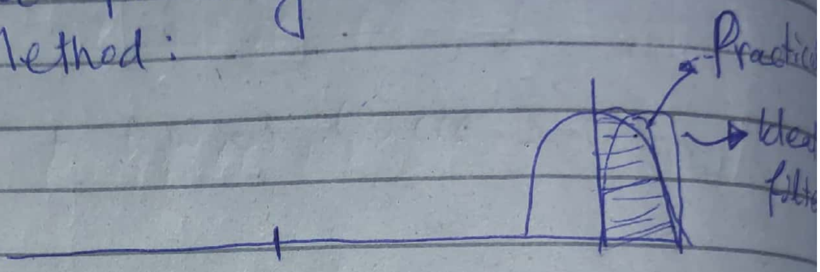
Com Sys After Mids:-

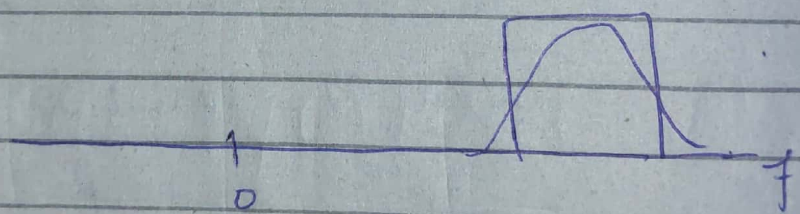
29/11/2023

★ Generation of SSB

(i) Selective Filtering Method:



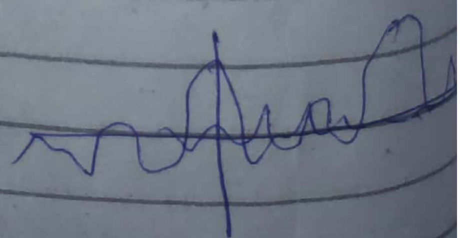
Can we do this?
No.
Why?



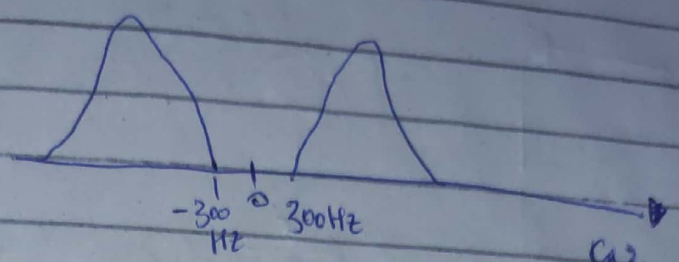
Butterworth filter

$$H(\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi B}\right)^{2n}}}$$

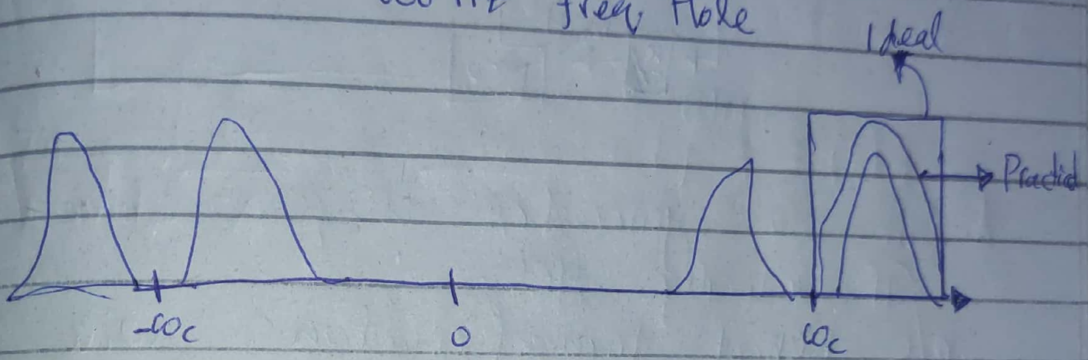
$$\text{rect}\left(\frac{\omega}{2W}\right)$$



★ Speech Signal 0.3 kHz to 3.4 kHz



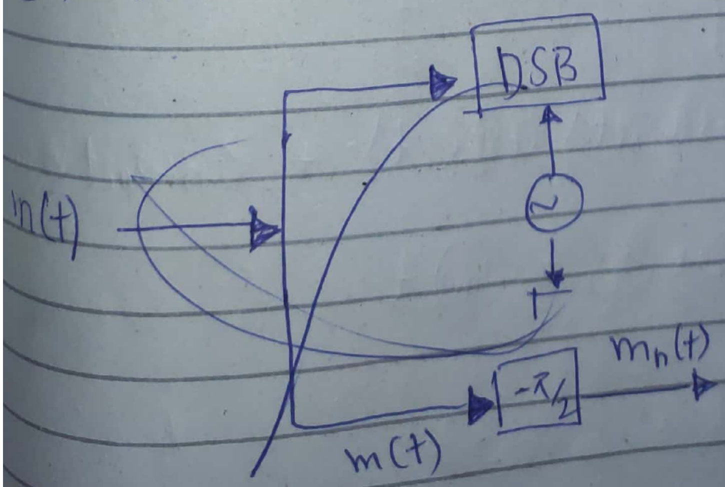
600 Hz freq. Hole

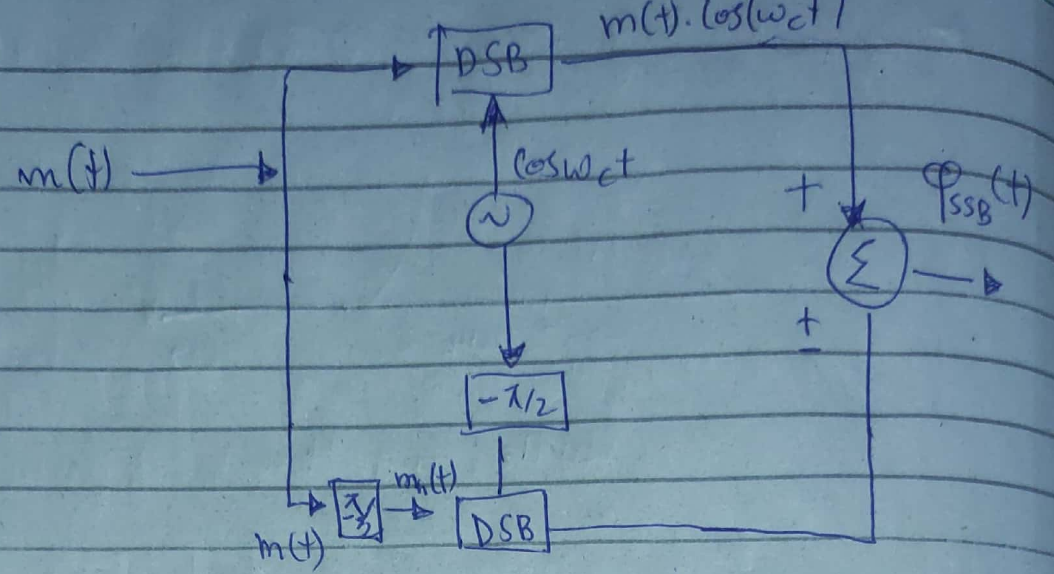


Speech signal can be isolated by practical filter bcoz freq. Hole

Now possible method.

(ii) Phase Shift Method :-



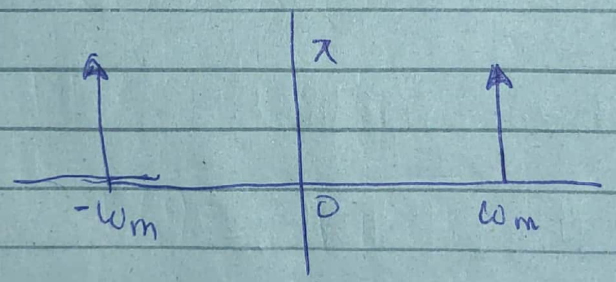


★ Can we generate using above diagram

Theoritically yes.

b/c hilbert transformation will applying constant transformation on every freq. component (ideal case).

$m(t) = \cos \omega_m t$

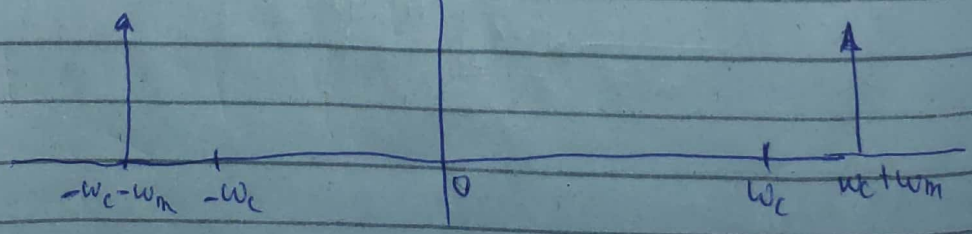


$m_h(t) = \cos(\omega_m t - \frac{\pi}{2})$

$= \sin \omega_m t$

$\varphi_{SSB}(t) = \cos(\omega_m t)\cos(\omega_c t) + \sin \omega_m t \sin \omega_c t$

$\quad '' \quad = \cos(\omega_c \pm \omega_m) t$



Multiplying $\varphi_{SSB}(t)$ with $\cos\omega_c t$

$$\varphi_{SSB}(t) = m(t)\cos^2\omega_c t + m_h(t).\sin\omega_c t\cos\omega_c t$$

$$\varphi_{SSB}(t) = \frac{m(t)}{2} + \frac{m(t)\cos(2\omega_c t)}{2} + \frac{m_h(t).\sin 2\omega_c t}{2}$$

$$= M(\omega)$$

Low Pass filter, Coherent Detection

Now we will send carrier along.

$$\varphi_{SSB+C}(t) = \underline{A}.\cos\omega_c t + m(t).\cos\omega_c t + m_h(t).\sin\omega_c t$$

$$'' = \left(A + m(t)\right).\cos\omega_c t + m_h(t).\sin\omega_c t$$

$$'' = E(t).\cos(\omega_c t + \theta)$$

as $\dfrac{a+ib}{\sqrt{a^2+b^2}}$

Now if $E(t) = \sqrt{[A+m(t)]^2 + m_h^2(t)}$

Similar to

$$a_n \cdot \cos n\omega_0 t + b_n \cdot \sin n\omega_0 t$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

Now $E(t) = \left\{A^2 + m^2(t) + 2 \cdot A \cdot m(t) + m_h^2(t)\right\}^{1/2}$

$$E(t) = A\left[1 + \frac{2\,m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]^{1/2}$$

if $m(t) << A$

then

$$E(t) \simeq A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

Now using binomial theorem

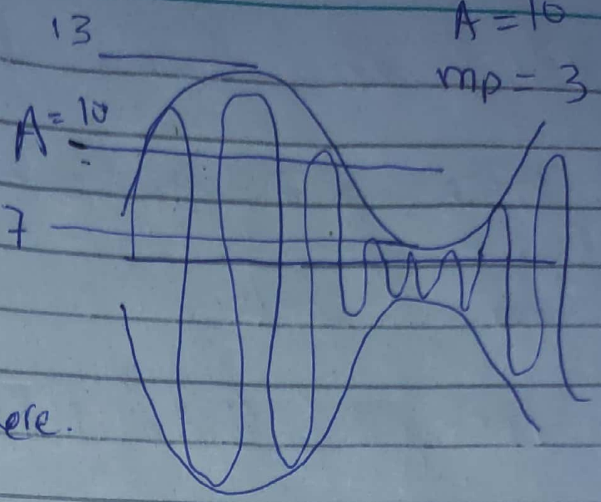
$$E(t) \approx A\left[1 + \frac{1}{\cancel{2}} \times \frac{\cancel{2}\,m(t)}{A}\right]$$

$$E(t) \simeq A + m(t)$$

Now put in above equations

$$\varphi_{\substack{SSB\\+C}}(t) = [A + m(t)]\cos(\omega_c t + \theta)$$

Scanned with CamScanner



We are using extra power here.

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{1}{3} \times 100 = 33.33\%$$

8/12/2023

$$\varphi_{DSB-SC}(t) = m(t) \cdot \cos(\omega_c t)$$

$$\varphi_{DSB+C}(t) = [A + m(t)] \cos(\omega_c t)$$

2B Hz

$$\varphi_{QAM}(t) = m_1(t) \cos(\omega_c t) + m_2(t) \sin(\omega_c t)$$

$$\varphi_{SSB}(t) = m(t) \cdot \cos(\omega_c t) \mp m_h(t) \sin \omega_c t$$

i) Selective Filtering Method
ii) Phase shift "

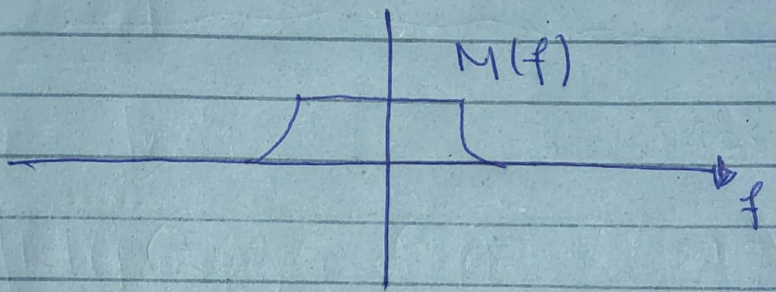
We have two extremes here.

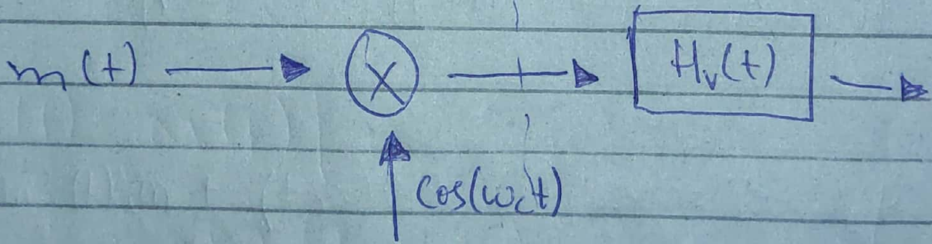
DSB $\rightarrow$ 2 B Hz SSB $\rightarrow$ B Hz

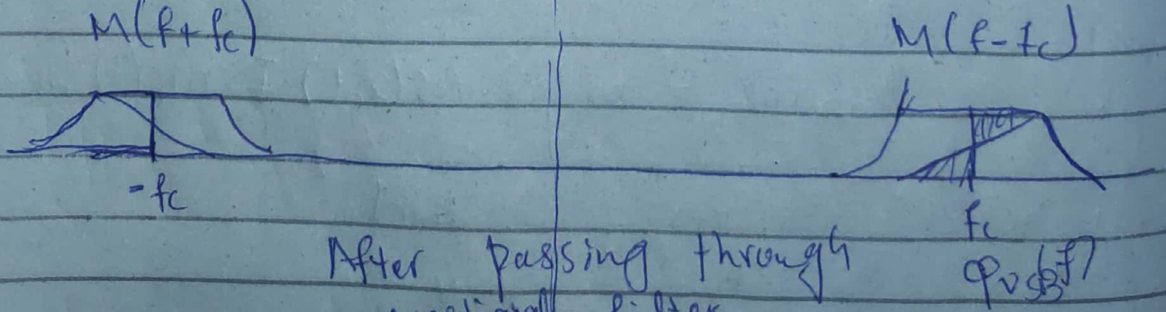
Now comes:

★ Vestigial Side Band Modulation:-

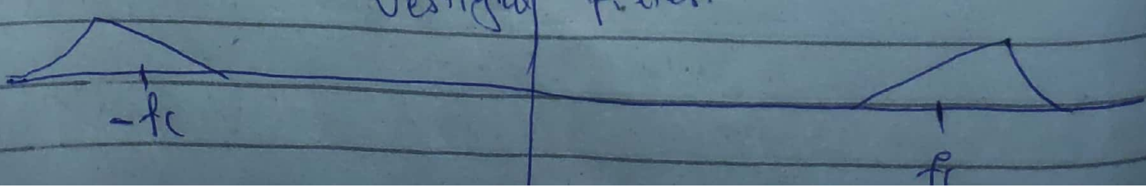
$$B_{Hz} < B_T < 2B\ Hz$$







After passing through vestigial filter.



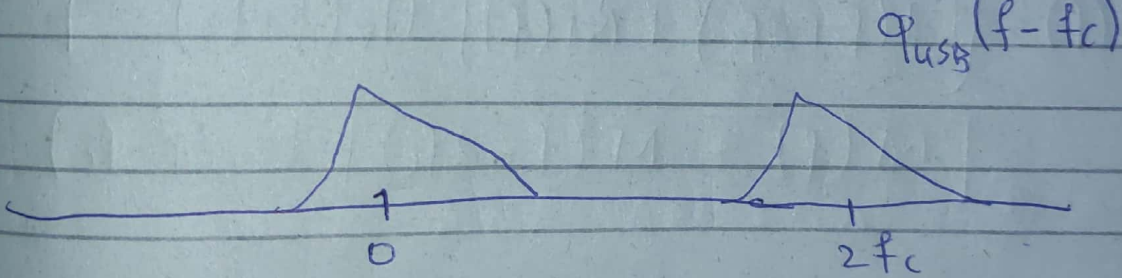
Demodulation

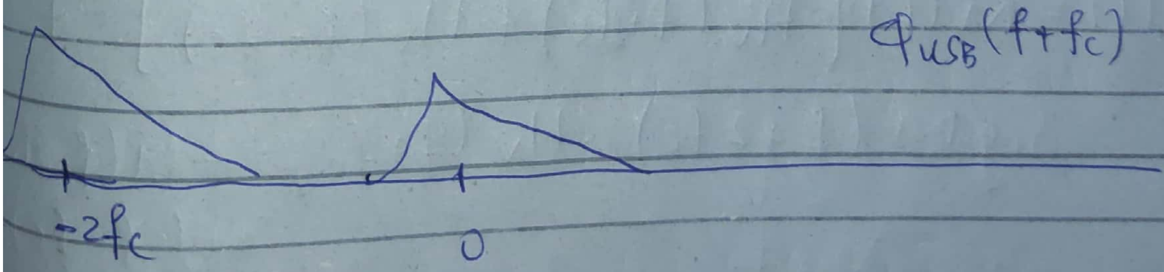
$\varphi(t)_{DSB-SC} = m(t).\cos(\omega_c t)$] Modulation

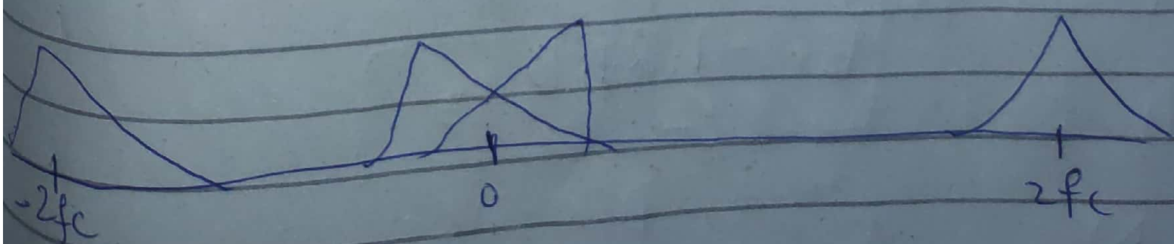
Multipley $\cos(\omega_c t)$ at reciever's end.

$s(t) = \varphi_{VSB}(t).\cos(\omega_c t)$] demodulation.

$S(f) = \varphi_{VSB}(f - f_c) + \varphi_{USB}(f + f_c)$







Now at sender's end we have

$$\varphi_{VSB}(f) = [M(f+f_c) + M(f-f_c)] H_v(f)$$

Now
$$\begin{cases} \varphi_{VSB}(f+f_c) = \{M(f+2f_c) + M(f)\} H_v(f+f_c) \\ \varphi_{VSB}(f-f_c) = \{M(f) + M(f-2f_c)\} H_v(f-f_c) \end{cases}$$

Now if we add these two, get

$$S(f) = \varphi_{VSB}(f+f_c) + \varphi_{VSB}(f-f_c)$$

$$S(f) = [M(f+2f_c) + M(f)] H_v(f+f_c) +$$
$$[M(f) + M(f-2f_c)] H_v(f-f_c)$$

$$S(f) = M(f)[H_v(f+f_c) + H_v(f-f_c)] +$$
$$M(f+2f_c) H_v(f+f_c) + M(f-2f_c) H_v(f-f_c).$$

Typical value fo Bandwidth is;

$$\boxed{B_T = 1.25B \text{ Hz}}$$

Now to recover $M(f)$ we

will pass it through low pass filter.

$$M(f) = S(f).H_{LPF}(f)$$

Hence.

$$M(f) = \cancel{M(f)}\left[H_v(f+f_c) + H_v(f-f_c)\right] H_{LPF}(f)$$

$$0 = H_v(f+f_c) +$$

$$H_{LPF}(f) = \frac{1}{H_v(f+f_c) + H_v(f-f_c)}$$

* Carrier Acquisition:-

$$m(t).\cos w_c t. \cos\left([w_c + \Delta w]t + \delta\right)$$

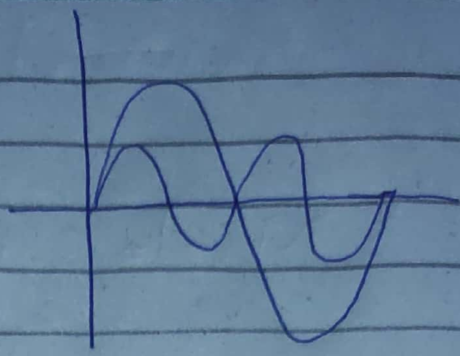
Now $\cos\alpha.\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$$m(t).\cos(\Delta wt + \delta) + m(t).\cos(2w_c t + \Delta wt + \delta)$$

By passing through LPF

$$m(t).\cos(\Delta wt + \delta)$$

$f = \frac{d\phi(t)}{dt}$



case
(1) Now if $\Delta\omega = 0$, $\delta = \text{const.}$

$m(t) \cdot \cos(\delta)$

eg

$0.8 \cdot m(t)$

Hence, shape of $m(t)$ will not be changed. Hence it is distortionless.

Case (2) $\Delta\omega =$ same value, $\delta = 0$

$m(t) \cdot \cos(\Delta\omega t)$

Signal will be distorted.

So, we need carrier aquisition.

Carrier Aquisition :-

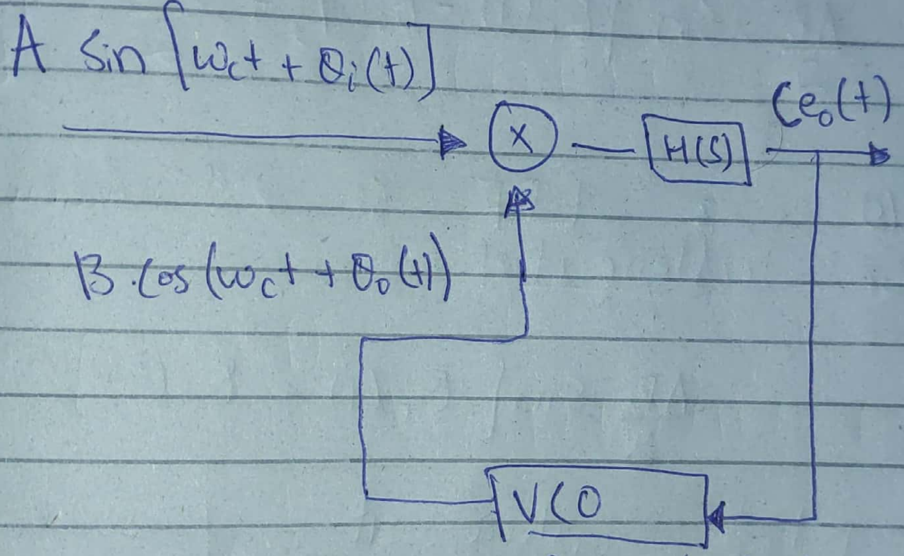
(i) Pilot Signal :-

DSB-WC

$\{A + m(t)\} \cos \omega_c t$

(ii) Phase Loop Lock:-
(iii) Squaring Method:-

Phase Loop Lock:-
This is a feedback system.



VCO: Voltage Control Oscillator.

$[\theta_i(t) - \theta_o(t)] \propto \underbrace{e_o}$ will create voltage

$$\omega_i = \omega_c + C.\, e_o(t) \quad -(i)$$

$$\varphi(t) = \omega_c t + \theta_o(t)$$

$$\frac{d}{dt}\varphi(t) = \frac{d}{dt}(\omega_c t) + \frac{d}{dt}(\theta_o(t))$$

$$\omega_i = \omega_c + \dot{\theta}_o(t) \text{ --- (ii)}$$

Compare (i) and (ii)

$$\dot{\theta}_o(t) = C \cdot e_o(t)$$

Now output of multiplier B:

$$A \cdot \sin(\omega_c t + \theta_i(t)) \cdot B \cdot \cos[\omega_c t + \theta_o(t)]$$

$$\frac{AB}{2} \cdot \sin[\theta_i(t) - \theta_o(t)] + \frac{AB}{2} \sin[2\omega_c t + \theta_i(t) + \theta_o(t)]$$

H(s) is LPF so 2nd term will be attenuated.

$$\frac{AB}{2} \sin[\theta_i(t) - \theta_o(t)]$$

Now when it passes through h(t).

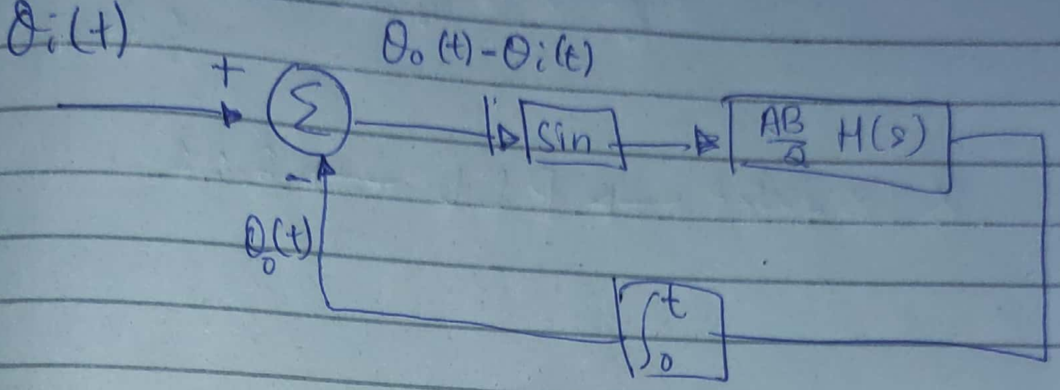
$$\frac{AB}{2}[h(t) * \sin[\theta_i(t) - \theta_o(t)]]$$

$$Ce_o(t) = \frac{AB}{2} \int_0^t h(t-x) \cdot \sin[\theta_i(x) - \theta_o(x)]\, dx$$

Put $Ce_o(t) = \dot{\theta}_o(t)$

$$\dot{\theta}_o(t) = \frac{AB}{2} \int_0^t \_\_\_\_\_\_\_\_ \, dx$$

$\theta_o(t) =$



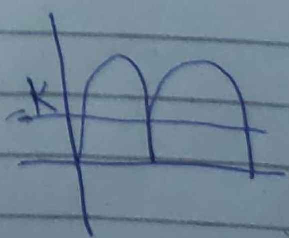
(iii) Squaring Method:-

$$(m(t))^2 (\cos w_c t)^2$$

$$m^2(t) \cdot \left[\frac{1 + \cos(2 w_c t)}{2} \right]$$

$$\frac{m^2(t)}{2} + \frac{m^2(t) \cos(2 w_c t)}{2}$$

Let $\frac{m^2(t)}{2} = \underbrace{K}_{\boxed{\text{Average value}}} + \phi(t)$



$$\frac{m^2(t)}{2} + \frac{[K + \phi(t)] \cos(2 w_c t)}{2}$$

$m(t) \rightarrow B\ Hz$

$m^2(t) \rightarrow 2B\ Hz$

$$\frac{m^2(t)}{2} + \frac{K.\cos 2\omega_c t}{2} + \frac{\varphi(t)\cos 2\omega_c t}{2}$$

High Quality Q   Narrowband Filter

$K \cos 2\omega_c t \rightarrow PLL \rightarrow 2:1$

$$\frac{m^2(t)}{2} + \frac{K.\cos 2\omega_c t}{2} + \frac{\varphi(t)\cos 2\omega_c t}{2}$$

High Quality Q   Narrowband Filter

$K\cos 2\omega_c t \longrightarrow$ PLL $\longrightarrow 2:1$

Date 13/12/2023

Angle Modulation :-

f   φ

$A\cos\theta(t)$ → May or May not be linear.

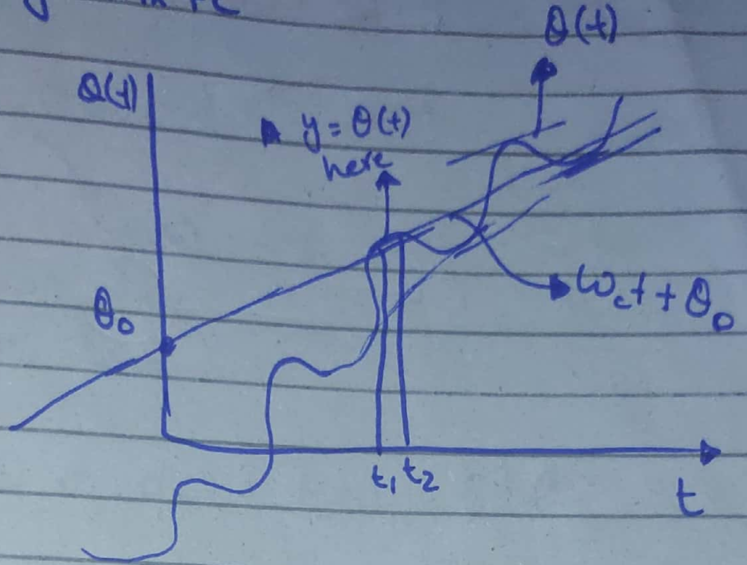
$\varphi_{EM}(t)$ — PM, FM

A.M $\longrightarrow$ Linear $\longrightarrow$ No Additional Components
$\longrightarrow$ No loss in shape
$\longrightarrow$ Bandwidth is limited.

PM, FM $\longrightarrow$ Non-Linear
→ Some addition components will be present.
→ Bandwidth is unlimited

$A.\cos(\omega_c t + \theta_0)$ → Linear

$y = mx + c$



$$\theta(t) = \omega_c t + \theta_0 \quad t_1 \le t \le t_2$$

At every instance of time, we have instantenous frequency.

$$\omega_i(t) = \frac{d\,\theta(t)}{dt}$$

$$d.\theta(t) = \omega_i(t)\,dt$$

PM & FM are closely related.

Taking integral.

$$\int d.\theta(t) = \int \omega_i(t).dt$$

Generalized Curve

$$\boxed{\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha).d\alpha}$$

$$\varphi_{PM}(t) = A \cdot \cos(\omega_c t + \theta_0 + k_p m(t))$$

Now $\quad \theta(t) = \omega_c t + \underbrace{\theta_0}_{} + k_p . m(t)$

Zyada
role nahi iska
and we can remove it

$$\theta(t) = \omega_c t + K_p . m(t)$$

Now derivate it to get
instantenous freq.

$$\omega_i = \frac{d\theta(t)}{dt} = \omega_c + k_p . m'(t)$$

$$m(t) \longrightarrow \boxed{PM} \xrightarrow{\varphi_{PM}(t)}$$

$$\varphi_{PM}(t) = A . \cos(\omega_c t + k_p . m(t))$$

$$m(t) \longrightarrow \boxed{d/dt} \xrightarrow{m'(t)} \boxed{FM} \xrightarrow{\varphi_{PM}}$$

$$\frac{2\pi f_i}{2\pi} = \frac{2\pi f_c + k_p . m'(t)}{2\pi}$$

$$f_i = f_c + \frac{k_p . m'(t)}{2\pi}$$

FM :-

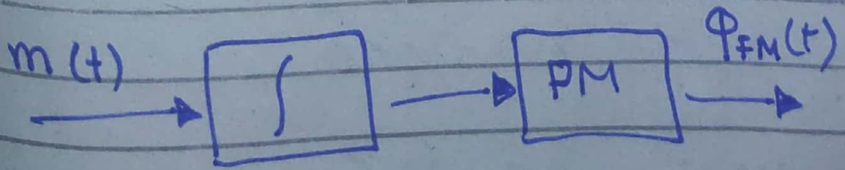
$$\omega_i = \omega_c + k_f . m(t)$$

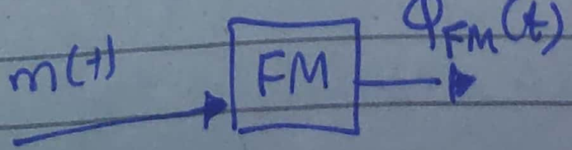
Now

$$\frac{d\,\theta(t)}{dt} = \omega_c + k_f . m(t)$$

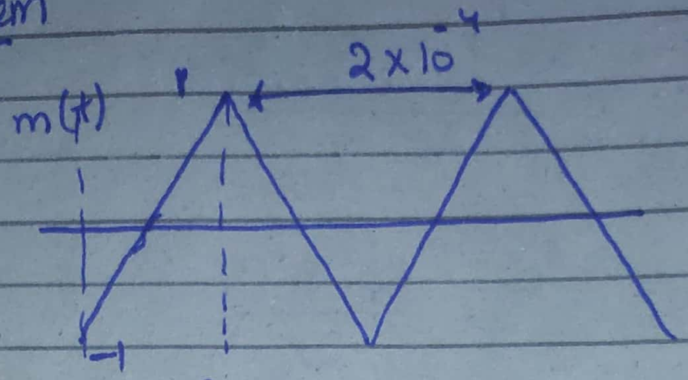
$$\theta(t) = \omega_c t + k_f . \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

$$\varphi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) . d\alpha\right]$$





★ Problem



$f_c = 100\ MHz$

$K_p = 10\pi$

$K_f = 2\pi\times10^{5}$

PM:-

for 1 slope:- $\omega_i = \omega_c + K_p . m'(t)$

$$f_i = f_c + \frac{K_p}{2\pi} . m'(t)$$

$$m'(t) = \frac{2}{1\times10^{-4}} = \boxed{20,000}$$

$$f_i = 100\ MHz + \frac{\overset{5}{\cancel{10}}\cancel{\pi}}{\underset{1}{\cancel{2}}\cancel{\pi}} \times 20,000$$

$$f_i = 100\ MHz + 0.1\ MHz$$

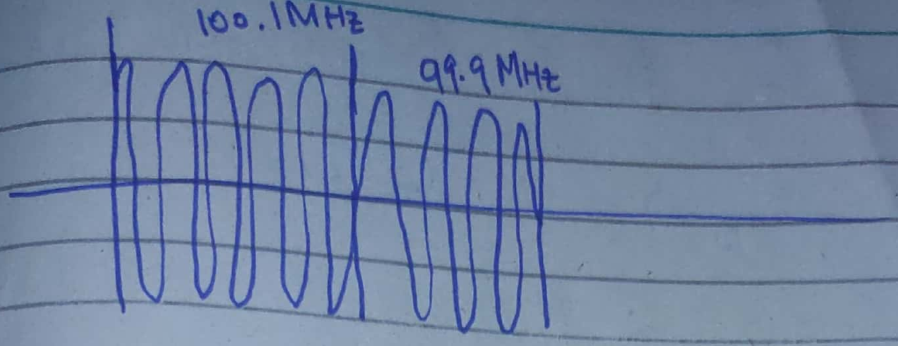
$$f_{i_{max}} = 100.1\ MHz$$

For

$$f_{i_{min}} = f_c - 5\times 20,000$$

$$f_{i_{min}} = 100\ MHz - 0.1\ MHz$$

$$f_{i_{min}} = 99.9\ MHz$$



→ Frequency isn't changing at every instance hence it is PM.

FM :-

$$\omega_i = \omega_c + k_f \cdot m(t)$$

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

take $m(t) = -1$

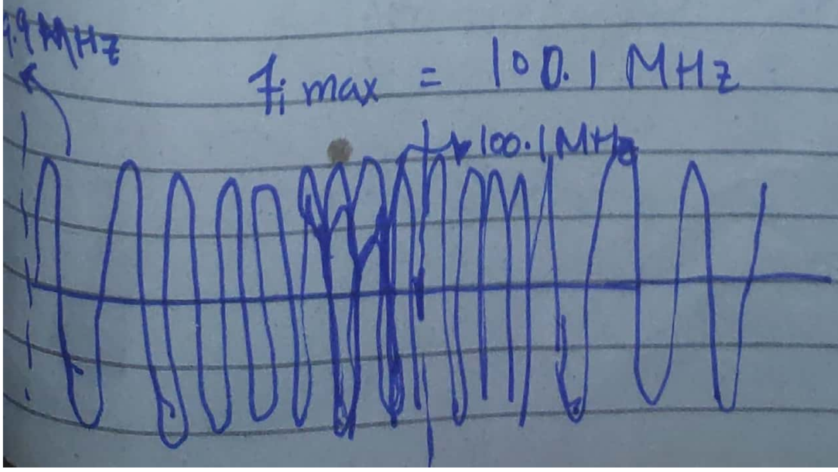
$$f_{i_{min}} = 100\text{ MHz} + \frac{2\pi \times 10^5 (-1)}{2\pi}$$

$$f_{i_{min}} = 100\text{ MHz} - 0.1\text{ MHz}$$

$$f_{i_{min}} = 99.9\text{ MHz}$$

Now for $f_{i\,max}$, $m(t) = 1$

$$f_{i\,max} = 100.1\text{ MHz}$$



At every instance ω_i is changing

POWER OF ANGLE MODULATED SIGNAL :-

$$\varphi_{EM}(t) = A.\cos(\theta(t))$$

$$P_{EM} = A^2/2$$

BANDWIDTH OF EM SIGNAL :-

$$\varphi_{FM}(t) = A\cos\left(\omega_c t + k\int_{-\infty}^{t} m(\alpha)\,d\alpha\right)$$

for convenience take

$$\int_{-\infty}^{t} m(\alpha)\,d\alpha \equiv a(\alpha)$$

$$\varphi_{FM}(t) = A.\cos(\omega_c t + k_f\,a(\alpha))$$

$$\hat{\varphi}_{FM}(t) = A.e^{j(\omega_c t + k_f . a(\alpha))}$$

+ X imaginary Part

Now $e^{j\theta} = \cos\theta + j\sin\theta$

$$\varphi_{FM}(t) = Re\{\hat{\varphi}_{FM}(t)\}$$

$$\hat{\varphi}_{FM}(t) = A.e^{j\omega_c t} . e^{jk_f . a(\alpha)}$$

Isse Bandwidth ka pata chalega

open it using power series

$$\hat{\varphi}_{FM}(t) = A.e^{j\omega_c t}\left[1 + j\,k_f.a(t) - k_f^2.a^2(t) + \cdots\cdots \frac{j^n.k_f^n.a^n(t)}{n!}\right]$$

$$\hat{\varphi}_{FM}(t) = A\left[\cos(\omega_c t) + j\sin\omega_c t\right]\left[\quad // \quad\right]$$

Now $\varphi_{FM}(t) = R_e\{\hat{\varphi}_{FM}(t)\}$

So we ignore terms with j.

$$\varphi_{FM}(t) = R_e\{\hat{\varphi}_{FM}(t)\} = A\cos\omega_c t - A\,k_f\,a(t).\sin\omega_c t - A\frac{k_f^2.a^2(t)}{2!}.\cos\omega_c t + \cdots\cdots \cdots \frac{A.k_f^n.a^n(t)}{n!}\cos\omega_c t$$

$m(t) \longrightarrow B\ Hz$

$m^2(t) \longrightarrow 2B\ Hz$

$m^3(t) \longrightarrow 3B\ Hz$

$\vdots$

$m^n(t) \longrightarrow n\,B\ Hz$

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Now if $|k_f . a(t)| << 1$

We can ignore the nth terms.

$\varphi_{FM}(t) = A . \cos w_c t - A . k_f . a(t) . \sin w_c t$

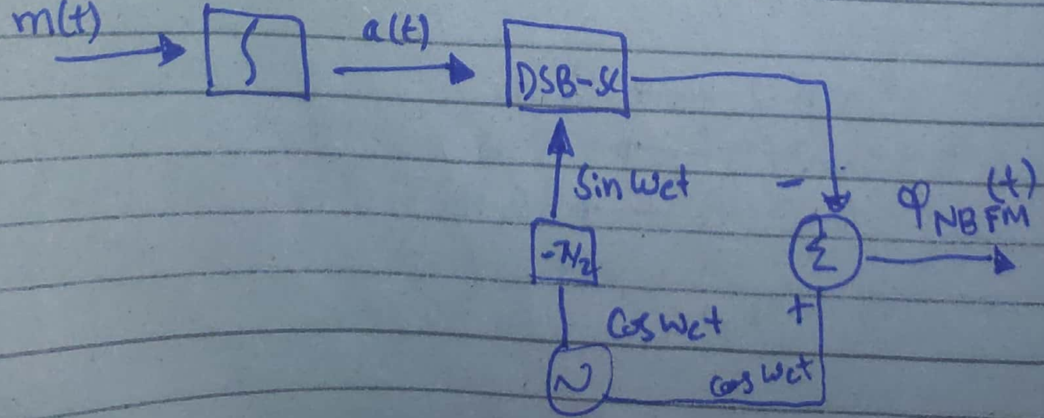
This is called narrow band FM

It's max band is $2B$ Hz
Here kam se kam (Minimum) band is $2B$ Hz
where as in DSB-SC Max Band was $2B$ Hz

We can compare it with.

$\varphi_{DSB+SC}(t) = A . \cos w_c t + m(t) . \cos w_c t$

Now we generate it using DSB-SC generator.



Date 14/12/2023

$|k_f . a(t)| << 1 \longrightarrow$ Wide Band doesn't fullfill this criteria

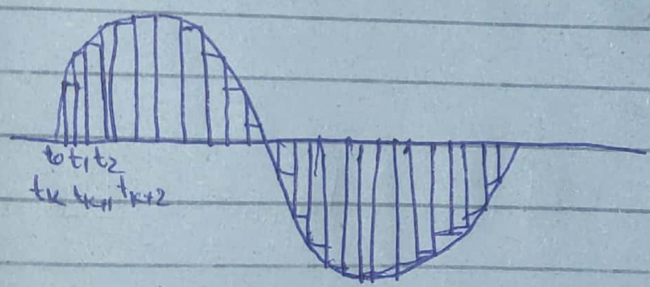
$$\varphi_{NBFM}(t) = A[\cos w_c t - k_f . a(t) . \sin w_c t]$$

$$\varphi_{NBPM}(t) = A[\cos w_c t - k_p . m(t) . \sin w_c t]$$

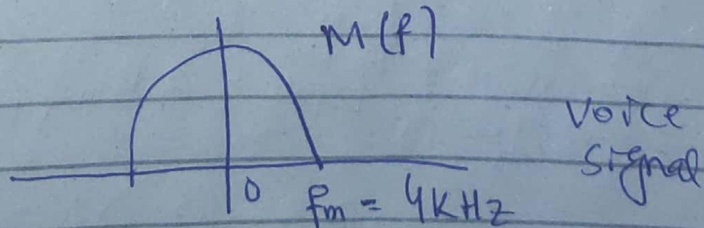
★ Wide Band Angle Modulated Signal:-

$$\omega_i = \omega_c + k_f . m(t)$$

$$f_i = f_c + \frac{k_f}{2\pi} . m(t)$$



Sampling Theorem

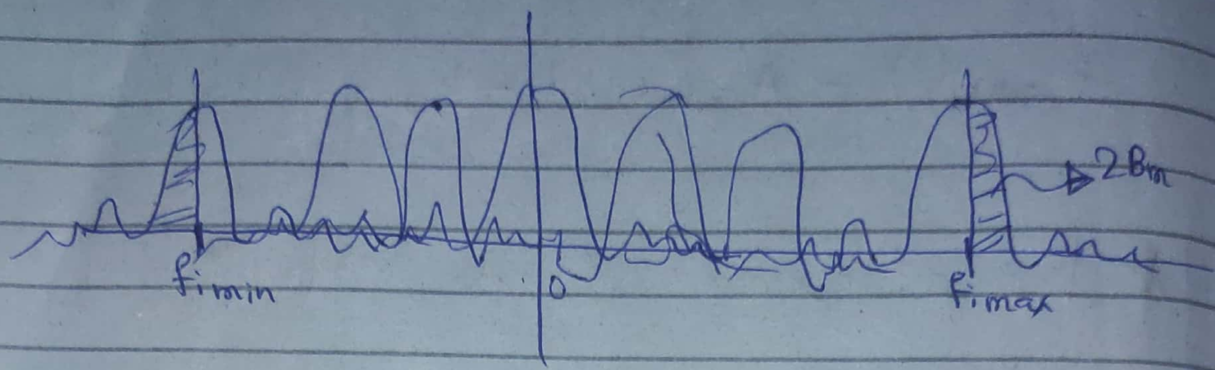


$R_s = 2B_m$ $R_s = 2 \times 4K$

$= 8K$ Samples/sec

Nyquist Criteria

$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \cdot \text{Sinc}\left(\frac{\omega\tau}{2}\right)$$



$$f_{i\,max} = f_c + \frac{k_f}{2\pi} \cdot m_p$$

$$f_{i\,min} = f_c - \frac{k_f}{2\pi} m_p$$

$$B_{FM} = 2B_m + f_{i\,max} - f_{i\,min} = 2\frac{k_f}{2\pi} m_p + 4B_m$$

$$B_{FM} = 2\Delta f + 4B_m$$

$$B_{FM} = 2B_m\left[\frac{\Delta f}{B_m} + 2\right]$$

not generilized expression ~~fallacy~~ ~~was~~ $\boxed{B_{FM} = 2B_m[\beta + 2]}$

$$\beta = \frac{\Delta f}{B_m}$$

~~If we take $\Delta f = 0$ in~~

~~$B_{FM} = 2[\Delta f + 2B_m]$~~

$B_{FM} = 4B_m$

Then Carson presented Carson's Rule to find Bandwidth and remove 2 in expression

$$B_{FM} = 2[\Delta f + B_m]$$

★ $m(t) = \alpha . \cos \omega_m t$

$$\varphi_{FM}(t) = A . \cos \omega_c t \left[\omega_c t + k_f . \int_{-\infty}^{t} m(\alpha) \, d\alpha \right]$$

$$= A \cos(\theta(t))$$

$$\omega_i = \omega_c + k_f . m(t)$$

$$\frac{d\,\theta(t)}{dt} = \omega_c + k_f . m(t).$$

$$\theta(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) . d\alpha$$

$$\varphi_{FM}(t) = A \cos \left[\omega_c t + k_f . a(t) \right]$$

$$\hat{\varphi}_{FM}(t) = A . e^{j\omega_c t} . e^{jk_f . a(t)}$$

$$a(t) = \int_{-\infty}^{t} \alpha . \cos \omega_m t \, dt$$

$$a(t) = \frac{\alpha}{\omega_m} \sin \omega_m t$$

$$\hat{\varphi}_{FM}(t) = A \cdot e^{j\omega_c t} \cdot e^{j\frac{k_f \alpha}{\omega_m} \cdot \sin\omega_m t}$$

$$\Delta f = \frac{k_f m_p}{2\pi}$$

$m_p = \alpha$

$$\beta = \frac{\Delta f}{B_m} = \frac{k_f \cdot \alpha}{\omega_m} = \frac{\Delta f}{B_m}$$

$$\hat{\varphi}_{FM}(t) = A \cdot \underbrace{e^{j\omega_c t}}_{\text{Constant term}} \cdot \underbrace{e^{j\beta \cdot \sin\omega_m t}}_{\text{variations in here}}$$

Fourier Series Expansion

$$e^{j\beta \sin\omega_m t} = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_m t}$$

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} g_{T_0}(t) \cdot e^{-jn\omega_0 t}\, dt$$

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{jB \sin\omega_m t} \cdot e^{-jn\omega_m t}\, dt$$

$$T = \frac{2\pi}{\omega_m}$$

$$\omega_m = \frac{2\pi}{T}$$

$$D_n = \frac{\omega_m}{2\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} \cdot e^{-jn\omega_m t}\, dt$$

Let $\omega_m t = x$

$$\omega_m = \frac{dx}{dt}$$

$$dt = \frac{dx}{\omega_m}$$

$$D_n = \frac{\cancel{\omega_m}}{2\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j(\beta \cdot \sin x - nx)} \cdot \frac{dx}{\cancel{\omega_m}}$$

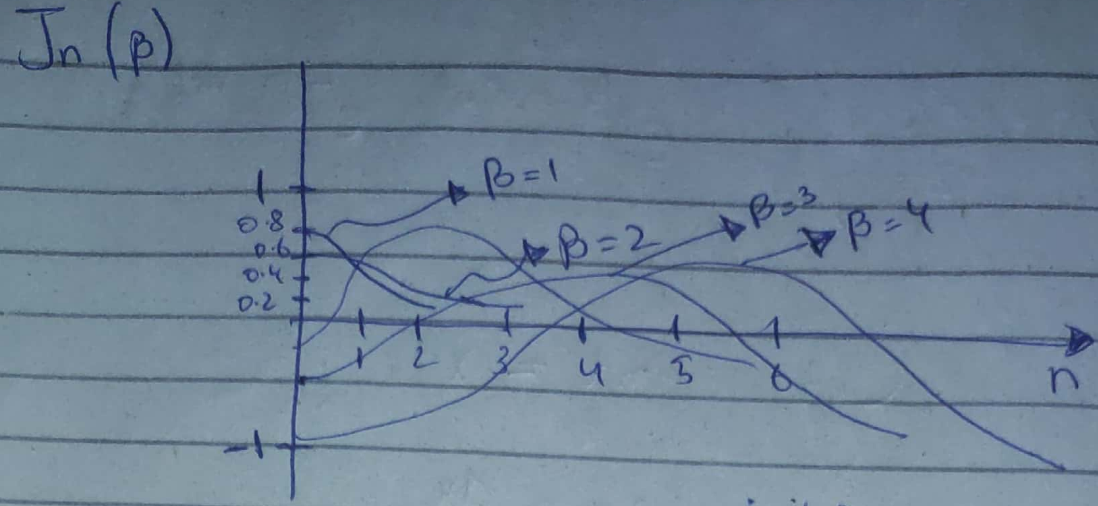
$$D_n = \frac{1}{2\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j(\beta \cdot \sin x - nx)}\, dx$$

Scale changed here, So our period is now 2π

$$D_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \cdot \sin x - nx)}\, dx$$

This integration is very complex,
So, we use bessel's functions.

$J_n(\beta)$



$n = \beta + 1 \qquad j_n(\beta) \approx 0$

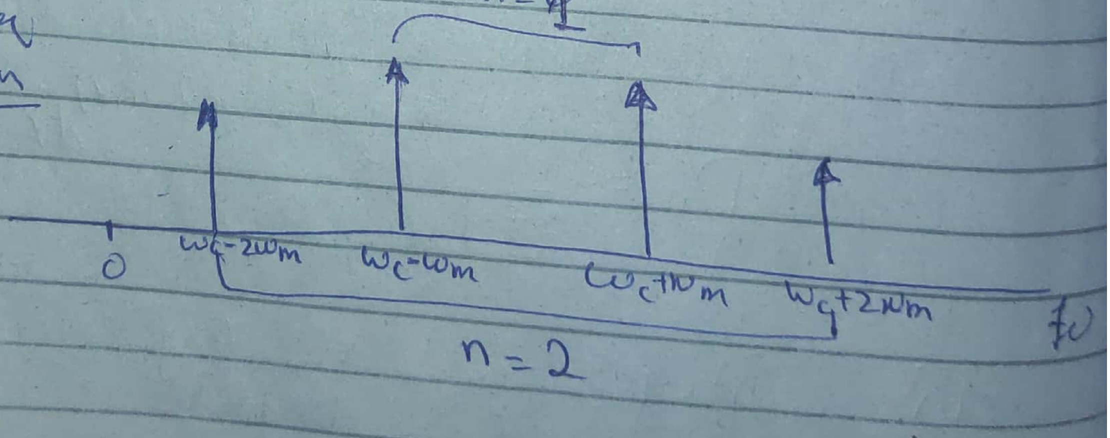
$$\hat{\varphi}_{FM}(t) = A \cdot e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) \, e^{jn\omega_m t}$$

$$\varphi_{FM}(t) = R_e\left[\hat{\varphi}_{FM}(t)\right] = A \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{j(\omega_c + n\omega_m}$$

$n = \beta + 1 \approx 0$

$$= A \cdot \sum_{n=-\infty}^{\infty} j_n(\beta) \cdot \cos(\omega_c + n\omega_m)$$

in freq Domain



$2 \cdot B_m(n)$

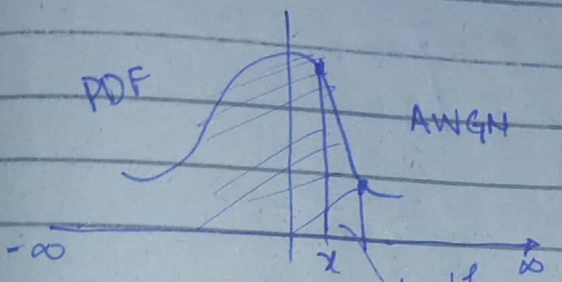
$B_{FM} = 2 \cdot B_m(\beta + 1)$

$\quad = 2 \cdot B_m\left(\frac{\Delta f}{B_m} + 1\right)$

$B_{FM} = 2(\Delta f + B_m)$

Date 20/12/2023

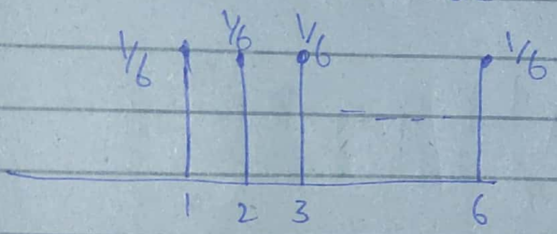
PDF



AWGN

$$\frac{1}{\sqrt{2\pi \sigma_x}} \int_{-\infty}^{t} e^{-\frac{(x-u_x)^2}{2\sigma_x}} dx$$

If we integrate it from $-\infty$ to $+\infty$, then it will be 1.



$+ = 1$

Q function Table

$$\hat{\varphi}_{FM}(t) = e^{j\omega_c t} \cdot e^{j\frac{K_f A}{\omega_m}\sin\omega_m t}$$

$$|K_f\,\alpha(t)| << 1$$

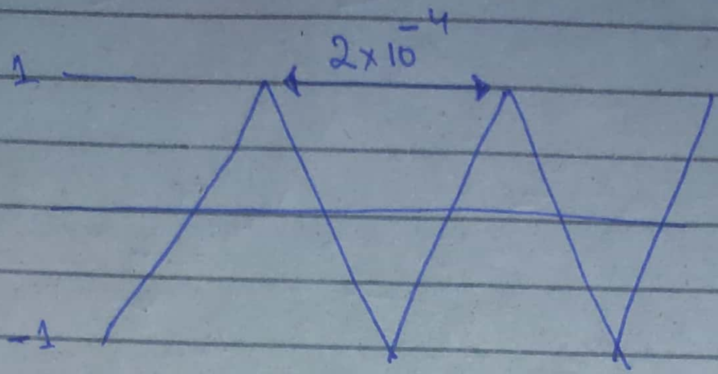
$2B_m$

$B_{FM} = 2(\Delta f + B_m)$

Carson's Rule

Ex

$K_f = 2\pi \times 10^5$
$K_p = 5\pi$



$B_{FM} = ?$, $B_{PM} = ?$

$B_{FM} = 2[\Delta f + B_m]$

$= 2\left[\frac{K_f m_p}{2\pi} + B_m\right]$

for this we need to calc. it's fourier series.
Will be given directly

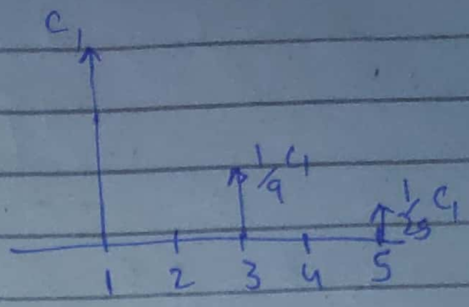
$$\sum_{n=-\infty}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$$

$$C_n = \begin{cases} \frac{8}{n^2\pi^2} & \text{for } n = \text{odd} \\ 0 & \text{for } n = \text{even} \end{cases}$$

$C_1 = 8/\pi^2$

$C_3 = \frac{1}{9} \cdot 8/\pi^2 = \frac{1}{9} C_1$

$C_5 = \frac{1}{25} C_1$



Mag. is shrinking very much.

$P_1 = \frac{1}{2}\left(8/\pi^2\right)^2$

$P_3 = \frac{1}{2} \cdot \frac{1}{81} C_1^2$

So, it's good to take very few samples here. We take the significant band. (e.g upto C_3)

So $B_m = 3 f_m$ → fundamental frequency

$B_m = 3 \cdot \frac{1}{T} = \frac{3}{2} \times 10^4 = 15\ KHz$

$B_{FM} = 2\left[\frac{2\pi \times 10^5}{2\pi}(1) + 15 \times 10^3\right]$

$= 2\left[100 \times 10^3 + 15 \times 10^3\right]$

$= 2\left[115 \times 10^3\right]$

$= 230 \times 10^3\ Hz$

$B_{pm} = ?$

$$B_{PM} = 2(\Delta f + B_m)$$

But Δf is changed here

$$\Delta f = \frac{k_p \cdot \dot{m}_p}{2\pi}$$

$$\Delta f = \frac{5\pi \times \cancel{20000}^{10000}}{\cancel{2\pi}_{1}}$$

$$\Delta f = 50{,}000 = 50\ kHz$$

Now repeat it :
1. Doubling the amplitude
2. D. // // period.

Ex

$$\varphi_{FM}(t) = 10 \cdot \cos[\omega_c t + 5 \sin 3000t + 10 \sin 2000\pi t]$$

$P = ?$

$\Delta f = ?$

$\beta = ?$

$\Delta\varphi = ?$

$B_{FM} = ?$

$$\omega_c = 2\pi \times 10^5$$

$$\omega_i = \frac{d}{dt}\theta(t)$$

$$\boxed{P = 50\ \text{watts}}$$

$$\omega_i = \omega_c + 15000 \cos 3000t + 20000\pi \cos 2000\pi t$$

$$\Delta\omega = 15000 + 20000\pi$$

```
li $V0, 4
la $a0, —
syscall

li $V0, 5
syscall
move $t2, $V0

li $V0, 1
move $a0, $t2
syscall
```

$$\Delta f = \frac{15000 + 20000\pi}{2\pi}$$

$$\beta = \frac{\Delta f}{B_m}$$

$\omega_m = 2000\pi$

So $f_m = \frac{2000\pi}{2\pi} = 1000Hz$

$\beta =$

Now for phase angle,

$$\Delta\varphi = 5(\sin 3000t)_{max} + 10(\sin 2000\pi t)_{max}$$

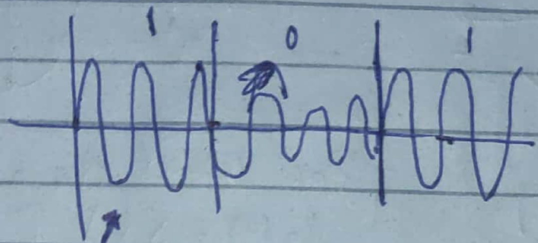
$$\Delta\varphi = 5 + 10 = 15 \text{ radians}$$

$$B_{FM} = 2(\Delta f + B_m)$$

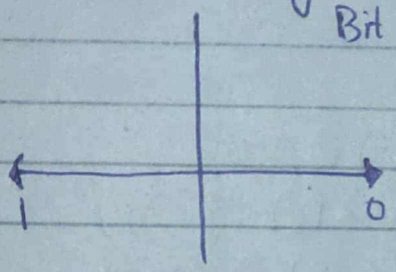
Com Sys Part (2) :-

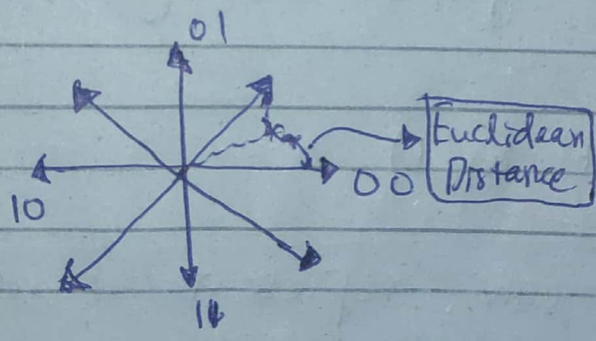
Date 27/12/2023

Analog to Digital Conversion:-



Symbol → Symbol rate]
Bit rate] → per second





Now at receivers end, it's difficult to determine the signals.

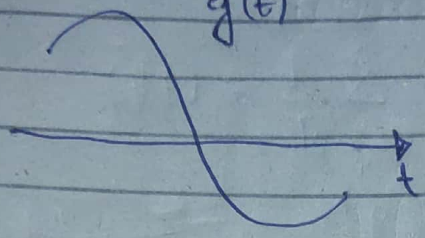
By increasing symbols, error rate also increases.

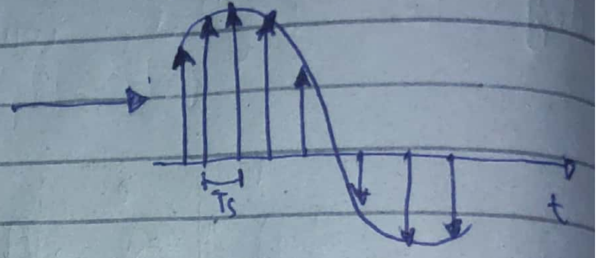
Digital Information:-

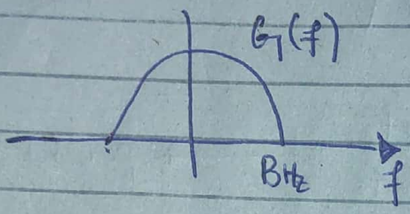
- can be stored
- can be compressed

★ **Sampling:-** $\boxed{f_s \geq 2B}$

↳ Then signal can be reconstructed (ideal channel)







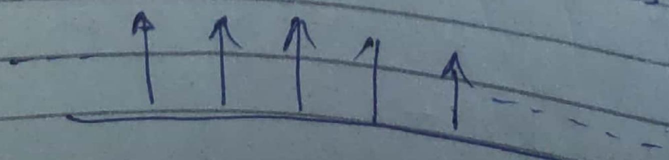
For ideal channel
$h(t) = \delta(t)$

$$g(t) \cdot \delta(t) = g(0)$$
$$g(t) \cdot \delta(t - T_s) = g(T_s)$$
$$g(t) \cdot \delta(t - nT_s) = g(nT_s)$$

$$g(t) \cdot \delta_{T_s}(t)$$

Now we can represent it as fourier series.

$$\delta_{T_s}(t) = \frac{1}{T_s}\left(1 + 2\cos \omega_s + 2\cos 2\omega_s t + + 2\cos 3\omega_s t + \ldots\right)$$



$$\delta_{T_s}(t) = \frac{1}{T_s}\sum_n e^{jn\omega_s t}$$

Nyquist Criteria :-

$$f_s = 2B$$

$$f_s = 1/T_s = 2B$$

$$\boxed{T_s = 1/2B}$$

Now we multiply $g(t)$ with $\delta_{T_s}(t)$.

$$\bar{g}(t) = g(t) \cdot \delta_{T_s}(t)$$

$$= g(t) \cdot \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} g(t) \cdot e^{jn\omega_s t}$$

or we can write as

$$\bar{g}(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} g(nT_s) \, \delta(t - nT_s)$$

$$g(t) \Longleftrightarrow G(f)$$

$$g(t) \cdot e^{jn\omega_s t} \Longleftrightarrow G(f - f_s)$$

$$g(t) \cdot e^{-jn\omega_s t} \Longleftrightarrow G(f + f_s)$$

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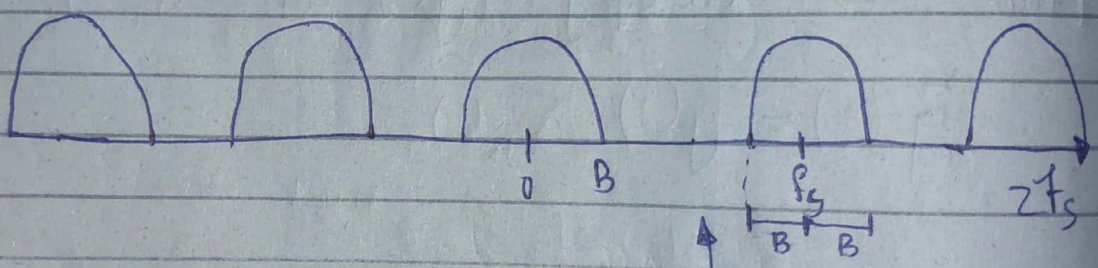
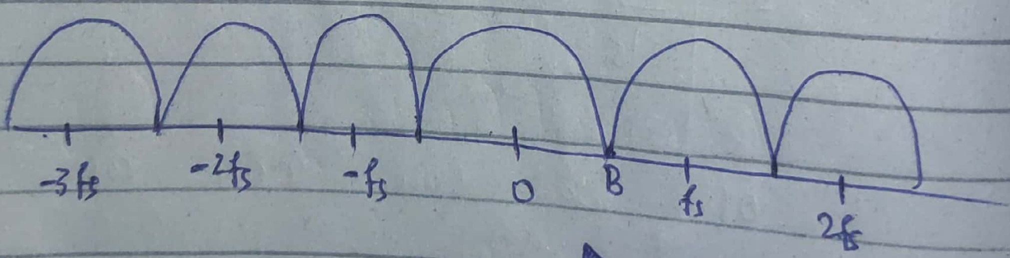
$$\boxed{\bar{g}(t) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} g(t)\, e^{jn\omega_s t}}$$

$$\bar{G}(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(f-nf_s)$$

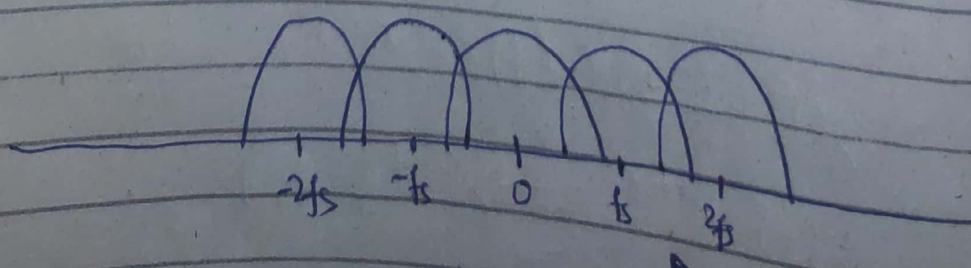
Now in frequency domain



$f_s > 2B$



$f_s = 2B$ Nyquist



$f_s < 2B$

Signal Reconstruction:-

Use Low Pass Filter

$$H(f) = T_s \, rect\left(\frac{f}{2B}\right)$$

or

$$H(\omega) = T_s \cdot rect\left(\frac{\omega}{4\pi B}\right)$$

From table,

$$\frac{W}{\pi} Sinc(Wt) \Longleftrightarrow rect\left(\frac{\omega}{2W}\right)$$

$$W = 2\pi B$$

$$\frac{2\pi B}{\pi} \cdot Sinc(2\pi Bt) \Longleftrightarrow rect\left(\frac{2\pi f}{4\pi B}\right)$$

$$2B \cdot Sinc(2\pi Bt) \Longleftrightarrow rect\left(\frac{f}{2B}\right)$$

Now $h(t) = T_s \cdot 2B \cdot Sinc(2\pi Bt)$

$\hookrightarrow$ $\boxed{\text{Low Pass Filter Response}}$

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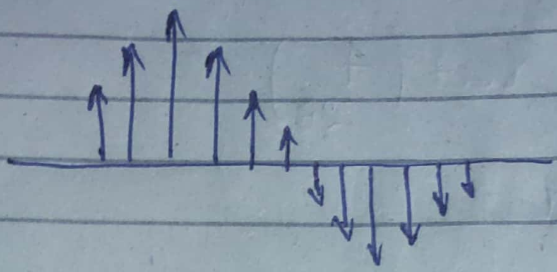
$$g(t) = \sum_{k=-\infty}^{\infty} g(kT_s)\,.\,h(t-kT_s)$$



Zero of Sinc :-

$$h(t) = 2BT_s \,\mathrm{Sinc}(2\pi Bt)$$

$$2\pi Bt = \pm n\pi$$

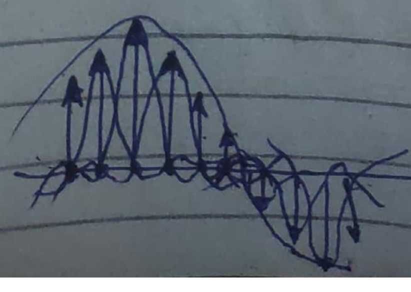
$$t = \frac{\pm n\pi}{2\pi B} = \pm n\,.\,\frac{1}{2B}$$

Interpolation Formula

$$g(t) = \sum_{k=-\infty}^{\infty} g(kT_s)\,.\,h(t-kT_s)$$

$$= \boxed{\sum_{k=-\infty}^{\infty} g(kT_s)\,.\,\mathrm{Sinc}\left(2\pi B\left(t-kT_s\right)\right)}$$

Nyquist



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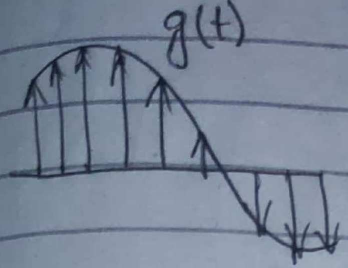


$f_s = 1/T_s$

$f_s \geq 2B$ Hertz

$$g(t) \cdot \delta(t) = g(0)$$

$$g(t) \cdot \delta_{T_s}(t)$$

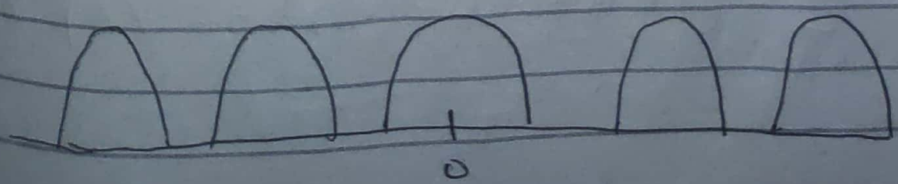
$$\bar{g}(t) = \sum_{n} g(kT_s) \cdot \delta(t - nT_s)$$

$$\bar{g}(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} g(t) \cdot e^{jn\omega_s t}$$

$$g(t) \Longleftrightarrow G(f)$$

$$g(t) \cdot e^{jn\omega_s t} \Longleftrightarrow G(f - nf_s)$$

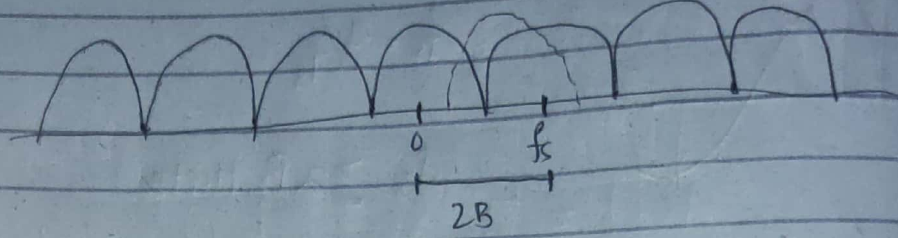
$$\bar{G}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$



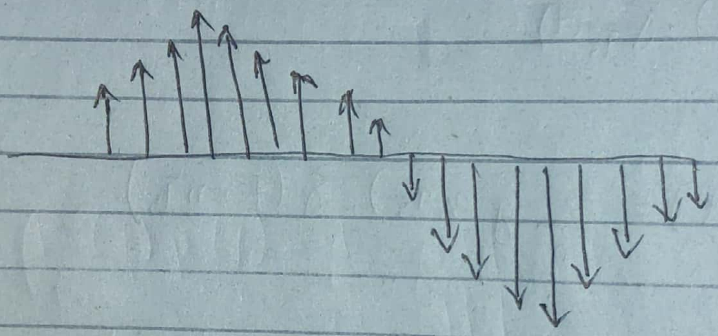
$f_s < 2B$

$f_s > 2B$ Hz

$f_s = 2B$ Hz Nyquist Criteria



$$\text{Sinc}(2\pi Bt) \Longleftrightarrow \text{rect}\left(\frac{f}{2B}\right)$$

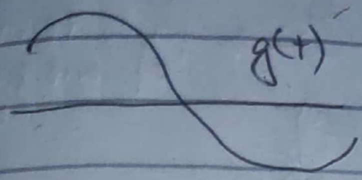


$$\sum_{K=-\infty}^{\infty} g(KT_s)\, h(t-KT_s)$$

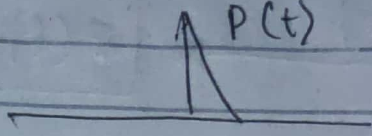
$$\sum_{K=-\infty}^{\infty} g(KT_s) \cdot \text{Sinc}\left(2\pi B\left(t-KT_s\right)\right)$$

$$\sum_{K=-\infty}^{\infty} g(KT_s) \cdot \text{Sinc}(2\pi Bt - K\pi)$$

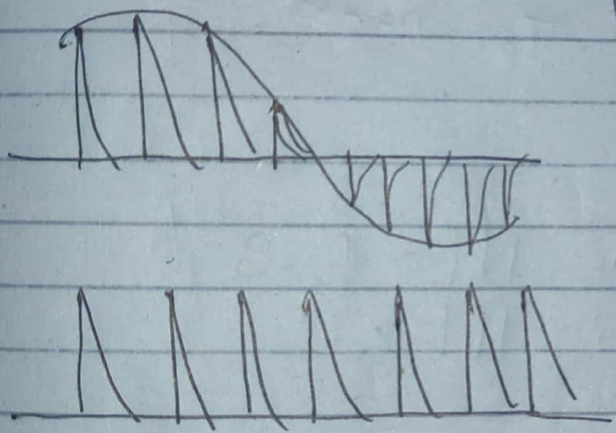
Practical Issues:-

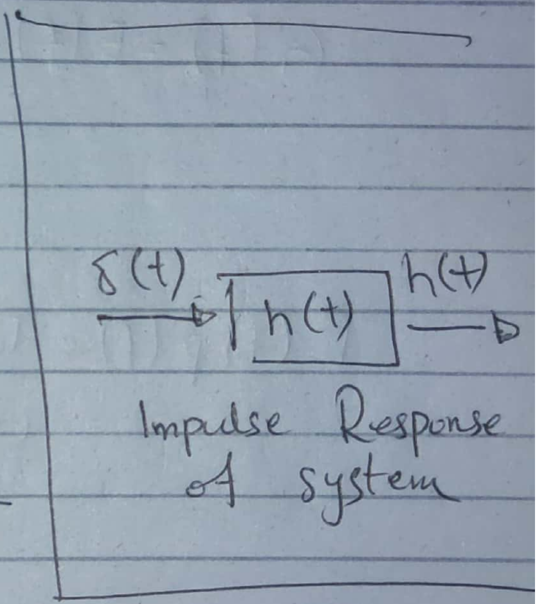


$$g(t).\delta(t) = g(0)$$



$$\tilde{g}(t) = \sum_{k=-\infty}^{\infty} g(kT_s).p(t-kT_s)$$



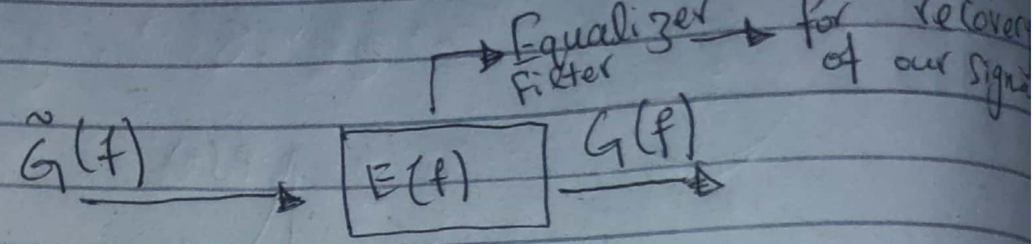


$$\tilde{g}(t) = p(t) * \underbrace{\sum_{k=-\infty}^{\infty} g(kT_s).\delta(t-kT_s)}_{\text{mathematical representation}}$$

$$\tilde{G}(f) = p(f) \cdot \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f-nf_s)]$$ Practical case

Now we want to recover our signal.

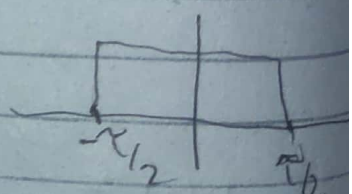
Reconstruction of our Signal :-

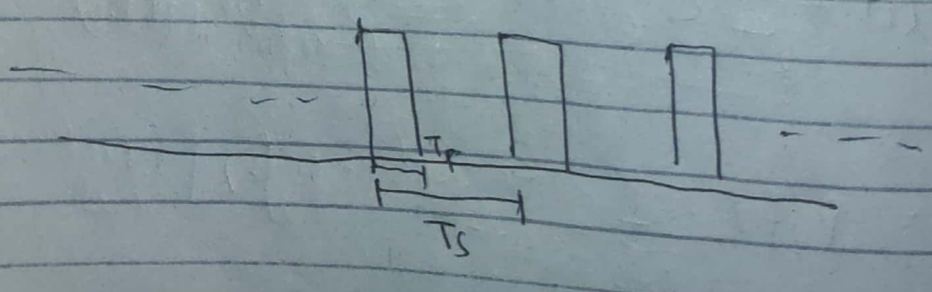


$$G(f) = E(f) \cdot \tilde{G}(f)$$

$$G(f) = E(f) \cdot \frac{P(f)}{T_s} \sum_{n=-\infty}^{\infty} G(f - n f_s)$$

$$E(f)P(f) = \begin{cases} T_s & |f| = f_s - B \\ 0 & |f| > B \end{cases}$$





$$\text{rect}\left(\frac{t - 0.5T_p - nT_s}{T_p}\right)$$

All signals are time limited.
Hence, it cannot be band limited

Both at same time not possible.

Oppossite of ~~above~~ statement 1
B possible.