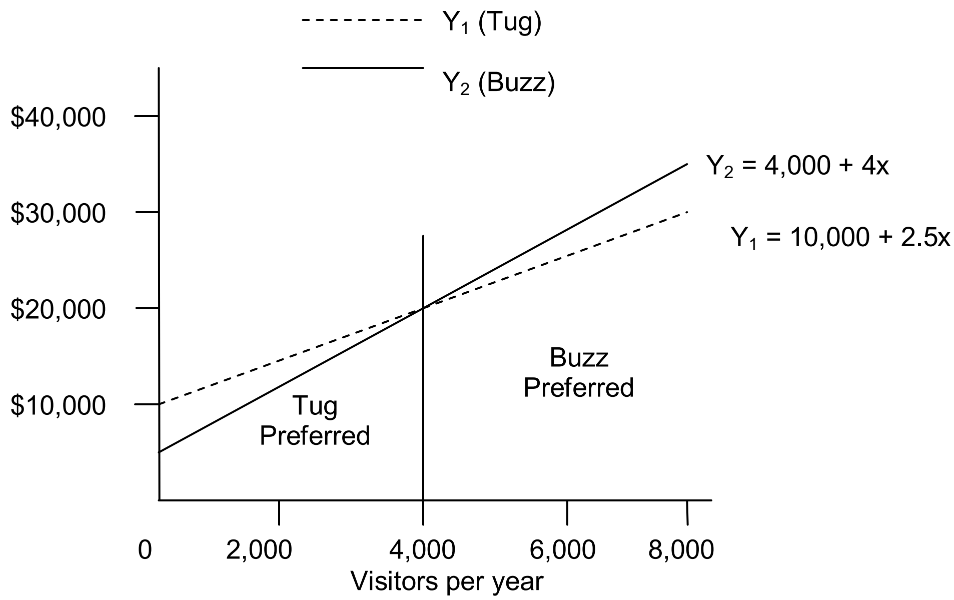
(c) capacity is 200 campers
80% of capacity is 160 campers
@ 160 campers x = 160
Total Cost = $48,000 + $80 (12) (160) = $201,600
Total Revenue= $120 (12) (160) = $230,400
Profit = Revenue – Cost = $230,400 - $201,600 = $28,800

2-7

(a) x = number of visitors per year
Break-even when: Total Costs (Tugger) = Total Costs (Buzzer)
$10,000 + $2.5 x = $4,000 + $4.00 x
x = 400 visitors is the break-even quantity

(b) See the figure below:

X	Y1 (Tug)	Y2 (Buzz)
0	10,000	4,000
4,000	20,000	20,000
8,000	30,000	36,000



2-8

x = annual production

(a) Total Revenue= ($200,000/1,000) x = $200 x

(b) Total Cost = $100,000 + ($100,000/1,000)x = $100,000 + $100 x

(c) Set Total Cost = Total Revenue
$200 x = $100,000 + $100 x
x = 1,000 units per year
The student can visually verify this from the figure.

(d) Total Revenue= $200 (1,500) = $300,000
Total Cost = $100,000 + $100 (150 = $250,000
Profit = $300,000 - $250,000 = $50,000

2-9

x = annual production

Let's look at the graphical solution first, where the cost equations are:
Total Cost (A) = $20 x + $100,000
Total Cost (B) = $5 x + $200,000
Total Cost (C) = $7.5 x + $150,000

[See graph below]

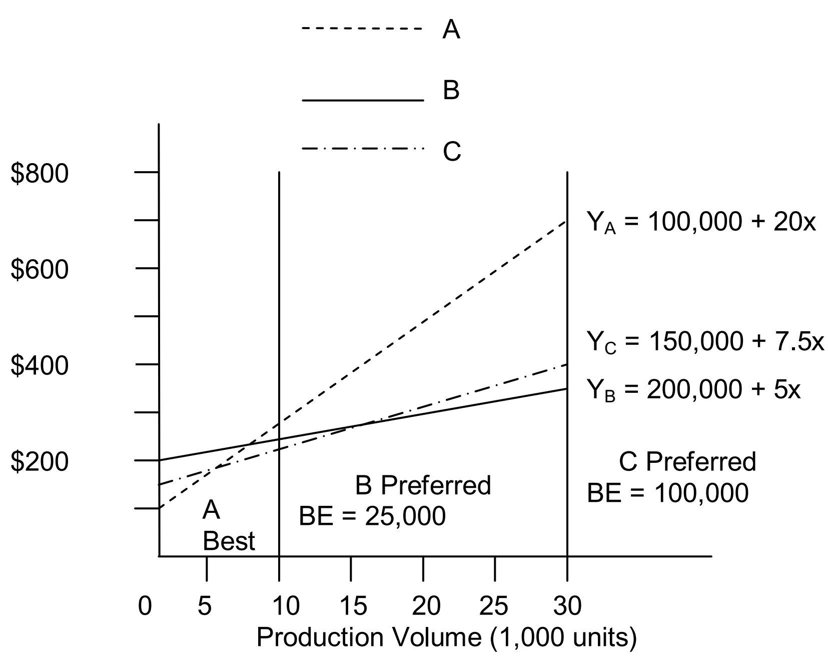
Quatro Hermanas wants to minimize costs over all ranges of x. From the graph we see that there are three break-even points: A & B, B & C, and A & C. Only A & C and B & C are necessary to determine the minimum cost alternative over x. Mathematically the break-even points are:

A & C: \$20 x + \$100,000 = \$7.5 x + \$150,000 x = 4,000
B & C: \$5 x + \$200,000 = \$8.5 x + \$150,000 x = 20,000

Thus our recommendation is, if:

$0 \leq x < 4,000$ choose Alternative A
$4,000 < x < 20,000$ choose Alternative C
$20,000 \leq x$ 30,000 choose Alternative B

X	A	B	C
0	100	200	150
10	300	250	225
20	500	300	300
30	700	350	375



2-10

x = annual production rate

(a) There are three break-even points for total costs for the three alternatives
A & B: \$20.5 x + \$100,000 = \$10.5 x + \$350,000
x = 25,000
B & C: \$10.5 x + \$350,000 = \$8 x + \$600,000
x = 100,000

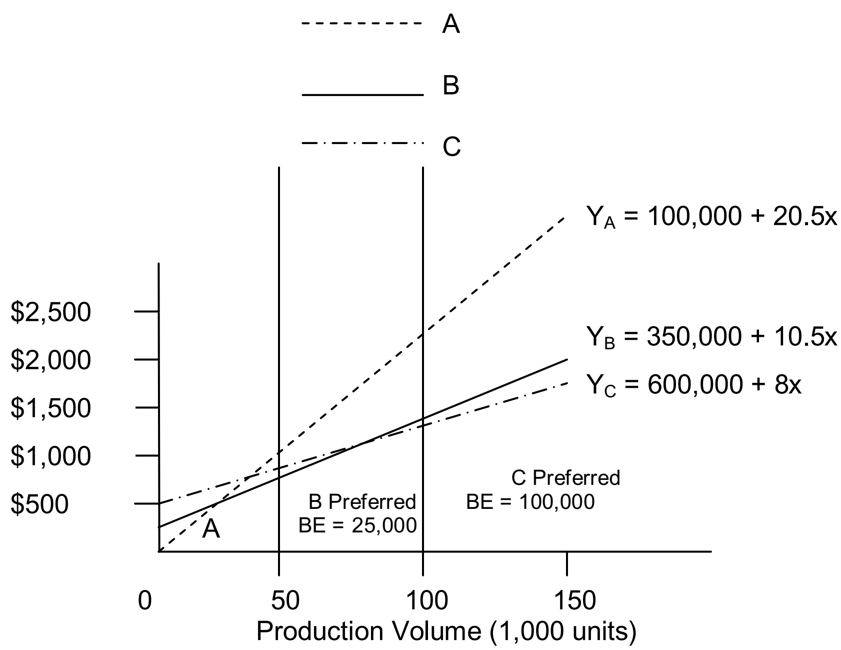
A & C: $20 x + $100,000 = $8 x + $600,000
x = 40,000

We want to minimize costs over the range of x, thus the A & C break-even point is not of interest. Sneaking a peak at the figure below we see that if:

$0 < x < 25{,}000$	choose A
$25{,}000 \le x < 80{,}000$	choose B
$80{,}000 \le x < 100{,}000$	choose C

(b) See graph below for Solution:

X	A	B	C
0	100	350	600
50	1,125	875	1,000
100	2,150	1,400	1,400
150	3,175	1,925	1,800



2-11

x = annual production volume (demand) = D

(a) Total Cost = $10,875 + $20 x
Total Revenue= (price per unit) (number sold)
= ($0.25 D + $250) D and if D = x
= -$0.25 x^2 + $250 x

(b) Set Total Cost = Total Revenue
$10,875 + $20 x = -$0.25 x^2 + $250 x
-$0.25 x^2 + $230 x - $10,875 = 0
This polynomial of degree 2 can be solved using the quadratic formula:

There will be two solutions:

$$x = (-b \pm (b^2 - 4ac)^{1/2})/2a = (-\$230 \pm \$205)/-0.50$$

Thus x = 870 and x = 50. There are two levels of x where TC = TR.

(c) To maximize Total Revenue we will take the first derivative of the Total Revenue equation, set it equal to zero, and solve for x:

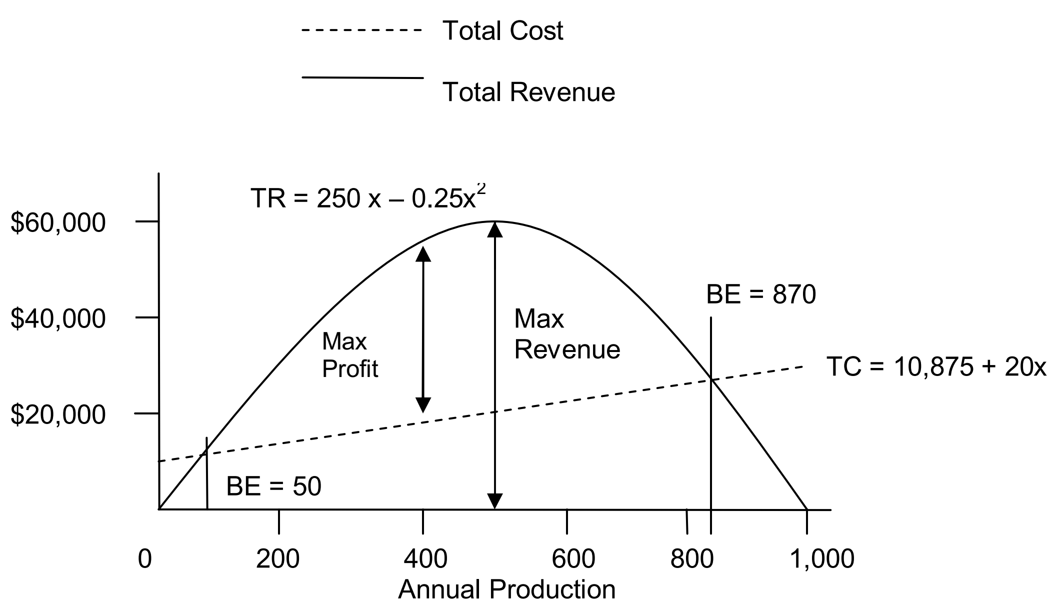
$TR = -\$0.25\ x^2 + \$250\ x$
$dTR/dx = -\$0.50\ x + \$250 = 0$
$x = 500$ is where we realize maximum revenue

(d) Profit is revenue – cost, thus let's find the profit equation and do the same process as in part (c).

$$\begin{aligned} \text{Total Profit} &= (-\$0.25\ x^2 + \$250\ x) - (\$10{,}875 + \$20\ x) \\ &= -\$0.25\ x^2 + \$230\ x - \$10{,}875 \\ dTP/dx &= -\$0.50\ x + \$230 = 0 \\ x &= 460 \text{ is where we realize our maximum profit} \end{aligned}$$

(e) See the figure below. Your answers to (a) – (d) should make sense now.

X	Total Cost	Total Revenue
0	\$10,875	\$0
250	\$15,875	\$46,875
500	\$20,875	\$62,500
750	\$25,875	\$46,875
1,000	\$30,875	\$0



2-12

x = units/year

By hand = Painting Machine
$1.40 x = $15,000/4 + $0.20
x = $5,000/1.20 = $4,167 units

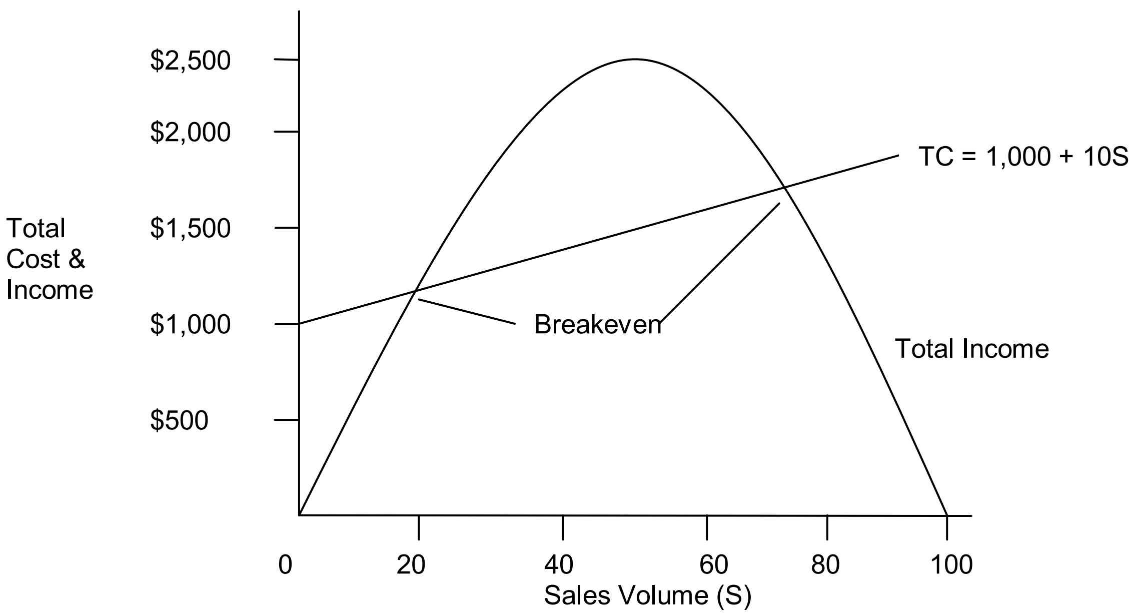
2-13

x = annual production units

Total Cost to Company A = Total Cost to Company B
$15,000 + $0.002 x = $5,000 + $0.05 x
x = $10,000/$0.048 = 208,330 units

2-14

(a)



Profit = S ($100 – S) - $1,000 - $10 S = $-S^2$ + $90 S - $1,000

(b) For break-even, set Profit = 0
$-S^2$ + $90S - $1,000 = $0
S = $(-b \pm (b^2 - 4ac)^{1/2})/2a$ = (-$90 $\pm$ ($$90^2$ – (4) (-1) (-1,000))$^{1/2}$)/-2
= 12.98, 77.02

(c) For maximum profit
dP/dS = -$2S + $90 = $0
S = 45 units

Answers: Break-even at 14 and 77 units. Maximum profit at 45 units.

Alternative Solution: Trial & Error

Price	Sales Volume	Total Income	Total Cost	Profit
$20	80	$1,600	$1,800	-$200
$23	77	$1,771	$1,770	$0 (Break-even)
$30	70	$2,100	$1,700	$400
$50	50	$2,500	$1,500	$1,000
$55	45	$2,475	$1,450	$1,025
$60	40	$2,400	$1,400	$1,000
$80	20	$1,600	$1,200	$400
$87	13	$1,131	$1,130	$0 (Break-even)
$90	10	$900	$1,100	-$200

2-15

In this situation the owners would have both recurring costs (repeating costs per some time period) as well as non-recurring costs (one time costs). Below is a list of possible recurring and non-recurring costs. Students may develop others.

Recurring Costs	Non-recurring costs
- Annual inspection costs	- Initial construction costs
- Annual costs of permits	- Legal costs to establish rental
- Carpet replacement costs	- Drafting of rental contracts
- Internal/external paint costs	- Demolition costs
- Monthly trash removal costs	
- Monthly utilities costs	
- Annual costs for accounting/legal	
- Appliance replacements	
- Alarms, detectors, etc. costs	
- Remodeling costs (bath, bedroom)	
- Durable goods replacements (furnace, air-conditioner, etc.)	

2-16

A cash cost is a cost in which there is a cash flow exchange between or among parties. This term derives from ‘cash’ being given from one entity to another (persons, banks, divisions, etc.). With today’s electronic banking capabilities cash costs may or may not involve ‘cash.’ ‘Book costs’ are costs that do not involve an exchange of ‘cash’, rather, they are only represented on the accounting books of the firm. Book costs are not represented as before-tax cash flows.

Engineering economic analyses can involve both cash and book costs. Cash costs are the before-tax cash flows usually estimated for a project (such as initial costs, annual costs, and retirement costs) as well as costs due to financing (payments on principal and interest debt) and taxes. Cash costs are important in such cases. For the engineering economist the primary book cost that is of concern is equipment depreciation, which is accounted for in after-tax analyses.

2-17

Here the student may develop several different thoughts as it relates to life-cycle costs. By life-cycle costs the authors are referring to any cost associated with a product, good, or service from the time it is conceived, designed, constructed, implemented, delivered, supported and retired. Firms should be aware of and account for all activities and liabilities associated with a product through its entire life-cycle. These costs and liabilities represent real cash flows for the firm — either at the time or some time in the future.

2-18

Figure 2-4 illustrates the difference between 'dollars spent' and 'dollars committed' over the life cycle of a project. The key point being that most costs are committed early in the life cycle, although they are not realized until later in the project. The implication of this effect is that if the firm wants to maximize value-per-dollar spent, the time to make important design decisions (and to account for all life cycle effects) is early in the life cycle. Figure 2-5 demonstrates 'ease of making design changes' and 'cost of design changes' over a project's life cycle. The point of this comparison is that the early stages of the design cycle are the easiest and least costly periods to make changes. Both figures represent important effects for firms.

In summary, firms benefit from spending time, money and effort early in the life cycle. Effects resulting from early decisions impact the overall life cycle cost (and quality) of the product, good, or service. An integrated, cross-functional, enterprise-wide approach to product design serves the modern firm well.

2-19

In this chapter, the authors list the following three factors as creating difficulties in making cost estimates: One-of-a-Kind Estimates, Time and Effort Available, and Estimator Expertise. Each of these factors could influence the estimate, or the estimating process, in different scenarios in different firms. One-of-a-kind estimating is a particularly challenging aspect for firms with little corporate-knowledge or suitable experience in an industry. Estimates, bids and budgets could potentially vary greatly in such circumstances. This is perhaps the most difficult of the factors to overcome. Time and effort can be influenced, as can estimator expertise. One-of-a-kind estimates pose perhaps the greatest challenge.

2-20

Total Cost = Phone unit cost + Line cost + One Time Cost
= ($100/2) 125 + $7,500 (100) + $10,000

= \$766,250

Cost to State = \$766,250 (1.35) = \$1,034,438

2-21

Cost (total) = Cost (paint) + Cost (labour) + Cost (fixed)

Number of Cans needed = (6,000/300) (2) = 40 cans

Cost (paint)	= (10 cans) \$15	= \$150.00
	= (15 cans) \$10	= \$150.00
	= (15 cans) \$7.50	= \$112.50
	Total	= \$412.50
Cost (labour)	= (5 painters) (10 hrs/day) (4.5 days/job) (\$8.75/hr)	
	= \$1,968.75	
Cost (total)	= \$412.50 + \$1,968.75 + \$200	= \$2,581.25

2-22

(a) Unit Cost = \$150,000/2,000 = \$75/ft^2

(bi) If all items change proportionately, then:
Total Cost = (\$75/ft^2) (4,000 ft^2) = \$300,000

(bii) For items that change proportionately to the size increase we multiply by: 4,000/2,000 = 2.0 all the others stay the same.
[See table below]

Cost item	2,000 ft^2 House Cost	Increase	4,000 ft^2 House Cost
1	(\$150,000) (0.08) = \$12,000	x 1	\$12,000
2	(\$150,000) (0.15) = \$22,500	x 1	\$22,500
3	(\$150,000) (0.13) = \$19,500	x 2	\$39,000
4	(\$150,000) (0.12) = \$18,000	x 2	\$36,000
5	(\$150,000) (0.13) = \$19,500	x 2	\$39,000
6	(\$150,000) (0.20) = \$30,000	x 2	\$60,000
7	(\$150,000) (0.12) = \$18,000	x 2	\$36,000
8	(\$150,000) (0.17) = \$25,500	x 2	\$51,000
		Total Cost	= \$295,500

2-23

(a) Unit Profit = \$410 (0.30) = \$123 or
= Unit Sales Price – Unit Cost
= \$410 (1.3) - \$410 = \$533 - \$410 = \$123

(b) Overall Batch Cost = \$410 (10,000) = \$4,100,000

(c) Of the 10,000 batch:
1. (10,000) (0.01) = 100 are scrapped in mfg.
2. (10,000 – 100) (0.03) = 297 of finished product go unsold
3. (9,900 – 297) (0.02) = 192 of sold product are not returned
Total = 589 of original batch are not sold for profit

Overall Batch Profit = (10,000 – 589) \$123 = \$1,157,553

(d) Unit Cost = 112 (\$0.50) + \$85 + \$213 = \$354
Batch Cost with Contract = 10,000 (\$354) = \$3,540,000

Difference in Batch Cost:
= BC without contract- BC with contract = \$4,100,000 - \$3,540,000
= \$560,000
SungSam can afford to pay up to \$560,000 for the contract.

2-24

$C_A/C_B = I_A/I_B$
$C_{50\text{ YEARS AGO}}/C_{\text{TODAY}} = AFCI_{50\text{ YEARS AGO}}/AFCI_{\text{TODAY}}$
$C_{\text{TODAY}} = (\$2{,}050/112)\ (55) = \$1{,}007$

2-25

$I_{\text{TODAY}} = (72/12)\ (100) = 600$
$C_{\text{LAST YEAR}} = (525/600)\ (72) = \$63$

2-26

Equipment	**Cost of New Equipment minus**	**Trade-In Value**	**= Net Cost**
Varnish Bath	$(75/50)^{0.80}$ (3,500) = \$4,841	\$3,500 (0.15)	= \$4,316
Power Scraper	$(1.5/0.75)^{0.22}$ (250) = \$291	\$250 (0.15)	= \$254
Paint Booth	$(12/3)^{0.6}$ (3,000) = \$6,892	\$3,000 (0.15)	= \$6,442
		Total	\$11,012

2-27

Equipment	**Cost of New Equipment minus**	**Trade-In Value**	**= Net Cost**
Varnish Bath	4,841 (171/154) = \$5,375	\$3,500 (0.15)	= \$4,850

Power Scraper	291 (900/780) = \$336	\$250 (0.15)	= \$298
Paint Booth	6892 (76/49) = \$10,690	\$3,000 (0.15)	= \$10,240
		Total	\$15,338

2-28

Scaling up cost:

Cost of 4,500 g/hr centrifuge = $(4{,}500/1{,}500)^{0.75}$ (40,000)= \$91,180

Updating the cost:

Cost of 4,500 model = \$91,180 (300/120) = \$227,950

2-29

Cost of VMIC – 50 today = 45,000 (214/151) = \$63,775

Using Power Sizing Model:

$(63{,}775/100{,}000) = (50/100)^x$

$\log(0.63775) = x \log(0.50)$

$x = 0.65$

2-30

(a) Gas Cost: (800 km) (11 litre/100 km) (\$0.75/litre) = \$66

Wear and Tear: (800 km) (\$0.05/km) = \$40

Total Cost = \$66 + \$40 = \$104

(b) (75 years) (365 days/year) (24 hours/day) = 657,000 hrs

(c) Miles around equator = 2 Π (4,000/2) = 12,566 mi

2-31

$T(7) = T(1) \times 7^b$

$60 = (200) \times 7^b$

$0.200 = 7^b$

$\log 0.30 = b \log(7)$

$b = \log(0.30)/\log(7) = -0.62$

b is defined as log (learning curve rate)/ log 20

b = [log (learning curve rate)/lob 2.0] = -0.62

log (learning curve rate) = -0.187

learning curve rate = $10^{(-0.187)}$ = .650 = 65%

2-32

Time for the first pillar is:

$T(10) = T(1) \times 10^{\log(0.75)/\log(2.0)}$

$T(1)$ = 676 person hours

Time for the 20th pillar is:

$T(20) = 676\ (20^{\log(0.75)/\log(2.0)})$
$= 195$ person hours

2-33

80% learning curve in use of SPC will reduce costs after 12 months to:

Cost in 12 months $= (x)\ 12^{\log(0.80)/\log(2.0)} = 0.45\ x$

Thus costs have been reduced:

$[(x - 0.45)/x]$ times 100% $= 55\%$

2-34

$T(25) = 0.60\ (25^{\log(0.75)/\log(2.0)}) = 0.16$ hours/unit

Labor Cost	= ($20/hr) (0.16 hr/unit)	= $3.20/unit
Material Cost	= ($43.75/25 units)	= $1.75/unit
Overhead Cost	= (0.50) ($3.20/units)	= $1.60/unit
Total Mfg. Cost		= $6.55/unit
Profit	= (0.20) ($7.75/unit)	= $1.55/unit
Unit Selling Price		= $8.10/unit

2-35

The concepts, models, effects, and difficulties associated with 'cost estimating' described in this chapter all have a direct (or near direct) translation for 'estimating benefits.' Differences between cost and benefit estimation include: (1) benefits tend to be over-estimated, whereas costs tend to be under-estimated, and (2) most costs tend to occur during the beginning stages of the project, whereas benefits tend to accumulate later in the project life comparatively.

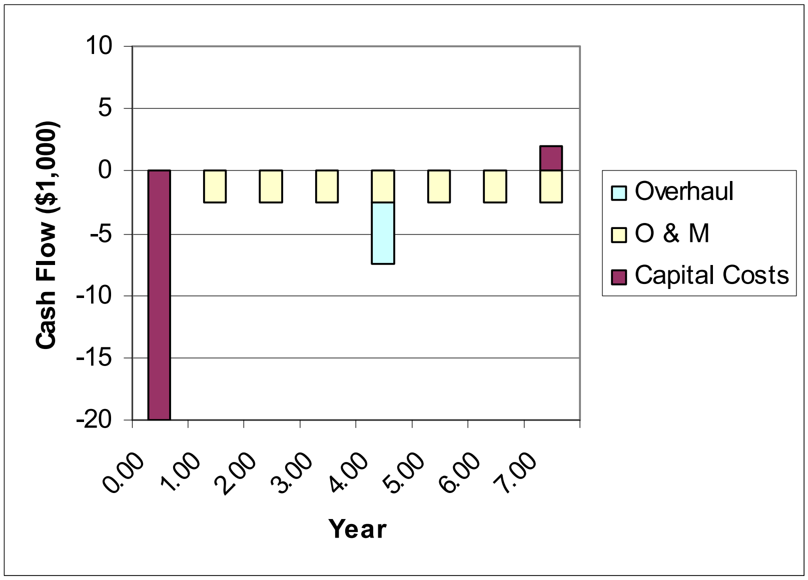
2-36

Time	Purchase Price	Maintenance	Market Value	Total
0	-$5,000	$0	$0	-$5,000
1	-$6,000	-$1,000	$0	-$7,000
2	-$6,000	-$2,000	$0	-$8,000
3	-$6,000	-$2,000	$0	-$8,000
4	$0	-$2,000	$7,000	+$5,000

2-37

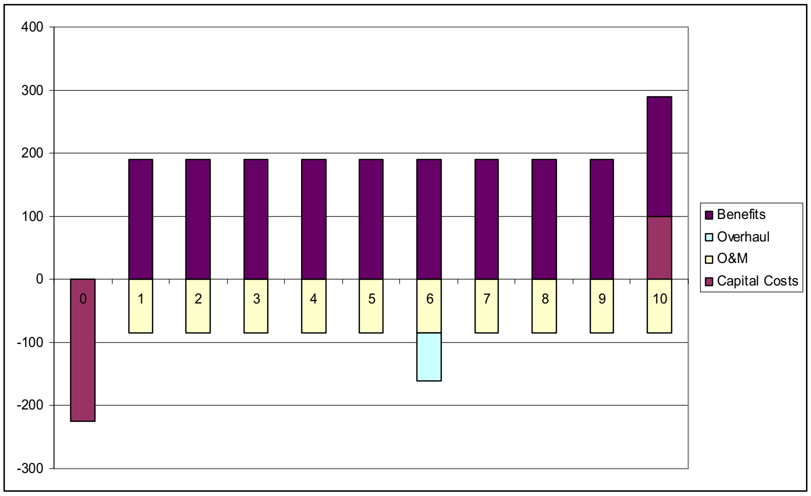
Year	**Capital Costs**	**O & M**	**Overhaul**
0.00	-20	0	0
1.00	0	-2.5	0
2.00	0	-2.5	0

3.00	0	-2.5	0
4.00	0	-2.5	-5
5.00	0	-2.5	0
6.00	0	-2.5	0
7.00	2	-2.5	0



2-38

Year	CapitalCosts	O&M	Overhaul	Benefits
0	-225			
1		-85		190
2		-85		190
3		-85		190
4		-85		190
5		-85		190
6		-85	-75	190
7		-85		190
8		-85		190
9		-85		190
10	100	-85		190



2-39

Each student's answers will be different depending on their university and life situation.

As an example:

First Costs: tuition costs, fees, books, supplies, board (if paid ahead)
O & M Costs: monthly living expenses, rent (if applicable)
Salvage Value: selling books back to student union, etc.
Revenues: wages & tips, etc.
Overhauls: periodic (random or planned) mid-term expenses

The cash flow diagram is left to the student.

Chapter 3: Interest and Equivalence

3-1

Time value of money means 'money has value over time.' Money has value, of course, because of what it can purchase. However, the time value of money means that ownership of money is valuable, and it is valuable because of the interest dollars that can be earned/gained due to its ownership. Understanding interest and its impact is important in many life circumstances. Examples could include some of the following:

- ✓ Selecting the best loans for homes, boats, jewellery, cars, etc.
- ✓ Many aspects involved with businesses ownership (payroll, taxes, etc.)
- ✓ Using the best strategies for paying off personal loans, credit cards, debt
- ✓ Making investments for life goals (purchases, retirement, college, weddings, etc.)
- ✓ Etc.

3-2

It is entirely possible that different decision makers will make a different choice in this situation. The reason this is possible (that there is not a RIGHT answer) is that Magdalen, Miriam, and Mary all could be using a different *discounting rate* (interest rate or investment rate) as they consider the choice of \$500 today versus \$1,000 three years from today.

We find the interest rate at which the two cash flows are equivalent by:

P=\$500, F=\$1000, n=3 years, i=unknown

So, F = P(1+i%)^n and, i% = {(F/P) ^ (1/n)} –1

Thus, i% = {(1000/500)^(1/3)}-1 = 26%

In terms of an explanation, Magdalen wants the \$500 today because she knows that she can invest it at a rate above 26% and thus have more than \$1000 three years from today. Miriam, on the other hand could know that she does not have any investment options that would come close to earning 26% and thus would be happy to pass up on the \$500 today to accept the \$1000 three years from today. Mary, on the other hand, could be indifferent because she has another investment option that earns exactly 26%, the same rate the \$500 would grow at if not accepted now. Thus, as a decision maker she would be indifferent.

Another aspect that may explain Magdalen's choice might have nothing to do with interest rates at all. Perhaps she simply needs \$500 right now to make a purchase or pay off a debt. Or, perhaps she is a pessimist and isn't convinced the \$1000 will be there in three years (a bird in hand idea).

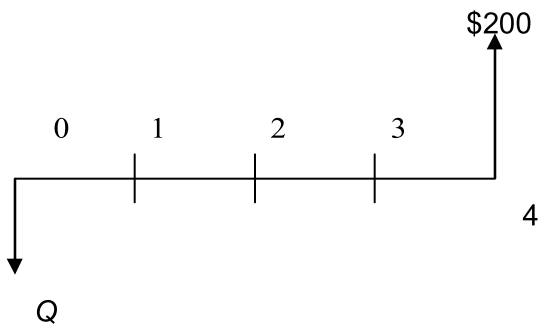
3-3

$\$2,000 + \$2,000\ (0.10 \times 3) = \$2,600$

3-4

$^{(\$5,350\ -\ \$5,000)}/_{(0.08 \times \$5,000)} = {}^{\$350}/_{\$400} = 0.875$ years= 10.5 months

3-5



$$
\begin{aligned}
Q &= \$200\ (P/F, 10\%, 4) \\
&= \$200\ (0.683) \\
&= \$136.60
\end{aligned}
$$

3-6

$$
\begin{aligned}
P &= \$1,400\ (P/A, 10\%, 5) - \$80\ (P/G, 10\%, 5) \\
&= \$1,400\ (3.791) - \$80\ (6.862) \\
&= \$4,758.44
\end{aligned}
$$

Using single payment factors:

$$
\begin{aligned}
P &= \$1400\ (P/F, 10\%, 1) + \$1,320\ (P/F, 10\%, 2) + \$1,240\ (P/F, 10\%, 3) + \$1,160\ (P/F, 10\%, 4) + \$1,080\ (P/F, 10\%, 5) \\
&= \$1,272.74 + \$1,090.85 + \$931.61 + \$792.28 + \$670.57 \\
&= \$4,758.05
\end{aligned}
$$

3-7

P=\$750, n =3 years, i =8%, F =?

$F = P\ (1+ i)^n = \$750\ (1.08)^3 = \$750\ (1.260)$

$= \$945$

Using interest tables:

$F = \$750\ (F/P, 8\%, 3) = \$750\ (1.360)$
$= \$945$

3-8

F =$8,250, *n* = 4 semi-annual periods, *i* =4%, *P* =?

$P = F\ (1+i)^{-n} = \$8,250\ (1.04)^{-4} = \$8,250\ (0.8548)$
$= \$7,052.10$

Using interest tables:

$P = F\ (P/F, 4\%, 4) = \$8,250\ (0.8548)$
$= \$7,052.10$

3-9

Local Bank
$F = \$3,000\ (F/P, 5\%, 2) = \$3,000\ (1.102)$
$= \$3,306$

Out of Town Bank
$F = \$3,000\ (F/P, 1.25\%, 8) = \$3,000\ (1.104)$
$= \$3,312$

Additional Interest = $6

3-10

P = $1	*n* = unknown number of semiannual periods	*i* = 2%	*F* = 2

$F = P\ (1 + i)^n$

$2 = 1\ (1.02)^n$

$2 = 1.02^n$

$n = \log(2) / \log(1.02)$
$= 35$

Therefore, the money will double in 17.5 years.

3-11

Lump Sum Payment = $350 (F/P, 1.5%, 8) = $350 (1.126)

= $394.10

Alternate Payment = $350 (F/P, 10%, 1) = $350 (1.100)
= $385.00

Choose the alternate payment plan.

3-12

Repayment at 4 ½% = $1 billion (F/P, 4 ½%, 30)
= $1 billion (3.745)
= $3.745 billion

Repayment at 5 ¼% = $1 billion $(1 + 0.0525)^{30}$
= $4.62 billion

Saving to foreign country = $897 million

3-13

Calculator Solution

1% per month F = $1,000 $(1 + 0.01)^{12}$ = $1,126.83

12% per year F = $1,000 $(1 + 0.12)^{1}$ = $1,120.00

Savings in interest = $6.83

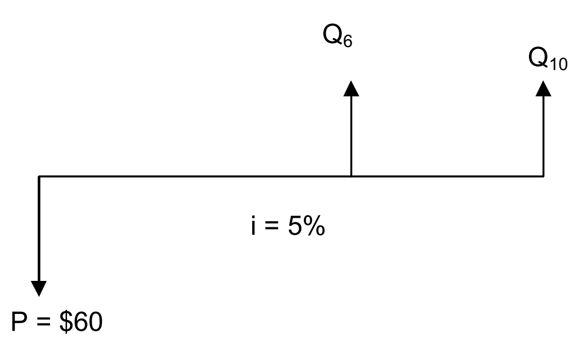
Compound interest table solution

1% per month F = $1,000 (1.127) = $1,127.00

12% per year F = $1,000 (1.120) = $1,120.00

Savings in interest = $7.00

3-14



Either:

Q_{10} = Q_6 (F/P, 5%, 4) (1)

Q_{10} = P (F/P, 5%, 10) (2)

Since P is between and Q_6 is not, solve Equation (2),

Q_{10} = $60 (1.629)
= <u>$97.74</u>

3-15

P = \$600
F = \$29,152,000
n = 92 years

$F = P(1 + i)^n$

$\$29,152,000/\$600 = (1 + i)^{92} = \$45,587$

$(1 + i) = (\$48,587)^{(1/92)} = \$48,587$

$i^* = 0.124 = 12.4\%$

3-16

(a) Interest Rates

(i) Interest rate for the past year = (\$100 - \$90)/\$90 = \$10/\$90
= 0.111 or 11.1%

(ii) Interest rate for the next year = (\$110 - \$100)/\$100
= 0.10 or 10%

(b) \$90 (F/P, i%, 2) = \$110
(F/P, i%, 2) = \$110/\$90= 1.222
So, $(1 + i)^2 = 1.222$
i = 1.1054 – 1 = 0.1054 = <u>10.54%</u>

3-17

n = 63 years
i = 7.9%
F = \$175,000

$P = F(1 + i)^{-n}$
$= \$175,000\ (1.079)^{-63}$
= <u>\$1,454</u>

3-18

$F = P(1 + i)^n$

Solve for P: $P = F/(1 + i)^n$
$P = F(1 + i)^{-n}$
$P = \$150,000\ (1 + 0.10)^{-5} = \$150,000\ (0.6209) = \$93,135$

3-19

The garbage company sends out bills only six times a year. Each time they collect one month's bills one month early.

100,000 customers x $6.00 x 1% per month x 6 times/yr = $36,000

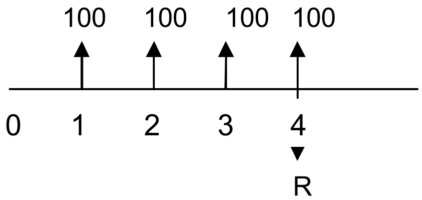
3-20

Year	Cash Flow
0	-$2,000
1	-$4,000
2	-$3,625
3	-$3,250
4	-$2,875

Chapter 4: More Interest Formulas

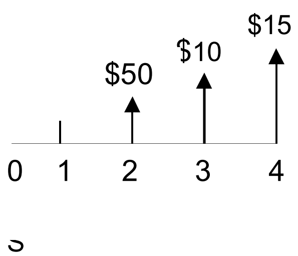
4-1

(a)



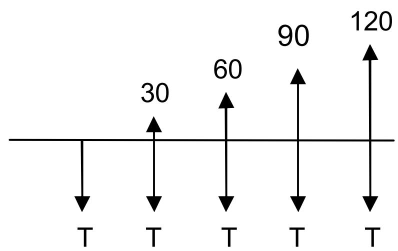
R = \$100(F/A, 10%, 4) = \$100(4.641)
= \$464.10

(b)



S = 50 (P/G, 10%, 4) = 50 (4.378)
= 218.90

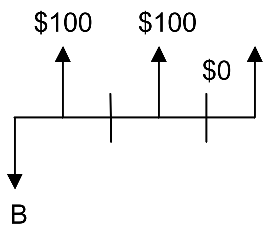
(c)



T = 30 (A/G, 10%, 5) = 30 (1.810)
= 54.30

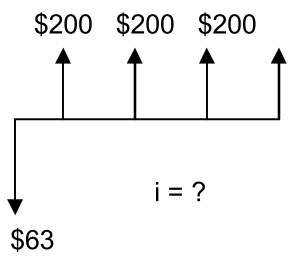
4-2

(a)



$$
\begin{aligned}
B &= \$100\,(P/F,\ 10\%,\ 1) + \$100\,(P/F,\ 10\%,\ 3) + \$100\,(P/F,\ 10\%,\ 5) \\
&= \$100\,(0.9091 + 0.7513 + 0.6209) \\
&= \$228.13
\end{aligned}
$$

(b)

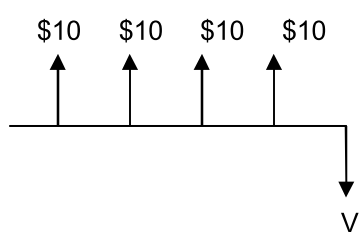


$\$634 = \$200\,(P/A,\ \text{i}\%,\ 4)$

$(P/A,\ i\%,\ 4) = \$634/\$200 = 3.17$

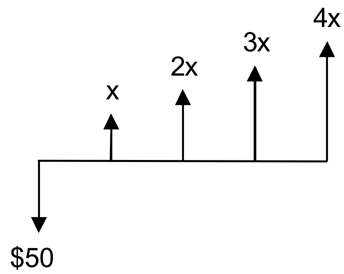
From compound interest tables, i = 10%.

(c)



$$
\begin{aligned}
V &= \$10\,(F/A,\ 10\%,\ 5) - \$10 \\
&= \$10\,(6.105) - \$10 \\
&= \$51.05
\end{aligned}
$$

(d)

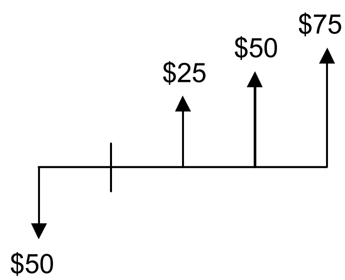


$500 = *x* (*P/A*, 10%, 4) + *x* (*P/G*, 10%, 4)
$500 = *x* (3.170 + 4.378)

x = $500/7.548
= $66.24

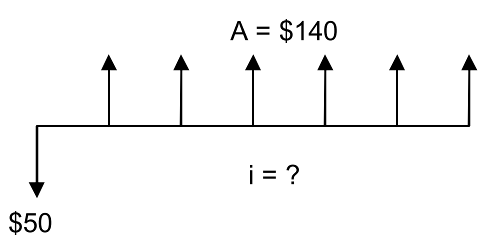
4-3

(a)



C = $25 (*P/G*, 10%, 4)
= $25 (4.378)
= $109.45

(b)



$500 = $140 (*P/A*, i%, 6)

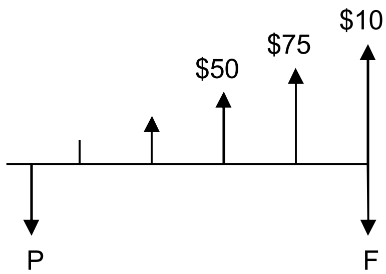
(*P/A*, *i*%, 6) = $500/$140 = 3.571

Performing linear interpolation:

$(P/A, i\%, 6)$	i
3.784	15%
3.498	18%

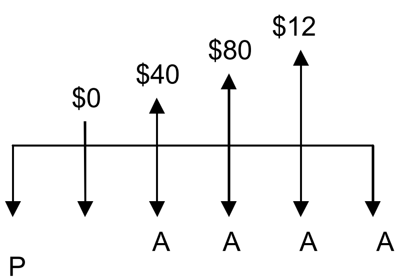
$$i = 15\% + (18\% - 15\%)\ ((3.487 - 3.571)/(3.784 - 3.498)$$
$$= 17.24\%$$

(c)



$$F = \$25\ (P/G, 10\%, 5)\ (F/P, 10\%, 5)$$
$$= \$25\ (6.862)\ (1.611)$$
$$= \$276.37$$

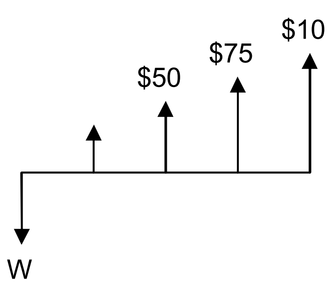
(d)



$$A = \$40\ (P/G, 10\%, 4)\ (F/P, 10\%, 1)\ (A/P, 10\%, 4)$$
$$= \$40\ (4.378)\ (1.10)\ (0.3155)$$
$$= \$60.78$$

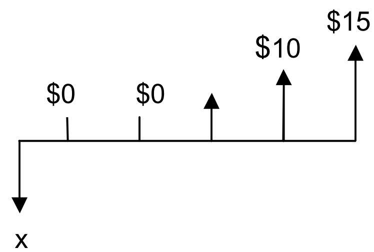
4-4

(a)



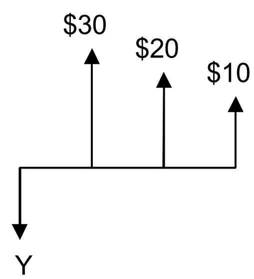
W $= \$25\ (P/A, 10\%, 4) + \$25\ (P/G, 10\%, 4)$
$= \$25\ (3.170 + 4.378)$
$= \$188.70$

(b)



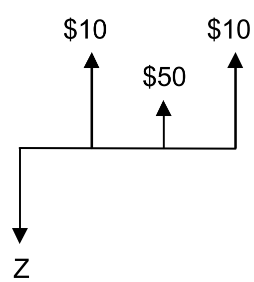
x $= \$100\ (P/G, 10\%, 4)\ (P/F, 10\%, 1)$
$= \$100\ (4.378)\ (0.9091)$
$= \$398.00$

(c)



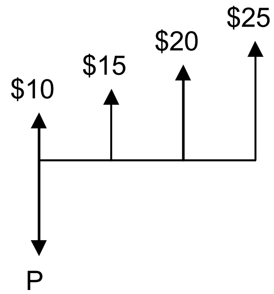
Y $= \$300\ (P/A, 10\%, 3) - \$100\ (P/G, 10\%, 3)$
$= \$300\ (2.487 – 2.329)$
$= \$513.20$

(d)



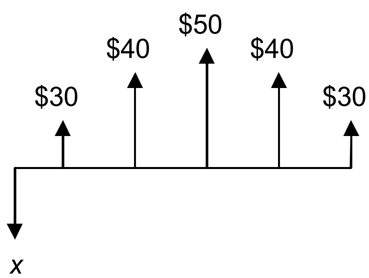
Z $= \$100\ (P/A, 10\%, 3) - \$50\ (P/F, 10\%, 2)$
$= \$100\ (2.487) - \$50\ (0.8264)$
$= \$207.38$

4-5



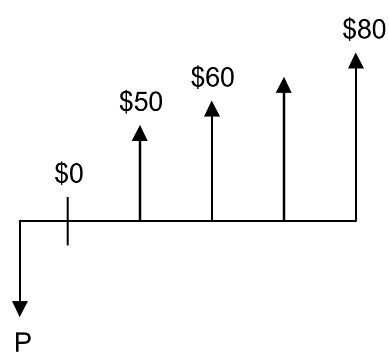
$$\begin{aligned} P &= \$100 + \$150\ (P/A, 10\%, 3) + \$50\ (P/G, 10\%, 3) \\ &= \$100 + \$150\ (2.487) + \$50\ (2.329) \\ &= \$589.50 \end{aligned}$$

4-6



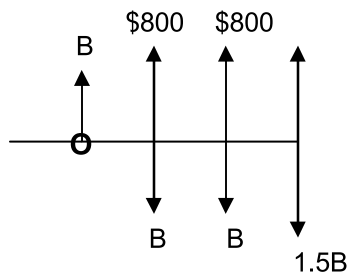
$$\begin{aligned} x &= \$300\ (P/A, 10\%, 5) + \$100\ (P/G, 10\%, 3) + \$100\ (P/F, 10\%, 4) \\ &= \$300\ (3.791) + \$100\ (2.329) + \$100\ (0.6830) \\ &= \$1{,}438.50 \end{aligned}$$

4-7



$$\begin{aligned} P &= \$10\ (P/G, 15\%, 5) + \$40\ (P/A, 15\%, 4)(P/F, 15\%, 1) \\ &= \$10\ (5.775) + \$40\ (2.855)\ (0.8696) \\ &= \$157.06 \end{aligned}$$

4-8



Receipts (upward) at time **O**:

PW = B + $800 (P/A, 12%, 3) = B + $1,921.6

Expenditures (downward) at time **O**:

PW= B (P/A, 12%, 2) + 1.5B (P/F, 12%, 3) = 2.757B

Equating:

B + $1,921.6 = 2.757B

B = $1,921.6/2.757
= $1,093.70

4-9

F = A (F/A, 10%, n)
$35.95 = 1 (F/A, 10%, n)
(F/A, 10%, n) = 35.95

From the 10% interest table, n = 16.

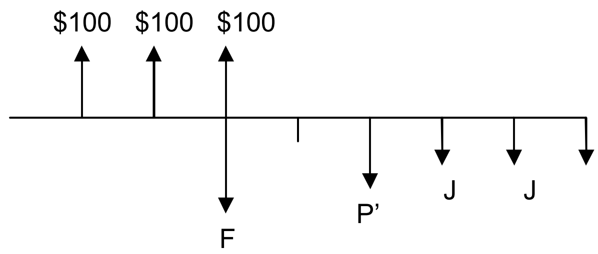
4-10

P = A (P/A, 3.5%, n)
$1,000 = $50 (P/A, 3.5%, n)

(P/A, 3.5%, n) = 20

From the 3.5% interest table, n = 35.

4-11



F = \$100 (F/A, 10%, 3) = \$100 (3.310)= \$331

P' = \$331 (F/P, 10%, 2) = \$331 (1.210)= \$400.51

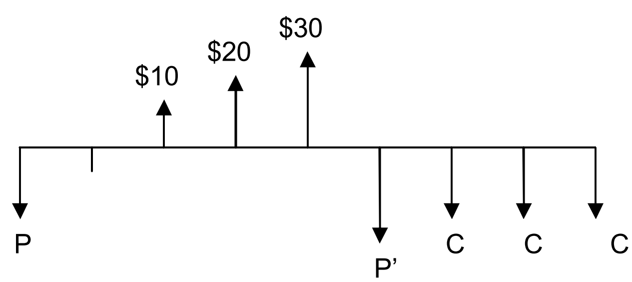
J = \$400.51 (A/P, 10%, 3) = \$400.51 (0.4021) = \$161.05

Alternate Solution:

One may observe that *J* is equivalent to the future worth of \$100 after five interest periods, or:

J = \$100 (F/P, 10%, 5) = \$100 (1.611)= \$161.10

4-12

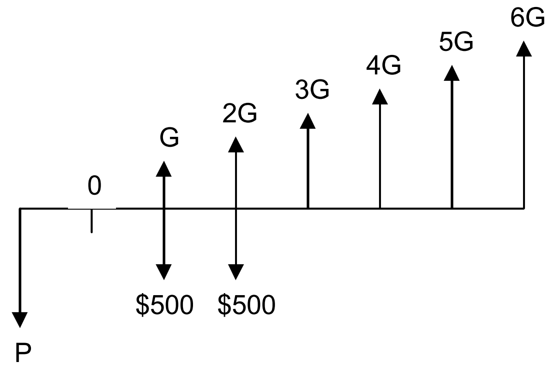


P = \$100 (P/G, 10%, 4) = \$100 (4.378)= \$437.80

P' = \$437.80 (F/P, 10%, 5) = \$437.80 (1.611) = \$705.30

C = \$705.30 (A/P, 10%, 3) = \$705.30 (0.4021) = \$283.60

4-13



Present Worth P of the two $500 amounts:

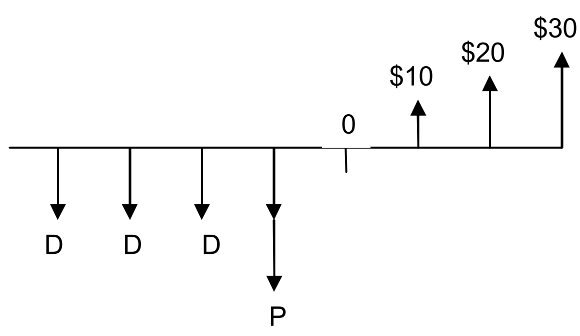
P = $500 (P/F, 12%, 2) + $500 (P/F, 12%, 1)
= $500 (0.7972) + $500 (0.7118)
= $754.50

Also:

P = G (P/G, 12%, 7)
$754.50 = G (P/G, 12%, 7)
= G (11.644)

G = $754.50/11.644
= $64.80

4-14

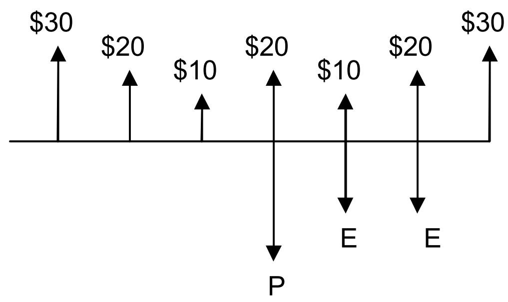


Present Worth of gradient series:

P = $100 (P/G, 10%, 4) = $100 (4.378)= $437.80

D = $437.80 (A/F, 10%, 4) = $4.7.80 (0.2155) = $94.35

4-15



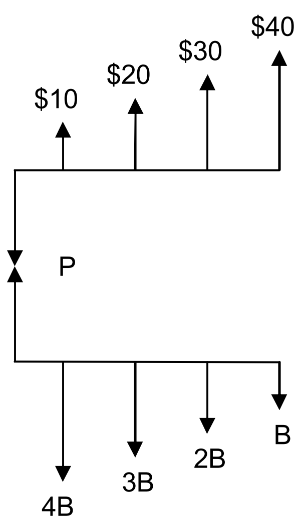
P = \$200 + \$100 (P/A, 10%, 3) + \$100 (P/G, 10%, 3) + \$300 (F/P, 10%, 3) + \$200 (F/P, 10%, 2) + \$100 (F/P, 10%, 1)

= \$200 + \$100 (2.487) + \$100 (2.329) + \$300 (1.331) + \$200 (1.210) + \$100 (1.100)

= \$1,432.90

E = \$1,432.90 (A/P, 10%, 2) = \$1,432.90 (0.5762) = \$825.64

4-16



P = \$100 (P/A, 10%, 4) + \$100 (P/G, 10%, 4)
= \$100 (3.170 + 4.378)
= \$754.80

Also:

P = 4B (P/A, 10%, 4) – B (P/G, 10%, 4)

Thus, $4B(3.170) - B(4.378) = \$754.80$

$B = \$754.80/8.30 = \$90.94$

4-17

$$P = \$1,250\ (P/A, 10\%, 8) - \$250\ (P/G, 10\%, 8) + \$3,000 - \$250\ (P/F, 10\%, 8)$$
$$= \$1,250\ (5.335) - \$250\ (16.029) + \$3,000 - \$250\ (0.4665)$$
$$= \$5,545$$

4-18

Cash flow number 1:

$P_0^1 = A\ (P/A, 12\%, 4)$

Cash flow number 2:

$P_0^2 = \$150\ (P/A, 12\%, 5) + \$150\ (P/G, 12\%, 5)$

Since $P_0^1 = P_0^2$,

$A\ (3.037) = \$150\ (3.605) + \$150\ (6.397)$

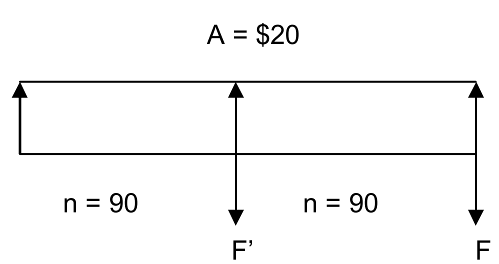
$A = (540.75 + 959.55)/3.037$
$= \$494$

4-19

$F = ?$ $n = 180$ months $i = 0.50\%$ /month $A = \$20.00$

$F = A\ (F/A, 0.50\%, 180)$

Since the ½% interest table does not contain n = 180, the problem must be split into workable components. On way would be:



F = \$20 (F/A, ½%, 90) + \$20 (F/A, ½%, 90)(F/P, ½%, 90)
= \$5,817

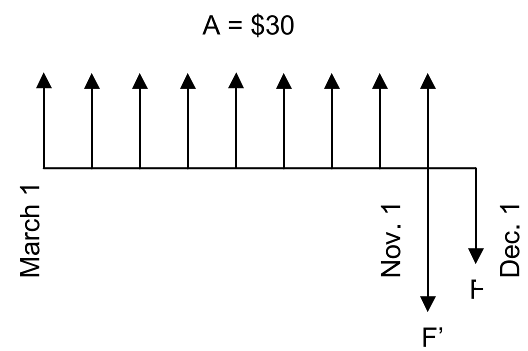
Alternate Solution

Perform linear interpolation between n = 120 and n = 240:

F = \$20 ((F/A, ½%, 120) – (F/A, ½%, 240))/2
= \$6,259

Note the inaccuracy of this solution.

4-20



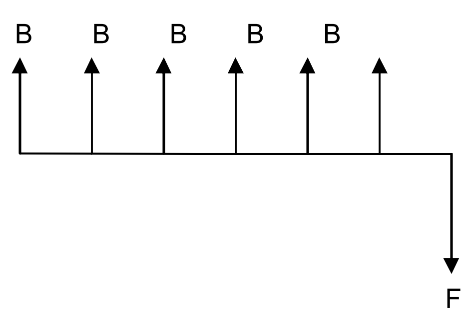
Amount on Nov 1:

F' = \$30 (F/A, ½%, 9) = \$30 (9.812) = \$275.46

Amount on Dec 1:

F = \$275.46 (F/P, ½%, 1) = \$275.46 (1.005) = 276.84

4-21



The solution may follow the general approach of the end-of-year derivation in the book.

(1) F $= B(1+i)^n + \ldots + B(1+i)^1$

Divide equation (1) by (1 + *i*):

(2) $F(1+i)^{-1} = B(1+i)^{n-1} + B(1+i)^{n-2} + \ldots + B$

Subtract equation (2) from equation (1):

(1) – (2) $F - F(1+i)^{-1} = B[(1+i)^n - 1]$

Multiply both sides by (1 + *i*):

$$F(1+i) - F = B[(1+i)^{n+1} - (1+i)]$$

So the equation is:

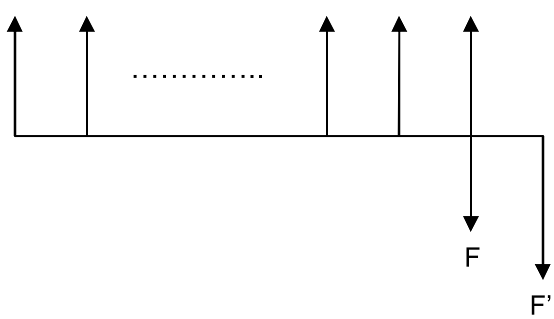
$$F = B[(1+i)^{n+1} - (1+i)]/i$$

Applied to the numerical values:

$$F = 100/0.08\,[(1+0.08)^7 - (1.08)]$$
$$= \$792.28$$

4-22

B = \$200 *i* = 7%
n = 15



F = \$200 (F/A, *i*%, *n*) = \$200 (F/A, 7%, 15) = \$200 (25.129)
= \$5,025.80

F' = F (F/P, *i*%, *n*) = \$5,025.80 (F/P, 7%, 1) = \$5,025.80 (1.07)
= \$5,377.61

4-23

F = \$2,000 (F/A, 8%, 10) (F/P, 8%, 5)
= \$2,000 (14.487) (1.469)
= \$42,560

4-24

A = \$300 i = 5.25% P = ? n = 10 years

$$
\begin{aligned}
P &= A\,(P/A, 5.25\%, 10) \\
&= A\,[(1 + i)^n - 1]/[i(1 + i)^n] \\
&= \$300\,[(1.0525)^{10} - 1]/[0.0525\,(1.0525)^{10}] \\
&= \$300\,(7.62884) \\
&= \$2{,}289
\end{aligned}
$$

4-25

P = \$10,000 i = 12% F = \$30,000 n = 4

\$10,000 (F/P, 12%, 4) + A (F/A, 12%, 4) = \$30,000

\$10,000 (1.574) + A (4.779) = \$30,000

A = \$2,984

4-26

Let **X** = toll per vehicle. Then:

A = 20,000,000 **X** i = 10% F = \$25,000,000 n = 3

20,000,000 **X** (F/A, 10%, 3) = \$25,000,000

20,000,000 **X** (3.31) = \$25,000,000

X = \$0.38 per vehicle

4-27

From compound interest tables, using linear interpolation:

(P/A, i%, 10)	i
7.360	6%
7.024	7%

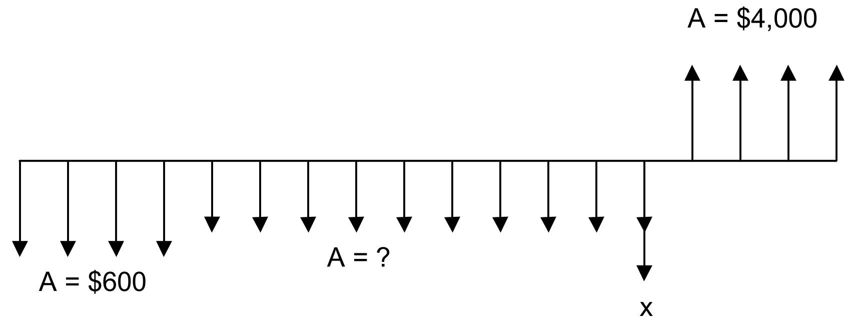
(P/A, 6.5%, 10) = ½ (7.360 – 7.024) + 7.024
= 7.192

Exact computed value:

(P/A, 6.5%, 10) = 7.189

Why do the values differ? Since the compound interest factor is non-linear, linear interpolation will not produce an exact solution.

4-28



To have sufficient money to pay the four $4,000 disbursements,

x = $4,000 (P/A, 5%, 4) = $4,000 (3.546)
= $14,184

This $14,184 must be accumulated by the two series of deposits.

The four $600 deposits will accumulate by x (17th birthday):

F = $600 (F/A, 5%, 4) (F/P, 5%, 10)
= $600 (4.310) (1.629)
= $4,212.59

Thus, the annual deposits between 8 and 17 must accumulate a future sum:
= $14,184 - $4,212.59
= $9,971.41

The series of ten deposits must be:

A = $9,971.11 (A/F, 5%, 10) = $9,971.11 (0.0745)
= $792.73

4-29

P = A (P/A, 1.5%, n)
$525 = $15 (P/A, 1.5%, n)

(P/A, 1.5%, n) = 35

From the 1.5% interest table, n = 50 months.

4-30

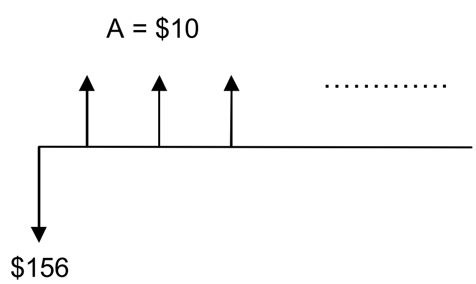
$P = 1$ $F = 2$ $i = 1\%$ $n = ?$

$2 = $1 (F/P, 1%, n)
(F/P, 1%, n) = 2

From the 1%, table:

n = 70 months

4-31



P = $156 n = ? i = 1.5% A = $10

$156 = $10 (P/A, 1.5%, n)

(P/A, 1.5%, n) = $156/$10
= 15.6

From the 1.5% interest table, n is between 17 and 18. Therefore, it takes 18 months to repay the loan.

4-32

A = $500 ($A/P$, 1%, 16) = $500 (0.0679)
= $33.95

4-33

This problem may be solved in several ways. Below are two of them:

Alternative 1:

$5000 = $1,000 (P/A, 8%, 4) + x (P/F, 8%, 5)
= $1,000 (3.312) + x (0.6806)
= $3,312 + x (0.6806)

x = ($5,000 - $3,312)/0.6806

= $2,480.16

Alternative 2:

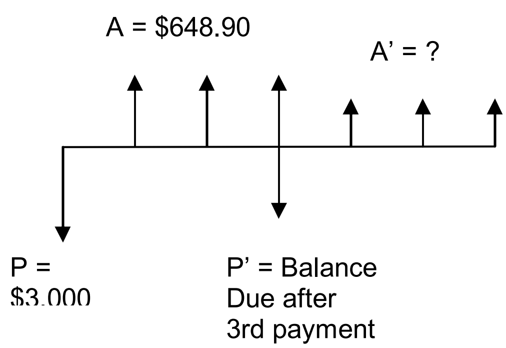
P = $1,000 (P/A, 8%, 4)
= $1,000 (3.312)
= $3,312

($5,000 - $3,312) (F/P, 8%, 5) = $2,479.67

4-34

A = P (A/P, 8%, 6)
= $3,000 (0.2163)
= $648.90

The first three payments were $648.90 each.



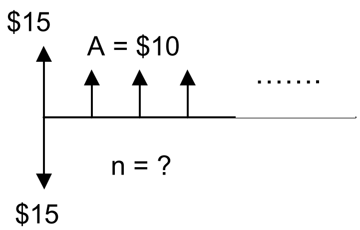
Balance due after 3rd payment equals the Present Worth of the originally planned last three payments of $648.90.

P' = $648.90 (P/A, 8%, 3) = $648.90 (2.577)
= $1,672.22

Last three payments:

A' = $1,672.22 (A/P, 7%, 3) = $1,672.22 (0.3811)
= $637.28

4-35



($150 - $15) = $10 (P/A, 1.5%, n)

(P/A, 1.5%, n) = $135/$10 = 13.5

From the 1.5% interest table we see that *n* is between 15 and 16. This indicates that there will be 15 payments of $10 plus a last payment of a sum less than $10.

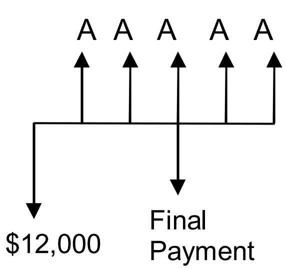
Compute how much of the purchase price will be paid by the fifteen $10 payments:

P = $10 (P/A, 1.5%, 15) = $10 (13.343)
= $133.43

Remaining unpaid portion of the purchase price:
= $150 - $15 - $133.43 = $1.57

16th payment = $1.57 (F/P, 1.5%, 16)
= $1.99

4-36

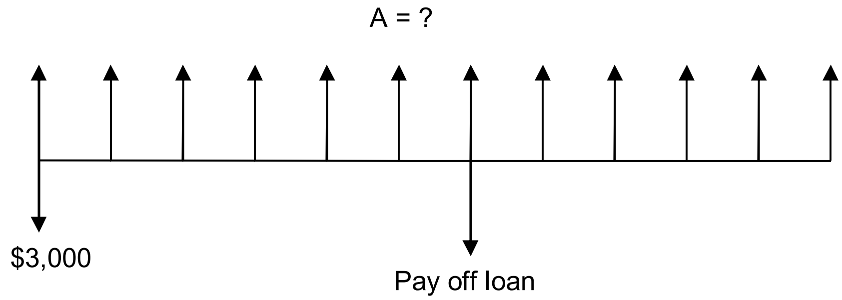


A = $12,000 (A/P, 4%, 5)
= $12,000 (0.2246)
= $2,695.20

The final payment is the present worth of the three unpaid payments.

Final Payment = $2,695.20 + $2,695.20 (P/A, 4%, 2)
= $2,695.20 + $2,695.20 (1.886)
= $7,778.35

4-37



Compute monthly payment:

$$
\begin{aligned}
\$3,000 &= A + A\,(P/A, 1\%, 11) \\
&= A + A\,(10.368) \\
&= 11.368\,A \\
A &= \$3,000/11.368 \\
&= \$263.90
\end{aligned}
$$

Car will cost new buyer:

$$
\begin{aligned}
&= \$1,000 + 263.90 + 263.90\,(P/A, 1\%, 5) \\
&= \$1263.90 + 263.90\,(4.853) \\
&= \$2,544.61
\end{aligned}
$$

4-38

(a)

A = ? i = 8% P = \$120,000 n = 15 years

P = \$150,000 - \$30,000 = \$120,000

$$
\begin{aligned}
A &= P\,(A/P, i\%, n) \\
&= \$120,000\,(A/P, 8\%, 15) \\
&= \$120,000\,(0.11683) \\
&= \$14,019.55
\end{aligned}
$$

R_Y = Remaining Balance in any year, Y

$R_Y = A\,(P/A, i\%, n - Y)$

$$
\begin{aligned}
R_7 &= \$14,019.55\ (P/A, 8\%, 8) \\
&= \$14,019.55\,(5.747) \\
&= \$80,570.35
\end{aligned}
$$

(b) The quantities in Table 4-38 below are computed as follows:
Column 1 shows the number of interest periods.

Column 2 shows the equal annual amount as computed in part (a) above.

The amount $14,019.55 is the total payment which includes the principal and interest portions for each of the 15 years. To compute the interest portion for year one, we must first multiply the interest rate in decimal by the remaining balance:

Interest Portion = (0.08) ($120,000) = $9,600

TABLE 4-38: SEPARATION OF INTEREST AND PRINCIPAL

YEAR	ANNUAL PAYMENT	INTEREST PORTION	PRINCIPAL PORTION	REMAINING BALANCE
0				$120,000.00
1	$14,019.55	$9,600	$4,419.55	$115,580.45
2	$14,019.55	$9,246.44	$4,773.11	$110,807.34
3	$14,019.55	$8,864.59	$5,154.96	$105,652.38
4	$14,019.55	$8,452.19	$5,567.36	$100,085.02
5	$14,019.55	$8,006.80	$6,012.75	$94,072.27
6	$14,019.55	$7,525.78	$6,493.77	$87,578.50
7*	$14,019.55	$7,006.28	$7,013.27	$80,565.23
8	$14,019.55	$6,445.22	$7,574.33	$72,990.90
9	$14,019.55	$5,839.27	$8,180.28	$64,810.62
10	$14,019.55	$5,184.85	$8,834.70	$55,975.92
11	$14,019.55	$4,478.07	$9,541.48	$46,434.44
12	$14,019.55	$3,714.76	$10,304.79	$36,129.65
13	$14,019.55	$2,890.37	$11,129.18	$25,000.47
14	$14,019.55	$2,000.04	$12,019.51	$12,981.00
15	$14,019.55	$1,038.48	$12,981.00	0

Subtracting the interest portion of $9,600 from the total payment of $14,019.55 gives the principal portion to be $4,419.55, and subtracting it from the principal balance of the loan at the end of the previous year (y) results in the remaining balance after the first payment is made in year 1 (y_1), of $115,580.45. This completes the year 1 row. The other row quantities are computed in the same fashion. The interest portion for row two, year 2 is:

(0.08) ($115,580.45) = $9,246.44

*NOTE: Interest is computed on the remaining balance at the end of the preceding year and not on the original principal of the loan amount. The rest of the calculations proceed as before. Also, note that in year 7, the remaining balance as shown on Table 4-38 is approximately equal to the value calculated in (a) using a formula except for round off error.

4-39

Determine the required present worth of the escrow account on January 1, 1998:

A = \$8,000 i = 5.75% PW = ? n = 3 years

PW= A (P/A, i%, n)
= \$8,000 + \$8,000 (P/A, 5.75%, 3)
= \$8,000 + \$8,000 $[(1 + i)^n – 1]/[i(1 + i)^n]$
= \$8,000 + \$8,000 $[(1.0575)^3 – 1]/[0.0575(1.0575)^3]$
= \$29,483.00

It is necessary to have \$29,483 at the end of 1997 in order to provide \$8,000 at the end of 1998, 1999, 2000, and 2001. It is now necessary to determine what yearly deposits should have been over the period 1981–1997 to build a fund of \$29,483.

A = ? i = 5.75% F = \$29,483 n = 18 years

A = F (A/F, i%, n) = \$29,483 (A/F, 5.75%, 18)

= \$29,483 $(i)/[(1 + i)^n – 1]$
= \$29,483 $(0.575)/[(1.0575)^{18} – 1]$
= \$29,483 (0.03313)
= \$977

4-40

Amortization schedule for a \$4,500 loan at 6%
Paid monthly for 24 months

P = \$4,500 i = 6%/12 mo = 1/2% per month

Pmt. #	Amt. Owed BOP	Int. Owed (this pmt.)	Total Owed (EOP)	Principal (This pmt)	Monthly Pmt.
1	4,500.00	22.50	4,522.50	176.94	199.44
2	4,323.06	21.62	4,344.68	177.82	199.44
3	4,145.24	20.73	4,165.97	178.71	199.44
4	3,966.52	19.83	3,986.35	179.61	199.44
5	3,786.91	18.93	3,805.84	180.51	199.44
6	3,606.41	18.03	3,624.44	181.41	199.44
7	3,425.00	17.13	3,442.13	182.32	199.44
8	3,242.69	16.21	3,258.90	183.23	199.44
9	3,059.46	15.30	3,074.76	184.14	199.44
10	2,875.32	14.38	2,889.69	185.06	199.44
11	2,690.25	13.45	2,703.70	185.99	199.44
12	2,504.26	12.52	2,516.79	186.92	199.44
13	2,317.35	11.59	2,328.93	187.85	199.44
14	2,129.49	10.65	2,140.14	188.79	199.44
15	1,940.70	9.70	1,950.40	189.74	199.44
16	1,750.96	8.75	1,759.72	190.69	199.44
17	1,560.28	7.80	1,568.08	191.64	199.44
18	1,368.64	6.84	1,375.48	192.60	199.44

19	1,176.04	5.88	1,181.92	193.56	199.44
20	982.48	4.91	987.40	194.53	199.44
21	787.96	3.94	791.90	195.50	199.44
22	592.46	2.96	595.42	196.48	199.44
23	395.98	1.98	397.96	197.46	199.44
24	198.52	0.99	199.51	198.45	199.44
TOTALS		286.63		4499.93	

B12 = $4,500.00 (principal amount)

B13 = B12 - E12 (amount owed BOP- principal in this payment)

Column C = amount owed BOP * 0.005
Column D = Column B + Column C (principal + interest)
Column E = Column F - Column C (payment - interest owed)
Column F = Uniform Monthly Payment (from formula for A/P)

4-41

Amortization schedule for a $4,500 loan at 6%
Paid monthly for 24 months

P = $4,500 i = 6%/12 mo = 1/2% per month

Pmt. #	Amt. Owed BOP	Int. Owed (this pmt.)	Total Owed (EOP)	Principal (This pmt)	Monthly Pmt.
1	4,500.00	22.50	4,522.50	176.94	199.44
2	4,323.06	21.62	4,344.68	177.82	199.44
3	4,145.24	20.73	4,165.97	178.71	199.44
4	3,966.52	19.83	3,986.35	179.61	199.44
5	3,786.91	18.93	3,805.84	180.51	199.44
6	3,606.41	18.03	3,624.44	181.41	199.44
7	3,425.00	17.13	3,442.13	182.32	199.44
8	3,242.69	16.21	3,258.90	483.79	500.00
9	2,758.90	13.79	2,772.69	185.65	199.44
10	2,573.25	12.87	2,586.12	267.13	280.00
11	2,306.12	11.53	2,317.65	187.91	199.44
12	2,118.21	10.59	2,128.80	188.85	199.44
13	1,929.36	9.65	1,939.01	189.79	199.44
14	1,739.57	8.70	1,748.27	190.74	199.44
15	1,548.83	7.74	1,556.57	191.70	199.44
16	1,357.13	6.79	1,363.92	192.65	199.44
17	1,164.48	5.82	1,170.30	193.62	199.44
18	970.86	4.85	975.71	194.59	199.44
19	776.27	3.88	780.15	195.56	199.44
20	580.71	2.90	583.61	196.54	199.44
21	384.18	1.92	386.10	197.52	199.44
22	186.66	0.93	187.59	186.66	187.59

23	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00	0.00
TOTALS		256.95		4500.00	

B12 = $4,500.00 (principal amount)

B13 = B12 - E12 (amount owed BOP- principal in this payment)

Column C = amount owed BOP * 0.005
Column D = Column B + Column C (principal + interest)
Column E = Column F - Column C (payment - interest owed)
Column F = Uniform Monthly Payment (from formula for A/P)

Payment 22 is the final payment. Payment amount = $187.59

4-42

Interest Rate per Month = 0.07/12 = 0.00583/month

Interest Rate per Day = 0.07/365 = 0.000192/day

Payment $= P[i(1 + i)^n]/[(1 + i)^n - 1]$
$= \$80,000\ [0.00583\ (1.00583)^{12}]/[(1.00583)^{12} - 1]$
$= \$532.03$

Principal in 1st payment = $532.03 – $80,000 (0.00583)
= $65.63

Loan Principal at beginning of month 2 = $80,000 - $65.63
= $79.934.37

Interest for 33 days $= Pin$ = $79,934 (33) (0.000192) = $506.46

Principal in 2nd payment = $532.03 – 506.46 = $25.57

4-43

(a) $F_{16} = \$10,000\ (1 + 0.055/4)^{16}$
$= \$12,442.11$

$F_{10} = \$12,442.11\ (1 + 0.065/4)^{24}$
$= \$18,319.24$

(b) $\$18,319.24 = (1 + i)^{10}\ (\$10,000)$

$(1 + i)^{10} = \$18,319.24/\$10,000 = 1.8319$

$10 \ln (1 + i) = \ln (1.8319)$

ln (1 + *i*) = (ln (1.8319))/10
= 0.0605

(1 + *i*) = 1.0624

i = 0.0624 = 6.24%

Alternative Solution

$18,319.24 = $10,000 (F/P, *i*, 10)

(F/P, *i,* 10) = 1.832
Performing interpolation:

(F/P, i%, 10)	i
1.791	6%
1.967	7%

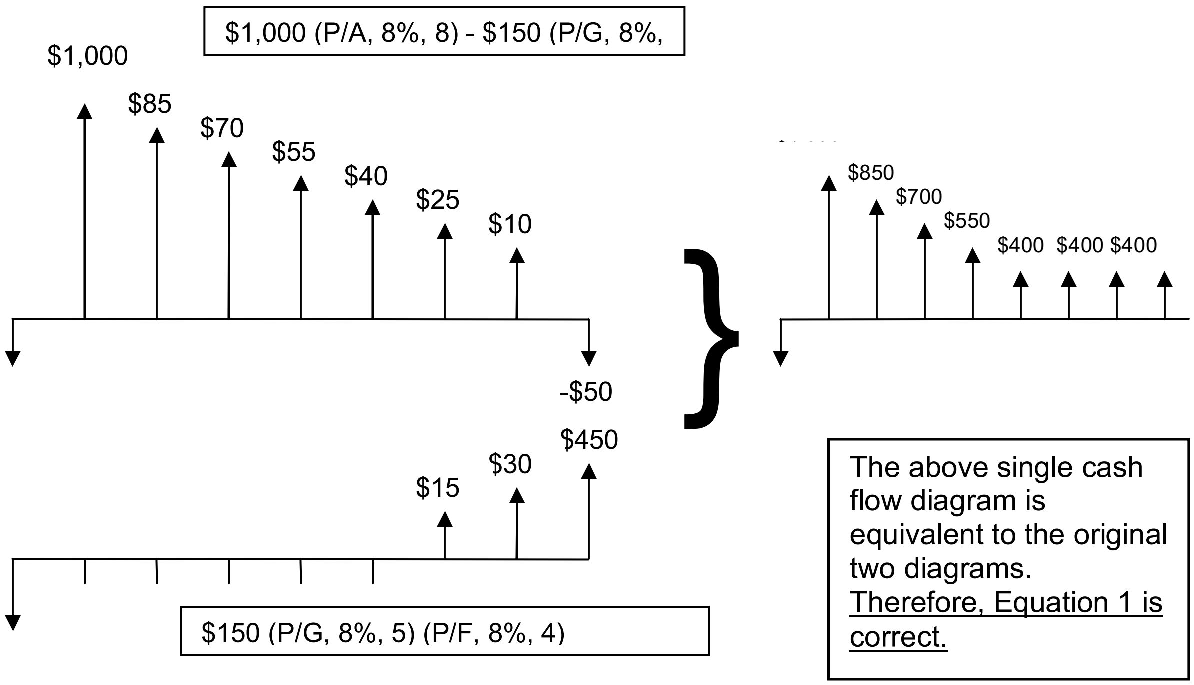
i = 6% + [(1.832 – 1.791)/(1.967 – 1.791)] = 6.24%

4-44

Correct equation is (2).

$50 (P/A, i%, 5) + $10 (P/G, i%, 5) + $50 (P/G, i%, 5) = 1
100

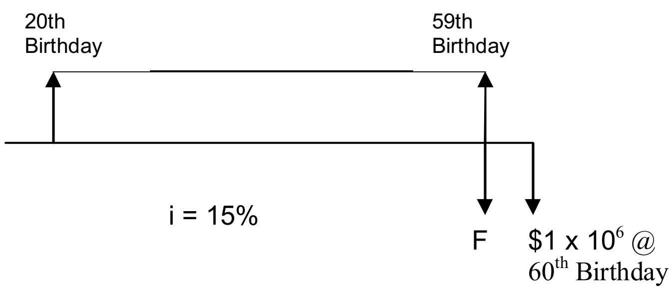
4-45



4-46

P = \$40 (P/A, 5%, 7) + \$10 (P/G, 5%, 7)
= \$40 (5.786) + \$10 (16.232)
= \$231.44 + \$162.32
= \$393.76

4-47



Number of yearly investments = (59 – 20 + 1) = 40

The diagram indicates that the problem is not in the form of the uniform series compound amount factor. Thus, find F that is equivalent to \$1,000,000 one year hence:

F = \$1,000,000 (P/F, 15%, 1) = \$1,000,000 (0.8696)
= \$869,600

A = \$869,600 (A/F, 15%, 40) = \$869,600 (0.00056)
= \$486.98

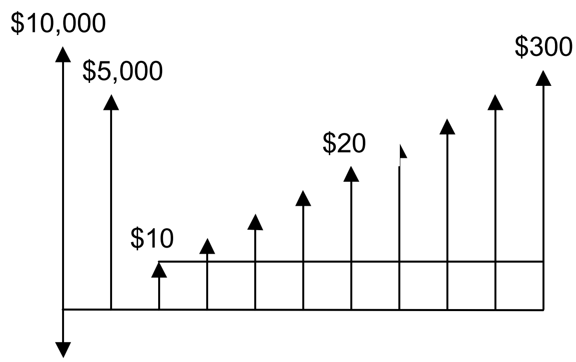
This result is very sensitive to the sinking fund factor. (A/F, 15%, 40) is actually 0.00056208 which makes A = \$488.78.

4-48

This problem has a declining gradient.

P = \$85,000 (P/A, 4%, 5) - \$10,000 (P/G, 4%, 5)
= \$85,000 (4.452) - \$10,000 (8.555)
= \$292,870

4-49

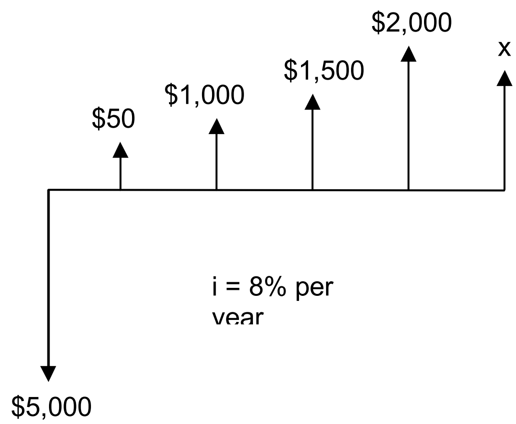


P = \$10,000 + \$500 (P/F, 6%, 1) + \$100 (P/A, 6%, 9) (P/F, 6%, 1)
+ \$25 (P/G, 6%, 9) (P/F, 6%, 1)

= \$10,000 + \$500 (0.9434) + \$100 (6.802) (0.9434) + \$25 (24.577) (0.9434)

= \$11,693.05

4-50



The first four payments will repay a present sum:

P = \$500 (P/A, 8%, 4) + \$500 (P/G, 8%, 4)
= \$500 (3.312) + \$500 (4.650)
= \$3,981

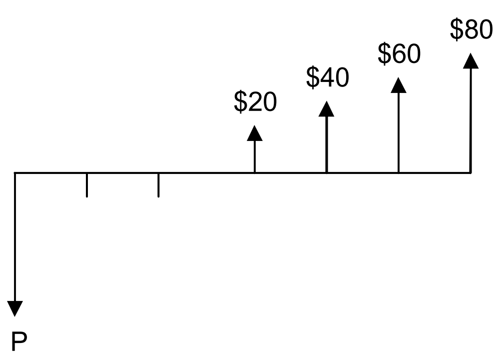
The unpaid portion of the \$5,000 is:

\$5,000 - \$3,981 = \$1,019

Thus:

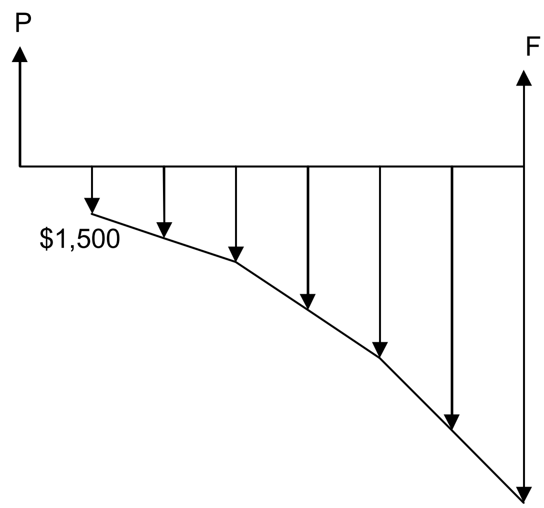
x = \$1,019 (F/P, 8%, 5)
= \$1,019 (1.469)
= \$1,496.91

4-51



P = \$20 (P/G, 8%, 5) (P/F, 8%, 1)
= \$20 (7.372) (0.9529)
= \$136.51

4-52



(a) Since the book only gives a geometric gradient to present worth factor, we must first solve for P and then F.

P = ? *n* = 6 *i* = 10% *g* = 8%

P = A_1 (P/A, g%, i%, n)

(P/A, g%, i%, n) $= [(1 - (1 + g)^n (1 + i)^{-n})/(i - g)]$
$= [(1 - (1.08)^6 (1.10)^{-6})/(0.10 - 0.08)]$
= 5.212

P = $1,500 (5.212) = $7,818

F = P (F/P, i%, n) = $7,818 (F/P, 10%, 6) = $13,853

As a check, solve with single payment factors:

$1,500.00 (F/P, 10%, 5)	= $1500.00 (1.611)	= $2,413.50
$1,620.00 (F/P, 10%, 4)	= $1,620.00 (1.464)	= $2,371.68
$1,749.60 (F/P, 10%, 3)	= $1,749.60 (1.331)	= $2,328.72
$1,889.57 (F/P, 10%, 2)	= $1,898.57(1.210)	= $2,286.38
$2,040.73 (F/P, 10%, 1)	= $2,040.73 (1.100)	= $2,244.80
$2,203.99 (F/P, 10%, 0)	= $2,203.99 (1.000)	= $2,203.99

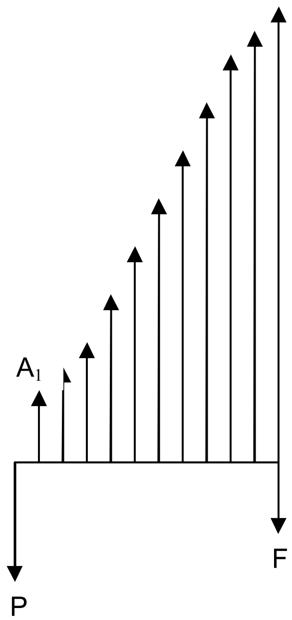
Total Amount =$13,852.07

(b) Here, i% = g%, hence the geometric gradient to present worth equation is:

$P = A_1 n (1 + i)^{-1} = \$1,500 (6) (1.08)^{-1} = \$8,333$

$F = P (F/P, 8\%, 6) = \$8,333 (1.587) = \$13,224$

4-53

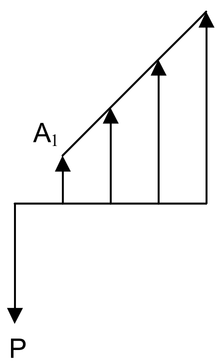


A = 5% (\$52,000) n = 20 $i = g = 8\%$ F = ?
= \$2,600

$$P = A_1 n (1 + i)^{-1} = \$2,600 (20) (1 + 0.08)^{-1} = \$48,148$$

$$F = P (F/P, i\%, n) = \$48,148 (1 + 0.08)^{20} = \$224,416$$

4-54



A_1 = 2nd year salary P = ? i = 12% g = 8% n = 4
= 1.08 ($225,000)
= $243,000

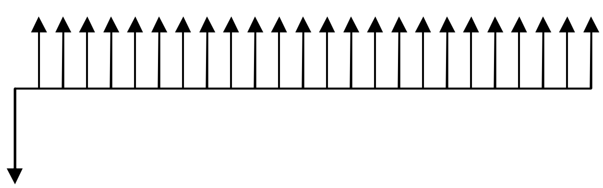
$$P = A_1 [(1- (1 + g)^n (1 + i)^{-n})/(i - g)]$$
$$= \$243{,}000[(1 - (1.08)^4 (1.12)^{-4})/0.04]$$
$$= \$243{,}000 [0.135385/0.04]$$
$$= \$822{,}462$$

4-55

P = 2000 cars/day n = 2 i = 5% F_2 = ? cars/day

$F_2 = P e^{in} = 2000\ e^{(0.05)(2)} = 2{,}210$ cars/day

4-56



P = $1,000 n = 24 months i = ? A = $47.50

P = A (P/A, i%, n)
$1,000 = $47.50 (P/A, i%, n)

(P/A, i%, 24) = $1,000/$47.50 = 21.053

Performing linear interpolation using interest tables:

(P/A, i%, 24)	i
21.243	1%
20.624	1.25%

i = 1% + 0.25% ((21.243 – 21.053)/(21.243 – 20.624))
= 1.077%/mo

Nominal Interest Rate = 12 months/year (1.077%/month)
= 12.92%/year

4-57

P = $2,000 *n* = 50 months *i* = ? *A* = $51.00

A = *P* (*A/P*, i%, n)
$51.00 = $2,000 (*A/P*, *i*%, 50)

(*A/P*, i%, 50) = $51.00/$2,000
= 0.0255

From interest tables:

i = 1% / month

Nominal Interest Rate = 12 months/ year (1% / month)
= 12% / year

Effective Interest Rate = $(1 + i)^m - 1$ = $(1.01)^{12} - 1$
= 12.7% / year

4-58

P = $1,000 Interest Payment = $10.87 / month *i* = ? *n = 12 months*

Nominal Interest Rate = 12 ($10.87)/$1,000
= 0.13 = 13%

4-59

i = 1% / month

Effective Interest Rate = $(1 + i)^m - 1$ = $(1.01)^{12} - 1$
= 0.127 = 12.7%

4-60

Nominal Interest Rate = 12 (1.5%) = 18%
Effective Interest Rate = $(1 + 0.015)^{12}$ = 0.1956 = 19.56%

4-61

(a) Effective Interest Rate $= (1 + i)^m - 1 = (1 + 0.025)^4 - 1 = 0.1038$
$= 10.38\%$

(b) Since the effective interest rate is 10.38%, we can look backwards to compute an equivalent i for 1/252 of a year.

$(1 + i)^{252} - 1 = 0.1038$

$(1 + i)^{252} = 1.1038$

$(1 + i) = 1.1038^{1/252} = 1.000392$

Equivalent i = 0.0392% per 1/252 of a year

(c) Subscriber's Cost per Copy:

$A = P\ (A/P, i\%, n) = P\ [(i\ (1 + i)^n)/((1 + i)^n - 1)]$

$A = \$206\ [\ (0.000392\ (1 + 0.000392)^{504})/(1 + 0.000392)^{504} - 1)]$
$= \$206\ (0.002187)$
$= \$0.45$ = 45 cents per copy

To check:

Ignoring interest, the cost per copy = \$206/(2(252)) = 40.8 cents per copy

Therefore, the answer of 45 cents per copy looks reasonable.

4-62

(a) $r = i \times m$
$= (1.25\%)\ (12)$
$= 15\%$

(b) $i_a = (1 + 0.0125)^{12} - 1$
$= 16.08\%$

(c) $A = \$10,000\ (A/P, 1.25\%, 48)$
$= \$10,000\ (0.0278)$
$= \$278$

4-63

(a)
P = \$1,000 A = \$90.30 i = ? m = 12 months

\$1,000 = \$90.30 (P/A, i%, 12)

$(P/A, i\%, 12) = \$1{,}000/\$90.30 = 11.074$

$i = 1.25\%$

(b) $r = (1.25\%)(12)$
$= 15\%$

(c) $i_a = (1 + 0.0125)^{12} - 1$
$= 16.08\%$

4-64

Effective interest rate $= (1 + i)^m - 1 =$

$1.61 = (1 + i)^{12}$

$(1 + i) = 1.61^{0.0833} = 1.0125$

$i = .0125 = 1.25\%$

4-65

Effective interest rate $= (1 + i_{mo})^m - 1 = 0.18$

$(1 + i_{mo}) = (1 + 0.18)^{1/12} = 1.01389$

$i_{mo} = 0.01388 = 1.388\%$

4-66

Effective Interest Rate $= (1 + i)^m - 1 = (1 + (0.07/365))^{365} - 1$
$= 0.0725 = 7.25\%$

4-67

$P = A (P/A, i\%, n)$

$\$1{,}000 = \$91.70 (P/A, i\%, 12)$

$(P/A, i\%, 12) = \$1{,}000/\$91.70 = 10.91$

From compound interest tables, $i = 1.5\%$

Nominal Interest Rate $= 1.5\% (12) = 18\%$

4-68

$F = P (1 + i)^n$

$\$85 = \$75 (1 + i)^1$

$(1 + i) = \$85/\$75 = 1.133$

$i = 0.133 = 13.3\%$

Nominal Interest Rate = 13.3% (2) = 26.6%

Effective Interest Rate = $(1 + 0.133)^2 - 1$ = 0.284 = 28.4%

4-69

Effective Interest Rate = $(1 + 0.0175)^{12} - 1$ = 0.2314 = 23.14%

4-70

Nominal Interest Rate = 1% (12) = 12%

Effective Interest Rate = $(1 + 0.01)^{12} - 1$ = 0.1268 = 12.7%

4-71

Effective Interest Rate = $(1 + i)^m - 1$

$0.0931 = (1 + i)^4 - 1$

$1.0931 = (1 + i)^4$

$1.0931^{0.25} = (1 + i)$

$1.0225 = (1 + i)$

$i = 0.0225$

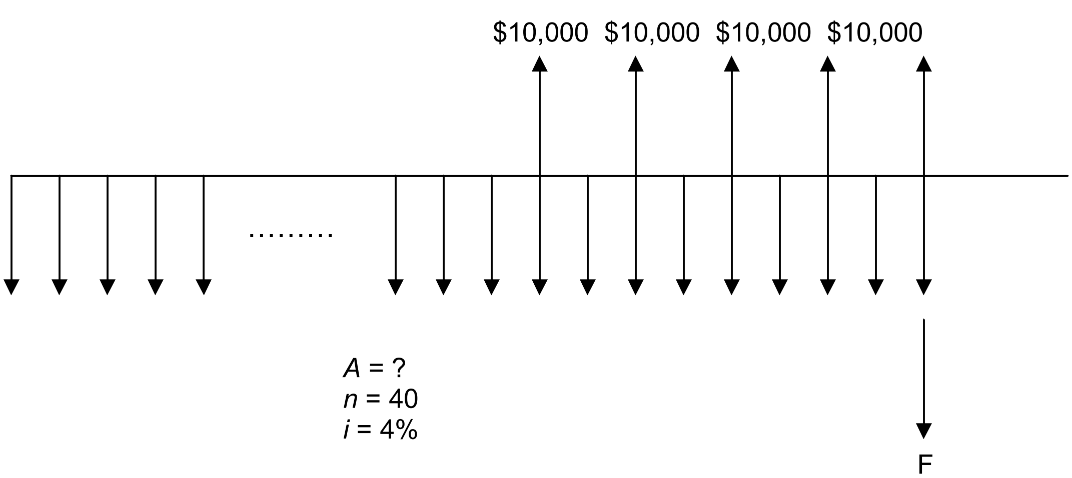
= 2.25% per quarter

= 9% per year

4-72

Effective Interest Rate = $(1 + i)^m - 1$ = $(1.03)^4 - 1$ = 0.1255 = 12.55%

4-73



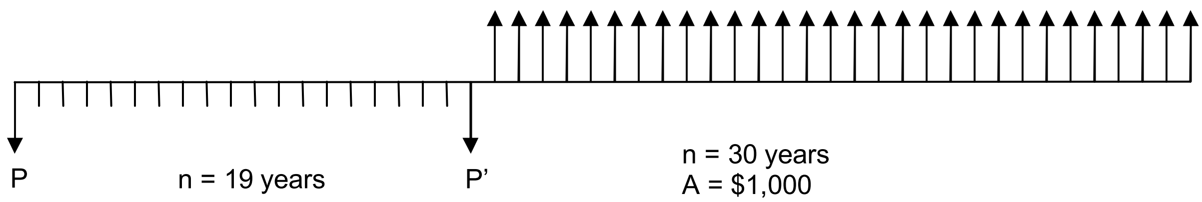
Compute F equivalent to the five $10,000 withdrawals:

F = $10,000 [(F/P, 4%, 8) + (F/P, 4%, 6) + (F/P, 4%, 4) + (F/P, 4%, 2) + 1]
= $10,000 [1.369 + 1.265 + 1.170 + 1.082 + 1]
= $58,850

Required series of 40 deposits:

A = F (A/F, 4%, 40) = $58,850 (0.0105) = $618

4-74



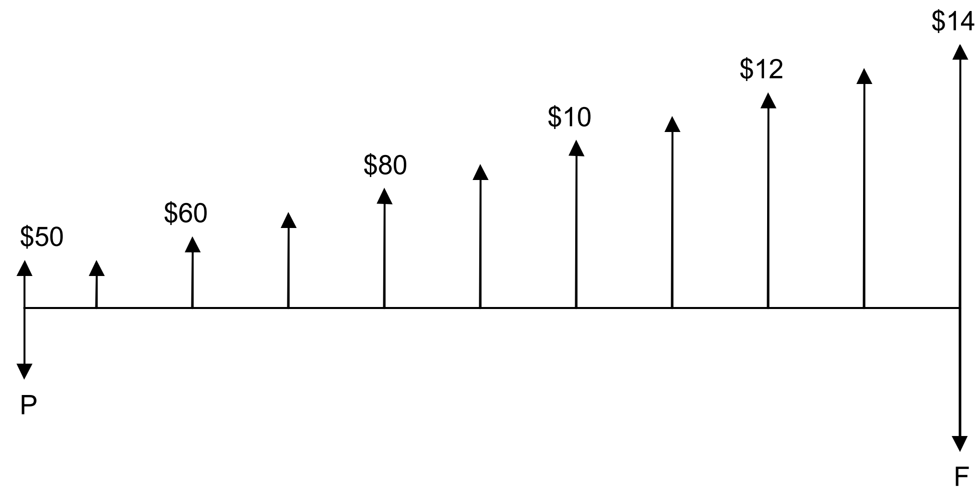
Note: There are 19 interest periods between P(40th birthday) and P' (6 months prior to 50th birthday)

P' = $1,000 (P/A, 2%, 30) = $1,000 (22.396)
= $22,396

P = P' (P/F, 2%, 19) = $22,396 (0.6864)
= $15,373 [Cost of Annuity]

4-75

The series of deposits are beginning-of-period deposits rather than end-of-period. The simplest solution is to draw a diagram of the situation and then proceed to solve the problem presented by the diagram.

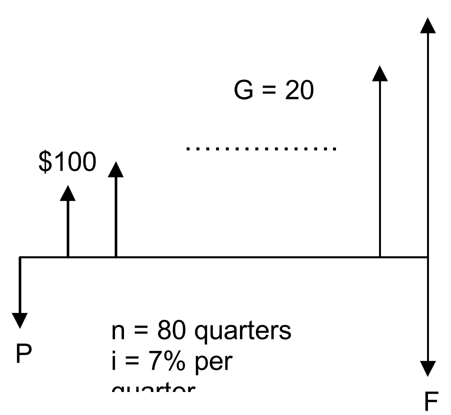


The diagram illustrates a problem that can be solved directly.

P = \$50 + \$50 (P/A, 3%, 10) + \$10 (P/G, 3%, 10)
= \$50 + \$50 (8.530) + \$10 (36.309)
= \$839.59

F = P (F/P, 3%, 10)
= \$839.59 (F/P, 3%, 10)
= \$839.59 (1.344)
= \$1,128.41

4-76



P = \$100 (P/A, 7%, 80) + \$20 (P/G, 7%, 80) = \$5,383.70
F = \$5,383.70 (F/P, 7%, 80) = \$1,207,200.00

Alternate Solution:

$$F = [\$100 + \$20\ (A/G, 7\%, 80)]\ (F/A, 7\%, 80)$$
$$= [\$100 + \$20\ (13.927)]\ (3189.1)$$
$$= \$1{,}207{,}200.00$$

4-77

Since there are annual deposits, but quarterly compounding, we must first compute the effective interest rate per year.

Effective interest rate $= (1 + i)^m - 1 = (1.02)^4 - 1 = 0.0824 = 8.24\%$

Since F = \$1,000,000 we can find the equivalent P for $i = 8.24\%$ and n = 40.

$$P = F\ (P/F, 8.24\%, 40)$$
$$= \$1{,}000{,}000\ (1 + 0.0824)^{-40}$$
$$= \$42{,}120$$

Now we can insert these values in the geometric gradient to present worth equation:

$$P = A_1\ [(1 - (1 + g)^n\ (1 + i)^{-n})/(i - g)]$$

$$\$42{,}120 = A_1\ [(1 - (1.07)^{40}\ (1.0824)^{-40})/(0.0824\text{-}0.0700)]$$
$$= A_1\ (29.78)$$

The first RRSP deposit, $A_1 = \$42{,}120/29.78 = \$1{,}414$

4-78

$i = 14\%$
$n = 19$ semiannual periods

$i_{qtr} = 0.14 / 4 = 0.035$
$i_{semiannual} = (1 + 0.035)^2 - 1 = 0.071225$

Can either solve for P or F first. Let's solve for F first:

$$F_{1/05} = A\ (F/A, i\%, n)$$
$$= \$1{,}000\ [(1 + 0.071225)^{19} - 1]/0.071225$$
$$= \$37{,}852.04$$

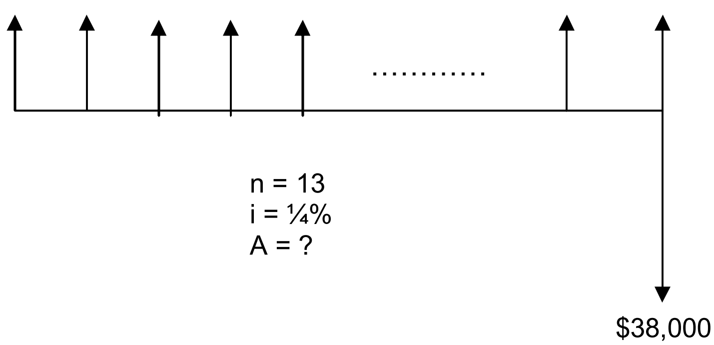
Now, we have the Future Worth at January 1, 2005. We need the Present Worth at April 1, 1998. We can use either interest rate, the quarterly or the semiannual. Let's use the quarterly with $n = 27$.

$$P = F\ (1 + i)^{-n}$$
$$= \$37{,}852.04\ (1.035)^{-27}$$
$$= \$14{,}952$$

This particular example illustrates the concept of these problems being similar to putting a puzzle together. There was no simple formula, or even a complicated formula, to arrive at the solution. While the actual calculations were not difficult, there were several steps required to arrive at the correct solution.

4-79

i = interest rate/interest period = 0.13/52 = 0.0025 = 0.25%
Paco's Account: 63 deposits of $38,000 each, equivalent weekly deposit



$$
\begin{aligned}
A &= F\,(A/F,\ i\%,\ n)\\
&= \$38{,}000\,(A/F,\ 0.25\%,\ 13)\\
&= \$38{,}000\,(0.0758)\\
&= \$2{,}880.40
\end{aligned}
$$

For 63 deposits:

$$
\begin{aligned}
F &= \$2{,}880.40\,(F/A,\ 0.25\%,\ 63\text{x}13)\\
&= \$2{,}880.40\,[((1.0025)^{819} – 1)/0.0025]\\
&= \$2{,}880.40\,(2691.49)\\
&= \$7{,}752{,}570 \text{ at } 4/1/2012
\end{aligned}
$$

Amount at 1/1/ 2007 = $7,742,570 (P/F, 0.25%, 273)
= $7,742,570 (0.50578)
= $3,921,000

Tisha's Account: 18 deposits of $18,000 each

Equivalent weekly deposit:

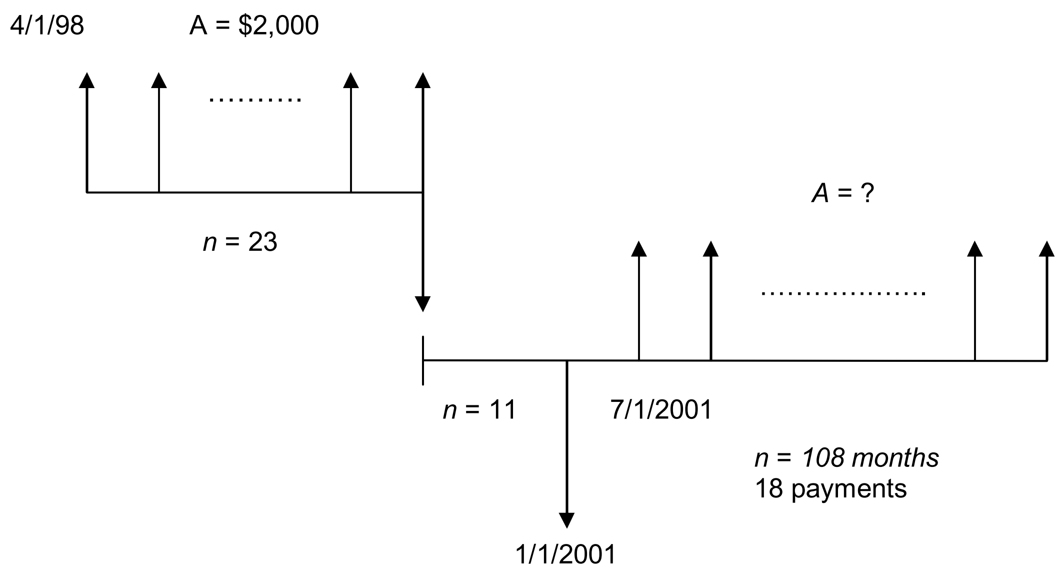
$$
\begin{aligned}
A &= \$18{,}000\,(A/F,\ 0.25\%,\ 26)\\
&= \$18{,}000\,(0.0373)\\
&= \$671.40
\end{aligned}
$$

Present Worth $P_{1/1/2006}$= \$671.40 (P/A, 0.25%, 18x26)
= \$671.40 [((1.0025)^{468} – 1)/275.67]
= \$185,084

Amount at 1/1/2007 = \$185,084 (F.P, 0.25%, 52)
= \$185,084 (1.139)
= \$211,000

Sum of both accounts at 1/1/2007 = \$3,921,000 + \$211,000 = \$4,132,000

4-80



Monthly cash flows:

$F_{2/1/2000}$ = \$2,000 (F/A, 1%, 23) = \$2,000 (25.716) = \$51,432

$F_{2/1/2001}$ = \$51,432 (F/P, 1%, 11) = \$51,432 (1.116) = \$57,398

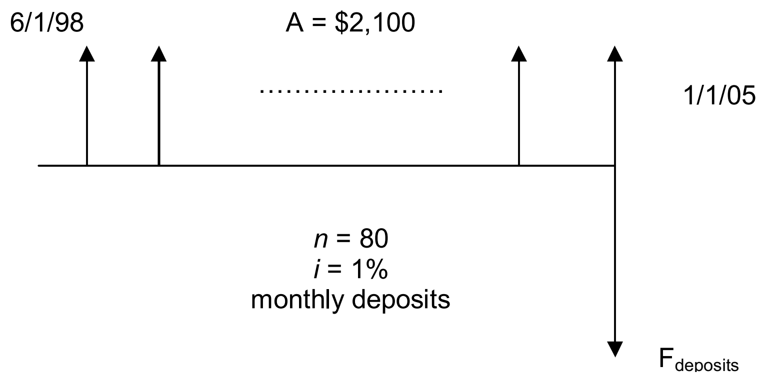
Equivalent A from 2/1/2001 through 1/1/2010 where *n* = 108 and *i* = 1%

A_{equiv} = \$57,398 (A/P, 1%, 108) = \$57,398 (0.01518)
= \$871.30

Equivalent semiannual payments required from 7/1/2001 through 1/1/2010:
$A_{semiann}$= \$871.30 (F/A, 1%, 6) = \$871.30 (6.152)
= \$5,360

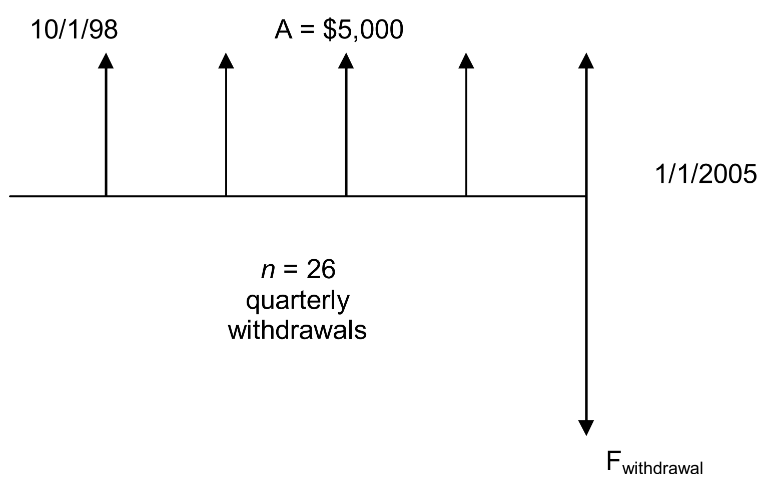
4-81

Deposits



$F_{deposits}$ = \$2,100 (F/A, 1%, 80)
= \$255,509

Withdrawals:



Equivalent quarterly interest $i_{quarterly}$ = $(1.01)^3 - 1$
= 0.0303 = 3.03%

$F_{withdrawals}$ = \$5,000 (F/A, 3.03%, 26)
= \$5,000 $[((1.0303)^{26} - 1)/0.0303]$
= \$193,561

Amount remaining in the account on January 1, 2005:
= \$255,509 - \$193,561
= \$61,948

4-82

A = 3($100) = $300 $\quad$ i = 1.5% per quarter year $\quad$ F = ? $\quad$ n = 12 quarterly periods (in 3 years)

F = A (F/A, i%, n) = $300 (F/A, 1.5%, 12) = $300 (13.041)
= $3,912.30

Note that this is no different from Ann's depositing $300 at the end of each quarter, as her monthly deposits do not earn any interest until the subsequent quarter.

4-83

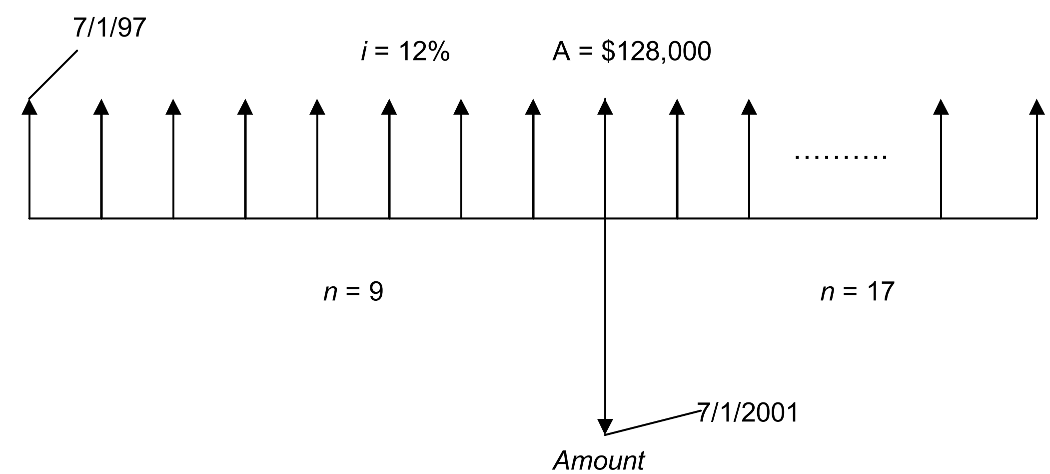
Compute the effective interest rate per quarterly payment period:

i_{qtr} = $(1 + 0.10/12)^3 - 1$ = 0.0252 = 2.52%

Compute the present worth of the 32 quarterly payments:

P = A (P/A, 2.52%, 32)
= $3,000 $[(1.0252)^{12} - 1]/[0.0252(1.0252)^{12}]$
= $3,000 (21.7878)
= $65,363

4-84



Amount$_{7/1/2001}$ = $128,000 (F/A, 6%, 9) + $128,000 (P/A, 6%, 17)
= $128,000 [(11.491) + (10.477)]
= $2,811,904

4-85

P = $3,000 $\quad$ A = ? $\quad$ i = 1% /month $\quad$ n = 30 months

A = P (A/P, i%, n)

A = \$3,000 (A/P, 1%, 30)
= \$3,000 (0.0387)
= \$116.10

4-86

(a) Bill's monthly payment = 2/3 (\$4,200) (A/P, 0.75%, 36)
= \$2,800 (0.0318)
= \$89.04

(b) Bill owed the October 1 payment plus the present worth of the 27 additional payments.

Balance = \$89.04 + \$89.04 (P/A, 0.75%, 27)
= \$89.04 (1 + 24.360)
= \$2,258.05

4-87

Amount of each payment = \$1,000 (A/P, 4.5%, 4) = \$1,000 (0.2787)
= \$278.70

Effective interest rate = $(1 + i)^m - 1$ = $(1.045)^4 - 1$ = 0.19252
= 19.3%

4-88

Monthly Payment = \$10,000 (A/P, 0.75%, 12) = \$10,000 (0.0875)
= \$875.00

Total Interest Per Year = \$875.00 x 12 - \$10,000 = \$500.00

Rule of 78s

With early repayment:

Interest Charge = ((12 + 11 + 10) / 78) (\$500) = \$211.54

Additional Sum (in addition to the 3rd \$875.00 payment)

Additional Sum = \$10,000 + \$211.54 interest – 3 (\$875.00) = \$7,586.54

Exact Method

Additional Sum equals present worth of the nine future payments that would have been made:

Additional Sum = \$875.00 (P/A, 0.75%, 9) = \$875.00 (8.672) = \$7,588.00

4-89

P = \$25,000 n = 60 months i = 18% per year
= 1.5% per month

(a) A = \$25,000 (A/P, 1.5%, 60)
= \$635

(b) P = \$25,000 (0.98) = \$24,500

\$24,500 = \$635 (P/A, i%, 60)

(P/A, i%, 60) = \$24,500/\$635 = 38.5827

Performing interpolation using interest tables:

(P/A, i%, 60)	i
39.380	1.50%
36.964	1.75%

i% = 0.015 + (0.0025) [(39.380 – 38.5827)/(39.380 – 36.964)]
= 0.015 + 0.000825
= 0.015825
= 1.5825% per month

i_a = $(1 + 0.015825)^{12} - 1$
= 0.2073
= 20.72%

4-90

i = 18%/12 = 1.5% /month

(a) A = \$100 (A/P, 1.5%, 24) = \$100 (0.0499) = \$4.99

(b) P_{13} = \$4.99 + \$4.99 (P/A, 1.5%, 11)
= (13^{th} payment) + (PW of future 11 payments)
= \$4.99 + \$4.99 (10.071)
= \$55.24

4-91

i = 6%/12 = ½% per month

(a) P = \$10 (P/A, 0.5%, 48) = \$10 (42.580)= \$425.80

(b) P_{24} = \$10 (P/A, 0.5%, 24) = \$10 (22.563)= \$225.63

(c) P = $\$10\ [(e^{(0.005)(48)} – 1)/(e^{(0.005)(48)}(e^{0.005} – 1))]$
= \$10 (42.568) = \$425.68

4-92

(a)

P = \$500,000 - \$100,000 = \$400,000 *n = 360* *i* = r/m = 0.09/12 = 0.75% *A* = ?

A = \$400,000 (A/P, 0.75%, 360) = \$400,000 (0.00805)
= \$3,220

(b) P = A (P/A, 0.75%, 240) = \$3,220 (111.145)
= \$357,887

(c) A = $\$400{,}000\ [(e^{(0.06/12)(360)})(e^{(0.06/12)} – 1)/(e^{(0.06/12)(360)} – 1)]$
= \$400,000 [(6.05)(0.005)/(5.05)]
= \$2,396

4-93

P = \$3,000 + \$280 (P/A, 1%, 60) = \$3,000 + \$280 (44.955)
= \$15,587

4-94

5% compounded annually

F = \$5,000 (F/P, 5%, 3) = \$5,000 (1.158)
= \$5,790

5% compounded continuously

F = Pe^{rn} = $\$5{,}000\ (e^{0.05\ (3)})$ = \$5,000 (1.1618)
= \$5,809

4-95

Compute effective interest rate for each alternative

(a) 4.375%

(b) $(1 + 0.0425/4)^4 – 1$ = $(1.0106)^4 – 1$ = 0.0431 = 4.31%

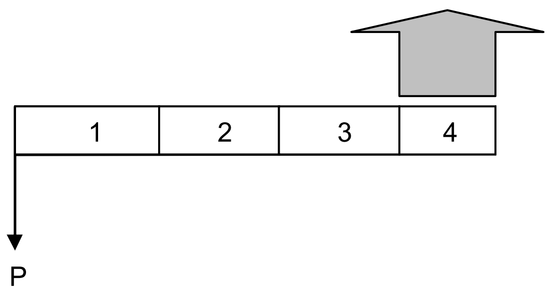
(c) $e^{rn} – 1 = e^{0.04125} – 1$ = 0.0421 = 4.21%

The 4 3/8% interest (a) has the highest effective interest rate.

4-96

$F = Pe^{rn} = \$100\ e^{0.04\,(5)} = \$100\ (1.2214) = \$122.14$

4-97



$P = F\ [(e^{r} - 1)/(r\ e^{rn})] = \$40{,}000\ [(e^{0.07} - 1)/(0.07\ e^{(0.07)(4)})]$
$= \$40{,}000\ (0.072508/0.092619)$
$= \$31{,}314.53$

4-98

(a) Interest Rate per 6 months $= \$20{,}000/\$500{,}000 = 0.0400 = 4\%$

Effective Interest Rate per yr. $= (1 + 0.04)^{2} - 1 = 0.0816$
$= 8.16\%$

(b) For continuous compounding:

$F = Pe^{rn}$

$\$520{,}000 = \$500{,}000\ e^{r(1)}$

$r = \ln\ (\$520{,}000/\$500{,}000) = 0.0392$
$= 3.92\%$ per 6 months

Nominal Interest Rate (per year) $= 3.92\%\ (2) = 7.84\%$ per year

4-99

$P = \$10{,}000$ $F = \$30{,}000$ $i = 5\%$ $n = ?$

$F = P\ e^{rn}$
$\$30{,}000 = \$10{,}000\ e^{(0.05)n}$

$0.05\ n = \ln\ (\$30{,}000/\$10{,}000) = 1.0986$

$n = 1.0986/0.05$
$= 21.97$ years

4-100

(a) Effective Interest Rate $= (1 + i)^m - 1 = (1.025)^4 - 1 = 0.1038$
$= 10.38\%$

(b) Effective Interest Rate $= (1 + i)^m - 1 = (1 + (0.10/365))^{365} - 1$
$= 0.10516 = 10.52\%$

(c) Effective Interest Rate $= e^r - 1 = e^{0.10} - 1 = 0.10517$
$= 10.52\%$

4-101

(a) $P = Fe^{-rn}$
$= \$8,000\ e^{-(0.08)(4.5)}$
$= \$5,581.41$

(b) $F = Pe^{rn}$
$F/P = e^{rn}$
$\ln(F/P) = rn$

$r = (1/n) \ln (F/P)$
$= (1/4.5) \ln(\$8,000/\$5000)$
$= 10.44\%$

4-102

(1) 11.98% compounded continuously
$F = \$10,000\ e^{(0.1198)(4)}$
$= \$16,147.82$

(2) 12% compounded daily
$F = \$10,000\ (1 + 0.12/365)^{365x4}$
$= \$16,159.47$

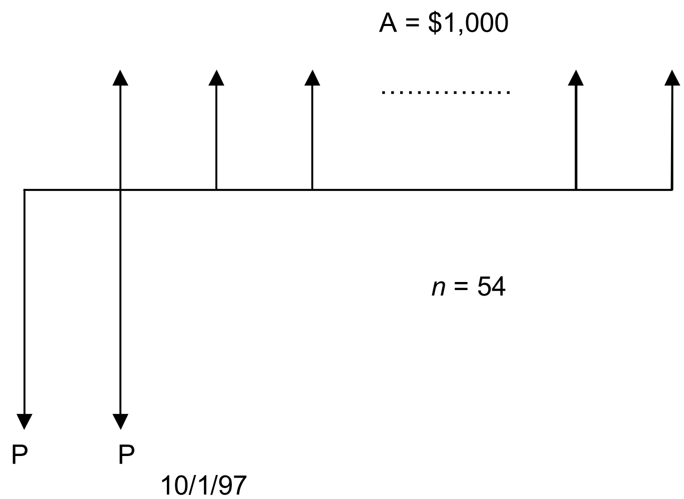
(3) 12.01% compounded monthly
$F = \$10,000\ (1 + 0.1201/12)^{12x4}$
$= \$16,128.65$

(4) 12.02% compounded quarterly
$F = \$10,000\ (1 + 0.1202/4)^{4x4}$
$= \$16,059.53$

(5) 12.03% compounded yearly
$F = \$10,000\ (1 + 0.1203)^4$
$= \$15,752.06$

Decision: Choose Alternative (2)

4-103



Continuous compounding

Effective interest rate/ quarter year $= e^{(0.13/4)} - 1 = 0.03303$
$= 3.303\%$

Solution One

$$
\begin{aligned}
P_{10/1/97} &= \$1,000 + \$1,000\ (P/A, 3.303\%, 53) \\
&= \$1,000 + \$1,000\ [((1.03303)^{53} - 1))/(0.03303(1.03303)^{53})] \\
&= \$25,866
\end{aligned}
$$

Solution Two

$$
\begin{aligned}
P_{10/1/97} &= \$1,000\ (P/A, 3.303\%, 54)\ (F/P, 3.303\%, 1) \\
&= \$1,000\ [((1.03303)^{54} - 1)/(0.03303\ (1.03303)^{54})]\ (1.03303) \\
&= \$25,866
\end{aligned}
$$

4-104

(a) Effective Interest Rate

$$
\begin{aligned}
i_a &= (1 + r/m)^m - 1 \\
&= (1 + 0.06/2)^2 - 1 \\
&= 0.0609 \\
&= 6.09\%
\end{aligned}
$$

Continuous Effective Interest Rate

$$
\begin{aligned}
i_a &= e^r - 1 \\
&= e^{0.06} - 1 \\
&= 0.0618 \\
&= 6.18\%
\end{aligned}
$$

(b) The future value of the loan, one period (6 months) before the first repayment:

= $2,000 (F/P, 3%, 5)
= $2,000 (1.159)
= $2318

The uniform payment:
= $2,318 (A/P, 3%, 4)
= $2,318 (0.2690)
= $623.54 every 6 months

(c) Total interest paid:
= 4 ($623.54) - $2,000
= $494.16

4-105

$P = Fe^{-rn} = \$6{,}000\ e^{-(0.12)(2.5)} = \$6{,}000\ (0.7408) = \$4{,}444.80$

4-106

Nominal Interest Rate = (1.75%) 12 = 21%

Effective Interest Rate $= e^{rn} - 1 = e^{(0.21\text{x}1)} - 1 = 0.2337 = 23.37\%$

4-107

P = $4,500 F = $10,000 i = ? n = 1 six month interest period

$F = P\ (1 + i)$

$(1 + i) = F/P = \$10{,}000/\$4{,}500 = 1.0526$

$i = .0526 = 5.26\%$

Nominal Interest Rate = 5.26% (2) = 10.52%

Effective Interest Rate $= (1 + .0526)^2 - 1 = 0.10797 = 10.80\%$

4-108

West Bank

$F = P\ (1 + i)^n = \$10{,}000\ (1 + (0.065/365))^{365} = \$10{,}671.53$

East Bank

$F = P e^{rn} = \$10{,}000\ e^{(.065x1)} = \$10{,}671.59$

Difference $= \$0.06$

4-109

$P = F e^{-rn} = \$10{,}000\ e^{-(0.08)(0.5)} = \$10{,}000\ e^{-0.04} = \$10{,}000\ (0.9608)$
$= \$9608$

4-110

(a) Continuous cash flow – continuous compounding (one period)

$$
\begin{aligned}
F &= P\hat{}\ [(e^{r} - 1)\ (e^{rn})/re^{r}] \\
&= \$1 \times 10^{9}\ [(e^{0.005} - 1)\ (e^{(0.005)(1)})/(0.005\ e^{0.005})] \\
&= \$1 \times 10^{9}\ [(e^{0.005} - 1)/0.005] \\
&= \$1 \times 10^{9}\ (0.00501252/0.005) \\
&= \$1{,}002{,}504{,}000
\end{aligned}
$$

Thus, the interest is \$2,504,000.

(b) Deposits of A = \$250 x 10^{6} occur four times a month

Continuous compounding

$$
\begin{aligned}
r &= \text{nominal interest rate per ¼ month} \\
&= 0.005/4 \quad = 0.00125 \quad = 0.125\%
\end{aligned}
$$

$$
\begin{aligned}
F &= A\ [(e^{rn} - 1)/(e^{r} - 1)] \\
&= \$250{,}000{,}000\ [(e^{(0.00125)(4)} - 1)/(e^{(0.00125)} - 1)] \\
&= \$250{,}000{,}000\ [0.00501252/0.00125078] \\
&= \$1{,}001{,}879{,}000
\end{aligned}
$$

Here, the interest is \$1,879,000.

So it pays \$625,000 a month to move quickly!

4-111

$$
\begin{aligned}
P &= F\hat{}\ \ [(e^{r} - 1)/re^{rn}] \\
&= \$15{,}000\ [(e^{0.08} - 1)/((0.08)(e^{(0.08)(6)})] \\
&= \$15{,}000\ [0.083287/0.129286] \\
&= \$9{,}663
\end{aligned}
$$

4-112

P = \$29,000 n = 3 years F = ?

(a) i_a = 0.13

$F = P(1 + i)^n$ = \$29,000 $(1.13)^3$ = \$41,844

(b) r = 0.1275

$F = P e^{rn}$ = \$29,000 $e^{(0.1275)(3)}$ = \$29,000 (1.4659) = \$42,511

We can see that although the interest rate was less with the continuous compounding, the future amount is greater because of the increased compounding periods (an infinite number of compounding periods). Thus, the correct choice for the company is to choose the 13% interest rate and discrete compounding.

4-113

A = \$1,200 r = 0.14/12 = *0.01167* n = 7 x 12 = 84 compounding periods

$F = A [(e^{rn} – 1)/(e^r – 1)]$

= \$1,200 $[(e^{(0.01167)(84)} – 1)/(e^{0.01167} – 1)]$

= \$1,200 [1.66520/0.011738]

= \$170,237

4-114

First Bank- Continous Compounding

Effective interest rate $i_a = e^r – 1 = e^{0.045} – 1$ = 0.04603 = 4.603%

Second Bank- Monthly Compounding

Effective interest rate $i_a = (1 + r/m)^m – 1 = (1 + 0.046/12)^{12} – 1$ = 0.04698 = 4.698%

No, Barry should have selected the Second Bank.

4-115

A = P (A/P, i%, 24)

(A/P, i%, 24) = A/P = 499/10,000 = 0.499

From the compound interest tables we see that the interest rate per month is exactly 1.5%.

4-116

Common Stock Investment

P = \$1,000 n = 20 quarters i = ? F = \$1,307

F	= P (F/P, i%, n)
\$1,307	= \$1,000 (F/P, i%, 20)
(F/P, i%, 20)	= \$1,307/\$1,000
	= 1.307

Performing linear interpolation using interest tables:

(P/A, i%, 20)	i
1.282	1.25%
1.347	1.50%

i = 1.25% + 0.25% ((1.307 – 1.282)/(1.347 – 1.282))
= 1.25% + 0.10%
= 1.35%

Nominal Interest Rate = 4 quarters / year (1.35% / quarter)
= 5.40% / year

Effective Interest Rate = $(1 + i)^m - 1$ = $(1.0135)^4 - 1$
= 5.51% / year

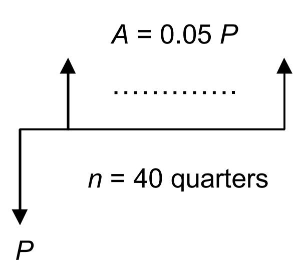
4-117

$F = P(1 + i)^n \quad = 0.98F(1 + i)^1$

i = (1.00/0.98) – 1
= 0.0204 = 2.04%

$i_{eff} = (1 + i)^m - 1 \quad = (1.0204)^{365/20} - 1$
= 0.4456 = 44.6%

4-118



$P \quad = 0.05\, P\, (P/A,\, i\%,\, 40)$

$(P/A,\, i\%,\, 40) \quad = 1/0.05 \quad = 20$

From interest tables:

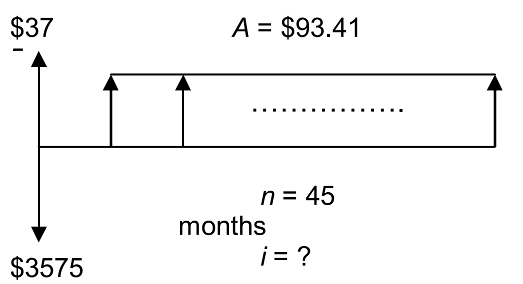
$(P/A, i\%, 40)$	i
21.355	3.5%
19.793	4.0%

Performing linear interpolation:

$$
\begin{aligned}
i &= 3.5\% + 0.5\%\, ((21.355 - 20)/(21.355 - 19.793)) \\
&= 3.5\% + 0.5\%\, (1.355/1.562) \\
&= 3.93\% \text{ per quarter year}
\end{aligned}
$$

Effective rate of interest $= (1 + i)^m - 1 \quad = (1.0393)^4 - 1$
$= 0.1667 \quad = 16.67\%$ per year

4-119



$\$3{,}575 \quad = \$375 + \$93.41\, (P/A,\, i\%,\, 45)$
$(P/A,\, i\%,\, 45) \quad = (\$3{,}575 - \$375)/\$93.41$
$= 34.258$

From compound interest tables, $i = 1.25\%$ per month
For an \$800 down payment, unpaid balance is \$2775.

$P = \$2{,}775 \qquad n = 45$ months $\qquad i = 1.25\% \qquad A = ?$

$$
\begin{aligned}
A &= \$2{,}775\, (A/P,\, 1.25\%,\, 45)^* \\
&= \$2{,}775\, (0.0292) \\
&= \$81.03
\end{aligned}
$$

Effective interest rate $= (1 + i)^{12} - 1 \quad = (1.0125)^{12} - 1$
$= 0.161 \quad = 16.1\%$ per year

* Note that no interpolation is required as $(A/P,\, 1.25\%,\, 45) = 1/(P/A,\, i\%,\, 45)$
$= 1/34.258 = 0.0292$

4-120

(a) Future Worth

$71 million = $165,000 (F/P, i%, 61)

(F/P, i%, 61) = $71,000,000/$165,000
= 430.3

From interest tables:

(*P/A*, *i*%, 61)	*i*
341.7	10%
1,034.5	12%

Performing linear interpolation:

i = 10% + (2%) ((430.3 – 341.7)/(1034.5 – 341.7))
= 10.3%

(b) In 1929, the Consumer Price Index was 17 compared to about 126 in 1990. So $165,000 in 1929 dollars is roughly equivalent to $165,000 (126/17) = $1,223,000 in 1990 dollars. The real rate of return is closer to 6.9%.

4-121

FW = FW
$1000 (F/A, *i*%, 10) (F/P, *i*%, 4) = $28,000

By trial and error:

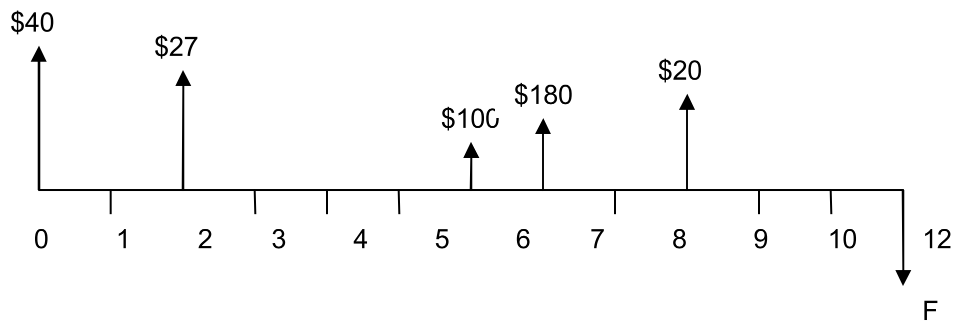
Try *i* = 12%	$1,000 (17.549) (1.574)	= $27,622	*i* too low
i = 15%	$1,000 (20.304) (1.749)	= $35,512	*i* too high

Using Interpolation:
i = 12% + 3% (($28,000 – $27,622)/($35,512 - $27,622))
= 12.14%

4-122

Since (A/P, *i*%, *n*) = (A/F, *i*%, *n*) + *i* (Shown on page 78)
0.1728 = 0.0378 + *i*
i = 13.5%

4-123



i = NIR/m = 9%/12 = 0.75%/mo

F_{12} = $400 (F/P, 0.75%, 12) + $270 (F/P, 0.75%, 10) + $100 (F/P, 0.75%, 6) + $180 (F/P, 0.75%, 5) + $200 (F/P, 0.75%, 3)
= $400 (1.094) + $270 (1.078) + $100 (1.046) + $180 (1.038) + $200 (1.023)
= $1,224.70 (same as above)

4-124

PW = $6.297m

Year	Cash Flows ($K) – 15%	PW Factor 10%	PW ($K)
1	$2,000	0.9091	$1,818
2	$1,700	0.9264	$1,405
3	$1,445	0.7513	$1,086
4	$1,228	0.6830	$839
5	$1,044	0.6209	$648
6	$887	0.5645	$501
		Total PW	= $6,297

4-125

Year	Cash Flows ($K) – 8%	PW Factor 6%	PW ($K)
1	$10,000	0.9434	$9,434
2	$10,800	0.8900	$9,612
3	$11,664	0.8396	$9,793
4	$12,597	0.7921	$9,978
		Total PW	= $38,817

4-126

Year	Cash Flows ($K) – 15%	PW Factor 10%	PW ($K)
1	$30,000	0.9091	$27,273
2	$25,500	0.9264	$21,074
3	$21,675	0.7513	$16,285
4	$18,424	0.6830	$12,584
5	$15,660	0.6209	$9,724
6	$13,311	0.5645	$7,514
		Total PW	= $94,453

4-127

Payment = 11K (A/P, 1%, 36) = 11K (0.0332) = $365.2
($365.357 for exact calculations)

Month	1% Interest	$365.36 Principal	Balance Due
0			$11,000.00
1	$110.00	$255.36	10,744.64
2	107.45	257.91	10,486.73
3	104.87	260.49	10,226.24
4	102.26	263.09	9,963.15
5	99.63	265.73	9697.41
6	96.97	268.38	9429.04
7	64.29	271.07	9157.97
8	91.58	273.78	8884.19
9	88.84	276.52	8607.68
10	86.08	279.28	8328.40
11	83.28	282.07	8046.32
12	80.46	284.89	7761.43
13	77.61	287.74	7473.69
14	74.74	290.62	7183.07
15	71.83	293.53	6889.54
16	68.90	296.46	6593.08
17	65.93	299.43	6293.65
18	62.94	302.42	5991.23
19	59.91	305.45	5685.74
20	56.86	308.50	5377.28
21	53.77	311.58	5065.70
22	50.66	314.70	4751.00
23	47.51	317.85	4433.15
24	44.33	321.03	4113.13
25	41.12	324.24	3787.89
26	37.88	327.48	3460.41
27	34.60	330.75	3129.66
28	31.30	334.06	3795.60
29	27.96	337.40	3458.20
30	24.58	340.78	2117.42
31	21.17	344.18	1773.24
32	17.73	347.63	1425.61

33	14.26	351.10	1074.51
34	10.75	354.61	719.90
35	7.20	358.16	361.74
36	3.62	361.74	0.00

4-128

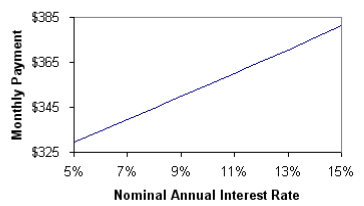
Payment = 17K (A/P, 0.75%, 60) = 17K (0.0208) = $353.60
($352.892 for exact calculations)

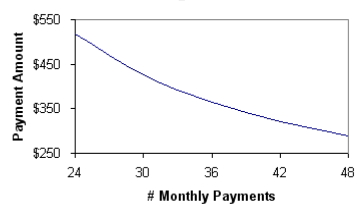
Month	0.75% Interest	$352.89 Principal	Balance Due	Month	0.75% Interst	$358.89 Principal	Balance Due
0			$17,000.00	30			$9,448.71
1	$127.50	$225.39	$16,774.61	31	$70.87	$282.03	9,166.68
2	125.81	227.08	16,547.53	32	68.75	284.14	8,882.54
3	124.11	228.79	16,318.74	33	66.62	286.27	8,596.27
4	122.39	230.50	16,088.24	34	64.47	288.42	8,307.85
5	120.66	232.23	15,856.01	35	62.31	290.58	8,017.27
6	118.92	233.97	15,622.04	36	60.13	292.76	7,724.51
7	117.17	235.73	15,386.31	37	57.93	294.96	7,429.55
8	115.40	237.49	15,148.81	38	55.72	297.17	7,132.38
6	113.62	239.28	14,909.54	39	53.49	299.40	6,832.98
10	111.82	241.07	14,668.48	40	51.25	301.64	6,531.33
11	110.01	242.88	14,425.59	41	48.98	303.91	6,227.43
12	108.19	244.70	14,180.89	42	46.71	306.19	5,921.24
13	106.36	246.54	13,934.35	43	44.41	308.48	5,612.76
14	104.51	278.38	13,685.97	44	42.10	310.80	5,301.96
15	102.64	250.25	13,435.72	45	39.76	313.13	4,988.83
16	100.77	252.12	13,183.60	46	37.42	315.48	4,673.36
17	98.88	254.02	12,929.58	47	35.05	317.84	4,355.52
18	96.97	255.92	12,673.66	48	32.67	320.23	4,035.29
19	95.05	257.84	12,415.82	49	30.26	322.63	3,712.66
20	93.12	259.77	12,156.05	50	27.84	325.05	3,387.62
21	91.17	261.72	11,894.33	51	25.41	327.48	6,030.13
22	89.21	263.68	11,630.64	52	22.95	329.94	2,730.19
23	87.23	265.66	11,364.98	53	20.48	332.45	2,397.77
24	85.24	237.65	11,097.33	54	17.98	334.91	2,062.86
25	83.23	269.66	10,827.67	55	15.47	337.42	1,725.44
26	81.21	271.68	10,555.98	56	12.94	339.95	1,385.49
27	79.17	273.72	10,282.26	57	10.39	342.50	1,042.99
28	77.12	275.78	10,006.48	58	7.82	345.07	697.92
29	75.05	277.84	9,728.64	59	5.23	347.66	350.27
30	72.96	279.93	9,448.71	60	2.63	350.27	0.00

4-129

See Excel output below:

	A	B	C
1	11000	loan amount	
2	12	# payments/year	
3	3	# years of payments	
4	12.00%	nominal annual interest rate	
5	Calculated data		
6	1.00%	interest rate per period	
7	36	# payments	
8	$365.36	payment	=PMT(A6,A7,-A1)
9			
10	interest	payment	
11	5%	$329.68	=PMT(A11/12,$A$7,-$A$1)
12	6%	$334.64	
13	7%	$339.65	
14	8%	$344.70	
15	9%	$349.80	
16	10%	$354.94	
17	11%	$360.13	
18	12%	$365.36	
19	13%	$370.63	
20	14%	$375.95	
21	15%	$381.32	
26	# payment	payment	
27	24	$517.81	=PMT($A$6,A27,-$A$1)
28	30	$426.23	
29	36	$365.36	
30	42	$322.03	
31	48	$289.67	

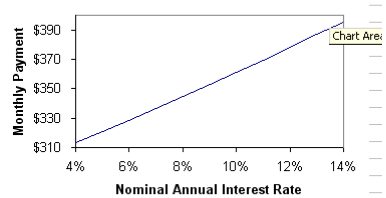


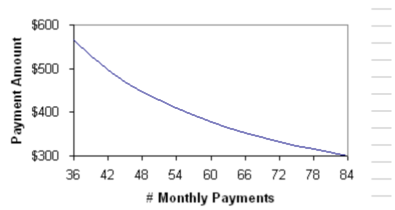


4-130

See Excel output below:

	A	B	C
1	17000	loan amount	
2	12	# payments/year	
3	5	# years of payments	
4	12.00%	nominal annual interest rate	
5	Calculated data		
6	1.00%	interest rate per period	
7	60	# payments	
8	$378.16	payment	=PMT(A6,A7,-A1)
9			
10	interest	payment	
11	4%	$313.08	=PMT(A11/12,$A$7,-$A$1)
12	5%	$320.81	
13	6%	$328.66	
14	7%	$336.62	
15	8%	$344.70	
16	9%	$352.89	
17	10%	$361.20	
18	11%	$369.62	
19	12%	$378.16	
20	13%	$386.80	
21	14%	$395.56	
26	# payment	payment	
27	36	$564.64	=PMT($A$6,A27,-$A$1)
28	42	$497.69	
29	48	$447.68	
30	54	$408.96	
31	60	$378.16	
32	66	$353.10	
33	72	$332.35	
34	78	$314.92	
35	84	$300.10	





4-131

Year	5% Salary	6% Interest	10% Deposit	Total
1	$50,000.00		$5,000.00	$5,000.00
2	52,500.00	$300.00	5,250.00	10,550.00
3	55,125.00	633.00	5,512.50	16,695.50
4	57,881.25	1,001.73	5,788.13	23,485.36
5	60,775.31	1,409.12	6,077.53	30,972.01
6	63,814.08	1,858.32	6,381.41	39,211.74
7	67,004.78	2,352.70	6,700.48	48,264.92
8	70,355.02	2,895.90	7,035.50	58,196.32
9	73,872.77	3,491.78	7,387.28	69,075.37
10	77,566.41	4,144.52	7,756.64	80,976.53
11	81,444.73	4,858.59	8,144.47	93,979.60
12	85,516.97	5,638.78	8,551.70	108,170.07
13	89,792.82	6,490.20	8,979.28	123,639.56
14	94,282.46	7,418.37	9,428.25	140,486.18
15	98,996.58	8,429.17	9,899.66	158,815.01
16	103,946.41	9,528.90	10,394.64	107,243.13
17	109,143.73	6,434.59	10,914.37	124,592.09
18	114,600.92	7,475.53	11,460.09	143,527.71
19	120,330.96	8,611.66	12,033.10	164,172.47
20	126,347.51	9,850.35	12,634.75	186,657.57
21	132,664.89	11,199.45	13,266.49	211,123.51
22	139,298.13	12,667.41	13,929.81	237,720.73
23	146,263.04	14,263.24	14,626.30	266,610.28
24	153,576.19	15,996.62	15,357.62	297,964.51
25	161,255.00	17,877.87	16,125.50	331,967.88
26	169,317.75	19,918.07	16,931.77	368,817.73
27	177,783.63	22,129.06	17,778.36	408,725.16
28	186,672.82	24,523.51	18,667.28	451,915.95
29	196,006.46	27,114.96	19,600.65	498,631.55
30	205,806.78	29,917.89	20,580.68	549,130.13
31	216,097.12	32,947.81	21,609.71	603,687.64
32	226,901.97	36,221.26	22,690.20	662,599.10
33	238,247.07	39,755.95	23,824.71	726,179.75
34	250,159.43	43,570.79	25,015.94	794,766.48
35	262,667.40	47,685.99	26,266.74	868,719.21
36	275,800.77	52,123.15	27,580.08	948,422.44
37	289,590.81	56,905.35	28,959.08	1,034,286.87
38	304,070.35	62,057.21	30,407.03	1,126,751.11
39	319,273.86	67,605.07	31,927.39	1,226,283.57
40	335,237.56	73,577.01	33,523.76	1,333,384.34

4-132

Year	$200,000 15%	Potential Lost Profit -3%	Incremental Cash Flow (B (1 – C)	PW (10%)
1	$200,000	1.00	$0.00	$0.00
2	230,000	0.9700	6,900.00	5,702.48
3	264,500	0.9409	15,631.95	11,744.52
4	304,175	0.9127	26,562.69	18,142.67
5	349,801	0.8853	40,124.72	24,914.29
6	402,271	0.8587	56,827.27	32,077.51
7	462,612	0.8330	77,269.18	39,651.31
8	532,004	0.8080	102,153.89	47,655.54
9	611,805,	0.7837	132,333.42	56,111.00
10	703,575	0.7602	168,695.49	65,039.42
			PW_5	= $60,503.96
			PW_{10}	= $301,038.74

4-133

Payment = 120K ≅ (A/P,10/12%,360) = 120K ≅ .00877572 = $1053.08

Month	0.83% Interest	$1,053.09 Principal	Balance Due	Month	0.83% Interest	$1,053.09 Principal	Balance Due
0			$120,000.00	50			$116,723.88
1	$1,000.00	$53.09	119,946.91	51	$972.70	$80.39	116,643.49
2	999.56	53.53	119,893.399373	52	972.03	81.06	116,562.43
3	999.11	53.97	119,839.411424	53	971.35	81.73	116,480.70
4	998.66	54.42	119,784.99	54	970.67	82.41	116,398.29
5	998.21	54.88	119,730.11	55	969.99	83.10	116,315.19
6	997.75	55.33	119,674.77	56	969.29	83.79	116,231.40
7	997.29	55.80	119,618.98	57	968.60	84.49	116,146.91
8	996.82	56.26	119,562.72	58	967.89	85.20	116,061.71
9	996.36	56.73	119,505.99	59	967.18	85.91	115,975.81
10	995.88	57.20	119,448.79	60	966.47	86.62	115,889.18
11	995.41	57.68	119,391.11	61	965.74	87.34	115,801.84
12	994.93	58.16	119,332.95	62	965.02	88.07	115,713.77
13	994.44	58.64	119,274.30	63	964.28	88.80	115,624.97
14	993.95	59.13	119,215.17	64	963.54	89.54	115,535.42
15	993.46	59.63	119,155.54	65	962.80	90.29	115,445.13
16	992.96	60.12	119,095.42	66	962.04	91.04	115,354.09
17	992.46	60.62	119,034.79	67	961.28	91.80	115,262.29
18	991.96	61.13	118,973.67	68	960.52	92.57	115,169.72
19	991.45	61.64	118,912.03	69	959.75	93.34	115,076.38
20	990.93	62.15	118,849.87	70	958.97	94.12	114,982.27
21	990.42	62.67	118,787.20	71	958.19	94.90	114,887.37
22	989.89	63.19	118,724.01	72	957.39	95.69	114,791.67
23	989.37	63.72	118,660.29	73	956.60	96.49	114,695.19
24	988.84	64.25	118,596.04	74	955.79	97.29	114,597.89
25	988.30	64.79	118,531.26	75	954.98	98.10	114,499.79
26	987.76	65.33	118,465.93	76	954.16	98.92	114,400.87
27	987.22	65.87	118,400.06	77	953.34	99.75	114,301.12
28	986.67	66.42	118,333.64	78	952.51	100.58	114,200.55
29	986.11	66.97	118,266.67	79	951.67	101.41	114,099.13
30	985.56	67.53	118,199.14	80	950.83	102.26	113,996.87
31	984.99	68.09	118,131.05	81	949.97	103.11	113,893.76
32	984.43	68.66	118,062.39	82	949.11	103.97	113,789.79
33	983.85	69.23	117,993.15	83	948.25	104.84	113,684.95
34	983.28	69.81	117,923.34	84	947.37	105.71	113,579.24
35	982.69	70.39	117,852.95	85	946.49	106.59	113,472.65
36	982.11	70.98	117,781.98	86	945.61	107.48	113,365.17
37	981.52	71.57	117,710.41	87	944.71	108.38	113,256.79
38	980.92	72.17	117,638.24	88	943.81	109.28	113,147.51

39	980.32	72.77	117,565.47	89	942.90	110.19	113,037.32
40	979.71	73.37	117,492.10	90	941.98	111.11	112,926.21
41	979.10	73.99	117,418.11	91	941.05	112.03	112,814.18
42	978.48	74.60	117,343.51	92	940.12	112.97	112,701.21
43	977.86	75.22	117,268.29	93	939.18	113.91	112,587.30
44	977.24	75.85	117,192.44	94	938.23	114.86	112,472.44
45	976.60	76.48	117,115.96	95	937.27	115.82	112,356.63
46	975.97	77.12	117,038.84	96	936.31	116.78	112,239.85
47	975.32	77.76	116,961.07	97	935.33	117.75	112,122.09
48	974.68	78.41	116,882.66	98	934.35	118.74	112,003.36
49	974.02	79.06	116,803.60	99	933.36	119.72	111,883.63
50	973.36	79.72	116,723.88	100	932.36	120.72	111,762.91
100			$111,762.91	150			$104,250.62
101	$931.36	$121.73	111,641.18	151	$868.76	$184.33	104,066.29
102	930.34	122.74	111,518.44	152	867.22	185.87	103,880.42
103	929.32	123.77	111,394.68	153	865.67	187.42	103,693.01
104	928.29	124.80	111,269.88	154	864.11	188.98	103,504.03
105	927.25	125.84	111,144.04	155	862.53	190.55	103,313.48
106	926.20	126.89	111,017.16	156	860.95	192.14	103,121.34
107	925.14	127.94	110,889.21	157	859.34	193.74	102,927.60
108	924.08	129.01	110,760.20	158	857.73	195.36	102,732.24
109	923.00	130.08	110,630.12	159	856.10	196.98	102,535.26
110	921.92	131.17	110,498.95	160	854.46	198.63	102,336.63
111	920.82	132.26	110,366.69	161	852.81	200.28	102,136.35
112	919.72	133.36	110,233.33	162	851.14	201.95	101,934.40
113	918.61	134.47	110,098.85	163	849.45	203.63	101,730.77
114	917.49	135.60	109,963.26	164	847.76	205.33	101,525.44
115	916.36	136.73	109,826.53	165	846.05	207.04	101,318.40
116	915.22	137.86	109,688.67	166	844.32	208.77	101,109.63
117	914.07	139.01	109,549.65	167	842.58	210.51	100,899.13
118	912.91	140.17	109,409.48	168	840.83	212.26	100,686.87
119	911.75	141.34	109,268.14	169	839.06	214.03	100,472.84
120	910.57	142.52	109,125.62	170	837.27	215.81	100,257.03
121	909.38	143.71	108,981.92	171	835.48	217.61	100,039.42
122	908.18	144.90	108,837.01	172	833.66	219.42	99,819.99
123	906.98	146.11	108,690.90	173	831.83	221.25	99,598.74
124	905.76	147.33	108,543.58	174	829.99	223.10	99,375.64
125	904.53	148.56	108,395.02	175	828.13	224.96	99,150.69
126	903.29	149.79	108,245.23	176	826.26	226.83	98,923.86
127	902.04	151.04	108,094.18	177	824.37	228.72	98,695.14
128	900.78	152.30	107,941.88	178	822.46	230.63	98,464.51
129	899.52	153.57	107,788.31	179	820.54	232.55	98,231.96
130	898.24	154.85	107,633.46	180	818.60	234.49	97,997.48
131	896.95	156.14	107,477.32	181	816.65	236.44	97,761.04
132	895.64	157.44	107,319.88	182	814.68	238.41	97,522.62
133	894.33	158.75	107,161.13	183	812.69	240.40	97,282.23
134	893.01	160.08	107,001.05	184	810.69	242.40	97,039.83
135	891.68	161.41	106,839.64	185	808.67	244.42	96,795.41
136	890.33	162.76	106,676.88	186	806.63	246.46	96,548.95
137	888.97	164.11	106,512.77	187	804.57	248.51	96,300.44
138	887.61	165.48	106,347.29	188	802.50	250.58	96,049.85
139	886.23	166.86	106,180.43	189	800.42	252.67	95,797.18
140	884.84	168.25	106,012.18	190	798.31	254.78	95,542.41
141	883.43	169.65	105,842.53	191	796.19	256.90	95,285.51
142	882.02	171.06	105,671.47	192	794.05	259.04	95,026.47
143	880.60	172.49	105,498.98	193	791.89	261.20	94,765.27
144	879.16	173.93	105,325.05	194	789.71	263.38	94,501.90
145	877.71	175.38	105,149.67	195	787.52	265.57	94,236.33
146	876.25	176.84	104,972.84	196	785.30	267.78	93,968.54
147	874.77	178.31	104,794.52	197	783.07	270.01	93,698.53
148	873.29	179.80	104,614.72	198	780.82	272.26	93,426.26
149	871.79	181.30	104,433.43	199	778.55	274.53	93,151.73
150	870.28	182.81	104,250.62	200	776.26	276.82	92,874.91
200			$92,874.91	250			$75,648.89
201	$773.96	$279.13	92,595.78	251	$630.41	$422.68	75,226.21
202	771.63	281.45	92,314.32	252	626.89	426.20	74,800.01
203	769.29	283.80	92,030.52	253	623.33	429.75	74,370.26
204	766.92	286.16	91,744.36	254	619.75	433.33	73,936.92
205	764.54	288.55	91,455.81	255	616.14	436.94	73,499.98
206	762.13	290.95	91,164.86	256	612.50	440.59	73,059.39

207	759.71	293.38	90,871.48	257	608.83	444.26	72,615.14
208	757.26	295.82	90,575.65	258	605.13	447.96	72,167.18
209	754.80	298.29	90,277.37	259	601.39	451.69	71,715.48
210	752.31	300.77	89,976.59	260	597.63	455.46	71,260.03
211	749.80	303.28	89,673.31	261	593.83	459.25	70,800.77
212	747.28	305.81	89,367.50	262	590.01	463.08	70,337.70
213	744.73	308.36	89,059.14	263	586.15	466.94	69,870.76
214	742.16	310.93	88,748.22	264	582.26	470.83	69,399.93
215	739.57	313.52	88,434.70	265	578.33	474.75	68,925.17
216	736.96	316.13	88,118.57	266	574.38	478.71	68,446.46
217	734.32	318.76	87,799.81	267	570.39	482.70	67,963.77
218	731.67	321.42	87,478.39	268	566.36	486.72	67,477.04
219	728.99	324.10	87,154.29	269	562.31	490.78	66,986.27
220	726.29	326.80	86,827.49	270	558.22	494.87	66,491.40
221	723.56	329.52	86,497.96	271	554.10	498.99	65,992.41
222	720.82	332.27	86,165.69	272	549.94	503.15	65,489.26
223	718.05	335.04	85,830.65	273	545.74	507.34	64,981.92
224	715.26	337.83	85,492.82	274	541.52	511.57	64,470.35
225	712.44	340.65	85,152.18	275	537.25	515.83	63,954.52
226	709.60	343.48	84,808.69	276	532.95	520.13	63,434.38
227	706.74	346.35	84,462.35	277	528.62	524.47	62,909.92
228	703.85	349.23	84,113.11	278	524.25	528.84	62,381.08
229	700.94	352.14	83,760.97	279	519.84	533.24	61,847.84
230	698.01	355.08	83,405.89	280	515.40	537.69	61,310.15
231	695.05	358.04	83,047.86	281	510.92	542.17	60,767.98
232	692.07	361.02	82,686.84	282	506.40	546.69	60,221.30
233	689.06	364.03	82,322.81	283	501.84	551.24	59,670.06
234	686.02	367.06	81,955.74	284	497.25	555.84	59,114.22
235	682.96	370.12	81,585.62	285	492.62	560.47	58,553.75
236	679.88	373.21	81,212.42	286	487.95	565.14	57,988.61
237	676.77	376.32	80,836.10	287	483.24	569.85	57,418.77
238	673.63	379.45	80,456.65	288	478.49	574.60	56,844.17
239	670.47	382.61	80,074.04	289	473.70	579.38	56,264.79
240	667.28	385.80	79,688.23	290	468.87	584.21	55,680.57
241	664.07	389.02	79,299.22	291	464.00	589.08	55,091.49
242	660.83	392.26	78,906.96	292	459.10	593.99	54,497.50
243	657.56	395.53	78,511.43	293	454.15	598.94	53,898.56
244	654.26	398.82	78,112.61	294	449.15	603.93	53,294.63
245	650.94	402.15	77,710.46	295	444.12	608.96	52,685.67
246	647.59	405.50	77,304.96	296	439.05	614.04	52,071.63
247	644.21	408.88	76,896.08	297	433.93	619.16	51,452.47
248	640.80	412.29	76,483.80	298	428.77	624.32	50,828.16
249	637.37	415.72	76,068.08	299	423.57	629.52	50,198.64
250	633.90	419.19	75,648.89	300	418.32	634.76	49,563.88
300			$49,563.88	330			$27,851.01
301	$413.03	$640.05	48,923.82	331	$232.09	$820.99	27,030.01
302	407.70	645.39	48,278.43	332	225.25	827.84	26,202.18
303	402.32	650.77	47,627.67	333	218.35	834.73	25,367.44
304	396.90	656.19	46,971.48	334	211.40	841.69	24,525.75
305	391.43	661.66	46,309.82	335	204.38	848.70	23,677.05
306	385.92	667.17	45,642.65	336	197.31	855.78	22,821.27
307	380.36	672.73	44,969.92	337	190.18	862.91	21,958.36
308	374.75	678.34	44,291.59	338	182.99	870.10	21,088.26
309	369.10	683.99	43,607.60	339	175.74	877.35	20,210.91
310	363.40	689.69	42,917.91	340	168.42	884.66	19,326.25
311	357.65	695.44	42,222.47	341	161.05	892.03	18,434.22
312	351.85	701.23	41,521.24	342	153.62	899.47	17,534.75
313	346.01	707.08	40,814.16	343	146.12	906.96	16,627.79
314	340.12	712.97	40,101.20	344	138.56	914.52	15,713.27
315	334.18	718.91	39,382.29	345	130.94	922.14	14,791.12
316	328.19	724.90	38,657.39	346	123.26	929.83	13,861.30
317	322.14	730.94	37,926.45	347	115.51	937.58	12,923.72
318	316.05	737.03	37,189.41	348	107.70	945.39	11,978.33
319	309.91	743.17	36,446.24	349	99.82	953.27	11,025.07
320	303.72	749.37	35,696.87	350	91.88	961.21	10,063.86
321	297.47	755.61	34,941.26	351	83.87	969.22	9,094.64
322	291.18	761.91	34,179.35	352	75.79	977.30	8,117.34
323	284.83	768.26	33,411.09	353	67.64	985.44	7,131.90
324	278.43	774.66	32,636.43	354	59.43	993.65	6,138.25

325	271.97	781.12	31,855.32	355	51.15	1001.93	5,136.31
326	265.46	787.62	31,067.69	356	42.80	1010.28	4,126.03
327	258.90	794.19	30,273.50	357	34.38	1018.70	3,107.33
328	252.28	800.81	29,472.70	358	25.89	1027.19	2,080.13
329	245.61	807.48	28,665.22	359	17.33	1035.75	1,044.38
330	238.88	814.21	27,851.01	360	8.70	1044.38	0.00

4-134

There are several ways to solve this, but one of the easiest is to simply calculate the PW for years 0 to 1, 0 to 2, 0 to 3, etc. This is the cumulative PW in the last column below. Note that if the average monthly cash flow savings of $85 are used, the furnace is paid off sooner, since the savings occur throughout the year rather than at the end of the year. The period with monthly figures is 34 months rather than the 35 months indicated below.

Year	Cash Flow	PW 9%	Cumulative PW
0	-$2,500	-$2,500	-$2,500
1	$1,020.00	$935.78	-$1,564.22
2	$1,020.00	$858.51	-$705.71
3	$1,020.00	$787.63	$81.92

4-135

(a) See Excel output below:

	A	B	C	D
1	15000	loan amount		
2	12	# payments/year		
3	3	# years of payments		
4	8.90%	nominal annual interest rate		
5	Calculated data			
6	0.74%	interest rate per period		
7	36	# payments		
8	$476.30	payment		
9				
10	period	interest	principal	rem.bal.
11	0			$ 15,000.00
12	1	$ 111.25	$365.05	$14,634.95
13	2	108.54	367.76	14,267.20
14	3	105.82	370.48	13,896.71
15	4	103.07	373.23	13,523.48
16	5	100.30	376.00	13,147.48
17	6	97.51	378.79	12,768.70
18	7	94.70	381.60	12,387.10
19	8	91.87	384.43	12,002.67
20	9	89.02	387.28	11,615.39
21	10	86.15	390.15	11,225.24
22	11	83.25	393.04	10,832.20
23	12	80.34	395.96	10,436.24
24	13	77.40	398.90	10,037.34
25	14	74.44	401.85	9,635.49
26	15	71.46	404.83	9,230.65
27	16	68.46	407.84	8,822.82
28	17	65.44	410.86	8,411.95
29	18	62.39	413.91	7,998.04
30	19	59.32	416.98	7,581.06
31	20	56.23	420.07	7,160.99
32	21	53.11	423.19	6,737.80
33	22	49.97	426.33	6,311.48
34	23	46.81	429.49	5,881.99
35	24	43.62	432.67	5,449.32
36	25	40.42	435.88	5,013.43
37	26	37.18	439.12	4,574.32
38	27	33.93	442.37	4,131.95
39	28	30.65	445.65	3,686.29
40	29	27.34	448.96	3,237.34
41	30	24.01	452.29	2,785.05
42	31	20.66	455.64	2,329.41
43	32	17.28	459.02	1,870.38
44	33	13.87	462.43	1,407.96
45	34	10.44	465.86	942.10
46	35	6.99	469.31	472.79
47	36	3.51	472.79	(0.00)

(b) See Excel output below:

	A	B	C	D
1	15000	loan amount		
2	12	# payments/year		
3	5	# years of payments		
4	8.90%	nominal annual interest rate		
5	Calculated data			
6	0.74%	interest rate per period		
7	60	# payments		
8	$310.65	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$15,000.00
12	1	$ 111.25	$199.40	$14,800.60
13	2	109.77	200.88	14,599.73
14	3	108.28	202.37	14,397.36
15	4	106.78	203.87	14,193.49
16	5	105.27	205.38	13,988.11
17	6	103.75	206.90	13,781.21
18	7	102.21	208.44	13,572.77
19	8	100.66	209.98	13,362.79
20	9	99.11	211.54	13,151.25
21	10	97.54	213.11	12,938.14
22	11	95.96	214.69	12,723.45
23	12	94.37	216.28	12,507.17
35	24	74.31	236.34	9,783.19
47	36	52.40	258.25	6,806.65
59	48	28.45	282.20	3,554.12
71	60	2.29	308.36	(0.00)

4-136

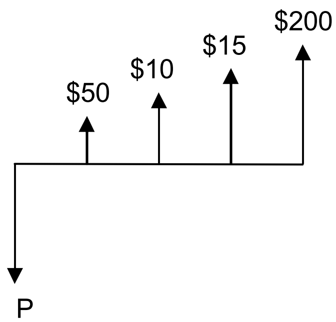
See Excel output below:

	A	B	C	D
3	15	# years of payments		
4	7.50%	nominal annual interest rate		
5	Calculated data			
6	0.63%	interest rate per period		
7	180	# payments		
8	$927.01	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$100,000.00
12	1	$ 625.00	$302.01	$99,697.99
13	2	623.11	303.90	99,394.09
14	3	621.21	305.80	99,088.29
15	4	619.30	307.71	98,780.58
16	5	617.38	309.63	98,470.94
17	6	615.44	311.57	98,159.38
18	7	613.50	313.52	97,845.86
19	8	611.54	315.48	97,530.38
20	9	609.56	317.45	97,212.94
21	10	607.58	319.43	96,893.50
22	11	605.58	321.43	96,572.08
23	12	603.58	323.44	96,248.64
35	24	578.47	348.55	92,206.05
47	36	551.41	375.60	87,849.63
59	48	522.25	404.76	83,155.00
71	60	490.83	436.19	78,095.92
83	72	456.96	470.05	72,644.09
95	84	420.47	506.54	66,769.01
107	96	381.15	545.86	60,437.85
119	108	338.77	588.24	53,615.17
131	120	293.10	633.91	46,262.84
143	132	243.89	683.12	38,339.72
155	144	190.86	736.15	29,801.51
167	156	133.71	793.30	20,600.46
179	168	72.12	854.89	10,685.11
191	180	5.76	921.25	0.00

	A	B	C	D
1	100000	loan amount		
2	12	# payments/year		
3	30	# years of payments		
4	7.50%	nominal annual interest rate		
5	Calculated data			
6	0.63%	interest rate per period		
7	360	# payments		
8	$699.21	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$ 100,000.00
12	1	$ 625.00	$74.21	$99,925.79
13	2	624.54	74.68	99,851.11
14	3	624.07	75.15	99,775.96
15	4	623.60	75.61	99,700.35
16	5	623.13	76.09	99,624.26
17	6	622.65	76.56	99,547.70
18	7	622.17	77.04	99,470.66
19	8	621.69	77.52	99,393.13
20	9	621.21	78.01	99,315.13
21	10	620.72	78.49	99,236.63
22	11	620.23	78.99	99,157.64
23	12	619.74	79.48	99,078.17
35	24	613.57	85.65	98,084.77
47	36	606.92	92.30	97,014.25
59	48	599.75	99.46	95,860.62
71	60	592.03	107.19	94,617.44
83	72	583.71	115.51	93,277.74
95	84	574.74	124.47	91,834.04
107	96	565.08	134.14	90,278.26
119	108	554.66	144.55	88,601.70
131	120	543.44	155.77	86,794.99
143	132	531.35	167.87	84,848.01
155	144	518.32	180.90	82,749.89
167	156	504.27	194.94	80,488.89
179	168	489.14	210.07	78,052.35
191	180	472.83	226.38	75,426.67
203	192	455.26	243.96	72,597.14
215	204	436.32	262.90	69,547.95
227	216	415.91	283.31	66,262.04
239	228	393.91	305.30	62,721.04
251	240	370.21	329.00	58,905.15
263	252	344.67	354.54	54,793.01
275	264	317.15	382.07	50,361.64
287	276	287.49	411.73	45,586.25
299	288	255.52	443.69	40,440.14
311	300	221.08	478.14	34,894.52
323	312	183.96	515.25	28,918.37
335	324	143.96	555.25	22,478.29
347	336	100.85	598.36	15,538.24
359	348	54.40	644.81	8,059.42
371	360	4.34	694.87	0.00

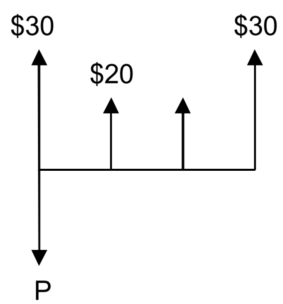
Chapter 5: Present Worth Analysis

5-1



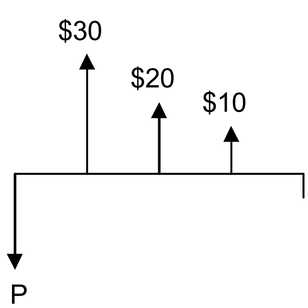
P = $50 (P/A, 10%, 4) + $50 (P/G, 10%, 4)
= $50 (3.170) + $50 (4.378)
= $377.40

5-2



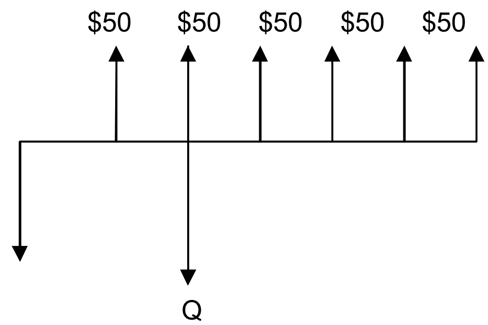
P = $30 + $20 (P/A, 15%, 2) + $30 (P/F, 15%, 3)
= $30 + $20 (1.626) + $30 (0.6575)
= $82.25

5-3



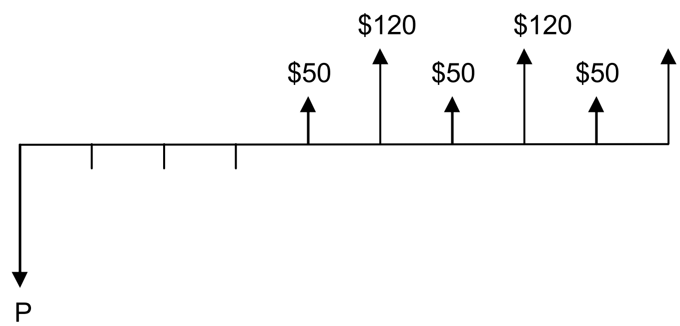
P = $300 (P/A, 12%, 3) - $100 (P/G, 12%, 3)
= $300 (2.402) - $100 (2.221)
= $498.50

5-4



Q = \$50 (P/A, 12%, 6) (F/P, 12%, 2)
= \$50 (4.111) (1.254)
= \$257.76

5-5`

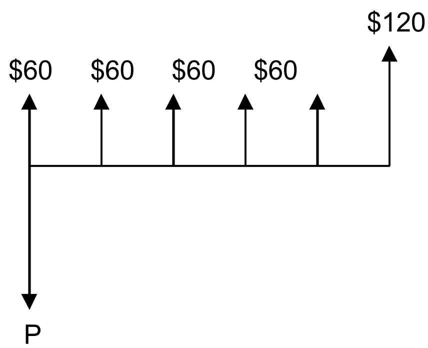


P = \$50 (P/A, 10%, 6) (P/F, 10%, 3) + \$70 (P/F, 10%, 5) + \$70 (P/F, 10%, 7)
+ \$70 (P/F, 10%, 9)
= \$50 (4.355) (0.7513) + \$70 (0.6209 + 0.5132 + 0.4241)
= \$272.67

Alternative Solution

P = [\$50 (P/A, 10%, 6) + \$70(P/F, 10%, 2) + \$70 (P/F, 10%, 4)
+ \$70 (P/F, 10%, 6)](P/F, 10%, 3)
= [\$50 (4.355) + \$70 (0.8264 + 0.6830 + 0.5645)] (0.7513)
= \$272.66

5-6

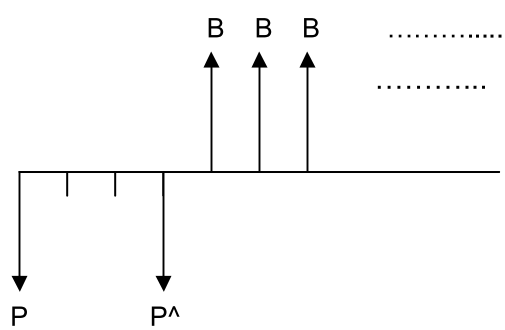


P = \$60 + \$60 (P/A, 10%, 4) + \$120 (P/F, 10%, 5)
= \$60 + \$60 (3.170) + \$120 (0.6209)
= \$324.71

5-7

P = A_1 (P/A, q, i, n)
= A_1 [(1 – $(1.10)^4$ $(1.15)^{-4}$)/(0.15 – 0.10)]
= \$200 (3.258)
= \$651.60

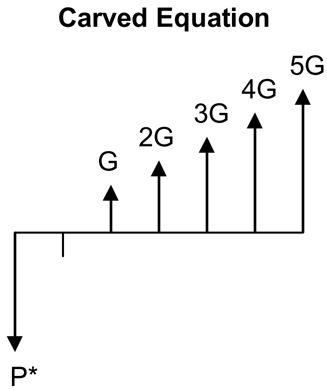
5-8

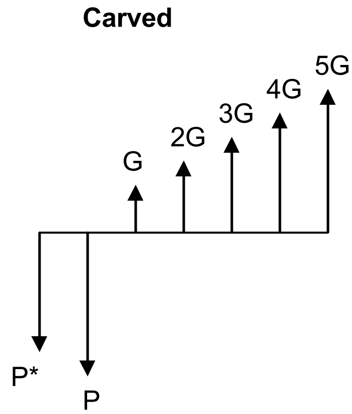


P^ = B/0.10 = 10 B

P = P^ (P/F, 10%, 3) = 10 B (0.7513) = 7.51 B

5-9





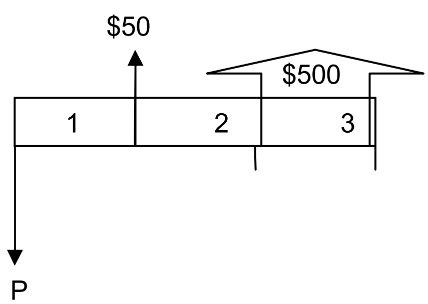
P* = G (P/G, *i*%, 6)

P = P* (F/P, *i*%, 1)

Thus:

P = G (P/G, *i*%, 6) (F/P, *i*%, 1)

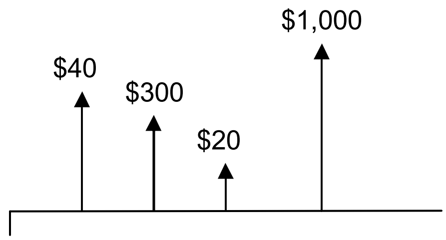
5-10



$$
\begin{aligned}
P &= F\,e^{-rn} + F^* [(e^r - 1)/(re^{rn})] \\
&= \$500\ (0.951229) + \$500\ [0.051271/0.058092] \\
&= \$475.61 + 441.29 \\
&= \$916.90
\end{aligned}
$$

5-11

The cycle repeats with a cash flow as below:



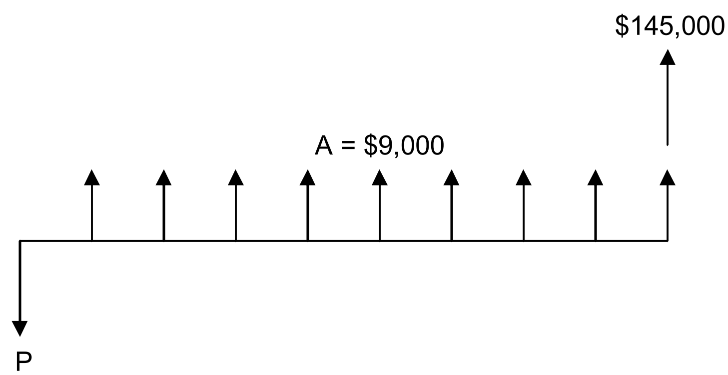
P = {[$400 - $100 (A/G, 8%, 4) + $900 (A/F, 8%, 4)]/0.08 + $1,000}
{P/F, 8%, 5}
= {[$400 - $100 (1.404) + $900 (0.2219)]/0.08 + $1,000} {0.6806}
= $4,588

Alternative Solution: An alternate solution may be appropriate if one assumes that the $1,000 cash flow is a repeating annuity from time 13 to infinity (rather than indicating the repeating decreasing gradient series cycles).

In this case P is calculated as:

P = [$500 - $100 (A/G, 8%, 4)](P/A, 8%, 8)(P/F, 8%, 4) + $500 (P/F, 8%, 5)
+ $500 (P/F, 8%, 9) + $1,000 (P/A, 8%, ∞) (P/F, 8%, 12)
= $7,073

5-12

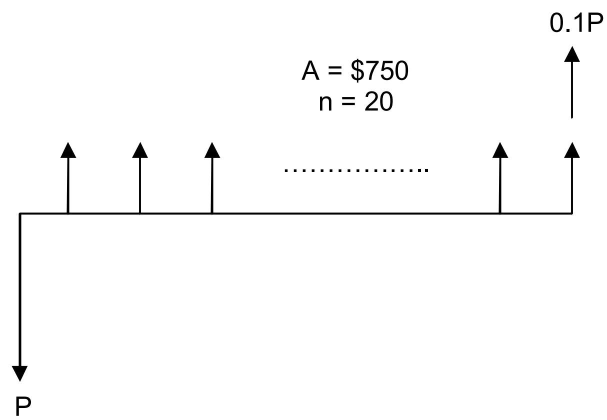


P = $9,000 (P/A, 18%, 10) + $145,000 (P/F, 18%, 10)
= $9,000 (4.494) + $145,000 (0.1911)
= $68,155.50

5-13

P = $100 (P/A, 6%, 6) + $100 (P/G, 6%, 6)
= $100 (4.917) + $100 (11.459)
= $1,637.60

5-14



PW of Cost = PW of Benefits

P = $750 (P/A, 7%, 20) + 0.1P (P/F, 7%, 20)
= $750 (10.594) + 0.1P (0.2584)
= $7945 + 0.02584P

P = $7945/(1-0.02584)
= $7945/0.97416
= $8156

5-15

Determine the cash flow:

Year	Cash Flow
0	-$4,400
1	$220
2	$1,320
3	$1,980
4	$1,540

NPW = PW of Benefits – PW of Cost
= $220 (P/F, 6%, 1) + $1,320 (P/F, 6%, 2) + $1,980 (P/F, 6%, 3)
+ $1,540 (P/F, 6%, 4) - $4,400
= $220 (0.9434) + $1,320 (0.8900) + $1,980 (0.8396)
+ $1,540 (0.7921) - $4,400
= -$135.41

NPW is negative. Do not purchase equipment.

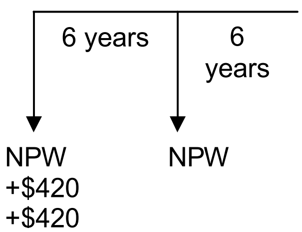
5-16

For end-of-year disbursements,

PW of wage increases = (\$0.40 x 8 hrs x 250 days) (P/A, 8%, 10)
+ (\$0.25 x 8 hrs x 250 days) (P/G, 8%, 10)
= \$800 (6.710) + \$500 (25.977)
= \$18,356

This \$18,356 is the increased justifiable cost of the equipment.

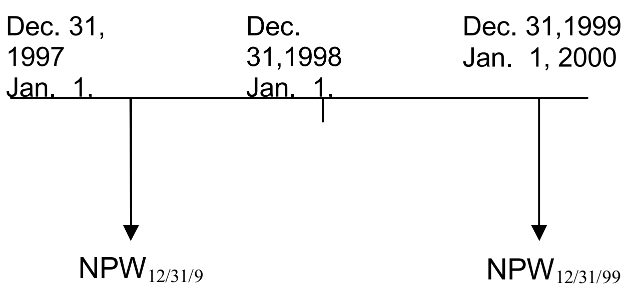
5-17



For the analysis period, the NPW of the new equipment = +\$420 as the original equipment.

$NPW_{12\ years}$ = \$420 + \$420 (P/F, 10%, 6)
= +\$657.09

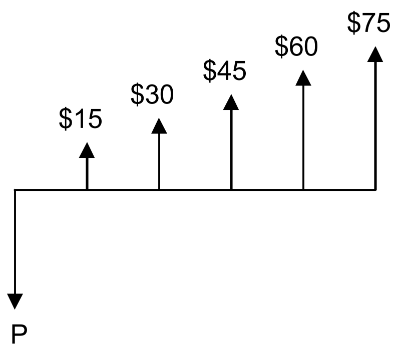
5-18



$NPW_{12/31/00}$ = -\$140

$NPW_{12/31/97}$ = -\$140 (P/F, 10%, 2) = -\$140 (0.8264) = -\$115.70

5-19



P = \$150 (P/A, 3%, 5) + \$150 (P/G, 3%, 5)
= \$150 (4.580) + \$150 (8.889)
= \$2,020.35

5-20

(a) PW of Cost = (\$26,000 + \$7,500) (P/A, 18%, 6)
= \$117,183

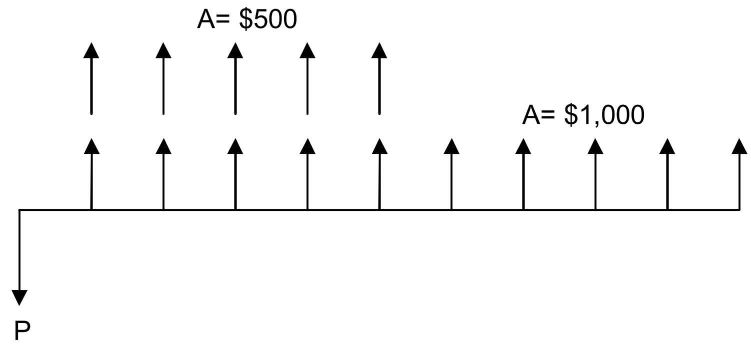
(b) PW of Cost = [(\$26,000 + \$7,500)/12] (P/A, 1.5%, 72)
= \$122,400

(c) PW of Cost = F Σ (n= 1 to 6) $[(e^r - 1)/(re^{rn})]$
= \$35,500 $[((e^{0.18} - 1)/(0.18e^{(0.18)(1)})) + ((e^{0.18} - 1)/(0.18e^{(0.18)(2)}) + ...]$
= \$35,000 [(0.1972/0.2155) + (0.1972/0.2580) +

(0.1972/0.3089) + (0.1972/0.3698) + (0.1972/0.4427) + (0.1972/0.5300)]
= \$33,500 (3.6686)
= \$122,897

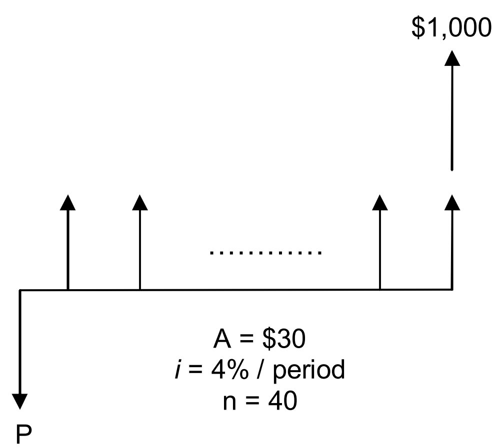
(d) Part (a) assumes end-of-year payments. Parts (b) and (c) assume earlier payments, hence their PW of Cost is greater.

5-21



Maximum investment = Present Worth of Benefits
= \$1,000 (P/A, 4%, 10) + \$500 (P/A, 4%, 5)
= \$1,000 (8.111) + \$500 (4.452)
= \$10, 337

5-22



P = \$30 (P/A, 4%, 40) + \$1,000 (P/F, 4%, 40)
= \$30 (19.793) + \$1,000 (0.2083)
= \$802

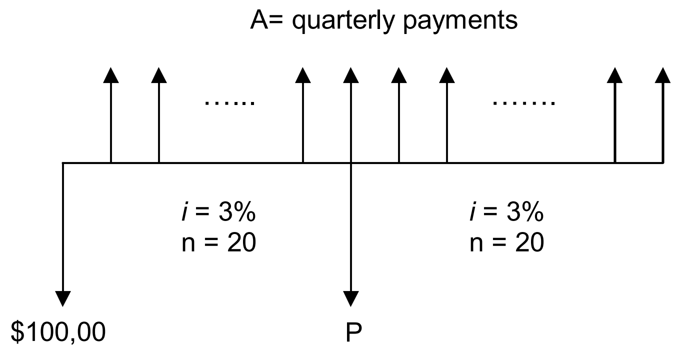
5-23

The maximum that the contractor would pay equals the PW of Benefits:

= (\$5.80 - \$4.30) (\$50,000) (P/A, 10%, 5) + \$40,000(P/F, 10%, 5)
= (\$1.50) (\$50,000) (3.791) + \$40,000 (0.6209)
= \$309,200

5-24

(a)



A = \$100,000 (A/P, 3%, 40) = \$100,000 (0.0433) = \$4,330

P = \$4,330 (P/A, 3%, 20) = \$4,330 (14.877) = \$64,417

(b) Service Charge = 0.05 P

Amount of new loan = 1.05 (\$64,417) = \$67,638

Quarterly payment on new loan = \$67,638 (A/P, 2%, 80)
= \$67,638 (0.0252)
= \$1,704

Difference in quarterly payments = \$4,330 - \$1,704 = \$2,626

5-25

The objective is to determine if the Net Present Worth is non-negative.

NPW of Benefits = \$50,000 (P/A, 10%, 10) + \$10,000 (P/F, 10%, 10)
= \$50,000 (6.145) + \$10,000 (0.3855)
= \$311,105

PW of Costs = \$200,000 + \$9,000 (P/A, 10%, 10)
= \$200,000 + \$9,000 (6.145)
= \$255, 305

NPW = \$311,105 - \$255,305 = \$55,800

Since NPW is positive, the process should be automated.

5-26

(a) PW Costs = \$700,000,000 + \$10,000,000 (P/A, 9%, 80)
= \$811,000,000

PW Receipts = (\$550,000) (90) (P/A, 9%, 10) + (\$50,000) (90) (P/G, 9%, 10) + (\$1,000,000) (90) (P/A, 9%, 70) (P/F, 9%, 10)
= \$849,000,000

NPW = \$849,000,000 - \$811,000,000 = \$38,000,000

This project meets the 9% minimum rate of return as NPW is positive.

(b) Other considerations:

Engineering feasibility
Ability to finance the project
Effect on trade with Brazil
Military/national security considerations

5-27

P = ? n = 36 months i = 1.50% /month A = \$250

P = \$250 (P/A, 1.5%, 36) = \$250 (27.661) = \$6,915

5-28

P = \$12,000 n = 60 months i = 1.0% /month A = ?

A = \$12,000 (A/P, 1%, 60) = \$12,000 (0.0222) = \$266

\$266 > \$250 and therefore she cannot afford the new car.

5-29

Find i:

(A/P, i, 60) = A/P = \$250/\$12,000 = 0.0208

From tables, i = ¾% per month = 9% per year

5-30

i_{month} = $(1 + (0.045/365))^{30} - 1$ = 0.003705

$$P = A[((1 + i)^n - 1)/(i(1 + i)^n)]$$
$$= \$199\ [((1.003705)^{60} - 1)/(0.003705\ (1.003705)^{60})]$$
$$= \$10,688$$

5-31

P = the first cost = \$980,000
F = the salvage value = \$20,000
AB = the annual benefit = \$200,000

Remember our convention of the costs being negative and the benefits being positive. Also, remember the *P* occurs at time = 0.

NPW = - P + AB (P/A, 12%, 13) + F (P/F, 12%, 13)
= -\$980,000 + \$200,000 (6.424) + \$20,000 (0.2292)
= \$309,384

Therefore, purchase the machine, as NPW is positive.

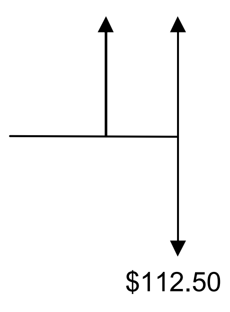
5-32

The market value of the bond is the present worth of the future interest payments and the face value on the current 6% yield on bonds.

A = \$1,000 (0.08%)/(2 payments/year) = \$40
P = \$40 (P/A, 3%, 40) + \$1,000 (P/F, 3%, 40)
= \$924.60 + \$306.60
= \$1,231.20

5-33

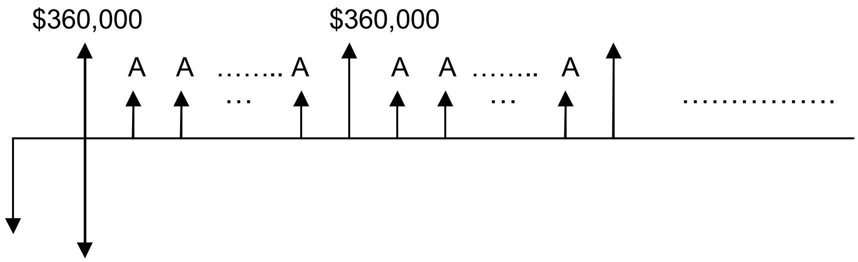
The interest the investor would receive is:
i = \$5,000 (0.045/2) = \$112.50 per 6 months
Probably the simplest approach is to resolve the \$112.50 payments every 6 months into equivalent payments every 3 months:



A = \$112.50 (A/F, 2%, 2) = \$112.50 (0.4951) = \$55.70

PW of Bond = \$55.70 (P/A, 2%, 40) + \$5,000 (P/F, 2%, 40)
= \$55.70 (27.355) + \$5,000 (0.4529) = \$3,788

5-34



P' = present worth of an inifinite series = A/i

A = 6 ($60,000) (A/F, 4%, 25)
= $360,000 (0.0240)
= $8640

P' = A/i
= $8640/0.04
= $216,000

P = ($216,000 + $360,000) (P/F, 4%, 10)
= $576,000 (0.6756)
= $389,150

5-35

P = A/*i* = $67,000/0.08 = $837,500

5-36

Two assumptions are needed:
1) Value of an urn of cherry blossoms (plus the cost to have the bank administer the trust) – say $50.00 / year

2) A "conservative" interest rate—say 5%

P = A/*i* = $50.00/0.05 = $1,000

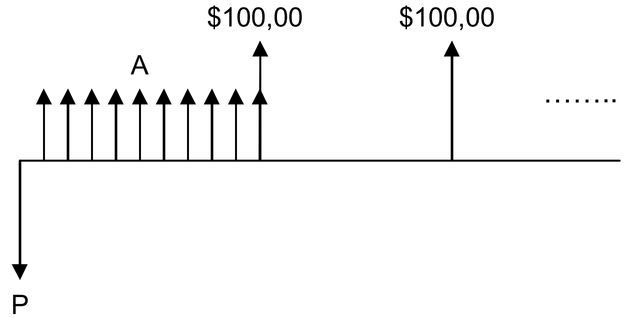
5-37

Capitalized Cost = PW of an infinite analysis period

When n = ∞ or P = A/*i*

PW = $5,000/0.08 + $150,000 (A/P, 8%, 40)/0.08
= $62,500 + $150,000 (0.0839)/0.08
= $219,800

5-38



Compute an A that is equivalent to $100,000 at the end of 10 years.

A = $100,000 (A/F, 5%, 10) = $100,000 (0.0795) = $7,950

For an infinite series,

P = A/i = $7,950/0.05 = $159,000

5-39

To provide $1,000 a month she must deposit:

P = A/i = $1,000/0.005 = $200,000

5-40

The amount of money needed now to begin the perpetual payments is:

P' = A/i = $10,000/0.08 = $125,000

The amount of money that would need to have been deposited 50 years ago at 8% interest is:

P = $125,000 (P/F, 8%, 50) = $125,000 (0.0213) = $2,662

5-41

Capitalized Cost = $2,000,000 + $15,000/0.05 = $2.3 million

5-42

Effective annual interest rate = $(1.025)^2 - 1$
= 0.050625
= 5.0625%

Annual Withdrawal A = Pi = $25,000 (0.05062) = $1,265.60

5-43

The trust fund has three components:

(1) P $= \$1$ million

(2) For $n = \infty$ $P = A/i = \$150,000/0.06 = \$2.5$ million

(3) $100,000 every 4 years: First compute equivalent A. Solving one portion of the perpetual series for A:

$$A = \$100,000\ (A/F, 6\%, 4) = \$100,000\ (0.2286) = \$22,860$$

$$P = A/i = \$22,860/0.06 = \$381,000$$

Required money in trust fund

$= \$1$ million $+ \$2.5$ million $+ \$381,000 = \$3.881$ million

5-44

$i = 5\%$

$$P = \$50/0.05 + [\$500\ (A/F, 5\%, 5)]/0.08$$
$$= \$50/0.05 + [\$500\ (0.1810)]/0.08$$
$$= \$2,810$$

5-45

(a) $P = \$5,000 + \$200/0.08 + \$300\ (A/F, 8\%, 4)/0.08$
$= \$5,000 + \$2,500 + \$300\ (0.1705)/0.08$
$= \$8,139$

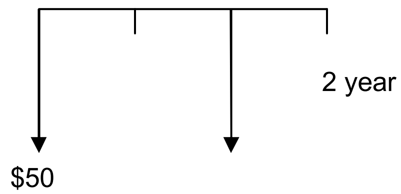
(b) $P = \$5,000 + \$200\ (P/A, 8\%, 75) + \$300\ (A/F, 8\%, 5)\ (P/A, 8\%, 75)$
$= \$5,000 + \$200\ (12.461) + \$300\ (0.1705)\ (12.461)$
$= \$8,130$

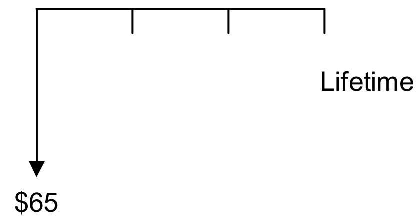
5-46

$P = ?$ $n = \infty$ $i = 10\%$ $A = \$100,000$

$$P = A/i = \$100,000/0.10 = \$1,000,000$$

5-47





By buying the "lifetime" muffler the car owner will avoid paying $50 two years hence. Compute how much he is willing to pay now to avoid the future $50 disbursement.

P = $50 (P/F, 20%, 2) = $50 (0.6944)= $34.72

Since the lifetime muffler costs an additional $15, it appears to be the desirable alternative.

5-48

Compute the PW of Cost for a 25-year analysis period.
Note that in both cases the annual maintenance is $100,000 per year after 25 years. Thus after 25 years all costs are identical.

Single Stage Construction

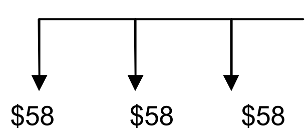
PW of Cost = $22,400,000 + $100,000 (P/A, 4%, 25)
= $22,400,000 + $100,000 (15.622)
= $23,962,000

Two Stage Construction

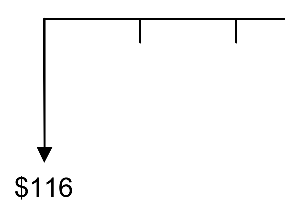
PW Cost =$14,200,000 + $75,000 (P/A, 4%, 25)
+ $12,600,000 (P/F, 4%, 25)
= $14,200,000 + $75,000 (15.622) + $12,600,000 (0.3751)
= $20,098,000

Choose two stage construction.

5-49



or



Three One-Year Subscriptions

PW of Cost = \$58 + \$58 (P/F, 20%, 1) + \$58 (P/F, 20%, ,2)
= \$58 (1 + 0.8333 + 0.6944)
= \$146.61

One Three-Year Subscription

PW of Cost = \$116

Choose the three-year subscription.

5-50

NPW = PW of Benefits – PW of Cost

NPW of 8 years of alternate A

= \$1,800 (P/A, 10%, 8) - \$5,300 - \$5,300 (P/F, 10%, 4)
= \$1,800 (5.335) - \$5,300 - \$5,300 (0.6830)
= \$683.10

NPW of 8 years of alternate B

= \$2,100 (P/A, 10%, 8) - \$10,700
= \$2,100 (5.335) - \$10,700
= \$503.50

Select Alternate A.

5-51

PW of Cost_A = \$1,300
PW of Cost_B = \$100 (P/A, 6%, 5) + \$100 (P/G, 6%, 5)
= \$100 (4.212 + 7.934)
= \$1,215

To minimize PW of Cost, choose B.

5-52

The revenues are common; the objective is to minimize cost.

(a) Present Worth of Cost for Option 1:

PW of Cost = \$200,000 + \$15,000 (P/A, 10%, 30)
= \$341, 400

Present Worth of Cost for Option 2:

PW of Cost = \$150,000 + \$150,000 (P/F, 10%, 10) + \$10,000 (P/A, 10%, 30) + \$10,000 (P/A, 10%, 20) (P/F, 10%, 10)

$= \$150,000 + \$150,000\ (0.3855) + \$10,000\ (9.427) + \$10,000\ (8.514)\ (0.3855)$
$= \$334,900$

Select option 2 because it has a smaller Present Worth of Cost.

(b) The cost for option 1 will not change. The cost for option 2 will now be higher.

PW of Cost $= \$150,000 + \$150,000\ (P/F, 10\%, 5) + \$10,000\ (P/A, 10\%, 30) + \$10,000\ (P/A, 10\%, 25)\ (P/F, 10\%, 5)$
$= \$394,300$

Therefore, the answer will change to option 1.

5-53

PW of $Cost_{wheel}$ $= \$50,000 - \$2,000\ (P/F, 8\%, 5) = \$48,640$

PW of $Cost_{track}$ $= \$80,000 - \$10,000\ (P/F, 8\%, 5) = \$73,190$

The wheel mounted backhoe, with its smaller PW of Cost, is preferred.

5-54

$NPW_A = -\$50,000 - \$2,000\ (P/A, 9\%, 10) + \$9,000\ (P/A, 9\%, 10) + \$10,000\ (P/F, 9\%, 10)$
$= -\$50,000 - \$2,000\ (6.418) + \$9,000\ (6.418) + \$10,000\ (0.4224)$
$= -\$850$

$NPW_B = -\$80,000 - \$1,000\ (P/A, 9\%, 10) + \$12,000\ (P/A, 9\%, 10) + \$30,000\ (P/F, 9\%, 10)$
$= -\$80,000 - \$1,000\ (6.418) + \$12,000\ (6.418) + \$30,00O\ (0.4224)$
$= +\$3,270$

(a) Buy Model B because it has a positive NPW.

(b) The NPW of Model A is negative; therefore, it is better to do nothing or look for more alternatives.

5-55

Machine A

NPW = - First Cost + Annual Benefit (P/A, 12%, 5) – Maintenance & Operating Costs (P/A, 12%, 5) + Salvage Value (P/F, 12%, 5)
$= -\$250,000 + \$89,000\ (3.605) - \$4,000\ (3.605) + \$15,000\ (0.5674)$
$= \$64,936$

Machine B

NPW = - First Cost + Annual Benefit (P/A, 12%, 5) – Maintenance & Operating Costs (P/A, 12%, 5) + Salvage Value (P/F, 12%, 5)
= -$205,000 + $86,000 (3.605) - $4,300 (3.605) + $15,000 (0.5674)
= $98,040

Choose Machine B because it has a greater NPW.

5-56

Since the necessary waste treatment and mercury recovery is classed as “Fixed Output,” choose the alternative with the least Present Worth of Cost.

Foxhill
PW of Cost = $35,000 + ($8,000 - $2,000) (P/A, 7%, 20) - $20,000 (P/F, 7%, 20)
= $35,000 + $6,000 (10.594) - $20,000 (0.2584)
= $93,396

Quicksilver
PW of Cost = $40,000 + ($7,000 - $2,200) (P/A, 7%, 20)
= $40,000 + $4,800 (10.594)
= $90,851

Almeden
PW of Cost = $100,000 + ($2,000 - $3,500) (P/A, 7%, 20)
= $100,000 - $1,500 (10.594)
= $84,109

Select the Almaden bid.

5-57

Use a 20-year analysis period:

Alt. A NPW = $1,625 (P/A, 6%, 20) - $10,000 - $10,000 (P/F, 6%, 10)
= $1,625 (11.470) - $10,000 - $10,000 (0.5584)
= $3,055

Alt. B NPW = $1,530 (P/A, 6%, 20) - $15,000
= $1,530 (11.470) - $15,000
= $2,549

Alt. C NPW = $1,890 (P/A, 6%, 20) - $20,000
= $1,890 (11.470) - $20,000
= $1,678

Choose Alternative A.

5-58

Fuel	Installed Cost	Annual Fuel Cost
Natural Gas	$30,000	$7,500 > Fuel Oil
Fuel Oil	$55,000	
Coal	$180,000	$15,000 > Fuel Oil

For fixed output, minimize PW of Cost:

Natural Gas

PW of Cost = $30,000 + $7,500 (P/A, 8%, 20) + PW of Fuel Oil Cost
= $30,000 + $7,500 (9.818) + PW of Fuel Oil Cost
= $103,635 + PW of Fuel Oil Cost

Fuel Oil

PW of Cost = $55,000 + PW of Fuel Oil Cost

Coal

PW of Cost = $180,000 - $15,000 (P/A, 8%, 20) + PW of Fuel Oil Cost
= $180,000 - $15,000 (9.818) + PW of Fuel Oil Cost
= $32,730 + PW of Fuel Oil Cost

Install the coal-fired steam boiler.

5-59

Company A

NPW = -$15,000 + ($8,000 - $1,600)(P/A, 15%, 4) + $3,000 (P/F, 15%, 4)
= -$15,000 + $6,400 (2.855) + $3,000 (0.5718)
= $4,987

Company B

NPW = -$25,000 + ($13,000 - $400) (P/A, 15%, 4) + $6,000 (P/F, 15%, 4)
= -$25,000 + $12,600 (2.855) + $6,000 (0.5718)
= $14,404

Company C

NPW = -$20,000 + ($11,000 - $900) (P/A, 15%, 4) + $4,500 (P/F, 15%, 4)
= -$20,000 + $10,100 (2.855) + $4,500 (0.5718)
= $11,409

To maximize NPW select Company B's office equipment.

5-60

The least common multiple life is 12 years, so this will be used as the analysis period.

Machine A

NPW_4 = -\$52,000 + (\$38,000 - \$15,000)(P/A, 12%, 4) + \$13,000(P/F, 12%, 4)
= -\$52,000 + \$69,851 + \$8,262
= \$26,113

NPW_{12}= NPW_4 [1 + (P/F, 12%, 4) + (P/F, 12%, 8)]
= \$26,113 [1 + $(1.12)^{-4}$ + $(1.12)^{-8}$]
= \$53,255

Machine B

NPW_6 = -\$63,000 + (\$31,000 - \$9,000)(P/A, 12%, 6) + \$19,000(P/F, 12%, 6)
= -\$63,000 + \$90,442 + \$9,625
= \$37,067

NPW_{12} = NPW_6 [1 + (P/F, 12%, 6)]
= \$37,067 [1 + $(1.12)^{-6}$]
= \$55,846

Machine C

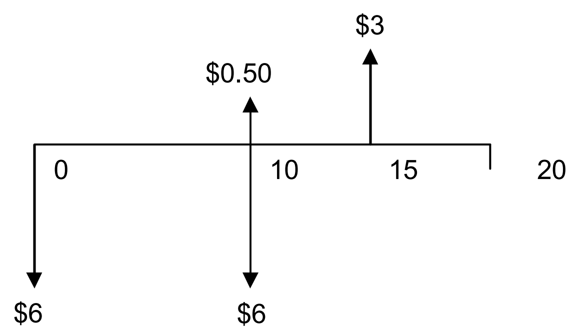
NPW_{12} =-\$67,000+(\$37,000 - \$12,000)(P/A, 12%, 12)+\$22,000(P/F, 12%, 12)
= -\$67,000 + \$154,850 + \$5,647
= \$93,497

Machine C is the correct choice.

5-61

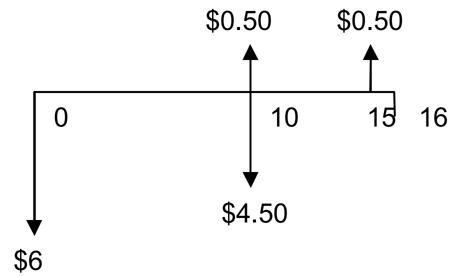
It appears that there are four alternative plans for the ties:

1) Use treated ties initially and as the replacement



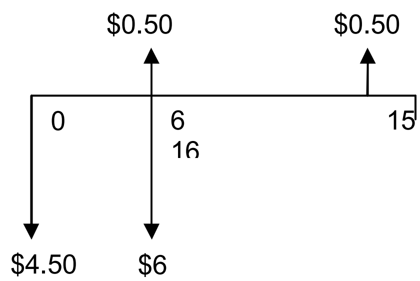
PW of Cost = \$6 + \$5.50 (P/F, 8%, 10) - \$3 (P/F, 8%, 15)
= \$6 + \$5.50 (0.4632) - \$3 (0.3152)
= \$7.60

2) Use treated ties initially. Replace with untreated ties.



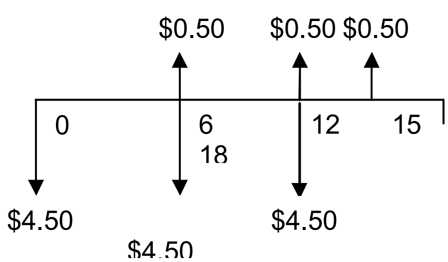
PW of Cost = $6 + $4 (P/F, 8%, 10) – $0.50 (P/F, 8%, 15)
= $6 + $4 (0.4632) - $0.50 (0.3152)
= $7.70

3) Use untreated ties initially. Replace with treated ties.



PW of Cost = $4.50 + $5.50 (P/F, 8%, 6) - $0.50 (P/F, 8%, 15)
= $4.50 + $5.50 (0.6302) - $0.50 (0.3152)
= $7.81

4) Use untreated ties initially, then two replacements with untreated ties.



PW of Cost = $4.50 + $4 (P/F, 8%, 6) + $4 (P/F, 8%, 12) - $0.50 (P/F, 8%, 15)
= $4.50 + $4 (0.6302) + $4 (0.3971) - $0.50 (0.3152)
= $8.45

Choose Alternative 1 to minimize cost.

5-62

This is a situation of Fixed Input. Therefore, maximize PW of benefits. By inspection, one can see that C, with its greater benefits, is preferred over A and B. Similarly, E is preferred over D. The problem is reduced to choosing between C and E.

Alternative C

PW of Benefits = \$100 (P/A, 10%, 5) + \$110 (P/A, 10%, 5) (P/F, 10%, 5)
= \$100 (3.791) + \$110 (3.791) (0.6209)
= \$638

Alternative E

PW of Benefits = \$150 (P/A, 10%, 5) + \$50 (P/A, 10%, 5) (P/F, 10%, 5)
= \$150 (3.791) + \$50 (3.791) (0.6209)
= \$686.40

Choose Alternative E.

5-63

Compute the Present Worth of Benefit for each share.

From the 10% interest table: (P/A, 10%, 4) = 3.170
(P/F, 10%, 4) = 0.683

	PW of Future Price	PW of Dividends		PW of Benefit
Western House	\$32 x 0.683	+ 1.25 x 3.170	= 21.86 + 3.96	= \$25.82
Fine Foods	\$45 x 0.683	+ 4.50 x 3.170	= 30.74 + 14.26	= \$45.00
Mobile Motors	\$42 x 0.683	+ 0 x 3.170	= 28.69 + 0	= \$28.69
Spartan Products	\$20 x 0.683	+ 0 x 3.170	= 13.66 + 0	= \$13.66
U.S. Tire	\$40 x 0.683	+ 2.00 x 3.170	= 27.32 + 6.34	= \$33.66
Wine Products	\$60 x 0.683	+ 3.00 x 3.170	= 40.98 + 9.51	= \$50.49

	PW of Benefit	PW of Cost	NPW per share	NPW per \$1 invested
Western House	\$25.82	\$23.75	+2.07	+0.09
Find Foods	\$45.00	\$45.00	0	0
Mobile Motors	\$28.69	\$30.62	-1.93	-0.06
Spartan Products	\$13.66	\$12.00	+1.66	+0.14
U.S. Tire	\$33.66	\$33.37	+0.29	+0.01
Wine Products	\$50.49	\$52.50	- 2.01	-0.04

In this problem, choosing to Maximize NPW per share leads to Western House. But the student should recognize that this is a faulty criterion.

An investment of some lump sum of money (like $1,000) will purchase different numbers of shares of the various shares. It would buy 83 shares of Spartan Products, but only 42 shares of Western House. The criterion, therefore, is to maximize NPW for the amount invested. This could be stated as Maximize NPW per $1 invested.

Buy Spartan Products.

5-64

NPW_A = $6.00 (P/A, 8%, 6) - $20 = +$7.74
NPW_B = $9.25 (P/A, 8%, 6) - $35 = +$7.76
NPW_C = $13.38 (P/A, 8%, 6) - $55 = +$6.86
NPW_D = $13.78 (P/A, 8%, 6) - $60 = +$3.70
NPW_E = $24.32 (P/A, 8%, 6) - $80 = +$32.43
NPW_F = $24.32 (P/A, 8%, 6) - $100 = +$12.43

Choose E.

5-65

Eight mutually exclusive alternatives:

Plan	Initial Cost	Net Annual Benefit x (P/A, 10%, 10) 6.145	PW of Benefit	NPW = PW of Benefit minus Cost
1	$265	$51	$313.40	$48.40
2	$220	$39	$239.70	$19.70
3	$180	$26	$159.80	-$20.20
4	$100	$15	$92.20	-$7.80
5	$305	$57	$350.30	$45.30
6	$130	$23	$141.30	$11.30
7	$245	$47	$288.80	$43.80
8	$165	$33	$202.80	$37.80

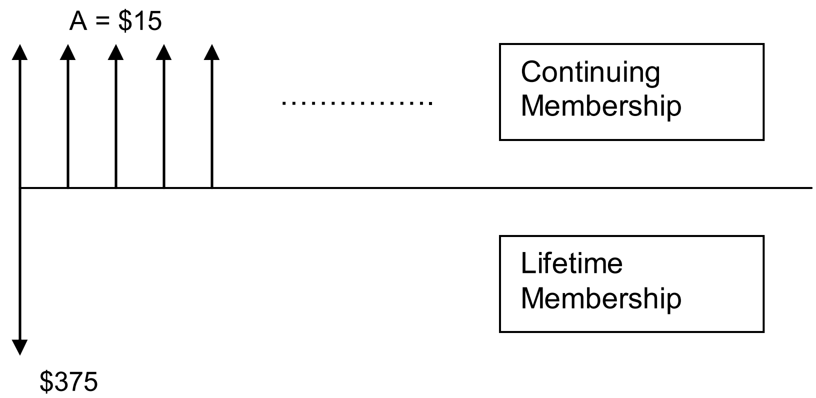
To maximize NPW, choose Plan 1.

5-66

$375 invested at 4% interest produces a perpetual annual income of $15.

$A = Pi$ = $375 (0.04) = $15

But this is not quite the situation here.



An additional $360 now instead of *n* annual payments of $15 each. Compute *n*.

$$P = A\ (P/A, 4\%, n)$$
$$\$360 = \$15\ (P/A, 4\%, n)$$

$$(P/A, 4\%, n) = \$360/\$15 = 24$$

From the 4% interest table, *n* = 82.

Lifetime (patron) membership is not economically sound unless one expects to be active for 82 + 1 = 83 years. (But that's probably not why people buy patron memberships or avoid buying them.)

5-67

$$\text{Cap. Cost}_A = \$500{,}000 + \$35{,}000/0.12 + [\$350{,}000(A/F, 12\%, 10)]/0.12$$
$$= \$500{,}000 + \$35{,}000/0.12 + [\$350{,}000\ (0.0570)]/.12$$
$$= \$957{,}920$$

$$\text{Cap. Cost}_B = \$700{,}000 + \$25{,}000/0.12 + [\$450{,}000\ (A/F, 12\%, 15)]/0.12$$
$$= \$700{,}000 + \$25{,}000/0.12 + [\$450{,}000\ (0.0268)]/0.12$$
$$= \$1{,}008{,}830$$

Type A with its smaller capitalized cost is preferred.

5-68

Full Capacity Tunnel

$$\text{Capitalized Cost} = \$556{,}000 + (\$40{,}000\ (A/F, 7\%, 10))/0.07$$
$$= \$556{,}000 + (\$40{,}000\ (0.0724))/0.07$$
$$= \$597{,}400$$

First Half Capacity Tunnel

Capitalized Cost = \$402,000 + [(\$32,000 (0.0724))/0.07] + [\$2,000/0.07]
= \$463,700

Second Half-Capacity Tunnel

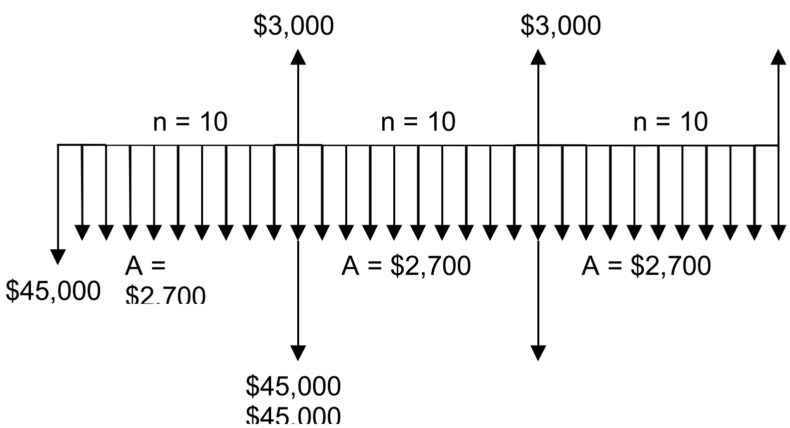
20 years hence the capitalized cost of the second half-capacity tunnel equals the present capitalized cost of the first half.

Capitalized Cost = \$463,700 (P/F, 7%, 20)
= \$463,700 (0.2584)
= \$119,800

Capitalized Cost for two half-capacity tunnels = \$463,700 + \$119,800
= \$583,500

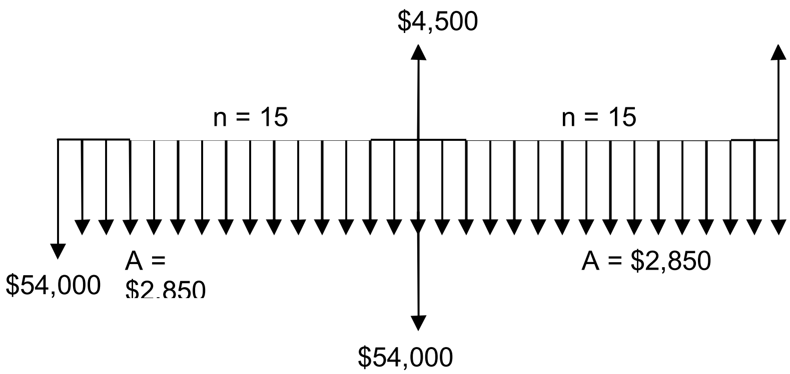
Build the full capacity tunnel.

5-69



PW of Cost of 30 years of Westinghome

= \$45,000 + \$2,700 (A/P, 10%, 30) + \$42,000 (P/F, 10%, 10) + \$42,000 (P/F, 10%, 20) - \$3,000 (P/F, 10%, 30)
= \$45,000 + \$2,700 (9.427) + \$42,000 (0.3855) + \$42,000 (0.1486) –
\$3,000 (0.0573)
= \$92,713



PW of Cost of 30 years of Itis

= $54,000 + $2,850 (P/A, 10%, 30) + $49,500 (P/F, 10%, 15)
- $4,500 (P/F, 10%, 30)
= $54,000 + $2,850 (9.427) + $49,500 (0.2394) - $4,500 (0.0573)
= $92,459

The Itis bid has a slightly lower cost.

5-70

For fixed output, minimize the Present Worth of Cost.

Quick Paving

PW of Cost = $42,500 + $21,250 (P/F, 1%, 6) + $21,250 (P/F, 1%, 12)
= $42,500 + $21,250 (0.9420) + $21,250 (0.8874)
= $81,375

Tartan Paving

PW of Cost = $82,000

Faultless Paving

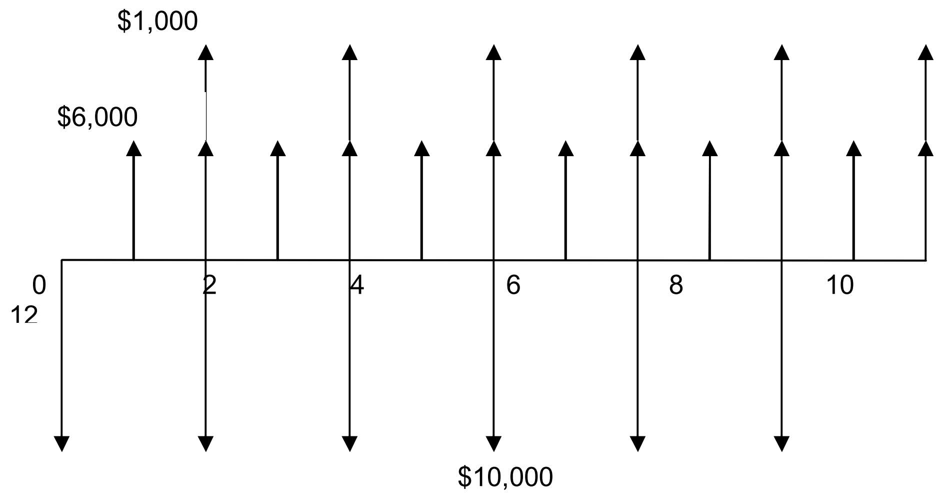
PW of Cost = $21,000 + $63,000 (P/F, 1%, 6)
= $21,000 + $63,000 (0.9420)
= $80,346

Award the job to Faultless Paving.

5-71

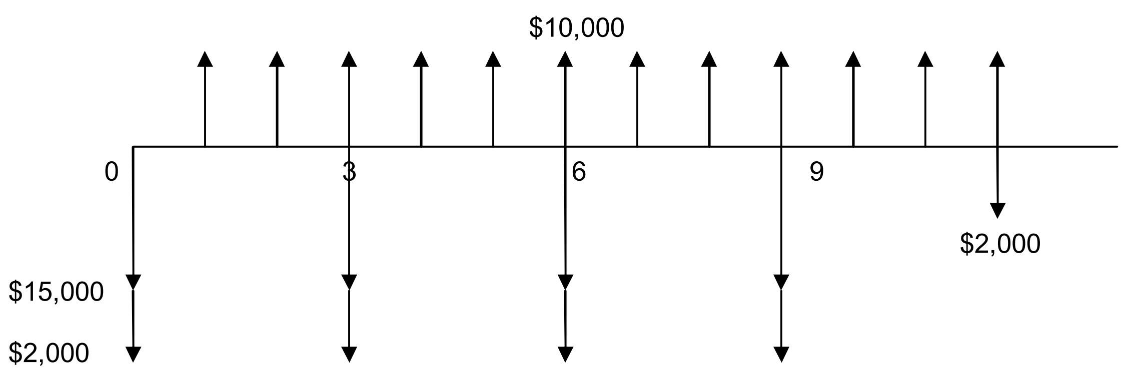
Using the PW Method the study period is a common multiple of the lives of the alternatives. Thus we use 12 years and assume repeatability of the cash flows.

Alternative A



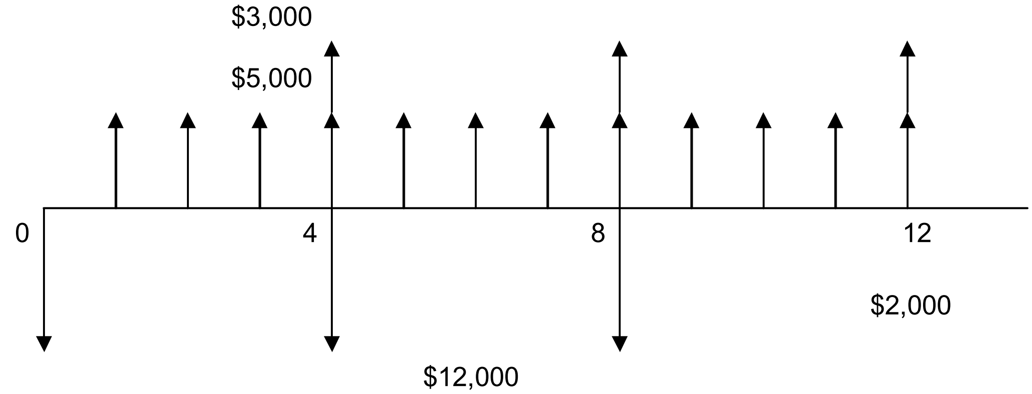
NPW = $6,000 (P/A, 10%, 12) + $1,000 (P/G, 10%, 12) - $10,000 – ($10,000 - $1,000) [(P/F, 10%, 2) + (P/F, 10%, 4) + (P/F, 10%, 6) + (P/F, 10%, 8) + (P/F, 10%, 10)]
= $40,884 + $319 - $10,000 - $26,331
= $4,872

Alternative B



NPW = $10,000 (P/A, 10%, 12) - $2,000 (P/F, 10%, 12) - $15,000 – ($15,000 + $2,000)[(P/F, 10%, 3) + (P/F, 10%, 6) + (P/F, 10%, 9)]
= $68,140 - $637 - $15,000 - $29,578
= $22,925

Alternative C



NPW = \$5,000 (P/A, 10%, 12) + \$3,000 (P/F, 10%, 12) - \$12,000 – (\$12,000 - \$3,000) [(P/F, 10%, 4) + (P/F, 10%, 8)]
= \$34,070 + \$956 - \$12,000 - \$10,345
= \$12,681

Choose Alternative B.

5-72

NPW = PW of Benefits – PW of Cost

NPW_A	= 0		
NPW_B	= \$12 (P/A, 10%, 5) - \$50	= \$12 (3.791) - \$50	= -\$4.51
NPW_C	= \$4.5(P/A, 10%, 10) - \$30	= \$4.5 (6.145) - \$30	= -\$2.35
NPW_D	= \$6 (P/A, 10%, 10) - \$40	= \$6 (6.145) - \$40	= -\$3.13

Select alternative A with NPW = 0.

5-73

Choose the alternative to maximize NPW.

(a) 8% interest

NPW_1 = \$135 (P/A, 8%, 10) - \$500 - \$500 (P/F, 8%, 5)
= +\$65.55
NPW_2 = (\$100 + \$250) (P/A, 8%, 10) - \$600 - \$350 (P/F, 9%, 5)
= -\$51.41
NPW_3 = \$100 (P/A, 8%, 10) - \$700 + \$180 (P/F, 8%, 10)
= +\$54.38
NPW_4 = \$0

Choose Alternative 1.

(b) 12% interest

NPW_1 = \$135 (P/A, 12%, 10) - \$500 - \$500 (P/F, 12%, 5)
= -\$20.95
NPW_2 = (\$100 + \$250) (P/A, 12%, 10) - \$600 - \$350 (P/F, 12%, 5)
= -\$153.09
NPW_3 = \$100 (P/A, 12%, 10) - \$700 + \$180 (P/F, 12%, 10)
= -\$77.04
NPW_4 = \$0

Choose Alternative 4.

5-74

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	**B**
1	Payment	\$500
2	N	48
3	Interest rate	0.50%
4	PW	\$21,290

5-75

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	**B**
1	Payment	\$6,000
2	N	4
3	Interest rate	6%
4	PW	\$20,791

5-76

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	**B**
1	Payment	\$6,000
2	N	4
3	Interest rate	6.168%
4	PW	\$20,711

5-77

Problem 5-74 will repay the largest loan because the payments start at the end of the month, rather than waiting until the end of the year.

Problem 5-76 has the same effective interest rate as 5-74, but the rate on 5-75 is lower.

5-78

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	B
1	Payment	1000
2	N	360
3	Interest rate	0.50%
4	PW	$166,792

5-79

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	B
1	Payment	12000
2	N	30
3	Interest rate	6%
4	PW	$165,178

5-80

Using the Excel function = -PV (B3,B2,B1) for Present Worth obtain:

	A	B
1	Payment	12000
2	N	30
3	Interest rate	6.168%
4	PW	$162,251

5-81

Problem 5-78 will repay the largest loan because the payments start at the end of the first month, rather than waiting until the end of the year.

Problem 5-80 has the same effective interest rate as 5-77, but the rate on 5-79 is lower.

5-82

At a 15% rate of interest, use the excel function = PV ($A$1, A3,,-1) for Present Worth.

	A	B	C	D
1	Year	Net Cash	(P/F,i,n)	PW
2	0	0	1.0000	0
3	1	-120000	0.8696	-104348
4	2	-60000	0.7561	-45369
5	3	20000	0.6575	13150
6	4	40000	0.5718	22870
7	5	80000	0.4972	39774
8	6	100000	0.4323	43233
9	7	60000	0.3759	22556
10			total	-8133

So don't do. This problem can also be solved by using NPV function:
PW = -$8,133 = NPV (A1, B4:B10) + B3

Notice that NPV function starts with year 1, and year 0 is added in separately.

5-83

Using a 10% interest rate, solve for PW using the function
= PV ($A$1, A3,,-1)

	A	B	C	D	E	F	G
1	year	annual sales	Cost/unit	Price/unit	Net revenue	(P/F,i,n)	PW
2	0	0			-42000	1.0000	-42000
3	1	5000	3.50	6.00	12500	0.9091	11364
4	2	6000	3.25	5.75	15000	0.8264	12397
5	3	9000	3.00	5.50	22500	0.7513	16905
6	4	10000	2.75	5.25	25000	0.6830	17075
7	5	8000	2.50	4.50	16000	0.6209	9935
8	6	4000	2.25	3.00	3000	0.5645	1693
9						total	27368

So do.
Can also solve without P/F column by using NPV function:
PW = $27,368 = NPV(A1, E4:E9) + E3
Notice that NPV function starts with year 1, and year 0 is added in separately.

5-84

Using interest=15%, solve for PW using the function = PV ($A$1, A3,,-1)

	A	B	C	D	E	F	G
1	year	annual prod.	Cost/unit	Price/unit	Net revenue	(P/F,i,n)	PW
2	0	0			-8000000	1.0000	-8000000
3	1	70000	25	35	700000	0.8696	608696
4	2	90000	20	34	1260000	0.7561	952741
5	3	120000	22	33	1320000	0.6575	867921
6	4	100000	24	34	1000000	0.5718	571753
7	5	80000	26	35	720000	0.4972	357967
8	6	60000	28	36	640000	0.4323	276690
9	7	40000	30	37	420000	0.3759	157894
10						total	-4206338

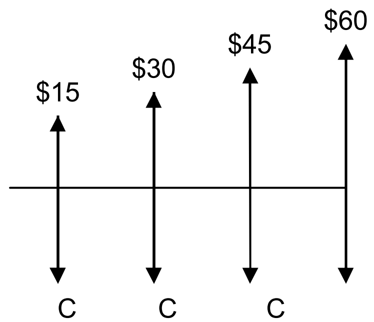
So do.
Can also solve without P/F column by using NPV function:

PW = -$4,206,338 = NPV(A1, E4:E10) + E3

Notice that NPV function starts with year 1, and year 0 is added in separately.

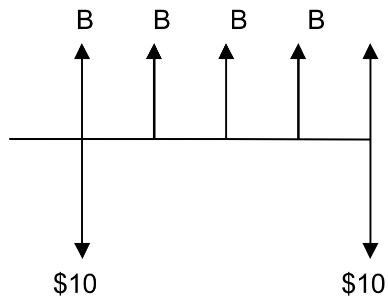
Chapter 6: Annual Cash Flow Analysis

6-1



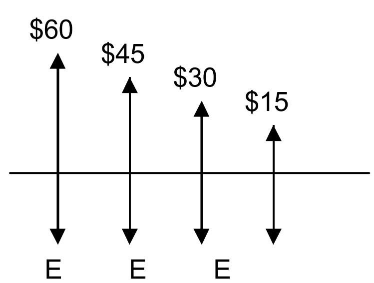
C = \$15 + \$15 (A/G, 10%, 4)
= \$15 + \$15 (1.381) = \$35.72

6-2



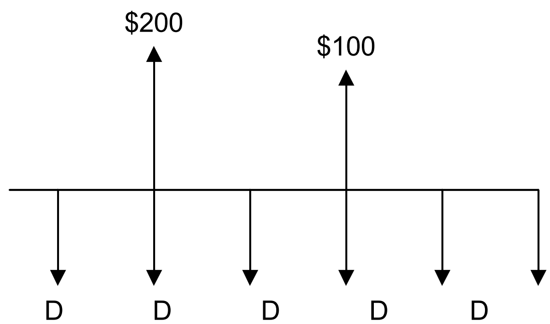
B = [\$100 + \$100 (F/P, 15%, 4)] (A/F, 15%, 5)
= [\$100 + \$100 (1.749)] (0.1483) = \$40.77

6-3



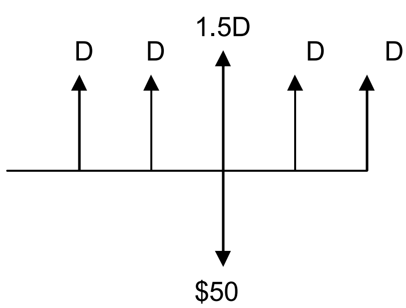
E = \$60 - \$15 (A/G, 12%, 4)
= \$60 - \$15 (1.359) = \$39.62

6-4



D = [$100 (F/P, 6%, 2) + $200 (F/P, 6%, 4)] (A/F, 6%, 6)
= [$100 (1.124) + $200 (1.262)] (0.1434)
= $52.31

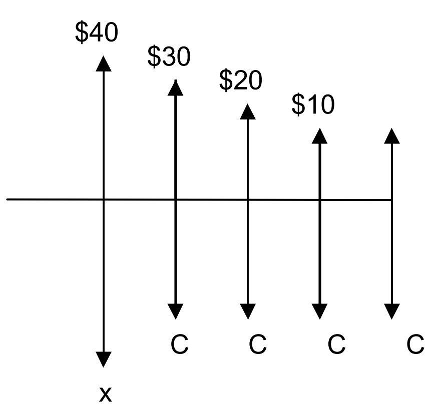
6-5



$500 = D (F/A, 12%, 3) + 0.5D + D (P/A, 12%, 2)
= D (3.374 + 0.5 + 1.690)

D = $500/5.564
= $89.86

6-6

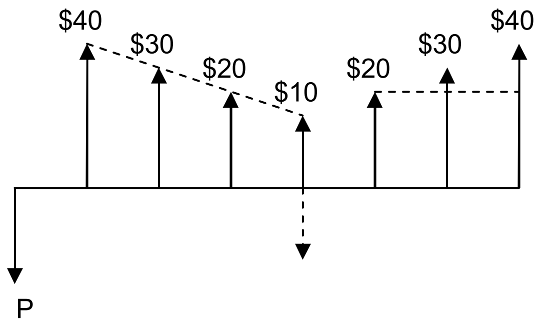


x = $40 + $10 (P/A, 10%, 4) + $20 (P/F, 10%, 1) + $10 (P/F, 10%, 2)

= \$40 + \$10 (3.170) + \$20 (0.9091) + \$10 (0.8264)
= \$98.15

C = \$98.15 (A/P, 10%, 4)
= \$98.15 (0.3155)
= \$30.97

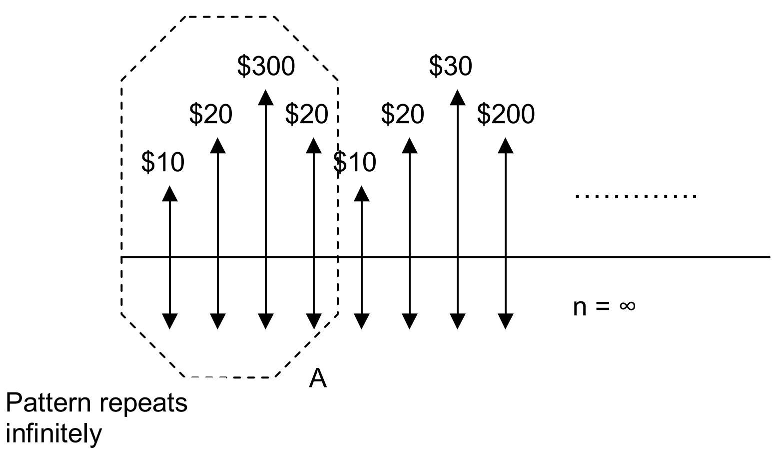
6-7



P = \$40 (P/A, 10%, 4) - \$10 (P/G, 10%, 4) + [\$20 (P/A, 10%, 3) + \$10 (P/G, 10%, 3)] (P/F, 10%, 4
= \$40 (3.170) - \$10 (4.378) + [\$20 (2.487) + \$10 (2.329)] (0.6830)
= \$132.90

A = \$132.90 (A/P, 10%, 7)
= \$132.90 (0.2054)
= \$27.30

6-8

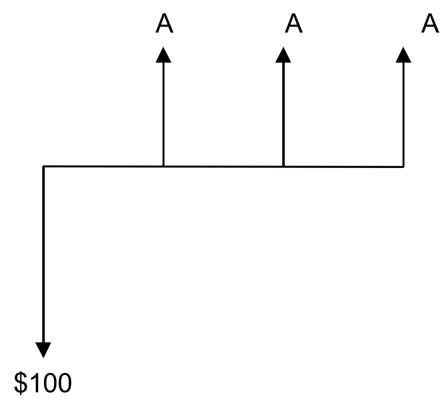


There is a repeating series:; 100 – 200 – 300 – 200. Solving this series for A gives us the A for the infinite series.

A = \$100 + [\$100 (P/F, 10%, 2) + \$200 (P/F, 10%, 3)

+ $100 (P/F, 10%, 4)] (A/P, 10%, 4)
= $100 + [$100 (0.8254) + $200 (0.7513) + $100 (0.6830)] (0.3155)
= $100 + [$301.20] (0.3155)
= $195.03

6-9



A = $100 (A/P, 3.5%, 3)
= $100 (0.3569)
= $35.69

6-10

EUAC = $60,000 (0.10) + $3,000 + $1,000 (P/F, 10%, 1) (A/P, 10%, 4)
= $6,000 + $3,000 + $1,000 (0.9091) (0.3155)
= $9,287

This is the relatively unusual situation where Cost = Salvage Value. In this situation the annual capital recovery cost equals interest on the investment. If anyone doubts this, they should compute:

$60,000 (A/P, 10%, 4) - $60,000 (A/F, 10%, 2).

This equals P*i = $60,000 (0.10) = $6,000.

6-11

Prospective Cash Flow:

Year	Cash Flow
0	-$30,000
1-8	+A
8	+$35,000

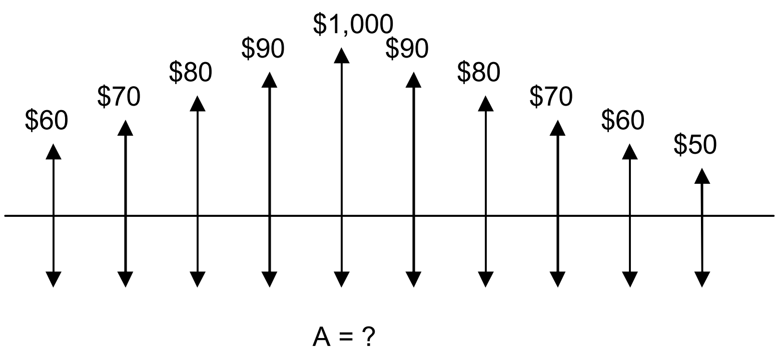
EUAC = EUAB

$30,000 (A/P, 15%, 8) = A + $35,000 (A/F, 15%, 8)

$30,000 (0.2229) = A + $35,000 (0.0729)
$6,687 = A + $2,551.50

A = $4,135.50

6-12



This problem is much harder than it looks!

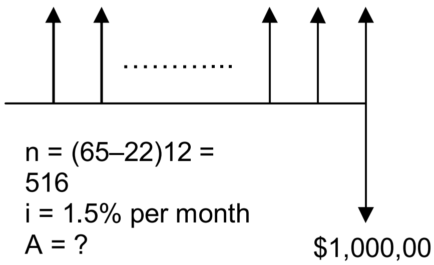
EUAC = {$600 (P/A, 8%, 5) + $100 (P/G, 8%, 5) + [$900 (P/A, 8%, 5) – $100 (P/G, 8%, 5)][(P/F, 8%, 5)]}{(A/P, 8%, 10)}
= {$600 (3.993) + $100 (7.372) + [$900 (3.993) - $100 (7.372)][0.6806]}{0.1490}
= $756.49

6-13

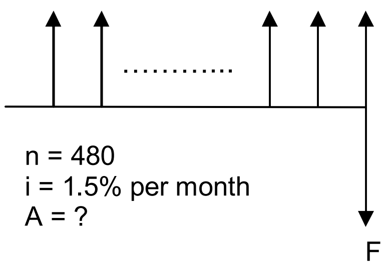
EUAC = $30,000 (A/P, 8%, 8) - $1,000 - $40,000 (A/F, 8%, 8)
= $30,000 (0.1740) - $1,000 - $40,000 (0.0940)
= $460

The equipment has an annual cost that is $460 greater than the benefits. The equipment purchase did not turn out to be desirable.

6-14



The 1.5% interest table does not contain n = 516. The problem must be segmented to use the 1.5% table.

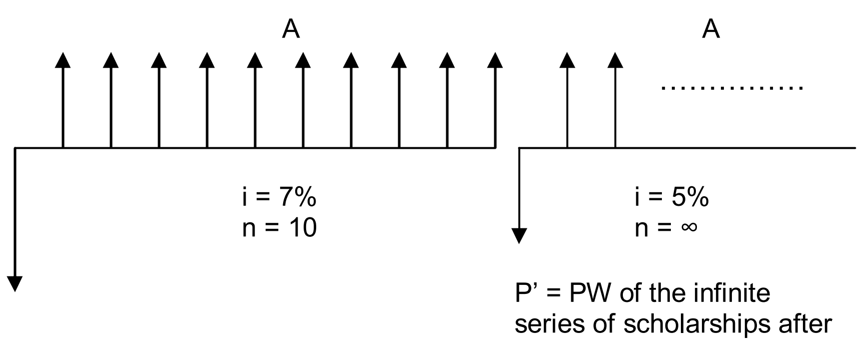


Compute the future value F of a series of A's for 480 interest periods.
F = A (F/A, 1.5%, 480) = A (84,579) = 84,579 A

Then substitute 84,579 A for the first 480 interest periods and solve for A.
84,579 A (F/P, 1.5%, 36) + A (F/A, 1.5%, 36) = $1,000,000
84,579 A (1.709) + A (42.276) = $1,000,000

A = $6.92 monthly investment

6-15



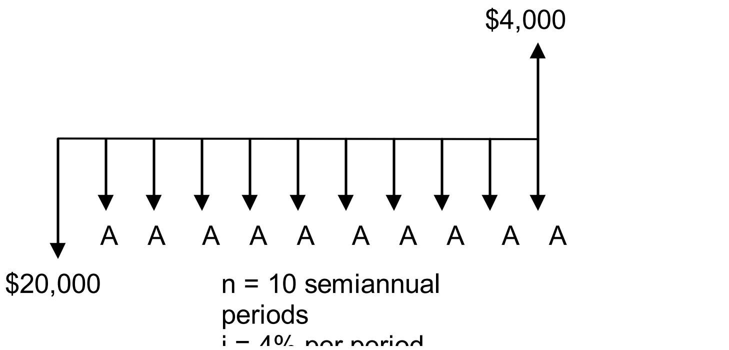
P' = A/I = A/0.05

$30,000 = PW of all future scholarships

= A (P/A, 7%, 10) + P'(P/F, 7%, 10)
= A (7.024) + A(0.5083/0.05)

A = \$30,000/17.190
= \$1,745.20

6-16



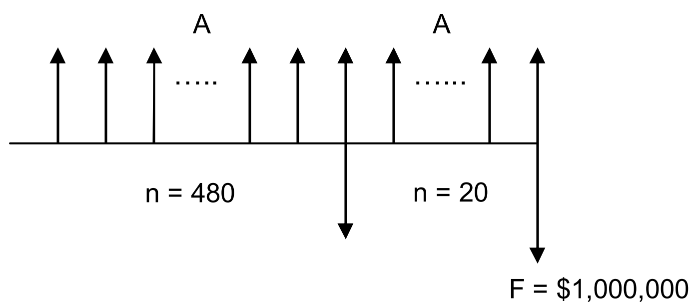
First, compute A:

A = (\$20,000 - \$4,000) (A/P, 4%, 10) + \$4,000 (0.04)
= \$16,000 (0.1233) + \$160
= \$2,132.80 per semiannual period

Now, compute the equivalent uniform annual cost:

EUAC = A (F/A, i%, n)
= \$2,132.80 (F/A, 4%, 2)
= \$2,132.80 (2.040)
= \$4,350.91

6-17



F = A (F/A, 1.25%, 480) (F/P, 1.25%, 20) + A (F/A, 1.25%, 20)
= A [(31,017) (1.282) + 22.6]
= A (39,786)

A = \$1,000,000/39,786
= \$25.13

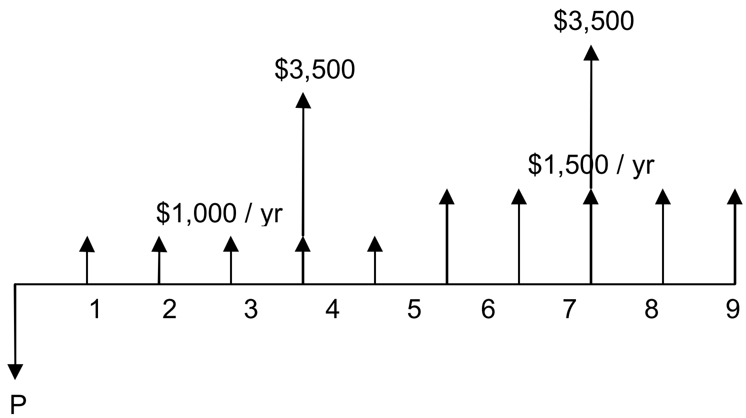
6-18

(a) EUAC $= \$6{,}000\ (A/P, 8\%, 30) + \$3{,}000$ (labor) $+ \$200$ (material)
$-\ 500$ bales (\$2.30/bale) – 12 (\$200/mo trucker)
$= \$182.80$

Therefore, bailer is not economical.

(b) The need to recycle materials is an important intangible consideration. While the project does not meet the 8% interest rate criterion, it would be economically justified at a 4% interest rate. The bailer probably should be installed.

6-19



$$
\begin{aligned}
P &= \$1{,}000\ (P/A, 6\%, 5) + \$3{,}500\ (P/F, 6\%, 4) \\
&\quad + \$1{,}500\ (P/A, 6\%, 5)\ (P/F, 6\%, 5) + \$3{,}500\ (P/F, 6\%, 8) \\
&= \$1{,}000\ (4.212) + \$3{,}500\ (0.7921) + \$1{,}500\ (4.212)\ (0.7473) \\
&\quad + \$3{,}500\ (0.6274) \\
&= \$4{,}212 + \$2{,}772 + \$4{,}721 + \$2{,}196 \\
&= \$13{,}901
\end{aligned}
$$

Equivalent Uniform Annual Amount $= \$13{,}901\ (A/P, 6\%, 10) = \$1{,}889$

6-20

$$
\begin{aligned}
A &= F[(e^{r} - 1)/(e^{rn} - 1)] \\
&= \$5 \times 10^{6}\ [(e^{0.15} - 1)/(e^{(0.15)(40)} - 1)] \\
&= \$5 \times 10^{6}\ [0.161834/402.42879] \\
&= \$2{,}011
\end{aligned}
$$

6-21

(a) EUAC $= \$2{,}500 + \$5{,}000\ (A/F, 8\%, 4)$
$= \$2{,}500 + \$5{,}000\ (0.2219)$
$= \$3{,}609.50$

(b) P $= A/i$

$= \$3,609.50/0.08$
$= \$45,119$

6-22

(a) EUAC $= \$5,000 + \$35,000\ (A/P, 6\%, 20)$
$= \$5,000 + \$35,000\ (0.0872)$
$= \$8,052$

(b) Since the EUAC of the new pipeline is less than the $5,000 annual cost of the existing pipeline, it should be constructed.

6-23

Given:
$P = -\$150,000$
$A = -\$2,500$
$F_4 = -\$20,000$
$F_5 = -\$45,000$
$F_8 = -\$10,000$
$F_{10} = +\$30,000$

$$
\begin{aligned}
EUAC &= \$150,000(A/P, 5\%, 10) + \$2,500 + \$20,000(P/F, 5\%, 4)(A/P, 5\%, 10) \\
&\quad + \$45,000(P/F, 5\%, 5)(A/P, 5\%, 10) \\
&\quad + \$10,000(P/F, 5\%, 8)\ (A/P, 5\%, 10) - \$30,000\ (A/F, 5\%, 10) \\
&= \$19,425 + \$2,500 + \$2,121 + \$4,566 + \$876 - \$2,385 \\
&= \$27,113
\end{aligned}
$$

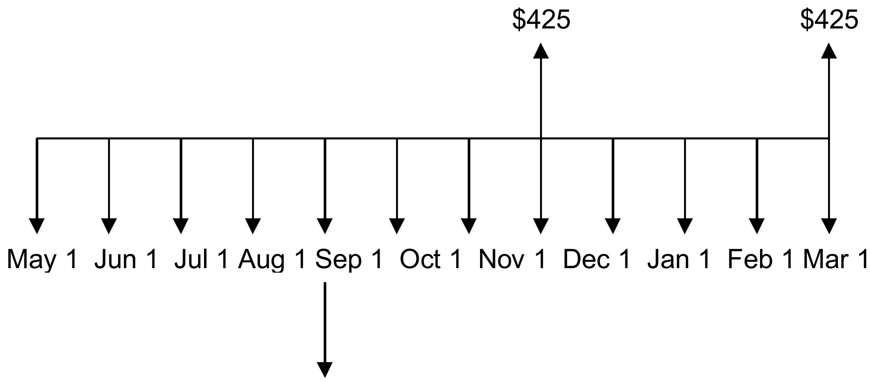
6-24

$i_{month} = (1 + (0.1075/52))^4 - 1 \qquad = 0.008295$

$P = 0.9\ (\$178,000) \qquad = \$160,200$

$$
\begin{aligned}
A &= P\ [(i\ (1 + i)^n)/((1 + i)^n - 1)] \\
&= \$160,200\ [(0.008295\ (1.008295)^{300})/((1.008295)^{300} - 1)] \\
&= \$1,450.55
\end{aligned}
$$

6-25



Equivalent total taxes if all were paid on April 1st:

= $425 + $425 (F/P, ¾%, 4)
= $425 + $425 (1.030)
= $862.75

Equivalent uniform monthly payment:

= $862.75 (A/P, ¾%, 12)
= $862.75 (0.0800)
= $69.02

Therefore the monthly deposit is $69.02.

Amount to deposit September 1:
= Future worth of 5 months deposits (May – Sep)
= $69.02 (F/A, ¾%, 5)
= $69.02 (5.075)
= $350.28

Notes:
1. The fact that the tax payments are for the fiscal year, July 1 Through June 30, does not affect the computations.
2. Quarterly interest payments to the savings account could have an impact on the solution, but they do not in this problem.
3. The solution may be verified by computing the amount in the savings account on Dec. 1 just before making the payment (about $560.03) and the amount on April 1 after making that payment ($0).

6-26

Compute equivalent uniform monthly cost for each alternative.

(a) Purchase for cash

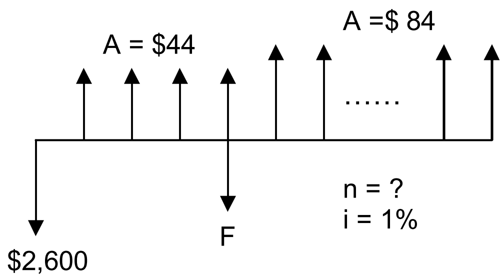
Equivalent Uniform Monthly Cost = (\$13,000 - \$4,000) (A/P, 1%, 36) +
\$4,000 (0.01)
= \$338.80

(b) Lease at a monthly cost = \$350.00

(c) Lease with repurchase option= \$360.00 - \$500 (A/F, 1%, 36)
= \$348.40

Alternative (a) has the least equivalent monthly cost, but non-monetary considerations might affect the decision.

6-27



Compute the equivalent future sum for the \$2,600 and the four \$44 payments at F.

F = \$2,600 (F/P, 1%, 4) - \$44 (F/A, 1%, 4)
= \$2,600 (1.041) - \$44 (4.060) = \$2,527.96
This is the amount of money still owed at the end of the four months. Now solve for the unknown n.

\$2,527.96 = \$84 (P/A, 1%, n)

(P/A, 1%, n) = \$2,572.96/\$84 = 30.09

From the 1% interest table n is almost exactly 36. Thus 36 payments of \$84 will be required.

6-28

Original Loan

Annual Payment = \$80,000 (A/P, 10%, 25) = \$8,816
Balance due at end of 10 years:

Method 1: Balance = \$8,816 (P/A, 10%, 15) = \$67,054
Method 2: The payments would repay:
= \$8,816 (P/A, 10%, 10) = \$54,170
making the unpaid loan at Year 0:
= \$80,000 - \$54,170 = \$25,830

At year 10 this becomes:
= \$25,830 (F/P, 10%, 10) = \$67,000

Note: The difference is due to four place accuracy in the compound interest tables. The exact answer is \$67,035.80

New Loan

(Using \$67,000 as the existing loan)
Amount = \$67,000 + 2% (\$67,000) + \$1,000 = \$69,340
New Pmt. = \$69,340 (A/P, 9%, 15) = \$69,340 (0.1241) = \$8,605

New payment < Old payment, therefore refinancing is desirable.

6-29

Provide Autos
P = \$18,000 F = \$7,000 A = \$600/yr + 0.075/km n = 4 years

Pay Salesmen
0.1875 x where x = km driven

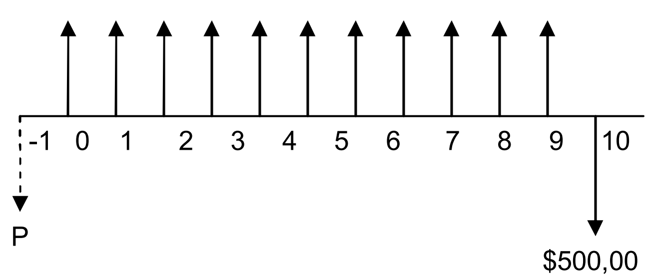
0.1875 x = (\$18,000 - \$7,000)(A/P, 10%, 4) + \$7,000(0.10) + \$600 + \$0.075 x

0.1125 x = (\$11,000) (0.3155) + \$700 + \$600
= \$4,770

Kilometres Driven (x) = 4,770 / 0.1125 = 42,400

6-30

A diagram is essential to properly see the timing of the 11 deposits:



These are beginning of period deposits, so the compound interest factors must be adjusted for this situation.

P_{now-1} = \$500,000 (P/F, 1%, 12) = \$500,000 (0.8874) = \$443,700
A = P_{now-1} (A/P, 1%, 11) = \$443,700 (0.0951) = \$42,196

Quarterly beginning of period deposit = \$42,196

6-31

New Machine
EUAC = \$3,700 (A/P, 8%, 4) - \$500 - \$200
= \$3,700 (0.3019) - \$700
= \$417.03

Existing Machine
EUAC = \$1,000 (A/P, 8%, 4)
= \$1,000 (0.3019)
= \$301.90

The new machine should not be purchased.

6-32

	Around the Lake	**Under the Lake**
First Cost	\$75,000	\$125,000
Maintenance	\$3,000/yr	\$2,000/yr
Annual Power Loss	\$7,500/yr	\$2,500/yr
Property Taxes	\$1,500/yr	\$2,500/yr
Salvage Value	\$45,000	\$25,000
Useful Life	15 years	15 years

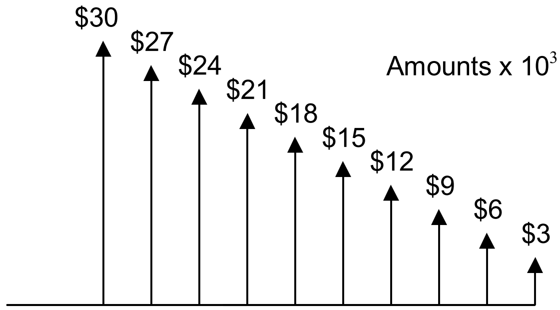
Around the Lake
EUAC = \$75,000 (A/P, 7%, 15) + \$12,000 - \$45,000 (A/F, 7%, 15)
= \$75,000 (0.1098) + \$12,000 - \$45,000 (0.0398)
= \$18,444

Under the Lake
EUAC = \$125,000 (A/P, 7%, 15) + \$7,000 - \$25,000 (A/F, 7%, 15)
= \$125,000 (0.1098) + \$7,000 - \$25,000 (0.0398)
= \$19,730

Go around the lake.

6-33

Engineering Department Estimate

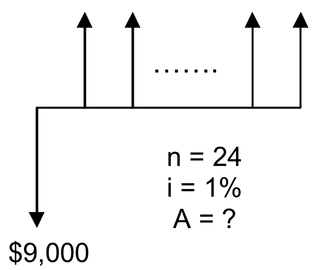


EUAC = $30,000 - $3,000 (A/G, 8%, 10)
= $30,000 - $3,000 (3.871)
= $18,387

Hyro-clean's offer of $15,000/yr is less costly.

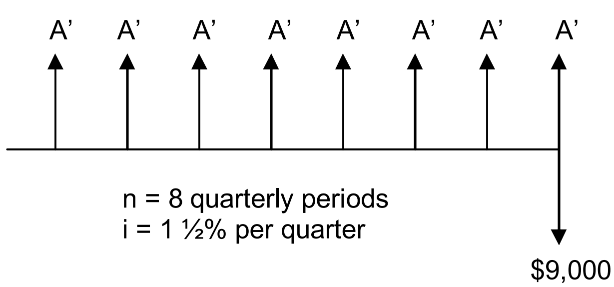
6-34

(a)



A = $9,000 (A/P, 1%, 24)
= $9,000 (0.0471)
= $423.90/month

(b)



Note that interest is compounded quarterly

A' = \$9,000 (A/F, 1.5%, 8)
= \$9,000 (0.1186)
= \$1,067.40

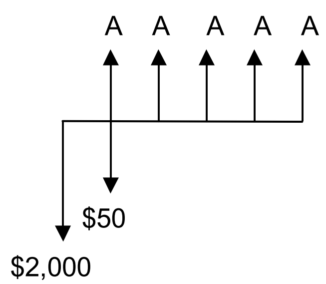
Monthly Deposit = ½ of A' = (\$1,067.40)/3 = \$355.80/mo

(c) In part (a) Bill Anderson's monthly payment includes an interest payment on the loan. The sum of his 24 monthly payments will exceed \$9,000

In part (b) Doug James' savings account monthly deposit earns interest for him that helps to accumulate the \$9,000. The sum of Doug's 24 monthly deposits will be less than \$9,000.

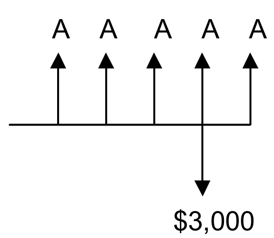
6-35

Alternative A



EUAC = A = [\$2,000 + \$500 (P/F, 12%, 1)] (A/P, 12%, 5)
= [\$2,000 + \$500 (0.8929)] (0.2774)
= \$678.65

Alternative B



EUAC = A = \$3,000 (F/P, 12%, 1) (A/F, 12%, 5)
= \$3,000 (1.120) (0.1574)
= \$528.86

To minimize EUAC, select B.

6-36

With neither input nor output fixed, maximize (EUAB – EUAC)

Continuous compounding capital recovery:

$A = P [(e^{rn} (e^{r} – 1))/(e^{rn} – 1)]$

For r = 0.15 and n = 5,

$[(e^{rn} (e^{r} – 1))/(e^{rn} – 1)] = [(e^{(0.15)(5)} (e^{0.15} – 1))/(e^{(0.15)(5)} – 1)]$
$= 0.30672$

Alternative A
EUAB – EUAC = \$845 - \$3,000 (0.30672) = -\$75.16

Alternative B
EUAB – EUAC = \$1,400 - \$5,000 (0.30672) = -\$133.60

To maximize (EUAB – EUAC) choose alternative A, (less negative value).

6-37

Machine X
EUAC = \$5,000 (A/P, 8%, 5)
= \$5,000 (0.2505)
= \$1,252

Machine Y
EUAC = (\$8,000 - \$2,000) (A/P, 8%, 12) + \$2,000 (0.08) + \$150
= \$1,106

Select Machine Y.

6-38

Annual Cost of Diesel Fuel = [\$50,000km/(35 km/l)] x \$0.48/l = \$685.71
Annual Cost of Gasoline = [\$50,000km/(28 km/l)] x \$0.51/l = \$910.71

$EUAC_{diesel}$ = (\$13,000 - \$2,000) (A/P, 6%, 4) + \$2,000 (0.06)
+ \$685.71 fuel + \$300 repairs + \$500 insurance
= \$11,000 (0.2886) + \$120 + \$1,485.71
= \$4,780.31

$EUAC_{gasoline}$ = (\$12,000 - \$3,000) (A/P, 6%, 3) + \$3,000 (0.06)
+ \$910.71 fuel + \$200 repairs + \$500 insurance
= \$5,157.61

The diesel taxi is more economical.

6-39

Machine A

EUAC $= \$1,000 + P_i$
$= \$1,000 + \$10,000\ (A/P, 10\%, 4) - \$10,000\ (A/F, 10\%, 4)$
$= \$1,000 + \$1,000$
$= \$2,000$

Machine B

EUAC $= (\$20,000 - \$10,000)\ (A/P, 10\%, 10) + \$10,000\ (0.10)$
$= \$1,627 + \$1,000$
$= \$2,627$

Choose Machine A**.**

6-40

It is important to note that the customary "identical replacement" assumption is not applicable here.

Alternative A

EUAB – EUAC $= \$15 - \$50\ (A/P, 15\%, 10) = \$15 - \$50\ (0.1993)$
$= +\$5.04$

Alternative B

EUAB – EUAC $= \$60\ (P/A, 15\%, 5)\ (A/P, 15\%, 10) - \$180\ (A/P, 15\%, 10)$
$= +\$4.21$

Choose A.

Check solution using NPW:

Alternative A

NPW $= \$15\ (P/A, 15\%, 10) - \$50 = +\$25.28$

Alternative B

NPW $= \$60\ (P/A, 15\%, 5) - \$180 = +\$21.12$

6-41

Because we may assume identical replacement, we may compare 20 years of B with an infinite life for A by EUAB – EUAC.

Alternative A

EUAB – EUAC (for an inf. period) $= \$16 - \$100\ (A/P, 10\%, \infty)$
$= \$16 - \$100\ (0.10)$
$= +\$6.00$

Alternative B

EUAB – EUAC (for 20 yr. period) $= \$24 - \$150\ (A/P, 10\%, 20)$
$= \$24 - \$150\ (0.1175)$
$= +\$6.38$

Choose Alternative B.

6-42

Seven-year analysis period:

Alternative A

EUAB – EUAC = \$55 – [\$100 + \$100 (P/F, 10%, 3) + \$100 (P/F, 10%, 6)] (A/P, 10%, 7)
= \$55 – [\$100 + \$100 (0.7513) + \$100 (0.5645)] (0.2054)
= +\$7.43

Alternative B

EUAB – EUAC = \$61 – [\$150 + \$150 (P/F, 10%, 4)] (A/P, 10%, 7)
= \$61 – [\$150 + \$150 (0.683)] (0.2054)
= +\$9.15

Choose B.

Note: The analysis period is seven years, hence one cannot compare three years of A vs. four years of B, If one does, the problem is constructed so he will get the wrong answer.

6-43

EUAC_{gas} = (P – S) (A/P, *i*%, n) + SL + Annual Costs
= (\$2,400 - \$300) (A/P, 10%, 5) + \$300 (0.10) + \$1,200 + \$300
= \$2,100 (0.2638) + \$30 + \$1,500
= \$2,084

$\text{EUAC}_{\text{electr}}$ = (\$6,000 - \$600) (A/P, 10%, 10) + \$600 (0.10) + \$750 + \$50
= \$5,400 (0.1627) + \$60 + \$800
= \$1,739

Select the electric motor.

6-44

EUAC Comparison

Gravity Plan

Initial Investment:	= \$2.8 million (A/P, 10%, 40)	
	= \$2.8 million (0.1023)	= \$286,400
Annual Operation and maintenance		= \$10,000
Annual Cost		= \$296,400

Pumping Plan

Initial Investment:	= \$1.4 million (A/P, 10%, 20)		
	= \$1.4 million (0.1023)		= \$143,200
Additional investment in 10th year:			
	= \$200,000 (P/F, 10%, 10) (A/P, 10%, 40)		
	= \$200,000 (0.3855) (0.1023)	= \$7,890	
Annual Operation and maintenance			= \$25,000
Power Cost:	= \$50,000 for 40 years		= \$50,000

Additional Power Cost in last 30 years:

= $50,000 (F/A, 10%, 30) (A/F, 10%, 40)
= $50,000 (164.494) (0.00226) = $18,590

Annual Cost = $244,680

Select the Pumping Plan.

6-45

Use 20 year analysis period.

Net Present Worth Approach

$NPW_{Mas.}$ = -$250 – ($250 - $10) [(P/F, 6%, 4) + (P/F, 6%, 8) + (P/F, 6%, 12) + (P/F, 6%, 16)] + $10 (P/F, 6%, 20) - $20 (P/A, 6%, 20)
= -$250 – $240 [0.7921 + 0.6274 + 0.4970 + 0.3936) + $10 (0.3118) - $20 (11.470)
= -$1,031

NPW_{BRK} = -$1,000 - $10 (P/A, 6%, 20) + $100 (P/F, 6%, 20)
= -$1,000 - $10 (11.470) + $100 (0.3118)
= -$1,083

Choose Masonite to save $52 on Present Worth of Cost.

Equivalent Uniform Annual Cost Approach

$EUAC_{Mas.}$ = $20 + $250 (A/P, 6%, 4) - $10 (A/F, 6%, 4)
= $20 + $250 (0.2886) - $10 (0.2286)
= $90

$EUAC_{BRK}$ = $10 + $1,000 (A/P, 6%, 20) - $100 (A/F, 6%, 20)
= $10 + $1,000 (0.872) - $100 (0.0272)
= $94

Choose Masonate to save $4 per year.

6-46

Machine A

EUAB – EUAC = - First Cost (A/P, 12%, 7) - Maintenance & Operating Costs + Annual Benefit + Salvage Value (A/F, 12%, 7)
= -$15,000 (0.2191) - $1,600 + $8,000 + $3,000 (0.0991)
= $3,411

Machine B

EUAB – EUAC = - First Cost (A/P, 12%, 10) - Maintenance & Operating Costs + Annual Benefit + Salvage Value (A/F, 12%, 10)
= -$25,000 (0.1770) - $400 + $13,000 + $6,000 (0.0570)
= $8,517

Choose Machine B to maximize (EUAB – EUAC).

6-47

Machine A

EUAB – EUAC = -\$700,000 (A/P, 15%, 10) - \$18,000 + \$154,000
- \$900 (A/G, 15%, 20) + \$210,000 (A/F, 15%, 20)
= -\$271,660 - \$29,000 + \$303,000 - \$4,024 + \$2,050
= \$466

Machine B

EUAB – EUAC = -\$1,700,000 (A/P, 15%, 20) - \$29,000 + \$303,000
- \$750 (A/F, 15%, 20) + \$210,000 (A/F, 15%, 20)
= -\$271,660 - \$29,000 + \$303,000 - \$4,024 + \$2,050
= \$366

Thus, the choice is Machine A but note that there is very little difference between the alternatives.

6-48

Choose alternative with minimum EUAC.

(a) 12 month tire	EUAC	= \$39.95 (A/P, 10%, 1)	= \$43.95
(b) 24 month tire	EUAC	= \$59.95 (A/P, 10%, 2)	= \$34.54
(c) 36 month tire	EUAC	= \$69.95 (A/P, 10%, 3)	= \$28.13
(d) 48 month tire	EUAC	= \$90.00 (A/P, 10%, 4)	= \$28.40

Buy the 36-month tire.

6-49

Alternative A

EUAB – EUAC = \$10 - \$100 (A/P, 8%, ∞) = \$10 - \$100 (0.08)
= +\$2.00

Alternative B

EUAB – EUAC = \$17.62 - \$15 (A/P, 8%, 20) = \$17.62 - \$150 (0.1019)
= +\$2.34

Alternative C

EUAB – EUAC = \$55.48 - \$200 (A/P, 8%, 5) = \$55.48 - \$200 (0.2505)
= +\$5.38

Select C.

6-50

Payment = PMT (0.75%, 48, -12000) = \$298.62
Owed = PV (0.75%, 18, -298.62) = \$5,010.60

6-51

Payment = PMT (0.75%, 60, -15000) = \$311.38
Owed = PV (0.75%, 48, -311.38) = \$12,512.74
She will have to pay \$513 more than she receives for the car.

6-52

DATA		
9.00%	Canadian conventional mortgage rate	
0.7363%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 78,000.00	Principal	
360	Months in amortization period	
$ 618.41	Monthly Payment	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))

Month	**Interest**	**Principal Payment**	**Ending Balance**
0			$ 78,000.00
1	$ 574.32	$ 44.09	$ 77,955.91
2	$ 574.00	$ 44.41	$ 77,911.50
3	$ 573.67	$ 44.74	$ 77,866.77
4	$ 573.34	$ 45.07	$ 77,821.70
5	$ 573.01	$ 45.40	$ 77,776.30
6	$ 572.68	$ 45.73	$ 77,730.57
7	$ 572.34	$ 46.07	$ 77,684.50
8	$ 572.00	$ 46.41	$ 77,638.09
9	$ 571.66	$ 46.75	$ 77,591.34
10	$ 571.31	$ 47.09	$ 77,544.24
11	$ 570.97	$ 47.44	$ 77,496.80
12	$ 570.62	$ 47.79	$ 77,449.01
13	$ 570.27	$ 48.14	$ 77,400.87
14	$ 569.91	$ 48.50	$ 77,352.37
15	$ 569.56	$ 48.85	$ 77,303.52

(a) 618.41
(b) $77,449.01
(c) $570.27

6-53

	DATA		
	9.00%	Canadian conventional mortgage rate	
	0.7363%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$	92,000.00	Principal	
	360	Months in amortization period	
$	729.41	Monthly Payment	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))

Month	Interest	Principal Payment	Ending Balance
0			$ 92,000.00
1	$ 677.41	$ 52.00	$ 91,948.00
2	$ 677.02	$ 52.38	$ 91,895.62
3	$ 676.64	$ 52.77	$ 91,842.85
24	$ 667.85	$ 61.56	$ 90,640.43
25	$ 667.40	$ 62.01	$ 90,578.42
26	$ 666.94	$ 62.47	$ 90,515.95
117	$ 607.63	$ 121.78	$ 82,401.17
118	$ 606.73	$ 122.68	$ 82,278.50
119	$ 605.83	$ 123.58	$ 82,154.92
120	$ 604.92	$ 124.49	$ 82,030.43
121	$ 604.00	$ 125.41	$ 81,905.02

(a) $729.41
(b)
(c) $82,030.43
(d) $667.40 / $62.01

6-54

	DATA		
	9.00%	Canadian conventional mortgage rate	
	0.7363%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$	95,000.00	Principal	
	360	Months in amortization period	
$	753.19	Monthly Payment	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))

(a) $753.19

DATA		
9.00%	Canadian conventional mortgage rate	
0.7363%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 95,000.00	Principal	
360	Months in amortization period	
$ 753.19	Monthly Payment	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))
1000	Accelerated Payment	

Month	Interest	Principal Payment	Ending Balance
0			$ 95,000.00
1	$ 699.50	$ 300.50	$ 94,699.50
2	$ 697.28	$ 302.72	$ 94,396.78
3	$ 695.06	$ 304.94	$ 94,091.84
4	$ 692.81	$ 307.19	$ 93,784.65
160	$ 35.21	$ 964.79	$ 3,817.71
161	$ 28.11	$ 971.89	$ 2,845.82
162	$ 20.95	$ 979.05	$ 1,866.77
163	$ 13.75	$ 986.25	$ 880.51
164	$ 6.48	$ 993.52	$ (113.00)

(b) The mortgage would be paid off after the 164th payment

DATA		
9.00%	Canadian conventional mortgage rate	
0.7363%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 95,000.00	Principal	
360	Months in amortization period	
$ 753.19	Monthly Payment	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))
$ 1,506.38	Double Payments	

Month	Interest	Principal Payment	Ending Balance
0			$ 95,000.00
1	$ 699.50	$ 806.89	$ 94,193.11
2	$ 693.56	$ 812.83	$ 93,380.29
3	$ 687.57	$ 818.81	$ 92,561.48
4	$ 681.54	$ 824.84	$ 91,736.64
84	$ 23.00	$ 1,483.38	$ 1,639.93
85	$ 12.07	$ 1,494.31	$ 145.62
86	$ 1.07	$ 1,505.31	$ (1,359.69)

(c) The mortgage would be paid off after the 86th payment

6-55

DATA		
6.00%	Canadian conventional mortgage rate	
0.4939%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 145,000.00	Principal	
360	Months in amortization period	
$ 862.49	**Monthly Payment**	=$A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))

(a) $862.49

DATA		
6.00%	Canadian conventional mortgage rate	
0.4939%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 145,000.00	Principal	
360	Months in amortization period	
$ 1,000.00	**Accelerated Payment**	

Month	Interest	Principal Payment	Ending Balance
0			$ 145,000.00
1	$ 716.10	$ 283.90	$ 144,716.10
2	$ 714.70	$ 285.30	$ 144,430.80
3	$ 713.29	$ 286.71	$ 144,144.09
253	$ 17.51	$ 982.49	$ 2,562.85
254	$ 12.66	$ 987.34	$ 1,575.51
255	$ 7.78	$ 992.22	$ 583.29
256	$ 2.88	$ 997.12	$ (413.83)

(b) The mortgage would be paid off after the 256 payment

DATA		
6.00%	Canadian conventional mortgage rate	
0.4939%	effectively monthly interest	=((1+$A$2/2)^(1/6))-1
$ 145,000.00	Principal	
360	Months in amortization period	
$ 1,034.99	Monthly Payment x 120%	=120% * $A$4*(($A$3*(1+$A$3)^$A$5)/((1+$A$3)^$A$5-1))

Month	Interest	Principal Payment	Ending Balance
0			$ 145,000.00
1	$ 716.10	$ 318.89	$ 144,681.11
2	$ 714.53	$ 320.47	$ 144,360.64
3	$ 712.94	$ 322.05	$ 144,038.59
234	$ 30.01	$ 1,004.98	$ 5,072.55
235	$ 25.05	$ 1,009.94	$ 4,062.61
236	$ 20.06	$ 1,014.93	$ 3,047.68
237	$ 15.05	$ 1,019.94	$ 2,027.74
238	$ 10.01	$ 1,024.98	$ 1,002.76
239	$ 4.95	$ 1,030.04	$ (27.28)

(c) The mortgage would be paid off after the 239 payment

6-56

	Around the Lake	Under the Lake
2ALTEUAW (modified)	MARR	7.00%
Length, km	16	5
First Cost/km	$5,000	$25,000
Maintenance/km/yr	$200	$400
Yearly power loss/km	$500	$500
Salvage Value/km	$3,000	$5,000

Property tax/0.02*first cost/yr	$1,500	$2,500
USEFUL LIFE	15	15
INITIAL COST	$75,000	$125,000
ANNUAL COSTS	$12,000	$7,000
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$45,000	$25,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$18,444	$19,729
EUAW	-$18,444	-$19,279

Input Data in Shaded Cells **Breakeven Analysis**

	Around the Lake	Under the Lake
2ALTEUAW (modified)	MARR	7.00%
Length, km	15	5
First Cost/km	$5,000	$23,019
Maintenance/km/yr	$200	$400
Yearly power loss/km	$500	$500
Salvage Value/km	$3,000	$5,000
Property tax/0.02*first cost/yr	$1,500	$2,302
USEFUL LIFE	15	15
INITIAL COST	$75,000	$115,095
ANNUAL COSTS	$12,000	$6,802
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$45,000	$25,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$18,444	$18,444
EUAW	-$18,444	-$18,444

6-57

Input Data in Shaded Cells

	Diesel	Gasoline
2ALTEUAW (modified)	MARR	6.00%
Km per year	50,000	50,000
First Cost	$13,000	$12,000
Fuel Cost per liter	$0.48	$0.51
Mileage, km/liter	35	28
Annual Repairs	$300	$200
Annual Insurance Premium	$500	$500
USEFUL LIFE	4	3
INITIAL COST	$13,000	$12,000
ANNUAL COSTS	$1,486	$1,611
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$2,000	$3,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$4,780	$5,158
EUAW	-$4,780	-$5,158

Mileage (km)		
10,000	$4,232	$4,429
20,000	$4,369	$4,611
40,000	$4,643	$4,976
60,000	$4,917	$5,340
80,000	$5,192	$5,704

6-58

Input Data in Shaded Cells

MARR	8.00%
Current Trucking Cost Per Month	$200.00
Labor Cost per year	$3,000
Strapping material cost per bale	$0.40
Revenue per bale	$2.30
Bales per year produced	500
USEFUL LIFE	30
Initial Cost for Bailer	$6,000
ANNUAL COSTS	$3,200
Annual Benefits	$3,550
SALVAGE VALUE	$0
Salvage value as a reduced Cost	
EUAB	3,550
EUAC (CR) + EUAC (O%M)	$3,733
EUAW	-$183

6-59

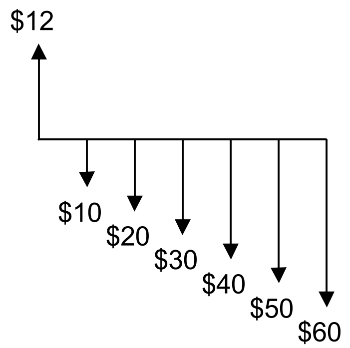
Input Data in Shaded Cells

	MARR	10.00%	Breakeven	
	Gravity Plan	Pumping	Pumping	
USEFUL LIFE	40	40	40	
COMMON MULTIPLE	40	40	40	
INITIAL COST	$2,800,000.00	$1,400,000.00	$1,400,000.00	
ANNUAL COSTS	$10,000.00	$75,000.00	$75,000.00	
Additional Cost, 10th Year		$200,000.00	$1,541,798.00	
Additional Power Cost, yr 11-40		$50,000.00	$50,000.00	
ANNUAL REVENUE	$0.00	$0.00	$0.00	
SALVAGE VALUE	$0.00	$0.00	$0.00	
NET ANNUAL CASH FLOW	-$10,000.00	-$75,000.00	-$75,000.00	
Net Annual Cash Flow (NACF) Method			Difference	
PWB	$0.00	$0.00	$0.00	Drive to zero using solver
PWC	$2,896,765.00	$2,379,444.00	$0.00	

NPW = PWB – PWC		- $2,896,765.00	- $2,379,444.00	- $2,896,765.00
EUAC		$296,222.00	$243,321.00	$296,222.00
		A	B	C
		NPW	NPW	NPW
		-2896765	-2379444	-2896765
Repeating Cash Flow		-2810000	-1475000	-1475000
Year 0		-10000	-140000	-1400000
	1	-10000	-75000	-75000
	2	-10000	-75000	-75000
	3	-10000	-75000	-75000
	4	-10000	-75000	-75000
	5	-10000	-75000	-75000
	6	-10000	-75000	-75000
	7	-10000	-75000	-75000
	8	-10000	-75000	-75000
	9	-10000	-75000	-75000
	10	-10000	-275000	-1616798
	11	-10000	-125000	-125000
	12	-10000	-125000	-125000
	13	-10000	-125000	-125000
	14	-10000	-125000	-125000
	15	-10000	-125000	-125000
	16	-10000	-125000	-125000
	17	-10000	-125000	-125000
	18	-10000	-125000	-125000
	19	-10000	-125000	-125000
	20	-10000	-125000	-125000
	21	-10000	-125000	-125000
	22	-10000	-125000	-125000
	23	-10000	-125000	-125000
	24	-10000	-125000	-125000
	25	-10000	-125000	-125000
	26	-10000	-125000	-125000
	27	-10000	-125000	-125000
	28	-10000	-125000	-125000
	29	-10000	-125000	-125000
	30	-10000	-125000	-125000
	31	-10000	-125000	-125000
	32	-10000	-125000	-125000
	33	-10000	-125000	-125000
	34	-10000	-125000	-125000
	35	-10000	-125000	-125000
	36	-10000	-125000	-125000
	37	-10000	-125000	-125000
	38	-10000	-125000	-125000
	39	-10000	-125000	-125000
	40	-10000	-125000	-125000

Chapter 7: Rate of Return Analysis

7-1



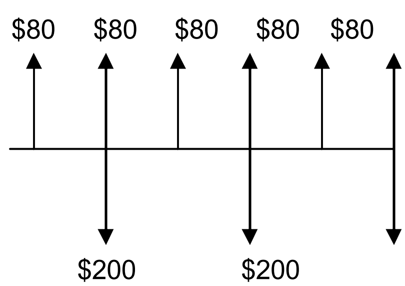
$125 = $10 (P/A, i%, 6) + $10 (P/G, i%, 6)

at 12%, $10 (4.111) + $10 (8.930) = $130.4

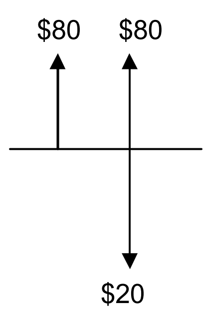
at 15%, $10 (3.784) + $10 (7.937) = $117.2

i* = 12% + (3%) ((130.4 – 125).(130.4-117.2)) = 13.23%

7-2



The easiest solution is to solve one cycle of the repeating diagram:



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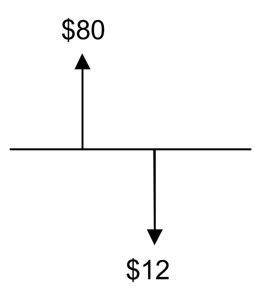


$120 = $80 (F/P, i%, 1)
$120 = $80 (1 + i)

$(1 + i) = \$120/\$80 = 1.50$

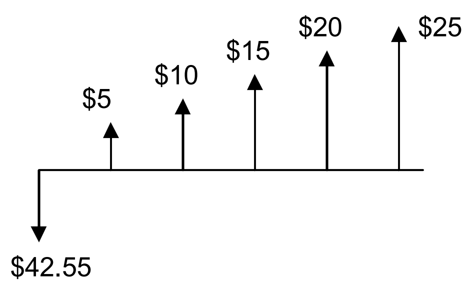
$i^* = 0.50 \quad = 50\%$

Alternative Solution:
EUAB = EUAC
$\$80 = [\$200\ (P/F, i\%, 2) + \$200\ (P/F, i\%, 4) + \$200\ (P/F, i\%, 6)]\ (A/P, i\%, 6)$

Try i = 50%
$\$80 = [\$200\ (0.4444) + \$200\ (0.1975) + \$200\ (0.0878)]\ (0.5481) \quad = \$79.99$

Therefore $i^* = 50\%$.

7-3



$\$42.55 = \$5\ (P/A, i\%, 5) + \$5\ (P/G, i\%, 5)$

Try i = 15%, $\$5\ (3.352) + \$5\ (5.775) = \$45.64 > \$42.55$

Try i = 20%, $\$5\ (2.991) + \$5\ (4.906) = \$39.49 < \$42.55$

Rate of Return $= 15\% + (5\%)\ [(\$45.64 - \$42.55)/(\$45.64 - \$39.49)]$
$= 17.51\%$

Exact Answer: 17.38%

7-4

For infinite series: A = Pi

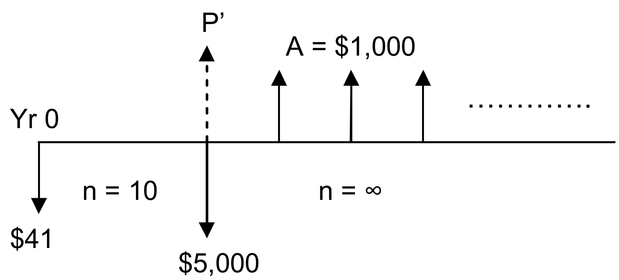
EUAC= EUAB
$\$3{,}810\ (i) = \$250 + \$250\ (F/P, i\%, 1)\ (A/F, i\%, 2)^*$
Try i = 10%
$\$250 + \$250\ (1.10)\ (0.4762) \quad = \$381$
$\$3{,}810\ (0.10) \quad = \$381$

i = 10%

*Alternate Equations:
$\$3{,}810\ (i) \quad = \$250 + \$250\ (P/F, i\%, 1)\ (A/P, i\%, 2)$

$3,810 (j) = $500 - $250 (A/G, i%, 2)

7-5



At Year 0, PW of Cost = PW of Benefits
$412 + $5,000 (P/F, i%, 10) = ($1000/i) (P/F, i%, 10)

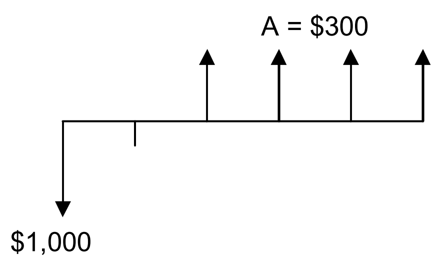
Try i = 15%
$412 + $5,000 (0.2472) = ($1,000/0.15) (0.2472)
$1,648 = $1,648

ROR = 15%

7-6

The algebraic sum of the cash flows equals zero. Therefore, the rate of return is 0%.

7-7



Try i = 5%
$1,000 =(?) $300 (3.546) (0.9524)
=(?) $1,013.16

Try i = 6%
$1,000 =(?) $300 (3.465) (0.9434)
=(?) $980.66

Performing Linear Interpolation:

i* = 5% + (1%) (($1,013.6 - $1,000)/($1,013.6 - $980.66))
= 5.4%

7-8

$400 = [$200 (P/A, i%, 4) - $50 (P/G, i%, 4)] (P/F, i%, 1)

Try i = 7%
[$200 (3.387) - $50 (4.795)] (0.9346) = 409.03

Try i = 8%
[$200 (3.312) - $50 (4.650)] (0.9259) = $398.08

i* = 7% + (1%) [($409.03 - $400)/($409.03 - $398.04)]
= 7.82%

7-9

$100 = $27 (P/A, i%, 10)
(P/A, i%, 10) = 3.704

Performing Linear Interpolation:

(*P/A*, *i*%, 10)	*i*
4.192	20%
3.571	25%

Rate of Return = 20% + (5%) [(4.192 – 3.704)/(4.912 – 3.571)]
= 23.9%

7-10

Year	Cash Flow
0	-$500
1	-$100
2	+$300
3	+$300
4	+$400
5	+$500

$500 + $100 (P/F, i%, 1)= $300 (P/A, i%, 2) (P/F, i%, 1)
+ $400 (P/F, i%, 4) + $500 (P/F, i%, 5)

Try i = 30%
$500 + $100 (0.7692) = $576.92
$300 (1.361) (0.7692) + $400 (0.6501) + $500 (0.2693) = $588.75
Δ = 11.83

Try i = 35%
$500 + $100 (0.7407) = $574.07
$300 (1.289) (0.7407) + $400 (0.3011) + $500 (0.2230) = $518.37

Δ = 55.70

Rate of Return = 30% + (5%) [11.83/55.70)
= 31.06%

Exact Answer: 30.81%

7-11

Year	Cash Flow
0	-$223
1	-$223
2	-$223
3	-$223
4	-$223
5	-$223
6	+$1,000
7	+$1,000
8	+$1,000
9	+$1,000
10	+$1,000

The rate of return may be computed by any conventional means. On closer inspection one observes that each $223 increases to $1,000 in five years.

$223 = $1,000 (P/F, i%, 5)

(P/F, i%, 5) = $223/$1,000 = 0.2230

From interest tables, Rate of Return = 35%

7-12

Year	Cash Flow
0	-$640
1	40
2	+$100
3	+$200
4	+$300
5	+$300

$640 = $100 (P/G, i%, 4) + $300 (P/F, i%, 5)

Try i = 9%
$100 (4.511) + $300 (0.6499) = $646.07 > $640

Try i = 10%
$100 (4.378) + $300 (0.6209) = $624.07 < $640

Rate of Return = 9% + (1%) [(%646.07 - $640)/($646.07 - $624.07)]

= 9.28%

7-13

Since the rate of return exceeds 60%, the tables are useless.

$F = P(1 + i)^n$
$\$4,500 = \$500(1 + i)^4$
$(1 + i)^4 = \$4,500/\$500 = 0$
$(1 + i) = 9^{1/4} = 1.732$
$i^* = 0.732 = 73.2\%$

7-14

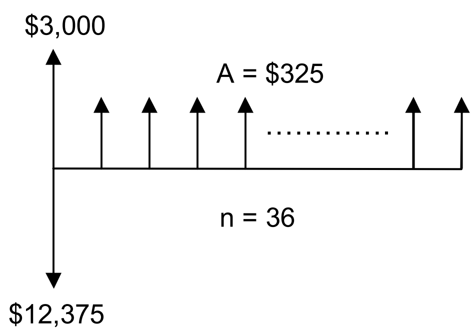
$\$3,000 = \$119.67\ (P/A, i\%, 30)$
$(P/A, i\%, 30) = \$3,000/\$119.67 = 25.069$
Performing Linear Interpolation:

(*P/A*, *i*%, 30)	*i*
25.808	1%
24.889	1.25%

$i = 1\% + (0.25\%)((25.808-25.069)/(25.808-24.889))$
$= 1.201\%$

(a) Nominal Interest Rate = 1.201 x 12 = 14.41%
(b) Effective Interest Rate = $(1 + 0.01201)^{12} - 1 = 0.154 = 15.4\%$

7-15



$\$9,375 = \$325\ (P/A, i\%, 36)$

$(P/A, i\%, 36) = \$9,375/\$325 = 28.846$

From compound interest tables, i = 1.25%

Nominal Interest Rate = 1.25 x 12 = 15%
Effective Interest Rate = $(1 + 0.0125)^{12} - 1 = 16.08\%$

7-16

1991 – 1626 = 365 years = n

F = P $(1 + i)^n$
12×10^9 = 24 $(1 + i)^{365}$

$(1 + i)^{365}$ = $12 \times 10^0/24$ = 5.00×10^8

This may be immediately solved on most hand calculators:
i* = 5.64%

Solution based on compound interest tables:

(F/P, i%, 365) = 5.00×10^8
= (F/P, i%, 100) (F/P, i%, 100) (F/P, i%, 100) (F/P, i%, 65)

Try i = 6%
(F/P, 6%, 365) = $(339.3)^3$ (44.14) = $17.24 \text{ X } 10^8$ (i too high)

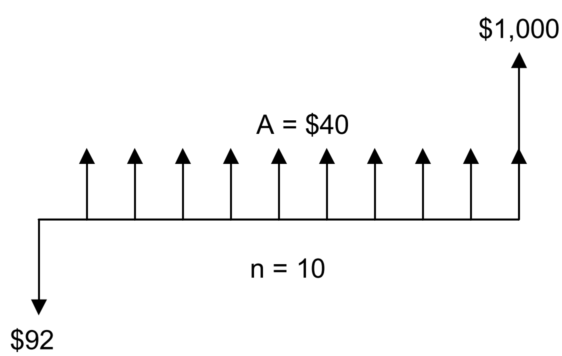
Try i = 5%
(F/P, 5%, 365) = $(131.5)^3$ (23.84) = $0.542 \text{ X } 10^8$ (i too low)

Performing linear interpolation:

i* = 5% + (1%) [((5 – 0.54) (10^8))/((17.24 – 0.54) (10^8))]
= 5% + 4.46/16.70
= 5.27%

The linear interpolation is inaccurate.

7-17



PW of Cost = PW of Benefits

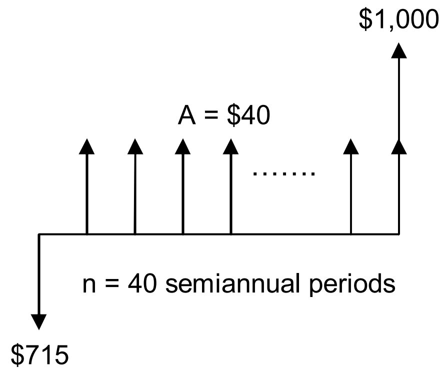
$925 = $40 (P/A, i%, 10) + $1,000 (P/F, i%, 10)

Try i = 5%
$925 = $40 (7.722) + $1,000 (0.6139) = $922.78 (i too high)

Try i = 4.5%
$925 = $40 (7.913) + $1,000 (0.6439) = $960.42 (i too low)

i* ≈ 4.97%

7-18



PW of Benefits – PW of Costs = 0
$20 (P/A, i%, 40) + $1,000 (P/F, i%, 40) - $715 = 0

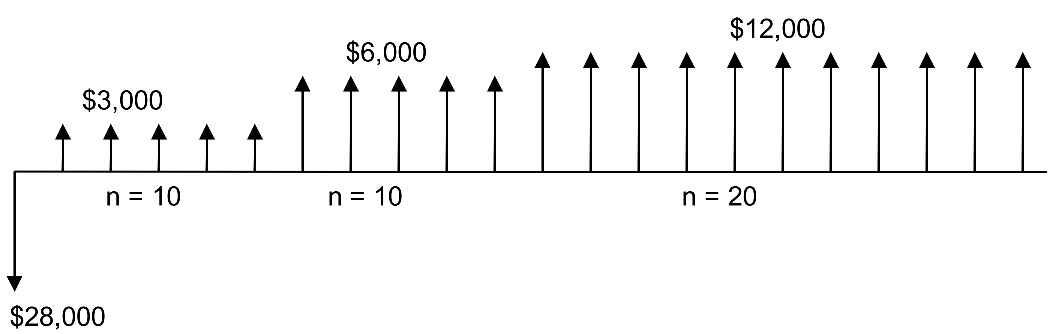
Try i = 3%
$20 (23.115) + $1,000 (0.3066) - $715 = $53.90 i too low

Try i = 3.5%
$20 (21.355) + $1,000 (0.2526) - $715 = -$35.30 i too high

Performing linear interpolation:
i* = 3% + (0.5%) [53.90/(53.90 – (-35.30))] = 3.30%

Nominal i* = 6.60%

7-19



PW of Cost = PW of Benefits

$28,000 = $3,000 (P/A, i%, 10) + $6,000 (P/A, i%, 10) (P/F, i%, 10)
+ $12,000 (P/A, i%, 20) (P/F, i%, 20)

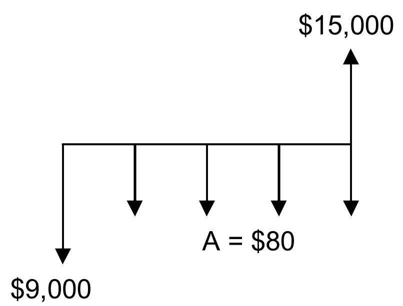
Try i = 12%
$3,000 (5.650) + $6,000 (5.650) (0.3220) + $12,000 (7.469) (0.1037)
= $37,160 > $28,000

Try i = 15%
$3,000 (5.019) + $6,000 (5.019) (0.2472) + $12,000 (6.259) (0.0611)
= $27,090 < $28,000

Performing Linear Interpolation:

i* = 15% - (3%) [($28,000 - $27,090)/($37,160 - $27,090)]
= 15% - (3%) (910/10,070)
= 14.73%

7-20



PW of Benefits – PW of Cost = $0
$15,000 (P/F, i%, 4) - $9,000 - $80 (P/A, i%, 4) = $0

Try i = 12%
$15,000 (0.6355) - $9,000 - $80 (3.037) = +$289.54

Try i = 15%
$15,000 (0.5718) - $9,000 - $80 (2.855) = -$651.40

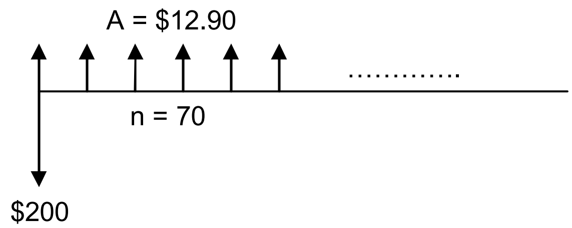
Performing Linear Interpolation:

i* = 12% + (3%) [289.54/(289.54 + 651.40)]
= 12.92%

7-21

The problem requires an estimate for n- the expected life of the infant. Seventy or seventy-five years might be the range of reasonable estimates. Here we will use 71 years.

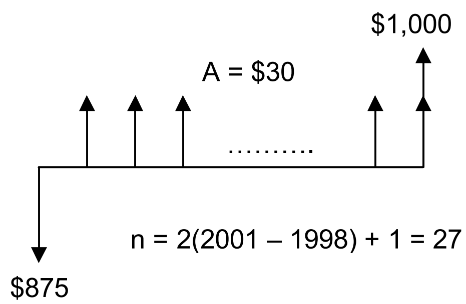
The purchase of a $200 life subscription avoids the series of *beginning-of-year* payments of $12.90. Based on 71 beginning-of-year payments,



$200 - $12.90 = $12,90 (P/A, i%, 70)
(P/A, i%, 70) = $187.10/$12.90 = 14.50

6% < i* < 8%, By Calculator: i* = 6.83%

7-22



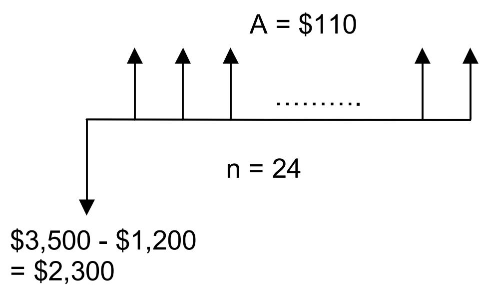
PW of Benefits – PW of Cost = $0
$30 (P/A, i%, 27) + $1,000 (P/F, i%, 27) - $875 = $0

Try i = 3 ½%
$30 (17.285) + $1,000 (0.3950) - $875 = $38.55 >$0

Try i = 4%
$30 (16.330) + $1,000 (0.3468) - $875 = -$38.30 < $0

i* = 3.75%
Nominal rate of return = 2 (3.75%) = 7.5%

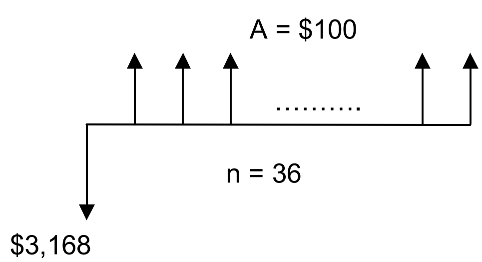
7-23



$2,300 = $110 (P/A, i%, 24)
(P/A, i%, 24) = $2,300/$110 = 20.91

From tables: 1% < i < 1.25%
On Financial Calculator: i = 1.13% per month
Effective interest rate $= (1 + 0.0113)^{12} – 1$ = 0.144 = 14.4%

7-24



PW of Cost = PW of Benefits
$100 (P/A, i%, 36)= $3,168
(P/A, i%, 36) = $3,168/$100 = 31.68

Performing Linear Interpolation:

(*P/A*, *i*%, 36)	*I*
32.871	½%
21.447	¾%

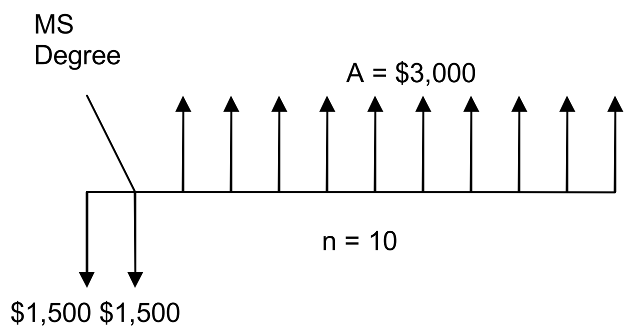
i* = (1/2%) + (1/4%) [(32.87 – 31.68)/(32.87 – 31.45)]
= 0.71%

Nominal Interest Rate = 12 (0.71%) = 8.5%

7-25

This is a thought-provoking problem for which there is no single answer. Two possible solutions are provided below.

A. Assuming the MS degree is obtained by attending graduate school at night while continuing with a full-time job:
Cost: $1,500 per year for 2 years
Benefit: $3,000 per year for 10 years

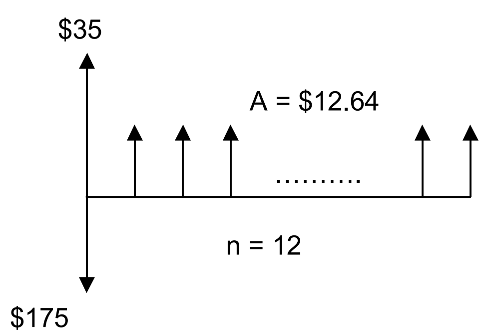


Computation as of award of MS degree:
$1,500 (F/A, i%, 2) = $3,000 (P/A, i%, 10)
i* > 60

B. Assuming the MS degree is obtained by one of year of full-time study
Cost: Difference between working & going to school. Whether working or at school there are living expenses. The cost of the degree might be $24,000
Benefit: $3,000 per year for 10 years

$24,000 = $3,000 (P/A, i%, 10)
i* = 4.3%

7-26



($175 - $35) = $12.64 (P/A, i%, 12)
(P/A, i%, 12) = $140/$12.64 = 11.08
i = 1 ¼%
Nominal interest rate = 12 (1 ¼%) = 15%

7-27

The rate of return exceeds 60% so the interest tables are not useful.

$F = P (1 + i)^n$
$\$25,000 = \$5,000 (1 + i)^3$
$(1 + i) = (\$25,000/\$5,000)^{1/3} = 1.71$
$i^* = 0.71$

Rate of Return = 71%

7-28

This is an unusual problem with an extremely high rate of return. Available interest tables obviously are useless.

One may write:
PW of Cost = PW of Benefits
$\$0.5 = \$3.5 (1 + i)^{-1} + \$0.9 (1 + i)^{-2} + \$3.9 (1 + i)^{-3} + \$8.6 (1 + i)^{-4} + \ldots$

For high interest rates only the first few terms of the series are significant:

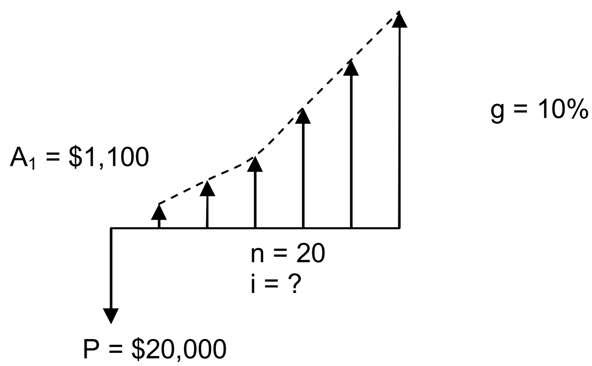
Try i = 650%
PW of Benefits $= \$3.5/(1 + 6.5) + \$0.9/(1 + 6.5)^2 + \$3.9/(1 + 6.5)^3 + \$8.6/(1 + 6.5)^4 + \ldots.$
$= 0.467 + 0.016 + 0.009 + 0.003$
$= 0.495$

Try i = 640%

PW of Benefits $= \$3.5/(1 + 6.4) + \$0.9/(1 + 6.4)^2 + \$3.9/(1 + 6.4)^3 + \$8.6/(1 + 6.4)^4 + \ldots.$
$= 0.473 + 0.016 + 0.010 + 0.003$
$= 0.502$

$i^* \approx 642\%$
(Calculator Solution: i = 642.9%)

7-29



The payment schedule represents a geometric gradient.
There are two possibilities:
$i \neq g$ and $i = g$
Try the easier $i = g$ computation first:
$P = A_1 n (1 + i)^{-1}$ where $g = i = 0.10$
$\$20{,}000 = \$1{,}100 (20) (1.10)^{-1} = \$20{,}000$
Rate of Return $i^* = g = 10\%$

7-30

(a) Using Equation (4-39):
$F = Pe^{rn}$
$\$4{,}000 = \$2{,}000e^{r(9)}$
$2 = e^{r(9)}$
$9r = \ln 2 = 0.693$
$r = 7.70\%$

(b) Equation (4-34)
$i_{eff} = e^r - 1 = e^{0.077} - 1 = 0.0800 \quad = 8.00\%$

7-31

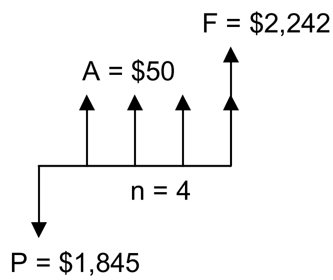
(a) When $n = \infty$, $i = A/P = \$3{,}180/\$100{,}000 \quad = 3.18\%$

(b) $(A/P, i\%, 100) = \$3180/\$100{,}000 \quad = 0.318$
From interest tables, $i^* = 3\%$

(c) $(A/P, i\%, 50) = \$3{,}180/\$100{,}000 \quad = 0.318$
From interest tables, $i^* = 2\%$

(d) The saving in water truck expense is just a small part of the benefits of the pipeline. Convenience, improved quality of life, increased value of the dwellings, etc., all are benefits. Thus, the pipeline appears justified.

7-32



Set PW of Cost = PW of Benefits

$1,845 = $50 (P/A, i%, 4) + $2,242 (P/F, i%, 4)

Try i = 7%
450 (3.387) + $2,242 (0.7629) = $1,879 > $1,845

Try i = 8%
450 (3.312) + $2,242 (0.7350) = $1,813 < $1,845

Rate of Return = 7% + (1%) [($1,879 - $1,845)/($1,879 - $1,813)]
= 7.52% for 6 months

Nominal annual rate of return = 2 (7.52%) = 15.0%
Equivalent annual rate of return = $(1 + 0.0752)^2 - 1$ = 15.6%

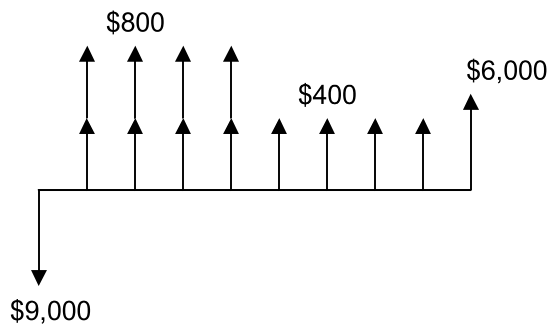
7-33

(a)
F = $5 *P* = $1 *n* = 5

F = $P (1 + i)^n$
$5 = $1 $(1 + i)^5$
(1 + i) = $5^{0.20}$ = 1.38
i* = 38%

(b) For a 100% annual rate of return
F = $1 $(1 + 1.0)^5$ = $32, not $5!
Note that the prices Diagonal charges do not necessarily reflect what anyone will pay a collector for his/her stamps.

7-34



Year	Cash Flow
0	-$9,000
1- 4	+$800
5- 8	+$400
9	+$6,000

PW of Cost = PW of Benefits
$9,000 = $400 (P/A, i%, 8) + $400 (P/A, i%, 4) + $6,000 (P/F, i%, 9)

Try i = 3%
$400 (7.020) + $400 (3.717) + $6,000 (0.7664) = $8,893 < $9,000

Try i = 2 ½%
$400 (7.170) + $400 (3.762) + $6,000 (0.8007) = $9,177 > $9,000

Rate of Return = 2 ½% + (1/2%) [($9,177 - $9,000)/($9,177 - $8,893)]
= 2.81%

7-35

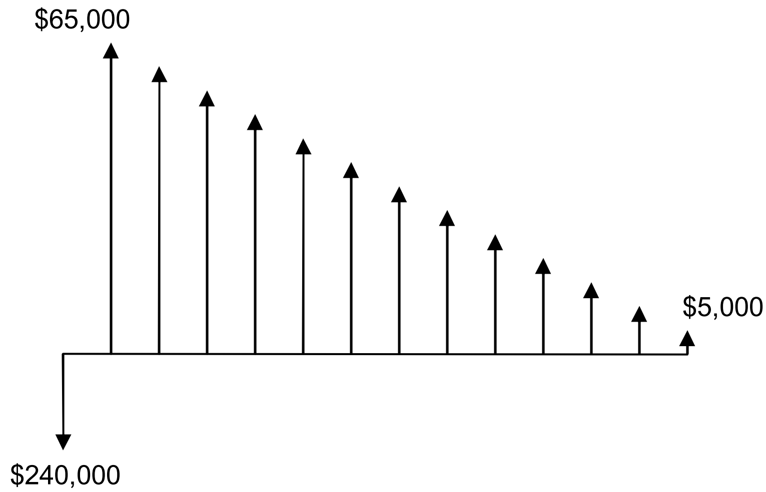
Year	Cash Flow
0	-$1,000
3	+$1,094.60
6	+$1,094.60

$1,000 = $1,094 [(P/F, i%, 6) + (P/F, i%, 9)]

Try i = 20%
$1,094 [(0.5787) + (0.3349)] = $1,000

Rate of Return = 20%

7-36



$240,000 = $65,000 (P/A, i%, 13) - $5,000 (P/G, i%, 13)

Try i = 15%
$65,000 (5.583) -$5,000 (23.135) = $247,220 > $240,000

Try i = 18%
$65,000 (4.910) -$5,000 (18.877) = $224,465 < $240,000

Rate of Return = 15% + 3% [($247,220 - $240,000)/($247,220 - $224,765)]
= 15.96%

7-37

3,000 = 30 (P/A, i*, 120)
(P/A, i*, 120) = 3,000/30 = 100

Performing Linear Interpolation:

(*P/A*, *i*%, 120)	*I*
103.563	¼%
100	i*
90.074	½%

i* = 0.0025 + 0.0025 [(103.562 – 100)/(103.562 – 90.074)]
= 0.00316 per month

Nominal Annual Rate = 12 (0.00316) = 0.03792 = 3.79%

7-38

(a) Total Annual Revenues = \$500 (12 months) (4 apt.) = \$24,000
Annual Revenues – Expenses = \$24,000 - \$8,000 = \$16,000
To find Internal Rate of Return the Net Present Worth must be \$0.

NPW = \$16,000 (P/A, i*, 5) + \$160,000 (P/F, i*, 5) - \$140,000

At i = 12%, NPW = \$8,464
At i = 15%, NPW = -\$6,816

IRR = 12% + (3%) [\$8,464/(\$8,464 + \$6,816)]
= 13.7%

(b) At 13.7% the apartment building is more attractive than the other options.

7-39

NPW = -\$300,000 + \$20,000 (P/F, i*, 10)
+ (\$67,000 - \$3,000) (P/A, i*, 10) - \$600 (P/G, i*, 10)

Try i = 10%
NPW= -\$300,000 + \$20,000 (0.3855) + (\$64,000) (6.145)
- \$600 (22.891)
= \$87,255 > \$0 The interest rate is too low.

Try i = 18%
NPW = -\$300,000 + \$20,000 (0.1911) + (\$64,000) (4.494)
- \$600 (14.352)
= -\$17,173 < \$0 The interest rate is too high.

Try i = 15%
NPW = -\$300,000 + \$20,000 (0.2472) + (\$64,000) (5.019)
- \$600 (16.979)
= \$9,130 > \$0

Thus, the rate of return (IRR) is between 15% and 18%. By linear interpolation:

i* = 15% + (3%) [\$9,130/(\$9,130 - \$17,173)]
= 16.0%

7-40

(a) First payment = \$2, Final payment = 132 x \$2 = \$264
Average Payment = (\$264 + \$2)/2 = \$133
Total Amount = 132 payments x \$133 average pmt = \$17,556

Alternate Solution: The payments are same as sum-of-years digits form
SUM = (n/2) (n + 1) = (132/2) (133) = 8,778

Total Amount = \$2 (8778) = \$17,556

(b) The bank will lend the present worth of the gradient series
Loan (P) = \$2 (P/G, 1%, 133)
Note: n- 1 = 132, so n = 133

By interpolation,
(P/G, 1%, 133) = 3,334.11 + (13/120) (6,878.6 – 3,334.1) = 3,718.1
Loan (P) = \$2 (3,718.1) = \$7,436.20

7-41

Year	Case 1 (incl. Deposit)
0	-\$39,264.00
1	+\$599.00
2	+\$599.00
3	+\$599.00
4	+\$599.00
5	+\$599.00
6	+\$599.00
7	+\$599.00
8	+\$599.00
9	+\$599.00
10	+\$599.00
11	+\$599.00
12	+\$599.00
…	+\$599.00
33	+\$599.00
34	+\$599.00
35	+\$599.00
36	+\$27,854.00 -\$625.00
	= +\$27,229.00

IRR = 0.86%
Nominal IRR = 10.32%
Effective IRR = 10.83%

7-42

MARR = 5% *P* = \$30,000 *n* = 35 years

Alternative 1: Withdraw \$15,000 today and lose \$15,000
Alternative 2: Wait, leave your fund in the system until retirement.

Equivalency seeks to determine what future amount is equal to \$15,000 now.

$$
\begin{aligned}
F &= P(1+i)^n \\
&= \$30{,}000\,(1.05)^{35} \\
&= \$30{,}000\,(5.516015) \\
&= \$165{,}480.46
\end{aligned}
$$

Therefore:

$\$15,000 = \$165,480.46 (1 + i)^{-35}$

$\$15,000 (1 + i)^{35} = \$165,480.46$

$(1 + i) = [(165,480.46/\$15,000)]^{1/35}$

$i = 1.071 – 1 = 7.1002\% > 5\%$

Unless $15,000 can be invested with a return higher than 7.1%, it is better to wait for 35 years for the retirement fund. $15,000 now is only equivalent to $165,480.46 35 years from now if the interest rate now is 7.1% instead of the quoted 5%.

7-43

$\$2,000 = \$91.05 (P/A, i^*, 30)$

$(P/A, i^*, 30) = \$2,000/\$91.05 = 21.966$

(*P/A*, *i*%, 30)	*i*
22.396	2
20.930	2 ½

$i_{mo} = 2\% + (1/2\%) [(22.396 – 21.966)/(22.396 – 20.930)]$

$= 2.15\%$ per month

Nominal ROR received by finance company = 12 (2.15%) = 25.8%

7-44

$\$3,000 = \$118.90 (P/A, i^*, 36)$

$(P/A, i^*, 36) = \$3,000/\$118.90 = 26.771$

(*P/A*, *i*%, 36)	*i*
27.661	1 ½%
26.543	1 ¾%

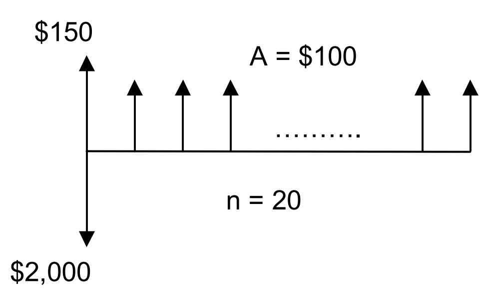
$i_{mo} = 1 \tfrac{1}{2}\% + \tfrac{1}{4}\% [(27.661 – 26.771)/(27.661 – 26.543)]$

$= 1.699\%$ per month

Nominal Annual ROR = 12 (1.699%) = 20.4%

7-45

(a)



($2,000 - $150)		= $100 (P/A, i%, 20)
(P/A, i%, 20)		= $1,850/$100 = 18.5
	i	= ¾% per month

The alternatives are equivalent at a nominal 9% annual interest.

(b) Take Alt 1- the $2,000- and invest the money at a higher interest rate.

7-46

Year	(A) Gas Station	(B) Ice Cream Stand	(B- A)
0	-$80,000	-$120,000	-$40,000
1- 20	+$8,000	+$11,000	+$3,000
Computed ROR	7.75%	6.63%	4.22%

The rate of return in the incremental investment (B- A) is less than the desired 6%. In this situation the lower cost alternative (A) Gas Station should be selected.

7-47

Year	A	B	(B- A)
0	-$2,000	-$2,800	-$800
1- 3	+$800	+$1,100	+$300
Computed ROR	9.7%	8.7%	6.1%

The rate of return on the increment (B- A) exceeds the Minimum Attractive Rate of Return (MARR), therefore the higher cost alternative B should be selected.

7-48

Year	X	Y	X- Y
0	-$100	-$50	-$50
1	+$35	+$16.5	+$18.5
2	+$35	+$16.5	+$18.5
3	+$35	+$16.5	+$18.5
4	+$35	+$16.5	+$18.5
Computed ROR	15.0%	12.1%	17.8%

The ∆ROR on X- Y is greater than 10%. Therefore, the increment is desirable. Select X.

7-49

The fixed output of +$17 may be obtained at a cost of either $50 or $53. The additional $3 for Alternative B does not increase the benefits. Therefore it is not a desirable increment of investment.

Choose A.

7-50

Year	A	B	(B- A)
0	-\$100.00	-\$50.00	-\$50.00
1- 10	+\$19.93	+\$11.93	+\$8.00
Computed ROR	15%	20%	9.61%

ΔROR = 9.61% > MARR.

Select A.

7-51

Year	X	Y	X- Y
0	-\$5,000	-\$5,000	\$0
1	-\$3,000	+\$2,000	-\$5,000
2	+\$4,000	+\$2,000	+\$2,000
3	+\$4,000	+\$2,000	+\$2,000
4	+\$4,000	+\$2,000	+\$2,000
Computed ROR	16.9%	21.9%	9.7%

Since X- Y difference between alternatives is desirable, select Alternative X.

7-52

(a) Present Worth Analysis- Maximize NPW

NPW_A = \$746 (P/A, 8%, 5) - \$2,500
= \$746 (3.993) – \$2,500 = +\$479

NPW_B = \$1,664 (P/A, 8%, 5) - \$6,000 = +\$644

Select B.

(b) Annual Cash Flow Analysis- Maximize (EUAB- EUAC)

$(EUAB\text{- }EAUC)_A$ = \$746 - \$2,500 (A/P, 8%, 5)
= \$746 - \$2,500 (0.2505)
= +\$120

$(EUAB – EUAC)_B$ = \$1,664 - \$6,000 (A/P, 8%, 5)
= +\$161

Select B.

(c) Rate of Return Analysis: Compute the rate of return on the B- A increment of investment and compare to 8% MARR.

Year	A	B	B- A
0	-\$2,500	-\$6,000	-\$3,500
1- 5	+\$746	+\$1,664	+\$918

\$3,500 = \$918 (P/A, i%, 5)

Try i = 8%, \$918 (3.993) = \$3,666 > \$3,500

Try i = 10%, $918 (3.791) = $3,480 < $3,500

Δ Rate of Return = 9.8%

Since ΔROR > MARR, B- A increment is desirable. Select B.

7-53

Using incremental analysis, computed the internal rate of return for the difference between the two alternatives.

Year	B- A
0	+$12,000
1	-$3,000
2	-$3,000
3	-$3,000
4	-$3,000
5	-$3,000
6	-$3,000
7	-$3,000
8	-$4,200

Note: Internal Rate of Return (IRR) equals the interest rate that makes the PW of costs minus the PW of Benefits equal to zero.

$12,000 - $3,000 (P/A, i*, 7) - $4,200 (P/F, i*, 8) = $0

Try i = 18%
$12,000 - $3,000 (3.812) - $4,200 (0.2660) = -$553 < $0

Try i = 17%
$12,000 - $3,000 (3.922) - $4,200 (0.2848) = $962 > $0

i* = 17% + (1%) [$962/($962 + $553)]
= 17.6%

The contractor should choose Alternative B and lease because 17.62% > 15% MARR.

7-54

	B	A	A- B
First Cost	$300,000	$615,000	$315,000
Maintenace & Operating Costs	$25,000	$10,000	-$15,000
Annual Benefit	$92,000	$158,000	$66,000
Salvage Value	-$5,000	$65,000	$70,000

NPW = -$315,000 + [$66,000 – (-$15,000)] (P/A, i*, 10) + $70,000 (P/F, i*, 10) = $0

Try i = 15%
-$315,000 + [$66,000 – (-$15,000)] (5.019) + $70,000 (0.2472) = $108,840

ΔROR > MARR (15%)

The higher cost alternative A is the more desirable alternative.

7-55

Year	A	B	A- B	NPW at 7%	NPW at 9%
0	-$9,200	-$5,000	-$4,200	-$4,200	-$4,200
1	+$1,850	+$1,750	+$100	+$93	+$92
2	+$1,850	+$1,750	+$100	+$87	+$84
3	+$1,850	+$1,750	+$100	+$82	+$77
4	+$1,850	+$1,750 -$5,000	+$5,100	+$3,891	+$3,613
5	+$1,850	+$1,750	+$100	+$71	+$65
6	+$1,850	+$1,750	+$100	+$67	+$60
7	+$1,850	+$1,750	+$100	+$62	+$55
8	+$1,850	+$1,750	+$100	+$58	+$50
			Sum	+$211	-$104

Δ ROR ≈ 8.3%
Choose Alternative A.

7-56

Year	Zappo	Kicko	Kicko – Zappo
0	-$56	-$90	-$34
1	-$56	$0	+$56
2	$0	$0	$0

Compute the incremental rate of return on (Kicko – Zappo)
PW of Cost = PW of Benefit
$34 = $56 (P/F, i%, 1)
(P/F, i%, 1) = $34/$56 = 0.6071

From interest tables, incremental rate of return > 60% (ΔROR = 64.7%), hence the increment of investment is desirable.

Buy Kicko.

7-57

Year	A	B	A- B
0	-$9,200	-$5,000	-$4,200
1	+$1,850	+$1,750	+$100
2	+$1,850	+$1,750	+$100
3	+$1,850	+$1,750	+$100
4	+$1,850	+$1,750 -$5,000	+$100 +$5,000
5	+$1,850	+$1,750	+$100
6	+$1,850	+$1,750	+$100
7	+$1,850	+$1,750	+$100
8	+$1,850	+$1,750	+$100
			Sum

Rates of Return

A: $9,200 = $1,850 (P/A, i%, 5)
Rate of Return = 11.7%
B: $5,000 = $1,750 (P/A, i%, 4)
Rate of Return = 15%
A- B: $4,200 = $100 (P/A, i%, 8) + $5,000 (P/F, i%, 4)
ΔROR_{A-B} = 8.3%

Select A.

7-58

Year	A	B	A- B
0	-$150	-$100	-$50
1- 10	+$25	+$22.25	+$2.75
11- 15	+$25	$0	+$25
15	+$20	$0	+$20
Computed ROR	14.8%	18%	11.6%

Rate of Return (A- B):
$50 = $2.75 (P/A, i%, 10) + $25 (P/A, i%, 5) (P/F, i%, 10) + $20 (P/F, i%, 15)
Rate of Return = 11.65

Select A.

Appendix 7A: Difficulties Solving for An Interest Rate

7A-1

Year	Cash Flow
0	+$4,000
1-9	+$4,000
10	-$71,000
11-19	+$4,000

There are 2 sign changes in the cash flow indicating there may be 2, 1, or zero positive interest rates.

At $i = 0\%$ NPW= +$5,000
At $i = \infty\%$ NPW =+$4,000

This suggests that the NPW plot may look like one of the following:

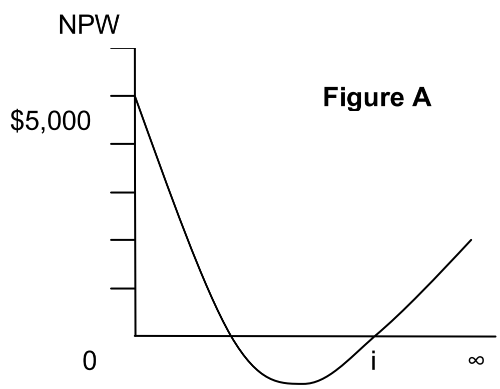


Figure A

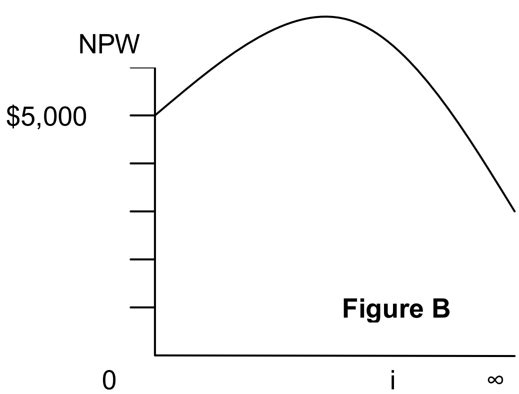


Figure B

After making a number of calculations, one is forced to conclude that Figure B is the general form of the NPW plot, and there is no positive interest rate for the cash flow.

There is external investment until the end of the tenth year. If an external interest rate (we will call it e* in Chapter 18) is selected, we can proceed to solve for the interest rate i for the investment phase of the problem.

For external interest rate = 6%
Future worth of \$4,000 a year for 10 years (11 payments)
= \$4,000 (F/A, 6%, 11) = \$4,000 (14.972) = \$59,888
At year 10 we have +\$59,888 -\$75,000 = -\$15,112

The altered cash flow becomes:

Year	Cash Flow
0	0
1-9	0
10	-\$15,112
11-19	+\$4,000

At the beginning of year 10:
PW of Cost = PW of Benefits
\$15,112 = \$4,000 (P/A, i%, 9)

(P/A, i%, 9) = \$15,112/\$4,000 = 3.78

By linear interpolation from interest tables, i = 22.1%.

The internal interest rate is sensitive to the selected external interest rate:

For external Interest rate	Computed internal Interest rate
0%	3.1%
6%	22.1%
8%	45.9%

7A-2

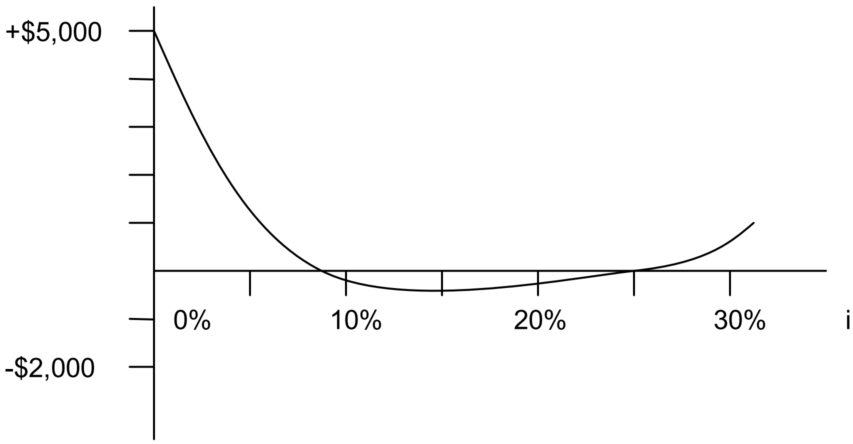
The problem statement may be translated into a cash flow table:

Year	Cash Flow
0	+\$80,000
1	-\$85,000
2	-\$70,000
3	0
4	+\$80,000

There are two sign changes in the cash flow indicating there may be as many as two positive rates of return.

To search for positive rates of return compute the NPW for the cash flow at several interest rates. This is done on the next page by using single payment present worth factors to compute the PW for each item in the cash flow. Then, their algebraic sum represents NPW at the stated interest rate.

Year	Cash Flow	PW at 0%	PW at 8%	PW at 9%	PW at 25%	PW at 30%
0	+$80,000	+$80,000	+$80,000	+$80,000	+$80,000	+$80,000
1	-$85,000	-$85,000	-$78,700	-$77,980	-$68,000	-$65,380
2	-$70,000	-$70,000	-$60,010	-$58,920	-$44,800	-$41,420
3	0	0	0	0	0	0
4	+$80,000	+$80,000	+$58,800	+$56,670	+$32,770	+$28,010
		+$5,000	+$90	-$230	-$30	+$1,210



The plow of NPW vs. i shows two positive interest rates: i ≈ 8.2% and i ≈ 25%

Using an external interest rate of 6%, the Year 0 cash flow is invested and accumulates to +$80,000 (1.06) = $84,800 at the end of Year 1. The revised cash flow becomes:

Year	Cash Flow
0	0
1	-$200
2	-$70,000
3	0
4	+$80,000

With only one sign change we know there no longer is more than one positive interest rate.

PW of Benefit = PW of Cost, or PW(Benefit) – PW(Cost) = 0

$80,000 (P/F, i%, 4) - $200 (P/F, i%, 1) - $70,000 (P/F, i%, 2) = 0

Try i = 7%
80,000 (0.7629) - $200 (0.9346) - $70,000 (0.8734) = -$293

Try i = 6%
$80,000 (0.7921) - $200 (0.9434) - $70,000 (0.8900) = +$879

By interpolation, i = 6.75%.

7A-3

(a)

Quarter	Quarterly Cash Flow	PW at 0%	PW at 20%	PW at 40%	PW at 45%
0	-$75	-$75	-$75.0	-$75.0	-$75.0
1	+$75	+$75	+$62.5	+$53.6	+$51.7
2	-$50	-$50	-$34.7	-$25.5	-$23.8
3	+$50	+$50	+$28.9	+$18.2	+$16.4
4	+$125	+$125	+$60.3	+$32.5	+$28.3
Sum		+$125	+$42.0	+$3.8	-$2.4

By interpolation, i ≈ 43% per quarter. The nominal rate of return = 4 (43%) = 172% per year.

(b)

Quarter	Quarterly Cash Flow	Transformed Cash Flow
0	-$75	-$75
1	+$75	+$26.46
2	-$50	0
3	+$50	+$50
4	+$125	+$125

Let X = amount required at end of quarter 1 to produce $50 at the end of 2 quarters:

X (1.03) = $50
X = $50/1.03 = $48.54

Solve the Transformed Cash Flow for the rate of return:

Quarter	Transformed Cash Flow	PW at 35%	PW at 40%
0	-$75	-$75.00	-$75.00
1	+$26.46	+$19.60	+$18.90
2	0	0	0
3	+$50	+$20.32	+$18.22
4	+$125	+$37.64	+$32.54
Sum		+$2.56	-$5.34

Rate of return = 35% + 5% (2.56/(2.56 – (-5.34)))
= 36.6% per quarter
Nominal annual rate of return = 36.6% x 4 = 146%

NOTE: Although there are three sign changes in the cash flow, the accumulated cash flow sign test, (described in Chapter 18) indicates there is only a single positive rate of return for the untransformed cash flow. It is 43%.

(c) In part (a) the required external investment in Quarter 1, for return in Quarter 2, is assumed to be at the internal rate of return (which we found is 43% per quarter).

In part (b) the required external investment is at 3% per quarter.

The "correct" answer is the one for the computation whose assumptions more closely fit the actual problem. Even though there is only one rate of return, there still exists the required external investment in Quarter 1 for Quarter 2. On this basis the Part (b) solution appears to have more realistic assumptions than Part (a).

7A-4

Year	Cash Flow
0	-$500
1	+$2,000
2	-$1,200
3	-$300
Sum	0

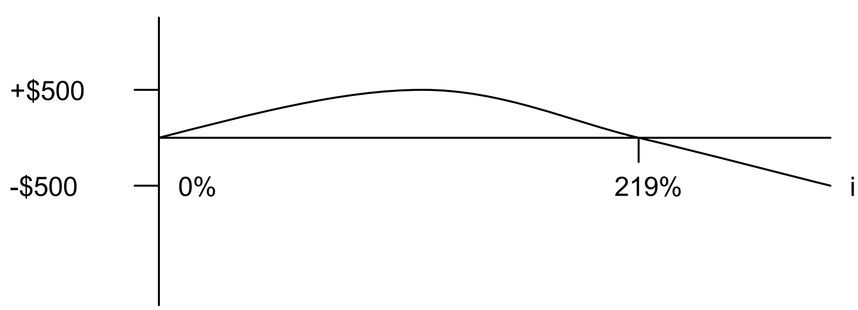
There are two sign changes in the cash flow indicating as many as two positive rates of return. The required disbursement in Year 2 & 3 indicate that money must be accumulated in an external investment to provide the necessary Year 2 & 3 disbursements.

Before proceeding, we will check for multiple rates of return. This, of course, is not necessary here.

Since the algebraic sum of the cash flow = $0, we know that NPW at 0% = 0, and 0% is a rate of return for the cash flow.

Looking for the other (possible) rate of return:

Year	Cash Flow	PW at 5%	PW at 50%	PW at 200%	PW at 219%	PW at 250%	PW at ∞%
0	-$500	-$500	-$500	-$500	-$500	-$500	-$500
1	+$2,000	+$1,905	+$1,333	+$667	+$627	+$571	$0
2	-$1,200	-$1,088	-$533	-$133	-$118	-$98	$0
3	-$300	-$259	-$89	-$11	-$9	-$7	$0
	NPW =	+$58	+$211	+$23	0	-$34	-$500



Solution using an external interest rate e* = 6%.

How much of the +$2,000 at Year 1 must be set aside in an external investment at 6% to provide for the Year 2 and Year 3 disbursements?

Amount to set aside = $1,200 (P/F, 6%, 1) + $300 (P/F, 6%, 2)

= $1,200 (0.94.4) + $300 (0.8900)
= $1,399.08

The altered cash flow becomes:

Year	Cash Flow
0	-$500
1	+$2,000 -$1,399.08 = +$600.92
2	0
3	0

Solve the altered cash flow for the unknown i:

$500 = $600.92 (P/F, i%, 1)
(P/F, i%, 1) = $500/$600.92 = 0.8321

From tables: 20.2%

7A-5

Year	Cash Flow		Altered Cash Flow	PW at 18%	PW at 20%
0	-$500		-$500		
1	+$2,000	$200(1.06)	0		
2	-$1,200	= +$212	-$288		
3	-$300		+$1,200		
			+$412	+$23	-$6

The rate of return is 18% + (2%) (23/(23 + 6)) = 19.6%

7A-6

Year	Cash Flow	PW at 20%	PW at 35%	PW at 50%
0	-$100	-$100	-$100	-$100
1	+$360	+$300	+$267	+$240
2	-$570	-$396	-$313	-$253
3	+$360	+$208	+$146	+$107
NPW=	+$50	+$12	0	-$6

There is a single positive rate of return at 35%.

Year	Cash Flow		Altered Cash Flow	PW at 12%	PW at 15%
0	-$100		-$100	-$100	-$100
1	+$360	$360(1.06)	0	0	0
2	-$570	= +$382	-$188	-$150	-$142
3	+$360		+$360	+$256	+$237
			+$72	+$6	-$5

Rate of return ≈ 13.6%.
For further computations, see the solution to Problem 18-4.

7A-7

Some money flowing out of the cash flow in Year 2 will be required for the Year 3 investment of $100. At 10% external interest, $90.91 at Year 2 becomes the required $100 at Year 3.

Year	Cash Flow		Transformed Cash Flow	NPW at 20%	NPW at 25%
0	-$110		-$110	-$110.00	-$110.00
1	-$500		-$500	-$416.65	-$400
2	+$300	-$90.91(1.10)	+$209.09	+$145.19	+$133.82
3	-$100	= +$100	0	0	0
4	+$400		+$400	+$192.92	+$163.84
5	+$500		+$500	+$200.95	+$163.85
				+$12.41	-$48.49

The rate of return on the transformed cash flow is 21%. (This is only slightly different from the 21.4% rate of return on the original cash flow because the external investment is small and of short duration.)

7A-8

Year	Cash Flow		Transformed Cash Flow	PW at 15%
0	-$50.0		-$50.0	-$50.0
1	+$20.0		0	0
2	-$40.0	$20(1.10)	-$18.0	-$13.6
3	+$36.8	= +$22	+$36.8	+$24.2
4	+$36.8		+$36.8	+$21.0
5	+$36.8		+$36.8	+$18.3
				-$0.1

From the computations we see that the rate of return on the internal investment is 15%.

7A-9

Year	Cash Flow		Transformed Cash Flow
0	-$15,000		-$15,000
1	+$10,000		+$8,980
2	+$6,000	X 1.12 = +$6,720	0
3	-$8,000		0
4	+$4,000	Y $(1.12)^2$ = +$1,280	+$4,000

5	+$4,000	Y = $1,020	+$4,000
6	+$4,000		+$4,000

Year	Transformed Cash Flow	PW at 10%	PW at 12%
0	-$15,000	-$15,000	-$15,000
1	+$8,980	+$8,164	+$8,018
2	0	0	0
3	0	0	0
4	+$4,000	+$2,732	+$2,542
5	+$4,000	+$2,484	+$2,270
6	+$4,000	+$2,258	+$2,026
Sum		+$638	-$144

Rate of Return = 10% + (2%) (638/(638+144)) = 11.6%

7A-10

The compound interest tables are for positive interest rates and are not useful here. (Tables could be produced, of course, for negative values.)

PW of Cost = PW of Benefits

$$\$50 = \$20\,(1 + i)^{-1} + \$20\,(1 + i)^{-2}$$

let $x = (1+ i)^{-1}$ thus, $\$50 = \$20x + \$20x^2$ or $x^2 + x - 2.50 = 0$

$$x = \frac{-1 + (1^2 - 4(-2.50))^{1/2}}{2} = \frac{-1+ (11)^{1/2}}{2} = +1.159, -2.158$$

Solving for i:

$x = (1 + i)^{-1} = +1.159$	$1 + i = 1/1.159 = 0.863$	$i = -0.137 = -13.7\%$
$x = (1 + i)^{-1} = -2.158$	$1 + i = 1/-2.158 = -0.463$	$i = -1.463 = -146\%$

7A-11

Year	Cash Flow		Transformed Cash Flow
0	0		0
1	0		0
2	-$20	X (1.15) = +$10	-$20
3	0	X = $10/1.15 = $8.7	0
4	-$10		-$10
5	+$20		+$11.3
6	-$10		0
7	+$100		+$100

Year	Transformed Cash Flow	PW at 35%	PW at 40%
0	0	0	0
1	0	0	0
2	-$20	-$11.0	-$10.2
3	0	0	0
4	-$10	-$3.0	-$2.6
5	+$11.3	+$2.5	+$2.1
6	0	0	0
7	+$100	+$12.2	+$9.5

Rate of return = 35% + 5% (0.7/(0.7+1.2)) = 36.8%

7A-12

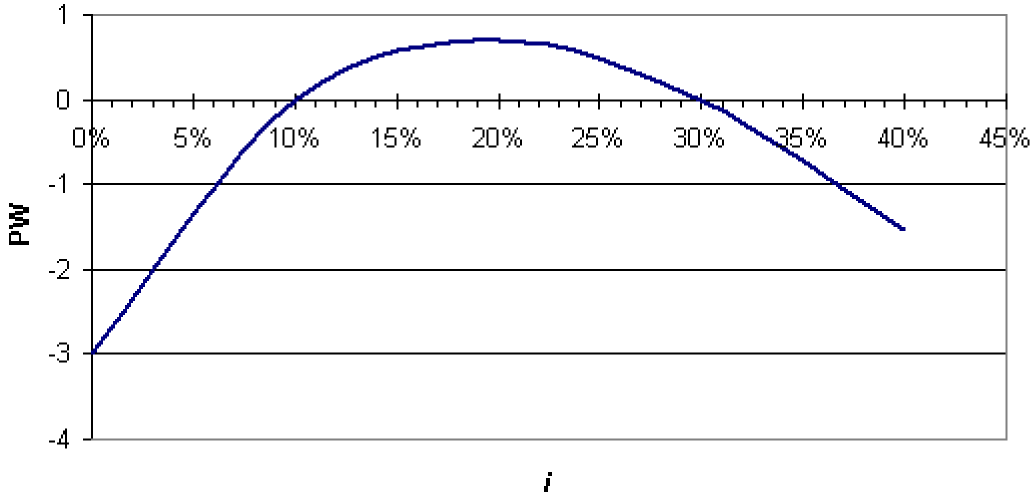
Year	Cash Flow		Transformed Cash Flow	PW at 25%
0	-$800		-$800	-$800
1	+$500		+$500	+$400
2	+$500	X (1.10) = +$300	+$227.27	+$145.5
3	-$300	X = $300/1.10 = $272.73	0	0
4	+$400		+$400	+$163.8
5	+$275		+$275	+$90.1
				-0.6

From the Present Worth computation it is clear that the rate of return is very close to 25% (Calculator solution says 24.96%).

7A-13

Year	Cash flow	I	PW		
0	-100	0%	-3	=$B$2+NPV(D2,$B$3:$B$4)	
1	240	10%	0		
2	-143	20%	1		
		30%	0		
		40%	-2	10.00%	root
				30.00%	root

2 sign changes => 2 roots possible

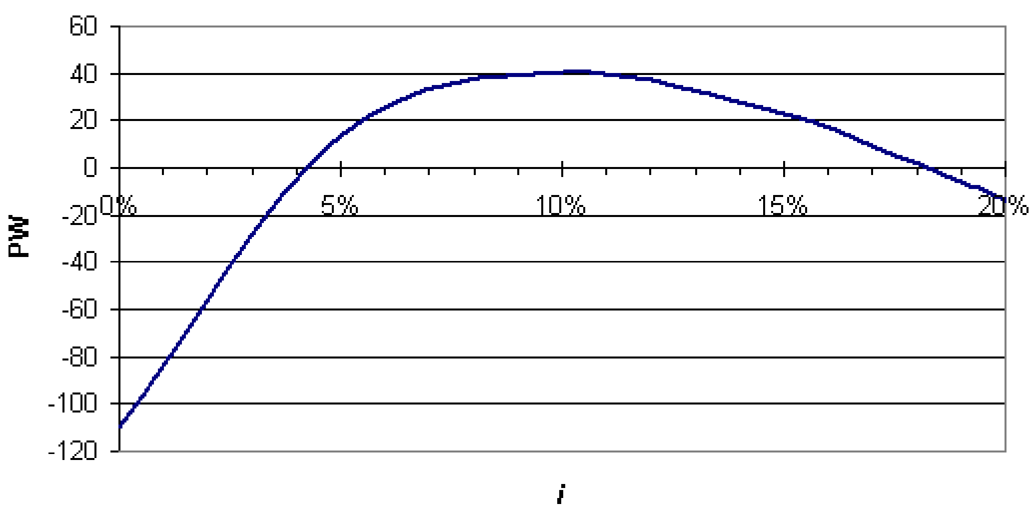


6%	external financing rate	
12%	external investing rate	
8.8%	MIRR	value is less than external investing rate => not attractive

7A-14

Year	Cash flow	I	PW			
0	-610	0%	-110	=$B$2+NPV(D2,$B$3:$B$12)		
1	200	5%	13			
2	200	10%	41			
3	200	15%	23			
4	200	17%	10		4.07%	root
5	200	19%	-6		18.29%	root
6	200	20%	-14			
7	200	25%	-57			
8	200					
9	200					
10	-1300					

2 sign changes => 2 roots possible

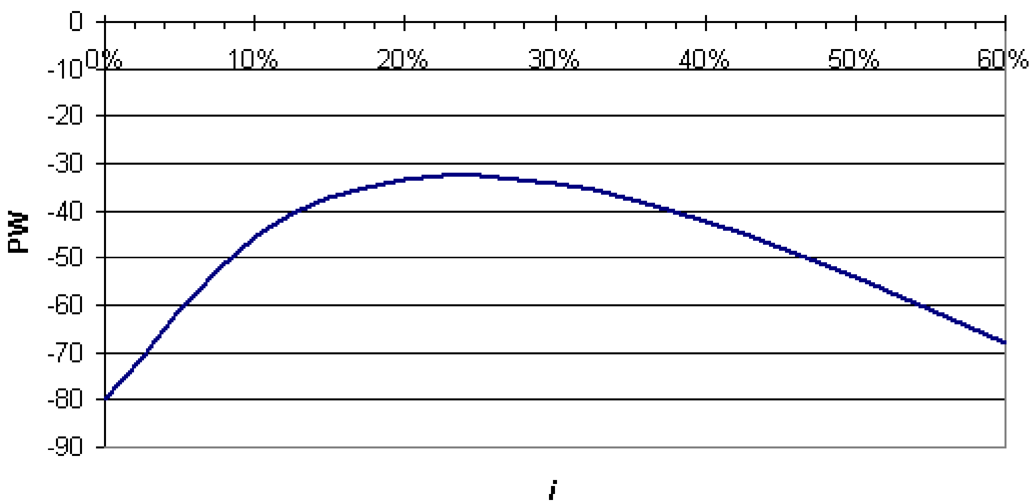


6%	external financing rate		
12%	external investing rate		
9.5%	MIRR	value is less than external investing rate => not attractive	

7A-15

Year	Cash flow	i	PW		
0	-500	0%	-80	=$B$2+NPV(D2,$B$3:$B$5)	
1	800	10%	-45		
2	170	20%	-34		
3	-550	30%	-34		
		40%	-42	#NUM!	root
		50%	-54	#NUM!	root
		60%	-68	No roots exist	

2 sign changes => 2 roots possible



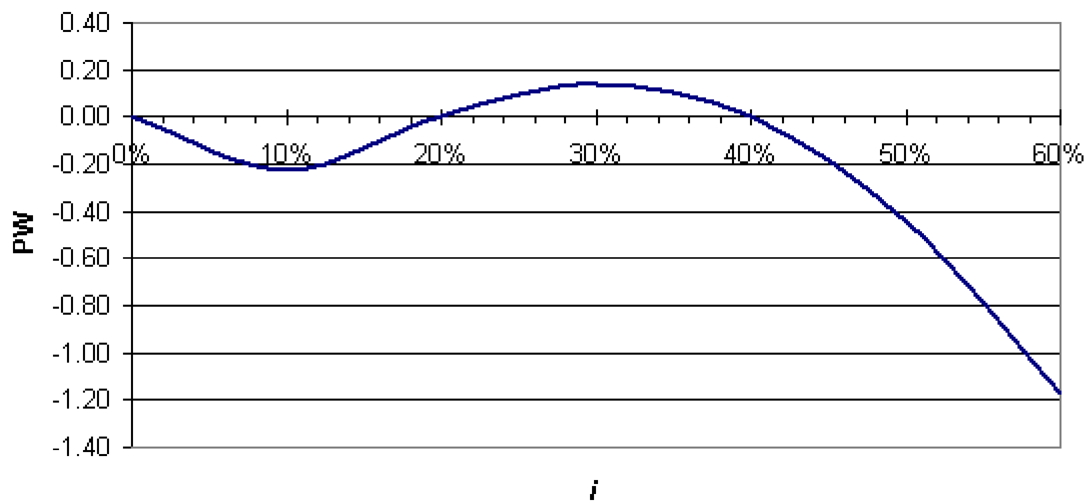
6%	external financing rate		
12%	external investing rate		
7.5%	MIRR	value is less than external investing rate => not attractive	

7A-16

Year	Cash flow	I	PW		
0	-100	0%	0.00	=$B$2+NPV(D2,$B$3:$B$5)	
1	360	10%	-0.23		
2	-428	20%	0.00		
3	168	30%	0.14		
		40%	0.00	0.00%	root
		50%	-0.44	20.00%	root
		60%	-1.17	40.00%	root

3 sign changes => 3 roots possible

All PW values = 0 given significant digits of cash flows

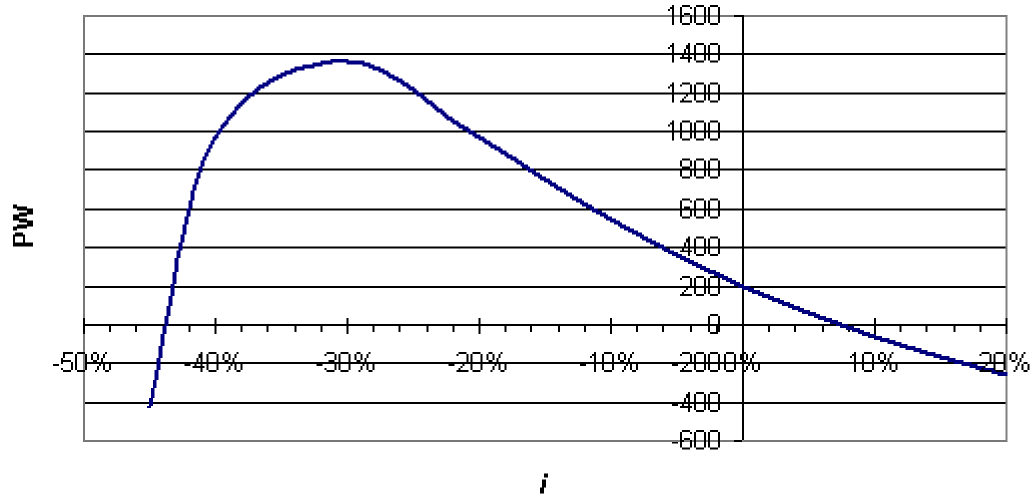


6%	external financing rate	
12%	external investing rate	
8.8%	MIRR	value is less than external investing rate => not attractive

7A-17

Year	Cash flow	i	PW		
0	-1200	-45%	-422	=$B$2+NPV(D2,$B$3:$B$8)	
1	358	-40%	970		
2	358	-30%	1358		
3	358	-20%	970		
4	358	-10%	541	7.22%	root
5	358	0%	196	-43.96%	root
6	-394	10%	-65		
		20%	-261		

2 sign changes => 2 roots possible

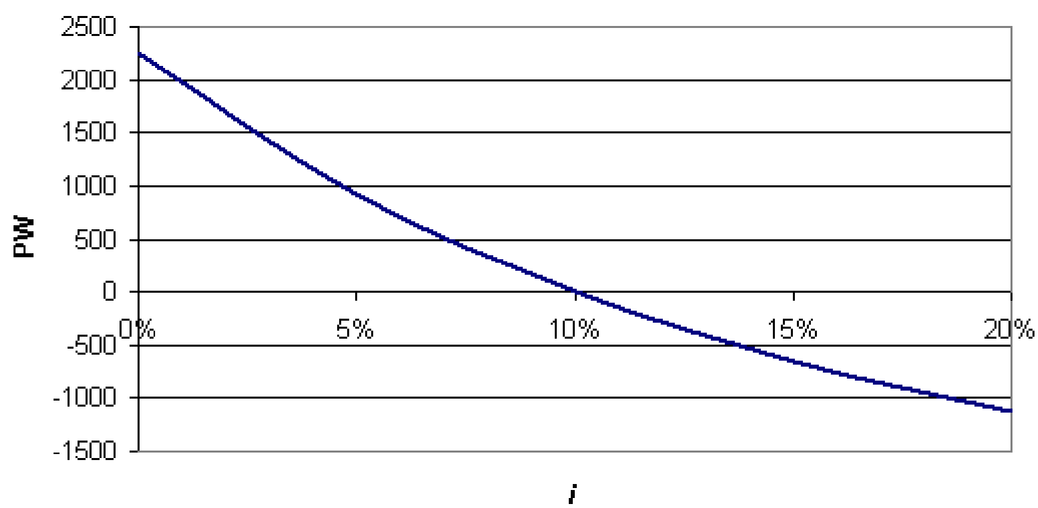


6%	external financing rate	
12%	external investing rate	
9.5%	MIRR	value is less than external investing rate => not attractive

7A-18

Year	Cash flow	i	PW	
0	-3570	0%	2260	=$B$2+NPV(D2,$B$3:$B$10)
1	1000	5%	921	
2	1000	10%	-1	
3	1000	15%	-651	
4	-3170	20%	-1120	10.00% IRR
5	1500			unique IRR
6	1500			
7	1500			
8	1500			

3 sign changes => 3 roots possible



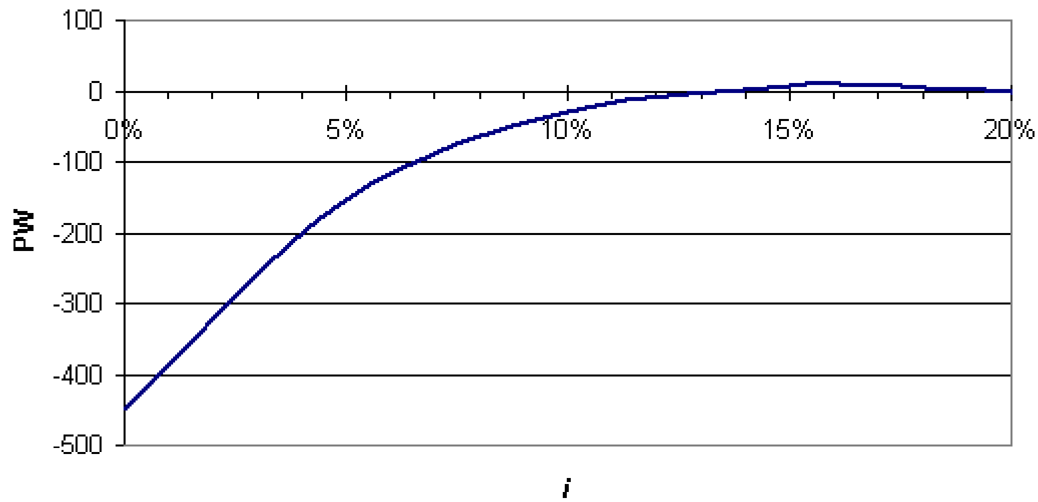
7A-19

800	down payment	
		1 sign change => 1 root possible
55	monthly payment	
40	# payment	
2500	final receipt	
-0.75%	IRR monthly	=RATE(A3,-A2,-A1,A4)
-8.62%	effective annual rate =(1+A6)^12-1	

7A-20

Year	Cash flow	i	PW			
0	-850	0%	-450	=$B$2+NPV(D2,$B$3:$B$12)		
1	600	5%	-153			
2	200	10%	-29			
3	200	15%	7			
4	200	17%	8		12.99%	root
5	200	19%	3		19.72%	root
6	200	20%	-1			
7	200	25%	-31			
8	200					
9	200					
10	-1800					

2 sign changes => 2 roots possible

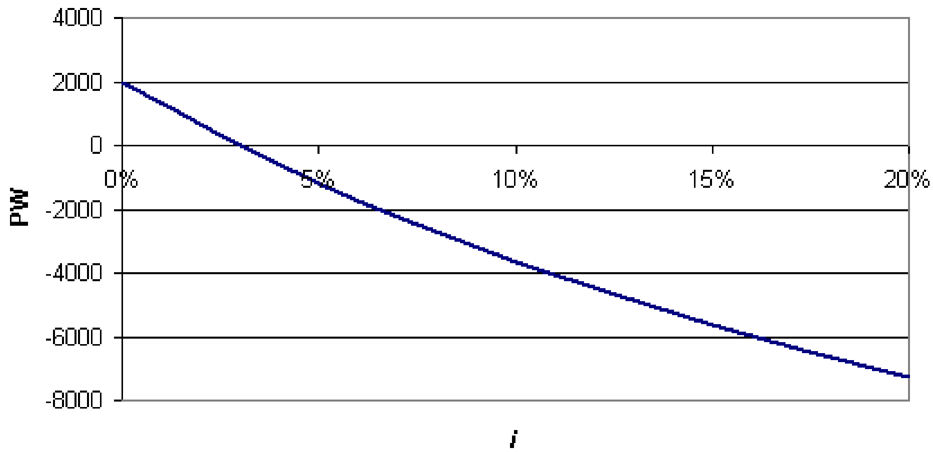


6%	external financing rate	
12%	external investing rate	
9.1%	MIRR	value is less than external investing rate => not attractive

7A-21

Year	Cash flow	i	PW			
0	-16000	0%	1950	=$B$2+NPV(D2,$B$3:$B$7)		
1	-8000	5%	-1158			
2	11000	10%	-3639			
3	13000	15%	-5644			
4	-7000	20%	-7284		3.00%	IRR
5	8950				unique IRR	

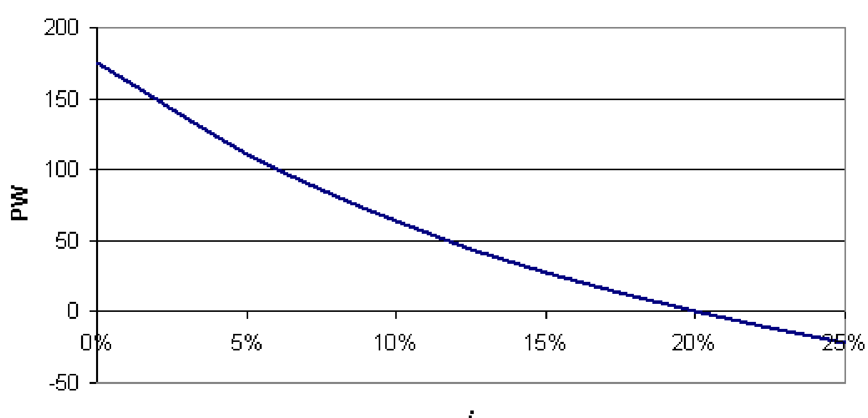
3 sign changes => 3 roots possible



7A-22

Year	Cash flow	i	PW			
0	-200	0%	176	=$B$2+NPV(D2,$B$3:$B$10)		
1	100	5%	111			
2	100	10%	63			
3	100	15%	27			
4	-300	20%	0		20.00%	IRR
5	100	25%	-21		unique IRR	
6	200					
7	200					
8	-124.5					

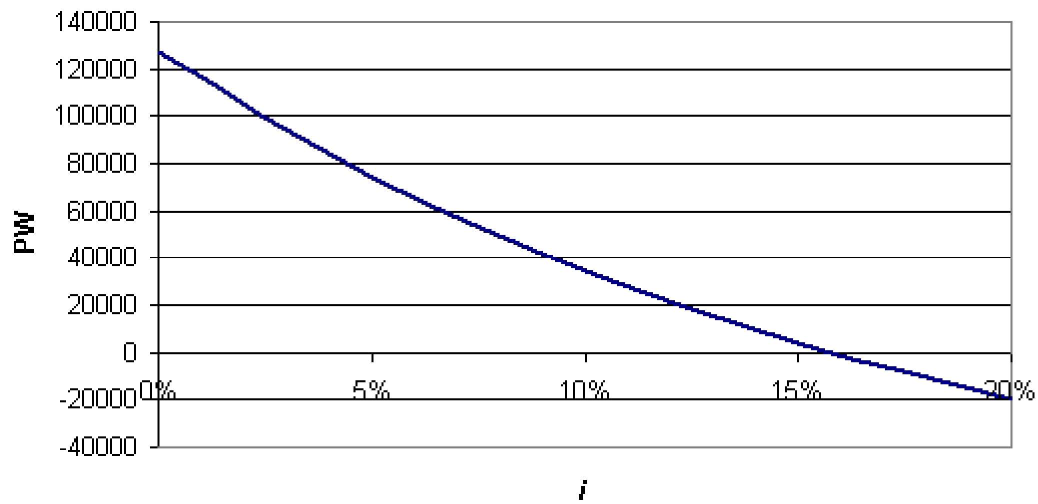
4 sign changes => 4 roots possible



7A-23

Year	Cash flow		i	PW		
0	-210000		0%	127000	=$B$2+NPV(D2,$B$3:$B$9)	
1	88000		5%	74284		
2	68000		10%	34635		
3	62000		15%	4110		
4	-31000		20%	-19899	15.78%	IRR
5	30000				unique IRR	
6	55000					
7	65000					

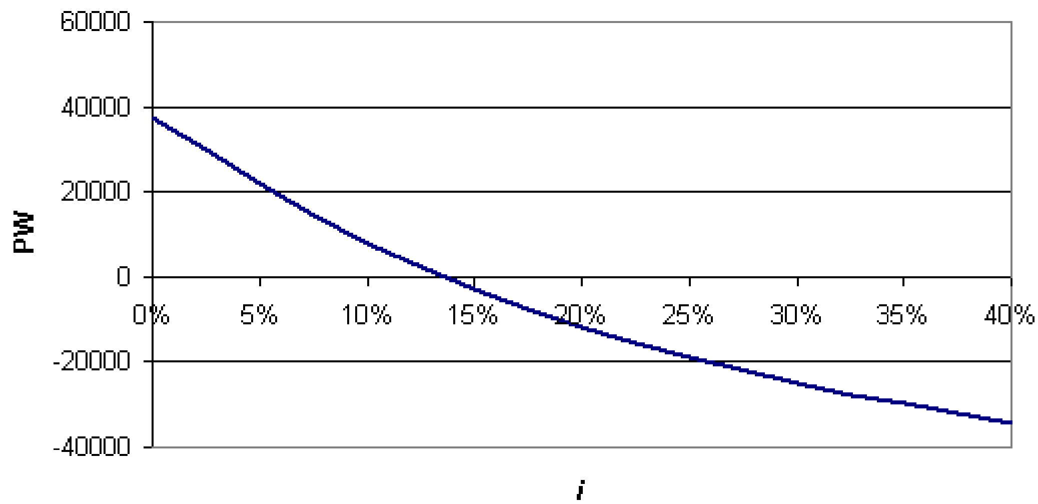
3 sign changes => 3 roots possible



7A-24

Year	Cash flow	i	PW		
0	-103000	0%	37400	=$B$2+NPV(D2,$B$3:$B$7)	
1	102700	10%	7699		
2	-87000	20%	-11676		
3	94500	30%	-25003		
4	-8300	40%	-34594	13.51%	IRR
5	38500			unique IRR	

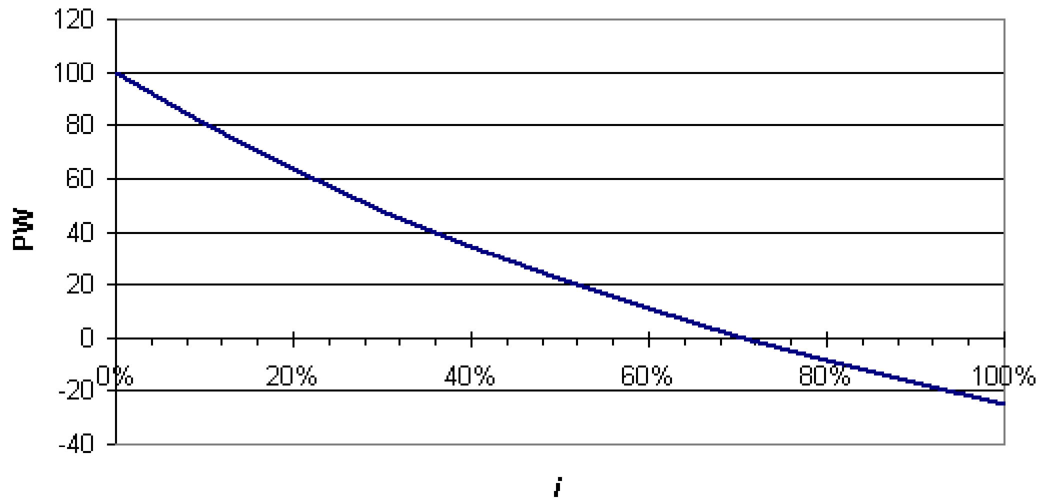
5 sign changes => 3 roots possible



7A-25

Year	Cash flow		i	PW	
0	-200		0%	100	=$B$2+NPV(D2,$B$3:$B$4)
1	400		20%	64	
2	-100		40%	35	
			60%	11	
			80%	-9	70.71% IRR
			100%	-25	unique IRR

2 sign changes => 2 roots possible

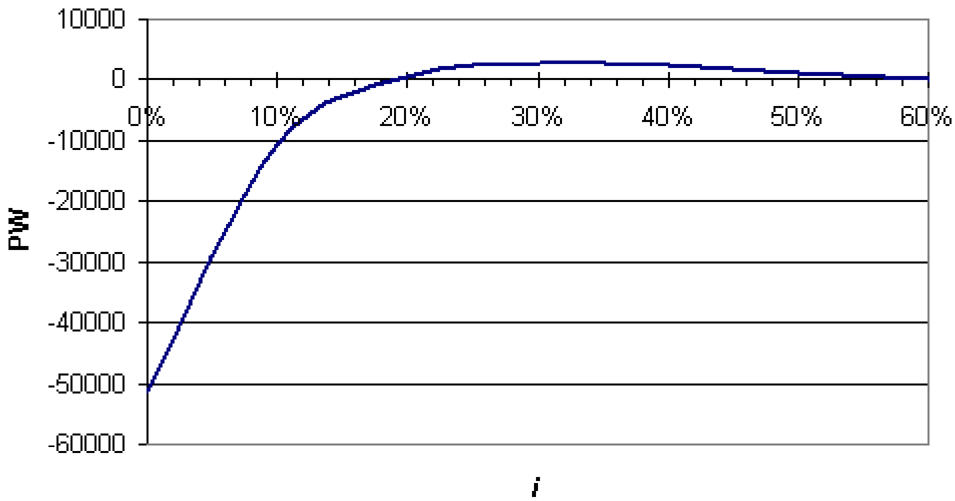


6%	external financing rate	
12%	external investing rate	
24.5%	MIRR	value is more than external investing rate => attractive

7A-26

5	6648	0	6,648	50%	1193	60.0%	root
6	6648	0	6,648	60%	-3		
7	6648	0	6,648	70%	-1086		
8	6648	0	6,648				
9	6648	0	6,648				
10	36648	138,000	- 101,352				
IRR	8.0%	11.0%	19.2%				

2 sign changes => 2 roots possible

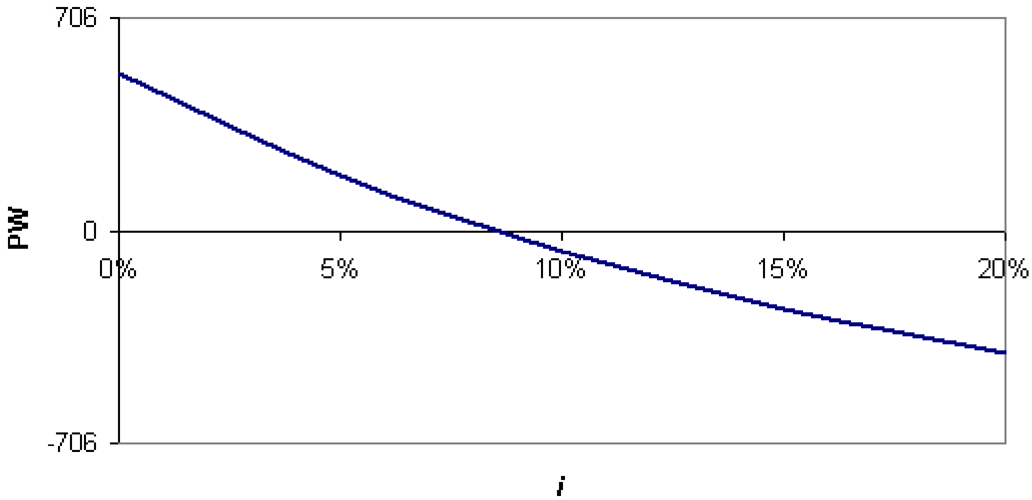


6%	external financing rate
12%	external investing rate
5.1%	MIRR for A-B — value is less than external investing rate => not attractive

7A-27

Year	Cash flow		i	PW	
0	-1000		0%	520	=$B$2+NPV(D2,$B$3:$B$7)
1	60		5%	181	
2	60		10%	-71	
3	-340		15%	-261	
4	0		20%	-406	8.44% IRR
5	1740				unique IRR

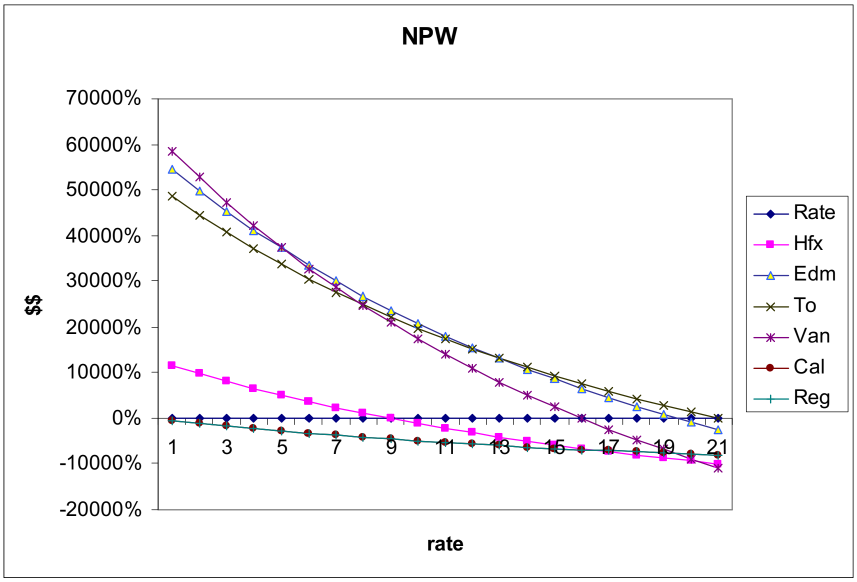
3 sign changes => 3 roots possible



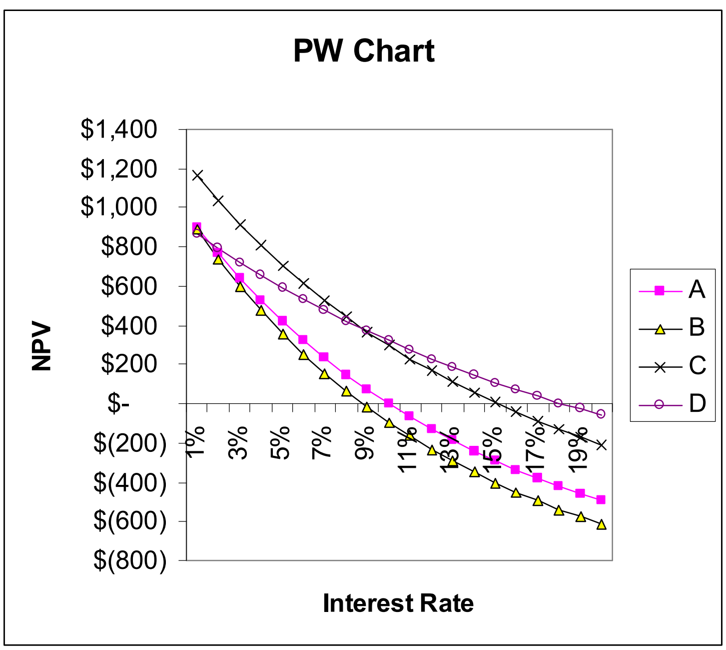
Chapter 8: Incremental Analysis

8-1

At 10%, select Edmonton.

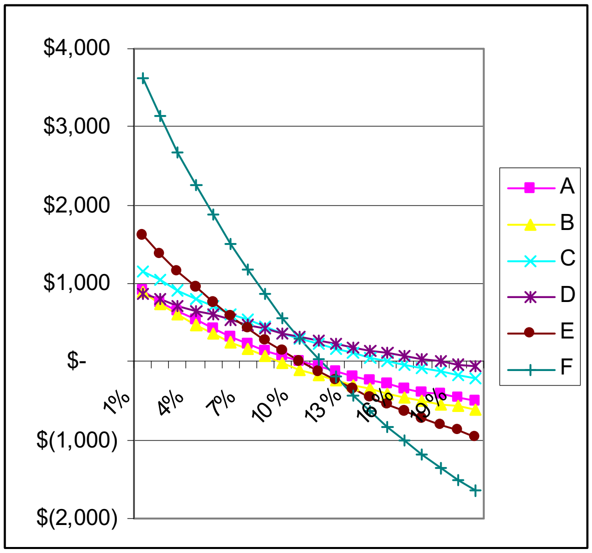


8-2



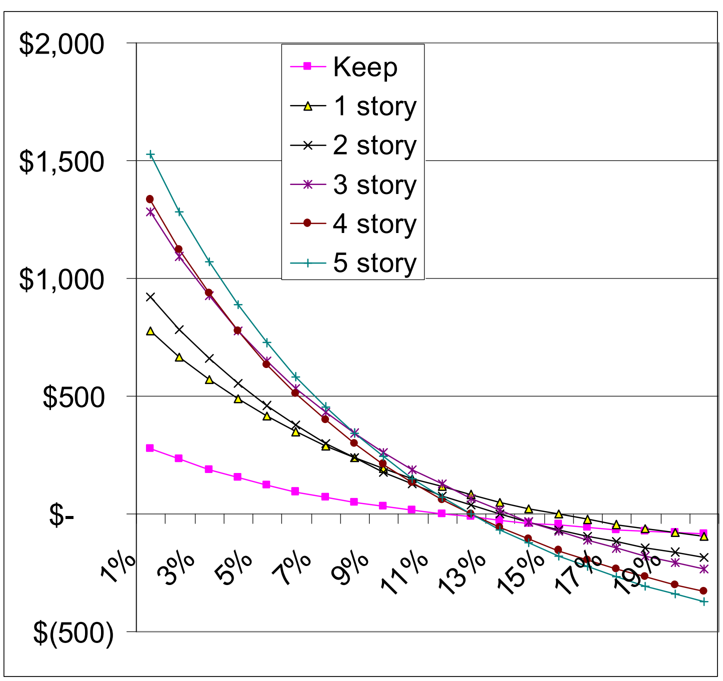
If		MARR >	18%	Choose	0
If	18%	>MARR>	9%	Choose	D
If	9%	>MARR>	0%	Choose	C

8-3



If		MARR >	18%	Choose	0
If	18%	>MARR>	10%	Choose	D
If	10%	>MARR>	0%	Choose	F

8-4



If		MARR >	15%	Choose	0
If	15%	>MARR>	12%	Choose	1 story
If	12%	>MARR>	8%	Choose	3 story
If	8%	>MARR>	0%	Choose	5 story

8-5

Plan	Cost of Improvements and Land	Net Annual Income	Salvage Value	Computed Rate of Return	Decision
A	$145,000	$23,300	$70,000	15%	Accept
B	$300,000	$44,300	$70,000	12.9%	Accept
C	$100,000	$10,000	$70,000	9%	Reject - fails to meet the 10% criterion
D	$200,000	$27,500	$70,000	12%	Accept

Rank the three remaining projects in order of cost and examine each separable increment of investment.

Plan D rather than Plan A

Δ Investment	Δ Annual Income	Δ Salvage Value
$55,000	$4,200	$0

$55,000 = $4,200 (P/A, *i*%, 15)

(P/A, i%, 15) = $55,000/$4,200 = 13.1

From interest tables: i = 1.75%

This is an unacceptable increment of investment. Reject D and retain A.

Plan B rather than Plan A

Δ Investment	Δ Annual Income	Δ Salvage Value
$155,000	$21,000	$0

$155,000 = $21,000 (P/A, i%, 15)
(P/A, i%, 15) = $155,000/$21,000 = 7.38

From interest tables: i = 10.5%

This is a desirable increment of investment. Reject A and accept B.

Conclusion: Select Plan B.

8-6

Year Plan A Cash Plan B Cash Plan B Plan C Cash Plan C

	Flow	Flow	Rather than Plan A	flow	rather than Plan B
0	-$10,000	-$15,000	-$5,000	-$20,000	-$5,0000
1-10	+$1,625	$1,625	$0	+$1,890	+$265
10	-$10,000	$0	+$10,000	$0	$0
11-20	+$1,625	+$1,625	$0	+$1,890	+$265
Rate of Return	10%*	8.8%	7.2%**	7%	0.6%***

*The computation may be made for a 10-year period:

$10,000 = $1,625 (P/A, i%, 10)   i = 10%

The second 10-year period has the same return.

**The computation is:

$5,000 = $10,000 (P/F, i%, 10)

(P/F, i%, 10) = $5,000/$10,000 = 0.5   i = 7.2%

***The computation is:

$5,000 = $265 (P/A, i%, 20) i = 0.6%

The table above shows two different sets of computations.

1. The rate of return for each Plan is computed.

Plan	Rate of Return
A	10%
B	8.8%
C	7%

2. Two incremental analyses are performed.

Increment	Rate of Return	
Plan B – Plan A	7.2%	A desirable increment. Reject Plan A. Retain Plan B.
Plan C – Plan B	0.6%	An undesirable increment. Reject Plan C. Retain Plan B.

Conclusion: Select Plan B.

8-7

Looking at Alternatives B & C it is apparent that B dominates C. Since at the same cost B produces a greater annual benefit, it will always be preferred over C. C may, therefore, be immediately discarded.

Alternative	Cost	Annual Benefit	Δ Cost	Δ Annual Benefit	Δ Rate of Return	Conclusion
B	$50	$12				
A	$75	$16	$25	$4	9.6%	This is greater than the 8% MARR.

D	$85	$17	$10	$1	0%	Retain A. < 8% MARR. Retain A.

Conclusion: Select Alternative A.

8-8

Like all situations where neither input nor output is fixed, the key to the solution is incremental rate of return analysis.

Alternative:	A	B	C
Cost	$200	$300	$600
Annual Benefit	$59.7	$77.1	$165.2
Useful Life	5 yr	5 yr	5 yr
Rate of Return	15%	9%	11.7%

	B – A	C – B	C – A
Δ Cost	$100	$300	$400
Δ Annual Benefit	$17.4	$88.1	$105.5
Δ Rate of Return	< 0%	14.3%	10%

Knowing the six rates of return above, we can determine the preferred alternative for the various levels of MARR.

MARR	Test: Alternative Rate of Return	Test: Examination of separable increments
0% < MARR < 9%	Reject no alternatives.	B – A increment unsatisfactory C- A increment satisfactory Choose C.
9% < MARR < 10%	Reject B.	C- A increment satisfactory. Choose C.
10% < MARR < 11.7%	Reject B.	C- A increment unsatisfactory. Choose A.
11.7% < MARR < 15%	Reject B & C	Choose A.

Therefore, Alternative C preferred when 0% < MARR < 10%.

8-9

Incremental Rate of Return Solution

	A	B	C	D	C - D	B – C	A- C
Cost	$1,000	$800	$600	$500	$100	$200	$400
Uniform Annual Benefit	$122	$120	$97	$122	-$25	$23	$25
Salvage Value	$750	$500	$500	$0	$500	$0	$250

Compute Incremental Rate of Return		10%	< 0%	1.8%

The C- D increment is desirable. Reject D and retain C.
The B- C increment is undesirable. Reject B and retain C.
The A- C increment is undesirable. Reject A and retain C.

Conclusion: Select alternative C.

Net Present Worth Solution

Net Present Worth = Uniform Annual Benefit (P/A, 8%, 8)
+ Salvage Value (P/F, 8%, 8) – First Cost

NPW_A = \$122 (5.747) + \$750 (0.5403) - \$1,000 = +\$106.36
NPW_B = \$120 (5.747) + \$500 (0.5403) - \$800 = +\$159.79
NPW_C = \$97 (5.747) + \$500 (0.5403) - \$600 = +\$227.61
NPW_D = \$122 (5.747) - \$500 = +\$201.13

8-10

Year	A	B	B- A	C	C-B
0	-\$1,000	-\$2,000	-\$1,000	-\$3,000	-\$1,000
1	+\$150	+\$150	\$0	\$0	-\$150
2	+\$150	+\$150	\$0	\$0	-\$150
3	+\$150	+\$150	\$0	\$0	-\$150
4	+\$150	+\$150	\$0	\$0	-\$150
5	+\$150 +\$1,000	+\$150	-\$1,000	\$0	-\$150
6		+\$150 +\$2,700	+\$2,850	\$0	-\$2,850
7				\$5,600	+\$5,600
Computed Incremental Rate of Return			9.8%		6.7%

The B – A incremental rate of return of 9.8% indicates a desirable increment of investment. Alternative B is preferred over Alternative A.

The C- B incremental rate of return of 6.7% is less than the desired 8% rate. Reject C.

Select Alternative B.

Check solution by NPW

NPW_A = \$150 (P/A, 8%, 5) + \$1,000 (P/F, 8%, 5) -\$1,000 = +\$279.55

NPW_B = \$150 (P/A, 8%, 6) + \$2,700 (P/F, 8%, 6) - \$2,000 = +\$397.99**
NPW_C = \$5,600 (P/F, 8%, 7) - \$3,000 = +\$267.60

8-11

Compute rates of return

Alternative X
$100 = \$31.5 (P/A, i%, 4)
(P/A, i%, 4) = \$100/\$31.5 = 3.17
ROR_X = 9.9%

Alternative Y
$50 = \$16.5 (P/A, i%, 4)
(P/A, i%, 4) = \$50/\$16.5 = 3.03
ROR_Y = 12.1%

Incremental Analysis

Year	X- Y
0	-$50
1-4	+$15

$50 = \$15 (P/A, i%, 4)
$\Delta ROR_{X\text{-}Y}$ = 7.7%

(a) At MARR = 6%, the X- Y increment is desirable. Select X.
(b) At MARR = 9%, the X- Y increment is undesirable. Select Y.
(c) At MARR = 10%, reject Alt. X as ROR_x < MARR. Select Y.
(d) At MARR = 14%, both alternatives have ROR < MARR. Do nothing.

8-12

Compute Rates of Return

Alternative A:
$100 = \$30 (P/A, i%, 5)
(P/A, i%, 5) = \$100/\$30 = 3.33
ROR_A = 15.2%

Alternative B:
$150 = \$43 (P/A, i%, 5)
(P/A, i%, 5) = \$150/\$43 = 3.49
ROR_A = 13.3%

Incremental Analysis

Year	B- A
0	-$50
1-5	+$13

$50 = \$13 (P/A, i%, 5)
$\Delta ROR_{B\text{-}A}$ = 9.4%

(a) At MARR = 6%, the B- A increment is desirable. Select B.
(b) At MARR = 8%, the B- A increment is desirable. Select B.
(c) At MARR = 10%, the B- A increment is undesirable. Select A.

8-13

Year	A	B	A- B
0	-$10,700	-$5,500	-$5,200
1-4	+$2,100	$1,800	+$300
4		-$5,500	+$5,500
5-8	+$2,100	+$1,800	+$300
Computed ROR	11.3%	11.7%	10.8%

Since ΔROR_{A-B} > MARR, the increment is desirable. Select A.

8-14

Year	A	B	C	B- C
0	-$300	-$600	-$200	-$400
1- 10	$41	$98	$35	$63
Computed ROR	6.1%	10.1%	11.7%	9.2%
Decision	ROR_A < MARR- reject.	Ok	Ok	$ROR_{\Delta B-C}$ > MARR. Select B.

8-15

Year	X	Y	Y- X
0	-$10	-$20	-$10
1	$15	$28	+$13
Computed ROR	50%	40%	30%

$ROR_{\Delta Y-X}$ = 30%, therefore Y is preferred for all values of MARR < 30%.
0 < MARR < 30%

8-16

Since B has a higher initial cost and higher rate of return, it dominates A with the result that there is no interest rate at which A is the preferred alternative. Assuming this is not recognized, one would first compute the rate of return on the increment B- A and then C- B. The problem has been worked out to make the computations relatively easy.

Year	A	B	B- A
0	-$770	-$1,406.3	-$636.30
1	+$420	+$420	$0
2	+$420	+$420	$0
	-$770	$0	+$770
3	+$420	+$420	$0
4	+$420	+$420	$0

Cash flows repeat for the next four years.

Rate of Return on B- A: $636.30 = $770 (P/F, i%, 2)
ΔROR_{B-A} = 10%

Year	B	C	C- B
0	-$1,406.3	-$2,563.3	-$1,157.0
1- 3	+$420	+$420	$0
4	+$420	+$420	$0
	-$1,406.3	$0	+$1,406.30
5-8	+$420	+$420	$0

Rate of Return on B- A: $1,157.00 = $1,406.30 (P/F, i%, 4)
ΔROR_{C-B} = 5%

Summary of Rates of Return

A	B-A	B	C-B	C	D
6.0%	10%	7.5%	5%	6.4%	0%

Value of MARR

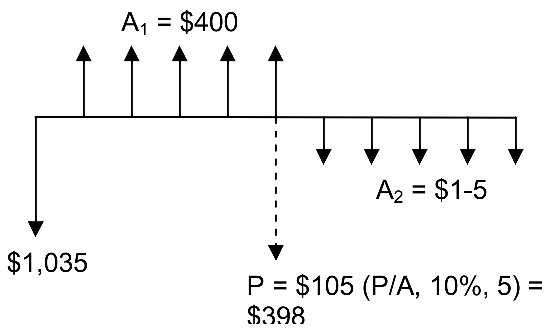
Value of MARR	Decision
0% - 5%	C is preferred
5% - 7.5%	B is preferred
> 7.5%	D is preferred

Therefore, B is preferred for values of MARR from 5% to 7.5%.

8-17

	A	B	A - B	C	C - B
Cost	-$1,500	-$1,000	-$500	-$2,035	-$1,035
Annual Benefit, first 5 years	+$250	+$250	$0	+$650	+$400
Annual Benefit, next 5 years	+$450	+$250	+$200	+$145	-$105
Computed rate of return	16.3%	21.4%	ΔROR = 9.3%	21.6%	Two sign changes in C – B cash flow. Transform.
Decision			Reject A. Keep B.		

C – B



Transformed Cash Flow

Year	C - B
0	-\$1,035
1-4	+\$400
5	+\$2

-\$1,035 = \$400 (P/A, i%, 4) + \$2 (P/F, i%, 5)

$\Delta ROR_{C\text{-}B}$ = 20%

This is a desirable increment. Select C.

8-18

Monthly payment on new warehouse loan = \$350,000 (A/P, 1.25%, 60)
= \$8,330

Month	Alt. 1	Alt. 2	1-2	Alt. 3	1-3
0	-\$100,000	-\$100,000	\$0	\$0	-\$100,000
1- 60	-\$8,330 +\$2,500 -\$1,000	-\$8,300 \$0 \$0	+\$1,500	-\$2,700	-\$4,130
60	+\$600,000	+\$600,000	\$0	\$0	+\$600,000
Decision			By inspection, this increment is desirable. Reject 2. Keep 1.		ΔROR = 1.34%/mo Nominal ROR = (1.34%)12 = 16.1% Effective ROR = $(1 + 0.0134)^{12} - 1$ = 17.3%

Being less desirable than Alternative 1, Alternative 2 may be rejected. The 1- 3 increment does not yield the required 20% MARR, so it is not desirable. Reject 1 and select 3 (continue as is).

8-19

Part One- Identical Replacements, Infinite Analysis Period

NPW_A = (UAB/i) – PW of Cost = \$10/0.08 - \$100 = +\$25.00

NPW_B:
EUAC = \$150 (A/P, 8%, 20) = \$15.29
EUAB = \$17.62 (Given)
NPW_B = (EUAB – EUAC)/i = (\$17.62 - \$15.29)/0.08 = +\$29.13

NPW_C uses same method as Alternative B:
EUAC = \$200 (A/P, 8%, 5) = \$50.10
NPW_C = (EUAB – EUAC)/i = (\$55.48 - \$50.10)/0.08 = +\$67.25

Select C.

Part Two- Replacements provide 8% ROR, Infinite Analysis Period

The replacements have an 8% ROR, so their NPW at 8% is 0.

NPW_A = (UAB/i) – PW of Cost = (\$10/0.08) - \$10 = +\$25.00
NPW_B = PW of Benefits – PW of Cost
= \$17.62 (P/A, 8%, 20) - \$150 + \$0 = +\$22.99
NPW_C = \$55.48 (P/A, 8%, 5) - \$200 + \$0 = +\$21.53

Select A.

8-20

Year	Pump 1	Pump 2	Increment 2- 1
0	-\$100	-\$110	-\$10
1	+\$70	+\$115	+\$45
2	\$70	\$30	-\$40

Transformation: x(1 + 0.10) = \$40
Solve for x: x = \$40/1.1 = \$36.36

Year	Transformed Increment 2 - 1
0	-\$10
1	+\$8.64
2	\$0

This is obviously an undesirable increment as ΔROR < 0%. Select Pump 1.

8-21

Year	A	B	C	A- B	C- B
0	-$20,000	-$20,000	-$20,000	$0	$0
1	$10,000	$10,000	$5,000	$0	-$5,000
2	$5,000	$10,000	$5,000	-$5,000	-$5,000
3	$10,000	$10,000	$5,000	$0	-$5,000
4	$6,000	$0	$15,000	$6,000	$15,000
Computed ROR	21.3%	23.4%	15.0%	9.5%	0%
Decision				Reject A	Reject C

Choose Alternative B.

8-22

	South End	Both Stores	North End
New Store Cost			-$500,000
Annual Profit	$170,000	$260,000	+$90,000
Salvage Value			+$500,000

Where the investment ($500,000) is fully recovered, as is the case here:
Rate of Return = A/P = $90,0000/$500,000 = 0.18 = 18%

Open The North End.

8-23

Year	Neutralization	Precipitation	Neut. – Precip.
0	-$700,000	-$500,000	-$200,000
1- 5	-$40,000	-$110,000	+$70,000
5	+$175,000	+$125,000	+$50,000

Solve (Neut. – Precip.) for rate of return.

$200,000 = $70,000 (P/A, i%, 5) + $50,000 (P/F, i%, 5)

Try i = 25%, $200,000 = $70,000 (2.689) + $50,000 (0.3277) = $204,615

Therefore, ROR > 25%. Computed rate of return = 26%

Choose Neutralization.

8-24

Year	Gen. Dev.	RJR	RJR – Gen Dev.
0	-$480	-$630	-$150
1- 15	+$94	+$140	+$46
15	+$1,000	+$1,000	$0

Computed ROR	21.0%	22.8%	30.0%

Neither bond yields the desired 25% MARR- so do nothing.

Note that simply examining the (RJR – Gen Dev) increment might lead one to the wrong conclusion.

8-25

The ROR of each alternative > MARR. Proceed with incremental analysis. Examine increments of investment.

	C	B	B- C
Initial Investment	\$15,000	\$22,000	\$7,000
Annual Income	\$1,643	\$2,077	\$434

\$7,000 = \$434 (P/A, i%, 20)
(P/A, i%, 20) = \$7,000/\$434 = 16.13
ΔROR_{B-C} = 2.1%
Since ΔROR_{B-C} < 7%, reject B.

	C	A	A- C
Initial Investment	\$15,000	\$50,000	\$35,000
Annual Income	\$1,643	\$5,093	\$3,450

\$35,000 = \$3,450 (P/A, i%, 20)
(P/A, i%, 20) = \$35,000/\$3,450 = 10.14
ΔROR_{A-C} = 7.6%
Since ΔROR_{A-C} > 7%, reject C.

Select A.

8-26

Since there are alternatives with ROR > 8% MARR, Alternative 3 may be immediately rejected as well as Alternative 5. Note also that Alternative 2 dominates Alternative 1 since its ROR > ROR Alt. 1. Thus ΔROR_{2-1} > 15%. So Alternative 1 can be rejected. This leaves alternatives 2 and 4. Examine (4-2) increment.

	2	4	4- 2
Initial Investment	\$130.00	\$330.00	\$200.00
Uniform Annual Benefit	\$38.78	\$91.55	\$52.77

\$200 = \$52.77 (P/A, i%, 5)
(P/A, i%, 5) = \$200/\$52.77 = 3.79
ΔROR_{4-2} = 10%

Since ΔROR_{4-2} > 8% MARR, select Alternative 4.

8-27

Check to see if all alternatives have a ROR > MARR.

Alternative A

NPW = $800 (P/A, 6%, 5) + $2,000 (P/F, 6%, 5) - $2,000
= $800 (4.212) + $2,000 (0.7473) - $2,000
= +$2,864 ROR > MARR

Alternative B

NPW = $500 (P/A, 6%, 6) + $1,500 (P/F, 6%, 6) - $5,000
= $500 (4.917) + $1,500 (0.7050) - $5,000
= -$1,484 ROR < MARR- Reject B

Alternative C

NPW = $400 (P/A, 6%, 7) + $1,400 (P/F, 6%, 7) - $4,000
= $400 (5.582) + $1,400 (0.6651) - $4,000
= -$610 ROR < MARR- Reject C

Alternative D

NPW = $1,300 (P/A, 6%, 4) + $3,000 (P/A, 6%, 4) - $3,000
= $1,300 (3.465) + $3,000 (0.7921) - $3,000
= +$3,881 ROR > MARR

So only Alternatives A and D remain.

Year	A	D	D- A
0	-$2,000	-$3,000	-$1,000
1	+$800	+$1,300	+$500
2	+$800	+$1,300	+$500
3	+$800	+$1,300	+$500
4	+$800	+$4,300	+$3,500
5	+$2,800		-$2,800

So the increment (D- A) has a cash flow with two sign changes.

Move the Year 5 disbursement back to Year 4 at MARR = 6%

Year 4 = +$3,500 - $2,800 (P/F, 6%, 1) = +$858

Now compute the incremental ROR on (D- A)

NPW = -$1,000 + $500 (P/A, i%, 3) + $858 (P/F, i%, 4)

Try i = 40%

NPW = -$1,000 + $500 (1.589) + $858 (0.2603) = +$18

So the ΔROR on (D- A) is slightly greater than 40%. Choose Alternative D.

8-28

Using Equivalent Uniform Annual Cost-

$EUAC_{Th}$ = -$5 - $20 (A/P, 12%, 3) = -$5 - $20 (0.4163) = -$13.33
$EUAC_{SL}$ = -$2 - $40 (A/P, 12%, 5) = -$2 - $40 (0.2774) = -$13.10

Fred should choose slate over thatch to save $0.23/yr.

To find incremental ROR, find *i* such that $EUAC_{SL} – EUAC_{TH} = 0$.

$0 = -$2 - $40 (A/P, i*, 5) – [-$5 - $20 (A/P, i*, 3)]
= $3 - $40 (A/P, i*, 5) + $20 (A/P, i*, 3)

At i = 12%
$3 - $40 (0.2774) + $20 (0.4163) = $0.23 > $0 12% too low

At i = 15%
$3 - $40 (0.2983) + $20 (0.4380) = -$0.172 < $015% too high
Using linear interpolation:
ΔROR = 12 + 3[0.23/(0.23 – (-0.172))] = 13.72%

8-29

(a) For the Atlas mower, the cash flow table is:

Year	Net Cash Flow (Atlas
0	-$6,700
1	$2,500
2	$2,500
3	$3,500

NPW = -$6,700 +$2,500 (P/A, i*, 2) + $3,500 (P/F, i*, 3) = $0
To solve for i*, construct a table as follows:

i	NPW
12%	+$16
i*	$0
15%	-$334

Use linear interpolation to determine ROR
ROR = 12% + 3% ($16 - $0)/($16 + $334) = 12.1%

(b) For the Zippy mower, the cash flow table is:

Year	Net Cash Flow (Zippy)
0	-$16,900
1-5	$3,300
6	$6,800

NPW = -$16,900 + $3,300 (P/A, i%, 5) + $6,800 (P/F, i%, 6)

At MARR = 8% NPW = -$16,900 + $3,300 (3.993) + $6,800 (0.6302)
= +$562

Since NPW is positive at 8%, the ROR > MARR.

(c) The incremental cash flow is:

Year	Net Cash Flow (Zippy)	Net Cash Flow (Atlas)	Difference (Zippy – Atlas)
0	-$16,900	-$6,700	-$10,200
1	$3,300	$2,500	+$800
2	$3,300	$2,500	+$800
3	$3,300	$2,500 - $6,700	+$6,500
4	$3,300	$2,500	+$800
5	$3,300	$2,500	+$800
6	$6,800	$3,500	+$3,300

NPW = -$10,200 +$800(P/A, i*, 5) + $5,700(P/F, i*, 3) + $3,300(P/F, i*, 6)

Compute the ΔROR

Try i = 6% NPW = -$10,200 +$800(4.212) + $5,700(0.8396) + $3,300(0.7050)
= +$282

Try i = 7% NPW = -$10,200 +$800(4.100) + $5,700(0.8163) + $3,300(0.6663)
= -$68

Using linear interpolation:
ΔROR = 6% + 1% ($282- $0)/($282 + $68) = 6.8%

The ΔROR < MARR, so choose the lower cost alternative, the Atlas.

8-30

(1) Arrange the alternatives in ascending order of investment.

	Company A	Company C	Company B
First Cost	$15,000	$20,000	$25,000

(2) Compute the rate of return for the least cost alternative (Company A) or at least insure that the ROR_A > MARR. At i = 15%:

NPW_A = -$15,000+($8,000 - $1,600)(P/A, 15%, 4) + $3,000(P/F, 15%, 4)
= -$15,000 + $6,400 (2.855) + $3,000 (0.5718) = $4,987

Since NPW_A at i = 15% is positive, ROR_A > 15%.

(3) Consider the increment (Company C – Company A)

	C- A
First Cost	$5,000
Maintenance & Operating Costs	-$700
Annual Benefit	$3,000
Salvage Value	$1,500

Determine whether the rate of return for the increment (C- A) is more or less than the 15% MARR. At i = 15%:

$NPW_{C\text{-}A}$ = -$5,000 +[$3,000 – (-$700)](P/A, 15%, 4) + $1,500(P/F, 15%, 4)
= -$5,000 + $3,700 (2.855) + $1,500(0.5718) = $6,421
Since $NPW_{C\text{-}A}$ is positive at MARR%, it is desirable. Reject Company A.

(4) Consider the increment (Company B – Company C)

	B- C
First Cost	$5,000
Maintenance & Operating Costs	-$500
Annual Benefit	$4,000
Salvage Value	$1,500

Determine whether the rate of return for the increment (B- C) is more or less than the 15% MARR. At i = 15%:

$NPW_{B\text{-}C}$ = -$5,000 +[$4,000 – (-$500)](P/A, 15%, 4) + $1,500(P/F, 15%, 4)
= -$5,000 + $4,500 (2.855) + $1,500(0.5718) = $6,421
Since $NPW_{C\text{-}A}$ is positive at MARR%, it is desirable. Reject Company A.

8-31

MARR = 10% n = 10 RANKING: 0 < Economy < Regular < Deluxe

Δ Economy – 0

NPW = -$75,000 + ($28,000 - $8,000) (P/A, i*, 10) + $3,000 (P/F, i*, 10)

i*	NPW
0	$128,000
0.15	$26,120
∞	-$75,000

i* > MARR so Economy is better than doing nothing.

Δ (Regular – Economy)

NPW = -($125,000 - $75,000) + [($43,000 - $28,000)
– ($13,000 - $8,000)](P/A, i*, 10) + ($6,900 - $3,000) (P/F, i*, 10)

i*	NPW
0	$53,900
0.15	$1,154
∞	-$50,000

i* > MARR so Regular is better than Economy.

Δ (Deluxe - Regular)

NPW = -($220,000 - $125,000) + [($79,000 - $43,000)
– ($38,000 - $13,000)](P/A, i*, 10) + ($16,000 - $6,900) (P/F, i*, 10)

i*	NPW
0	$24,100
0.15	-$37,540
∞	-$95,000

i* < MARR so Deluxe is less desirable than Regular.

The correct choice is the Regular model.

8-32

Put the four alternatives in order of increasing cost:
Do nothing < U-Sort-M < Ship-R < Sort-Of

U-Sort-M – Do Nothing

First Cost	$180,000
Annual Benefit	$68,000
Maintenance & Operating Costs	$12,000
Salvage Value	$14,400

$NPW_{15\%}$ = -$180,000 + ($68,000 - $12,000)(P/A, 15%, 7)
+ $14,400(P/F, 15%, 7)
= -$180,000 + $232,960 + $5,413
= $58,373

ROR > MARR- Reject Do Nothing

Ship-R – U-Sort-M

First Cost	$4,000
Annual Benefit	$23,900
Maintenance & Operating Costs	$7,300
Salvage Value	$9,000

$NPW_{15\%}$ = -$4,000 + ($23,900 - $7,300)(P/A, 15%, 7) + $9,000(P/F, 15%, 7)
= -$4,000 + $69,056 + $3,383
= $68,439

ROR > MARR- Reject U-Sort-M

Sort-Of – Ship-R

First Cost	$51,000
Annual Benefit	$13,700
Maintenance & Operating Costs	$0
Salvage Value	$5,700

$$NPW_{15\%} = -\$51,000 + (\$13,700)(P/A, 15\%, 7) + \$5,700(P/F, 15\%, 7)$$
$$= -\$51,000 + \$56,992 + \$2,143$$
$$= \$8,135$$

ROR > MARR- Reject Ship-R, Select Sort-of

8-33

Since the firm requires a 20% profit on each increment of investment, one should examine the B- A increment of $200,000. (With only a 16% profit rate, C is unacceptable.)

Alt.	Initial Cost	Annual Profit	Δ Cost	Δ Profit	Δ Profit Rate
A	$100,000	$30,000			
B	$300,000	$66,000	$200,000	$36,000	18%
C	$500,000	$80,000			

Alternative A produces a 30% profit rate. The $200,000 increment of investment of B rather than A, that is, B- A, has an 18% profit rate and is not acceptable. Looked at this way, Alternative B with an overall 22% profit rate may be considered as made up of Alternative A plus the B- A increment. Since the B-A increment is not acceptable, Alternative B should not be adopted.

Thus the best investment of $300,000, for example, would be Alternative A (annual profit = $30,000) plus $200,000 elsewhere (yielding 20% or $40,000 annually). This combination yields a $70,000 profit, which is better than Alternative B profit of $66,000. Select A.

8-34

Assume there is $30,000 available for investment. Compute the amount of annual income for each alternative situation.

	A	Elsewhere	B	Elsewhere	C	Elsewhere
Investment	$10,000	$20,000	$18,000	$12,000	$25,000	$5,000
Net Annual Income	$2,000	$3,000	$3,000	$1,800	$4,500	$750
Sum		= $5,000		= $4,800		= $5,250

Elsewhere is the investment of money in other projects at a 15% return. Thus if $10,000 is invested in A, then $20,000 is available for other projects.

In addition to the three alternatives above, there is Alternative D where the $30,000 investment yields a $5,000 annual income.

To maximize annual income, choose C.

An alternate solution may be obtained by examining each separable increment of investment.

Incremental Analysis

	A	B	B- A	A	C	C- A
Investment	$10,000	$18,000	$8,000	$10,000	$25,000	$15,000

Net Annual Income	$2,000	$3,000	$1,000	$2,000	$4,500	$2,500
Return on Increment of Investment			13% undesirable Reject B			17% OK Reject A

	C	D	D- C
Investment	$10,000	$20,000	$18,000
Net Annual Income	$2,000	$3,000	$3,000
Return on Increment of Investment			10% undesirable Reject D

Conclusion: Select Alternative C.

8-35

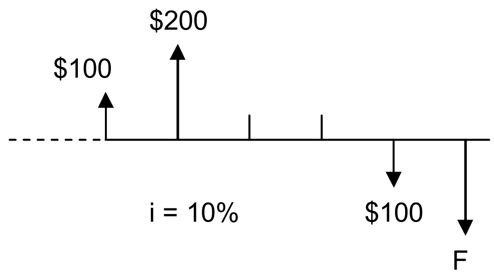
	A	B	C	D	E	F
1	interest rate	12.00%				
2						
3		Thatch	Slate			
4	First Cost	$ 20.00	$ 40.00			
5	Annual O&M	$ 5.00	$ 2.00			
6	Life (years)	3	5			
7	EAC	$ 13.33	$ 13.10	=PMT($B$1,C6,-C4)+C5		
8	EAC difference	$ 0.23				
9						
10	For the goal seek, cell B8 is set equal to 0 and cell B1 is changed.					
11	When B1=13.73%, both EACs equal $13.58, and the difference = 0.					
12						

8-36

	A	B	C	D	E	F
1	interest rate	8.00%				
2						
3		Atlas	Zippy			
4	First Cost	$6,700	$16,900			
5	Annual O&M	1500	1200			
6	Annual Benefit	4000	4500			
7	Salvage	1000	3500			
8	Life (years)	3	6			
9	IRR	12.1%	9.0%			
10	EAW	$ 208.21	$ 121.37	=-PMT($B$1,C8,-C4,C7)-C5+C6		
11	EAW differenc	$ 86.84				
12						
13	For the goal seek, cell B11 is set equal to 0 and cell B1 is changed.					
14	When B1=6.80%, both EAWs equal $267.85, and the difference = 0.					
15	So incremental IRR = 6.80%					
16						

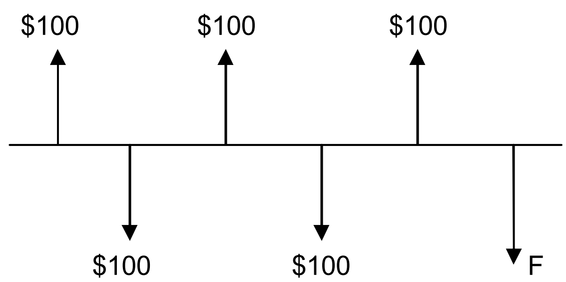
Chapter 9: Other Analysis Techniques

9-1



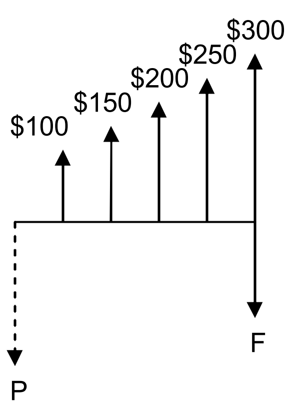
F = \$100 (F/P, 12%, 5) + \$200 (F/P, 12%, 4) - \$100 (F/P, 12%, 1)
= \$100 (1.762) + \$200 (1.574) - \$100 (1.120)
= 379.00

9-2



F = \$100 (F/P, 10%, 5) + \$100 (F/P, 10%, 3) + \$100 (F/P, 10%, 1)
- \$100 (F/P, 10%, 4) - \$100 (F/P, 10%, 2)
= \$100(1.611 + 1.331 + 1.100 – 1.464 – 1.210)
= \$136.80

9-3



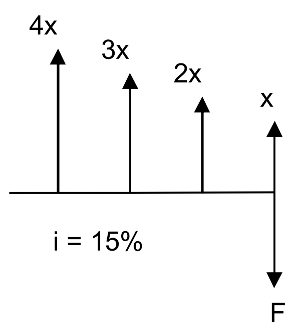
P = \$100 (P/A, 12%, 5) + \$50 (P/G, 12%, 5)
= \$100 (3.605) + \$50 (6.397)
= \$680.35

F = \$680.35 (F/P, 12%, 5)
= \$680.35 (1.762)
= \$1,198.78

Alternate Solution

F = [\$100 + \$50 (A/G, 12%, 5)] (F/A, 12%, 5)
= [\$100 + \$50 (1.775)] (6.353)
= 1,199.13

9-4

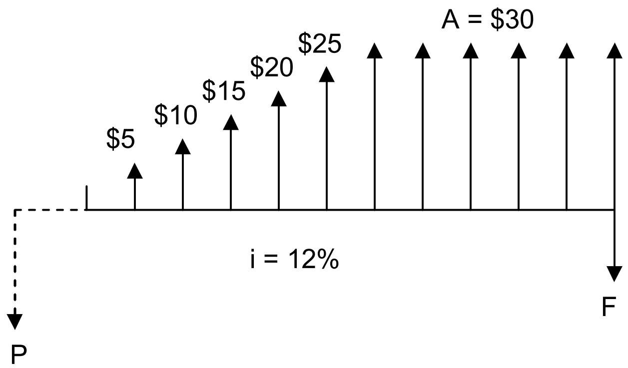


F = [4x – x(A/G, 15%, 4)] (F/A, 15%, 4)
= [4x – x(1.326)] (4.993)
= 13.35x

Alternate Solution

F = 4x (F/P, 15%, 3) + 3x (F/P, 15%, 2) + 2x (F/P, 15%, 1) + x
= 4x (1.521) + 3x (1.323) + 2x (1.150) + x
= 13.35x

9-5



$$\begin{aligned} F &= \$5\ (P/G,\ 10\%,\ 6)\ (F/P,\ 10\%,\ 12) + \$30\ (F/A,\ 10\%,\ 6) \\ &= \$5\ (6.684)\ (3.138) + \$30\ (7.716) \\ &= 383.42 \end{aligned}$$

9-6

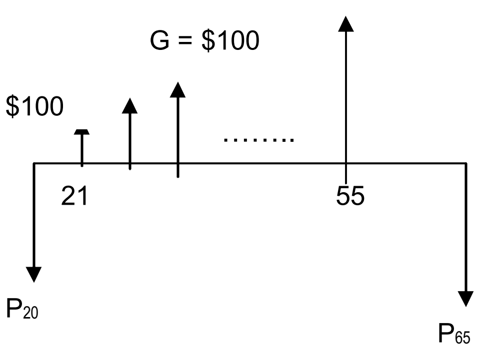
$P_{\text{system 1}}$ = A (P/A, 12%, 10) = \$15,000 (5.650) = \$84,750
$P_{\text{system 2}}$ = G (P/G, 12%, 10) = \$1,200 (20.254) = \$24,305
P_{Total} = \$84,750 + \$24,305 = \$109,055
F_{Total} = P_{Total} (F/P, 12%, 10)= \$109,055 (3.106) = \$338,725

9-7

$$\begin{aligned} i_a &= (1 + r/m)^m - 1 \\ &= (1 + 0.10/48)^{48} - 1 \\ &= 0.17320 \end{aligned}$$

$$F = P\ (1 + i_a)^5 \quad = \$50{,}000\ (1 + 0.17320)^5 \quad = \$111{,}130$$

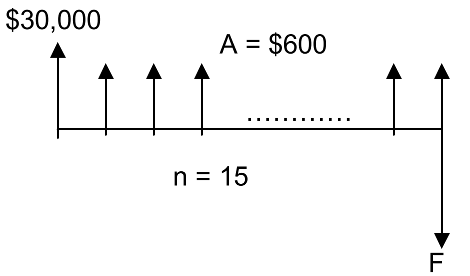
9-8



$$
\begin{aligned}
P_{20} &= \$100\,(P/A, 12\%, 35) + \$100\,(P/G, 12\%, 35) \\
&= \$100\,(8.176) + \$100\,(62.605) \\
&= \$7{,}078.10
\end{aligned}
$$

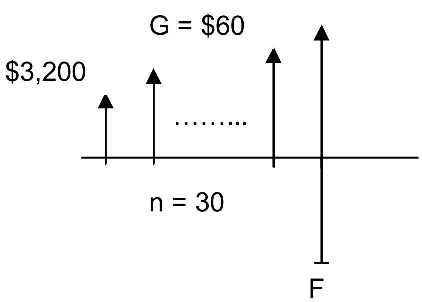
$$P_{65} = P_{20}\,(F/P, 12\%, 45) \qquad = \$7{,}078.10\,(163.988) \qquad = \$1{,}160{,}700$$

9-9



$$
\begin{aligned}
F &= \$30{,}000\,(F/P, 10\%, 15) + \$600\,(F/A, 10\%, 15) \\
&= \$30{,}000\,(4.177) + \$600\,(31.772) \\
&= \$144{,}373
\end{aligned}
$$

9-10

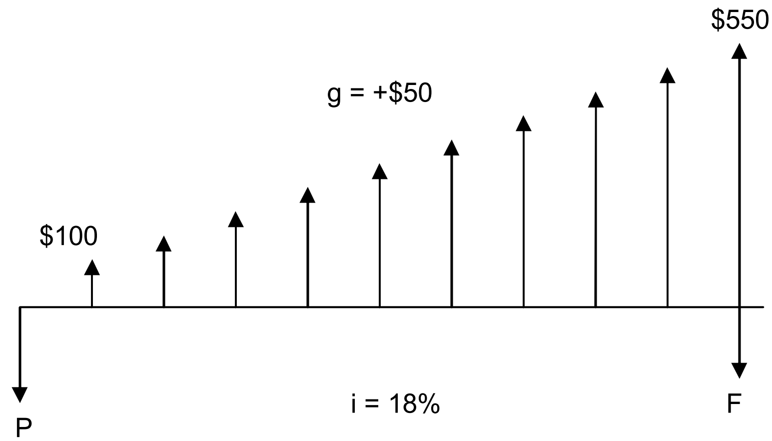


$$
\begin{aligned}
F &= \$3{,}200\,(F/A, 7\%, 30) + \$60\,(P/G, 7\%, 30)\,(F/P, 7\%, 30) \\
&= \$3{,}200\,(94.461) + \$60\,(120.972)\,(7.612) \\
&= \$357{,}526
\end{aligned}
$$

9-11

$$
\begin{aligned}
F &= \$100\,(F/A, \tfrac{1}{2}\%, 24)\,(F/P, \tfrac{1}{2}\%, 60) \\
&= \$100\,(25.432)\,(1.349) \\
&= \$3{,}430.78
\end{aligned}
$$

9-12



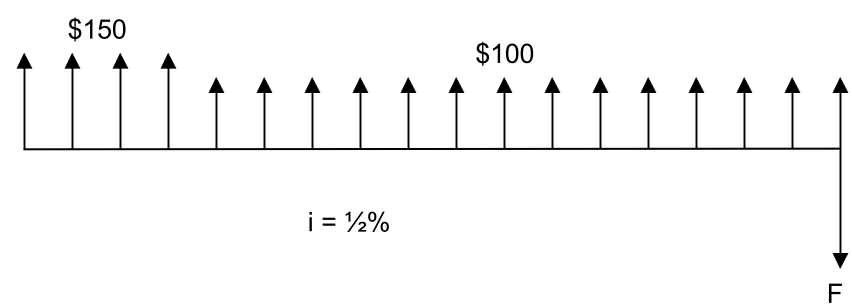
P = \$100 (P/A, 18%, 10) + \$50 (P/G, 18%, 10)
= \$100 (4.494) + \$50 (14.352)
= 1,167.00

F = \$1,167 (F/P, 18%, 10)
= \$1,167 (5.234)
= 6,108.08

9-13

F = £100 $(1 + 0.10)^{800}$ = £1.3 x 10^{35}

9-14



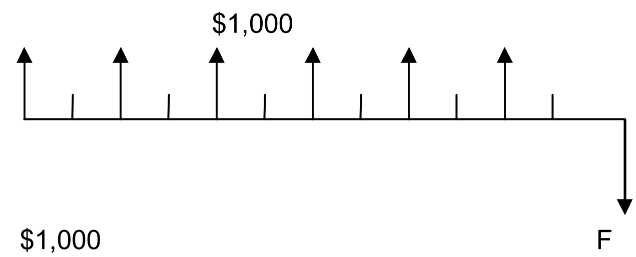
F = \$150 (F/A, ½%, 4) (F/P, ½%, 14) + \$100 (F/A, ½%, 14)
= \$150 (4.030) (1.072) + \$100 (14.464)
= 2,094.42

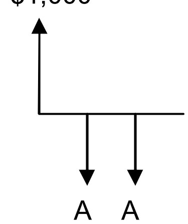
9-15

Using single payment compound amount factors

F = \$1,000 [(F/P, 4%, 12) + (F/P, 4%, 10) + (F/P, 4%, 8) + (F/P, 4%, 6)
+ (F/P, 4%, 4) + (F/P, 4%, 2)]
= \$1,000 [1.601 + 1.480 + 1.369 + 1.265 + 1.170 + 1.082]
= \$7,967

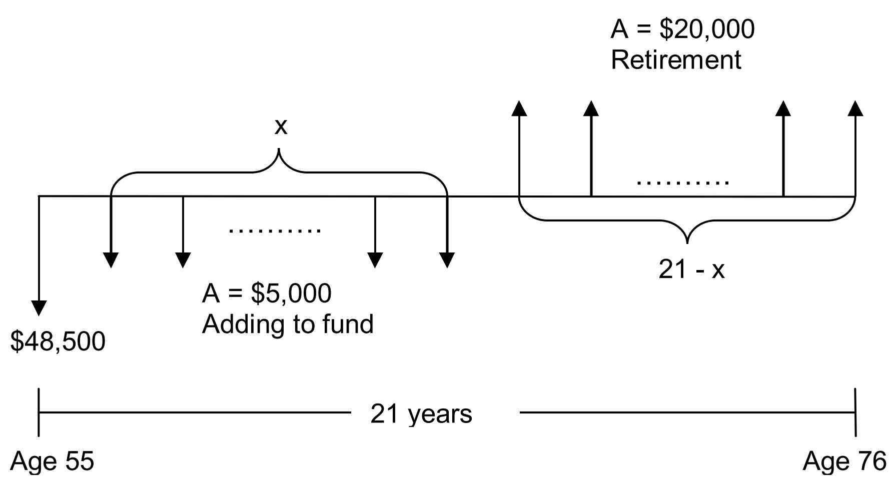
Alternate Solution





A = \$1,000 (A/P, 4%, 2) = \$1,000 (0.5302) = \$530.20
F = \$530.20 (F/A, 4%, 12) = \$530.20 (15.026) = \$7,966.80

9-16



x = years to continue working
age to retire = 55 + x

Amount at Retirement = PW of needed retirement funds
$48,500 (F/P, 12%, x) + $5,000 (F/A, 12%, x)
= $20,000 (P/A, 12%, 21- x)

Try x = 10
$48,500 (3.106) + $5,000 (17.549) = $238,386
$20,000 (5.938) = $118,760 so x can be < 10

Try x = 5
$48,500 (1.762) + $5,000 (6.353)= $117,222
$20,000 (6.974) = $139,480 so x > 5

Try x = 6
$48,500 (1.974) + $5,000 (8.115)= $136.314
$20,000 (6.811) = $136,220

Therefore, x = 6. The youngest age to retire is 55 + 6 = 61.

9-17

Geometric Gradient:

$n = 10$ $g = 100\%$ $A_1 = \$100$ $i = 10\%$

$$\begin{aligned} P &= A_1 [(1 - (1+g)^n (1 + i)^{-n})/(i - g)] \\ &= \$100 [(1 - (1 + 1.0)^{10} (1 + 0.10)^{-10})/(0.10 - 1.0)] \\ &= \$100 [(1 - (1{,}024) (0.3855))/(-0.90)] \\ &= \$43{,}755 \end{aligned}$$

Future Worth = $43,755 (F/P, 10%, 10) = $43,755 (2.594) = $113,500

9-18

$i = 0.0865/12 = 0.007208$
$n = 24$

$F = P (1 + i)^n = \$2{,}500 (1 + 0.007208)^{24} = \$2{,}970.30$

9-19

$$\begin{aligned} F_{56} &= \$25{,}000 \text{ (F/P, 6\%, 35)} + \$1{,}000 \text{ (F/A, 6\%, 35)} \\ &\quad + \$200 \text{ (P/G, 6\%, 35) (F/P, 6\%, 35)} * \\ &= \$25{,}000 (8.147) + \$1{,}000 (111.432) + \$200 (165.743) (7.686) \\ &= \$569.890 \end{aligned}$$

* The factor we want is (F/A, 6%, 35) but it is not tabulated in the back of the book. Instead we can substitute: (P/G, 6%, 35) (F/P, 6%, 35)

9-20

Assuming no disruption, the expected end-of-year deposits are:

A_1 = \$1,000,000 (A/F, 7%, 10)
= \$1,000,000 (0.0724)
= \$72,400

Compute the future worth of \$72,400 per year at the end of 7 years:

F_7 = \$72,400 (F/A, 7%, 7)
= \$626,550

Compute the future worth of \$626,550 in 3 years i.e. at the end of year 10:

F_{10} = \$626,550 (F/P, 7%, 3)
= \$767,524

Remaining two deposits = (\$1,000,000 - \$767,524) (A/F, 7%, 2)
= \$112,309

9-21

F = \$2,000 (F/A, 10%, 41) = \$2,000 (487.852) = \$975,704

Alternative solutions using interest table values:

F = \$2,000 (F/A, 10%, 40) + \$2,000 (F/P, 10%, 40)
= \$2,000 (45.259 + 442.593)
= \$975,704

9-22

FW (Costs) = \$150,000 (F/P, 10%, 10) + \$1,500 (F/A, 10%, 10)
+ \$500 (P/G, 10%, 7) (F/P, 10%, 7) – (0.05) (\$150,000)
= \$150,000 (2.594) + \$1,500 (15.937) + \$500 (12.763) (1.949)
- \$7,500
= \$417,940

9-23

Given: P = \$325,000
$A_{1\text{-}120}$ = \$1,200
$A_{84\text{-}120}$ = \$2,000 - \$1,200 = \$800
F_{60} = \$55,000 overhaul
n = 12 (10) = 120 months
i = 7.2/12 = 0.60% per month

Find: F_{120} = ?

F_P = (F/P, 0.60%, 120) ($325,000)
= $(1 + 0.0060)^{120}$ ($325,000)
= $666,250
F_{A1-120} = (F/A, 0.60%, 120) ($1,200)
= $[((1 + 0.006)^{120} – 1)/0.006]$ ($1,200)
= $210,000
$F_{A84-120}$ = (F/A, 0.60%, 36) ($800)
= $[((1 + 0.006)^{36} – 1)/0.006]$ ($800)
= $32,040
F_{60} = (F/P, 0.60%, 60) ($55,000)
= $(1 + 0.006)^{60}$ ($55,000)
= $78,750

F_{120} = $666,250 + $210,000 + $32,040 + $78,750
= $987,040

9-24

Find F.
F = $150 (F/A, 9%, 10) + $150 (P/G, 9%, 10) (F/P, 9%, 10)
= $10,933

Alternate Solution
Remembering that G must equal zero at the end of period 1, adjust the time scale where equation time zero = problem time – 1. Then:
F = $150 (F/G, 9%, 11) = $150 (P/G, 9%, 11) (F/P, 9%, 11)
= $150 (28.248) (2.580) = $10,932

9-25

$i_{semiannual}$ = $(1 + 0.192/12)^6 – 1$ = 0.10 = 10%

$F_{1/1/12}$ = $F_A + F_G$
From the compound interest tables (i = 10%, n = 31):

F_A = $5,000 (F/A, 10%, 31) = $5,000 (181.944) = $909,720
F_G = -$150 (P/G, 10%, 31) (F/P, 10%, 31)
= -$150 (78.640) (19.194)
= -$226,412

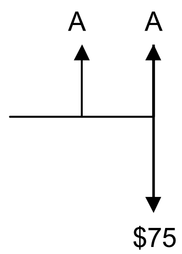
$F_{1/1/12}$ = $909,720 - $226,412 = $683,308
$F_{7/1/14}$ = $683,308 (F/P, 10%, 5) = $683,308 (1.611) = $1,100,809

9-26

The monthly deposits to the savings account do not match the twice a month compounding period. To use the standard formulas we must either
(1) compute an equivalent twice a month savings account deposit, or

(2) compute an equivalent monthly interest rate.

Method (1)



$n = 2$
$i = 0.045/24 = 0.001875$

Equivalent twice a month deposit (A) $= \$75\ (A/F, i\%, n)$
$= \$75\ [0.001875/((1 + 0.001875)^2 – 1)]$
$= \$37.4649$

Future Sum $F_{1/1/15} = A\ (F/A, i\%, 18 \times 24)$
$= \$37.4649\ [((1 + 0.001875)^{432} – 1)/0.001875]$
$= \$24{,}901$

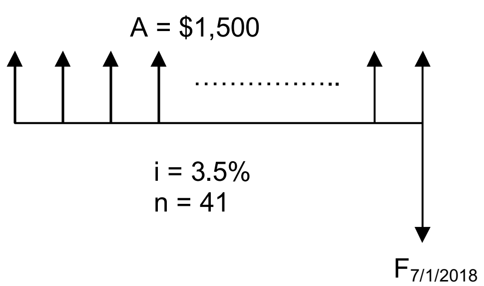
Method 2

Effective i per month (i_{month}) $= (1 + 0.045/24)^2 – 1 \quad = 0.0037535$

Future Sum $F_{1/1/15} = A\ (F/A, i_{month}, 18 \times 12)$
$= \$75\ [((1 + 0.0037535)^{216} – 1)/0.0037535]$
$= \$24{,}901$

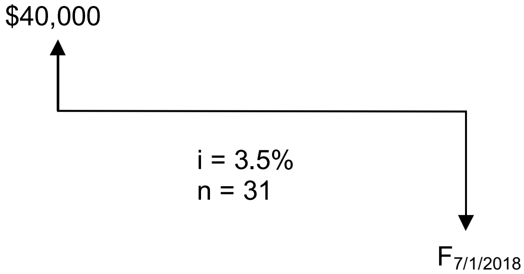
9-27

Bob's Plan



$F \quad = \$1{,}500\ (F/A, 3.5\%, 41) \qquad = \$1{,}500\ (86.437) \qquad = \$129{,}650$

Joe's Plan

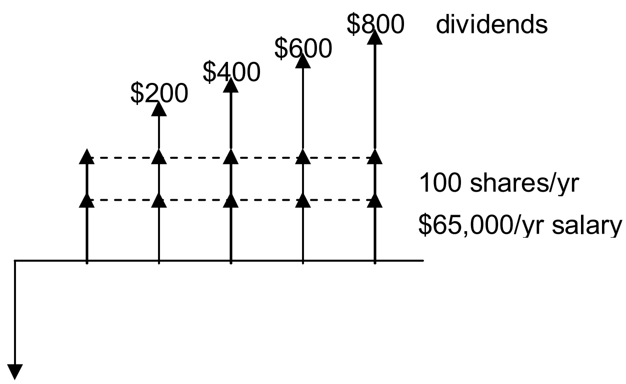


F = $40,000 (F/P, 3.5%, 31) = $40,000 (2.905) = $116,200

Joe's deposit will be insufficient. He should deposit:
$132,764 (P/F, 3.5%, 31) = $132,764 (0.3442) = $45,701

9-28

Fearless Bus



P = $65,000 (P/A, 9%, 5) + $200 (P/G, 9%, 5) + 500 shares of stock
= $65,000 (3.890) + $200 (7.111) + 500 shares of stock
= $254,272 + 500 shares of stock

F = $257,272 (F/P, 9%, 5) + $500 shares ($60/share)
= $257,272 (1.539) + $30,000
= $421,325

Generous Electric

F = $62,000 (F/A, 9%, 5) + 600 shares of stock
= $62,000 (5.985) + 600 shares of stock
= $371,070 + 600 shares of stock

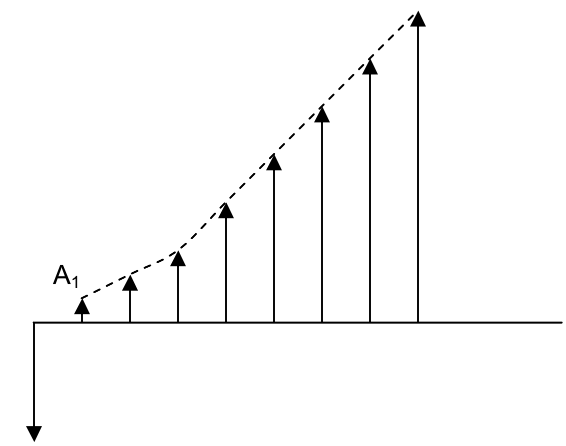
Set $F_{Fearless} = F_{GE}$

600 shares of GE stock + $371,070 = $421,325

Required Value of GE stock = ($421,325 - $371,070)/600

= $50,255/600
= $83.76/share

9-29



Geometric gradient at a 10% uniform rate.

A_1 = $10,000 i = 10% g = 1-% n = 8 yrs

Where i = g: $P = A_1 n\,(1 + i)^{-1}$
B/C = PW of Benefits/PW of Cost
= [$10,000 (8) $(1 + 0.10)^{-1}$]/$50,000
= 1.45

9-30

	A	B	C
Cost	$600	$500	$200
Uniform Annual Benefit	$158.3	$138.7	$58.3

$B/C_{OF\ A}$ = $158.3/[$600 (A/P, 10%, 5)] = 1.00
$B/C_{OF\ B}$ = $138.7/[$500 (A/P, 10%, 5)] = 1.05
$B/C_{OF\ C}$ = $58.3/[$200 (A/P, 10%, 5)] = 1.11

All alternatives have a B/C ratio > 1.00. Proceed with incremental analysis.

	B- C	A- B
Cost	$300	$100
Uniform Annual Benefit	$8034	$19.6

$B/C_{OF\ B\text{-}C}$ = $80.4/[$300 (A/P, 10%, 5)] = 1.02
Desirable increment. Reject C.

$B/C_{OF\ A\text{-}B}$ = \$19.6/[\$100 (A/P, 10%, 5)] = 0.74
Undesirable increment. Reject A.

Conclusion: Select B.

9-31

B/C_A = (\$142 (P/A, 10%, 10))/\$800 = 1.09
B/C_B = (\$60 (P/A, 10%, 10))/\$300 = 1.23
B/C_C = (\$33.5 (P/A, 10%, 10))/\$150 = 1.37

Incremental Analysis

B- C Increment

	B- C
Δ Cost	\$150
Δ UAB	\$26.5

$\Delta B/\Delta C$ = (\$26.5 (P/A, 10%, 10))/\$150 = 1.09

This is a desirable increment. Reject C.

A- B Increment

	A- B
Δ Cost	\$500
Δ UAB	\$82

$\Delta B/\Delta C$ = (\$82 (P/A, 10%, 10))/\$500 = 1.01
This is a desirable increment. Reject B.

Conclusion: Select A.

9-32

	2 Stories	5 Stories	10 Stories
Cost (including land)	\$500,000	\$900,000	\$2,200,000
Annual Income (A)	\$70,000	\$105,000	\$256,000
Salvage Value (F)	\$200,000	\$300,000	\$400,000

B/C Ratio Analysis

Cost	\$500,000	\$900,000	\$2,200,000
- PW of Salvage Value = F(P/F, 8%, 20) = 0.2145F	\$42,900	\$64,350	\$85,800
PW of Cost	\$457,100	\$835,650	\$2,114,200
PW of Benefit = A (P/A, 8%, 20) = 9.818A	\$687,260	\$1,030,890	\$2,513,410
B/C Ratio = PW of Benefit/PW of Cost	1.50	1.23	1.19

Incremental B/C Ratio Analysis

	5 Stories Rather than 2 Stories	10 Stories Rather than 2 Stories
Δ PW of Cost	\$835,650 - \$457,100 = \$378,550	\$2,114,200 - \$457,100 = \$1,657,100
ΔPW of Benefit	\$1,030,890 - \$687,260 = \$343,630	\$2,513,410 - \$687,260 = \$1,826,150
ΔB/ΔC = ΔPW of Benefits/ΔPW of Costs	\$343,630/\$378,550 = 0.91	\$1,826,150/\$1,657,100 = 1.10
Decision	<1 Undesirable increment. Reject 5 stories.	> 1 Desirable increment.

With ΔB/ΔC = 0.91, the increment of 5 stories rather than 2 stories is undesirable. The 10 stories rather than 2 stories is desirable.

Conclusion: Choose the 10-story alternative.

9-33

Note that the three alternatives have been rearranged below in order of increasing cost.

	C	B	A
First Cost	\$120	\$340	\$560
Uniform Annual Benefit	\$40	\$100	\$140
Salvage Value	\$0	\$0	\$40
Compute B/C Ratio	1.45	1.28	1.13

Incremental Analysis

	B- C	A- B
Δ First Cost	\$220	\$220
Δ Uniform Annual Benefit	\$60	\$40
Δ Salvage Value	\$0	\$40
Compute ΔB/ΔC value	1.19	0.88

Benefit- Cost Ratio Computations:

Alternative A: B/C = [\$140 (P/A, 10%, 6)]/[\$560 - \$40 (P/F, 10%, 6)]
= [\$140 (4.355)]/(\$560 - \$40 (0.5645)]
= 1.13

Alternative B: B/C = [\$100 (P/A, 10%, 6)]/\$340
= 1.28

Alternative C: B/C = [\$40 (P/A, 10%, 6)]/\$120
= 1.45

Incremental Analysis:

B- C $\Delta B/\Delta C$ = [$60 (P/A, 10%, 6)]/$220 = 1.19

B- C is a desirable increment.

A- B $\Delta B/\Delta C$ = [$40 (P/A, 10%, 6)/[$220 - $40 (P/F, 10%, 6)] = 0.88

A- B is an undesirable increment.

Conclusion: Choose B.

The solution may be checked by Net Present Worth or Rate of Return

NPW Solution

NPW_A = $140 (P/A, 10%, 6) + $40 (P/F, 10%, 6) - $560
= $140 (4.355) + $40 (0.5645) - $560
= +$72.28

NPW_B = $100 (P/A, 10%, 6) - $340
= +$95.50

NPW_C = $40 (P/A, 10%, 6) - $120
= +$54.20

Select B.

Rate of Return Solution

	B- C	A- B
Δ Cost	$220	$220
Δ Uniform Annual Benefit	$60	$40
Δ Salvage Value	$0	$40
Computed Δ ROR	16.2%	6.6%
Decision	> 10% Accept B. Reject C.	< 10% Reject A.

Select B.

9-34

This is an above-average difficulty problem. An incremental Uniform Annual Benefit becomes a cost rather than a benefit.

Compute B/C for each alternative
Form of computation used:

(PW of B)/(PW of C) = (UAB (P/A, 8%, 8))/(Cost – S (P/F, 8%, 8))

$= (UAB\ (5.747))/(Cost – S\ (0.5403))$

B/C_A	= (\$12.2 (5.747))/(\$100 - \$75 (0.5403))	= 1.18
B/C_B	= (\$12 (5.747))/(\$80 - \$50 (0.5403))	= 1.30
B/C_c	= (\$9.7 (5.747))/(\$60 - \$50 (0.5403))	= 1.69
B/C_d	= (\$12.2 (5.747))/\$50	= 1.40

All alternatives have B/C > 1. Proceed with Δ analysis.

Incremental Analysis

C- D Increment

	C- D
Δ Cost	\$10
Δ Uniform Annual Benefit	-\$2.5
Δ Salvage Value	\$50

The apparent confusion may be cleared up by a detailed examination of the cash flows:

Year	Cash Flow C	Cash Flow D	Cash Flow C- D
0	-\$60	-\$50	-\$10
1- 7	+\$9.7	+\$12.2	-\$2.5
8	+\$9.7 +\$50	+\$12.2	+\$47.5

B/C ratio = (\$47.5 (P/F, 8%, 8)/(\$10 + \$2.5 (P/A, 8%, 7))
= (\$47.5 (0.5403)/(\$10 + \$2.5 (5.206)
= 1.11

The C- D increment is desirable. Reject D.

B- C Increment

	B- C
Δ Cost	\$20.0
Δ Uniform Annual Benefit	\$2.3
Δ Salvage Value	\$0

B/C ratio = (\$2.3 (0.5403)/\$20
= 0.66

Reject B.

A- C Increment

	A- C
Δ Cost	\$40.0
Δ Uniform Annual Benefit	\$2.5
Δ Salvage Value	\$25.0

B/C ratio = ($2.5 (0.5403)/($40 - $25 (0.5403))
= 0.54

Reject A.

Conclusion: Select C.

9-35

	A	B	C	D	E
Cost	$100	$200	$300	$400	$500
UAB	$37	$69	$83	$126	$150
PW of Benefits = UAB (P/A, 15%, 5)	$124	$231.3	$278.2	$422.4	$502.8
B/C Ratio	1.24	1.16	0.93	1.06	1.01

	B- A	D- B	E- B
Δ Cost	$100	$200	$300
Δ UAB	$32	$57	$81
PW of Benefits	$107.3	$191.1	$271.5
$\Delta B/\Delta C$	1.07	0.96	0.91
Decision	Reject A.	Reject D.	Reject E.

Conclusion: Select B.

9-36

By inspection one can see that A, with its smaller cost and identical benefits, is preferred to F in all situations, hence F may be immediately rejected. Similarly, D, with greater benefits and identical cost, is preferred over B. Hence B may be rejected. Based on the B/C ratio for the remaining four alternatives, three exceed 1.0 and only C is less than 1.0. On this basis C may be rejected. That leaves A, D, and E for incremental B/C analysis.

	E- D	A- E
Δ Cost	$25	$50
Δ Benefits	$10	$16
PW of Benefits	$10 (P/A, 15%, 5) = $10 (3.352)	$16 (P/A, 15%, 5) = $16 (3.352)
$\Delta B/\Delta C$	$10 (3.352)/$25 = 1.34	$16 (3.352)/$50 = 1.07
Decision	Reject D.	Reject E.

Therefore, do A.

9-37

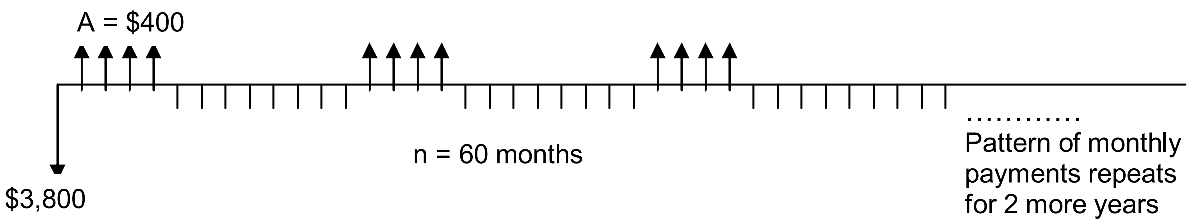
Investment= $67,000
Annual Benefit = $26,000/yr for 2 years

Payback period = $67,000/$26,000q= 2.6 years

Do not buy because total benefits (2) ($26,000) < Cost.

9-38

Payback Period = Cost/Annual Benefit = $3,800/(4*$400) = 2.4 years



$3,800= $400 (P/A, i%, 4) + $400 (P/A, i%, 4) (P/F, i%, 12)
+ $400 (P/A, i%, 4) (P/F, i%, 24) + $400 (P/A, i%, 4) (P/F, i%, 36)
+ $400 (P/A, i%, 4) (P/F, i%, 48)

$3,800= $400 (P/A, i%, 4) [1 + (P/F, i%, 12) + (P/F, i%, 24) + (P/F, i%, 36)
+ (P/F, i%, 48)]

Try i = 3%

P(3%) = $400 (3.717) [1 + 0.7014 + 0.4919 + 0.3450 + 0.2420]
= $1,486.80 [2.7803]
= $4,134 i too low

Try i = 4%

P(4%) = $400 (3.630) [1 + 0.6246 + 0.3901 + 0.2437 + 0.1522]
= $1,452 [2.4106]
= $3,500 i too high

Try i = 3.5%

P(3.5%) = $400 (3.673) [1 + 0.6618 + 0.4380 + 0.2898 + 0.1918]
= $1,469.20 [2.5814]
= $3,798

So i ≈ 3.5% per month

Nominal Rate of Return = 12 (3.5%) = 42%

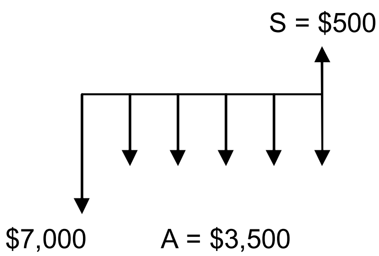
9-39

Costs = Benefits at end of year 8
Therefore, payback period = 8 years.

9-40

Lease: A = \$5,000/yr

Purchase:



(a) Payback Period
Cost = \$7,000
Benefit = \$1,500/yr + \$500 at any time
Payback = (\$7,000 - \$500)/\$1,500 = 4.3 years

(b) Benefit- Cost Ratio
B/C = EUAB/EUAC
= [\$1,500 + \$500 (A/F, 10%, 6)]/[\$7,000 (A/P, 10%, 6)]
= [\$1,500 + \$500 (0.1296)]/[\$7,000 (0.2296)]
= 0.97

9-41

(a) **Payback Periods**

	Alternative A		Alternative B	
Period	Cash Flow	Sum CF	Cash Flow	Sum CF
-2	-\$30	-\$30	-\$30	-\$30
-1	-\$100	-\$130	-\$100	-\$130
0	-\$70	-\$200	-\$70	-\$200
1	\$40	-\$160	\$32.5	-\$167.5
2	\$40	-\$120	\$32.5	-\$135
3	\$40	-\$80	\$32.5	-\$102.5
4	\$40	-\$40	\$32.5	-\$70
5	\$40	\$0	\$32.5	-\$37.5
6	\$40	\$40	\$32.5	-\$5
7	\$40	\$80	\$32.5	\$27.5

Payback_A = 5.0 years
Payback_B = 7 years (based on end of year cash flows)

(b) **Equivalent Investment Cost**

= \$30 (F/P, 10%, 2) + \$100 (F/P, 10%, 1) + \$70
= \$30 (1.210) + \$100 (1.100) + \$70
= \$216.3 million

(c) **Equivalent Uniform Annual Worth** = EUAB – EUAC

$EUAW_A$ = \$40 - \$216.3 (A/P, 10%, 10) = \$4.81 million
$EUAW_B$ = \$32.5 - \$216.3 (A/P, 10%, 20) = \$7.08 million

Since the EUAW for the Alternative B is higher, this alternative should be selected. Alternative A may be considered if the investor is very short of cash and the short payback period is of importance to him.

9-42

(a)

	Increment B- A
Δ Cost	\$300
ΔUAB	\$50

Incremental Payback = Cost/UAB = \$300/\$50 = 6 years

(b) $\Delta B/\Delta C$ = [\$50 (P/A, 12%, 8)]/\$300 = 0.83

Reject B and select A.

9-43

			|-----------------Part (a)	----------------------|
Year	Conventional	Solar	Solar – Conventional	Net Investment
0	-\$200	-\$1,400	-\$1,200	-\$1,200
1- 4	-\$230/yr	-\$60/yr	+\$170/yr	-\$520
4		-\$180	-\$180	-\$700
5- 8	-\$230/yr	-\$60/yr	+\$170/yr	-\$20
8		-\$180	-\$180	-\$200
9- 12	-\$230/yr	-\$60/yr	+\$170/yr	+\$480 ← Payback
12		- \$180	-\$180	+\$300

(a) Payback = 8 yrs + \$200/\$170 = 9.18 yrs

(b) The key to solving this part of the problem is selecting a suitable analysis method. The Present Worth method requires common analysis period, which is virtually impossible for this problem. The problem is easy to solve by Annual Cash Flow Analysis.

$EUAC_{conventional\text{- }20\ yrs}$ = \$200 (A/P, 10%, 20) + \$230 = \$253.50

$EUAC_{solar-\ n\ yrs}$ = \$1,400 (A/P, 10%, n) + \$60

For equal EUAC:
(A/P, 10%, n) = [\$253.50 - \$60]/\$1,400 = 0.1382

From the interest tables, n = 13.5 years.

9-44

(a) **Net Future Worth**

NFW_A = \$18.8 (F/A, 10%, 5) - \$75 (F/P, 10%, 5) = -\$6.06
NFW_B = \$13.9 (F/A, 10%, 5) - \$50 (F/P, 10%, 5) = +\$4.31
NFW_C = \$4.5 (F/A, 10%, 5) - \$15 (F/P, 10%, 5) = +\$3.31
NFW_D = \$23.8 (F/A, 10%, 5) - \$90 (F/P, 10%, 5) = +\$0.31

Select B.

(b) **Incremental Analysis**

	C	B	A	D
Cost	\$15.0	\$50.0	\$75.0	\$90.0
UAB	\$4.5	\$13.9	\$18.8	\$23.8
Computed Uniform Annual Cost (UAC)	\$3.96	\$13.19	\$19.78	\$23 74
B/C Ratio	1.14	1.05	0.95	1.00
Decision	Ok	Ok	Reject	Ok

	B- C	D- B
Δ UAB	\$9.40	\$9.90
Δ UAC	\$9.23	\$10.55
ΔB/ΔC	1.02	0.94
Decision	Reject C.	Reject D.

Conclusion: Select B.

(c) **Payback Period**

$Payback_A$ = \$75/\$18.8 = 4.0
$Payback_B$ = \$50/\$13.9 = 3.6
$Payback_C$ = \$15/\$4.5 = 3.3
$Payback_D$ = \$90/\$23.8 = 3.8

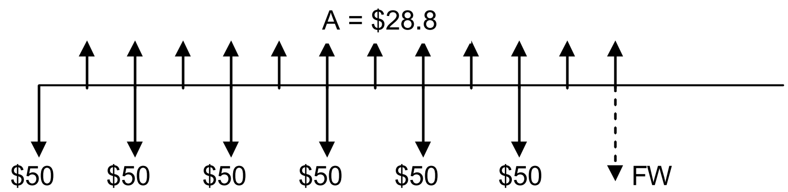
To minimize Payback, select C.

9-45

	A	B	C
Cost	\$50	\$150	\$110
Annual Benefit	\$28.8	\$39.6	\$39.6
Useful Life	2 yr	6 yr	4 yr

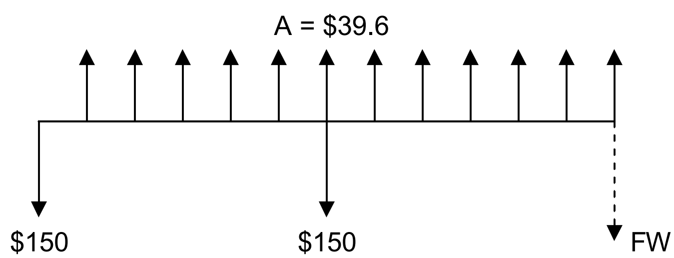
(a) Solve by Future Worth analysis. In future worth analysis there must be a common future time for all calculations. In this case 12 years hence is a practical future time.

Alternative A



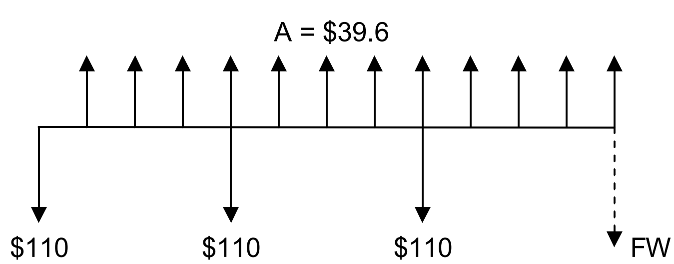
$$NFW_A = \$28.8\ (F/A, 12\%, 12) - \$50\ (A/P, 12\%, 2)\ (P/A, 12\%, 12)$$
$$= \$28.8\ (24.133) - \$50\ (0.5917)\ (24.133)$$
$$= -\$18.94$$

Alternative B



$$NFW_B = \$39.6\ (F/A, 12\%, 12) - \$150\ (F/P, 12\%, 6) - \$150\ (F/P, 12\%, 12)$$
$$= \$39.6\ (24.133) - \$150\ [1.974 + 3.896]$$
$$= +\$75.17$$

Alternative C



$$NFW_C = \$39.6\ (F/A, 12\%, 12) - \$110\ (F/P, 12\%, 4) - \$110\ (F/P, 12\%, 8) - \$110\ (F/P, 12\%, 12)$$

= \$39.6 (24.133) - \$110 [1.574 + 2.476 + 3.896]
= +\$81.61

Choose Alternative C because it maximizes Future Worth.

(b) Solve by Benefit-Cost ratio analysis

With neither input nor output fixed, incremental analysis is required.

Alternative C- Alternative A

Year	Alt. C	Alt. A	C- A
0	-\$110	-\$50	-\$60
1	+\$39.6	+\$28.8	+\$10.8
2	+\$39.6	+\$28.8 -\$50	+\$60.8
3	+\$39.6	+\$28.8	+\$10.8
4	+\$39.6	+\$28.8	+\$10.8

Four years is a suitable analysis period for Alternatives C and A.

For the increment C- A:
PW of Cost = \$60
PW of Benefits = \$10.8 (P/A, 12%, 4) + \$50 (P/F, 12%, 2)
= \$10.8 (3.037) + \$50 (0.7972)
= \$72.66

$\Delta B/\Delta C$ = PW of Benefits/PW of Cost = \$72.66/\$60 > 1

The increment of investment is acceptable and therefore Alternative C is preferred over Alternative A.

Increment B- C

Year	Alt. B	Alt. C	B- C
0	-\$150	-\$110	-\$40
1- 4	+\$39.6	+\$39.6	\$0
4	\$0	-\$110	+\$110
5-6	+\$39.6	+\$39.6	\$0
6	-\$150	\$0	-\$150
7- 8	+\$39.6	+\$39.6	\$0
8	\$0	-\$110	+\$110
9- 12	+\$39.6	+\$39.6	\$0

Twelve years is a suitable analysis period for Alternatives B and C.

For the increment B- C
Ignoring the potential difficulties signaled by 3 sign changes in the B- C cash flow:

PW of Cost = \$40 + \$150 (P/F, 12%, 6)
= \$40 + \$150 (0.5066)

= $115.99

PW of Benefits = $110 (P/F, 12%, 4) + $110 (P/F, 12%, 8)
= $110 (0.6355) + $110 (0.4039)
= $114.33

ΔB/ΔC = PW of Benefits/PW of Cost = $114.33/$115.99 < 1

The increment is undesirable and therefore Alternative C is preferred over Alternative B.

Alternative Analysis of the Increment B- C

An examination of the B- C cash flow suggests there is an external investment of money at the end of Year 4. Using an external interest rate (say 12%) the +$110 at Year 4 becomes:

+$110 (F/P, 12%, 2) = $110 (1.254) = $137.94 at the end of Yr. 6.

The altered cash flow becomes:

Year	B- C
0	-$40
1- 6	$0
6	-$150 +$137.94 = -$12.06
7-8	$0
8	+$110

For the altered B- C cash flow:

PW of Cost = $40 + $12.06 (P/F, 12%, 6)
= $40 + $12.06 (0.5066)
= $46.11

PW of Benefits = $110 (P/F, 12%, 8)
= $110 (0.4039)
= $44.43

ΔB/ΔC = PW of Benefits/PW of Cost = $44.43/$46.11 < 1

The increment is undesirable and therefore Alternative C is preferred to Alternative B.

Solutions for part (b): Choose Alternative C.

(c) Payback Period

Alternative A: Payback = $50/$28.8 = 1.74 yr
Alternative B: Payback = $150/$39.6 = 3.79 yr
Alternative C: Payback = $150/$39.6 = 2.78 yr

To minimize the Payback Period, choose Alternative A.

(d) Payback period is the time required to recover the investment. Here we have three alternatives that have rates of return varying from 10% to 16.4%. Thus each generates uniform annual benefits in excess of the cost, during the life of the alternative. From this is must follow that the alternative with a 2-year life has a payback period less than 2 years. The alternative with a 4-year life has a payback period less than 4 years, and the alternative with a 6-year life has a payback period less than 6 years.

Thus we see that the shorter-lived asset automatically has an advantage over longer-lived alternatives in a situation like this. While Alternative A takes the shortest amount of time to recover its investment, Alternative C is best for long-term economic efficiency.

9-46

(a) B/C of Alt. x = [$25 (P/A, 10%, 4)]/$100 = 0.79

(b) **Payback Period**

x = $100/$25 = 4 yrs
y = $50/$16 = 3.1 yrs
z = $50/$21 = 2.4 yrs

To minimize payback, select z.

(c) No computations are needed. The problem may be solved by inspection.

Alternative x has a 0% rate of return (Total benefits = cost).
Alternative z dominates Alternative y. (Both cost $50, but Alternative z yields more benefits).
Alternative z has a positive rate of return (actually 24.5%) and is obviously the best of the three mutually exclusive alternatives.

Choose Alternative z.

9-47

(a) **Payback Period**

Payback_A = 4 + $150/$350 = Year 4.4
Payback_B = Year 4
Payback_C = 5 + $100/$200 Year 5.5

For shortest payback, choose Alternative B.

(b) **Net Future Worth**

NFW_A = $200 (F/A, 12%, 5) + [$50 (P/F, 12%, 5) - $400] (F/P, 12%, 5) - $500 (F/P, 12%, 6)
= $200 (6.353) + [$50 (6.397) - $400] (1.762) - $500 (1.974)
= +$142.38

NFW_B = $350 (F/A, 12%, 5) + [-$50 (P/G, 12%, 5) - $300] (F/P, 12%, 5) - $600 (F/P, 12%, 6)
= $350 (6.353) + [-$50 (6.397) - $300] (1.762) - $600 (1.974)

= -$53.03

NFW_C = $200 (F/A, 12%, 5) - $900 (F/P, 12%, 6)
= $200 (6.353) - $900 (1.974)
= -$506.00

To maximize NFW, choose Alternative A.

9-48

(a) $Payback_A$ = 4 years
$Payback_B$ = 2.6 years
$Payback_C$ = 2 years

To minimize payback, choose C.

(b) B/C Ratios
B/C_A = ($100 (P/A, 10%, 6) + $100 (P/F, 10%, 1)/$500
= 1.05
B/C_B = ($125 (P/A, 10%, 5) + $75 (P/F, 10%, 1)/$400
= 1.36
B/C_C = ($100 (P/A, 10%, 4) + $100 (P/F, 10%, 1)/$300
= 1.36

Incremental Analysis

B- C Increment

Year	B- C
0	-$100
1	$0
2	+$25
3	+$25
4	+$25
5	+$125

$\Delta B/\Delta C_{B-C}$ = ($25 (P/A, 10%, 3)(P/F, 10%, 1) + $125 (P/F, 10%, 5))/$100
= 1.34

This is a desirable increment. Reject C.

A- B Increment

Year	A- B
0	-$100
1	$0
2	-$25
3	-$25
4	-$25
5	+$100

By inspection we see that $\Delta B/\Delta C < 1$

$\Delta B/\Delta C_{A\text{-}B}$ = (\$100 (P/F, 10%, 6))/(\$100 + \$25 (P/A, 10%, 4) (P/F, 10%, 1))
= 0.33

Reject A. Conclusion: Select B.

9-49

(a) **Future Worth Analysis at 6%**

NFW_E = \$20 (F/A, 6%, 6) - \$90 (F/P, 6%, 6) = +\$11.79
NFW_F = \$35 (F/A, 6%, 4) (F/P, 6%, 2) - \$110 (F/P, 6%, 6) = +\$16.02*
NFW_G = [\$10 (P/G, 6%, 6) - \$100] (F/P, 6%, 6) = +\$20.70
NFW_H = \$180 - \$120 (F/P, 6%, 6) = +\$9.72

To maximize NFW, select G.

(b) **Future Worth Analysis at 15%**

NFW_E = \$20 (F/A, 15%, 6) - \$90 (F/P, 15%, 6) = -\$33.09
NFW_F = [\$35 (P/A, 15%, 4) - \$110] (F/P, 15%, 6) = -\$23.30*
NFW_G = [\$10 (P/G, 15%, 6) - \$100] (F/P, 15%, 6) = -\$47.72
NFW_H = \$180 - \$120 (F/P, 15%, 6) = -\$97.56

* Note: Two different equations that might be used.

To maximize NFW, select F.

(c) **Payback Period**

$Payback_E$ = \$90/\$20 = 4.5 yr.
$Payback_F$ = \$110/\$35 = 3.1 yr
$Payback_G$ = 5 yr.
$Payback_H$ = 6 yr.

To minimize payback period, select F.

(d) B/C_G = PW of Benefits/PW of Cost = [\$10 (P/G, 7%, 6)]/\$100 = 1.10

9-50

$EUAC_{AMERICAN}$ =(\$8,900 - \$1,700) (A/P, 8%, 3)+\$1,700 (0.08)+12,000(0.09)
= \$4,010

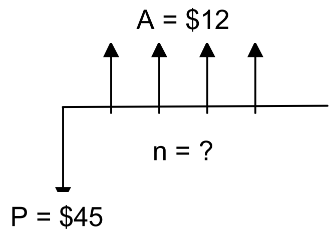
$EUAC_{FIASCO}$ =(\$8,000 – x) (A/P, 8%, 3) + x (0.08) + 12,000 (0.08)
= \$3,104 – 0.3880x + 0.08x + \$960

Set $EUAC_{AMERICAN}$ = $EUAC_{FIASCO}$

\$4,010 = \$4,064 – 0.308X

Minimum Fiasco Resale Value x = \$54/0.308 = \$175

9-51



$45 = $12 (P/A, i%, n)

(P/A, i%, n) = $45/$12 = 3.75

n	i%
4	2.6%
5	10.4%
6	15.3%
7	18.6%
8	20.8%
∞	A/P = $12/$45 = 26.7%

(b) For a 12% rate of return, the useful life must be 5.25 years.
(c) When n = ∞, rate of return = 26.7%

9-52

$(\text{EUAB} - \text{EUAC})_A$ = $230 - $800 (A/P, 12%, 5) = +$8.08

Set $(\text{EUAB} - \text{EUAC})_B$ = +$8.08 and solve for x.

$(\text{EUAB} - \text{EUAC})_B$ = $230 - $1,000 (A/P, 12%, x) = +$8.08

(A/P, 12%, x) = [$230 - $8.08]/$1,000 = 0.2219

From the 12% compound interest table, x = 6.9 yrs.

9-53

NPW_A = $40 (P/A, 12%, 6) + $100 (P/F, 12%, 6) - $150 = +$65.10

Set $\text{NPW}_B = \text{NPW}_A$
= $65 (P/A, 12%, 6) + $200 (P/F, 12%, 6) – x = +$65.10
$368.54 – x = +$65.10
x = 303.44

9-54

NPW Solution

$NPW_A = \$75/0.10 - \$500 = +\$250$

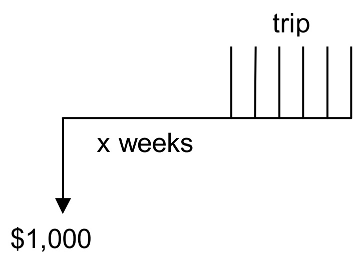
$NPW_B = \$75\ (P/A, 10\%, n) - \$300 = +\$250$

$(P/A, 10\%, n) = \$550/\$75 = 7.33$

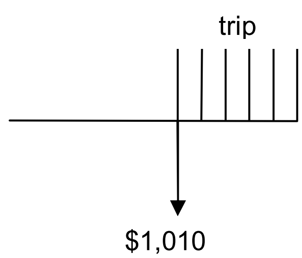
From the 10% table, n = 13.9 yrs.

9-55

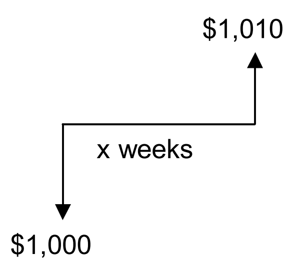
Alternative 1: Buy May 31st



Alternative 2: Buy just before trip



Difference between alternatives



i = ¼% per week

$\$1{,}000 = \$1{,}010\ (P/F, ¼\%, x \text{ weeks})$

$(P/F, ¼\%, x) = 0.9901$

x = 4 weeks

9-56

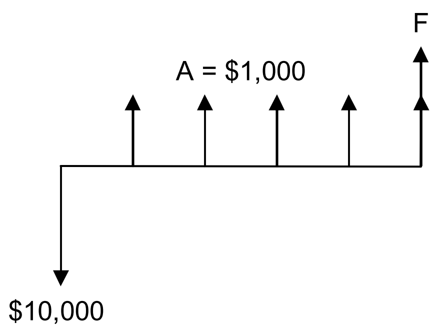
Untreated: EUAC = \$10.50 (A/P, 10%, 10) = \$10.50 (0.1627)
= \$1.71

Treated: EUAC = (\$10.50 + treatment) (A/P, 10%, 15)
= \$1.38 + 0.1315 (treatment)

Set $EUAC_{UNTREATED}$ = $EUAC_{UNTREATED}$
\$1.71 = \$1.38 + 0.1315 (treatment)
Treatment = (\$1.71 - \$1.38)/0.1315 = \$2.51

So, up to \$2.51 could be paid for post treatment.

9-57



F = \$10,000 (F/P, 10%, 5) - \$1,000 (F/A, 10%, 5)
= \$10,000 (1.611) - \$1,000 (6.105)
= \$10,000

9-58

Year	Cash Flow
0	-x
1	+\$8,400
2	+\$8,400
3	+\$8,400
4	+\$8,400
5	+\$8,400
6	+\$8,400
7	+\$8,400
8	+\$8,400
9	+\$8,400
10	+\$8,400
11	+\$8,400
12	+\$8,400
	+\$80,000

Where x = maximum purchase price,
x = (\$14,400 - \$6,000) (P/A, 7%, 12) + \$80,000 (P/F, 7%, 12)

= \$8,400 (7.943) + \$80,000 (0.4440)
= \$102,240

9-59

Since both motors have the same annual maintenance cost, it may be ignored in the computations. Here, however, we will include it.

Graybar

$EUAC_G$ = \$7,000 (A/P, 10%, 20) + \$300
+ [[(200 hp) (0.746 kw/hp) (\$0.072/kwhr)]/0.89 eff]
= \$7,000 (0.1175) + \$300 + 12.07 hrs
= \$1,122.50 + \$12.07/hr (hrs)

Blueball

$EUAC_B$= \$6,000 (A/P, 10%, 20) + \$300
+ [[(200 hp) (0.746 kw/hp) (\$0.072/kwhr)]/0.85 eff]
= \$6,000 (0.1175) + \$300 + 12.64 hrs
= \$1,005 + 12.64 hrs

Set $EUAC_B$ = $EUAC_G$
\$1,005 + 12.64 hrs = \$1,122.50 + \$12.07/hr (hrs)

The minimum number of hours the graybar, with its smaller power cost, must be used is:
(12.64 – 12.07) hrs = \$1,122.50 - \$1,005
hrs = \$117.50/\$0.57 = 206 hours

9-60

The difference between the alternatives is that Plan A requires \$20,000 extra now and Plan B requires \$40,000 extra n years hence.

At breakeven:
\$20,000 = \$40,000 (P/F, 8%, n)
(P/F, 8%, n) = 0.5

From the 8% interest table, n = 9 years.

9-61

The annual cost of the untreated part:
\$350 (A/P, 10%, 6) = \$350 (0.2296) = \$80.36

The annual cost of the treated part must be at least this low so:
\$80.36 = \$500 (A/P, 10%, n)
(A/P, 10%, n) = \$80.36/\$500 = 0.1607

So n = 10 yrs + (1) [(0.1627 – 0.1607)/(0.1627 – 0.1540)] = 10.2 years

9-62

(a) $\text{PW of Cost}_A = \text{PW of Cost}_B$

$\$55{,}000 + \$16{,}200\ (P/A, 10\%, n) = \$75{,}000 + \$12{,}450\ (P/A, 10\%, n)$

$(P/A, 10\%, n) = (\$75{,}000 - \$55{,}000)/(\$16{,}200 - \$12{,}450)$
$= 5.33$

From the 10% interest table, (P/A, 10%, 8) = 5.335 so the machines are equivalent at 8 years.

(b) At 0% interest, from (a):
$(P/A, 0\%, n) = 5.33$
Since (P/A, 0%, n) = n, the machines are equivalent at 5 1/3 years.

9-63

(a) **Payback Period**

At first glance, payback would appear to be
$\$5{,}240/\$1{,}000 = 5.24$ years
However, based on end-of-year benefits, as specified in the problem, the correct answer is:
Payback = 6 years

(b) **Breakeven Point (in years)**

Here interest is used in the computations. For continuous compounding:

$P = A[(e^{rn} – 1)/(e^{rn} (e^r – 1)]$

P = \$5,240 A = \$1,000 R = 0.10 n = ?

$\$5{,}240 = \$1{,}000[(e^{0.10n} – 1)/(e^{0.10n} (e^{0.10} – 1)]$
$= \$1{,}000 [(e^{0.10n} – 1)/(0.1052\ e^{.10n})]$

$[e^{0.10n} – 1] = 5.24 [0.1052\ e^{0.10n}]$
$e^{0.10n} [1- 0.5511] = 1$
$e^{0.10n} = 1/(1 – 0.5511) = 2.23$

Solving, n = 8 years.

(c) Both (a) and (b) are "correct." Since the breakeven analysis takes all eight years of benefits into account, as well as the interest rate, it is a better measure of long term economic efficiency.

Chapter 10: Uncertainty in Future Events

10-1

(a) Some reasons why a pole might be removed from useful service:
1. The pole has deteriorated and can no longer perform its function of safely supporting the telephone lines
2. The telephone lines are removed from the pole and put underground. The poles, no longer being needed, are removed.
3. Poles are destroyed by damage from fire, automobiles, etc.
4. The street is widened and the pole no longer is in a suitable street location.
5. The pole is where someone wants to construct a driveway.

(b) Telephone poles face varying weather and soil conditions; hence there may be large variations in their useful lives. Typical values for Pacific Telephone Co. in California are:

Optimistic Life:	59 years
Most Likely Life:	28 years
Pessimistic Life:	2.5 years

Recognizing there is a mortality dispersion it would be possible, but impractical, to define optimistic life as the point where the last one from a large group of telephone poles is removed (for Pacific Telephone this would be 83.5 years). This is *not* the accepted practice. Instead, the optimum life is where only a small percentage (often 5%) of the group remains in service. Similarly, pessimistic life is when, say, 5% of the original group of poles have been removed from the group.

10-2

If 16,000 km per year, then fuel cost = oil/tires/repair = \$990/year, and salvage value = 9,000 - 5x16,000x.05 = 9,000 – 4,000 = 5,000

$$\begin{aligned} EUAC_{16,000} &= 9{,}000(A/P,8\%,5) + 2\text{x}990 - 5{,}000(A/F,8\%,5) \\ &= 9{,}000\text{x}.2505 + 1{,}980 - 5{,}000\text{x}.1705 \\ &= 2{,}254.5 + 1{,}980 - 852.5 = \$3{,}382 \end{aligned}$$

Increasing annual mileage to 24,000 is a 50% increase so it increases operating costs by 50%. The salvage value drops by 5x8,000x.05 = 2,000

$$\begin{aligned} EUAC_{24,000} &= 9{,}000(A/P,8\%,5) + 2\text{x}1.5\text{x}990 - 3{,}000(A/F,8\%,5) \\ &= 9{,}000\text{x}.2505 + 1.5\text{x}1{,}980 - 3000\text{x}.1705 \\ &= 2{,}254.5 + 2{,}970 - 511.5 = \$4{,}713 \end{aligned}$$

Decreasing annual mileage to 8,000 is a 50% decrease so it decreases operating costs by 50%. The salvage value increases by 5x8,000x.05 = 2,000

$$\begin{aligned} EUAC_{8,000} &= 9{,}000(A/P,8\%,5) + 2\text{x}.5\text{x}990 - 7{,}000(A/F,8\%,5) \\ &= 9{,}000\text{x}.2505 + .5\text{x}1{,}980 - 7{,}000\text{x}.1705 \\ &= 2{,}254.5 + 990 - 1193.5 = \$2{,}051 \end{aligned}$$

10-3

Mean Life = (12 + 4 x 5 + 4)/6 = 6 years

PW of Cost = PW of Benefits
\$80,000 = \$20,000 (P/A, i%, 6)

Rate of Return is between 12% and 15%
Rate of Return ≈ 13%

10-4

Since the pessimistic and optimistic answers are symmetric about the most likely value of 16,000, the weighted average is 16,000 km. If 16,000 km per year, then fuel cost = oil/tires/repair = \$990/year, and salvage value = 8,000 - 5x16,000x.05 = 9,000 – 4,000 = 5,000

$$\begin{aligned} EUAC_{16,000} &= 9{,}000(A/P,8\%,5) + 2\text{x}990 - 5{,}000(A/F,8\%,5) \\ &= 9{,}000\text{x}.2505 + 1980 - 5{,}000\text{x}.1705 \\ &= 2{,}254.5 + 1{,}980 - 852.5 = \$3{,}382 \end{aligned}$$

10-5

There are six ways to roll a 7: 1 & 6, 2 & 5, 3 & 4, 4 & 3, 5 & 2, 6 & 1
There are two ways to roll an 11: 5 & 6 or 6 & 5

Probability of rolling a 7 or 11 = (6 + 2)/36 = 8/36

10-6

Since the *P*s must sum to 1: *P*(30K) = 1 - .2 - .3 = .5
E(savings) = .3(20K) + .5(30K) + .2(40K) = \$29K

10-7

Since the *P*s must sum to 1: *P*(20%) = 1 - 2/10 - 3/10 = .5
E(*i*) = .2(10%) + .3(15%) + .5(20%) = 16.5%

10-8

State of Nature	Completion Time	Probability
Sunny and Hot	250 Days	0.2
In Between Weather	300 Days	0.5 = 1 – 0.2 – 0.3
Cool and Damp	350 Days	0.3

E(days) = .20(250) + .5(300) + .3(350) = 305 days

10-9

If you have another accident or a violation this year, which has a .2 probability, it is assumed to occur near the end of the year so that it affects insurance rates for years 1-3. A violation in year 1 affects the rates in years 2 and 3 only if there was no additional violation in this year, which is P(none in 0)·P(occur in 1) = .8·.2 = .16. So the total probability of higher rates for year 2 is .2 + .16 or .36. This also equals 1 - P(no violation in 0 or 1) = 1 - $.8^2$.

For year 3, the result can be found as P(higher in year 2) + P(not higher in year 2)·P(viol. in year 2) = .36 + .64·.2 = .488. This also equals either 1 - P(no violation in 0 to 2) = 1 - $.8^3$.

Rates for Year	**0**	**1**	**2**	**3**
P($600)	0	.2	.36	.488

10-10

		Instructor A		Instructor B	
Grade		Grade Distribution	Expected Grade Point	Grade Distribution	Expected Grade Point
A	4.0	0.10	0.40	0.15	0.60
B	3.0	0.15	0.45	0.15	0.45
C	2.0	0.45	0.90	0.30	0.60
D	1.0	0.15	0.15	0.20	0.20
F	0	0.15	0	0.20	0
Sum		1.00	1.90	1.00	1.85

To minimize the Expected Grade Point, choose instructor A.

10-11

Expected outcome= $2,000 (0.3) + $1,500 (0.1) + $1,000 (0.2)
+ $500 (0.3) + $0 (0.1)
= $1,100

10-12

The sum of probabilities for all possible outcomes is one.

An inspection of the Regular Season situation reveals that the sum of the probabilities for the outcomes enumerated is 0.95. Thus one outcome (win less than three games), with probability 0.05, has not been tabulated. This is not a faulty problem statement. The student is expected to observe this difficulty.

Similarly, the complete probabilities concerning a post-season Bowl Game are:
Probability of playing = 0.10
Probability of not playing = 0.90

Expected Net Income for the team
= (0.05 + 0.10 + 0.15 + 0.20) ($250,000) + (0.15 + 0.15 + 0.10) ($400,000)

$+ (0.07 + 0.03) (\$600,000) + (0.10) (\$100,000)$
$= 0.50 (\$250,000) + 0.40 (\$400,000) + 0.10 (\$600,000) + 0.10 (\$100,000)$
$+ 0.90 (\$0)$
$= \$355.00$

10-13

Determine the different ways of throwing an 8 with a pair of dice.

Die 1	Die 2
2	6
3	5
4	4
5	3
6	2

The five ways of throwing an 8 have equal probability of 0.20.

The probability of winning is 0.20
The probability of losing is 0.80

The outcome of a \$1 bet = 0.20 (\$4) + 0.80 (\$0) = \$0.80
This means a \$0.20 loss.

10-14

$$E(PW_{\text{extra costs}}) = .2 \cdot 600(P/F,8\%,1) + .36 \cdot 600(P/F,8\%,2) + .488 \cdot 600(P/F,8\%,3)$$
$$= .2 \cdot 600 \cdot .9259 + .36 \cdot 600 \cdot .8573 + .488 \cdot 600 \cdot .7938 = \$528.7$$

10-15

Leave the Valve as it is

Expected PW of Cost = 0.60 (\$10,000) + 0.50 (\$20,000)
+ 0.40 (\$30,000)
= \$28,000

Repair the Valve

Expected PW of Cost = \$10,000 repair + 0.40 (\$10,000)
+ 0.30 (\$20,000) + 0.20 (\$30,000)
= \$26,000

Replace the Valve

Expected PW of Cost = \$20,000 replacement + 0.30 (\$10,000)
+ 0.20 (\$20,000) + 0.10 (\$30,000)
= \$30,000

To minimize Expected PW of Cost, repair the valve.

10-16

Expected number of wins in 100 attempts = 100/38 = 2.6316
Results of a win = 35 x \$5 + \$5 bet return = \$180.00
Expected winnings = \$180.00 (2.6313) = \$473.69
Expected loss = \$500.00 - \$473.69 = \$26.31

10-17

Do Nothing

EUAC = Expected Annual Damage
= 0.20 (\$10,000) + 0.10 (\$25,000) = \$4,500

\$15,000 Building Alteration

Expected Annual Damage = 0.10 (\$10,000) = \$1,000
Annual Floodproofing Cost = \$15,000 (A/P, 15%, 15) = \$2,565
EUAC = \$3,565

\$20,000 Building Alteration

Expected Annual Damage = \$0
Annual Floodproofing Cost = \$20,000 (A/P, 15%, 15) = \$3,420
EUAC = \$3,420

To minimize expected EUAC, recommend \$20,000 building alteration.

10-18

Height above roadway	Annual Probability of Flood Damage	x Damage	= Expected Annual Damage
2 m	0.333	\$300,000	= \$100,000
2.5 m	0.125	\$300,000	= \$37,500
3 m	0.04	\$300,000	=\$12,000
3.5 m	0.02	\$300,000	= \$6,000
4 m	0.01	\$300,000	= \$3,000

Height above roadway	Initial Cost	x (A/P, 12%, 50)	= EUAC of Embankment	Expected Annual Damage	Total Expected Annual Cost
2 m	\$100,000	0.1204	= \$12,040	\$100,000	\$112,040
2.5 m	\$165,000	0.1204	= \$19,870	\$37,500	\$53,370
3 m	\$300,000	0.1204	=\$36,120	\$12,000	\$48,120 □
3.5 m	\$400,000	0.1204	= \$48,160	\$6,000	\$54,160
4 m	\$550,000	0.1204	= \$66,220	\$3,000	\$69,220

Select the 3-metre embankment to minimize total Expected Annual Cost.

10-19

E(first cost) = 300,000(.2) + 400,000(.5) + 600,000(.3) = \$440K

E(net revenue)= 70,000(.3) + 90,000(.5) + 100,000(.2) = \$86K

E(PW) = -440K + 86K(P/A,12%,10) = \$45.9K, do the project

10-20

E(savings) = .2(18K) + .7(20K) + .10(22K) = \$19,800

E(life) = (1/6)(4) + (2/3)(5) + (1/6)(12) = 6 years

0 = -81,000 + 19,800(P/A,i,6)
(P/A,i,6) = 81,000/19,800 = 4.0909
(P/A,12%,6) = 4.111 & (P/A,13%,6) = 3.998

i = .12 + .01(4.111 - 4.0909)/(4.111 - 3.998) = 12.18%

Because (P/A,i,N) is a non-linear function of N, the use of 6 years for the expected value of N is an approximation. The discounting by i has much more impact in the 12 year life, so that we expect the true IRR to be less than 12.18%.

0 = -81K + 19.8K(1/6)(P/A,i,4) + 19.8K(2/3)(P/A,i,5) + 19.8K(1/6)(P/A,i,12)
$PW_{12\%}$ = -81K + 19.8K(1/6)· 3.037 + 19.8K(2/3)· 3.605 + 19.8K(1/6)· 6.194
= -2952
$PW_{11\%}$ = -81K + 19.8K(1/6)· 3.102 + 19.8K(2/3)· 3.696 + 19.8K(1/6)· 6.492
= -553
i = .11 - .01(553)/(553 + 2952) = 10.84%

10-21

Since \$250,000 of dam repairs must be done in all alternatives, this \$250,000 can be included or ignored in the analysis. Here it is ignored. (Remember, only the differences between alternatives are relevant.)

	Flood	Probability of damage in any year = 1/yr flood	Downstream Damage	Spillway Damage
	25 yr.	0.04	\$50,000	
	50 yr.	0.02	\$200,000	
For 10 yrs:	100 yr.	0.01	\$1,000,000	\$250,000
Thereafter:	100 yr.	0.01	\$2,000,000	\$250,000

Alternative I: Repair existing dam but make no other alterations

Spillway damage: Probability that spillway capacity equaled or exceeded in any year is 0.02. Damage if spillway capacity exceed: \$250,000

Expected Annual Cost of Spillway Damage = \$250,000 (0.02)
= \$5,000

Downstream Damage during next 10 years:

Flood	Probability that flow* will be equaled or exceeded	Damage	Δ Damage over more frequent flood	Annual Cost of Flood Risk
25 yr.	0.04	\$50,000	\$50,000	\$2,000
50 yr.	0.02	\$200,000	\$150,000	\$3,000
100 yr.	0.01	\$1,000,000	\$800,000	\$8,000

Next 10 year expected annual cost of downstream damage = \$13,000

Downstream Damage after 10 years: Following the same logic as above,
Expected annual cost of downstream damage
= \$2,000 + \$3,000 + 0.1 (\$2,000,000 - \$200,000)
= \$23,000

Present Worth of Expected Spillway and Downstream Damage
PW= \$5,000 (P/A, 7%, 50) + \$13,000 (P/A, 7%, 10)
+ \$23,000 (P/A, 7%, 40) (P/F, 7%, 10)
= \$5,000 (13.801) + \$13,000 (7,024) + \$23,000 (13.332) (0.5083)
= \$316,180

Equivalent Uniform Annual Cost
Annual Cost = \$316,180 (A/P, 7%, 50)
= \$316,180 (0.0725)
= \$22,920

* An N-year flood will be equaled or exceed at an average interval of N years.

Alternative II: Repair the dam and redesign the spillway

Additional cost to redesign/reconstruct the spillway = \$250,000

PW to Reconstruct Spillway and Expected Downstream Damage
Downstream Damage- same as alternative 1
PW = \$250,000 + \$13,000 (P/A, 7%, 10)
+ \$23,000 (P/A, 7%, 40) (P/F, 7%, 10)
= \$250,000 + \$13,000 (7.024) + \$23,000 (13.332) (0.5083)
= \$497,180

EUAC = \$497,180 (A/P, 7%, 50)
= \$497,180 (0.0725)
= \$36,050

Alternative III: Repair the dam and build flood control dam upstream

Cost of flood control dam = $1,000,000

EUAC = $1,000,000 (A/P, 7%, 50)
= $1,000,000 (0.7225)
= $72,500

Note: One must be careful not to confuse the frequency of a flood and when it might be expected to occur. The occurrence of a 100-year flood this year is no guarantee that it won't happen again next year. In any 50-year period, for example, there are 4 chances in 10 that a 100-year flood (or greater) will occur.

Conclusion: Since we are dealing with conditions of risk, it is not possible to make an absolute statement concerning which alternative will result in the least cost to the community. Using a probabilistic approach, however, Alternative I is most likely to result in the least equivalent uniform annual cost.

10-22

The $35K is a sunk cost and should be ignored.

a. E(PW) = $5951
b. P(PW<0) = .3 and σ = $65,686.

State	**Bad**	**OK**	**Great**		
Probability	.3	.5	.2		
Net Revenue	$-15,000	$15,000	$20,000		
Life (yrs)	5	5	10		
PW	-86,862	26,862	92,891	$5,951	E_{PW}
PW^2 · Prob	2,263,491,770	360,778,191	1,725,760,288	$65,686	σ_{PW}

10-23

a. The $35K is still a sunk cost and should be ignored. Note: P(PW<0) = .3 and *N* = 1 used for PW_{bad} since termination allowed here. This improves the E_{PW} by 18,918 - 5951 = $12,967. This also equals the E(PW) of the avoided negative net revenue in years 2 – 5, which equals .3 (1/1.1)x15,000(*P*/*A*,.1,4).

b. The P(loss) is unchanged at .3. However, the standard deviation improves by 65,686 - 47,957 = $17,709.

State	**Bad**	**OK**	**Great**		
Probability	.3	.5	.2		
Net Revenue	$-15,000	$15,000	$20,000		
Life (yrs)	1	5	10		
PW	-43,636	26,862	92,891	$18,918	E_{PW}
PW^2 · Prob	571,239,669	360,778,191	1,725,760,288	$47,957	σ_{PW}

10-24

Since the expected life is not an integer, it is easier to use a spreadsheet table to calculate each PW. For example, the first row's PW = -80K + 15K(*P*/*A*,9%,3).

Savings/ yr	***P***	**Life**	***P***	***P***	**PW**	***P*·PW**
15,000	.3	3	.6	0.18	-42,031	-7,566
15,000	.3	5	.4	0.12	-21,655	-2,599
30,000	.5	3	.6	0.30	-4,061	-1,218
30,000	.5	5	.4	0.20	36,690	7,338
45,000	.2	3	.6	0.12	33,908	4,069
45,000	.2	5	.4	0.08	95,034	7,603
				Expected Value		$7,627

10-25

If the savings were only $15K per year, spending $50K for 3 more years would not make sense. For the two or three shift situations, the table from 10-24 can be modified for 3 extra years, and to include the $50K at the end of 3 or 5 years. For example, the first and second rows' PWs are unchanged. The third row's PW = -80K + 15K(*P*/*A*,9%,6) - 50K(*P*/*F*,9%,3).

Savings/ yr	*P*	Life	*P*	*P*	PW	*P*·PW
15,000	.3	3	.6	0.18	-42,031	-7,566
15,000	.3	5	.4	0.12	-21,655	-2,599
30,000	.5	6	.6	0.30	15,968	4,791
30,000	.5	8	.4	0.20	53,548	10,710
45,000	.2	6	.6	0.12	83,257	9,991
45,000	.2	8	.4	0.08	136,570	10,926
				Expected Values		26,252

The option of extending the life is not used for single shift operations, but it increases the expected PW by 26,252 - 7,627 = $18,625.

10-26

Al's Score was x + (5/20) s = x + 0.25 s
Bill's Score was x + (2/4) s = x + 0.50 x

Therefore, Bill ranked higher in his class.

10-27

P	**.3**	**.5**	**.2**	***E*(*x*)**
PW	\$6570	\$8590	\$9730	\$8212
PW^2	43,164,900	73,788,100	94,672,900	68,778,100

$\sigma_{PW} = (68{,}778{,}100 - 8212^2)^{1/2} = \$1158$

10-28

$PW_1 = -25{,}000 + 7000(P/A,12\%,4) = -\$3739$
$PW_2 = -25{,}000 + 8500(P/A,12\%,4) = \$817$
$PW_3 = -25{,}000 + 9500(P/A,12\%,4) = \$3855$
From the table the E(PW) = \$361.9
$\sigma_{PW} = (8{,}918{,}228 - 361.9^2)^{1/2} = \$2964$

P	.3	.4	.3	*E*(*x*)
Annual Savings	\$7000	\$8500	\$9500	\$8,350
PW	-3739	817	3855	361.87
PW^2	13,976,790	668,256	14,859,628	8,918,228

10-29

To calculate the risk, it is necessary to state the outcomes based on the year in which the next accident or violation occurred.

year of 2nd offence	**0**	**1**	**2**	**ok**	
extra \$600 in years	1-3	2-3	3	none	
P	.2	.16	.128	.512	*E*(*x*)
PW	\$-1546	\$-991	\$-476	\$0	\$-529
PW^2	2,390,914	981,492	226,861	0	664,260

$\sigma_{PW} = (664{,}260 - 529^2)^{1/2} = \$620.0$

10-30

For example, the first row's PW = -300K + 70K(*P*/*A*,12%,10)

First Cost	*P*	Net Revenue	*P*	*P*	PW	*P*·PW	*P*·PW^2
-300	.2	70	.3	0.06	95.5	5.73	547
-300	.2	90	.5	0.10	208.5	20.85	4,347
-300	.2	100	.2	0.04	265.0	10.60	2,809
-400	.5	70	.3	0.15	-4.5	-0.68	3
-400	.5	90	.5	0.25	108.5	27.13	2,943
-400	.5	100	.2	0.10	165.0	16.50	2,723

-600	.3	70	.3	0.09	-204.5	-18.41	3,764
-600	.3	90	.5	0.15	-91.5	-13.73	1,256
-600	.3	100	.2	0.06	-35.0	-2.10	74
				Expected Values		45.90	18,468

Risk can be measured using the P(loss), range, or the standard deviation of the PWs.

P(loss) = .15 + .09 + .15 + .06 = .45

The range is -204.5K to $265K.

The standard deviation is $\sigma_{PW} = \sqrt{(18{,}468 - 45.90^2)}$ = $127.9K

10-31

a. The probability of a negative PW is .18 + .12 + .3 = .6

Savings/ yr	*P*	Life	*P*	*P*	PW	*P*·PW	*P*·PW^2
15,000	.3	3	.6	0.18	-42,031	-7,566	317,982,538
15,000	.3	5	.4	0.12	-21,655	-2,599	56,273,884
30,000	.5	3	.6	0.30	-4,061	-1,218	4,947,906
30,000	.5	5	.4	0.20	36,690	7,338	269,224,438
45,000	.2	3	.6	0.12	33,908	4,069	137,972,411
45,000	.2	5	.4	0.08	95,034	7,603	722,521,558
					Expected Values	7,627	1,508,922,738

Risk can also be measured using the standard deviation of the PWs. The standard deviation is $\sigma_{PW} = \sqrt{(1{,}508{,}922{,}738 - 7627^2)}$ = $38,089.

b. Extending the life for 2 & 3 shift operations reduces the probability of a negative PW by .3 to .3.

Savings/ yr	*P*	Life	*P*	*P*	PW	*P*·PW	*P*·PW^2
15,000	.3	3	.6	0.18	-42,031	-7,566	317,982,538
15,000	.3	5	.4	0.12	-21,655	-2,599	56,273,884
30,000	.5	6	.6	0.30	15,968	4,791	76,496,783
30,000	.5	8	.4	0.20	53,548	10,710	573,477,749
45,000	.2	6	.6	0.12	83,257	9,991	831,810,614
45,000	.2	8	.4	0.08	136,570	10,926	1,492,115,547
				Expected Values		26,252	3,348,157,118

Risk can also be measured using the standard deviation of the PWs. The standard deviation is increased by $13,477. This illustrates why standard deviation alone is not the best measure of risk. Extending the life makes the project more attractive, and increases the spread of the possible values. The standard deviation is higher, but the P(loss) has dropped by half.

$\sigma_{PW} = \sqrt{}(3{,}348{,}157{,}118 - 26{,}252^2) = \$51{,}565$

10-32

(a) Expected fire loss in any year= 0.010 ($10,000) + 0.003 ($40,000)
+ 0.001 ($200,000)
= $420.00

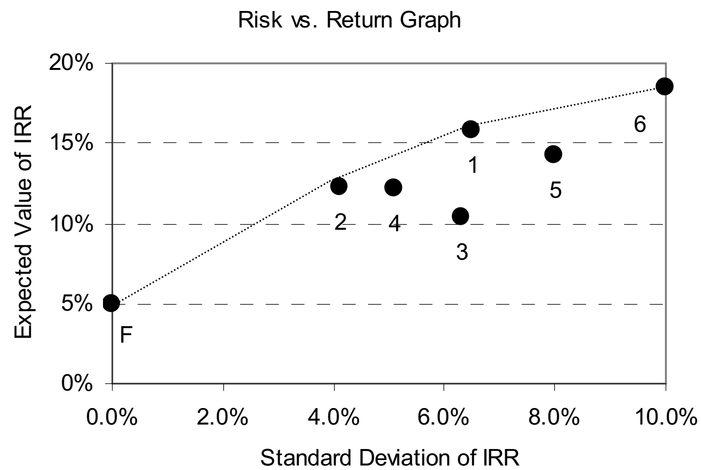
(b) The engineer buys the fire insurance because
1. a catastrophic loss is an unacceptable risk
or 2. he has a loan on the home and fire insurance is required by the lender.

10-33

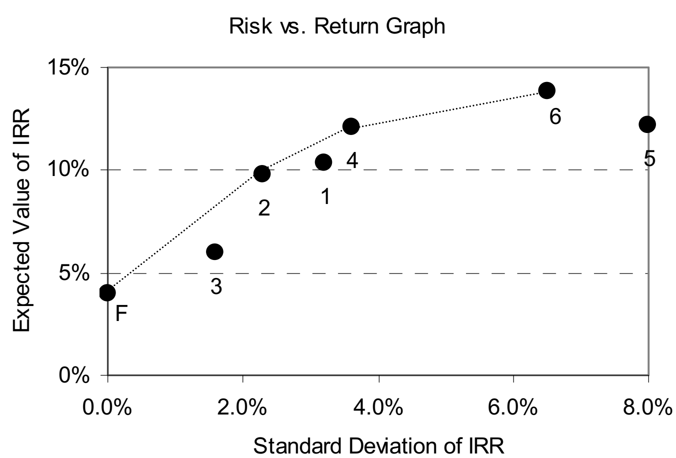
Project	IRR	Std.Dev.
1	15.8%	6.5%
2	12.3%	4.1%
3	10.4%	6.3%
4	12.1%	5.1%
5	14.2%	8.0%
6	18.5%	10.0%
F	5.0%	0.0%

10-34

Project	IRR	Std.Dev.
1	10.4%	3.2%
2	9.8%	2.3%
3	6.0%	1.6%
4	12.1%	3.6%
5	12.2%	8.0%
6	13.8%	6.5%
F	4.0%	0.0%



Risk vs. Return Graph
20%
15%
10%
5%
0%
Expected Value of IRR
F
2
4
3
1
5
6
0.0%
2.0%
4.0%
6.0%
8.0%
10.0%
Standard Deviation of IRR



Risk vs. Return Graph
15%
10%
5%
0%
Expected Value of IRR
F
3
2
1
4
6
5
0.0%
2.0%
4.0%
6.0%
8.0%
Standard Deviation of IRR

Chapter 11: Income, Depreciation, and Cash Flow

11-1

Year	SOYD	DDB
1	$2,400	$3,333
2	$2,000	$2,222
3	$1,600	$1,482
4	$1,200	$988
5	$800	$375*
6	$400	$0
Sum	$8,400	$8,400

*Computed $658 must be reduced to $375 to avoid depreciating the asset below its salvage value.

11-2

DDB Schedule is:

Year n	d(n)=(2/n)[P – sum d(n)]	DDB Depreciation
1	(2/6) ($1,000,000 - $0)	= $333,333
2	(2/6) ($1,000,000 - $333,333)	= $222,222
3	(2/6) ($1,000,000 - $555,555)	= $148,148
4	(2/6) ($1,000,000 - $703,703)	= $98,766
5	(2/6) ($1,000,000 - $802,469)	= $65,844
6	See below	= $56,687

If switch DDB to SL for year 5:

SL = ($1,000,000 - $802,469 - $75,000)/2 = $61,266

Do not switch.

If switch DDB to SL for year 6:

SL = ($1,000,000 - $868,313 - $75,000)/1 = $56,687

Do switch.

Sum-of-Years Digits Schedule is:

SOYD in N = [(Remain. useful life at begin. of yr.)/[(N/2)(N+1)]] (P – S)

1^{st} Year:	SOYD	= (6/21) ($1 mil - $75,000)	= $264,286
2^{nd} Year:		= (5/21) ($1 mil - $75,000)	= $220,238
3^{rd} Year:		= (4/21) ($1 mil - $75,000)	= $176,190
4^{th} Year:		= (3/21) ($1 mil - $75,000)	= $132,143
5^{rd} Year:		= (2/21) ($1 mil - $75,000)	= $ 88,095
6^{th} Year:		= (1/21) ($1 mil - $75,000)	= $ 44,048

Question: Which method is preferred?

Answer: It depends, on the MARR%, i% used by the firm (individual)

As an example:

If i% is=	PW of DDB is=	PW of SOYD is=	Preferred is
0%	$925,000	$925,000	Equal, same
2%	$881,211	$877,801	DDB
10%	$738,331	$724,468	DDB
25%	$561,631	$537,130	DDB

Thus, if MARR% is > 0%, DDB is best. One can also see this by inspection of the depreciation schedules above.

11-3

DDB Depreciation

Year		DDB Depreciation
1	(2/5) ($16,000 - $0)	= $6,400
2	(2/5) ($16,000 - $6,400)	= $3,840
3	(2/5) ($16,000 - $10,240)	= $2,304
4	(2/5) ($16,000 - $13,926)	= $830
Sum		$14,756

Converting to Straight Line Depreciation

If Switch for Year	Beginning of Yr Book Value	Remaining Life	SL = (Book – Salvage)/Remaining Life	Decision
2	$9,600	4 yrs	$2,400	Do not switch
3	$5,760	3 yrs	$1,920	Do not switch
4	$3,456	2 yrs	$1,728	Switch to SL
5	$2,074	1 yr	$2,074	

Resulting Depreciation Schedule:

Year	DDB with Conversion to Straight Line
1	$6,400
2	$3,840
3	$2,304
4	$1,728
5	$1,728
Sum	$16,000

11-4

P=$12,000 **S=$3,500** **N=4**

(a) **Straight Line Depreciation**

SL = -(P-S) / N =($12,000-$3,500) / 4
$2,125

(b) **Sum-of-Years Digits Depreciation**

SOYD in yr. N = [(Remain. useful life at begin. of yr.)/[(N/2)(N+1)]] (P – S)

1st Year:	SOYD	= (4/10) ($12,000 - $3,500)=	$3,400
2nd Year:		= (3/10) ($12,000 - $3,500)=	$2,550
3rd Year:		= (2/10) ($12,000 - $3,500)=	$1,700
4th Year:		= (1/10) ($12,000 - $3,500)=	$850

(c) **Double Declining Balance Depreciation**

DDB in any year = 2/N(BookValue)

DDB in any year = 2/N (Book Value)

1st Year:	DDB	= (2/4) ($12,000 - $0)	= $6,000
	2nd Year:	= (2/4) ($12,000 - $6,000)	= $3,000
	3rd Year:	= (2/4) ($12,000 - $9,000)	= $1,500
	4th Year:	= (2/4) ($12,000 - $10,500)	= $750

(d) **CCA**

Special handling equipment classifies as a Class 43 asset with a CCA rate of 30%

Year	UCC at Start of Year	CCA Rate	Depreciation Charge for year t = d_t	UCC at end of year t
1	$ 12,000	15%	$ 1,800	$ 10,200
2	$ 10,200	30%	$ 3,060	$ 7,140
3	$ 7,140	30%	$ 2,142	$ 4,998
4	$ 4,998	30%	$ 1,499	$ 3,499

11-5

The computations for the first three methods (SL, DB, SOYD) are similar to Problem 11-4.

(d) Furniture: Class 8 (CCA rate=20%)

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	8	0	50,000.00	0	50,000.00	25,000.00	25,000.00	20	5,000.00	45,000.00
2	8	45,000.00	0	0	45,000.00	0.00	45,000.00	20	9,000.00	36,000.00
3	8	36,000.00	0	0	36,000.00	0.00	36,000.00	20	7,200.00	28,800.00
4	8	28,800.00	0	0	28,800.00	0.00	28,800.00	20	5,760.00	23,040.00
5	8	23,040.00	0	0	23,040.00	0.00	23,040.00	20	4,608.00	18,432.00
6	8	18,432.00	0	0	18,432.00	0.00	18,432.00	20	3,686.40	14,745.60
7	8	14,745.60	0	0	14,745.60	0.00	14,745.60	20	2,949.12	11,796.48
8	8	11,796.48	0	0	11,796.48	0.00	11,796.48	20	2,359.30	9,437.18
9	8	9,437.18	0	0	9,437.18	0.00	9,437.18	20	1,887.44	7,549.75
10	8	7,549.75	0	0	7,549.75	0.00	7,549.75	20	1,509.95	6,039.80

Summary of Methods

Year	SL	DDB	SOYD	CCA
1	$5,000	$10,000	$9,091	$5,000.00
2	$5,000	$8,000	$8,182	$9,000.00
3	$5,000	$6,400	$7,273	$7,200.00
4	$5,000	$5,120	$6,364	$5,760.00
5	$5,000	$4,096	$5,455	$4,608.00
6	$5,000	$3,277	$4,545	$3,686.40
7	$5,000	$2,621	$3,636	$2,949.12
8	$5,000	$2,097	$2,727	$2,359.30
9	$5,000	$1,678	$1,818	$1,887.44
10	$5,000	$1,342	$909	$1,509.95

11-6

(a)

Year	SL	SOYD		DDB
1	$15,200	$25,333	(2/5) ($76,000 - $0)	= $30,400
2	$15,200	$20,267	(2/5) ($76,000 - $30,400)	= $18,240
3	$15,200	$15,200	(2/5) ($76,000 - $48,640)	= $10,944
4	$15,200	$10,133	(2/5) ($76,000 - $59,584)	= $6,566
5	$15,200	$5,067	(2/5) ($76,000 - $66,150)	= $3,940
Sum	$76,000	$76,000		$70,090

(b) By looking at the data in Part (a), some students may jump to the conclusion that one should switch from DDB to Straight Line depreciation at the beginning of Year 3. This mistaken view is based on the fact that in the table above the Straight Line depreciation for Year 3 is $15,2000, while the DDB depreciation is only $10,944. This is not a correct analysis of the situation.

This may be illustrated by computing the Straight Line depreciation for Year 3, if DDB depreciation had been used in the prior years.
With DDB depreciation for the first two years, the book value at the beginning of Year 3 = $76,000 - $30,400 - $18,240 = $27,360.

SL depreciation for subsequent years = ($27,360 - $0)/3 = $9,120.

Thus, the choice for Year 3 is to use DDB = $10,944 or SL = $9,120. One would naturally choose to continue with DDB depreciation.

For subsequent years:

If Switch for Year	Beginning of Yr Book Value	Remaining Life	SL = (Book – Salvage)/Remaining Life
4	$16,416	2 yrs	$8,208
5	$9,850	1 yr	$9,850

When SL is compared to DDB in Part (a), it is apparent that the switch should take place at the beginning of Year 4. The resulting depreciation schedule is:

Year	DDB with Conversion to Straight Line
1	$30,400
2	$18,240
3	$10,944
4	$8,208
5	$8,208
Sum	$76,000

(c) CCA rate =30%

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	43	0	76,000.00	0	76,000.00	38,000.00	38,000.00	20	7,600.00	68,400.00
2	43	68,400.00	0	0	68,400.00	0.00	68,400.00	20	13,680.00	54,720.00
3	43	54,720.00	0	0	54,720.00	0.00	54,720.00	20	10,944.00	43,776.00
4	43	43,776.00	0	0	43,776.00	0.00	43,776.00	20	8,755.20	35,020.80
5	43	35,020.80	0	0	35,020.80	0.00	35,020.80	20	7,004.16	28,016.64

Salvage value=0, therefore loss on disposal=$28,016.64

11-7

(a) Straight Line

SL depreciation in any year = ($45,000 - $0)/5 = $9,000

(b) SOYD

sum = (n/2) (n+1) = (5/2) (5) = 15

Depreciation in Year 1 = (5/15) ($45,000 - $0) = $15,000

Gradient = (1/15) ($45,000 - $0) = -$3,000

(c) DDB

Year		DDB
1	(2/5) ($45,000 - $0)	= $18,000
2	(2/5) ($45,000 - $18,000)	= $10,800
3	(2/5) ($45,000 - $28,800)	= $6,480
4	(2/5) ($45,000 - $35,280)	= $3,888
5	(2/5) ($45,000 - $39,168)	= $2,333

(d) CCA Class 8 asset (CCA rate=20%)

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	8	0	45,000.00	0	45,000.00	22,500.00	22,500.00	20	4,500.00	40,500.00
2	8	40,500.00	0	0	40,500.00	0.00	40,500.00	20	8,100.00	32,400.00
3	8	32,400.00	0	0	32,400.00	0.00	32,400.00	20	6,480.00	25,920.00
4	8	25,920.00	0	0	25,920.00	0.00	25,920.00	20	5,184.00	20,736.00
5	8	20,736.00	0	0	20,736.00	0.00	20,736.00	20	4,147.20	16,588.80

Salvage value=0, therefore loss on disposal=$16,588.80

Summary of Depreciation Schedules

Year	**SL**	**DDB**	**SOYD**	**CCA**
1	$9,000	$18,000	$15,000	$4,500.00
2	$9,000	$10,800	$12,000	$8,100.00
3	$9,000	$6,480	$9,000	$6,480.00
4	$9,000	$3,888	$6,000	$5,184.00
5	$9,000	$2,333	$3,000	$4,147.20
Sum	$45,000	$41,501	$45,000	$28,411.20

11-8

CCA rate=30%

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	8	0	6,500.00	0	6,500.00	3,250.00	3,250.00	30	975.00	5,525.00
2	8	5,525.00	0	0	5,525.00	0.00	5,525.00	30	1,657.50	3,867.50
3	8	3,867.50	0	0	3,867.50	0.00	3,867.50	30	1,160.25	2,707.25
4	8	2,707.25	0	0	2,707.25	0.00	2,707.25	30	812.18	1,895.08
5	8	1,895.08	0	0	1,895.08	0.00	1,895.08	30	568.52	1,326.55

Salvage value=$1,200, therefore loss on disposal=$126.55

Year	SL	SOYD	DDB	UOP*	CCA	Year
1	$1,060	$1,767	$2,600	$707	$975.00	1
2	$1,060	$1,413	$1,560	$1,178	$1,657.50	2
3	$1,060	$1,060	$936		$1,160.25	3
4	$1,060	$707	$204		$812.18	4
5	$1,060	$353	$0		$568.52	5
Sum	$5,300	$5,300	$5,300		$5,174.45	

*UOP Depreciation is based on actual production.

11-9

Class 9 asset (CCA rate=25%)

Year	Class No.	Undep. capital cost at beginning of year ($1,000)	Cost of acq. during the year ($1,000)	Proceeds of disp. during the year ($1,000)	Undep. capital cost ($1,000)	50% rule ($1,000)	Reduced undep. capital cost ($1,000)	CCA rate %	Capital cost allowance ($1,000)	Undep. Capital Cost at end of year ($1,000)
1	9	0	1,500.00	0	1,500.00	750.00	750.00	25	187.50	1,312.50
2	9	1,312.50	0	0	1,312.50	0.00	1,312.50	25	328.13	984.38
3	9	984.38	0	0	984.38	0.00	984.38	25	246.09	738.28
4	9	738.28	0	0	738.28	0.00	738.28	25	184.57	553.71
5	9	553.71	0	0	553.71	0.00	553.71	25	138.43	415.28

11-10

CCA class 8 (CCA rate 20%)

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	8	0	10,000.00	0	10,000.00	5,000.00	5,000.00	20	1,000.00	9,000.00
2	8	9,000.00	0	0	9,000.00	0.00	9,000.00	20	1,800.00	7,200.00
3	8	7,200.00	0	0	7,200.00	0.00	7,200.00	20	1,440.00	5,760.00
4	8	5,760.00	0	0	5,760.00	0.00	5,760.00	20	1,152.00	4,608.00
5	8	4,608.00	0	0	4,608.00	0.00	4,608.00	20	921.60	3,686.40
6	8	3,686.40	0	0	3,686.40	0.00	3,686.40	20	737.28	2,949.12
7	8	2,949.12	0	0	2,949.12	0.00	2,949.12	20	589.82	2,359.30
8	8	2,359.30	0	0	2,359.30	0.00	2,359.30	20	471.86	1,887.44
9	8	1,887.44	0	0	1,887.44	0.00	1,887.44	20	377.49	1,509.95
10	8	1,509.95	0	0	1,509.95	0.00	1,509.95	20	301.99	1,207.96

11-11

Class 43 asset (CCA rate=30%)

Year	Class No.	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	43	0	75,000.00	0	75,000.00	37,500.00	37,500.00	30	11,250.00	63,750.00
2	43	63,750.00	0	0	63,750.00	0.00	63,750.00	30	19,125.00	44,625.00
3	43	44,625.00	0	0	44,625.00	0.00	44,625.00	30	13,387.50	31,237.50
4	43	31,237.50	0	0	31,237.50	0.00	31,237.50	30	9,371.25	21,866.25
5	43	21,866.25	0	0	21,866.25	0.00	21,866.25	30	6,559.88	15,306.38
6	43	15,306.38	0	0	15,306.38	0.00	15,306.38	30	4,591.91	10,714.46
7	43	10,714.46	0	0	10,714.46	0.00	10,714.46	30	3,214.34	7,500.12
8	43	7,500.12	0	0	7,500.12	0.00	7,500.12	30	2,250.04	5,250.09
9	43	5,250.09	0	0	5,250.09	0.00	5,250.09	30	1,575.03	3,675.06
10	43	3,675.06	0	0	3,675.06	0.00	3,675.06	30	1,102.52	2,572.54

11-12

Initial Cost	145
Useful Life	5
Salvage Value	10
CCA Rate	0.3

Year	**SL**	**SOYD**	**150%DB**	**DDB**	**CCA**
1	$27.00	$45.00	$43.50	$58.00	$21.75
2	$27.00	$36.00	$30.45	$34.80	$36.98
3	$ 27.00	$27.00	$21.32	$20.88	$25.88
4	$27.00	$18.00	$14.92	$12.53	$18.12

5	$27.00	$9.00	$10.44	$7.52	$12.68
6	$---	$---	$---	$---	$---
7	$---	$---	$---	$---	$---
SV-UCC	$---	$---	$14.37	$1.28	$19.59
TOTAL	**$135.00**	**$135.00**	**$120.63**	**$133.72**	**$115.41**

Column B is probably a unit of production.

11-13

Initial Cost	1060
Useful Life	5
Salvage Value	90
CCA Rate	40%

Year	SL	SOYD	150%DB	DDB	CCA
1	$194.00	$323.33	$318.00	$424.00	$212.00
2	$194.00	$258.67	$222.60	$254.40	$339.20
3	$194.00	$194.00	$155.82	$152.64	$203.52
4	$194.00	$129.33	$109.07	$ 91.58	$122.11
5	$194.00	$ 64.67	$ 76.35	$ 54.95	$ 73.27
6	$---	$---	$---	$---	$---
7	$---	$---	$---	$---	$---
SV-UCC	$---	$---	$ 88.15	$ (7.57)	$ 19.90
TOTAL	**$970.00**	**$970.00**	**$881.85**	**$977.57**	**$950.10**

Column E is probably a unit of production.

11-14

Initial Cost	$8,000
Useful Life	6
Salvage Value	$600
CCA Rate	40%

Year	SL	SOYD	150%DB	DDB	CCA
1	$1,233	$2,114	$2,000	$2,667	$1,600
2	$1,233	$1,762	$1,500	$1,778	$2,560
3	$1,233	$1,410	$1,125	$1,185	$1,536
4	$1,233	$1,057	$ 844	$ 790	$ 922
5	$1,233	$ 705	$ 633	$ 527	$ 553
6	$1,233	$ 352	$ 475	$ 351	$ 332
7	$---	$---	$---	$---	$---
SV-UCC	$0	$---	$ 824	$ 102	$ (102)
TOTAL	**$7,400**	**$7,400**	**$6,576**	**$7,298**	**$7,502**

11-15

Comparison Worksheet

Initial Cost	$ 250,000
Useful Life	15
Salvage Value	0
CCA Rate	0.1

depreciable amount= $250,000

						Book Values				
Year	SL	SOYD	150%DB	DDB	CCA	SL	SOYD	150%DB	DDB	CCA
1	$16,667	$31,250	$25,000	$33,333	$12,500	$ 233,333	$218,750	$ 225,000	$ 216,667	$237,500
2	$16,667	$29,167	$22,500	$28,889	$23,750	$ 216,667	$189,583	$ 202,500	$ 187,778	$213,750
3	$16,667	$27,083	$20,250	$25,037	$21,375	$ 200,000	$162,500	$ 182,250	$ 162,741	$192,375
4	$16,667	$25,000	$18,225	$21,699	$19,238	$ 183,333	$137,500	$ 164,025	$ 141,042	$173,138
5	$16,667	$22,917	$16,403	$18,806	$17,314	$ 166,667	$114,583	$ 147,623	$ 122,236	$155,824
6	$16,667	$20,833	$14,762	$16,298	$15,582	$ 150,000	$ 93,750	$ 132,860	$ 105,938	$140,241
7	$16,667	$18,750	$13,286	$14,125	$14,024	$ 133,333	$ 75,000	$ 119,574	$ 91,813	$126,217
8	$16,667	$16,667	$11,957	$12,242	$12,622	$ 116,667	$ 58,333	$ 107,617	$ 79,571	$113,596
9	$16,667	$14,583	$10,762	$10,610	$11,360	$ 100,000	$ 43,750	$ 96,855	$ 68,962	$102,236
10	$16,667	$12,500	$ 9,686	$ 9,195	$10,224	$ 83,333	$ 31,250	$ 87,170	$ 59,767	$ 92,012
11	$16,667	$10,417	$ 8,717	$ 7,969	$ 9,201	$ 66,667	$ 20,833	$ 78,453	$ 51,798	$ 82,811
12	$16,667	$ 8,333	$ 7,845	$ 6,906	$ 8,281	$ 50,000	$ 12,500	$ 70,607	$ 44,892	$ 74,530
13	$16,667	$ 6,250	$ 7,061	$ 5,986	$ 7,453	$ 33,333	$ 6,250	$ 63,547	$ 38,906	$ 67,077
14	$16,667	$ 4,167	$ 6,355	$ 5,187	$ 6,708	$ 16,667	$ 2,083	$ 57,192	$ 33,719	$ 60,369
15	$16,667	$ 2,083	$ 5,719	$ 4,496	$ 6,037	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
16	$---	$---	$---	$---	$---	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
17	$---	$---	$---	$---	$---	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
18	$---	$---	$---	$---	$---	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
19	$---	$---	$---	$---	$---	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
20	$---	$---	$---	$---	$---	$ 0	$ (0)	$ 51,473	$ 29,223	$ 54,332
SV-UCC	$ 0	$---	$51,473	$29,223	$ 54,332					
TOTAL	$250,000	$250,000	$198,527	$220,777	$195,668					

i=	$ 0.2				
NPW of deprec=	$ 97,456.2	$ 127,119.9	$ 97,469.7	$ 115,957.0	$ 90,807.4
NPW of balance	$ 0.0	$ -	$ 6,325.7	$ 3,591.3	$ 6,677.2
Total NPW	**$ 97,456.2**	**$ 127,119.9**	**$ 103,795.4**	**$ 119,548.3**	**$ 97,484.6**

CCA gives greater NPW than SL

11-16

	Comparison Worksheet	
	Initial Cost	**$100,000**
	Useful Life	**5**
	Salvage Value	**$20,000**
	CCA Rate	**30%**

Year	**SL**	**SOYD**	**DDB**	**CCA**
1	$16,000	$26,667	$40,000	$15,000
2	$16,000	$21,333	$24,000	$25,500
3	$16,000	$16,000	$14,400	$17,850
4	$16,000	$10,667	$ 8,640	$12,495
5	$16,000	$ 5,333	$ 1,600	$ 8,747
SV-UCC	$---	$---		$ 409
TOTAL	**$80,000**	**$80,000**	**$80,000**	**$80,000**
i =	**10%**			
NPW of deprec=	$60,653	$64,491	$73,912	$62,087
NPW of balance	$---	$---		$ 254
Total NPW	**$60,653**	**$64,491**	**$73,912**	**$62,341**

The DDB method gives the highest NPW

11-17

SOYD Depreciation

Sum = (5/2) (5 + 1) = 15

Year		SOYD	PW at Yr 0 at 8%
1	(5/15) ($120,000)	= $40,000	$37,036
2	(4/15) ($120,000)	= $32,000	$27,434
3	(3/15) ($120,000)	= $24,000	$19,051
4	(2/15) ($120,000)	= $16,000	$11,760
5	(1/15) ($120,000)	= $8,000	$5,445
Sum		= $120,000	= $100,726

Unit of Production Depreciation

Year		UOP	PW at Yr 0 at 8%
1	($15,000/$40,000) ($120,000)	= $45,000	$41,666
2	($11,000/$40,000) ($120,000)	= $33,000	$28,291
3	($4,000/$40,000) ($120,000)	= $12,000	$9,526
4	($6,000/$40,000) ($120,000)	= $18,000	$13,230
5	($4,000/$40,000) ($120,000)	= $12,000	$8,167
Sum		= $120,000	= $100,880

To maximize PW at Year 0, choose UOP depreciation.

11-18

(a) DDB

Year		DDB
1	(2.4)($10,000 - $0)	= $5,000
2	(2/4)($10,000 - $5,000)	= $2,500

2nd year depreciation - $2,500

(b) SOYD
2nd year SOYD = (3/10) = $3,000

(c) CCA – Class 12 Assets are at 100% rate
Because of ½ year rule, this is really 50% per year

2nd year CCA = 50% x ($10,000) = $5,000

11-19

See solution to 11-8.

11-20

See solution to 11-15.

11-21

(1) Use Table 11-1 to find the MACRS GDS Property Class for each asset:
(a) CCA class 10 asset
(b) CCA class 43 asset
(c) CCA class 1 asset

(2) Depreciation in year 3
(a) Dep_3=$3,034.50
(b) Dep_3=$5,355.00
(c) Dep_3=$4,892.16

(3) Book Value = Cost Basis – Sum of Depreciation Charges
(a) $17,000 - $14,571.39 = $2,428.61
(b) $30,000 - $25,714.22 = $4,285.78
(c) $130,000 - $26,121.52 = $103,878.48

11-22

Year	Class No.	Undep. capital cost at beginning of year ($1,000)	Cost of acq. during the year ($1,000)	Proceeds of disp. during the year ($1,000)	Undep. capital cost ($1,000)	50% rule ($1,000)	Reduced undep. capital cost ($1,000)	CCA rate %	Capital cost allowance ($1,000)	Undep. Capital Cost at end of year ($1,000)
1	1	0	600.00	0	600.00	300.00	300.00	4	12.00	588.00
2	1	588.00	0	0	588.00	0.00	588.00	4	23.52	564.48
3	1	564.48	0	0	564.48	0.00	564.48	4	22.58	541.90
4	1	541.90	0	0	541.90	0.00	541.90	4	21.68	520.22
5	1	520.22	0	0	520.22	0.00	520.22	4	20.81	499.42

UCC at end of 5 years=$600,000-100,584.22=$499,416

Gain on disposal=$100,584

11-23

Year	Class No.	Undep. capital cost at beginning of year ($1,000)	Cost of acq. during the year ($1,000)	Proceeds of disp. during the year ($1,000)	Undep. capital cost ($1,000)	50% rule ($1,000)	Reduced undep. capital cost ($1,000)	CCA rate %	Capital cost allowance ($1,000)	Undep. Capital Cost at end of year ($1,000)
1	1	0	850.00	0	850.00	425.00	425.00	4	17.00	833.00
2	1	833.00	0	0	833.00	0.00	833.00	4	33.32	799.68
3	1	799.68	0	0	799.68	0.00	799.68	4	31.99	767.69
4	1	767.69	0	0	767.69	0.00	767.69	4	30.71	736.99

UCC at end of year 4=$850,000-113,015 = $736,985

11-24

Same as above.

11-25

(a) $EUAC_I$ = (P – S) (A/P, i%, n) + Si + Annual operating cost
= ($80,000 - $20,000) (A/P, 10%, 20) + $20,000 (0.10) + $18,000
= $60,000 (0.1175) + $2,000 + $18,000
= $27,050

$EUAC_{II}$ = ($100,000 - $25,000) (A/P, 10%, 25) + $25,000 (0.10) + $20,000
- $5,000 (P/A, 10%, 10) (A/P, 10%, 25)
= $75,000 (0.1102) + $2,500 + $20,000 - $5,000 (6.145) (0.1102)
= $27,380

To minimize EUAC, select Machine II.

(b) Capitalized Cost of Machine I = PW of an infinite life = EUAC/i
In part (a), EUAC = \$27,050, so:
Capitalized Cost = \$27,050/0.10 = \$270,500

(c) Fund to replace Machine I
Required future sum F = \$80,000 - \$20,000 = \$60,000
Annual Deposit A = \$60,000 (A/P, 10%, 20)
= \$60,000 (0.0175) = \$1,050

(d)

Year	Cash Flow
0	-\$80,000
1- 20	+\$28,000
	-\$18,000
20	+\$20,000

\$80,000 = (\$28,000 - \$18,000) (P/A, i%, 20) + \$20,000 (P/F, i%, 20)

Solve by trial and error:

Try i = 10%
(\$10,000) (8.514) + \$20,000 (0.1486) = \$88,112 ≠ \$80,000

Try i = 12%
(\$10,000) (7.469) + \$20,000 (0.1037) = \$76,764 ≠ \$80,000

Rate of Return = 10% + (2%) [(\$88,112 - \$80,000)/(\$88,112 - \$76,764)]
= 11.4%

(e) SOYD depreciation
Book value of Machine I after two periods

Dep. Charge in any year = (Remain. useful life at beginning of yr/SOYD for total useful life)(P – S)

Sum of years digits = (n/2) (n + 1) = 20/2 (20 + 1) = 210

1st Year depreciation = (20/210) (\$80,000 - \$20,000) = \$5,714
2nd Year depreciation = (19/210) (\$80,000 - \$20,000) = \$5,429
Sum = \$11,143

Book value = Cost – depreciation to date
= \$80,000 - \$11,143
= \$68,857

(f) DDB Depreciation
Book value of Machine II after three years

Depreciation charge in any year
= (2/n) (P – Depreciation charge to date)

1st Year Depreciation = (2/25) ($100,000 - $0) = $8,000
2nd Year Depreciation = (2/25) ($100,000 - $8,000) = $7,360
3rd Year Depreciation = (2/25) ($100,000 - $15,360) = $6,771
Sum = $22,131

Book Value = Cost – Depreciation to date
= $100,000 - $22,131
= $77,869

(g) CCA depreciation (class 43 asset)

Year	Class No.	Undep. capital cost at beginning of year ($1,000)	Cost of acq. during the year ($1,000)	Proceeds of disp. during the year ($1,000)	Undep. capital cost ($1,000)	50% rule ($1,000)	Reduced undep. capital cost ($1,000)	CCA rate %	Capital cost allowance ($1,000)	Undep. Capital Cost at end of year ($1,000)
1	43	0	100.00	0	100.00	50.00	50.00	30	15.00	85.00
2	43	85.00	0	0	85.00	0.00	85.00	30	25.50	59.50
3	43	59.50	0	0	59.50	0.00	59.50	30	17.85	41.65

11-26

Year	Undep. capital cost at beginning of year	Cost of acq. during the year	Proceeds of disp. during the year	Undep. capital cost	50% rule	Reduced undep. capital cost	CCA rate %	Capital cost allowance	Undep. Capital Cost at end of year
1	0	20,000.00	0	20,000.00	10,000.00	10,000.00	30	3,000.00	17,000.00
2	17,000.00	0	0	17,000.00	0.00	17,000.00	30	5,100.00	11,900.00

Loss on disposal = 17,000-14,000 = $3,000

11-27

		CCA	SL
	P=	$50,000	$50,000
	dep yr 1	$ 7,500	$6,250
	dep yr 2	$ 12,750	$6,250
UCC at end of 3 rd year (before 3rd yr CCA taken) =		**$29,750**	**$37,500**
	selling price	recapture (loss)	
	15000	**$(14,750)**	**$(22,500)**
	25000	**$(4,750)**	**$(12,500)**
	60000	**$20,250**	**$12,500**

11-28

Gross income from sand and gravel
$0.65/m^3$ (45,000 m^4) = \$29,250
To engineering student - \$2,500
Taxable Income inc. depletion = \$26,750

Percentage depletion = 5% (\$29,250) = \$1,462.50
Therefore, allowable depletion is \$1,462.50.

11-29

Mr. Salt's cost of depletion = \$45,000 (1,000 Bbl/15,000Bbl) = \$3,000

Percentage depletion = 15% (\$12,000) = \$1,800, but limited to 50% of taxable income before depletion or
= 50% (\$12,000 - \$3,000) = \$4,500

Therefore, allowable depletion = \$3,000.00.

11-30

(a) **SOYD Depreciation**

N = 8
SUM = (N/2) (N+1) = 36

1st Year SOYD Depreciation = (8/36) (\$600,000 - \$60,000)
= \$120,000

Subsequent years are a declining gradient:
G = (1/36) (\$600,000 - \$60,000)
= \$15,000

Year	SOYD Depreciation
1	\$120,000
2	\$105,000
3	\$90,000
4	\$75,000
5	\$60,000
6	\$45,000
7	\$30,000
8	\$15,000
Sum	\$540,000

(b) **Unit of Production (UOP) Depreciation**

Depreciation/hour = \$540,000/21,600 hours = \$25/hr

Year	Utilization hrs/yr	UOP Depreciation

1	6,000	$150,000
2	4,000	$100,000
3	4,000	$100,000
4	1,600	$40,000
5	800	$20,000
6	800	$20,000
7	2,200	$55,000
8	2,200	$55,000
Sum		$540,000

11-31

	A	B	C	D	E
1	1,000	First cost			
2	50	Salvage	(not used in MACRS calculations)		
3	5	Life			
4	10%	interest rate			
5					
6		Straight line		Sum-of-years digits	
7	Period	Depr	BookValue	Depr	BookValue
8	1	$190	810	$317	683
9	2	$190	620	$253	430
10	3	$190	430	$190	240
11	4	$190	240	$127	113
12	5	$190	50	$63	50
13	NetPW(10%)	$720		$766	
14					
15	Thus, SOYD is better than straight line, since the PW of				
16	the depreciation deduction for SOYD is $46 higher.				
17					

11-32

	A	B	C	D	E	F
1	12,000	First cost				
2	400	Salvage	(not used in MACRS calculations)			
3	5	Life				
4	200%	Factor				
5	7%	interest rate				
6						
7		Declining balance		Sum-of-years digits		
8	Period	Depr	BookValue	Depr	BookValue	
9	1	$4,800	7,200	$3,867	8,133	
10	2	$2,880	4,320	$3,093	5,040	
11	3	$1,728	2,592	$2,320	2,720	
12	4	$1,037	1,555	$1,547	1,173	
13	5	$622	933	$773	400	
14	NetPW(7%)	$9,647		$9,941		
15						
16	Without switch to straight-line, declining balance BV5 is $533 high					
17	Thus, SOYD is better than DB without switching since					
18	depreciation deduction for SOYD has a higher present worth.					
19						

11-33

See solution to problem 11.16

11-34

Comparison Worksheet	
Initial Cost	100000
Useful Life	6
Salvage Value	10000
CCA Rate	20%

Year	SL	BV	Sum Of Year	BV	CCA	UCC
1	$15,000	$85,000	$25,714	$74,286	$10,000	$90,000
2	$15,000	$70,000	$21,429	$52,857	$18,000	$72,000
3	$15,000	$55,000	$17,143	$35,714	$14,400	$57,600
4	$15,000	$40,000	$12,857	$22,857	$11,520	$46,080
5	$15,000	$25,000	$ 8,571	$14,286	$ 9,216	$36,864
6	$15,000	$10,000	$ 4,286	$10,000	$ 7,373	$29,491

11-35

Year	CCA
1	$175,000
2	$315,000
3	$252,000
4	$201,600
5	$161,280
6	$129,024
7	$103,219
8	$ 82,575
9	$ 66,060
10	$ 52,848

Chapter 12: After-Tax Cash Flows

12-1

(a) Federal Taxes

On the first $35,000	16%	$5,600.00
From $35,000 to $70,000	22%	$5,940.00
		$11,540.00

Taxable income $62,000	Federal taxes=$11,540
Non-refundable tax credit=$8,000×16%=	$1,280.00
Federal taxes payable=	**$10,260.00**

(b) Increase in federal taxes after increase of $16,000

On the first $35,000	16%	$5,600.00
From $35,000 to $70,000	22%	$7,700.00
From $70,001 to $113,804	26%	$2,079.74
		$13,300.00

Taxable income $78,000	Federal taxes=$15,380
Non-refundable tax credit=$8,000×16%=	$1,280.00
Federal taxes payable=	$14,100.00
Increase in federal taxes=	**$3,840.00**

12-2

(a) Alberta resident

Federal Taxes

On the first $35,000	16%	$5,600.00
From $35,000 to $70,000	22%	$2,200.00
		$7,800.00

Taxable income $62,000	Federal taxes=$7,800
Non-refundable tax credit=$6,000×16%=	$960.00
Federal taxes payable=	**$6,840.00**

Provincial taxes:
Taxable income=$45,000
Non-refundable tax credit=$6,000

On all income 10%	$4,500.00

Total taxes= $11,340.00

(b) Ontario resident

Federal taxes payable=$6,840

Provincial taxes:

On the first $33,375	6.05%	$2,019.19
From $33,375 to $66,752	9.15%	$1,063.69
		$3,082.88

Total taxes= $9,922.88

12-3

Problem 12.3

Federal Taxes:

On the first $35,000	16%	$1,248.00
		$1,248.00

Non-refundable tax credit=$6,000×16% $960.00

Federal taxes payable= $288.00

Provincial taxes:

On the first $30,544	10.90%	$850.20
		$850.20

Total taxes= $1,138.20

Net Income=$7,800-$1,138.2= $6,661.80

Upon increase in income of $2,600 (gross income=$7,800+$2,600=$10,400)

Federal Taxes:

On the first $35,000	16%	$1,664.00
		$1,664.00

Non-refundable tax credit=$6,000×16% $960.00

Federal taxes payable= $704.00

Provincial taxes:

On the first $30,544	10.90%	$1,133.60
		$1,133.60

Total taxes= $1,837.60

Net Income=$7,800+$2,600-$1,837.60
$8,562.40

Increase in net income=$8,562.40-$6,661.80
$1,900.60

12-4

Federal Taxes:

On the first $35,000	16%	$1,248.00
		$1,248.00

Non-refundable tax credit=$6,000×16%= $960.00

Federal taxes payable=		**$288.00**
Provincial taxes:		
On all income	10.00%	$780.00
		$780.00
Total taxes=	**$1,068.00**	
Net Income=$7,800 – $1,068.2=		$6,732.00
Upon increase in income of $2,600		(gross income=$7,800+$2,600=$10,400)
Federal Taxes:		
On the first $35,000	16%	$1,664.00
		$1,664.00
Non-refundable tax credit=$6,000×16%=		$960.00
Federal taxes payable=		**$704.00**
Provincial taxes:		
On all income	10.00%	$1,040.00
		$1,040.00
Total taxes=	**$1,744.00**	
Net Income=$7,800+$2,600 – $1,837.60		
	$8,656.00	
Increase in net income=$8,656.00 – $6,732.00		
	$1,924.00	

12-5

PROBLEM 12-5 **DATA**

Taxable Income	**$**	**65,000.00**
Non-refundable tax credit	**$**	**8,000.00**
Province		

2004 Federal Personal rates						Amount	Total per Level	Tax per Level
		On the first	$ 35,000.00	16.00%	$ 5,600.00	$ 5,600.00	**$5,600.00**
from	$ 35,000.00	to	$ 70,000.00	22.00%	$ 7,700.00	$ 13,300.00	**$ 6,600.00**
from	$ 70,001.00	to	$ 113,804.00	26.00%	$11,388.78	$ 24,688.78	**$ -**
above	$ 113,805.00			29.00%			**$ -**
							$12,200.00

Taxable Income	**$ 65,000.00**	Fed tax =	**$12,200.00**	Fed Avg Tax=	16.80%
Non-refundable tax credit	**$ 8,000.00**	x 16% =	$1,280.00	Fed Marg Tax=	22.00%
		Total Taxes Payable =	**$10,920.00**		

2004 Ontario Provincial Tax							Tax Amdt.
		On the first	$ 33,375.00	6.05%	$ 2,019.19	$ 2,019.19	**$ 2,019.19**
from	$ 33,376.00	to	$ 66,752.00	9.15%	$ 3,053.90	$ 5,073.09	**$ 2,893.60**
above	$ 66,753.00			11.2%			**$ -**
		surtax	Surtax on Prov Tax over				$ 4,912.78

		$3,685	20.0%			**$ 245.56**
		4864	36.0%			**$ 17.56**
	Ontario tax =	**$5,175.90**		Avg Ont Tax=	7.96%	
	Combined tax =	$16,095.90		Avg Combined Tax=	24.76%	
Taxable Income	$ 22,000.00					
Non-refundable tax credit	$ 8,000.00					
Province						

2004 Federal Personal rates

					Amount	Total per Level	Tax per Level
		On the first	$ 35,000.00	16.00%	$ 5,600.00	$ 5,600.00	**$3,520.00**
from	$ 35,000.00	to	$ 70,000.00	22.00%	$ 7,700.00	$ 13,300.00	**$ -**
from	$ 70,001.00	to	$ 113,804.00	26.00%	$11,388.78	$ 24,688.78	**$ -**
above	$ 113,805.00			29.00%			**$ -**
							$3,520.00

Taxable Income	**$ 22,000.00**	Fed tax =	**$3,520.00**	Fed Avg Tax=	10.18%
Non-refundable tax credit	**$ 8,000.00**	x 16% =	$1,280.00	Fed Marg Tax=	16.00%
		Total Taxes Payable =	**$2,240.00**		

2004 Ontario Provincial Tax

							Tax Amdt.
		On the first	$ 33,375.00	6.05%	$ 2,019.19	$ 2,019.19	**$ 1,331.00**
from	$ 33,376.00	to	$	9.15%	$ 3,053.90	$	**$ -**

		66,752.00		5,073.09		
above $ 66,753.00			11.2%		**$**	**-**
	surtax	Surtax on Prov Tax over			$ 1,331.00	
		$3,685	20.0%		**$**	**-**
		4864	36.0%		**$**	**-**

Ontario tax =	**$1,331.00**	Avg Ont Tax=	6.05%
Combined tax =	$3,571.00	Avg Combined Tax=	16.23%

Corporate Tax – ($65,000 - $22,000) x 18.6% = $7,998

	PaysPersonal Tax	Pays Corp Tax	Total
Jane as an individual=	$ 16,095.90	$ -	$ 16,095.90
Jane as corp =	$ 3,571.00	$ 7,998.00	$ 11,569.00

65000-22000=43000 @ Ontario Small Business Rates 18.60%

12-6

(i) As a proprietorship

Federal Taxes:		
On the first $35,000	16%	$5,600.00
From $35,000 to $70,000	22%	$6,600.00
		$12,200.00
Non-refundable tax credit=$8,000×16%=		$1,280.00
	Federal taxes payable=	**$10,920.00**
Provincial taxes:		
On all income	10.00%	$6,500.00
	Provincial taxes payable=	**$6,500.00**
Total taxes=	**$17,420.00**	

(ii) As a corporation

(ii-1) Personal income (income=$22,000)

Federal Taxes:		
On the first $35,000	16%	$3,520.00
	Non-refundable tax credit=$8,000*16%=	$1,280.00
	Federal taxes payable=	**$2,240.00**
Provincial taxes:		
On all income	10.00%	$2,200.00
	Taxes payable from personal income=	**$4,440.00**

(ii-2) Corporate income (income=$43,000)

Combined tax rate=16.1%

	Taxes payable from corporate income=	**$6,923.00**
Total taxes=	**$11,363.00**	

12-7

Taxable Income = Adjustable Gross Income – Allowable Deductions
= ($500,000 - $30,000) - $30,000
= $170,000

Tax Bill = 0.15 ($50,000) + 0.25 ($25,000) + 0.34 ($25,000)
+ 0.39 ($70,000) – tax credits
= $49,550 – 48,000
= $41,550

12-8

Generally all depreciation methods allocated the cost of the equipment (less salvage value) over some assigned useful life. While the depreciation charges in any year may be different for different methods, the sum of the depreciation charges will be the same. This will affect the amount of taxes paid in any year, but with a stable income tax rate, the total taxes paid will be the same. (The difference is not the amount of the taxes, but their timing.)

12-9

Let i_a = annual effective after-tax cost of capital.
XYZ, Inc. is paying:
(100%) (100% - 3%) – 1 = 0.030928 = 3.0928%
for use of the money for:
45 – 5 = 40 days.

Number of 40-day periods in 1 year = 365/40 = 9.125

i_a = $[1 + (0.030928) (1 – 0.4)^{9.125} – 1$ = 0.1827
= 18.27%

12-10

A = $5,000 (A/P, 15%, 4) = $5,000 (0.3503) = $1,751.50

	n = 0	1	2	3	4
Loan Balance	$5,000				
Interest Payment		$750.00	$599,80	$427.02	$228.35
Principal Payment		$1,001.50	$1,151.70	$1,324.48	$1,522.32
Loan Balance		$3,998.50	$2,846.80	$1,522.32	$0
Sum of Payments		$1,751.50	$1,751.50	$1,751.50	$1,751.50
Additional "Point" Interest		$75.00	$75.00	$75.00	$75.00
BTCF	+$4,700	-$1,751.50	-$1,751.50	-$1,751.50	-$1,751.50
Tax Benefit-Interest Deduction					
Interest		$825.00	$674.80	$502.02	$303.35
Tax Saving (Interest x 0.40)		+$330.00	+$269.90	+$200.80	+$121.30
ATCF	+$4,700	-$1,421.50	-$1,481.60	-$1,550.70	-$1,630.20

Solving the After-Tax Cash Flow, the after-tax interest rate is 10.9%.

12-11

Federal Tax = 22.1% ($150,000) = $33,150

Provincial Tax =16.0% ($150,000) = $24,000

Combined federal and provincial tax = $57,150

Combined incremental state and federal income tax rate = 22.1%+16%
= 38.1%

12-12

Combined incremental tax rate
= federal tax rate + provincial tax rate
= 26% + 13.7% =39.7%

12-13

(a) Bonds plus Loan

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes	After-Tax Cash Flow
0	-$75,000 +$50,000			-$25,000
1- 5	+$5,000 -$5,000	$0	$0	$0
5	+$100,000 -$50,000	$25,000* capital gain	-$5,000	+$45,000

* taxed at 20%, the capital gain rate.

After-Tax Rate of Return
$25,000 = $45,00 (P/F, i%, 5)
(P/F, i%, 5) = 0.5556, thus the Rate of Return = 12.47%

Note: The Tax Reform Act of 1986 permits interest paid on loans to finance investments to continue to be deductible, but only up to the taxpayer's investment income.

(b) Bonds but no loan

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes	After-Tax Cash Flow
0	-$75,000			-$75,000
1- 5	+$5,000	$5,000	-$2,500	$2,500
5	+$100,000	$25,000* capital gain	-$5,000	+$95,000

* Taxed at 20%

After-Tax Rate of Return
$75,000 = $2,500 (P/A, i%, 5) + $95,000 (P/F, i%, 5)

Try i = 7%, $2,500 (4.100) + $95,000 (0.7130) = $77,985

Try i = 8%, $2,500 (3.993) + $95,000 (0.6806) = $74,639

Using linear interpolation, Rate of Return = 7.9%

12-14

(a)

DATA	
Purchase Price P	$ 84,000.00
Land Purchase P	$ 9,000.00
n	20
SL (yrs)	20
DDB (%)	10%
SOTD (yrs)	
CCA rate	10%
Revenue Yr 1	$ 9,600.00
ann . Rev increase	
Expense yr 1	$ 600.00
ann. Exp inc	$ -
Tax Rate	38%
Land Salvage	$ 9,000.00
Selling Price S	$ -
MARR	10%

Depreciable Asset Net Salvage

UCC(n) =	$ 10,779.80
disposal tax effect =	$ 4,096.32
Net Salvage at Yr n =	**$ 4,096.32**

Land Net Salvage

Land Purchase P	$ 9,000.00
Land Salvage	$ 9,000.00
Capital Gain (Loss) x Tax Rate/2 =	$ -
Land Net Salvage at Yr n =	$ 9,000.00

IRR= **5.78%** **5.15%**

NPV= **($21,564.22)**

CCA

Year	Revenue	Outlay or Expense	Before-Tax Cash Flow	CCA	Taxable Income	Income Tax	After-Tax Cash Flow	Loans, Land & WC	Total After-Tax Cash Flow
									$ -
0		84,000	-84,000				-84,000	-9,000	-93,000
1	9,600	600	9,000	4,200	4,800	1,824	7,176		7,176
2	9,600	600	9,000	7,980	1,020	388	8,612		8,612
3	9,600	600	9,000	7,182	1,818	691	8,309		8,309
4	9,600	600	9,000	6,464	2,536	964	8,036		8,036
5	9,600	600	9,000	5,817	3,183	1,209	7,791		7,791
6	9,600	600	9,000	5,236	3,764	1,430	7,570		7,570
7	9,600	600	9,000	4,712	4,288	1,629	7,371		7,371
8	9,600	600	9,000	4,241	4,759	1,808	7,192		7,192
9	9,600	600	9,000	3,817	5,183	1,970	7,030		7,030
10	9,600	600	9,000	3,435	5,565	2,115	6,885		6,885
11	9,600	600	9,000	3,092	5,908	2,245	6,755		6,755
12	9,600	600	9,000	2,782	6,218	2,363	6,637		6,637
13	9,600	600	9,000	2,504	6,496	2,468	6,532		6,532
14	9,600	600	9,000	2,254	6,746	2,564	6,436		6,436
15	9,600	600	9,000	2,028	6,972	2,649	6,351		6,351
16	9,600	600	9,000	1,826	7,174	2,726	6,274		6,274
17	9,600	600	9,000	1,643	7,357	2,796	6,204		6,204
18	9,600	600	9,000	1,479	7,521	2,858	6,142		6,142
19	9,600	600	9,000	1,331	7,669	2,914	6,086		6,086
20	9,600	600	9,000	1,198	7,802	2,965	6,035	13,096	19,131

(b)

	DATA
Purchase Price P	$ 84,000.00
Land Purchase P	$ 9,000.00
n	20
SL (yrs)	20
DDB (%)	10%
SOTD (yrs)	
CCA rate	10%
Revenue Yr 1	$ 9,600.00
ann . Rev increase	
Expense yr 1	$ 600.00
ann. Exp inc	$ -
Tax Rate	38%
Land Salvage	$ 16,000.00
Selling Price S	$ 84,000.00
MARR	10%

Depreciable Asset Net Salvage

UCC(n) =	$ 10,779.80
disposal tax effect =	$ (27,823.68)
Net Salvage at Yr n =	**$ 56,176.32**

Land Net Salvage

Land Purchase P	$ 9,000.00
Land Salvage	$ 16,000.00
Capital Gain (Loss) x Tax Rate/2 =	$ 1,330.00
Land Net Salvage at Yr n =	$ 14,670.00

IRR= 5.78% **7.22%**

NPV= ($21,564.22)

CCA

Year	Revenue	Outlay or Expense	Before-Tax Cash Flow	CCA	Taxable Income	Income Tax	After-Tax Cash Flow	Loans, Land & WC	Total After-Tax Cash Flow
									$ -
0		84,000	-84,000				-84,000	-9,000	-93,000
1	9,600	600	9,000	4,200	4,800	1,824	7,176		7,176
2	9,600	600	9,000	7,980	1,020	388	8,612		8,612
3	9,600	600	9,000	7,182	1,818	691	8,309		8,309
4	9,600	600	9,000	6,464	2,536	964	8,036		8,036
5	9,600	600	9,000	5,817	3,183	1,209	7,791		7,791
6	9,600	600	9,000	5,236	3,764	1,430	7,570		7,570
7	9,600	600	9,000	4,712	4,288	1,629	7,371		7,371
8	9,600	600	9,000	4,241	4,759	1,808	7,192		7,192
9	9,600	600	9,000	3,817	5,183	1,970	7,030		7,030
10	9,600	600	9,000	3,435	5,565	2,115	6,885		6,885
11	9,600	600	9,000	3,092	5,908	2,245	6,755		6,755
12	9,600	600	9,000	2,782	6,218	2,363	6,637		6,637
13	9,600	600	9,000	2,504	6,496	2,468	6,532		6,532
14	9,600	600	9,000	2,254	6,746	2,564	6,436		6,436
15	9,600	600	9,000	2,028	6,972	2,649	6,351		6,351
16	9,600	600	9,000	1,826	7,174	2,726	6,274		6,274
17	9,600	600	9,000	1,643	7,357	2,796	6,204		6,204
18	9,600	600	9,000	1,479	7,521	2,858	6,142		6,142
19	9,600	600	9,000	1,331	7,669	2,914	6,086		6,086
20	9,600	600	9,000	1,198	7,802	2,965	6,035	70,846	76,881

12-15

SOYD Depreciation
N = 8 SUM = (N/2) (N + 1) = 36

1st Year Depreciation = (8/36) ($120,000 - $12,000) = $24,000
Annual Decline = (1/36) ($120,000 - $12,000) = $3,000

Year	Before-Tax Cash Flow	SOYD Deprec.	Taxable Income	Income Taxes at 48%	After-Tax Cash Flow
0	-$120,000				-$120,000
1	+$29,000	$24,000	$5,000	-$2,300	+$26,700
2	+$26,000	$21,000	$5,000	-$2,300	+$23,700
3	+$23,000	$18,000	$5,000	-$2,300	+$20,700
4	+$20,000	$15,000	$5,000	-$2,300	+$17,700
5	+$17,000	$12,000	$5,000	-$2,300	+$14,700
6	+$14,000	$9,000	$5,000	-$2,300	+$11,700
7	+$11,000	$6,000	$5,000	-$2,300	+$8,700
8	+$8,000	$3,000	$5,000	-$2,300	+$5,700
	+$12,000		$0	$0	+$12,000
Sum		$108,000			

Will the firm obtain a 6% after tax rate of return?
PW of Cost = PW of Benefits
$120,000 =$26,700(P/A, i%, 8)-$3,000(P/G, i%, 8)+$12,000(P/F, i%, 8)

At i = 6%
PW of Benefits =$26,700(6.210)-$3,000(19.842)+$12,000(0.6274)
= $113,810 i is too high
Therefore, the firm will not obtain a 6% after-tax rate of return.

Further calculations show actual rate of return to be approximately 4.5%.

12-16

Year	Before-Tax Cash Flow	SOYD Deprec.	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	-$50,000				-$50,000
1	+$20,000	$15,000	$5,000	-$1,000	+$19,000
2	+$17,000	$12,000	$5,000	-$1,000	+$16,000
3	+$14,000	$9,000	$5,000	-$1,000	+$13,000
4	+$11,000	$6,000	$5,000	-$1,000	+$10,000
5	+$8,000	$3,000	$5,000	-$1,000	+$7,000
	+$5,000 (salvage val.)		$0	$0	+$5,000
Sum		$45,000			

PW of Benefits – PW of Cost = 0
$19,000 (P/A, i%, 5) - $3,000 (P/G, i%, 5)
+ $5,000 (P/F, i%, 5) - $50,000 = 0

Try i = 15%

$19,000 (3.352) - $3,000 (5.775) + $5,000 (0.4972) - $50,000 = -$1,151

Try i = 12%

$19,000 (3.605) - $3,000 (6.397) + $5,000 (0.5674) - $50,000 = +$2,141

Using linear interpolation, find that i = 14%.

12-17

Year	Before-Tax Cash Flow	SL Deprec.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	-$20,000				-$20,000
1- 8	+$5,000	$2,500	$2,500	-$1,000	+$4,000
Sum		$20,000	$20,000	-$8,000	

(a) Before Tax Rate of Return

$20,000 = $5,000 (P/A, i%, 8)

(P/A, i%, 8) = $20,000/$5,000 = 4

i* = 18.6%

(b) After Tax Rate of Return

$20,000 = $4,000 (P/A, i%, 8)

(P/A, i%, 8) = $20,000/$4,000 = 5

i* = 11.8%

(c)

Year	Before-Tax Cash Flow	SL Deprec.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	-$20,000				-$20,000
1- 8	+$5,000	$1,000	$4,000	-$1,600	+$3,400
9- 20	$0	$1,000	-$1,000	+$400	+$400
Sum		$20,000	$20,000	-$8,000	

Note that the changed depreciable life does not change Total Depreciation, Total Taxable Income, or Total Income Taxes. It does change the timing of these items.

After-Tax Rate of Return

PW of Benefits – PW of Cost = 0

$400 (P/A, i%, 20) + $3,000 (P/A, i%, 8) - $20,000 = 0

Try i = 9%

$400 (9.129) + $3,000 (5.535) - $20,000 = +$256.60

Try i = 10%

$400 (8.514) + $3,000 (5.335) - $20,000 = -$589.40

Using linear interpolation, i* = 9.3%.

12-18

Year	Before-Tax Cash Flow	DDB Deprec.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow	NPW at 10%
0	-$1,000				-$1,000	-$1,000
1	+$500	$400	$100	-$34	+$466	$423.6

2	+$340	$240	$100	-$34	+$306	$252.9
3	+$244	$144	$100	-$34	+$210	$157.8
4	+$100	$86.4	$13.6	-$4.6	+$95.4	$65.2
5	+$100	$4.6*	$95.4	-$32.4	+$192.6	$119.6
	+$125					
Sum		$875				+$19.1

* Reduced to $4.60 so book value not less than salvage value.

At 10%, NPW = +$19.1
Thus the rate of return exceeds 10%. (Calculator solution is 10.94%)
The project should be undertaken.

12-19

Year	B_{n-1}	CCA Dep.	B_n
1	$100,000.00	$15,000.00	$85,000.00
2	$85,000.00	$25,500.00	$59,500.00
3	$59,500.00	$17,850.00	$41,650.00
4	$41,650.00	$12,495.00	$29,155.00
5	$29,155.00	$8,746.50	$20,408.50

Calculation of net Salvage	UCC at year 5=	$20,408.50
	Proceed S=	$35,000.00
	Gain on disp.=	$14,591.50
	Tax effect G=	$4,961.11
	Net S=	$30,038.89

Year	0	1	2	3	4	5
OR		$30,000.00	$30,000.00	$30,000.00	$30,000.00	$30,000.00
CCA		$15,000.00	$25,500.00	$17,850.00	$12,495.00	$8,746.50
BTCF		$15,000.00	$4,500.00	$12,150.00	$17,505.00	$21,253.50
Taxes		$5,100.00	$1,530.00	$4,131.00	$5,951.70	$7,226.19
Net Profit		$9,900.00	$2,970.00	$8,019.00	$11,553.30	$14,027.31
CCA		$15,000.00	$25,500.00	$17,850.00	$12,495.00	$8,746.50
Investment	-100,000					
Salvage						$30,038.89
Net ATCF	-100,000	$24,900.00	$28,470.00	$25,869.00	$24,048.30	$52,812.70

IRR= 15.07%

12-20

SOYD= 15

Year	B_{n-1}	Factor	Dep.	B_n
1	$120,000.00	0.33	$40,000.00	$80,000.00
2	$80,000.00	0.27	$32,000.00	$48,000.00
3	$48,000.00	0.20	$24,000.00	$24,000.00
4	$24,000.00	0.13	$16,000.00	$8,000.00
5	$8,000.00	0.07	$8,000.00	$0.00

Calculation of net Salvage	UCC at year 5=	$0.00
	Proceed S=	$40,000.00
	Gain on disp.=	$40,000.00
	Tax effect G=	$13,600.00
	Net S=	$26,400.00

Year	0	1	2	3	4	5
OR		$32,000.00	$32,000.00	$32,000.00	$32,000.00	$32,000.00
Dep.		$40,000.00	$32,000.00	$24,000.00	$16,000.00	$8,000.00
BTCF		-$8,000.00	$0.00	$8,000.00	$16,000.00	$24,000.00
Taxes		-$2,720.00	$0.00	$2,720.00	$5,440.00	$8,160.00
Net Profit		-$5,280.00	$0.00	$5,280.00	$10,560.00	$15,840.00
Dep.		$40,000.00	$32,000.00	$24,000.00	$16,000.00	$8,000.00
Investment	-$120,000					
Salvage						$26,400.00
Net ATCF	-$120,000	$34,720.00	$32,000.00	$29,280.00	$26,560.00	$50,240.00

IRR= 12.88% **IRR>MARR, therefore it was a good investment**

12-21

Double Declining Balance Depreciation

Year	B_{n-1}	D_n	B_n
1	$100,000.00	$50,000.00	$50,000.00
2	$50,000.00	$25,000.00	$25,000.00
3	$25,000.00	$12,500.00	$12,500.00
4	$12,500.00	$6,250.00	$6,250.00

Year	0	1	2	3	4	5	6
OR		$30,000.00	$30,000.00	$35,000.00	$40,000.00	$10,000.00	$10,000.00
Dep.		$50,000.00	$25,000.00	$12,500.00	$6,250.00	$0.00	$0.00
BTCF		-$20,000.00	$5,000.00	$22,500.00	$33,750.00	$10,000.00	$10,000.00
Taxes		-$9,200.00	$2,300.00	$10,350.00	$15,525.00	$4,600.00	$4,600.00
Net Profit		-$10,800.00	$2,700.00	$12,150.00	$18,225.00	$5,400.00	$5,400.00
Dep.		$50,000.00	$25,000.00	$12,500.00	$6,250.00	$0.00	$0.00
Investment	-$100,000						
Salvage							$6,250.00
Net ATCF	-$100,000	$39,200.00	$27,700.00	$24,650.00	$24,475.00	$5,400.00	$11,650.00

IRR= 11.61% **After tax rate of return=11.61%**

12-22

$25,240*
Loan Payment

Loan Repayment

Year	Payment	B_{n-1}	Interest	Principal Red.	Balance
1	$25,240.00	$80,000.00	$8,000.00	$17,240.00	$62,760.00
2	$25,240.00	$62,760.00	$6,276.00	$18,964.00	$43,796.00
3	$25,240.00	$43,796.00	$4,379.60	$20,860.40	$22,935.60
4	$25,240.00	$22,935.60	$2,293.56	$22,946.44	-$10.84

Year	0	1	2	3	4	5	6
OR		$30,000.00	$30,000.00	$35,000.00	$40,000.00	$10,000.00	$10,000.00
Dep.		$50,000.00	$25,000.00	$12,500.00	$6,250.00	$0.00	$0.00
Interest		$8,000.00	$6,276.00	$4,379.60	$2,293.56	$0.00	$0.00
BTCF		-$28,000.00	-$1,276.00	$18,120.40	$31,456.44	$10,000.00	$10,000.00
Taxes		-$12,880.00	-$586.96	$8,335.38	$14,469.96	$4,600.00	$4,600.00
Net Profit		-$15,120.00	-$689.04	$9,785.02	$16,986.48	$5,400.00	$5,400.00
Dep.		$50,000.00	$25,000.00	$12,500.00	$6,250.00	$0.00	$0.00
Investment	-$20,000	-$17,240.00	-$18,964.00	-$20,860.40	-$22,946.44	0	0
Salvage							$6,250.00
Net ATCF	-$20,000	$17,640.00	$5,346.96	$1,424.62	$290.04	$5,400.00	$11,650.00

IRR= 34.29%

(a) After-Tax Rate of Return = 34.3%

(b) The purchase of the special tools for $20,000 cash plus an $80,000 loan represents a leveraged situation.

Under the tax laws all the interest paid is deductible when computing taxable income, so the after-tax cost of the loan is not 10%, but 5.4%. The resulting rate of return on the $20,000 cash is therefore much higher in this situation.

Note, however, that the investment now is not just $20,000, but really $20,000 plus the obligation to repay the $80,000 loan.

12-23

SOYD Depreciation

SOYD=36

Year	Bn-1	Factor	Dep.	Bn
1	$108,000.00	0.22	$24,000.00	$84,000.00
2	$84,000.00	0.19	$21,000.00	$63,000.00
3	$63,000.00	0.17	$18,000.00	$45,000.00
4	$45,000.00	0.14	$15,000.00	$30,000.00
5	$30,000.00	0.11	$12,000.00	$18,000.00
6	$18,000.00	0.08	$9,000.00	$9,000.00
7	$9,000.00	0.06	$6,000.00	$3,000.00
8	$3,000.00	0.03	$3,000.00	$0.00

Initial investment=$108,000+$25000= $133,000

Year	0	1	2	3	4	5	6	7	8
OR		$24,000.00	$24,000.00	$24,000.00	$24,000.00	$24,000.00	$24,000.00	$24,000.00	$24,000.00
Dep.		$24,000.00	$21,000.00	$18,000.00	$15,000.00	$12,000.00	$9,000.00	$6,000.00	$3,000.00
BTCF		$0.00	$3,000.00	$6,000.00	$9,000.00	$12,000.00	$15,000.00	$18,000.00	$21,000.00
Taxes		$0.00	$1,020.00	$2,040.00	$3,060.00	$4,080.00	$5,100.00	$6,120.00	$7,140.00
Net Profit		$0.00	$1,980.00	$3,960.00	$5,940.00	$7,920.00	$9,900.00	$11,880.00	$13,860.00
Dep.		$24,000.00	$21,000.00	$18,000.00	$15,000.00	$12,000.00	$9,000.00	$6,000.00	$3,000.00
Investment	-$133,000								
Salvage									$25,000.00
Net ATCF	-$133,000	$24,000.00	$22,980.00	$21,960.00	$20,940.00	$19,920.00	$18,900.00	$17,880.00	$41,860.00

NPW at 15% is negative (-$29,862). Therefore the project should not be undertaken.

(Calculator solution: i = 8.05%)

12-24

Year	Bn-1	CCA Dep.	Bn
1	$12,000.00	$1,500.00	$10,500.00
2	$10,500.00	$2,625.00	$7,875.00
3	$7,875.00	$1,968.75	$5,906.25
4	$5,906.25	$1,476.56	$4,429.69
5	$4,429.69	$1,107.42	$3,322.27
6	$3,322.27	$830.57	$2,491.70

Calculation of net Salvage	UCC at year 6=	$2,491.70
	Proceed S=	$1,000.00
	Gain on disp.=	-$1,491.70
	Tax effect G=	-$507.18
	Net S=	$1,507.18

Year	0	1	2	3	4	5	6
OR		$1,727.00	$2,414.00	$2,872.00	$3,177.00	$3,358.00	$1,997.00
Dep.		$1,500.00	$2,625.00	$1,968.75	$1,476.56	$1,107.42	$830.57
BTCF		$227.00	-$211.00	$903.25	$1,700.44	$2,250.58	$1,166.43
Taxes		$77.18	-$71.74	$307.11	$578.15	$765.20	$396.59
Net Profit		$149.82	-$139.26	$596.15	$1,122.29	$1,485.38	$769.85
Dep.		$1,500.00	$2,625.00	$1,968.75	$1,476.56	$1,107.42	$830.57
Investment	-$12,000						
Salvage							$1,507.18
Net ATCF	-$12,000	$1,649.82	$2,485.74	$2,564.90	$2,598.85	$2,592.80	$3,107.59

AW=-$230.58
Since AW<0, the investment is not desirable.

12-25

(a) Payback = $500,000/($12,000,000 x (0.05 – 0.03)) = 2.08 years

(b) After-Tax Payback:

SOYD= 15

Year	B_{n-1}	Factor	Dep.	B_n
1	$500,000.00	0.33	$166,666.67	$333,333.33
2	$333,333.33	0.27	$32,000.00	$301,333.33
3	$301,333.33	0.20	$24,000.00	$277,333.33
4	$277,333.33	0.13	$16,000.00	$261,333.33
5	$261,333.33	0.07	$8,000.00	$253,333.33

Year	0	1	2	3	4	5
OR		$240,000.00	$240,000.00	$240,000.00	$240,000.00	$240,000.00
Dep.		$166,666.67	$32,000.00	$24,000.00	$16,000.00	$8,000.00
BTCF		$73,333.33	$208,000.00	$216,000.00	$224,000.00	$232,000.00
Taxes		$29,333.33	$83,200.00	$86,400.00	$89,600.00	$92,800.00

Net Profit		$44,000.00	$124,800.00	$129,600.00	$134,400.00	$139,200.00
Dep.		$166,666.67	$32,000.00	$24,000.00	$16,000.00	$8,000.00
Investment	-$500,000.00					
Net ATCF	-$500,000.00	$210,666.67	$156,800.00	$153,600.00	$150,400.00	$147,200.00

Payback period=2.86 years
IRR= 20.49%

12-26

SOYD= 28

Year	Bn-1	Factor	Dep.	Bn
1	$14,000.00	0.25	$3,500.00	$10,500.00
2	$10,500.00	0.21	$3,000.00	$7,500.00
3	$7,500.00	0.18	$2,500.00	$5,000.00
4	$5,000.00	0.14	$2,000.00	$3,000.00
5	$3,000.00	0.11	$1,500.00	$1,500.00
6	$1,500.00	0.07	$1,000.00	$500.00
7	$500.00	0.04	$500.00	$0.00

Year	0	1	2	3	4	5	6	7
OR		$3,600	$3,600	$3,600	$3,600	$3,600	$3,600	$3,600
Dep.		$3,500	$3,000	$2,500	$2,000	$1,500	$1,000	$500
BTCF		$100	$600	$1,100	$1,600	$2,100	$2,600	$3,100
Taxes		$47	$282	$517	$752	$987	$1,222	$1,457
Net Profit		$53	$318	$583	$848	$1,113	$1,378	$1,643
Dep.		$3,500	$3,000	$2,500	$2,000	$1,500	$1,000	$500
Investment	-$14,000							
Net ATCF	-$14,000	$3,553	$3,318	$3,083	$2,848	$2,613	$2,378	$2,143

IRR= 10.71%

12-27

GIVEN: First Cost = $18,600
Annual Cost = $16,000
Salvage Value = $3,600
Depreciation = S/L with n = 10, S = $3,600
Savings/bag = $0.030
Cartons/year = 200,000
Savings bag/carton = 3.5
Annual Savings = ($0.03) (3.5) (200,000) = $21,000
SL Depreciation = ($18,000 - $3,600)/10 = $1,500/year

Initial investment=$18,600-$1,860= $16,740 (10% tax credit)

Year	0	1	2	3	4	5	6	7	8	9	10
OR		$5,000	$5,000	$5,000	$5,000	$5,000	$5,000	$5,000	$5,000	$5,000	$5,000
Dep.		$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500
BTCF		$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500	$3,500
Taxes		$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750
Net Profit		$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750	$1,750
Dep.		$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500	$1,500
Investment	$16,740										
Salvage											$3,600
Net ATCF	$16,740	$3,250	$3,250	$3,250	$3,250	$3,250	$3,250	$3,250	$3,250	$3,250	$6,850

(a) PW = -$16,740 + $3,250 (P/A, 20%, 10) + $3,600 (P/F, 20%, 10)
= -$2,535

(b) Set PW = 0 at i* and solve for i*:
$0 = -$16,740 + $3,250 (P/A, i*, 10) + $3,600 (P/F, i*, 10)
by trial and error method, i* = 16% per year.

12-28

Fed + Alta rate = 29+ 10 = 39%

	DATA
Purchase Price P	$ 82,000.00
Land Purchase P	$ 30,000.00
n	5
SL (yrs)	
DDB (%)	
SOTD (yrs)	
CCA rate	4%
Revenue Yr 1	$ 9,000.00
ann . Rev increase	
Expense yr 1	
ann. Exp inc	
Tax Rate	39%
Land Salvage	$ 30,000.00
Selling Price S	$ 90,000.00
MARR	10%

Depreciable Asset Net Salvage

UCC(n) =	$ 68,253.49
disposal tax effect =	$ (6,921.14)
Net Salvage at Yr n =	**$ 83,078.86**

Land Net Salvage

Land Purchase P	$ 30,000.00
Land Salvage	$ 30,000.00
Capital Gain (Loss) x Tax Rate/2 =	$ -
Land Net Salvage at Yr n =	$ 30,000.00

IRR=	**-24.28%**	**9.98%**
NPV=	**($57,189.11)**	**($167.58)**

Year	Revenue	Outlay or Expense	Before-Tax Cash Flow	CCA	Taxable Income	Income Tax	After-Tax Cash Flow	Loans, Land & WC	Total After-Tax Cash Flow
									$ -
0		82,000	-82,000				-82,000	-30,000	-112,000
1	9,000	0	9,000	1,640	7,360	2,870	6,130		6,130
2	9,000	0	9,000	3,214	5,786	2,256	6,744		6,744
3	9,000	0	9,000	3,086	5,914	2,307	6,693		6,693
4	9,000	0	9,000	2,962	6,038	2,355	6,645		6,645
5	9,000	0	9,000	2,844	6,156	2,401	6,599	113,079	119,678

12-29

Year	B_{n-1}	CCA Dep.	B_n
1	$90,000.00	$4,500.00	$85,500.00
2	$85,500.00	$8,550.00	$76,950.00
3	$76,950.00	$7,695.00	$69,255.00
4	$69,255.00	$6,925.50	$62,329.50

Calculation of net Salvage	UCC at year 5=	$62,329.50
	Proceed S=	$90,000.00
	Gain on disp.=	$27,670.50
	Tax effect G=	$11,068.20
	Net S=	$78,931.80

Capital gain tax on land= $1,060

S= $92,871.80

Year	0	1	2	3	4
OR		$6,000.00	$6,000.00	$6,000.00	$6,000.00
CCA		$4,500.00	$8,550.00	$7,695.00	$6,925.50
BTCF		$1,500.00	-$2,550.00	-$1,695.00	-$925.50
Taxes		$600.00	-$1,020.00	-$678.00	-$370.20
Net Profit		$900.00	-$1,530.00	-$1,017.00	-$555.30
CCA		$4,500.00	$8,550.00	$7,695.00	$6,925.50
Investment	-$99,700				
Salvage					$92,871.80
Net ATCF	-$99,700	$5,400.00	$7,020.00	$6,678.00	$99,242.00

IRR= 4.78%

12-30

Year	B_{n-1}	CCA Dep.	B_n
1	$50,000.00	$7,500.00	$42,500.00
2	$42,500.00	$12,750.00	$29,750.00
3	$29,750.00	$8,925.00	$20,825.00
4	$20,825.00	$6,247.50	$14,577.50
5	$14,577.50	$4,373.25	$10,204.25
6	$10,204.25	$3,061.28	$7,142.98

Calculation of net Salvage	UCC at year 6=	$7,142.98
	Proceed S=	$0.00
	Gain on disp.=	-$7,142.98
	Tax effect G=	-$2,857.19
	Net S=	$2,857.19

Year	0	1	2	3	4	5	6
OR		$2,000.00	$8,000.00	$17,600.00	$13,760.00	$5,760.00	$2,880.00
CCA		$7,500.00	$12,750.00	$8,925.00	$6,247.50	$4,373.25	$3,061.28
BTCF		-$5,500.00	-$4,750.00	$8,675.00	$7,512.50	$1,386.75	-$181.28

Taxes		-$2,200.00	-$1,900.00	$3,470.00	$3,005.00	$554.70	-$72.51
Net Profit		-$3,300.00	-$2,850.00	$5,205.00	$4,507.50	$832.05	-$108.77
CCA		$7,500.00	$12,750.00	$8,925.00	$6,247.50	$4,373.25	$3,061.28
Investment	-$50,000						
Salvage							$2,857.19
Net ATCF	-$50,000	$4,200.00	$9,900.00	$14,130.00	$10,755.00	$5,205.30	$5,809.70

IRR= 0.00%

(b) Similarly, the before-tax rate of return equals 0%.

12-31

Year	B_{n-1}	CCA Dep.	B_n
1	$100,000.00	$10,000.00	$90,000.00
2	$90,000.00	$18,000.00	$72,000.00
3	$72,000.00	$14,400.00	$57,600.00
4	$57,600.00	$11,520.00	$46,080.00
5	$46,080.00	$9,216.00	$36,864.00
6	$36,864.00	$7,372.80	$29,491.20

Year	0	1	2	3	4	5	6
OR		$35,000.00	$35,000.00	$35,000.00	$35,000.00	$35,000.00	$35,000.00
CCA		$10,000.00	$18,000.00	$14,400.00	$11,520.00	$9,216.00	$7,372.80
BTCF		$25,000.00	$17,000.00	$20,600.00	$23,480.00	$25,784.00	$27,627.20
Taxes		$8,500.00	$5,780.00	$7,004.00	$7,983.20	$8,766.56	$9,393.25
Net Profit		$16,500.00	$11,220.00	$13,596.00	$15,496.80	$17,017.44	$18,233.95
CCA		$10,000.00	$18,000.00	$14,400.00	$11,520.00	$9,216.00	$7,372.80
Investment	-$100,000						
Salvage							
Net ATCF	-$100,000	$26,500.00	$29,220.00	$27,996.00	$27,016.80	$26,233.44	$25,606.75

Payback period=3.6 years

12-32

For 2-year payback, annual benefits must be ½ ($400) = $200

(a) Before-Tax Rate of Return
$400 = $200 (P/A, i%, 4)
(P/A, i%, 4) = 2
Before-Tax Rate of Return = 34.9%

(b) After-Tax Rate of Return

Year	B_{n-1}	CCA Dep.	B_n
1	$400.00	$200.00	$200.00
2	$200.00	$200.00	$0.00
3	$0.00	$0.00	$0.00
4	$0.00	$0.00	$0.00

Calculation of net Salvage		UCC at year 4=	$0.00
		Proceed S=	$0.00
		Gain on disp.=	$0.00
		Tax effect G=	$0.00
		Net S=	$0.00

Year	0	1	2	3	4
OR		$200.00	$200.00	$200.00	$200.00
CCA		$200.00	$200.00	$0.00	$0.00
BTCF		$0.00	$0.00	$200.00	$200.00
Taxes		$0.00	$0.00	$68.00	$68.00
Net Profit		$0.00	$0.00	$132.00	$132.00
CCA		$200.00	$200.00	$0.00	$0.00
Investment	-$400				
Salvage					$0.00
Net ATCF	-$400	$200.00	$200.00	$132.00	$132.00

IRR= 26.48%

12-33

Year	B_{n-1}	CCA Dep.	B_n
1	$14,000.00	$2,100.00	$11,900.00
2	$11,900.00	$3,570.00	$8,330.00
3	$8,330.00	$2,499.00	$5,831.00
4	$5,831.00	$1,749.30	$4,081.70

Calculation of net Salvage		UCC at year 4=	$4,081.70
		Proceed S=	$3,000.00
		Gain on disp.=	-$1,081.70
		Tax effect G=	-$486.77
		Net S=	$3,486.77

Year	0	1	2	3	4
OR		$5,000.00	$5,000.00	$5,000.00	$5,000.00
CCA		$2,100.00	$3,570.00	$2,499.00	$1,749.30
BTCF		$2,900.00	$1,430.00	$2,501.00	$3,250.70
Taxes		$1,305.00	$643.50	$1,125.45	$1,462.82
Net Profit		$1,595.00	$786.50	$1,375.55	$1,787.89
CCA		$2,100.00	$3,570.00	$2,499.00	$1,749.30
Investment	-$14,000				
Salvage					$3,486.77
Net ATCF	-$14,000	$3,695.00	$4,356.50	$3,874.55	$7,023.95

IRR= 11.98%

12-34

	DATA
Building Price P	$ 200,000.00
Land Purchase P	$ 50,000.00
n	5
SL (yrs)	
DDB (%)	
SOTD (yrs)	
Building CCA rate	4%
Machinery Purchase	$ 150,000.00
Machinery CCA rate	30%
Machinery Salvage	$ 111,530.00
Revenue Yr 1	$ 70,000.00
ann . Rev increase	
Expense yr 1	
ann. Exp inc	
Tax Rate	34%
Land Salvage	$ 50,000.00
Selling Price S	$ 166,470.00
MARR	15%

Building Net Salvage		***Machinery Net Salvage***
UCC(n) =	$ 166,471.93	$ 30,612.75
disposal tax effect =	$ 0.65	$ (27,511.87)
Net Salvage at Yr n =	**$ 166,470.65**	**$ 84,018.14**

Land Net Salvage

Land Purchase P	$ 50,000.00
Land Salvage	$ 50,000.00
Capital Gain (Loss) x Tax Rate/2 =	$ -
Land Net Salvage at Yr n =	$ 50,000.00

IRR=	**-6.78%**	**10.14%**
AC	**($47,533.55)**	**($17,882.17)**
NPV=	**($159,339.83)**	**($59,943.79)**

CCA

Year	Revenue	Outlay or Expense	Before-Tax Cash Flow	Building CCA	Machinery CCA	Taxable Income	Income Tax	After-Tax Cash Flow	Loans, Land & WC	Total After-Tax Cash Flow
										$ -
0		200,000	-200,000		-150,000			-350,000	-50,000	-400,000
1	70,000	0	70,000	4,000	22,500	43,500	14,790	55,210		55,210
2	70,000	0	70,000	7,840	38,250	23,910	8,129	61,871		61,871
3	70,000	0	70,000	7,526	26,775	35,699	12,138	57,862		57,862
4	70,000	0	70,000	7,225	18,743	44,032	14,971	55,029		55,029
5	70,000	0	70,000	6,936	13,120	49,944	16,981	53,019	300,489	353,508

12-35

Year	B_{n-1}	CCA Dep.	B_n
1	$55,000.00	$4,125.00	$50,875.00
2	$50,875.00	$7,631.25	$43,243.75
3	$43,243.75	$6,486.56	$36,757.19
4	$36,757.19	$5,513.58	$31,243.61
5	$31,243.61	$4,686.54	$26,557.07
6	$26,557.07	$3,983.56	$22,573.51

Calculation of net Salvage	UCC at year 6=	$22,573.51
	Proceed S=	$35,000.00
	Gain on disp.=	$12,426.49
	Tax effect G=	$4,225.01
	Net S=	$30,774.99

Year	0	1	2	3	4	5	6
OR		$10,000.00	$10,000.00	$10,000.00	$10,000.00	$10,000.00	$10,000.00
CCA		$4,125.00	$7,631.25	$6,486.56	$5,513.58	$4,686.54	$3,983.56
BTCF		$5,875.00	$2,368.75	$3,513.44	$4,486.42	$5,313.46	$6,016.44
Taxes		$1,997.50	$805.38	$1,194.57	$1,525.38	$1,806.58	$2,045.59
Net Profit		$3,877.50	$1,563.38	$2,318.87	$2,961.04	$3,506.88	$3,970.85
CCA		$4,125.00	$7,631.25	$6,486.56	$5,513.58	$4,686.54	$3,983.56
Investment	-$55,000						
Salvage							$30,774.99
Net ATCF	-$55,000	$8,002.50	$9,194.63	$8,805.43	$8,474.62	$8,193.42	$38,729.40

IRR= 9.62%

12-36

Year	B_{n-1}	CCA Dep.	B_n
1	$1,800,000.00	$351,000.00	$1,449,000.00
2	$1,449,000.00	$565,110.00	$883,890.00
3	$883,890.00	$344,717.10	$539,172.90
4	$539,172.90	$210,277.43	$328,895.47
5	$328,895.47	$128,269.23	$200,626.24
6	$200,626.24	$78,244.23	$122,382.00
7	$122,382.00	$47,728.98	$74,653.02
8	$74,653.02	$29,114.68	$45,538.34

Calculation of net Salvage	UCC at year 5=	$45,538.34
	Proceed S=	$0.00
	Gain on disp.=	-$45,538.34
	Tax effect G=	-$15,483.04
	Net S=	$15,483.04

Year	0	1	2	3	4	5	6	7	8
OR		$450,000.00	$450,000.00	$450,000.00	$450,000.00	$450,000.00	$450,000.00	$450,000.00	$450,000.00
CCA		$351,000.00	$565,110.00	$344,717.10	$210,277.43	$128,269.23	$78,244.23	$47,728.98	$29,114.68
BTCF		$99,000.00	-$115,110.00	$105,282.90	$239,722.57	$321,730.77	$371,755.77	$402,271.02	$420,885.32
Taxes		$33,660.00	-$39,137.40	$35,796.19	$81,505.67	$109,388.46	$126,396.96	$402,271.02	$420,885.32
Net Profit		$65,340.00	-$75,972.60	$69,486.71	$158,216.90	$212,342.31	$245,358.81	$136,772.15	$143,101.01
CCA		$351,000.00	$565,110.00	$344,717.10	$210,277.43	$128,269.23	$78,244.23	$265,498.87	$277,784.31
Investment	-$1,800,000							$47,728.98	$29,114.68
Salvage									
Net ATCF	-$1,800,000	$416,340.00	$489,137.40	$414,203.81	$368,494.33	$340,611.54	$323,603.04		$14,616.22

IRR= 14.03%

12-37

Year	B_{n-1}	CCA Dep.	B_n
1	$300,000.00	$45,000.00	$255,000.00
2	$255,000.00	$76,500.00	$178,500.00
3	$178,500.00	$53,550.00	$124,950.00
4	$124,950.00	$37,485.00	$87,465.00
5	$87,465.00	$26,239.50	$61,225.50

Calculation of net Salvage	UCC at year 5=	$61,225.50
	Proceed S=	$0.00
	Gain on disp.=	-$61,225.50
	Tax effect G=	-$23,877.95
	Net S=	$23,877.95

Year	0	1	2	3	4	5
OR		$150,000.00	$150,000.00	$150,000.00	$150,000.00	$150,000.00
CCA		$45,000.00	$76,500.00	$53,550.00	$37,485.00	$26,239.50
BTCF		$105,000.00	$73,500.00	$96,450.00	$112,515.00	$123,760.50
Taxes		$40,950.00	$28,665.00	$37,615.50	$43,880.85	$48,266.60
Net Profit		$64,050.00	$44,835.00	$58,834.50	$68,634.15	$75,493.91
CCA		$45,000.00	$76,500.00	$53,550.00	$37,485.00	$26,239.50
Investment	-$300,000					
Salvage						$23,877.95
Net ATCF	-$300,000	$109,050.00	$121,335.00	$112,384.50	$106,119.15	$125,611.35

(a) Payback period=2.62 years

(b)IRR= 26.13% (IRR>MARR, therefore it is a desirable investment)

12-38

Year	B_{n-1}	CCA Dep.	B_n
1	$10,000.00	$1,000.00	$9,000.00
2	$9,000.00	$1,800.00	$7,200.00
3	$7,200.00	$1,440.00	$5,760.00
4	$5,760.00	$1,152.00	$4,608.00
5	$4,608.00	$921.60	$3,686.40
6	$3,686.40	$737.28	$2,949.12
7	$2,949.12	$589.82	$2,359.30
8	$2,359.30	$471.86	$1,887.44
9	$1,887.44	$377.49	$1,509.95
10	$1,509.95	$301.99	$1,207.96

Calculation of net Salvage	UCC at year 10=	$1,207.96
	Proceed S=	$0.00
	Gain on disp.=	-$1,207.96
	Tax effect G=	-$483.18
	Net S=	$483.18

Year	0	1	2	3	4	5	6	7	8	9	10
OR		$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00
CCA		$1,000.00	$1,800.00	$1,440.00	$1,152.00	$921.60	$737.28	$589.82	$471.86	$377.49	$301.99
BTCF		$800.00	$0.00	$360.00	$648.00	$878.40	$1,062.72	$1,210.18	$1,328.14	$1,422.51	$1,498.01
Taxes		$320.00	$0.00	$144.00	$259.20	$351.36	$425.09	$484.07	$531.26	$569.01	$599.20
Net Profit		$480.00	$0.00	$216.00	$388.80	$527.04	$637.63	$726.11	$796.88	$853.51	$898.81
CCA		$1,000.00	$1,800.00	$1,440.00	$1,152.00	$921.60	$737.28	$589.82	$471.86	$377.49	$301.99
Investment	-$10,000										
Salvage											$483.18
Net ATCF	-$10,000	$1,480.00	$1,800.00	$1,656.00	$1,540.80	$1,448.64	$1,374.91	$1,315.93	$1,268.74	$1,230.99	$1,683.98

(b) IRR=8.14%

(c)

Year	B_{n-1}	CCA Dep.	B_n
1	$10,000.00	$1,000.00	$9,000.00
2	$9,000.00	$1,800.00	$7,200.00
3	$7,200.00	$1,440.00	$5,760.00
4	$5,760.00	$1,152.00	$4,608.00
5	$4,608.00	$921.60	$3,686.40

Calculation of net Salvage	UCC at year 5=	$3,686.40
	Proceed S=	$7,000.00
	Gain on disp.=	$3,313.60
	Tax effect G=	$1,325.44
	Net S=	$5,674.56

Year	0	1	2	3	4	5
OR		$1,800.00	$1,800.00	$1,800.00	$1,800.00	$1,800.00
CCA		$1,000.00	$1,800.00	$1,440.00	$1,152.00	$921.60
BTCF		$800.00	$0.00	$360.00	$648.00	$878.40
Taxes		$320.00	$0.00	$144.00	$259.20	$351.36
Net Profit		$480.00	$0.00	$216.00	$388.80	$527.04
CCA		$1,000.00	$1,800.00	$1,440.00	$1,152.00	$921.60
Investment	-$10,000					
Salvage						$5,674.56
Net ATCF	-$10,000	$1,480.00	$1,800.00	$1,656.00	$1,540.80	$7,123.20

IRR= 8.62% (Therefore, yes, IRR would be higher)

12-39

Year	B_{n-1}	CCA Dep.	B_n
1	$25,000.00	$3,750.00	$21,250.00
2	$21,250.00	$6,375.00	$14,875.00
3	$14,875.00	$4,462.50	$10,412.50
4	$10,412.50	$3,123.75	$7,288.75

Calculation of net Salvage	UCC at year 4=	$7,288.75
	Proceed S=	$5,000.00
	Gain on disp.=	-$2,288.75
	Tax effect G=	-$915.50
	Net S=	$5,915.50

Year	0	1	2	3	4
OR		$8,000.00	$8,000.00	$8,000.00	$8,000.00
CCA		$3,750.00	$6,375.00	$4,462.50	$3,123.75
BTCF		$4,250.00	$1,625.00	$3,537.50	$4,876.25
Taxes		$1,700.00	$650.00	$1,415.00	$1,950.50
Net Profit		$2,550.00	$975.00	$2,122.50	$2,925.75
CCA		$3,750.00	$6,375.00	$4,462.50	$3,123.75
Investment	-$25,000				
Salvage					$5,915.50

Net ATCF	-$25,000	$6,300.00	$7,350.00	$6,585.00	$11,965.00

IRR= 9.87% (IRR<MARR, therefore she should not buy the churn)

12-40

(a)

Year	B_{n-1}	CCA Dep.	B_n
1	$60,000.00	$3,000.00	$57,000.00
2	$57,000.00	$5,700.00	$51,300.00
3	$51,300.00	$5,130.00	$46,170.00
4	$46,170.00	$4,617.00	$41,553.00
5	$41,553.00	$4,155.30	$37,397.70

(b) Capital Gain on land = $20,000 - $10,000 =$10,000
Tax on Cap Gain = 0.135 ($10,000) = $1,350

(c)
Calculation of net Salvage

House	UCC at year 5=	$37,397.70
	Proceed S=	$60,000.00
	Gain on disp.=	$22,602.30
	Tax effect G=	$6,102.62
	Net S=	$53,897.38
Land	Cap. Gain Tax=	$1,350

Year	0	1	2	3	4	5
OR		$12,250.00	$12,250.00	$12,250.00	$12,250.00	$12,250.00
CCA		$3,000.00	$5,700.00	$5,130.00	$4,617.00	$4,155.30
BTCF		$9,250.00	$6,550.00	$7,120.00	$7,633.00	$8,094.70
Taxes		$2,497.50	$1,768.50	$1,922.40	$2,060.91	$2,185.57
Net Profit		$6,752.50	$4,781.50	$5,197.60	$5,572.09	$5,909.13
CCA		$3,000.00	$5,700.00	$5,130.00	$4,617.00	$4,155.30
Investment	-$70,000					
Salvage						$72,547.38
Net ATCF	-$70,000	$9,752.50	$10,481.50	$10,327.60	$10,189.09	$82,611.81

Solving by trial and error, annual rent should be $12,210 or greater to obtain MARR 15%

12-41

Year	B_{n-1}	CCA Dep.	B_n
1	$20,000.00	$3,000.00	$17,000.00
2	$17,000.00	$5,100.00	$11,900.00
3	$11,900.00	$3,570.00	$8,330.00

Calculation of net Salvage

UCC at year 3=	$8,330.00
Proceed S=	$10,000.00
Gain on disp.=	$1,670.00

Tax effect G=	$751.50
Net S=	$9,248.50

Year	0	1	2	3
OR		$8,000.00	$8,000.00	$8,000.00
CCA		$3,000.00	$5,100.00	$3,570.00
BTCF		$5,000.00	$2,900.00	$4,430.00
Taxes		$2,250.00	$1,305.00	$1,993.50
Net Profit		$2,750.00	$1,595.00	$2,436.50
CCA		$3,000.00	$5,100.00	$3,570.00
Investment	-$20,000			
Salvage				$9,248.50
Net ATCF	-$20,000	$5,750.00	$6,695.00	$15,255.00

Year	BTCF	CCA	Taxable income	Income tax	ATCF	PW(12%)
0	-$20,000	$0.00	0	0	-20,000	-$20,000
1	$8,000	$3,000.00	$5,000.00	$2,250.00	$5,750.00	$5,133.93
2	$8,000	$5,100.00	$2,900.00	$1,305.00	$6,695.00	$5,337.21
3	$18,000	$3,570.00	$4,430.00	$1,993.50	$15,255.00	$10,858.21
					NPW=	$1,329

12-42

Year	Principal pymt	Rem. Balance	Interest
1	$2,500	$7,500	$1,000
2	$2,500	$5,000	$750
3	$2,500	$2,500	$500
4	$2,500	$0	$250

Year	B_{n-1}	CCA Dep.	B_n
1	$14,000.00	$2,100.00	$11,900.00
2	$11,900.00	$3,570.00	$8,330.00
3	$8,330.00	$2,499.00	$5,831.00
4	$5,831.00	$1,749.30	$4,081.70
Calculation of net Salvage		UCC at year 4=	$4,081.70
		Proceed S=	$3,000.00
		Gain on disp.=	-$1,081.70
		Tax effect G=	-$486.77
		Net S=	$3,486.77

Year	0	1	2	3	4
OR		$5,000.00	$5,000.00	$5,000.00	$5,000.00
CCA		$2,100.00	$3,570.00	$2,499.00	$1,749.30
Interest		$1,000.00	$750.00	$500.00	$250.00
BTCF		$1,900.00	$680.00	$2,001.00	$3,000.70
Taxes		$855.00	$306.00	$900.45	$1,350.32
Net Profit		$1,045.00	$374.00	$1,100.55	$1,650.39
CCA		$2,100.00	$3,570.00	$2,499.00	$1,749.30
Investment	-$4,000	-$2,500	-$2,500	-$2,500	-$2,500
Salvage					$3,486.77

Net ATCF	-$4,000	$645.00	$1,444.00	$1,099.55	$4,386.45

IRR= 22.86%

(b) This problem illustrates the leverage that a loan can produce. The cash investment is greatly reduced. Since the truck rate of return (12.5% in Problem 12-33) exceeds the loan interest rate (10%), combining the two increased the overall rate of return.

Two items worth noting:

1. The truck and the loan are independent decisions and probably should be examined separately.

2. There is increased risk when investments are leveraged.

12-43

Year	Before-Tax Cash Flow	Deprec.	Δ Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	- x - $5,500		-$3,000	+$1,200	- x - $4,300
1	+$7,000		$7,000	-$2,800	+$4,200
2	+$7,000		$7,000	-$2,800	+$4,200
…	…		…	…	…
…	…		…	…	…
9	+$7,000		$7,000	-$2,800	+$4,200
10	+$7,000		$7,000	-$2,800	+$4,200
	+x + $2,500		$0	$0	+x + $2,500

Where x = maximum purchase price for old building and lot.

PW of benefits – PW of cost = 0

$4,200 (P/A, i%, 10) + (+x + $2,500) (P/F, i%, 10) – x - $4,300 = 0

At the desired i = 15%

$4,200 (5.019) + (+x + $2,500) (0.2472) – x - $4,300 = 0

$21,080 + 0.2472x + $618 – x - $4,300 = 0

x = ($21,080 + $618 - $4,300)/0.7528 = $23,100

12-44

Year	Before-Tax Cash Flow	Deprec.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	- P				- P
1	+$87,500 – 0.065P	0.0667P	+$87,500 – 0.1317P	-$35,000 + 0.0527 P	+$52,500 – 0.0123 P
2	…	….	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
15	+$87,500 – 0.065P	0.0667P	+$87,500 – 0.1317P	-$35,000 + 0.0527 P	+$52,500 – 0.0123 P

Where P = maximum expenditure for new equipment.

Solve the after-tax cash flow for P
PW of Cost = PW of Benefits

P = ($52,500 – 0.0123P) (P/A, 8%, 15)
= ($52,500 – 0.0123P) (8.559)
= $449,348 – 0.1053 P
= $449,348/1.1053 = $406,500

12-45

Let x = number of days/year that the trucks are used.
Annual Benefit of truck ownership = ($83 - $35)x - $1,100 = $48x - $1,100

Year	Before-Tax Cash Flow	Deprec.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	-$13,000				-$13,000
1	$48x-$1,100	$1,429	$48x-$2,529	-$24x+$1,264	$24x+$164
2	…	….	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
7	$48x-$1,100	$1,429	$48x-$2,529	-$24x+$1,264	$24x+$164
	+$3,000		$0	$0	$3,000

Set PW of Cost = PW of Benefits
$13,000 = ($24x + $164) (P/A, 10%, 7) + $3,000 (P/F, 10%, 7)
= ($24x + $164) (4.868) + $3,000 (0.5132)
= $116.8x + $798 + $1,540
x = ($13,000 - $798 - $1,540)/116.8 = 91.5 days

Alternate Analysis

An alternate approach is to compute the after-tax cash flow of owning the truck. From this the after-tax EUAC may be calculated (= $2,189 + $17.5x).

In a separate calculation the after-tax EUAC of hiring a truck is determined (= $41.5x). By equating the EUAC for the alternatives we get:

$2,189 + $17.5 x = $41.5 x
x = 91.2 which is approximately equal to 9.5 days.

12-46

SOYD Depreciation
N = 5 SUM = (N/2) (N + 1) = (5/2) (6) = 15

1st year depreciation = (5/15) ($20,000 - $5,000) = $5,000
Annual decline = (1/15) ($20,000 - $5,000) = $1,000

Year	Before-Tax Cash Flow	Deprec.	Taxable Income	Income Taxes at 50%	After-Tax Cash Flow
0	-$20,000				-$20,000

1	+A	$5,000	A - $5,000	-0.5A + $2,500	0.5A + $2,500
2	+A	$4,000	A - $4,000	-0.5A + $2,000	0.5A + $2,000
3	+A	$3,000	A - $3,000	-0.5A + $1,500	0.5A + $1,500
4	+A	$2,000	A - $2,000	-0.5A + $1,000	0.5A + $1,000
5	+A	$1,000	A - $1,000	-0.5A + $500	0.5A + $500
	+$5,000		$0	$0	+$5,000

A = Before-Tax Annual Benefit

After Tax Cash flow computation:

$20,000 = (0.5A + $2,500) (P/A, 8%, 5) -$500 (P/G, 8%, 5)
+ $5,000 (P/F, 8%, 5)
= (0.5A + $2,500) (3.993) - $500 (7.372) + $5,000 (0.6806)
A = ($20,000 - $9,983 + $3,686 - $3,403)/2 = $5,150

Required Before-Tax Annual Benefit = $5,150

12-47

	DATA
Building Price P	$ 110,000.00
Land Purchase P	$ 45,000.00
n	10
SL (yrs)	27.5
DDB (%)	
SOTD (yrs)	
Building CCA rate	
Machinery Purchase	
Machinery CCA rate	
Machinery Salvage	
Revenue Yr 1	$ 12,000.00
ann . Rev increase	
Expense yr 1	
ann. Exp inc	
Tax Rate	27%
Land Salvage	$ 75,000.00
Selling Price S	
MARR	10%

Land Purchase P	$ 45,000.00
Land Salvage	$ 75,000.00
Capital Gain (Loss) x Tax Rate/2 =	$ 4,050.00
Land Net Salvage at Yr n =	$ 70,950.00

IRR=	**10.01%**
AC	
NPV=	**$76.79**

				CCA		Straight Line			
Year	**Revenue**	**Outlay or Expense**	**Before-Tax Cash Flow**	**Taxable Income**	**Income Tax**	**SL Dep**	**Taxable Income**	**Income Tax**	**After-Tax Cash Flow**
0		110,000	-110,000						$ (155,000.00)
1	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
2	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
3	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
4	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
5	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
6	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
7	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
8	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
9	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 9,840.00
10	12,000	0	12,000	12,000	3,240	4,000	8,000	2,160	$ 255,245.00
11	0	0	0	0	0				

Let P = selling Price of house
Net receipts = P-($40000 x 27%)- (P-110000)*13.5%

P= 197000 Net= 174455

12-48

This problem is similar to 12-44

Yr	Before-Tax Cash Flow	SOYD Depr.*	Taxable Income	Income Taxes at 50%	After-Tax Cash Flow
0	- P				- P
1	\$110,000	(6/21) P	\$110,000 – (6/21) P	-(\$55,000 – (3/21) P)	+\$55,000 + (3/21) P
2	\$110,000	(5/21) P	\$110,000 – (5/21) P	-(\$55,000 – (2.5/21) P)	+\$55,000 + (2.5/21) P
3	\$110,000	(4/21) P	\$110,000 – (4/21) P	-(\$55,000 – (2/21) P)	+\$55,000 + (2/21) P
4	\$110,000	(3/21) P	\$110,000 – (3/21) P	-(\$55,000 – (1.5/21) P)	+\$55,000 + (1.5/21) P
5	\$110,000	(2/21) P	\$110,000 – (2/21) P	-(\$55,000 – (1/21) P)	+\$55,000 + (1/21) P
6	\$110,000	(1/21) P	\$110,000 – (1/21) P	-(\$55,000 – (0.5/21) P)	+\$55,000 + (0.5/21) P

* Sum = (N/2) (N+1) = (6/2) (7) = 21

Write an equation for the After-Tax Cash Flow:

$$
\begin{aligned}
P &= (\$55{,}000 + (3/21)\,P)\,(P/A, 15\%, 6) - (0.5/21)\,(P/G, 15\%, 6) \\
&= (\$55{,}000 + (3/21)\,P)\,(3.784) - (0.5/21)\,(7.937) \\
&= \$208{,}120 + 0.541\,P - 0.189\,P \\
&= \$208{,}120/0.648 \qquad = \$321{,}173
\end{aligned}
$$

12-49

Problem can only be solved through trial and error

Year	B_{n-1}	CCA Dep.	B_n
1	\$14,500.00	\$2,175.00	\$12,325.00
2	\$12,325.00	\$3,697.50	\$8,627.50
3	\$8,627.50	\$2,588.25	\$6,039.25

Calculation of net Salvage		
	UCC at year 4=	\$6,039.25
	Proceed S=	\$5,000.00
	Gain on disp.=	-\$1,039.25
	Tax effect G=	-\$311.78
	Net S=	\$5,311.78

Year	0	1	2	3
OR		\$6,637.00	\$6,637.00	\$6,637.00
CCA		\$2,175.00	\$3,697.50	\$2,588.25
M & I		-\$1,000.00	-\$1,500.00	-\$2,000.00
BTCF		\$3,462.00	\$1,439.50	\$2,048.75
Taxes		\$1,038.60	\$431.85	\$614.63
Net Profit		\$2,423.40	\$1,007.65	\$1,434.13
CCA		\$2,175.00	\$3,697.50	\$2,588.25
Investment	-\$14,500			
Salvage				\$5,311.78

Net ATCF	-$14,500	$4,598.40	$4,705.15	$9,334.15
PW (12%)=	$0.49			

Number of days car is to be rented= 221.23
Therefore 222 days or more

12-50

Solving by trial and error

Year	B_{n-1}	CCA Dep.	B_n
1	$52,559.00	$7,883.85	$44,675.15
2	$44,675.15	$13,402.55	$31,272.61
3	$31,272.61	$9,381.78	$21,890.82
4	$21,890.82	$6,567.25	$15,323.58

Calculation of net Salvage	UCC at year 4=	$15,323.58
	Proceed S (0.2 P)=	$10,511.80
	Gain on disp.=	-$4,811.78
	Tax effect G=	-$1,924.71
	Net S=	$12,436.51

Year	0	1	2	3	4
OR		$10,000.00	$15,000.00	$20,000.00	$25,000.00
CCA		$7,883.85	$13,402.55	$9,381.78	$6,567.25
BTCF		$2,116.15	$1,597.46	$10,618.22	$18,432.75
Taxes		$846.46	$638.98	$4,247.29	$7,373.10
Net Profit		$1,269.69	$958.47	$6,370.93	$11,059.65
CCA		$7,883.85	$13,402.55	$9,381.78	$6,567.25
Investment	-$52,559				
Salvage					$12,436.51
Net ATCF	-$52,559	$9,153.54	$14,361.02	$15,752.71	$30,063.41

PW(10%)= $0

12-51

X cost=	$ 248,751	CCA rate=	0.25	CTF(1/2)=	0.761364	BVof X(11)=	12257.04	X Ann Cost	9980
Y cost=	$ 264,500	MARR=	0.1	CTF=	0.75	BVof Y(11)=	13033.06	Y Ann Cost	5120
		t=	0.35						

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$ 189,390	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ 6,487	$ (2,706)
$ 201,381	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ 3,328	$ (6,447)

PW of X=	$ 228,301	AE of X=	$ 35,150
PW of Y=	$ 219,570	AE of Y=	$ 33,806
PW of Y-X=	$ (8,731)	AEof Y-X=	$ 1,344

Select Lowest Cost which is Y

12-52

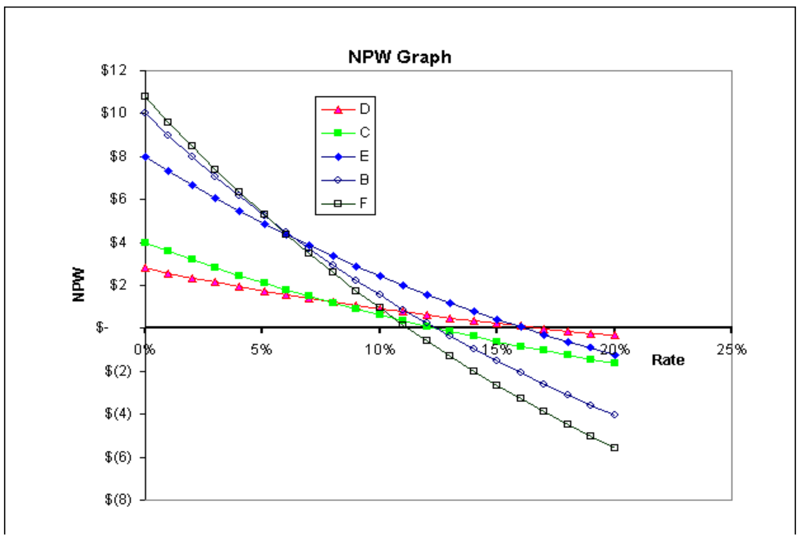
Taxation rate= 20%

yrs. of Life = 5

	Alternatives				
	B	**C**	**D**	**E**	**F**
Cost	$ (25.00)	$ (10.00)	$ (5.00)	$ (15.00)	$ (30.00)
BTCF	$ 7.50	$ 3.00	$ 1.70	$ 5.00	$ 8.70
SL Dep	$ (5.00)	$ (2.00)	$ (1.00)	$ (3.00)	$ (6.00)
TI	$ 2.50	$ 1.00	$ 0.70	$ 2.00	$ 2.70
IT	$ 0.50	$ 0.20	$ 0.14	$ 0.40	$ 0.54
ATCF	$ 7.00	$ 2.80	$ 1.56	$ 4.60	$ 8.16
IRR=	12.38%	12.38%	16.92%	16.18%	11.21%

Rate		B		C		D		E		F		Maximum
				NPW								
0%	$	10	$	4	$	3	$	8	$	11	$	11
1%	$	9	$	4	$	3	$	7	$	10	$	10
2%	$	8	$	3	$	2	$	7	$	8	$	8
3%	$	7	$	3	$	2	$	6	$	7	$	7
4%	$	6	$	2	$	2	$	5	$	6	$	6
5%	$	5	$	2	$	2	$	5	$	5	$	5
6%	$	4	$	2	$	2	$	4	$	4	$	4
7%	$	4	$	1	$	1	$	4	$	3	$	4
8%	$	3	$	1	$	1	$	3	$	3	$	3
9%	$	2	$	1	$	1	$	3	$	2	$	3
10%	$	2	$	1	$	1	$	2	$	1	$	2
11%	$	1	$	0	$	1	$	2	$	0	$	2
12%	$	0	$	0	$	1	$	2	$	(1)	$	2
13%	$	(0)	$	(0)	$	0	$	1	$	(1)	$	1
14%	$	(1)	$	(0)	$	0	$	1	$	(2)	$	1
15%	$	(2)	$	(1)	$	0	$	0	$	(3)	$	0
16%	$	(2)	$	(1)	$	0	$	0	$	(3)	$	0
17%	$	(3)	$	(1)	$	(0)	$	(0)	$	(4)	$	(0)
18%	$	(3)	$	(1)	$	(0)	$	(1)	$	(4)	$	(0)
19%	$	(4)	$	(1)	$	(0)	$	(1)	$	(5)	$	(0)
20%	$	(4)	$	(2)	$	(0)	$	(1)	$	(6)	$	(0)

If MARR>17% do nothing
If 17%>MARR>6% Select E
If 6%>MARR Select F



12-53

Alternative 1

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$10,000				-$10,000
1- 10	$4,500	$1,000	$3,500	-$1,190	$3,310
11- 20	$0	$0			$0

Year	ATCF	PW(10%)	EUAB-EUAC	FW(10%)
0	-$10,000	-$10,000	-$1,175	-$67,270
1- 10	$3,310	+$20,340	+$2,390	+$136,836
11- 20	$0	$0	$0	$0
Sum		+$10,340	+$1,215	+$69,566

Alternative 2

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$20,000				-$20,000
1- 10	$4,500	$2,000	$2,500	-$850	$3,650
11- 20	$4,500	$0	$4,500	-$1,530	$2,970

Year	ATCF	PW(10%)	EUAB-EUAC	FW(10%)
0	-$20,000	-$20,000	-$2,350	-$134,540
1- 10	$3,650	+$22,429	+$2,635	+$150,902
11- 20	$2,970	+$7,036	+$827	+$47,338
Sum		+$9,465	+$1,112	+$63,700

Increment 2- 1 After-Tax Cash Flow

Year	Alt. 1	Alt. 2	Alt. 2 – Alt. 1
0	-$10,000	-$20,000	-$10,000
1- 10	$3,310	$3,650	+$340
11- 20	$0	$2,970	+$2,970
Rate of Return	30.9%	10%	9.2%
B/C Ratio	2.03	1.47	0.91

(a) To maximize NPW, choose Alternative 1 with a total present worth of $10,340.

(b) To maximize (EUAB – EUAC), choose Alternative 1 with (EUAB – EUAC) = $1,215.

(c) Based on the rate of return of 9.2% from investing in Alt. 2 instead of 1, note that the increment is unacceptable. Choose Alternative 1.

(d) To maximize Net Future Worth, choose Alternative 1.

(e) Because the 2- 1 increment has a B/C ratio less than 1, reject the increment and select Alternative 1.

12-54

Alternative A

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$3,000				-$3,000
1	$1,000	$1,000	$0	$0	$1,000
2	$1,000	$800	$200	-$68	$932
3	$1,000	$600	$400	-$136	$864
4	$1,000	$400	$600	-$204	$796
5	$1,000	$200	$800	-$272	$728

Alternative B

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$5,000				-$5,000
1	$1,000	$1,000	$0	$0	$1,000
2	$1,200	$1,000	$200	-$68	$1,132
3	$1,400	$1,000	$400	-$136	$1,264
4	$2,600	$1,000	$1,600	-$544	$2,056
5	$2,800	$1,000	$1,800	-$612	$2,188

Alternative B- Alternative A

Year	B- A ATCF	PW at 15%	PW at 12%
0	-$2,000	-$2,000	-$2,000
1	$0	$0	$0
2	$200	$151	$159
3	$400	$263	$285
4	$1,260	$720	$801
5	$1,460	$726	$828
Sum		-$140	+$73

The B- A has a desirable 13% rate of return. Choose B.

12-55

Alternative A

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$11,000				-$11,000
1	$3,000	$3,000	$0	$0	$3,000
2	$3,000	$3,000	$0	$0	$3,000
3	$3,000	$3,000	$0	$0	$3,000
4	$3,000	$0	$3,000	-$1,020	$1,980
5	$3,000	$0	$3,000	-$1,020	$3,980
	$2,000		$0		

NPW(12%) = -$278

Alternative B

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	-$33,000				-$33,000
1	$9,000	$12,000	-$3,000	+$1,020	$10,202
2	$9,000	$9,000	$0	$0	$9,000
3	$9,000	$6,000	$3,000	-$1,020	$7,980
4	$9,000	$3,000	$6,000	-$2,040	$6,960
5	$9,000	$0	$9,000	-$3,060	$10,260
	$5,000		$2,000	-$680	

NPW(12%) = -$953

Neither A nor B meet the 12% criterion. By NPW one can see that A is the better of the two undesirable alternatives.

Select Alternative A.

Chapter 13: Replacement Analysis

13-1

For the Replacement Analysis Decision Map, the appropriate analysis method is a function of the cash flows and assumptions made regarding the defender and challenger assets.

Thus, the answer would be the last it depends on the data and the assumptions

13-2

The replacement decision is a function of both the defender and the challenger.

The statement is false.

13-3

The book value of the equipment describes past actions or a *sunk cost* situation. The answer is the last it should be ignored in this *before-tax* analysis.

13-4

With no resale value, and maintenance costs that are expected to be higher in the future, EUAC would be a minimum for one year. (This is such a common situation that the early versions of the MAPI replacement analysis model were based on a one year remaining life for the defender.)

The answer is one year.

13-5

The EUAC of installed cost will decline as the service life increases. The EUAC of maintenance is constant. Thus total EUAC is declining over time.

Answer: For minimum EUAC, keep the bottling machine indefinitely.

13-6

The value to use is the present market value of the defender equipment. (The book indicates that trade-in value may be purposely inflated as a selling strategy, hence it may or may not represent market value.)

13-7

(a) Expected good performance, productivity, energy efficiency, safety, long service life. Retraining in operation and maintenance may be required. High comfort of operation. High purchase price. May not be immediately available. Sales taxes to be paid. Can be depreciated. Supplier warranty and spare parts backup available.

(b) All as in (a) except for lower price and probably faster delivery.

(c) All as in (a) except for still lower cost, lost production during the rebuild period, and that the rebuild costs can be expensed, at least partially. No sales tax applies.

(d) Performance and productivity may not be as good as in option (c). Retraining in operation and maintenance is not required. Production will be lost during the rebuilding period. Cost may be substantially lower than in previous options. The rebuild costs can be expensed. No sales tax applies.

(e) Performance, productivity, service life, energy efficiency, safety, reliability may be significantly lower than in the other options. Retraining in operation and maintenance may be required if the new unit is different from the previous one. Cost may be only 20-50% of the new equipment. Immediate delivery is a possibility. The sales tax applies. Equipment can be depreciated.

13-8

Looking at Figure 13-1: For this problem marginal cost data is available, and is not strictly increasing. This would lead to the use of Replacement Analysis Technique #2. In this case we compute the minimum cost life of the defender and compare the EUAC at that life against the EUAC of the best available challenger. We chose the options with the smallest EUAC.

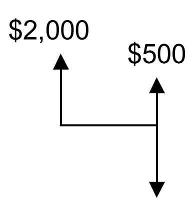
13-9

EUAC of Capital Recovery

In this situation P = S = $15,000
So EUAC of Capital Recovery = $15,000 (0.15) = $2,250 for all useful lives.

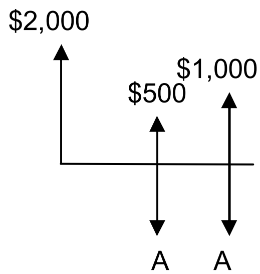
EUAC of Maintenance

For a **1-year useful life**



EUAC = $2,000 $(1 + 0.15)^1$ + $500 = $2,800

For a **2-year useful life**

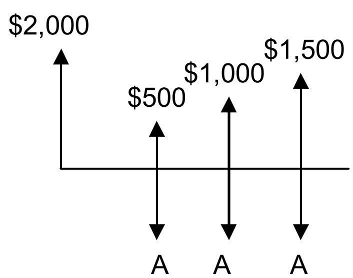


$FW_{yr\ 2}$ = \$2,000 (F/P, 15%, 2) + \$500 (F/P, 15%, 1) + \$1,000
= \$4,220

A = \$4,220 (A/F, 15%, 2) = \$1,963

EUAC = A = \$1,963

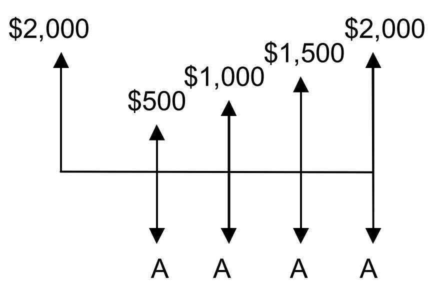
For **3-year useful life**



$FW_{yr\ 3}$ = \$2,000 (F/P, 15%, 3) + \$500 (F/P, 15%, 2) + \$1,000 (F/P, 15%, 1)
+ 1,500
= \$6,353
A = \$6,353 (A/F, 15%, 3) = \$1,829

EUAC = A = \$1,829

For a 4-year useful life



$FW_{yr\ 4}$ = \$2,000 (F/P, 15%, 4) + \$500 (P/G, 15%, 5) (F/P, 15%, 5)
= \$9,305
A = \$9,305 (A/F, 15%, 4) = \$1,864
EUAC = A = \$1,864

Alternate computation of maintenance in any year N:

$EUAC_N = A = \$2,000 (A/P, 15\%, N) + \$500 + \$500 (A/G, 15\%, N)$

(a) Total EUAC = \$2,250 + EUAC of Maintenance

Therefore, to minimize Total EUAC, choose the alternative with minimum EUAC of maintenance.

Economical life = 3 years

(b) The stainless steel tank will always be compared with the best available replacement (the challenger). If the challenger is superior, then the defender tank probably will be replaced.

It will cost a substantial amount of money to remove the existing tank from the plant, sell it to someone else, and then buy and install another one. As a practical matter, it seems unlikely that this will be economical.

13-10

Year	Salvage Value	Maintenance	Year	Salvage Value	Maintenance
0	P = \$10,000		4	\$4,500	\$600
1	\$3,000	\$300	5	\$5,000	\$1,200
2	\$3,500	\$300	6	\$5,500	\$2,400
3	\$4,000	\$300	7	\$6,000	\$4,800

EUAC of Maintenance

$EUAC_1 = EUAC_2 = EUAC_3 = \$300$

$EUAC_4 = \$300 + \$300 (A/F, 15\%, 4) = \$360$

$EUAC_5 = \$300 + [\$300 (F/P, 15\%, 1) + \$900] (A/F, 15\%, 5) = \$485$

$EUAC_6 = \$300 + [\$300 (F/P, 15\%, 2) + \$900 (F/P, 15\%, 1) + \$2,100] (A/F, 15\%, 6)$
$= \$703$

$EUAC_7 = \$300 + [\$300 (F/P, 15\%, 3) + \$900 (F/P, 15\%, 2) + \$2,100 (F/P, 15\%, 1) + \$4,500] (A/F, 15\%, 7)$
$= \$1,074$

EUAC of Installed Cost

Year	(P – S) (A/P, i%, n) + (S) (i)	= EUAC of Installed Cost
1	(\$10,000 - \$3,000) (A/P, 15%, 1) + \$3,000 (0.15)	= \$8,500
2	(\$10,000 - \$3,500) (A/P, 15%, 2) + \$3,500 (0.15)	= \$4,523
3	(\$10,000 - \$4,000) (A/P, 15%, 3) + \$4,000 (0.15)	= \$3,228
4	(\$10,000 - \$4,500) (A/P, 15%, 4) + \$4,500 (0.15)	= \$2,602
5	(\$10,000 - \$5,000) (A/P, 15%, 5) + \$5,000 (0.15)	= \$2,242
6	(\$10,000 - \$5,500) (A/P, 15%, 6) + \$5,500 (0.15)	= \$2,014
7	(\$10,000 - \$6,000) (A/P, 15%, 7) + \$6,000 (0.15)	= \$1,862

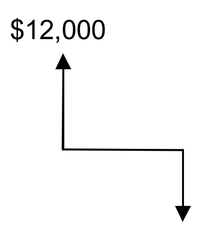
Year	EUAC of Installed Cost +	EUAC of Maintenance	= Total EUAC
1	$8,500	$300	= $8,800
2	$4,523	$300	= $4,823
3	$3,228	$300	= $3,528
4	$2,602	$360	= $2,962
5	$2,242	$485	= $2,727
6	$2,014	$703	= $2,717
7	$1,862	$1,074	= $2,936

The Economical Life is 6 years because this life has the smallest total EUAC.

13-11

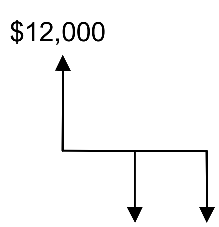
For various lives, determine the EUAC for the challenger assuming it is retired at the end of the period. The best useful life will be the one in which EUAC is a minimum.

Useful Life- 1 year



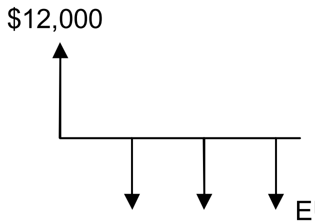
EUAC = $12,000 (F/P, 10%, 1) = $13,200

Useful Life- 2 years



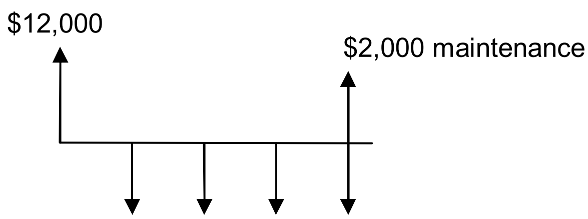
EUAC = $12,000 (A/P, 10%, 2) = $6,914

Useful Life- 3 years



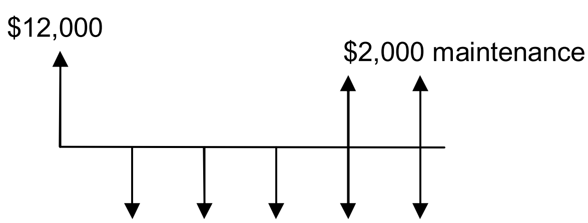
EUAC = $12,000 (A/P, 10%, 3) = $4,825

Useful Life- 4 years



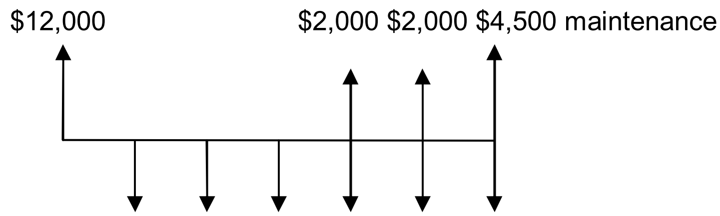
EUAC = $12,000 (A/P, 10%, 4) + $2,000 (A/F, 10%, 4)
= $12,000 (0.3155) + $2,000 (0.2155)
= $4,217

Useful Life- 5 years



EUAC = $12,000 (A/P, 10%, 5) + [$2,000 (1 + (F/P, 10%, 1)] (A/F, 10%, 5)
= $12,000 (0.2638) + [$2,000 (1 + (1.100)] (0.1638)
= $3,854

Useful Life- 6 years



EUAC = [$12,000 (F/P, 10%, 5) + $2,000 (F/A, 10%, 3) + $2,500](A/F, 10%, 6)
= [$12,000 (1.772) + $2,000 (3.310) + $2,500](0.1296)
= $3,938

Summary

Useful Life	EUAC
1 yr.	$13,200
2 yr.	$6,914
3 yr.	$4,825
4 yr.	$4,217
5 yr.	$3,854 Best Useful Life is 5 years
6 yr.	$3,938

13-12

First Cost = $1,050,000
Salvage Value = $225,000
Maintenance & Operating Cost = $235,000
Maintenance & Operating Gradient = $75,000
MARR = 10%

EUAB – EAUC = $1,050,000 (A/P, 10%, n) + $225,000 (A/F, 10%, n)
- $235,000 - $75,000 (A/G, 10%, n)

Try n = 4 years:

EUAB – EAUC = $331,275 + $48,488 - $235,000 - $103,575 = -$621,362

Try n = 5 years:

EUAB – EUAC = -$276,990 + $36,855 - $235,000 - $135,750 = -$610,885

Try n = 6 years:

EUAB – EUAC = -$241,080 + $29,160 - $235,000 - $166,800 = -$613,720

Thus, year 5 has the minimum EUAB – EUAC, hence the most economic life is 5 years.

13-13

A tabulation of the decline in resale value plus the maintenance is needed to solve the problem.

Age	Value of Car	Decline in Value for the Year	Maintenance for the Year	Sum of Decline in Value + Maintenance
New	$11,200			
1 yr	$8,400	$2,800	$50	$2,850
2	$6,300	$2,100	$150	$2,250
3	$4,725	$1,575	$180	$1,755
4	$4,016	$709	$200	$909
5	$3,414	$602	$300	$902
6	$2,902	$512	$390	$902
7	$2,466	$536	$500	$936

From the table it appears that minimum cost would result from buying a 3-year-old car and keeping it for three years.

13-14

Find: $NPW_{OVERHAUL}$ and $NPW_{REPLACE}$
Note: All costs which occur before today are *sunk costs* and are irrelevant.

$$NPW_{OVERHAUL} = -\$1,800 - \$800\ (P/A, 5\%, 2)$$
$$= -\$1,800 - \$800\ (1.859) = -\$3,287$$
$$NPW_{REPLACE} = +\$1,500 – (\$2,500 + \$300)\ (P/A, 5\%, 2)$$
$$= +\$1,500 - \$2,800\ (1.859) = -\$3,705$$

Since the PW of Cost of the overhaul is less than the PW of Cost of the replacement car, the decision is to overhaul the 1988 auto.

13-15

In a before-tax computation the data about depreciation is unneeded.

Year	Reconditioned Equipment	New Equipment	New vs. reconditioned	PW at 12%	PW at 15%
0	-$35,000 -$10,000	-$85,000	-$40,000	-$40,000	-$40,000
1- 10		+$7,000	+$7,000	+$39,555	$35,133
10	+$10,000	+$15,000	+$5,000	+$1,610	+$1,236
Sum				= +$1,165	=-$3,631

By linear interpolation, the incremental before-tax rate of return is 12.7%.

The 12.7% rate of return on the increment is unsatisfactory, so reject the increment and recondition the old tank car.

13-16

(a) Before-Tax Analysis

Year	New Machine BTCF	Existing Machine BTCF	New Machine rather than Existing Machine BTCF
0	-\$3,700	-\$1,000	-\$2,700
1	+\$900	\$0	+\$900
2	+\$900	\$0	+\$900
3	+\$900	\$0	+\$900
4	+\$900	\$0	+\$900

Compute Rate of Return
PW of Cost = PW of Benefit
\$2,700 = \$900 (P/A, i%, 4)
(P/A, i%, 4) = \$2,700/\$900 = 3.0

Rate of return = 12.6%

(b) After-Tax Analysis

New Machine

Year	BTCF	SOYD Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-\$3,700				-\$3,705
1	+\$900	\$1,480	-\$580	+\$232	+\$1,132
2	+\$900	\$1,110	-\$210	+\$84	+\$984
3	+\$900	\$740	+\$160	-\$64	+\$836
4	+\$900	\$370	\$530	-\$212	+\$688

SOYD Deprec

Sum = (4/2) (5) = 10
1st Year SOYD = (4/10) (\$3,700 - \$0)
Annual Decline = (1/10) (\$3,700 - \$0) - \$370

Existing Machine

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-\$1,000		\$1,000*	-\$200**	-\$1,200
1	\$0	\$500	-\$500	+\$200	+\$200
2	\$0	\$500	-\$500	+\$200	+\$200
3	\$0	\$500	-\$500	+\$200	+\$200
4	\$0	\$500	-\$500	+\$200	+\$200

* Long term capital loss foregone by keeping machine:
\$2,000 Book Value - \$1,000 Selling price = \$1,000 capital loss

** The $1,000 long term capital loss foregone would have offset $1,000 of long term capital gains elsewhere in the firm. The result is a tax saving of 20% ($1,000) = $200 is foregone.

New Machine rather than Existing Machine

Year	New Tool ATCF	Existing Tool ATCF	New- Existing ATCF	PW AT 5%	PW AT 6%
0	-$3,705	-$1,200	-$2,500	-$2,500	-$2,500
1	+$1,132	+$200	$932	$888	$879
2	+$984	+$200	$784	$711	$698
3	+$836	+$200	$636	$549	$534
4	+$688	+$200	$488	$400	$387
			Sum	= +$50	-$2

Δ After-Tax rate of return = 5.96%

13-17

Alternative I: Retire the 4 old machines and buy 6 new machines.

Initial Cost:

6 new machines at $32,000 each		$192,000
Training Program at 6 x $700	+$4,200	
Total		= $196,200

Savings:

Annual Labor Saving		$12,000
Less Maintenance	$3,600	
Total		$8,400

Compute Equivalent Uniform Annual Cost (EUAC)

Initial Cost: $196,000 (A/P, 9%, 8)	= $196,000 (0.1807)	= $35,453
Less Salvage Value: (6 x $750) (A/F, 9%, 8)	= $4,500 (0.0907)	= -$408
Less Net Annual Benefit:		= -$8,400
		EUAC = $26,645

Alternative II: Keep 4 old machines and buy 3 new ones

Initial Cost:

Value of 4 old machines 4 x $2,000	$8,000	
3 new machines at $32,000 each	$96,000	
Training Program at 3 x $700	$2,100	
Total		= $106,100

Annual Maintenance= 4 old x $1,500 + 3 new x $600 = $7,800 per year

Salvage Value 8 years hence= 4 old x $500 + 3 new x $750 = $4,250

Compute Equivalent Uniform Annual Cost (EUAC)

Initial Cost: $106,100 (A/P, 9%, 8) = $106,100 (0.1807) = $19,172

Less Salvage Value: (\$4,250) (A/F, 9%, 8) = \$4,250 (0.0907) = -\$385
Add Annual Maintenance: = +\$7,800
EUAC = \$26,587

Decision: Choose Alternative II with its slightly lower EUAC.

13-18

For this problem we have marginal cost data for the defender, so we will check to see if that data is strictly increasing.

Defender
Current Market Value = \$25,000 $(0.90)^5$ = \$14,762

Year	Time Line	Market Value (n)	Loss in MV (n)	Annual Costs (n)	Lost Interest in (n)	Total Marg. Cost
0		\$25,000				
1	-5	\$22,500	\$2,500	\$1,250	\$2,000	\$5,750
2	-4	\$20,250	\$2,250	\$1,750	\$1,800	\$5,800
3	-3	\$18,225	\$2,025	\$2,250	\$1,620	\$5,895
4	-2	\$16,403	\$1,823	\$2,750	\$1,458	\$6,031
5	-1	\$14,762	\$1,640	\$3,250	\$1,312	\$6,202
6	1	\$13,286	\$1,476	\$3,750	\$1,181	\$6,407
7	2	\$11,957	\$1,329	\$4,250	\$1,063	\$6,641
8	3	\$10,762	\$1,196	\$4,750	\$957	\$6,902
9	4	\$9,686	\$1,076	\$5,250	\$861	\$7,187
10	5	\$8,717	\$969	\$5,750	\$775	\$7,493

We see that this data is strictly increasing from the Time Line of today □ onward (year 6 of the original life). Thus we use Replacement Analysis Technique #1 and compare the marginal cost data of the defender against the min. EUAC of the challenger. Let's find the Challenger's min. EUAC at its 5-year life.

Challenger
Challenger's min. cost life is given at 5 years in the problem.

EUAC = \$27,900 (A/P, 8%, 5) = \$6,989

From this we would recommend that we keep the Defender for three more years and then replace it with the Challenger. This is because after three years the marginal costs of the Defender become greater than the min. EUAC of the Challenger.

13-19

For this problem we have marginal cost data for the defender, so we will check to see if that data is strictly increasing.

Defender:
Current Market Value = \$25,000 $(0.70)^5$ = \$4,202

Year	Time	Market	Loss in	Annual	Lost	Total

	Line	Value (n)	MV (n)	Costs (n)	Interest in (n)	Marg. Cost
0		$25,000				
1	-5	$17,500	$7,500	$3,000	$2,000	$12,500
2	-4	$12,250	$5,250	$3,300	$1,400	$9,950
3	-3	$8,575	$3,675	$3,630	$980	$8,285
4	-2	$6,003	$2,573	$3,993	$686	$7,252
5	-1	$4,202	$1,801	$4,392	$480	$6,673
6	1	$2,941	$1,261	$4,832	$336	$6,428
7	2	$2,059	$882	$5,315	$235	$6,432
8	3	$1,441	$618	$5,846	$165	$6,629
9	4	$1,009	$532	$6,431	$115	$6,978
10	5	$706	$303	$7,074	$81	$7,457

Again here the marginal costs of the Defender are strictly increasing from the Time Line of today □ onward (year 6 of the original life). Thus, we use Replacement Analysis Technique #1 and compare the marginal cost data of the defender against the min. EUAC of the challenger.

From the previous problem the Challenger's min. EUAC at its 5-year life is:
EUAC = $27,900 (A/P, 8%, 5) = $6,989

From this we would recommend that we keep the Defender for four more years and then replace it with the Challenger. This is because after three years the marginal costs of the Defender become greater than the min. EUAC of the Challenger.

13-20

Year	Time Line	Salv.	Oper.	Insur.	Maint.	Lost Interest	Lost MV	Total Marg. Cost
1	-5	$80,000	$16,000	$17,000	$5,000	$31,250	$45,000	$114,250
2	-4	$78,000	$20,000	$16,000	$10,000	$20,000	$2,000	$68,000
3	-3	$76,000	$24,000	$15,000	$15,000	$19,500	$2,000	$75,500
4	-2	$74,000	$28,000	$14,000	$20,000	$19,000	$2,000	$83,000
5	-1	$72,000	$32,000	$13,000	$25,000	$18,500	$2,000	$90,500
6	1	$70,000	$36,000	$12,000	$30,000	$18,000	$2,000	$98,000
7	2	$68,000	$40,000	$11,000	$35,000	$17,500	$2,000	$105,500
8	3	$66,000	$44,000	$10,000	$40,000	$17,000	$2,000	$113,000
9	4	$64,000	$48,000	$10,000	$45,000	$16,500	$2,000	$121,500
10	5	$62,000	$52,000	$10,000	$50,000	$16,000	$2,000	$130,000

(a) Total marginal cost for this previously implemented asset is given above.

(b) In looking at the table above one can see that the marginal cost data of the defender is strictly increasing over the next five year period. Thus the Replacement Decision Analysis Map would suggest that we use Replacement Analysis Technique #1. We compare the defender marginal cost data against the challenger's minimum EUAC. We would keep the defender asset for two more years and then replace it with the new automated shearing equipment. After two years the MC (def) > Min. EUAC (chal): $113,000 > $110,000

13-21

In this case we first compute the total marginal costs of the defender asset. From Figure 13-1 the marginal cost data is available, and it is not strictly increasing (see Total MC column in the table below). Thus, we use Replacement Analysis Technique #2, comparing minimum EUAC defender against minimum EUAC of challenger. In the table below the minimum EUAC is at year 5 for the old paver (five years from today), the value is $59,703. We compare this value to the minimum EUAC for the challenger of $62,000. Thus, we recommend keeping the defender for at least one more year and reviewing the data for changes.

MARR%	20%
First Cost	120000

YEAR (n)	OPER Cost	MAINT Cost	MV in (n)	Lost MV (n)	Lost Int. (n)	Total MC (n)	NPW (1-->n)	EUAC (1-->n)
1	15000	9000	85000	35000	24000	83000	$69,166.67	$83,000.00
2	15000	10000	65000	20000	7000	52000	$105,277.78	$68,909.09
3	17000	12000	50000	15000	4000	48000	$133,055.56	$63,164.84
4	20000	18000	40000	10000	3000	51000	$157,650.46	$60,898.66
5	25000	20000	35000	5000	2000	52000	$178,548.10	$59,702.86
6	30000	25000	30000	5000	1000	61000	$198,976.87	$59,833.49
7	35000	30000	25000	5000	1000	71000	$218,791.67	$60,698.04

13-22

(a) The minimum cost life is where the EUAC of ownership is minimized for the number of years held. This would occur at 4 years for the defender where EUAC = $4,400.

(b) The minimum cost life of the challenger is 5 years where the EUAC = $6,200.

(c) Using Replacement Analysis Technique #3: Assuming that the defender and challenger costs do not change over the next 4 years we should keep the defender for four years and then reevaluate the costs with challengers at that time. Here we are comparing the min. EUAC (def) vs. min. EUAC (challenger) and $4,400 < $6,200 thus we keep the defender.

13-23

(a) The minimum cost life is where the EUAC of ownership is minimized for the number of years held. This would occur at 1 year for the defender, where EUAC = $4,000.

(b) The minimum cost life of the challenger is 4 years where the EUAC = $3,300.

(c) Using Replacement Analysis Technique #3: Given these costs for the defender and challenger we should replace the defender with the challenger asset now. This is because the min. EUAC (def) > min. EUAC (challenger): $4,000 > $3,300.

13-24

Here we use Replacement Analysis Technique #3. Because the remaining life of the defender and the life of the challenger are both 10 years we can use either the “opportunity cost” or “cash flow” approach to setting the first cost of each option (keep defender or replace with challenger). Let’s show each solution:

Opportunity Cost Approach

EUAC (def)	= 4 ($600) (A/P, 25%, 10)	= $672
EUAC (chal)	= $5,000 (A/P, 25%, 10) - $10,000 (0.075)	= $650

Cash Flow Cost Approach

EUAC (def)	= $0.00	
EUAC (chal)	= ($5,000 - $2,400) (A/P, 255, 10) - $10,000 (0.075)	= -$22

In either case we recommend that the new high efficiency machine be implemented today.

13-25

From the facts stated, we see that if the old forklift is retained the EUAC is minimum for a one year useful life. The problem says the challenger economic life is 10 years. (Using the data provided this fact could be verified, but that is not part of the problem.)

Annual Cash-Flow Analysis:

Keep Old Forklift Another Year

Year	BTCF	Deprec.	Taxable Income	40% Income Taxes	ATCF
0	$0				$0
1	$400	$0	-$400	+$160	-$240

EUAC for one more year with old forklift = $240

Buy New Forklift

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-$6,500				-$6,500
1- 10	-$50	$650	-$700	+$280	+$230

EUAC = $6,500 (A/P, 8%, 10) - $230
= $6,500 (0.1490) - $230
= $738.50

Decision: Choose the alternative with the minimum EUAC. Keep the old forklift another year.

13-26

The problem, with a 7-year analysis period, may be solved in a variety of ways. A first step is to compute an after-tax cash flow for each alternative.

Alternative A

Year	BTCF	Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-$44,000		-$44,000	+$17,600	-$26,400
1- 7	$0		$0		$0

Alternative B

This alternative is less desirable than Alternative D and may be immediately rejected.

Alternative C

Year	BTCF	SOYD Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-$56,000				-$56,000
1	$12,000	$14,000	-$2,000	+$800	+$12,800
2	$12,000	$12,000	$0	$0	+$12,000
3	$12,000	$10,000	$2,000	-$800	+$11,200
4	$12,000	$8,000	$4,000	-$1,600	+$10,400
5	$12,000	$6,000	$6,000	-$2,400	+$9,600
6	$12,000	$4,000	$8,000	-$3,200	+$8,800
7	$12,000	$2,000	$10,000	-$4,000	+$8,000

Alternative D

Year	BTCF	Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-$49,000				-$49,000
1- 7	$7,000	$7,000	$0	$0	+$7,000

Alternative E (Do Nothing)

Year	BTCF	Deprec.	Taxable Income	40% Income Taxes	ATCF
0	$0				$0
1- 7	-$8,000	$0	-$8,000	+$3,200	-$4,800

A NPW solution is probably easiest to compute:

NPW_A = -$26,400
NPW_C = -$56,000 + $12,800 (P/A, 10%, 7) - $800 (P/G, 10%, 7)
= -$56,000 + $12,800 (4.868) - $800 (12.763)
= -$3,900
NPW_D = -$49,000 + $7,000 (P/A, 10%, 7)
= -$49,000 + $7,000 (4.868)
= -$14,924
NPW_E = -$4,800 (P/A, 10%, 7)
= -$4,800 (4.868)
= -$23,366

Choose the solution that maximizes NPW. Choose Alternative C.

Rate of Return Solution

Alternative A rather than Alternative E (Do nothing)

Year	Alt. A ATCF	Alt. E ATCF	(A- E) ATCF
0	-$26,400	$0	-$26,400
1- 7	$0	-$4,800	+$4,800

ΔROR = 6.4%
Reject Alternative A.

Alternative D rather than Alternative E

Year	Alt. D ATCF	Alt. E ATCF	(D- E) ATCF
0	-$49,000	$0	-$49,000
1- 7	+$7,000	-$4,800	+$11,800

ΔROR = 12.8%
Reject Alternative E.

Alternative C rather than Alternative D

Year	Alt. C ATCF	Alt. D ATCF	(C- D) ATCF
0	-$56,000	-$49,000	-$7,000
1	+$12,800	$7,000	$5,800
2	+$12,000	$7,000	$5,000
3	+$11,200	$7,000	$4,200
4	+$10,400	$7,000	$3,400
5	+$9,600	$7,000	$2,600
6	+$8,800	$7,000	$1,800
7	+$8,000	$7,000	$1,000

$7,000 = $5,800 (P/A, i%, 7) - $800 (P/G, i%, 7)

ΔROR > 60%
(Calculator Solution: ΔROR = 65.9%)
Reject D.

Conclusion: Choose Alternative C.

13-27

Book value of Machine A now = Cost – Depreciation to date
= $54,000 – (9/12) ($54,000 - $0)
= $13,500
Recaptured Deprec. If sold now = $30,000 - $13,500
= $16,500
Machine A annual depreciation = (P – S)/n = ($54,000 - $0)/12 = $4,500
Machine B annual depreciation = (P – S)/n = ($42,000 - $0)/12 = $3,500

Alternate 1: Keep A for 12 more years

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF

0	-$30,000*		-$16,500	+$6,600	-$23,400
1	$0	$4,500	-$4,500	+$1,800	+$1,800
2	$0	$4,500	-$4,500	+$1,800	+$1,800
3	$0	$4,500	-$4,500	+$1,800	+$1,800
4- 12	$0	$0	$0	$0	$0

* If A were sold the Year 0 entries would be:

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF
0	+$30,000		$16,500	-$6,600	+$23,400

If A is kept, the entries are just the reverse.

After-Tax Annual Cost
= [$23,400 - $1,800 (P/A, 10%, 4)] (A/P, 10%, 12)
= [$23,400 - $1,800 (2.487)] (0.1468)
= $2,778

The cash flow in year 0 reflects the loss of income after Recaptured Depreciation tax from not selling Machine A. This is the preferred way to handle the current market value of the "defender."

Alternate 2: Buy Machine B

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF
0	-$42,000				-$42,000
1- 12	+$2,500	$3,500	-$1,000	+$400	+$2,900

After-Tax Annual Cost
= $42,000 (A/P, 10%, 12) - $2,900
= $42,000 (0.1468) - $2,900
= $3,266

Choose the alternative with the smaller annual cost. Keep Machine A.

13-28

(a) SONAR

SOYD = (8/2) (9) = 36
ΔD/yr = (1/36) ($18,000 - $3,600) = $400

	Original Year j	SOYD Deprec.	Book Value	
	1	$3,200	$14,800	
	2	$2,800	$12,000	
	3	$2,400	$9,600	
	4	$2,000	$7,600	
Now →	5	$1,600	$6,000	← BV_5
	6	$1,200	$4,800	
	7	$800	$4,000	

8	$400	$3,600

Orig. Year	Analysis Year	BTCF	SOYD Deprec.	Δ Tax Income	Δ Tax	ATCF
5	0	-$7,000		-$1,000*	+$400	-$6,600
6	1		$1,200	-$1,200	+$480	+$480
7	2		$800	-$800	+$320	+$320
8	3	$1,600	$400	-$400	+$160	+$2,560
				$2,000**	+$800	

* Foregone recaptured depreciation is $7,000 – BV_5 = $1,000
** Loss is $1,600 – BV_5 = -$2,000

(b)

Year	BTCF	CCA Depreciation (30%	Δ Tax Income	Δ Tax (40%)	ATCF (SHSS)	ATCF (Sonar)
0	($10,000)				($10,000.0)	($6,600)
1	$500	$1,500	($1,000)	($400.0)	$900.0	$480
2	$500	$2,550	($2,050)	($820.0)	$1,320.0	$320
3	$500	$1,785	($1,285)	($514.0)	$1,014.0	$2,560
	$4,000		($165)	($66.0)	$4,066.0	

Recaptured depreciation (loss) = SV-BV
BV(3)= $4,165

(c)

Year	ΔATCF = $ATCF_{SHSS}$ – $ATCF_{Sonar}$
0	($3,400)
1	$420
2	$1,000
3	$2,520

(d) Compute the NPW of the difference between alternatives

NPW @ 20% = $897.22

13-29

Here we use the Opportunity Cost Approach for finding the first costs.

(a) **Problem as given**
Defender: SL depreciation = ($50,000 - $15,000)/10
= $3,500 per year

MV today = $30,000

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$30,000		$4,500*	-$2,025	$27,975
Keep	0	-$30,000		-$4,500	+$2,025	-$27,975

* TI = Taxable Inc. = Recaptured Deprec.
= $30,000 – [$50,000 – 7 ($3,500)]
= $4,500

b) Defender Market value = $25,500
Defender: SL Depr = ($50,000 - $15,000)/10 = $3,500 per year
MV (today) = $30,000

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$30,000		$0*	$0	$30,000
Keep	0	-$30,000		$0	$0	-$30,000

* Recaptured Depr. = $25,500 – [$50,000 – 7 ($3,500)] = $0

Challenger

Year	BTCF	Depr.	TI	IT	ATCF
0	-$85,000			+$8,500	-$76,500

(c) Defender Market Value = $18,000
Defender: SL Depr = ($50,000 - $15,000)/10 = $3,500 per year
MV (today) = $18,000

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$30,000		-$7,500*	+$3,375	$33,375
Keep	0	-$30,000		+$7,500	-$3,375	-$33,375

* Loss = $18,000 – [$50,000 – 7 ($3,500)] = -$7,500

Challenger

Year	BTCF	Depr.	TI	IT	ATCF
0	-$85,000			+$8,500	-$76,500

13-30

Challenger

Year	BTCF	30% Depr.	TI	35% IT	ATCF	18% PV
0	($10,000)				($10,000)	($10,000)
1	($100)	$1,500	($1,600)	$560	$460	$390

2	(\$150)	\$2,550	(\$2,700)	\$945	\$795	\$571
3	(\$200)	\$1,785	(\$1,985)	\$695	\$495	\$301
4	(\$250)	\$1,250	(\$1,500)	\$525	\$275	\$142
5	(\$300)	\$875	(\$1,175)	\$411	\$111	\$49
6	(\$350)	\$612	(\$962)	\$337	(\$13)	(\$5)
6	\$1,000		(\$429)	\$150	\$1,275	\$472
		\$8,571			**NPV=**	**(\$8,080)**
	BV(yr 6) =	**\$1,429**			**AW=**	**\$2,310.26**

$$CCA_1 = P(\frac{d}{2}) \qquad for \quad n=1$$

$$CCA_n = Pd(1-\frac{d}{2})(1-d)^{n-2} \quad for \ n \geq 2$$

$$UCC_n = P(1-\frac{d}{2})(1-d)^{n-1} \quad for \ n \geq 2$$

13-31

Solution three years into purchase:

15000	First cost
10000	Initial salvage
-1000	Salvage gradient
1000	Initial O&M
1000	O&M gradient
35%	Tax Rate
30%	CCA rate
25%	Interest rate

Year	Capital Cost	CCA	Book Value	AT Salvage	O&M cash flow	Taxable Income	PW Sum O&M - tax	EAC
0	-15000							
1		2250						
2		\$ 3,825	\$ 8,925					
3		\$ 2,678	\$ 6,248					

1	4	10000	$ 1,874	$ 4,373	$ 8,031	-1000	$ (2,874)	$	5	$11,823
2	5	9000	$ 1,312	$ 3,061	$ 6,921	-2000	$ (3,312)	$ (533)		$8,143
3	6	8000	$ 918	$ 2,143	$ 5,950	-3000	$ (3,918)	$ (1,367)		$7,054
4	7	7000	$ 643	$ 1,500	$ 5,075	-4000	$ (4,643)	$ (2,340)		$6,602
5	8	6000	$ 450	$ 1,050	$ 4,268	-5000	$ (5,450)	$ (3,353)		$6,397
6	9	5000	$ 315	$ 735	$ 3,507	-6000	$ (6,315)	$ (4,347)		$6,308
7	10	4000	$ 221	$ 515	$ 2,780	-7000	$ (7,221)	$ (5,285)		$6,279
8	11	3000	$ 154	$ 360	$ 2,076	-8000	$ (8,154)	$ (6,148)		$6,353

=NPV($A$8,$G$14:G22)-$A$6*NPV($A$8,$H$14:H22)

=PMT($A$8,A22,I22+$C$11,F23)

Solution three years into purchase - 7 yr min EAC

13-32

125000	First cost
80000	Initial salvage
-2000	Salvage gradient
	Initial O&M
	O&M gradient
35%	Tax Rate
30%	CCA rate
25%	Interest rate

	Year	Capital Cost	CCA	Book Value	AT Salvage	Operating	Main.	Insurance	O&M cash	Taxable Income	PW Sum O&M - tax	EAC
	0	-125000		125000								
1	1	80000	18750	106250	89188	16000	5000	17000	-38000	-56750	-14510	$85,200
2	2	78000	31875	74375	76731	20000	10000	16000	-46000	-77875	-26506	$71,110
3	3	76000	22313	52063	67622	24000	15000	15000	-54000	-76313	-40479	$67,037
4	4	74000	15619	36444	60855	28000	20000	14000	-62000	-77619	-54747	$65,557
5	5	72000	10933	25511	55729	32000	25000	13000	-70000	-80933	-68402	$65,126
6	6	70000	7653	17857	51750	36000	30000	12000	-78000	-85653	-80991	$65,197
7	7	68000	5357	12500	48575	40000	35000	11000	-86000	-91357	-92320	$65,525
8	8	66000	3750	8750	45963	44000	40000	10000	-94000	-97750	-102351	$65,980
9	9	64000	2625	6125	43744	48000	45000	10000	-103000	-105625	-111214	$66,513
10	10	62000	1838	4288	41801	52000	50000	10000	-112000	-113838	-118962	$67,070
11	11	60000	1286	3001	40050	56000	55000	10000	-121000	-122286	-125679	$67,618

Chapter 14: Inflation and Price Change

14-1

During times of inflation, the purchasing power of a monetary unit is reduced. In this way the currency itself is less valuable on a per unit basis. In the USA, what this means is that during inflationary times our dollars have less purchasing power, and thus we can purchase less products, goods and services with the same $1, $10, or $100 dollar bill as we did in the past.

14-2

Actual dollars are the cash dollars that we use to make transactions in our economy. These are the dollars that we carry around in our wallets and purses, and have in our savings accounts. Real dollars represent dollars that do not carry with them the effects of inflation, these are sometimes called "inflation free" dollars. Real dollars are expressed as of purchasing power base, such as Year-2000-based-dollars.

The inflation rate captures the loss in purchasing power of money in a percentage rate form. The real interest rate captures the growth of purchasing power, it does not include the effects of inflation is sometimes called the "inflation free" interest rate. The market interest rate, also called the combined rate, combines the inflation and real rates into a single rate.

14-3

There are a number of mechanisms that cause prices to rise. In the chapter the authors talk about how *money supply, exchange rates, cost-push,* and *demand pull* effects can contribute to inflation.

14-4

Yes. Dollars, and interest rates, are used in engineering economic analyses to evaluate projects. As such, the purchasing power of dollars, and the effects of inflation on interest rates, are important.

The important principle in considering effects of inflation is not to mix-and-match dollars and interest rates that include, or do not include, the effect of inflation. A constant dollar analysis uses real dollars and a real interest rate, a then-current (or actual) dollar analysis uses actual dollars and a market interest rate. In much of this book actual dollars (cash flows) are used along with a market interest rate to evaluate projects — this is an example of the later type of analysis.

14-5

The Consumer Price Index (CPI) is a composite price index that is managed by the US Department of Labor Statistics. It measures the historical cost of a bundle of "consumer goods" over time. The goods included in this index are those commonly purchased by consumers in the US economy (e.g. food, clothing, entertainment, housing, etc.).

Composite indexes measure a collection of items that are related. The CPI and Producers Price Index (PPI) are examples of composite indexes. The PPI measures the cost to produce goods and services by companies in our economy (items in the PPI include materials, wages, overhead, etc.). Commodity specific indexes track the costs of specific and individual items, such as a labor cost index, a material cost index, a "football ticket" index, etc.

Both commodity specific and composite indexes can be used in engineering economic analyses. Their use depends on how the index is being used to measure (or predict) cash flows. If, in the analysis, we are interested in estimating the labor costs of a new production process, we would use a specific labor cost commodity index to develop the estimate. Much along the same lines, if we wanted to know the cost of treated lumber 5 years from today, we might use a commodity index that tracks costs of treated lumber. In the absence of commodity indexes, or in cases where we are more interested in capturing aggregate effects of inflation (such as with the CPI or PPI) one would use a composite index to incorporate/estimate how purchasing power is affected.

14-6

The stable price assumption is really the same as analyzing a problem in Year 0 dollars, where all the costs and benefits change at the same rate. Allowable depreciation charges are based on the original equipment cost and do not increase. Thus the stable price assumption may be suitable in some before-tax computations, but is not satisfactory where depreciation affects the income tax computations.

14-7

F = P (F/P, *f*%, 10 yrs) = \$10 (F/P, 7%, 10) = \$10 (1.967) = \$19.67

14-8

$i_{equivalent} = i'_{inflation\ corrected} + f\% + (i'_{inflation\ corrected})(f\%)$

In this problem: $i_{equivalent} = 5\%$
$f\% = +2\%$
$i_{inflation\ corrected} = \text{unknown}$

$0.05 = i'_{inflation\ corrected} + 0.02 + (i'_{inflation\ corrected})(0.02)$

$i'_{inflation\ corrected} = (0.05 - 0.02)/(1 + 0.02) = 0.02941 = 2.941\%$

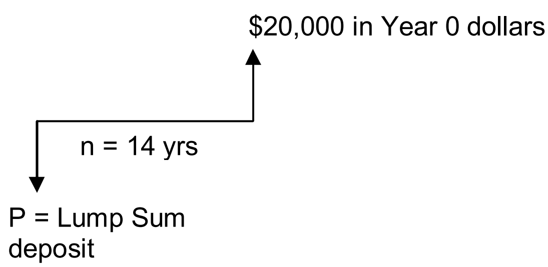
That this is correct may be proved by the year-by-year computations.

Year	Cash Flow	$(1 + f)^{-n}$ (P/F, *f*%, n)	Cash Flow in Year 0 dollars	PW at 2.941%
0	-\$1,000	0	-\$1,000.00	-\$1,000.00

1	+$50	0.9804	+$49.02	+$47.62
2	+$50	0.9612	+$48.06	+$45.35
3	+$50	0.9423	+$47.12	+$43.20
4	+$50	0.9238	+$46.19	+$41.13
5	+$50	0.9057	+$45.29	+$39.18
6	+$50	0.8880	+$44.40	+$37.31
7	+$50	0.8706	+$43.53	+$35.54
8	+$50	0.8535	+$42.68	+$33.85
9	+$50	0.8368	+$41.84	+$32.23
10	+$50	0.8203	+$41.02	+$30.70
11	+$50	0.8043	+$40.22	+$29.24
12	+$50	0.7885	+$39.43	+$27.85
13	+$50	0.7730	+$38.65	+$26.52
14	+$50	0.7579	+$37.90	+$25.26
15	+$50	0.7430	+$37.15	+$24.05
16	+$50	0.7284	+$36.42	+$22.90
17	+$50	0.7142	+$35.71	+$21.82
18	+$50	0.7002	+$35.01	+$20.78
19	+$50	0.6864	+$34.32	+$19.79
20	+$1,000	0.6730	+$706.65	+$395.76
				+$0.08

Therefore, $i_{\text{inflation corrected}}$ = 2.94%.

14-9



Actual Dollars 14 years hence $= \$20,000\ (1 + f\%)^{n}$
$= \$20,000\ (1 + 0.08)^{14}$
$= \$58,744$

At 5% interest:

$P\ = F\ (1 + i)^{-n}\quad = \$58,744\ (1 + 0.05)^{-14}\qquad = \$29,670$

Since the inflation rate (8%) exceeds the interest rate (5%), the money is annual losing purchasing power.

Deposit $29,670.

14-10

To buy $1 worth of goods today will require:

$F \quad = P\ (F/P,\ f\%,\ n) \qquad$ n years hence.
$F \quad = \$1\ (1 + 0.05)^5 \qquad = \$1.47 \qquad$ 5 years hence.

For the subsequent 5 years the amount required will increase to:
$\$1.47\ (F/P,\ f\%,\ n) \quad = \$1.47\ (1 + 0.06)^5 \quad = \$1.97$

Thus for the ten year period $1 must be increased to $1.97. The average price change per year is:
($1.97 - $1.00)/10 yrs = 9.7% per year

14-11

$(1 + f)^5 \quad = 1.50$
$(1 + f) \quad = 1.50^{1/5} \quad = 1.0845$
$f \quad = 0.845 \quad = 8.45\%$

14-12

Number of dollars required five years hence to have the buying power of one dollar today = $1 (F/P, 7%, 5) = $1.403

Number of cruzados required five years hence to have the buying power of 15 cruzados today = 15 (F/P, 25%, 5) = 45.78 cruzados.

Combining: $1.403 = 45.78 cruzados
$1 = 32.6 cruzados (Brazil uses cruzados.)

14-13

Price increase $= (1 + 0.12)^8 = 2.476$ x present price
Therefore, required fuel rating = 10 x 2.476 = 24.76 km/liter

14-14

P= 1.00 *F = 1.80* *n = 10* *f = ?*

1.80 = 1.00 (F/P, *f*%, 10)
(F/P, *f*%, 10) = 1.80

From tables, *f* is slightly greater than 6%. (*f* = 6.05% exactly).

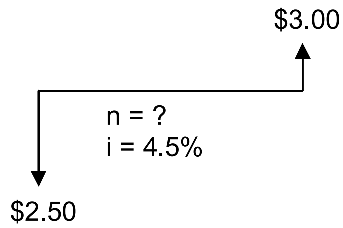
14-15

$i \quad = i' + f + (i')(f)$
$0.15 = i' + 0.12 + 0.12\ (i')$
$1.12\ i' = 0.03$
$i' \quad = 0.03/1.12 \quad = 0.027 \quad = 2.7\%$

14-16

Compute equivalent interest/3 mo. = x

$$
\begin{aligned}
i_{eff} &= (1 + x)^n - 1 \\
0.1925 &= (1 + x)^4 - 1 \\
(1 + x) &= 1.1925^{0.25} = 1.045 \\
x &= 0.045 = 4.5\%/3 \text{ mo.}
\end{aligned}
$$



$$
\begin{aligned}
\$2.50 &= \$3.00\ (P/F, 4.5\%, n) \\
(P/F, 4.5\%, n) &= \$2.50/\$3.00 = 0.833
\end{aligned}
$$

n is slightly greater than 4.

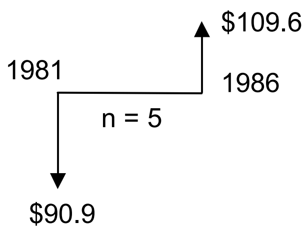
So purchase pads of paper- one for immediate use plus 4 extra pads.

14-17

f = 0.06
i' = 0.10
$i = 0.10 + 0.06 + (0.10)(0.06) = 16.6\%$

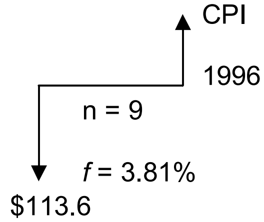
14-18

(a)



$$
\begin{aligned}
\$109.6 &= \$90.9\ (F/P, f\%, 5) \\
(F/P, f\%, 5) &= \$109.6/\$90.9 = 1.2057 \\
f\% &= 3.81\%
\end{aligned}
$$

(b)



$$CPI_{1996} = \$113.6\ (F/P, 3.81\%, 9)$$
$$= \$113.6\ (1 + 0.0381)^9 = \$159.0$$

14-19

$F = \$20{,}000\ (F/P, 4\%, 10) = \$29{,}600$

14-20

Compute an equivalent i:

$$i_{equivalent} = i' + f + (i')\ (f)$$
$$= 0.05 + 0.06 + (0.05)\ (0.06)$$
$$= 0.113$$
$$= 11.3\%$$

Compute the PW of Benefits of the annuity:

$$\text{PW of Benefits} = \$2{,}500\ (P/A, 11.3\%, 10)$$
$$= \$2{,}500\ [((1.113)^{10} - 1)/(0.113\ (1.113)^{10})]$$
$$= \$14{,}540$$

Since the cost is $15,000, the benefits are less than the cost computed at a 5% real rate of return. Thus the actual real rate of return is less than 5% and the annuity should not be purchased.

14-21

$$1 = 0.20\ (1.06)^n$$
$$\log (1/0.20) = n \log (1.06)$$

$$n = 27.62 \text{ years}$$

14-22

Use $\$97{,}000\ (1 + f\%)^n$, where f%=7% and n=15

$$\$97{,}000\ (1 + 0.07)^{15} = \$97{,}000\ (F/P, 7\%, 15)$$

$= \$97,000\ (2.759)$
$= \$268,000$

If there is 7% inflation per year, a \$97,000 house today is equivalent to \$268,000 15 years hence. But will one have "profited" from the inflation?

Whether one will profit from owning the house depends somewhat on an examination of the alternate use of the money. Only the differences between alternatives are relevant. If the alterate is a 5% savings account, neglecting income taxes, the profit from owning the house, rather than the savings account, would be: \$268,000 - \$97,000 (F/P, 5%, 15) = \$66,300.

On the other hand, compared to an alternative investment at 7%, the profit is \$0. And if the alternative investment is at 9% there is a loss. If "profit" means an enrichment, or being better off, then multiplying the price of everything does no enrich one in real terms.

14-23

Let x = selling price
Then long-term capital gain = x- \$18,000
Tax = 0.15 (x - \$18,000)
After-Tax cash flow in year 10 = x – 0.15 (x - \$18,000) = 0.85x + \$2,700

Year	ATCF	Multiply by	Year 0 $ ATCF
0	-\$18,000	1	-\$18,000
10	+0.85x + \$2,700	1.06^{-10}	0.4743x + \$1,508

For a 10% rate of return:
\$18,000 = (0.4746x + \$1,508) (P/F, 10%, 10)
= 0.1830 x + \$581
x = \$95,186

Alternate Solution using an equivalent interest rate
$i_{equiv} = i' + f + (i')\,(f) = 0.10 + 0.06 + (0.10)\,(0.06) = 0.166$

So $\$18,000\ (1 + 0.166)^{10} = 0.85x + \$2,700$
\$83,610 = 0.85x + \$2,700

Selling price of the lot = x = (\$83,610 - \$2,700)/0.85 = \$95,188

14-24

Cash Flow:

Year	$500 Kit	$900 Kit
0	-\$500	-\$900
5	-\$500	\$0

(a) $PW_{\$500\ kit}$ = \$500 + \$500 (P/F, 10%, 5) = \$810
$PW_{\$900\ kit}$ = \$900

To minimize PW of Cost, choose \$500 kit.

(b) Replacement cost of \$500 kit, five years hence
= \$500 (F/P, 7%, 5) = \$701.5

$PW_{\$500\text{ kit}}$ = \$500 + \$701.5 (P/F, 10%, 5) = \$935.60
$PW_{\$900\text{ kit}}$ = \$900

To minimize PW of Cost, choose \$900 kit.

14-25

If one assumes the 5-year hence cost of the Filterco unit is:
\$7,000 (F/P, 8%, 5) = \$10,283
in Actual Dollars and \$7,000 in Yr 0 dollars, the year 0 \$ cash flows are:

Year	Filterco	Duro	Duro – Filterco
0	-\$7,000	-\$10,000	-\$3,000
5	-\$7,000	\$0	+\$7,000

ΔROR = 18.5%
Therefore, buy Filterco.

14-26

Year	Cost to City (Year 0 \$)	Benefits to City	Description of Benefits
0	-\$50,000		
1- 10	-\$5,000/yr	+ A	Fixed annual sum in then-current dollars
10		+\$50,000	In then-current dollars

i = i' + *f* + i'*f*
= 0.03 + 0.07 + 0.03 (0.07)
= 0.1021 = 10.21%

PW of Cost = PW of Benefits
\$50,000 + \$5,000 (P/A, 3%, 10) = A(P/A, 10.21%, 10) +\$50,000 (P/F,10.21%,10)
\$50,000 + \$5,000 (8.530) = A (6.0895*) + \$50,000 (0.3783*)
\$92,650 = 6.0895A + \$18,915

A = (\$92,650 - \$18,915)/6.0895
= \$12,109

* Computed on hand calculator

14-27

Month	BTCF
0	\$0
1- 36	-\$1,000
36	+\$40,365

$1,000 (F/A, i%, 36 mo) = $40,365
(F/A, i%, 36) = 40.365

Performing linear interpolation:

(F/A, i%, 36))	*i*
41.153	¾%
39.336	½%

i = 0.50% + 0.25% [(40.365 – 39.336)/(41.153 – 39.336)]
= 0.6416% per month

Equivalent annual interest rate

i per year = $(1 + 0.006416)^{12} – 1$ = 0.080 = 8%

So, we know that i = 8% and *f* = 8%. Find i'.
i = i' + *f* + (i') (*f*)
0.08 = i' + 0.08 + (i') (0.08)
i' = 0% Thus, before-Tax Rate of Return = 0%

14-28

Actual Dollars: F= $10,000 (F/P, 10%, 15) = $41,770

Real Dollars:

Year	Inflation
1- 5	3%
6- 10	5%
11- 15	8%

R$ in today's base = $41,770 (P/F, 8%, 5) (P/F, 5%, 5) (P/F, 3%, 5)
= $18,968

Thus, the real growth in purchasing power has been:
$\$18,968 = \$10,000 (1 + i^*)^{15}$
$i^* = 4.36\%$

14-29

(a) F = $\$2,500 (1.10)^{50}$ = $293,477 in A$ today

(b) R$ today in (-50) purchasing power = $293,477 (P/F, 4%, 50)
= $41,296

14-30

(a) PW $= \$2,000\ (P/A, i_c, 8)$

$i_{combined} = i_{real} + f + (i_{real})(f) = 0.03 + 0.05 + (0.03)(0.05)$
$= 0.0815$

PW $= \$2,000\ (P/A, 8.15\%, 9) = \$11,428$

(b) PW $= \$2,000\ (P/A, 3\%, 8) = \$14,040$

14-31

Find PW of each plan over the next 5-year period.

$i_r = (i_c - f)/(1 + f) = (0.08 - 0.06)/1.06 = 1.19\%$

PW(A) $= \$50,000\ (P/A, 11.5\%, 5) = \$236,359$
PW(B) $= \$45,000\ (P/A, 8\%, 5) + \$2,500\ (P/G, 8\%, 5) = \$198,115$
PW(C) $= \$65,000\ (P/A, 1.19, 5)\ (P/F, 6\%, 5) = \$229,612$

Here we choose Company A's salary to maximize PW.

14-32

(a) R today \$ in year 15 $= \$10,000\ (P/F, i_r\%, 15)$

$i_r = (0.15 - 0.08)/1.08 = 6.5\%$

R today \$ in year 15 $= \$10,000\ (1.065)^{15} = \$25,718$

(b) $i_c = 15\%$ $f = 8\%$
$F = \$10,000\ (1.15)^{15} = \$81,371$

14-33

No Inflation Situation

Alternative A: PW of Cost $= \$6,000$
Alternative B: PW of Cost $= \$4,500 + \$2,500\ (P/F, 8\%, 8)$
$= \$4,500 + \$2,500\ (0.5403)$
$= \$5,851$
Alternative C: PW of Cost $= \$2,500 + \$2,500\ (P/F, 8\%, 4) + \$2,500\ (P/F, 8\%, 8)$
$= \$2,500\ (1 + 0.7350 + 0.5403)$
$= \$5,688$

To minimize PW of Cost, choose Alternative C.

For *f* = +5% (Inflation)

Alternative A: PW of Cost = $6,000

Alternative B: PW of Cost = $4,500 + $2,500 (F/P, 5%, 8) (P/F, 8%, 8)
= $4,500 + $2,500 $(1 + f\%)^8$ (P/F, 8%, 8)
= $4,500 + $2,500 (1.477) (0.5403)
= $6,495

Alternative C: PW of Cost = $2,500 + $2,500 (F/P, 5%, 4) (P/F, 8%, 4)
+ $2,500 (F/P, 5%, 8) (P/F, 8%, 8)
= $2,500 + $2,500 (1.216) (0.7350)
+ $2,500 (1.477) (0.5403)
= $6,729

To minimize PW of Cost in year 0 dollars, choose Alternative A.

This problem illustrates the fact that the prospect of future inflation encourages current expenditures to be able to avoid higher future expenditures.

14-34

Alternative I: Continue to Rent the Duplex Home

Compute the Present Worth of renting and utility costs in Year 0 dollars.

Assuming end-of-year payments, the Year 1 payment is:
= ($750 + $139) (12) = $7,068
The equivalent Year 0 payment in Year 0 dollars is:
$\$7{,}068\ (1 + 0.05)^{-1} = \$6{,}713.40$

Compute an equivalent i
$i_{equivalent} = i' + f + (i')(f)$
Where i' = interest rate without inflation = 15.5%
f = inflation rate = 5%
$i_{equivalent} = 0.155 + 0.05 + (0.155)(0.05)$
= 0.21275
= 21.275%

PW of 10 years of rent plus utilities:
= $6,731.40 (P/A, 21.275%, 10)
$= \$6{,}731.40\ [(1 + 0.21275)^{(10-1)})/(0.21275\ (1 + 0.21275)^{10})]$
= $6,731.40 (4.9246)
= $33,149

An Alternative computation, but a lot more work:
Compute the PW of the 10 years of inflation adjusted rent plus utilities using 15.5% interest.
$PW_{year\ 0} = 12[\$589\ (1 + 0.155)^{-1} + \$619\ (1 + 0.155)^{-2} + \ldots + \$914\ (1 + 0.155)^{-10}]$
= 12 ($2,762.44)
= $33,149

Alternative II: Buying a House

$3,750 down payment plus about $750 in closing costs for a cash requirement of $4,500.
Mortgage interest rate per month = $(1+I)^6$ = 1.04   I = 0.656%
n = 30 years x 12 = 360 payments

Monthly Payment: A = ($75,000 - $3,750) (A/P, 0.656%, 360)
= $71,250 [(0.00656 $(1.00656)^{360}$)/($(1.00656)^{360}$ – 1)]
= -$516.36

Mortgage Balance After the 10-year Comparison Period:
A' = $523 (P/A, 0.656%, 240)
= $523 [($(1.00656)^{240}$ – 1)/(0.00656 $(1.00656)^{240}$)]
= $62,335

Thus:

$523 x 12 x 10	= $62,760	total payments
$71,250 - $62,504	= $8,746	principal repayments (12.28% of loan)
	= $54,014	interest payments

Sale of the property at 6% appreciation per year in year 10:
F = $75,000 $(1.06)^{10}$ = $134,314
Less 5% commission = -$6,716
Less mortgage balance = -$62,335
Net Income from the sale = $65,263

Assuming no capital gain tax is imposed, the Present Worth of Cost is:
PW= $4,500 [Down payment + closing costs in constant dollars]
+ $516.36 x 12 (P/A, 15.5%, 10) [actual dollar mortgage]
+ $160 x 12 (P/A, 10%, 10) [constant dollar utilities]
+ $50 x 12 (P/A, 10%, 10) [constant dollar insurance & maint.]
- $65,094 (P/F, 15.5%, 10) [actual dollar net income from sale]
PW= $4,500 + $516.36 x 12 (4.9246) + $160 x 12 (6.145)
+ $50 x 12 (6.145)- $65,263 (0.2367)
= $35,051

The PW of Cost of owning the house for 10 years = $35,051 in Year 0 dollars. Thus $33,149 < $35,329 and so buying a house is the more attractive alternative.

14-35

Year	Cost- 1	Cost- 2	Cost- 3	Cost- 4	TOTAL	PW-TOTAL
1	$4,500	$7,000	$10,000	$8,500	$30,000	$24,000
2	$4,613	$7,700	$10,650	$8,288	$31,250	$20,000
3	$4,728	$8,470	$11,342	$8,080	$32,620	$16,702
4	$4,846	$9,317	$12,079	$7,878	$34,121	$13,976
5	$4,967	$10,249	$12,865	$7,681	$35,762	$11,718
6	$5,091	$11,274	$13,701	$7,489	$37,555	$9,845
7	$5,219	$12,401	$14,591	$7,302	$39,513	$8,286
8	$5,349	$13,641	$15,540	$7,120	$41,649	$6,988

9	\$5,483	\$15,005	\$16,550	\$6,942	\$43,979	\$5,903
10	\$5,620	\$16,506	\$17,626	\$6,768	\$46,519	\$4,995

PW = -\$60,000 – (\$24,000 + \$20,000 + \$16,702 + … +\$4,995)
+ \$15,000 (P/F, 25%, 10)

= \$180,802

14-36

(a) Unknown Quantities are calculated as follows:

% change = [(\$100 - \$89)/\$89] x 100% = 12.36%
PSI = 100 (1.04) = 104
% change = (\$107 - \$104)/\$104 = 2.88%
% change = (\$116 - \$107)/\$107 = 8.41%
PSI = 116 (1.0517) = 122

(b) The base year is 1993. This is the year of which the index has a value of 100.

(c)

(i) PSI (1991) = 82
PSI (1995) = 107
h = 4 years
i* = ?
i* = $(107/82)^{0.25} - 1$ = 6.88%

(ii) PSI (1992) = 89
PSI (1998) = 132
n = 6 years
i* = ?
i* = $(132/89)^{(1/6)} - 1$ = 6.79%

14-37

(a) LCI(-1970) = 100
LCI(-1979) = 250
n = 9
i* = ?
i* = $(250/100)^{(1/9)} - 1$ = 10.7%

(b) LCI(1980) = 250
LCI(1989) = 417
n = 9
i* = ?
i* = $(417/250)^{(1/9)} - 1$ = 5.85%

(c) LCI(1990) = 417
LCI(1998) = 550
n = 8
i* = ?
i* = $(550/417)^{(1/8)} - 1$ = 3.12%

14-38

(a) Overall LCI change = [(250 – 100)/100] x 100% = 150%
(b) Overall LCI change = [(415 – 250)/250] x 100% = 66.8%
(c) Overall LCI change = [(650 – 417)/417] x 100% = 31.9%

14-39

(a) CPI (1978) = 43.6
CPI (1982) = 65.3
n = 4
i* = ?
$i^* = (65.3/43.6)^{(1/4)} - 1 = 9.8\%$

(b) CPI (1980) = 52.4
CPI (1989) = 89.0
n = 9
i* = ?
$i^* = (89.0/52.4)^{(1/9)} - 1 = 6.1\%$

(c) CPI (1985) = 75.0
CPI (1997) = 107.6
n = 12
i* = ?
$i^* = (107.6/75.0)^{(1/12)} - 1 = 3.1\%$

14-40

(a)

Year	Brick Cost	CBI
1970	2.10	442
1998	X	618

x/2.10 = 618/442
x = $2.94

Total Material Cost = 800 x $2.94 = $2,350

(b) Here we need f% of brick cost
CBI(1970) = 442
CBI(1998) = 618
n = 18
i* = ?
$i^* = (618/442)^{(1/18)} - 1 = 1.9\%$

We assume the past average inflation rate continues for 10 more years.

Brick Unit Cost in 2008 = 2.94 (F/P, 1.9%, 10)= $3.54

Total Material Cost = 800 x $3.54 = $2.833

14-41

EAT(today) = $330 (F/P, 12$, 10) = $1,025

14-42

Item	Year 1	Year 2	Year 3
Structural	$125,160	$129,165	$137,690
Roofing	$14,280	$14,637	$15,076
Heat etc.	$35,560	$36,306	$37,614
Insulating	$9,522	$10,093	$10,850
Labor	$89,250	$93,266	$97,463
Total	$273,772	$283,467	$298,693

(a) $89,250; $93,266; $97,463

(b) PW = $9,522 (P/F, 25%, 1) + $10,093 (P/F, 25%, 2)
+ $10,850 (P/F, 25%, 3)
= $19,632

(c) FW = ($9,522 + $89,250) (F/P, 25%, 2) + ($10,093
+ $93,266) (F/P, 25%, 1) + ($10,850 + $97,463)
= $391,843

(d) PW = $273,772 (P/F, 25%, 1) + $283,467 (P/F, 25%, 2)
+ $298,693 (P/F, 25%, 3)
= $553,367

14-43

The total cost of the bike 10 years from today would be $2,770

Item	**Current Cost**	**Inflation**	**Future Cost**
Frame	800	2.0%	975.2
Wheels	350	10.0%	907.8
Gearing	200	5.0%	325.8
Braking	150	3.0%	201.6
Saddle	70	2.5%	89.6
Finishes	125	8.0%	269.9
Sum=	1695	**Sum=**	2769.8

14-44

To minimize purchase price Mary Clare should select the vehicle from company X.

Car	Current Price	Inflation	Future Price
X	27500	4.0%	30933.8
Y	30000	1.5%	31370.4
Z	25000	8.0%	31492.8
		Min=	30933.8

14-45

$F_{YEAR\ 5}$ = \$100 (F/A, 12/4=3%, 5 x 4=20) = \$2,687

$F_{YEAR\ 10}$ = \$2,687 (F/P, 4%, 20) + \$100 (F/A, 4%, 20) = \$8,865

$F_{YEAR\ 15\ (TODAY)}$ = \$8,865 (F/P, 2%, 20) + \$100 (F/A, 2%, 20) = \$15,603

14-46

To pay off the loan Andrew will need to write a check for \$ 18,116

Year	Amt due Begin yr	Inflation	Amt due End yr
1	15000	5.0%	15750.0
2	15750.0	6.5%	16773.8
3	16773.8	8.0%	18115.7
		Due=	18115.7

14-47

See the table below for (a) through (e)

Year	Ave Price	Inflation for year
5 years ago	165000.0	(a) = 1.2%
4 years ago	167000.0	(b) = 3.0%
3 years ago	172000.0	(c) = 4.7%
2 years ago	180000.0	(d) = 1.7%
last year	183000.0	(e) = 3.8%
This year	190000.0	(f) see below

One could predict the inflation (appreciation) in the home prices this year using a number of approaches. One simple rule might involve using the average of the last 5 years inflation rates. This rate would be (1.2+3+4.7+1.7+3.8)/5 = 2.9%.

14-48

Depreciation charges that a firm makes in its accounting records allow a profitable firm to have that amount of money available for replacement equipment without any deduction for income taxes.

If the money available from depreciation charges is inadequate to purchase needed replacement equipment, then the firm may need also to use after-tax profit for this purpose.

Depreciation charges produce a tax-free source of money; profit has been subjected to income taxes. Thus substantial inflation forces a firm to increasingly finance replacement equipment out of (costly) after-tax profit.

14-49

(a)

Year	BTCF	SL Deprec.	TI	34% Income Taxes	ATCF
0	-$85,000				-$85,000
1	$8,000	$1,500	$6,500	-$2,210	$5,790
2	$8,000	$1,500	$6,500	-$2,210	$5,790
3	$8,000	$1,500	$6,500	-$2,210	$5,790
4	$8,000	$1,500	$6,500	-$2,210	$5,790
5	$8,000	$1,500	$6,500	-$2,210	$83,290
	$77,500		$0		
Sum		$7,500			

SL Depreciation = (\$67,500 - \$0)/45 = \$1,500
Book Value at end of 5 years = \$85,000 – 5 (\$1,500) = \$77,500

After-Tax Rate of Return = 5.2%

(b)

Year	BTCF	SL Deprec.	TI	34% Income Taxes	Actual Dollars ATCF
0	-$85,000				-$85,000
1	$8,560	$1,500	$7,060	-$2,400	$6,160
2	$9,159	$1,500	$7,659	-$2,604	$6,555
3	$9,800	$1,500	$8,300	-$2,822	$6,978
4	$10,486	$1,500	$8,986	-$3,055	$7,431
5	$11,220	$1,500	$9,720	-$3,305	$131,913
	$136,935*			-$16,242**	
Sum		$7,500			

*Selling Price = \$85,000 (F/P, 10%, 5) = \$85,000 (1.611) = \$136,935
** On disposal, there are capital gains and depreciation recapture
Capital Gain = \$136,935 - \$85,00 = \$51,935
Tax on Cap. Gain = (20%) (\$51,935) = \$10,387
Recaptured Depr. = \$85,000 - \$77,500 = \$7,500
Tax on Recap. Depr. = (34%)(\$7,500) = \$2,550

Total Tax on Disposal = $10,387 + $2,550 = $12,937

After Tax IRR = 14.9%

After-Tax Rate of Return in Year 0 Dollars

Year	Actual Dollars ATCF	Multiply by	Year 0 $ ATCF
0	-$85,000	1	-$85,000
1	$6,160	1.07^{-1}	$5,757
2	$6,555	1.07^{-2}	$5,725
3	$6,978	1.07^{-3}	$5,696
4	$7,431	1.07^{-4}	$5,669
5	$131,913	1.07^{-5}	$94,052
Sum			

In year 0 dollars, After-Tax Rate of Return = 7.4%

14-50

Year	BTCF	TI	42% Income Taxes	ATCF	Multiply by	Year 0 $ ATCF
0	-$10,000			-$10,000	1	-$10,000
1	$1,200	$1,200	-$504	$696	1.07^{-1}	$650
2	$1,200	$1,200	-$504	$696	1.07^{-2}	$608
3	$1,200	$1,200	-$504	$696	1.07^{-3}	$568
4	$1,200	$1,200	-$504	$696	1.07^{-4}	$531
5	$1,200	$1,200	-$504	$10,696	1.07^{-5}	$7,626
	$10,000					
					Sum	-$17

(a) **Before-Tax Rate of Return ignoring inflation**
Since the $10,000 principal is returned unchanged,
i = A/P = $1,200/$10,000 = 12%

If this is not observed, then the rate of return may be computed by conventional means.

$10,000 = $1,200 (P/A, i%, 5) + $10,000 (P/F, i%, 5)
Rate of Return = 12%

(b) **After-Tax Rate of Return ignoring inflation**
Solved in the same manner as Part (a):
i = A/P = $696/$10,000 = 6.96%

(c) **After-Tax Rate of Return after accounting for inflation**
An examination of the Year 0 dollars after-tax cash flow shows the algebraic sum of the cash flow is -$17. Stated in Year 0 dollars, the total receipts are less than the cost, hence there is no positive rate of return.

14-51

Now:

Taxable Income = $60,000
Income Taxes = $35,000x0.16+25,000x0.22 = $11,100
After-Tax Income = $60,000 - $11,100 = $48,900

Twenty Years Hence: To have some buying power, need:

After-Tax Income = $\$48{,}900(1.07)^{20}$ = $189,227.60
= Taxable Income – Income Taxes

Income Taxes = $24,689.04+0.29(Taxable Income - $113,804)

Taxable Income = After-Tax Income + Income Taxes
= $189,227.60+$24,689.04+0.22(TI - $113,804)
= $242,153.50

14-52

P=	$10,000
CCA=	30%
t =	50%
S =	0
***f* =**	**7%**

Year	**Actual $s Received**	**Actual CCA**	**Actual $s Tax**	**Net Salvage**	**Actual $s ATCF**	**Real $s ATCF**
0	-$10,000				-$10,000	-$10,000
1	$2,000	-$1,500	$250		$1,750	$1,636
2	$3,000	-$2,550	$225		$2,775	$2,424
3	$4,000	-$1,785	$1,108		$2,893	$2,361
4	$5,000	-$1,250	$1,875		$3,125	$2,384
5	$6,000	-$875	$2,563		$3,437	$2,451
6	$7,000	-$612	$3,194		$3,806	$2,536
7	$8,000	-$429	$3,786	$500	$4,714	$2,936
					IRR=	**13.71%**

Net Salvage Calculation from Equation 12-7, 12-8, & 11-8

UCC_7 = $1,000 =$B$1*(1-$B$2/2)*(1-$B$2)^(7-1)

Net Salvage end of yr 7 = $500

=$B$4+$B$3*IF($B$4>$B$1,$C$18-$B$1,$C$18-$B$4)-1/2*$B$3*MAX(($B$4-$B$1),0)

14-53

P=	$103,500
CCA=	10%
t =	35%
S =	$ 103,500
***f* =**	**0%**

Year	Actual $s Received	Actual CCA	Actual $s Tax	Net Salvage	Actual $s ATCF	Real $s ATCF
0	-$103,500			-$46,500	-$150,000	-$150,000
1	$15,750	-$5,175	$3,701		$12,049	$12,049
2	$15,750	-$9,833	$2,071		$13,679	$13,679
3	$15,750	-$8,849	$2,415		$13,335	$13,335
4	$15,750	-$7,964	$2,725		$13,025	$13,025
5	$15,750	-$7,168	$3,004	$136,354	$149,100	$149,100
						7.06% **=IRR**

Net Salvage Calculation from Equation 12-7, 12-8, & 11-8

UCC_5 =$64,511 =$B$1*(1-$B$2/2)*(1-$B$2)^(5-1)

Net Salvage end of yr 5 = $89,854

=$B$4+$B$3*IF($B$4>$B$1,$C$18-$B$1,$C$18-$B$4)-1/2*$B$3*MAX(($B$4-$B$1),0)

14-54

P=	$103,500
CCA=	10%
t =	35%
S =	$ 103,500
***f* =**	**10%**

Year	Actual $s Received	Actual CCA	Actual $s Tax	Net Salvage	Actual $s ATCF	Real $s ATCF
0	-$103,500			-$46,500	-$150,000	-$150,000
1	$12,000	-$5,175	$2,389		$9,611	$8,738
2	$13,440	-$9,833	$1,263		$12,177	$10,064
3	$15,053	-$8,849	$2,171		$12,882	$9,678
4	$16,859	-$7,964	$3,113		$13,746	$9,389
5	$18,882	-$7,168	$4,100	$230,694	$245,476	$152,421
					16.01%	**5.46%** **=IRR**

Net Salvage Calculation from Equation 12-7, 12-8, & 11-8

UCC_5 = $64,511 =$B$1*(1-$B$2/2)*(1-$B$2)^(5-1)

Net Salvage end of yr 5 = $89,854

=$B$4+$B$3*IF($B$4>$B$1,$C$18-$B$1,$C$18-$B$4)-1/2*$B$3*MAX(($B$4-$B$1),0)

Market Value in 5 years = 150000 x (1+12%)^5		$264,351.25
	house value	$103,500.00
	land sale	$160,851.25
	capital gain's tax	$ 20,011.47
	net land salvage	**$140,839.78**

14-55

Alternative A

Year	Cash Flow in Year 0 $	Cash Flow in Actual $	SL Deprec.	TI	25% Income Tax	ATCF in Actual $	ATCF in Year 0 $
0	-$420	-$420				-$420	-$420
1	$200	$210	$140	$70	-$17.5	$192.5	$183.3
2	$200	$220.5	$140	$80.5	-$20.1	$200.4	$181.8
3	$200	$231.5	$140	$91.5	-$22.9	$208.6	$180.2

Alternative B

Year	Cash Flow in Year 0 $	Cash Flow in Actual $	SL Deprec.	TI	25% Income Tax	ATCF in Actual $	ATCF in Year 0 $
0	-$300	-$300				-$300	-$300
1	$150	$157.5	$100	$57.5	-$14.4	$143.1	$136.3
2	$150	$165.4	$100	$65.4	-$16.4	$149.0	$135.1
3	$150	$173.6	$100	$73.6	-$18.4	$155.2	$134.1

Quick Approximation of Rates of Return:

Alternative A:

$420 = $182 (P/A, i%, 3)
(P/A, i%, 3) = $420/$182 = 2.31
12% < ROR < 15%
(Actual ROR = 14.3%)

Alternative B:

$300 = $135 (P/A, i%, 3)
(P/A, i%, 3) = $300/$135 = 2.22
15% < ROR < 18%
(Actual ROR = 16.8%)

Incremental ROR Analysis for A- B

Year	A	B	A- B
0	-$420	-$300	-$120
1	$183.3	$136.3	$47

2	$181.8	$135.1	$46.7
3	$180.2	$134.1	$46.1

Try i = 7%

NPW = -$120 + $47 (P/F, 7%, 1) + $46.7 (P/F, 7%, 2)
+ $46.1 (P/F, 7%, 3)
= +$2.3

So the rate of return for the increment A- B is greater than 7% (actually 8.1%). Choose the higher cost alternative: choose Alternative A.

Chapter 15: Selection of a Minimum Attractive Rate of Return

15-1

The interest rates on these securities vary greatly over time, making it impossible to predict rates. Three factors that distinguish the securities:

	Bond Duration	Bond Safety
Municipal Bond	20 years	Safe
Corporate Bond	20 years	Less Safe

The importance of the non-taxable income feature usually makes the municipal bond the one with the lowest interest rate. The corporate bond generally will have the highest interest rate.

15-2

As this is a situation of “neither input nor output fixed,” incremental analysis is required.

	C- D	B- C	B- D	D- A
Δ Cost	$25	$50	$75	$25
Δ Benefit	$4	$6.31	$10.31	$5.96
Δ Rate of Return	9.6%	4.5%	6.2%	20%

Using the incremental rates of return one may determine the preferred alternative at any interest rate.

For interest rates between:

$$0\% \;\underbrace{\text{————}}_{B}\; 4.5\% \;\underbrace{\text{————}}_{C}\; 9.6\% \;\underbrace{\text{————}}_{D}\; 20\% \;\underbrace{\text{————}}_{A}\; \infty$$

The problem here concerns Alternative C. C is preferred for 4.5% < Interest Rate < 9.6%.

15-3

Lease: Pay $267 per month for 24 months.
Purchase:
A = $9,400 (A/P, 1%, 24) = $9,400 (0.0471) = $442.74
Salvage (resale) value = $4,700

(a) **Purchase Rather than Lease**

ΔMonthly payment = $442.71 - $267 = $175.74
ΔSalvage value = $4,700 - $0 = $4,700

Δ Rate of Return
PW of Cost = PW of Benefit

$175.74 (P/A, i%, 24) = $4,700
(P/A, i%, 24) = $4,700/$175.74 = 26.74

i = 0.93% per month

Thus, the additional monthly payment of $175.74 would yield an 11.2% rate of return. Leasing is therefore preferred at all interest rates above 11.2%.

(b) Items that might make leasing more desirable:
1. One does not have, or does not want to spend, the additional $175.74 per month.
2. One can make more than 11.2% rate of return in other investment.
3. One does not have to be concerned about the resale value of the car at the end of two years.

15-4

Investment opportunities may include:
1. Deposit of the money in a Bank.
2. Purchase of common stock, US Treasury bonds, or corporate bonds.
3. Investment in a new business, or an existing business.
4. (and so on.)

Assuming the student has a single investment in which more than $2,000 could be invested, the MARR equals the projected rate of return for the investment.

15-5

Venture capital syndicates typically invest money in situations with a substantial amount of risk. The process of identifying and selecting investments is a time-consuming (and hence costly) process. The group would therefore only make a venture capital investment where (they think) the rate of return will be high- probably 25% or more.

15-6

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, -first cost, salvage value), and then the table is sorted with IRR as the key. Projects A and B are the top two projects, which fully utilize the $100,000 capital budget. The opportunity cost of capital is 12.0% if based on the first project rejected.

Project	IRR	First Cost	Annual Benefits	Life	Salvage Value
A	13.15%	$50,000	$13,500	5 yrs	$5,000
B	12.41%	$50,000	$9,000	10 yrs	$0
D	11.99%	$50,000	$9,575	8 yrs	$6,000
C	10.66%	$50,000	$13,250	5 yrs	$1,000

15-7

The IRR for each project is calculated using the Excel function = RATE (3, annual benefit, -first cost) since N = 3 for all projects. Then the table is sorted with IRR as the key. Do projects 3, 1 and 7 with a budget of $70,000. The opportunity cost of capital is 26.0% if based on the first project rejected.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit
3	36.31%	$10,000	$10,000	$6,000
1	29.92%	$30,000	$20,000	$11,000
7	26.67%	$70,000	$40,000	$21,000
5	26.01%	$95,000	$25,000	$13,000
4	20.71%	$100,000	$5,000	$2,400
2	18.91%	$130,000	$30,000	$14,000
6	18.91%	$145,000	$15,000	$7,000

15-8

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, -first cost, salvage value), and then the table is sorted with IRR as the key. With a budget of $500,000, the opportunity cost of capital is 19.36% if based on the first project rejected. Projects 3, 1, 4, and 6 should be done.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit	Life (years)
3	28.65%	$100,000	$100,000	$40,000	5
1	24.01%	$300,000	$200,000	$50,000	15
4	21.41%	$350,000	$50,000	$12,500	10
6	20.85%	$500,000	$150,000	$32,000	20
2	19.36%	$800,000	$300,000	$70,000	10
7	16.99%	$1,200,000	$400,000	$125,000	5
5	15.24%	$1,450,000	$250,000	$75,000	5

15-9

The IRR for each project is calculated using the Excel function = Rate (life, annual benefit, -first cost), and then the table is sorted with IRR as the key. The top 6 projects required $260K in capital funding, and the opportunity cost of capital based on the first rejected project is 8.0%.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit	Life (years)
E	15.00%	$40,000	$40,000	$11,933	5
H	13.44%	$100,000	$60,000	$12,692	8
C	12.00%	$130,000	$30,000	$9,878	4
G	10.97%	$165,000	$35,000	$6,794	8
I	10.00%	$240,000	$75,000	$14,058	8
B	9.00%	$260,000	$20,000	$6,173	4
D	8.00%	$285,000	$25,000	$6,261	5

A	7.01%	$300,000	$15,000	$4,429	4
F	5.00$	$350,000	$50,000	$11,550	5

15-10

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, -first cost, salvage value), and then the table is sorted with IRR as the key. With a budget of $100,000, the top 5 projects should be done (6, 5, 4, 1, and 7). The opportunity cost of capital based on the first rejected project is 16.41%.

Project	IRR	First Cost	Annual Benefits	Life (years)	Salvage Value
6	26.16%	$20,000	$5,800	10	$0
5	22.50%	$20,000	$4,500	25	-$20,000
4	21.25%	$20,000	$4,500	15	$0
1	19.43%	$20,000	$4,000	20	$0
7	19.26%	$20,000	$4,000	15	$10,000
3	16.41%	$20,000	$3,300	30	$10,000
2	16.00%	$20,000	$3,200	20	$20,000

Chapter 16: Economic Analysis in the Public Sector

16-1

Public decision-making involves the use of public money and resources to fund public projects. Often there are those who are advocating for particular projects, those who oppose projects, those who will be immediately affected by such project, and those who may be affected in the future. There are those who represent their own stated interests, and those who are representing others' interests. Thus the "multi-actor" aspect of the phrase refers to the varied and wide group of "stakeholders" who are involved with, affected by, or place some concern on the decision process.

16-2

Public decision-making is focused on *promoting the general welfare* of the aggregate public. There is an explicit recognition in promoting the good of the whole, in some cases, that individual's goals must be subordinate (e.g. eminent domain). Private decision making, on the other hand, is generally focused on increasing stakeholder wealth or investment. This is not to say that private decision-making is entirely focuses on financials, clearly private decision-making focuses on non-monetary issues. However, the goal and objective of the enterprise is economic survival and growth and thus the primary objective is financial in nature (for without success financially all other objectives are moot is the firm dissolves).

16-3

The general suggestion is that the viewpoint should be at least as broad as those who pay the costs and/or receive the benefits. This approach balances local decisions, which may sub-optimize decision making if not taken. Example 16-1 describes this dilemma for a municipal project funded partly by federal money (50%). In this example, it still made sense to approve the project from the municipality's viewpoint but not the federal government, after the benefit estimate was revised.

16-4

This phrase refers to the fact that most benefits are confined locally for government investments. As the authors state, "Other than investments in defense and social programs, most benefits provided by government are realized at the local or regional levels." This is true for projects funded with full or partial government money. The conflict arises when some regions, states, municipalities perceive that they are consistently passed over for projects that would benefit their region, state, municipality. Powerful members in congress, and state legislatures, with key committee/subcommittee appointments can influence government spending in their districts. Politics have an effect in this regard. However, many projects, including the US parks system, the interstate highway, and others reach many beyond even regional levels.

16-5

Students will pull elements from the discussion of this topic in the textbook. In the text the concepts discussed include (1) No Time Value of Money, (2) Cost of Capital, and (3) Opportunity Cost. The Recommended Concept is to select the largest of the cost of capital, the government opportunity cost, or the taxpayer opportunity cost.

16-6

The *conventional* benefit-cost ratio has net benefits to the users in the numerator and cost to the sponsor in the denominator. The *modified* B-C ratio takes the project operating and maintenance costs paid by the sponsor, and subtracts these from the net benefits to the users. This quantity is all in the numerator. These leaves only the projects initial costs in the denominator.

The *conventional* and *modified* versions of the B-C ratio use different algebra/math to calculate the ration, but the resulting recommendation will always be the same. That is, for any problem, both ratios will either be greater than or less than 1.0 at the same time.

16-7

This is a list of potential costs, benefits, and disbenefits for a nuclear power plant.

Costs	Benefits	Disbenefits
Land Acquisition Site Preparation Cooling System - Reservoir dams - Reservoir cooling Construction - Reactor vessel/core - Balance of plant - Spent fuel storage - Water cleaning	Environment - No greenhouse gas - No leakage - No combustion Jobs & Economy - At enrichment plants - At power plant - Increase tax base Increase Demand - Uranium plants	Fission product material to contend with forever Not in my backyard Risk of Reactor - Real - Psychological Loss to Economy - Coal - Electric

16-8

(a) The conventional and modified versions of the B/C Ratio will always give consistent recommendations in terms of “invest” or “do not invest”. However, the magnitude of the B/C Ratio will be different for the two methods. Advocates of a project may use the method with the larger ratio to bolster their advocacy.

(b) Larger interest rates raise the “cost of capital” or “lost interest” for public projects because of the sometimes quite expensive construction costs. A person favoring a $200 M turnpike project would want to use lower i% values in the B/C Ratio calculations to offset the large capital costs.

(c) A decision-maker in favor of a particular public project would advocate the use of a longer project in the calculation of the B/C ratio. Longer durations spread the large initial costs over a greater number of years.

(d) Benefits, costs and disbenefits are quantities that have various amounts of “certainty” associated with them. Although this is true for all engineering economy estimates it is particularly true for public projects. It is much easier to estimate labor savings in a production environment than it is to estimate the impact on local hotels of new signage along a major route through town. Because benefits, costs, and disbenefits tend to have more uncertainty it is therefore easier to manipulate their values to make a B/C Ratio indicate a decision with your position.

16-9

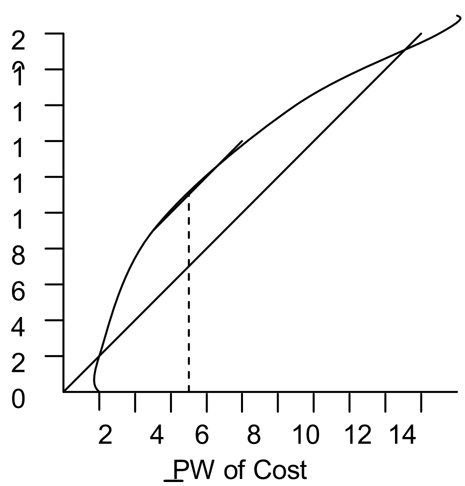
The time required to initiate, study, fund and construct public projects is generally several years (or even decades). Because of this it is not uncommon for there to be turnover in public policy makers. Politicians, who generally strive to maintain a positive public image, have been known to “stand up and gain political capital” from projects that originally began many years before they took office.

16-10

Benefit- Cost Ratio = PW of Benefits/PW of Cost
= [\$20,000 (P/A, 7%, 9) (P/F, 7%, 1)]/[\$100,000 + \$50,000 (P/F, 7%, 1)]
= [\$20,000 (6.515) (0.9346)]/[\$100,000 + \$50,000 (0.9346)]
= 0.83

16-11

The problem requires the student to use calculus. The text points out in Example 8-9 (of Chapter 8) that one definition of the point where $\Delta B = \Delta C$ is that of the slope of the benefits curve equals the slope of the NPW = 0 line.



Values for the graph:

PW of Cost (x)	PW of Benefits (y)
2	0
4	6.6

6	9.4
8	11.5
10	13.3
12	14.8
16	17.6
20	19.9

Let x = PW of Cost and y = PW of Benefits

$y^2 - 22x + 44 = 0$ or $y = (22x - 44)^{1/2}$

$dy/dx = ½ (22x - 44)^{(-1/2)} (22) = 1$

(Note that the slope of the NPW = 0 line is 1)

$22x - 44 = [(1/2) (22)]^2$

$x = (11^2 + 44)/22 = 7.5$ = optimum PW of cost

16-12

Since we have a 40-year analysis period, the problem could be solved by any of the exact analysis techniques. Here the problem specifies a present worth analysis. The annual cost solution, with a 10% interest rate, is presented in problem 6-44.

Gravity Plan

PW of Cost = \$2,800,000 + \$10,000 (P/A, 8%, 40)
= \$2,800,000 + \$10,000 (11.925) = \$2,919,250

Pumping Plan

PW of Cost = \$1,400,000 + \$200,000 (P/F, 8%, 10)
+ (\$25,000 + \$50,000) (P/A, 8%, 40)
+ \$50,000 (P/A, 8%, 30) (P/F, 8%, 10)
= \$1,400,000 + \$200,000 (0.4632)
+ (\$25,000 + \$50,000) (11.925)
+ \$50,000 (11.258) (0.4632)
= \$2,647,700

To minimize PW of Cost, choose pumping plan.

16-13

(a) Conventional B/C Ratio
= [PW (Benefits – Disabilities)]/[PW (1st Cost + Annual Cost)]
= [(\$500,000-\$25,000) (P/A, 10%, 35)]/[(\$1,200,000
+ \$125,000) (P/A, 10%, 35)]
= 1.9

(b) Modified B/C Ratio
= [PW (Benefits – Disbenefits – Cost)]/[PW (1st Cost)]
= [(\$500,000 - \$25,000 - \$125,000) (P/A, 10%, 35)]/\$1,200,000
= 2.8

16-14

Using the Conventional B/C Ratio

(i) Using PW B/C Ratio = 1.90 (as above)
(ii) Using AW B/C Ratio = ($500,000 - $25,000)/[$1,200,000 (A/P,10%,35) + $125,000]
= 1.90
(iii) Using FW B/C Ratio = [($500,000 - $25,000) (F/A,10%,35)] /[$1,200,000 (F/P, 10%, 35) + $125,000 (F/A, 10%, 35)]
= 1.90

16-15

(a) B/C Ratio = [($550 - $35) (P/A, 8%, 20)]/[($750 + $2,750) + $185 (P/A, 8%, 20)]
= 0.95

(b) Let's find the breakeven number of years at which B/C = 1.0
1.0= [($550 - $35) (P/A, 8%, x)]/[($750 + $2,750) + $185 (P/A, 8%, x)]

By trial and error:

x	*B/C ratio*
24 years	0.995
25 years	1.004
26 years	1.031

One can see how Big City Carl arrived at his value of "at least" 25 years for the project duration. This is the minimum number of years at which the B/C ratio is greater than 1.0 (nominally).

16-16

Annual Travel Volume = (2,500) (365) = 912,500 cars/year

The High Road

1st Cost	= $200,000 (35)	= $7,000,000
Annual Benefits	= 0.015 ($912,500) 35)	= $479,063
Annual O & M Cost	= $2,000 (35)	= $70,000

The Low Road

1st Cost	= $450,000 (10)	= $4,500,000
Annual Benefits	= 0.045 ($912,500) (10)	= $410,625
Annual O & M Cost	= $10,000 (10)	= $100,000

These are two mutually exclusive alternatives; we use an incremental analysis process.

Rank Order based on denominator = Low Road, High Road

	Do Nothing-vs.-Low	Low-vs.-High
Δ 1st Cost	\$4,500,000	\$2,500,000
Δ Annual Benefits	\$410,625	\$68,438
Δ Annual O & M Costs	\$100,000	-\$30,000
Δ B / Δ C	1.07[a]	0.61[b]
Justified?	Yes	No

Recommend investing in the Low road, it is the last justified increment.

[a] [(\$410,625 - \$100,000) (\$15,456)]/\$4,500,000 = 1.07
[b] [(\$68,438 + \$30,000) (\$15,456)]/\$2,500,000 = 0.61

16-17

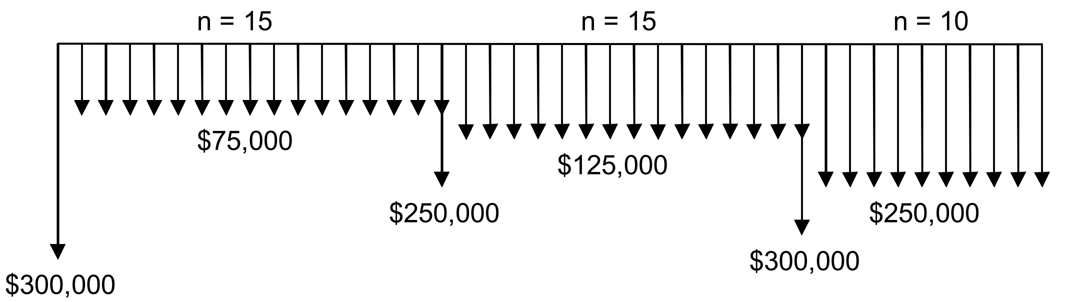
(a) PW of Benefits = \$60,000 (P/A, 5%, 10)
+ \$64,000 (P/A, 5%, 10) (P/F, 5%, 10)
+ \$66,000 (P/A, 5%, 20) (P/F, 5%, 20)
+ \$70,000 (P/A, 5%, 10) (P/F, 5%, 40)
= \$60,000 (7.722)
+ \$64,000 (7.722) (0.6139)
+ \$66,000 (12.462) (0.3769)
+ \$70,000 (7.722) (0.1420)
= \$1,153,468

For B/C ratio = 1, PW of Cost = PW of Benefits
Justified capital expenditure
= \$1,153,468 - \$15,000 (P/A, 5%, 5)
= \$1,153,468 - \$15,000 (18.256)
= \$879,628

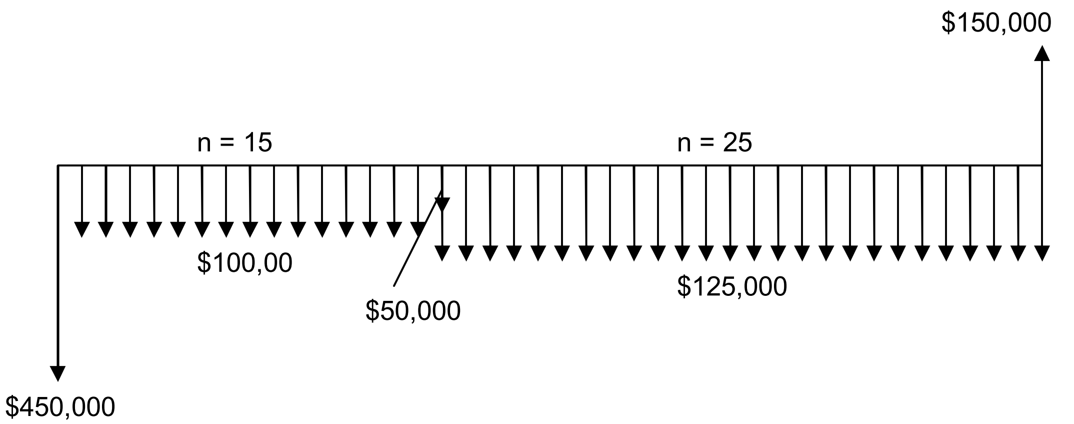
(b) Same equation as on previous page except use 8% interest
PW of Benefits = \$60,000 (6.710) + \$64,000 (6.710) (0.4632)
+ \$66,000 (9.818) (0.2145)
+ \$70,000 (6.710) (0.0460)
= \$762,116
Justified Capital Expenditure
= \$762,116 - \$15,000 (12.233)
= \$578,621

16-18

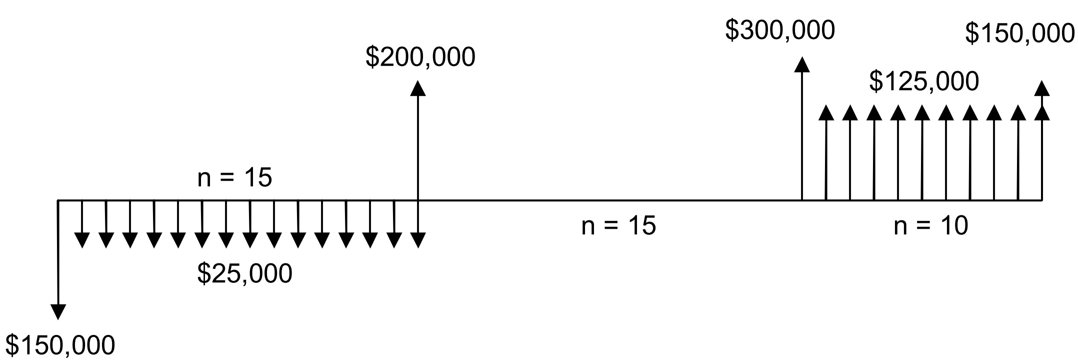
Plan A



Plan B



Differences between Alternatives A and B



An examination of the differences between the alternatives will allow us to quickly determine which plan is preferred.

Cash Flow			**Present Worth**		
Year	A	B	B- A	At 7%	At 5%
0	-$300	-$450	-$150	-$150	-$150

1- 15	-$75	-$100	-$25	-$228	-$259
15	-$250	-$50	+$200	+$72	+$96
16- 30	-$125	-$125	$0	$0	$0
30	-$300	$0	+$300	+$39	+$69
31- 40	-$250	-$125	+$125	+$115	+$223
40	$0	+$150	+$150	+$10	+$21
Sum			+$1,375*	-$142	$0

* This is sum of -$150 – 15 ($25) + $200 ….

(a) When the Present Worth of the B- A cash flow is computed at 7%, the NPW = -142. The increment is not desirable at i = 7%.

Choose Plan A.

(b) For Plan B to be chosen, the increment B- A must be desirable. The last column in the table above shows that the B- A increment has a 5% rate of return. In other words, at all interest rates at or below 5%, the increment is desirable and hence Plan B is the preferred alternative. The value of MARR would have to be 5% or less.

16-19

Overpass Cost = $1,800,000   Salvage Value = $100,000   n = 30   i = 6%

Benefits to Public

Time Saving   for 100 vehicles per day

400 trucks x (2/60) x ($10/hr) = $240 per day
600 others x (2/60) x ($5/hr) = $100 per day
Total = $340 per day

Benefits to the State

Saving in accident investigation costs= $2,000 per year

Combined Benefits

Benefits to the Public + Benefits to the State
= $340/day (365 days) + $6,000 = $130,100 per year

Benefits to the Railroad

Saving in crossing guard expense = $48,000 per year
Saving in accident case expense = $60,000 per year
Total = $108,000 per year

Should the overpass be built?

Benefit- Cost Ratio Analysis

Annual Cost (EUAC) = $1,700,000 (A/P, 6%, 30) + $100,000 (0.06)
= $1,700,000 (0.0726) + $6,000

= $129,420

Annual Benefit (EUAB) = $130,100 + $108,000
= $238,100

B/C= EUAB/EUAC = $238,100/$129,420 = 1.84

With a B/C ratio > 1, the project is economically justified.

Allocation of the $1,800,000 cost
The railroad should contribute to the project in proportion to the benefits received.

PW of Cost = $1,800,000 - $100,000 (P/F, 6%, 30)
= $1,800,000 - $100,000 (0.1741)
= $1,782,590

The railroad portion would be
($108,000/$238,100) ($1,782,590) = $808,570

The State portion would be
($130,100/$238,100) ($1,782,590) + $100,000 (P/F, 6%, 30)
= ($130,100/$238,100) ($1,782,590) + $100,000 (0.1741)
= $991,430

Note that $808,570 + $991,430 = $1,800,000

While this problem is a simplified representation of the situation, it illustrates a realistic statement of benefits and an economic analysis solution to the allocation of costs.

16-20

	Existing	Plan A	Plan B	Plan C
Length (miles)	10	10	10	10.3
Number of Lanes	2	4	4	4
Average ADT	20,000	20,000	20,000	20,000
Autos	19,000	19,000	19,000	19,000
Trucks	1,000	1,000	1,000	1,000
Time Savings (minutes)				
Autos				
Trucks		2	3	5
		1	3	4
Accident Rate/MVM	4.58	2.50	2.40	2.30
Initial Cost per mil (P)	-	$450,000	$650,000	$800,000
Annual Maintenance per lane per mile	$1,500	$1,250	$1,000	$1,000
Total Annual Maintenance	$30,000	$50,000	$40,000	$41,200
EUAC of Initial Cost = (P x miles) (A/P, 5%, 20)	$0	$360,900	$521,300	$660,850

Total Annual Cost of EUAC + Maintenance	$30,000	$410,900	$561,300	$702,050

Annual Incremental Operating Costs due to distance

None for Plans A and B, as they are the same length as existing road.

Plan C Autos	19,000 x 365 x 0.3 mi x $0.06	= $124,830
	Trucks 1,000 x 365 x 0.3 mi x $0.18	= $19,710
		Total = $144,540/yr

Annual Accident Savings compared to Existing Highway

Plan A: (4.58 – 2.50) (10^{-6}) (10 mi) (365 days) (20,000 ADT) ($1,200)
= $182,200

Plan B: (4.58 – 2.40) (10^{-6}) (10 mi) (365 days) (20,000 ADT) ($1,200)
= $190,790

Plan C: (4.58 – 2.30) (10^{-6}) (10.3 mi) (365 days) (20,000 ADT) ($1,200)
= $205,720

Time Savings Benefits to Road Users compared to Existing Highway

Plan A:

Autos	19,000 x 365 days x 2 min x $0.03	= $416,100
Trucks	1,000 x 365 days x 1 min x $0.15	= $54,750
	Total	= $470,850

Plan B:

Autos	19,000 x 365 days x 3 min x $0.03	= $624,150
Trucks	1,000 x 365 days x 3 min x $0.15	= $164,250
	Total	= $788,400

Plan C:

Autos	19,000 x 365 days x 5 min x $0.03	= $1,040,250
Trucks	1,000 x 365 days x 4 min x $0.15	= $219,000
	Total	= $1,259,250

Summary of Annual Costs and Benefits

	Existing	Plan A	Plan B	Plan C
Annual Highway Costs	$30,000	$410,900	$561,300	$702,050
Annual Benefits				
Accident Savings		$182,200	$190,970	$205,720
Time Savings		$470,850	$788,400	$1,259,250
Additional Operating Cost*				-$144,540
Total Annual Benefits		$653,050	$979,370	$1,320,430

* User costs are considered as disbenefits.

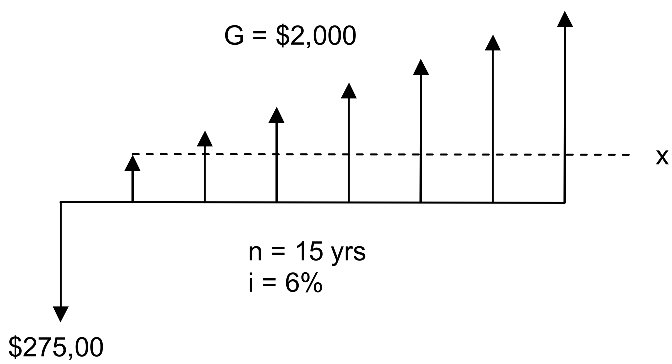
Benefit-Cost Ratios

A rather than Existing: B/C = $653,050/($410,900 - $30,000)
= 1.71
B rather than A: B/C = ($979,370 - $653,050)/($561,300 - $410,900)
= 2.17

C rather than B: B/C = ($1,320,430 - $979,370)/($702,050 - $561,300
= 2.42

Plan C is preferred.

16-21



Compute X for NPW = 0

NPW = PW of Benefits – PW of Costs
= X (P/A, 6%, 15) + $2,000 (P/G, 6%, 15) - $275,000 = $0
= X (9.712) + $2,000 (57.555) - $275,000 = $0

X = [$275,000 - $2,000 (57.555)]/9.712 = $16,463

Therefore, NPW at yr 0 turns positive for the first time when X is greater than $16,463. This indicates that construction should not be done prior to 19x5 as NPW is not positive. The problem thus reduces to deciding whether to proceed in 2005 or 2006. The appropriate criterion is to maximize NPW at some point. If we choose the beginning of 2005 for convenience,

Construct in 2005
NPW_{2005} = $18,000 (P/A, 6%, 15) + $2,000 (P/G, 6%, 15) - $275,000
= $18,000 (9.712) +$2,000 (57.555) - $275,000
= +$14,926

Construct in 2006
NPW_{2006} = [$20,000 (P/A, 6%, 15) + $2,000 (P/G, 6%, 15)

$- \$265{,}000]\ (P/F, 6\%, 1)$
$= [\$20{,}000\ (9.712) + \$2{,}000\ (57.555) - \$275{,}000]\ (0.9434)$
$= +\$32{,}406$

Conclusion: Construct in 2006.

16-22

It is important to recognize that if Net Present Worth analysis is done, then the criterion is to maximize NPW. But, of course, the NPWs must be computed at a common point in time, like Year 0.

Repair Now

$$
\begin{aligned}
NPW_{YEAR\ 0} &= \$5{,}000\ (P/F, 15\%, 1) + \$10{,}000\ (P/G, 15\%, 5) \\
&\quad + \$50{,}000\ (P/A, 15\%, 5)\ (P/F, 15\%, 5) - \$150{,}000 \\
&= \$5{,}000\ (0.8696) + \$10{,}000\ (5.775) \\
&\quad + \$50{,}000\ (3.352)\ (0.4972) - \$150{,}000 \\
&= -\$4{,}571
\end{aligned}
$$

Repair Two Years Hence

$$
\begin{aligned}
NPW_{YEAR\ 2} &= \$20{,}000\ (P/A, 15\%, 3) + \$10{,}000\ (P/G, 15\%, 3) \\
&\quad + \$50{,}000\ (P/A, 15\%, 7)\ (P/F, 15\%, 3) - \$150{,}000 \\
&= \$20{,}000\ (2.283) + \$10{,}000\ (2.071) \\
&\quad + \$50{,}000\ (4.160)\ (0.6575) - \$150{,}000 \\
&= +\$53{,}130
\end{aligned}
$$

$NPW_{YEAR\ 0} = \$53{,}130\ (P/F, 15\%, 2) \quad = \$53{,}130\ (0.756) \quad = +\$40{,}172$

Repair Four Years Hence

$$
\begin{aligned}
NPW_{YEAR\ 4} &= \$50{,}000\ (P/A, 15\%, 10) - \$10{,}000\ (P/F, 15\%, 1) - \$150{,}000 \\
&= \$50{,}000\ (5.019) - \$10{,}000\ (0.8696) - \$150{,}000 \\
&= +\$92{,}254
\end{aligned}
$$

$NPW_{YEAR\ 0} = \$92{,}254\ (P/F, 15\%, 4) \quad = \$92{,}254\ (0.5718) \quad = +\$52{,}751$

Repair Five Years Hence

$$
\begin{aligned}
NPW_{YEAR\ 5} &= \$50{,}000\ (P/A, 15\%, 10) - \$150{,}000 \\
&= \$50{,}000\ (5.019) - \$150{,}000 \\
&= +\$100{,}950
\end{aligned}
$$

$NPW_{YEAR\ 0} = \$100{,}950\ (P/F, 15\%, 5) \quad = \$100{,}950\ (0.4972) = +\$50{,}192$

To maximize NPW at year 0, repair the road four years hence.

It might be worth noting in this situation that since the benefits in the early years (Years 1, 2, and 3) are less than the cost times the interest rate ($150,000 x 0.15 = $22,500), delaying the project will increase the NPW at Year 0. In other words, we would not expected the project to be selected (if it ever would be) until the annual benefits are greater than $22,500.

If a “repair three years hence” alternative were considered, we would find that it has an NPW at year 0 of +$49,945. So the decision to repair the road four years hence is correct.

16-23

This problem will require some student though on how to structure the analysis. This is a situation of providing the necessary capacity when it is needed- in other words Fixed Output. Computing the cost is easy, but what is the benefit?

One cannot compute the B/C ratio for either alternative, but the incremental B/C ratio may be computed on the difference between alternatives.

Year	A Half capacity tunnel now plus half capacity tunnel in 20 years	B Full Capacity Tunnel	B- A Difference Between the alternatives
0	-\$300,000	-\$500,000	-\$200,000
10	-\$16,000	-\$20,000	-\$4,000
20	-\$16,000 -\$400,000	-\$20,000	+\$396,000
30	-\$32,000	-\$20,000	+\$12,000
40	-\$32,000	-\$20,000	+\$12,000
50	\$0	\$0	\$0

$$\begin{aligned}\Delta B/\Delta C &= [\$396{,}000\ (P/F, 5\%, 20) + \$12{,}000\ (P/F, 5\%, 30) \\ &\quad + \$12{,}000\ (P/F, 5\%, 40)]/[\$200{,}000 + \$4{,}000\ (P/F, 5\%, 10)] \\ &= \$153{,}733/\$202{,}456 \\ &= 0.76\end{aligned}$$

This is an undesirable increment of investment. Build the half-capacity tunnel now.

16-24

	Alt. A	**Alt. B**	**Alt. C**	**Alt. D**
First Cost	\$9,500	\$12,500	\$14,000	\$15,750
Annual O & M Costs	\$550	\$175	4325	\$145
Salvage Value	\$1,000	\$6,000	\$3,500	\$7,500
PW of Denominator	\$15,592	\$13,874	\$17,311	\$16,637
Annual Benefits	\$2,200	\$1,500	\$1,000	\$2,500
Annual Disbenefits	\$350	\$150	\$75	\$700
PW of Numerator	\$20,928	\$15,198	\$10,413	\$20,265
B/C Ratio	1.34	1.10	0.60	1.22

We eliminate Alternative C from consideration. Our rank order is B, A, D.

	Do nothing- B	**B- A**	**A- D**
Δ First Cost	\$12,500	-\$3,000	\$6,250
Δ Annual O & M Costs	\$175	\$375	-\$405
Δ Salvage Value	\$6,000	-\$5,000	\$6,500

PW of Δ Denominator	$13,874	$1,719	$1,045
Δ Annual Benefits	$1,500	$700	$300
Δ Annual Disbenefits	$150	$200	$350
PW of Δ Numerator	$15,198	$5,629	-$563
Δ B/C Ratio	1.10	3.28	-0.54
Justified?	Yes	Yes	No

Choose Alternative A because it is associated with the last justified increment of investment.

16-25

	1	**2**	**3**	**4**	**5**	**6**
AW Costs (sponsor)	15.5	13.7	16.8	10.2	17	23.3
AW Benefits (users)	20	16	15	13.7	22	25
B/C Ratio	1.29	1.17	0.89	1.34	1.29	1.07

We can eliminate project #3 from consideration. Our rank order is 4, 2, 1, 5, and 6.

	DN- 4	**4- 2**	**4- 1**	**1- 5**	**5- 6**
Δ AW Costs (sponsor)	10.2	3.5	5.3	1.5	6.3
Δ AW Benefits (users)	13.7	2.3	6.3	2	3
Δ B/C Ratio	1.34	0.66	1.19	1.33	0.48
Justified?	Yes	No	Yes	Yes	No

Choose Alternative 5 because it is associated with the last justified increment of investment.

16-26

	A	**B**	**C**
Initial Investment	$9,500	$18,500	$22,000
Annual Savings	$3,200	$5,000	$9,800
Annual Costs	$1,000	$2,750	$6,400
Salvage Value	$6,000	$4,200	$14,000

(a) Conventional B/C

	A	**B**	**C**
PW Numerator	$21,795	$34,054	$66,746
PW Denominator	$15,215	$36,463	$63,032
B/C Ratio	1.43	0.93	1.06

Here we eliminate Alternative B. Rank order is A, then C.

	Do Nothing- A	**A- C**
Δ Initial Investment	$9,500	$12,500
Δ Annual Savings	$3,200	$6,600
Δ Annual Costs	$1,000	$5,400

Δ Salvage Value	$6,000	$8,000
Δ PW Numerator	$21,795	$44,952
Δ PW Denominator	$15,215	$47,817
Δ B/C Ratio	1.43	0.94
Justified?	Yes	No

We recommend Alternative A.

(b) Modified B/C

	A	**B**	**C**
PW Numerator	$14,984	$15,324	$23,157
PW Denominator	$8,404	$17,733	$19,442
B/C Ratio	1.78	0.86	1.19

Here we eliminate Alternative B. Our rank order is A then C.

	Do Nothing- A	**A- C**
Δ Initial Investment	$9,500	$12,500
Δ Annual Savings	$3,200	$6,600
Δ Annual Costs	$1,000	$5,400
Δ Salvage Value	$6,000	$8,000
Δ PW Numerator	$14,984	$8,173
Δ PW Denominator	$8,404	$11,038
Δ B/C Ratio	1.78	0.74
Justified?	Yes	No

We recommend Alternative A.

(c) Present Worth

Year	**A**	**B**	**C**
0	-$9,500	-$18,500	-$22,000
1-14	$2,200	$2,250	$3,400
15	$8,200	$6,450	$17,400
Present Worth	$6,580	-$2,408	$3,715

We recommend Alternative A.

(d) IRR Method

	A	**B**	**C**
IRR	23%	10%	15%

Here we need the incremental analysis method. Eliminate Alternative B because IRR < MARR.

Year	**Do Nothing- A**	**A- C**
Δ 0	-$9,500	-$12,500

Δ 1 - Δ 14	$2,200	$1,200
Δ 15	$8,200	$9,200
Δ IRR	23%	8%
Justified?	Yes	No

We recommend Alternative A.

(e) Simple Payback

Year	**A**	**B**	**C**
0	-$9,500	-$18,500	-$22,000
1	-$7,300	-$16,250	-$18,600
2	-$5,100	-$14,000	-$15,200
3	-$2,900	-$11,750	-$11,800
4	-$700	-$9,500	-$8,400
5	$1,500	-$7,250	-$5,000
6	$3,700	-$5,000	-$1,600
7	$5,900	-$2,750	$1,800
8	$8,100	-$500	$5,200
9	$10,300	$1,750	$8,600
10	$12,500	$4,000	$12,000
11	$14,700	$6,250	$15,400
12	$16,900	$8,500	$18,800
13	$19,100	$10,750	$22,200
14	$21,300	$13,000	$25,600
15	$29,500	$19,450	$43,000

Alternative A (SPB)	= 4 + [$700/($700 + $2,200)]	= 4.32 years
Alternative B (SPB)	= 8 + [$500/($500 + $1,750)]	= 8.22 years
Alternative C (SPB)	= 6 + [$1,600/($1,600 + $1,800)]	= 6.47 years

Recommend Alternative A.

Chapter 17: Rationing Capital Among Competing Projects

17-1

(a) With no budget constraint, do all projects except Project #4.
Cost = $115,000

(b) Ranking the 9 projects by NPW/Cost

Project	Cost	Uniform Benefit	NPW at 12%	NPW/Cost
1	$5	$1.03	$0.82	0.16
2	$15	$3.22	$3.19	0.21
3	$10	$1.77	$0	0
5	$5	$1.19	$1.72	0.34
6	$20	$3.83	$1.64	0.08
7	$5	$1.00	$0.65	0.13
8	$20	$3.69	$0.85	0.04
9	$5	$1.15	$1.50	0.30
10	$10	$2.23	$2.60	0.26

Projects ranked in order of desirability

Project	Cost	NPW at 12%	NPW/Cost	Cumulative Cost
5	$5	$1.72	0.34	$5
9	$5	$1.50	0.30	$10
10	$10	$2.60	0.26	$20
2	$15	$3.19	0.21	$35
1	$5	$0.82	0.16	$40
7	$5	$0.65	0.13	$45
6	$20	$1.64	0.08	$65
8	$20	$0.85	0.04	$85
3	$10	$0	0	$95

(c) At $55,000 we have more money than needed for the first six projects ($45,000), but not enough for the first seven projects ($65,000). This is the "lumpiness" problem. There may be a better solution than simply taking the first six projects, with total NPW equal to 10.48. There is in this problem. By trial and error we see that if we forego Projects 1 and 7, we have ample money to fund Project 6. For this set of projects, Σ NPW = 10.65.

To maximize NPW the proper set of projects for $55,000 capital budget is:

Projects 5, 9, 10, 2, and 6

17-2

(a) Select projects, given MARR = 10%. Incremental analysis is required.

Project		Δ Cost	Δ Uniform Annual Benefit	Δ Rate of Return	Conclusion
1	Alt 1A- Alt. 1C	$15	$2.22	7.8%	Reject 1A
	Alt. 1B- Alt. 1C	$40	$7.59	13.7%	Reject 1C Select 1B
2	Alt. 2B- Alt. 2A	$15	$2.57	11.2%	Reject 2A Select 2B
3	Alt. 3A- Alt. 3B	$15	$3.41	18.6%	Reject 3B Select 3A
4		$10	$1.70	11%	Select 4

Conclusion: Select Projects 1B, 2B, 3A, and 4.

(b) Rank separable increments of investment by rate of return

Alternative	Cost or Δ Cost	Δ Rate of Return	For Budget of $100,000
1C	$10	20%	~~1C $10*~~
3A	$25	18%	3A $25
2A	$20	16%	2A $20
1B- 1C	$40	13.7%	1B $50
2B- 2A	$15	11.2%	-
4	$10	11%	-
			Σ = $95

* The original choice of 1C is overruled by the acceptable increment of choosing 1B instead of 1C.

Conclusion: Select Projects 3A, 2A, and 1B.

(c) The cutoff rate of return equals the cost of the best project foregone. Project 1B, with a Rate of Return of 13.7% is accepted and Project 2B with a Rate of Return of 11.2% is rejected. Therefore the cutoff rate of return is actually 11.2%, but could be considered as midway between 13.7% and 11.2% (12%).

(d) Compute NPW/Cost at i = 12% for the various alternatives

Project	Cost	Uniform Benefit	NPW	NPW/Cost
1A	$25	$4.61	$1.05	0.04
1B	$50	$9.96	$6.28	0.13
1C	$10	$2.39	$3.50	0.35
2A	$20	$4.14	$3.39	0.17
2B	$35	$6.71	$2.91	0.08
3A	$25	$5.56	$6.42	0.26
3B	$10	$2.15	$2.15	0.21
4	$10	$1.70	-$0.39	-0.03

Project Ranking

Project	Cost	NPW/Cost
1C	$10	0.35
3A	$25	0.26
3B	$10	0.21
2A	$20	0.17
1B	$50	0.13
2B	$35	0.08
1A	$25	0.04
4	$10	-0.03

(e) For a budget of $\$100 \times 10^3$, select:
3A($25) + 2A ($20) + 1B ($50) thus Σ = $95

17-3

(a) Cost to maximize total ohs - no budget limitation
Select the most appropriate gift for each of the seven people

Recipient	Gift	Oh Rating	Cost
Father	Shirt	5	$20
Mother	Camera	5	$30
Sister	Sweater	5	$24
Brother	Camera	5	$30
Aunt	Candy	5	$20
Uncle	Sweater	4	$24
Cousin	Shirt	4	$20
Total			$168

Cost of Best Gifts = $168

(b) This problem differs from those described in the book where a project may be rejected by selecting the do-nothing alternative. Here, each person must be provided a gift. Thus while we can move the gift money around to maximize "ohs", we cannot eliminate a gift. This constraint destroys the validity of the NPW- p (PW of Cost) or Ohs – P (Cost) technique.

The best solution is to simplify the problem as much as possible and then to proceed with incremental analysis. The number of alternatives may be reduced by observing that since the goal is to maximize "ohs," for any recipient one should not pay more than necessary for a given number of "ohs," or more dollars for less "ohs."

For example, for Mother the seven feasible alternatives (the three 0-oh alternatives are not feasible) are:

Alternative	Cost	Ohs
1	$20	4

4	$20	3
5	$24	4
6	$30	5
8	$16	3
9	$18	4
10	$16	2

Careful examination shows that for five ohs, one must pay $30, for four ohs, $18, and $16 for three ohs. The other three and four oh alternatives cost more, and the two alternative costs the same as the three oh alternatives.

Thus for Mother the three dominate alternatives are:

Alternative	Cost	Ohs
6	$30	5
9	$18	4
10	$16	2

All other alternatives are either infeasible or inferior.

If the situation is examined for each of the gift recipients, we obtain:

Ohs	Father		Mother		Sister		Brother	
	Cost	Δ Cost /oh	Cost	Δ Cost /oh	Cost	Δ Cost /oh	Cost	Δ Cost /oh
5	$20		$30		$24		$30	
		$4		$12		$8		$14
4	$16		$18		$16		$16	
		$4		$2				
3	$12		$16					
						$3.3		$1.3
2								
1					$6		$12	

Ohs	Aunt		Uncle		Cousin	
	Cost	Δ Cost /oh	Cost	Δ Cost /oh	Cost	Δ Cost /oh
5	$20					
		$2				
4	$18		$24		$20	
		$2		$8		$4
3	$16		$16		$16	
		$1.3				$4
2				$2	$12	$4.6
		$5				$6
1	$6		$12		$6	

In part (a) we found that the most appropriate gifts cost $168. This table confirms that the gifts with the largest oh for each person cost $20 + $30 + $24 + $30 + $20 +$24 +

$20 = $168. (This can be found by reading across the top of the table on the previous page.)

For a budget limited to $112 we must forego increments of Cost/Oh that consume excessive dollars. The best saving available is to go from a five-oh to a four-oh gift for Brother, thereby savings $14. This makes the cost of the seven gifts = $168 - $14 = $154. Further adjustments are required, first on Mother, then Sister, then Father and finally a further adjustment of Sister. The selected gifts are:

Recipient	Gift	Ohs	Cost
Father	Shirt	5	$20
Mother	Book	4	$18
Sister	Magazine	4	$16
Brother	Magazine	4	$16
Aunt	Candy	5	$20
Uncle	Necktie	3	$16
Cousin	Calendar	1	$6
Total		26	$112

(c) For a budget of $90 the process described above must be continued. The selected gifts are:

Recipient	Gift	Ohs	Cost
Father	Cigars	3	$12
Mother	Book	4	$18
Sister	Magazine	4	$16
Brother	Magazine	4	$16
Aunt	Calendar	1	$6
Uncle	Necktie	3	$16
Cousin	Calendar	1	$6
Total		20	$90

17-4

This problem is based on unlimited capital and a 12% MARR. Replacements (if needed) in the 16-year analysis period will produce a 12% rate of return.

In the Present Worth computations at 12%, the NPW of the replacements will be zero. In this situation the replacements do not enter into the computation of NPW.

See the data and computations of NPW for this problem. For each project select the alternative which maximizes NPW.

For Project	Select Alternative
1	B
2	A
3	F
4	A
5	A

Data for Problems 17-4, 17-5, and 17-7

Project	Cost	Useful Life	Prob. 17-4 Alternative at useful life NPW	Prob. 17-5 Alternative & identical replacements for 16 years NPW	Prob. 17-7 NPW (computed for Problem 17-5) and p = 0.20 NPW - P (Cost)
1A	$40	2	Negative	Negative	Negative
1B	$10	16	+$3.86	+$3.86	+$1.86
1C	$55	4	$0	$0	Negative
1D	$30	8	+$3.23	+$4.53	Negative
1E	$15	2	+$3.30	+$13.60	+$10.60
2A	$10	16	+$3.65	+$3.65	+$1.65
2B	$5	8	+$1.46	+$2.05	+$1.05
2C	$5	8	+$0.63	+$0.88	Negative
2D	$15	4	+$1.95	+$4.46	+$1.46
3A	$20	16	$0	$0	Negative
3B	$5	16	+$0.86	+$0.86	Negative
3C	$10	16	Negative	Negative	Negative
3D	$15	16	+$2.57	+$2.57	Negative
3E	$10	4	+$0.63	+$1.45	Negative
3F	$15	16	+$3.14	+$3.14	+$0.14
4A	$10	8	+$2.97	+$4.17	+$2.17
4B	$5	16	+$1.76	+$1.76	+$0.76
4C	$5	16	+$2.10	+$2.10	+$1.10
4D	$15	8	+$1.59	+$2.23	Negative
5A	$5	8	+$0.75	+$1.05	+$0.05
5B*	$10	4	+$0.63	+$1.45	Negative
5C	$15	8	$0	$0	Negative

*5B and 3E have the same parameters.

17-5

This problem is based on unlimited capital, a 12% MARR, and identical replacement throughout the 16-year analysis period. The NPW is computed for each alternatives together with any identical replacements. From the table above the alternatives that maximize NPW will be selected:

For Project	Select Alternative
1	E
2	D
3	F
4	A
5	B

17-6

To solve this problem with neither input nor output fixed, incremental analysis is required with rate of return methods. With 22 different alternatives, the problem could be lengthy. By careful examination, most of the alternatives may be eliminated by inspection.

Project 1

Reject 1A Rate of Return < MARR
Reject 1B Alt. 1E has a greater investment and a greater ROR

Reject 1D **1D- 1E Increment**

Year	Cash Flow 1D	Cash Flow 1E	Cash Flow 1D- 1E
0	-\$30	-\$15	-\$15
1- 8	+\$6.69	+\$3.75	+\$2.94
8	-\$30	\$0	-\$30
9- 16	+\$6.69	+\$3.75	+\$2.94

i* is very close to 1 ½%. By inspection we can see there must be an external investment prior to year 8 (actually in years 6 and 7). Assuming e* = 6%, i* will still be less than 12%. Therefore, Reject 1D

Reject 1C Higher cost alternative has ROR = MARR, and lower cost alternative has ROR > MARR. The increment between them must have a ΔROR < MARR.

Select Alternative 1E.

Project 2

Reject 2A The increment between 2D and 2A has a desirable ΔROR = 18%.
Reject 2C Higher cost alternative 2D has a higher ROR.

Reject 2B **Increment 2D- 2B**

Year	Cash Flow 1D	Cash Flow 1E	Cash Flow 1D- 1E
0	-\$15	-\$5	-\$10
1- 4	+\$5.58	+\$1.30	+\$4.28
4	-\$15	\$0	-\$15
5- 8	+\$5.58	+\$1.30	+\$4.28

(The next 8 years duplicate the first)

15% < i* < 18%. There is not net investment throughout the 8 years, but at e* = 6%, i* still appears to be > 12%.

Select Alternative 2D.

Project 3

Reject 3C	ROR < MARR
Reject 3B, 3D, and 3E	Alt. 3F with the same ROR has higher cost. Therefore, Δ ROR = 15%
Reject 3A	The increment 3A – 3F must have ΔROR < 12%

Select Alternative 3F.

Project 4

Reject 4B and 4C	These alternatives are dominated by Alternative 4A with its higher cost and greater ROR.
Reject 4D	**Increment 4D- 4A**

Year	Cash Flow 4D- 4A
0	-$5
1- 8	+$0.73

Computed i* = 3.6%. Reject 4D.

Select Alternative 4A.

Project 5

Reject 5A	Alternative 5B with the same ROR has a higher cost. Therefore, ΔROR = 15%.
Reject 5C	The increment 5C – 5B must have an ΔROR < 12%.

Select Alternative 5B.

Conclusion: Select 1E, 2D, 3F, 4A, AND 5B. (Note that this is also the answer to 17-5)

17-7

This problem may be solved by the method outlined in Figure 17-3.

With no budget constraint the best alternatives were identified in Problem 17-5 with a total cost of $65,000. here we are limited to $55,000.

Using NPW – p (cost), the problem must be solved by trial and error until a suitable value of p is determined. A value of p = 0.20 proves satisfactory. The computations for NPW – 0.20 (cost) is given in the table between Solutions 17-4 and 17-5.

Selecting the alternatives from each project with the largest positive NPW – 0.2 (cost) gives:

For Project	Select Alternative	Cost
1	E	$15,000

2	A	$10,000
3	F	$15,000
4	A	$10,000
5	A	$5,000
Total		$55,000

17-8

The solution will follow the approach of Example 17-5. The first step is to compute the rate of return for each increment of investment.

Project A1- no investment

Project A2 (A2- A1)

Year	Cash Flow	PW at 20%
0	-$500,000 (keep land)	-$500,000
1- 20	+$98,700	+$480,669
20	+$750,000	$15,000
Total		+$244

Therefore, Rate of Return ≈ 20%.

Project A3 (A3- A1)
Expected Annual Rental Income
= 0.1 ($1,000,000) + 0.3 ($1,100,000) + 0.4 ($1,200,000) + 0.2 ($1,900,000)
= $1,290,000

Year	Cash Flow	PW at 18%
0	-$5,000,000	-$5,000,000
1- 2	$0	$0
3- 20	+$1,290,000	+$4,885,200
20	+$3,000,000	+$109,000
Total		-$5,300

Therefore, Rate of Return ≈ 18%.

Project A3- Project A2

Year	Project A3	Project A2	A3- A2
0	-$5,000,000	-$500,000	-$4,500,000
1	$0	+$98,700	-$98,700
2	$0	+$98,700	-$98,700
3- 20	+$1,290,000	+$98,700	+$1,191,300
20	+$3,000,000	+$750,000	+$2,250,000

Year	A3- A2	PW at 15%	PW at 18%
0	-$4,500,000	-$4,500,000	-$4,500,000
1	-$98,700	-$85,830	-$83,650

2	-\$98,700	-\$74,630	-\$70,890
3- 20	+\$1,191,300	+\$5,519,290	+\$4,511,450
20	+\$2,250,000	+\$137,480	+\$82,120
Total		+\$996,310	-\$60,970

Δ Rate of Return ≈ 17.7% (HP-12C Answer = 17.8%)

Project B

Rate of Return $= i_{eff} = e^r - 1 = e^{0.1375} - 1 = 0.1474 = 14.74\%$

Project C

Year	Cash Flow	PW at 18%
0	-\$2,000,000	-\$2,000,000
1-10	+\$500,000	+\$1,785,500
10	+\$2,000,000	+\$214,800
Total		+\$300

Actually the rate of return is exactly \$500,000/\$2,000,000 = 25%.

Project D

Rate of Return = 16%

Project E

$i_{eff} = (1 + 0.1406/12)^{12} - 1 = 15.00\%$

Project F

Year	Cash Flow	PW at 18%
0	-\$2,000,000	-\$2,000,000
1	+\$1,000,000	+\$847,500
2	+\$1,604,800	+\$1,152,600
Total		+\$100

Rate of Return = 18%

Rank order of increments of investment by rate of return

Project	Increment	Rate of Return
C	\$2,000,000	25%
A2	\$500,000	20%
F	\$2,000,000	18%
A3- A2	\$4,500,000	17.7%
D	\$500,000	16%
E	Any amount > \$100,000	15%
B	Not stated	14.7%

Note that \$500,000 value of Project A land is included.

(a) Budget= \$4 million (or \$4.5 million including Project A land)
Go down the project list until the budget is exhausted

Choose Project C, A2, and F.
MARR = Cutoff rate of Return = Opportunity cost ≈ 17.7%- 18%

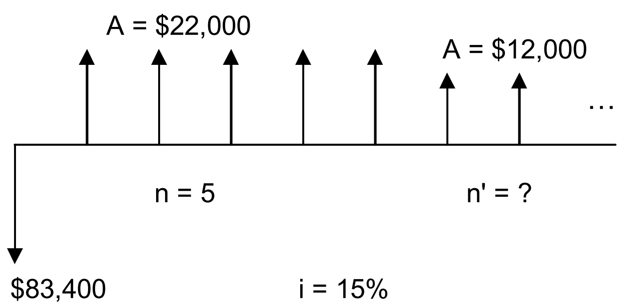
(b) Budget= $9 million (or $9.5 million including Project A land)
Again, go down the project list until the budget is exhausted.
Choose Projects C, F, A3, D.

Note that this would become a lumpiness problem at a capital budget of $5 million (or many other amounts).

17-9

Project I: Liquid Storage Tank
Saving at 0.1 cent per kg of soap:
First five years = $0.001 x 22,000 x 1,000 = $22,000
Subsequent years = $0.001 x 12,000 x 1,000 = $12,000

How long must the tank remain in service to produce a 15% rate of return?



$83,400 = $22,000 (P/A, 15%, 5) + $12,000 (P/A, 15%, n') (P/F, 15%, 5)
= $22,000 (3.352) + $12,000 (P/A, 15%, n') (0.4972)

(P/A, 15%, n') = 1.619
n' = 2 years (beyond the 5 year contract)

Thus the storage tank will have a 15% rate of return for a useful life of 7 years. This appears to be far less than the actual useful life of the tank to Raleigh.
Install the Liquid Storage Tank.

Project II: Another sulfonation unit

There is no alternative available, so the project must be undertaken to provide the necessary plant capacity.
Install Solfonation Unit.

Project III: Packaging department expansion

Cost = $150,000
Salvage value at tend of 5 years = $42,000

Annual saving in wage premium = $35,000

Rate of Return:
$150,000 - $42,000 (P/F, i%, 5) = $35,000 (P/A, i%, 5)

Try i = 12%

$150,000 - $42,000 (0.5674) = $35,000 (3.605)
$126,169 = $126,175

The rate of return is 12%.

Reject the packaging department expansion and plan on two-shift operation.

Projects 4 & 5: New warehouse or leased warehouse

Cash Flow

Year	Leased Warehouse	New Warehouse	New Rather than Leased
0	$0	-$225,000	-$225,000
1	-$49,000	-$5,000	+$44,000
2	-$49,000	-$5,000	+$44,000
3	-$49,000	-$5,000	+$44,000
4	-$49,000	-$5,000	+$44,000
5	-$49,000	-$5,000 +$200,000	+$244,000

Compute the rate of return on the difference between the alternatives.

$225,000 = $44,000 (P/A, i%, 5) + $200,000 (P/F, i%, 5)

Try i = 18%
$225,000 = $44,000 (3.127) + $200,000 (0.4371)
= $225,008

The incremental rate of return is 18%.

Build the new warehouse.

17-10

This is a variation of Problem 17-1.

(a) Approve all projects except D.

(b) Ranking Computations for NPW/Cost

Project	Cost	Uniform Benefit	NPW at 14%	NPW/Cost
A	$10	$2.98	$0.23	0.023
B	$15	$5.58	$4.16	0.277
C	$5	$1.53	$0.25	0.050
D	$20	$5.55	-$0.95	-0.048
E	$15	$4.37	$0	0
F	$30	$9.81	$3.68	0.123
G	$25	$7.81	$1.81	0.072
H	$10	$3.49	$1.98	0.198
I	$5	$1.67	$0.73	0.146
J	$10	$3.20	$0.99	0.099

Ranking:

Project	Cost	NPW/Cost	Cumulative Cost
B	$15	0.277	$15
H	$10	0.198	$25
I	$5	0.146	$30
F	$30	0.123	$60
J	$10	0.099	$70
G	$25	0.072	$95
C	$5	0.050	$100
A	$10	0.023	$110
E	$15	0	$125
D	$20	-0.048	$145

(c) Budget = $85,000
The first five projects (B, H, I, F, and J) equal $70,000. There is not enough money to add G, but there is enough to add C and A. Alternately, one could delete J and add G. So two possible selections are:

B H I F G	NPW(14%)	= $28.36
B H I F J C A	NPW(14%)	= $28.26

For $85,000, maximize NPW.

Choose: B, H, I, F, and G.

17-11

Project	Cost (P)	Annual Benefit (A)	(A/P, i%, 10)	ROR
1A	$5,000	$1,192.50	0.2385	20%
1B- 1A	$5,000	$800.50	0.1601	9.6%
2A	$15,000	$3,337.50	0.2225	18%

2B- 2A	$10,000	$1,087.50	0.1088	1.6%

(a) 1A
(b) 8%
(c) 1B and 2A

Chapter 18: Accounting and Engineering Economy

18-1

Engineers and managers make better decisions when they understand the "dollar" impact of their decisions. Accounting principles guide the reporting of cash flows for the firm. Engineers and managers can access this information through formal and informal education means, both within and outside the firm.

18-2

The accounting function is the economic analysis function within a company — it is concerned with the dollar impact of past decisions. It is important to understand, and account for, these past decisions from management, operational, and legal perspectives. Accounting data relates to all manner of activities in the business.

18-3

Balance Sheet – picture of the firm's financial worth at a specific point in time.

Income Statement – synopsis of the firm's profitability for a period of time.

18-4

Short-term liabilities represent expenses that are due within one year of the balance sheet, while long-term liabilities are payments due beyond one year of the balance sheet.

18-5

The two primary general accounting statements are the balance sheet and the income statement. Both serve useful and needed functions.

18-6

Today's weather is not a good basis to pack for a 3-month trip, and local and recent financial data is not a complete basis for judging a firm's performance. Historical and seasonal trends and a context of industry standards are also needed.

18-7

Not necessarily. The current ratio will provide insight into the firm's solvency over the short term and although a ratio of less than 2 historically indicates there could be problems, it doesn't mean the company will go out of business. The same is true with the acid-test ratio. If the company has a low ratio, then it probably doesn't have the ability to instantly pay off debt. That doesn't necessarily indicate the firm will go bankrupt. Both tests should be used as an indicator or warning sign.

18-8

Assets = $1,000,000
Total liabilities = $127,000 + 210,000 = $337,000
Equity = assets – liabilities = $1,000,000 – 337,000 = $663,000

18-9

6 days/week * 52 weeks/year = 312 days/year in operation
$1000 profit/day * 312 days/year = $312,000 profit/year
Revenues – expenses = $500,000 – 312,000 = $188,000

18-10

a. Working capital = current assets - current liabilities
= $5,000,000 – 2,000,000
= $3,000,000
b. Current ratio = (current assets / current liabilities)
= $5,000,000/2,000,000
= 2.5

18-11

Net profit (loss) = revenues – expenses = $100,000 – 60,000 = $40,000

18-12

a. Working capital = ($90,000 + 175,000 + 210,000) – (322,000 + 87,000)
= $475K – 409K = $66,000
b. Current ratio = ($475K/409K) = 1.161
c. Acid test ratio = ($90,000 + 175,000)/409,000 = 0.648

18-13

Operating Revenues and Expenses

	Revenue
Sales	30,000
Total	30,000
	Expenses
Administrative	2750
Cost of goods sold	18,000
Development	900
Selling	4500
Total	26,150
Total operating income	3,850

Non-operating revenues & expenses	
Interest paid	200
Income before taxes	3650
Taxes (@27%)	985.50
Net profit (loss)	2664.50

18-14

Assets = $100,000 + 45,000 + 150,000 + 200,000 + 8,000 = $503,000
Liabilities = $315,000 + 90,000 = $405,000

a. Working capital = $503,000 – 405,000 = $98,000
b. Current ratio = $495,000/405,000 = 1.22
c. Acid test ratio = $295,000/405,000 = 0.73

18-15

a. Working capital = current assets – current liabilities
= ($110K + 40K + 10K + 250K) – (442K) = $118,000
b. Current ratio = current assets / current liabilities
= $560K/442K = 1.27
c. Acid test ratio = quick assets / current liabilities
= $310K/442K = 0.701
A good current ratio is 2 or above, and a good acid test ratio is 1 or above. This company is in major trouble unless they move inventory quickly.

18-16

Total revenues = $51 + 35 = $86 million
Total expenses = $70 + 7 = $77 million

a. Net income before taxes = revenue – expenses = $86 – 77 = $9 million
b. Net profit = net income before taxes – taxes = $9 – 1 = $8 million

Interest coverage = (total revenues – total expenses) / interest
= ($86 – 70)/7 = 2.28
This interest coverage is not acceptable because it should be at least 3.0 for industrial firms.

18-17

Profit = $50,000 – 30,000 – 5,000 = $15,000
Net income = profit – taxes = $15,000 – 2,000 = $13,000

18-18

a. Current ratio = current assets / current liabilities = (1.5million)/50,000 =30
b. Acid test ratio = quick assets / current liabilities = (1.0 million)/50,000= 20

While it may be tempting to think that a higher ratio is better, this is not always the case. Such high ratios as these could mean that an excessive amount of capital is being kept on hand. Excess capital does very little for the company if it is just sitting in the bank – it could and/or should be used to make the company more profitable through investing, automation, employee training, etc.

18-19

Total current assets = $1740 + 900 + 2500 – 75 = $5065
Total current liabilities = $1050 + 500 + 125 = $1675
Current ratio = $5065/1675 = 3.0238
This company's financial standing is good because the current ratio is greater than 2.0.

18-20

a. Current ratio = current assets / current liabilities = $2670/1430 = 1.87

This is below the recommended ratio of 2.0 and may indicate that the firm is not solvent, especially since the height of the nursery business is the spring and summer and this is a June balance sheet.

b. Acid test ratio = (cash + accounts receivable) / current liabilities
= ($870 + 450)/1430 = 0.92

This indicates that 92% of the current liabilities could be paid out within the next thirty days, which is not a bad situation, although a little higher would be preferable.

18-21

a. Interest coverage = total income / interest payments
= ($455 – 394 + 22)/22 = 3.77

This is a good ratio, indicating the company's ability to repay its debts. It should be at least 3.0.

b. Net profit ratio = net profits / sales revenue = $31/(395 – 15) = 0.08

This is a very small ratio, indicating that the company needs to assess their ability to operate efficiently in order to increase profits. The company should compare itself to industry standards.

18-22

Activity	Model S	Model M	Model G
Direct material cost	$3,800,000	$1,530,000	$2,105,000
Direct labor cost	$600,500	$380,000	$420,000
Direct labor hours	64,000	20,000	32,000
Allocated overhead	64,000 X 137 = $8,768,000	20,000 X 137 = $2,740,000	32,000 X 137 = $4,384,000
Total costs	$131,685,000	$4.650,100	$6,909,000
Units produced	100,000	50,000	82,250
Cost per unit	$132	$93	$84

18-23

a. Total direct labor = 50,000 + 65,000 = $115,000
Allocation of overhead
$\text{Overhead}_{\text{Standard}}$ = (50,000/115,000)(35,000) = $15,217
$\text{Overhead}_{\text{Deluxe}}$ = (65,000/115,000)(35,000) = $19,783
$\text{Total Cost}_{\text{Standard}}$ = 50,000 + 40,000 + 15,217 = $105,217
$\text{Total Cost}_{\text{Deluxe}}$ = 65,000 + 47,500 + 19,783 = $132,283
$\text{Net Revenue}_{\text{Standard}}$ = 1800(60) - 105,217 = $2783
$\text{Net Revenue}_{\text{Deluxe}}$ = 1400(95) - 132,283 = $717

b. Total materials = 40,000 + 47,500 = $87,500
Allocation of overhead
$\text{Overhead}_{\text{Standard}}$ = (40,000/87,500)(35,000) = $16,000
$\text{Overhead}_{\text{Deluxe}}$ = (47,500/87,500)(35,000) = $19,000
$\text{Total Cost}_{\text{Standard}}$ = 50,000 + 40,000 + 16,000 = $106,000
$\text{Total Cost}_{\text{Deluxe}}$ = 65,000 + 47,500 + 19,000 = $131,500
$\text{Net Revenue}_{\text{Standard}}$ = 1800(60) - 106,000 = $2000
$\text{Net Revenue}_{\text{Deluxe}}$ = 1400(95) - 131,500 = $1500

In both cases the total net revenues equal $3500, but the deluxe bag appears far more profitable with materials-based allocation.

18-24

a. $60,000,000/12,000 hours = $5000/hour
b. Total cost = $1,000,000 + $600,000 + 200hours*$5000/hour = $2,600,000

18-25

RLW-II will use the ABC system to understand all of the activities that drive costs in their manufacturing enterprise. Based on the presence and magnitude of the activities, RLW-II will want to assign costs to each. In doing this, RLW-II will gain a more accurate view of the true costs of producing their products. Potential categories of indirect costs that RLW-II will want to account for include costs for: ordering from and maintaining a relationship with specific vendors/suppliers; shipping, receiving, and storing raw materials, components and sub-assemblies; retrieval and all material handling activities from receiving to final shipment; all indirect manufacturing and assembly activities that support the direct costs; activities related to requirements for specific and unique machinery, tools and fixtures, and engineering and technical support; all indirect quality related activities in areas such as testing, rework and scrap; activities related to packaging, documentation and final storage; shipping, distribution and warehousing activities, and customer support/service and warranty activities.

18-26

Direct Costs	Indirect Costs
Machine operator wages	Insurance costs
Machine labor	Utility costs
Overtime expenses	Material handling costs
Cost of materials	Engineering drawings
	Cost to market the product
	Cost of storage
	Cost of product sales force
	Support staff salaries
	Cost of tooling and fixtures
	Machine run costs