Multi-Path Fading Channel

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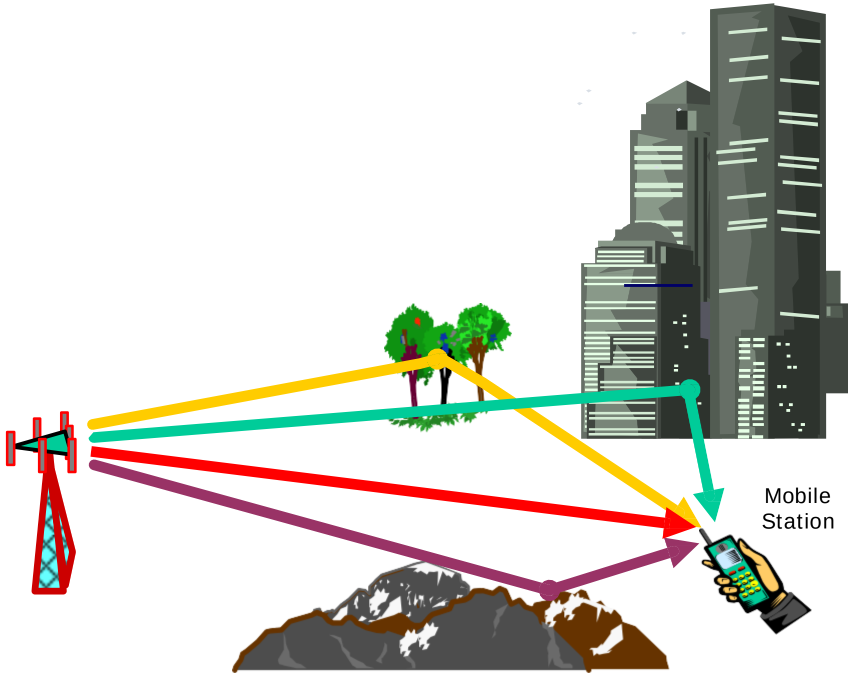
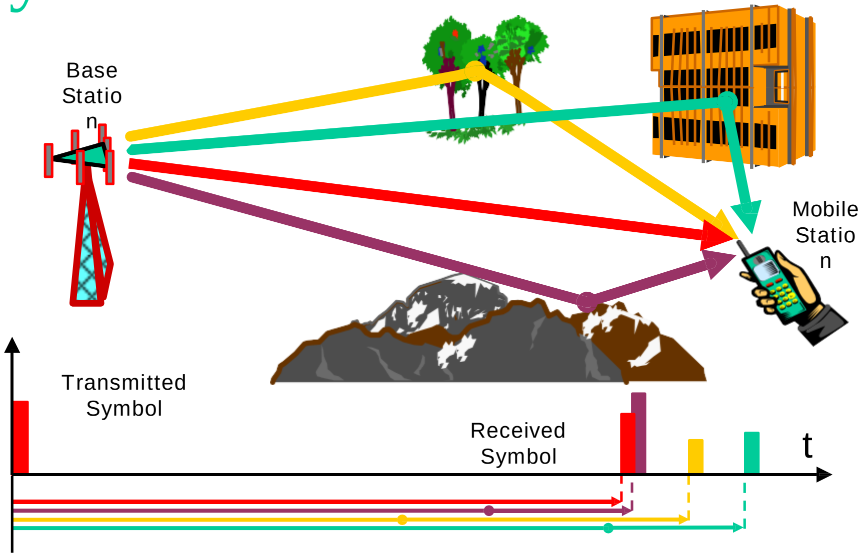


Mobile Channel Parameters

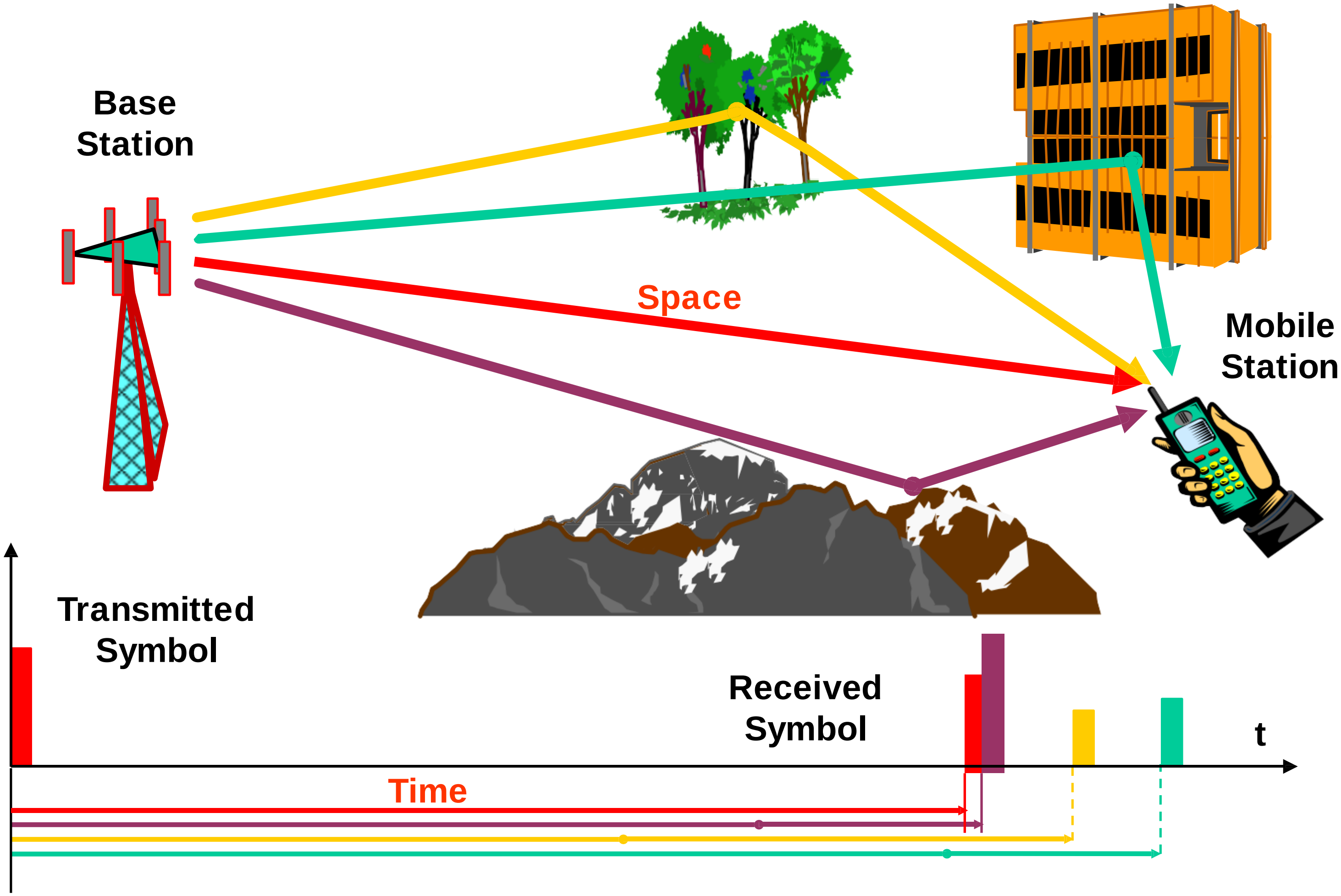
- Time delay spread |
- Coherence Bandwidth | -> ISI
- Doppler Spread |
- Coherence Time | -> Unstable channel
- Flat fading
- Frequency selective fading
- Fast fading
- Slow fading

Multi-path Propagation

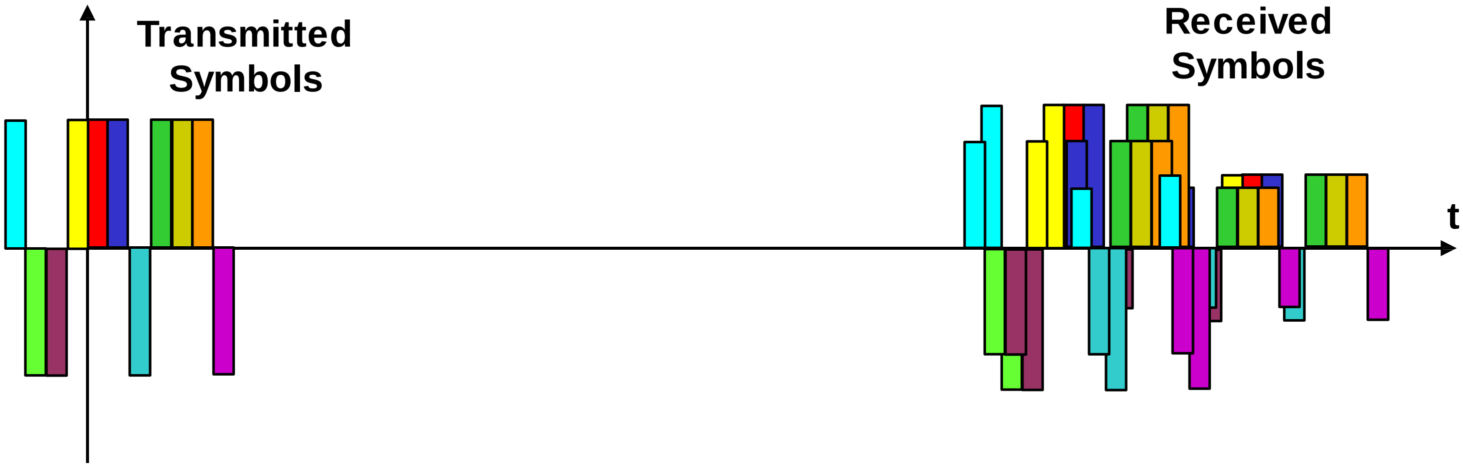
- Multi-path smears or spreads out the signal
  - delay spread
- Causes inter-symbol interference
  - limits the maximum symbol rate



Delay Spread



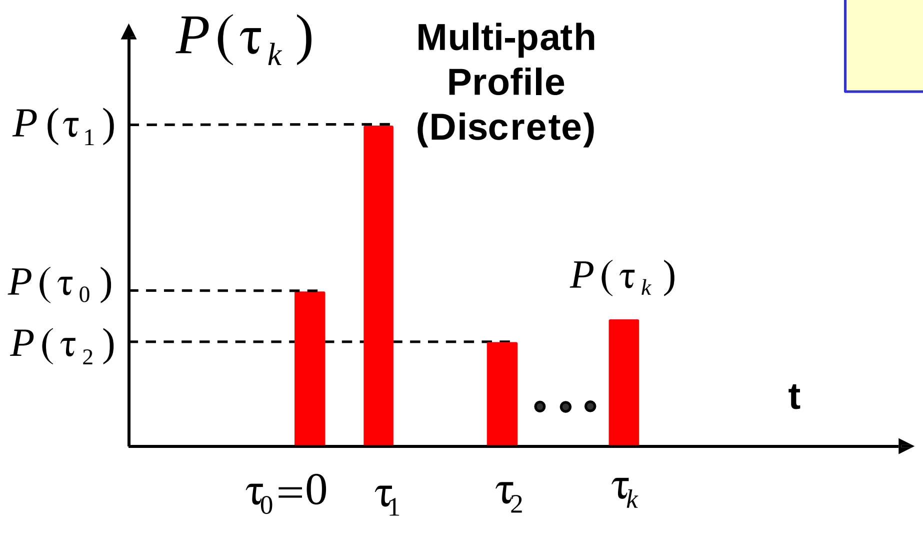
Intersymbol Interference



Average Delay Spread

- Average delay spread $\overline{\tau}$

$$\overline{\tau} = \frac{\sum_k |a_k|^2 \tau_k}{\sum_k |a_k|^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$



RMS Delay Spread (Discrete)

- RMS delay spread σ_τ

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$

$$\overline{\tau^2} = \frac{\sum_k |a_k|^2 \tau_k^2}{\sum_k |a_k|^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

Coherence Bandwidth

- Coherence bandwidth B_c is a range of frequencies over which the channel can be considered flat
  - passes all spectral components with approximately equal gain and liner phase
- Bandwidth where the correlation function $R_T(\omega)$ for signal envelopes is high
- Therefore two sinusoidal signals with frequencies that are farther apart than the coherence bandwidth will fade independently.

Coherence Bandwidth

- If $R_T(\omega) > 0.9$

$$B_C = \frac{1}{50\sigma_\tau}$$

- If $R_T(\omega) > 0.5$

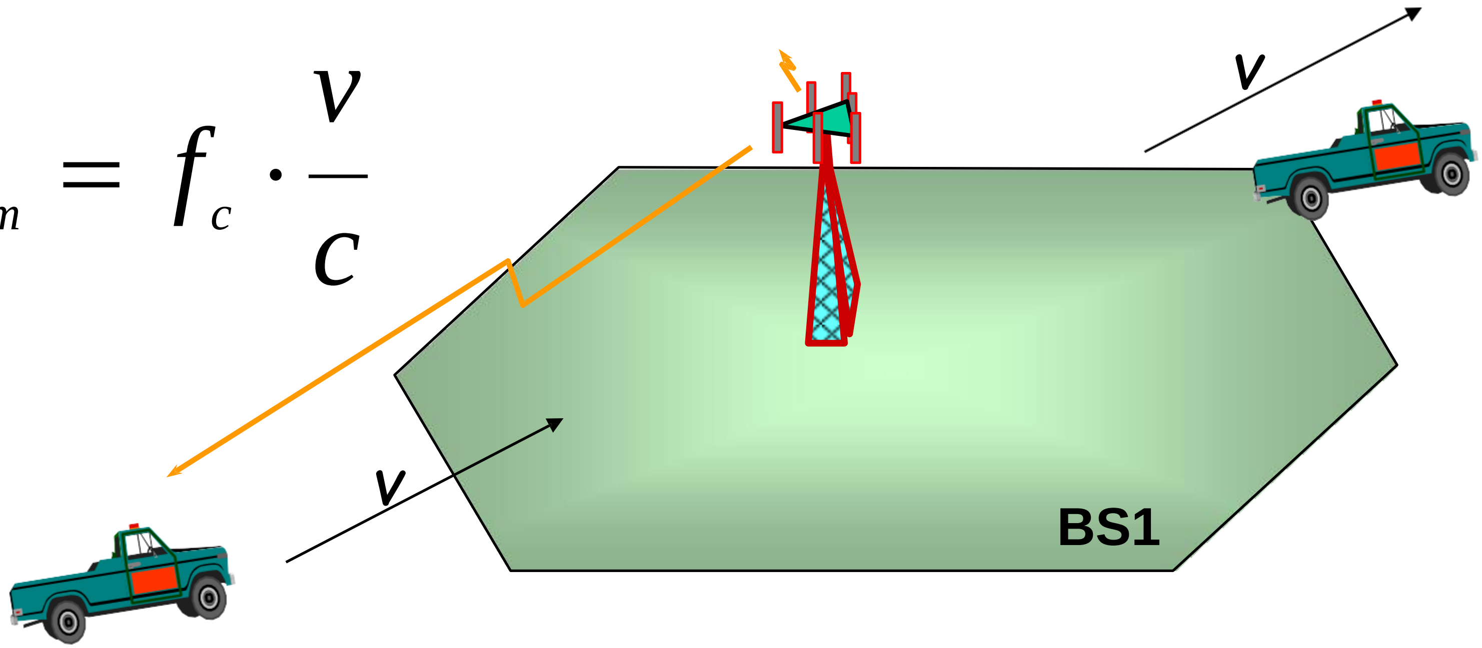
$$B_C = \frac{1}{5\sigma_\tau}$$

- An exact relationship between coherence bandwidth & delay spread does not exist

Doppler Shift

- f_c broadening from f_c to ($f_c + f_m$)

$$f_m = f_c \cdot \frac{v}{c}$$



Relativistic Doppler Frequency

The observed frequency is

$$f = f_c \cdot \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \qquad f_d = f - f_c \approx f_c \cdot \frac{v}{c}$$

where the relative velocity *v* is positive if the source is approaching and negative if receding.

f_c- carrier freq., c-speed of light, f_d-Doppler shift

Doppler Spread & Coherence Time

- Describes the time varying nature of the channel in a local area
- Doppler Spread B_D, is a measure of the spectral broadening caused by the time rate of change
- f_c broadening from $(f_c - f_m)$ to $(f_c + f_m)$
- If the base-band signal bandwidth is much greater than B_D, the effects of Doppler spread are negligible at the receiver

Coherence Time

- Coherence Time is the time domain dual of Doppler spread
- Doppler spread and coherence time are inversely proportional
- $T_C = 1/f_m$
- Statistical measure of the time duration over which the channel impulse response is invariant

Coherence Time

- If the coherence time is defined as the time over which the correlation function is above 0.5, then

$$T_C \approx \frac{9}{16\pi\ f_m}$$

- Rule of thumb for modern digital communication defines TC as the geometric mean of the above two expressions for TC

$$T_C = \sqrt{\frac{9}{16\pi\ f_m^2}}$$

Inter-symbol Interference

- For no Inter-symbol Interference the transmission rate R for a digital transmission is limited by delay spread and is represented by: $R < 1/2\sigma_\tau$;
- If $R > 1/2\sigma_\tau$ Inter-symbol Interference (ISI) occurs
- Need for ISI removal measures (Equalizers)

Types of Small-Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

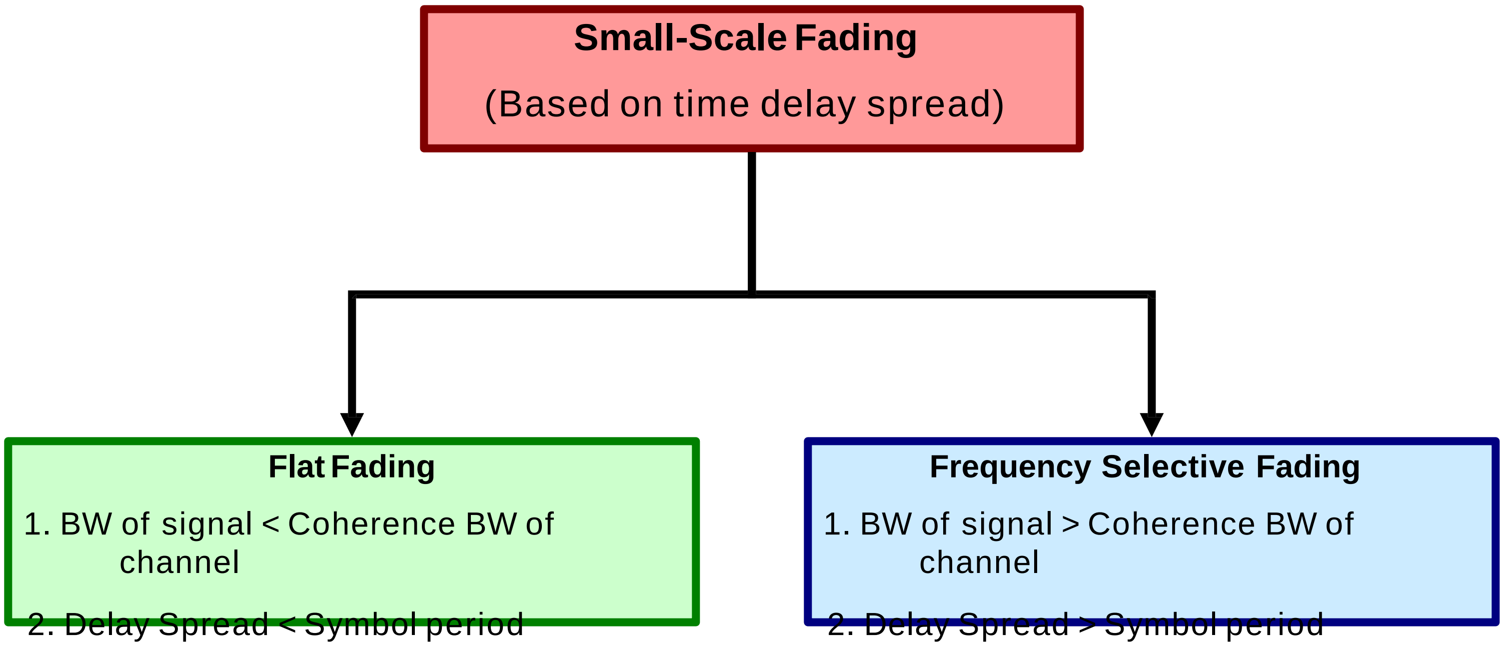
Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

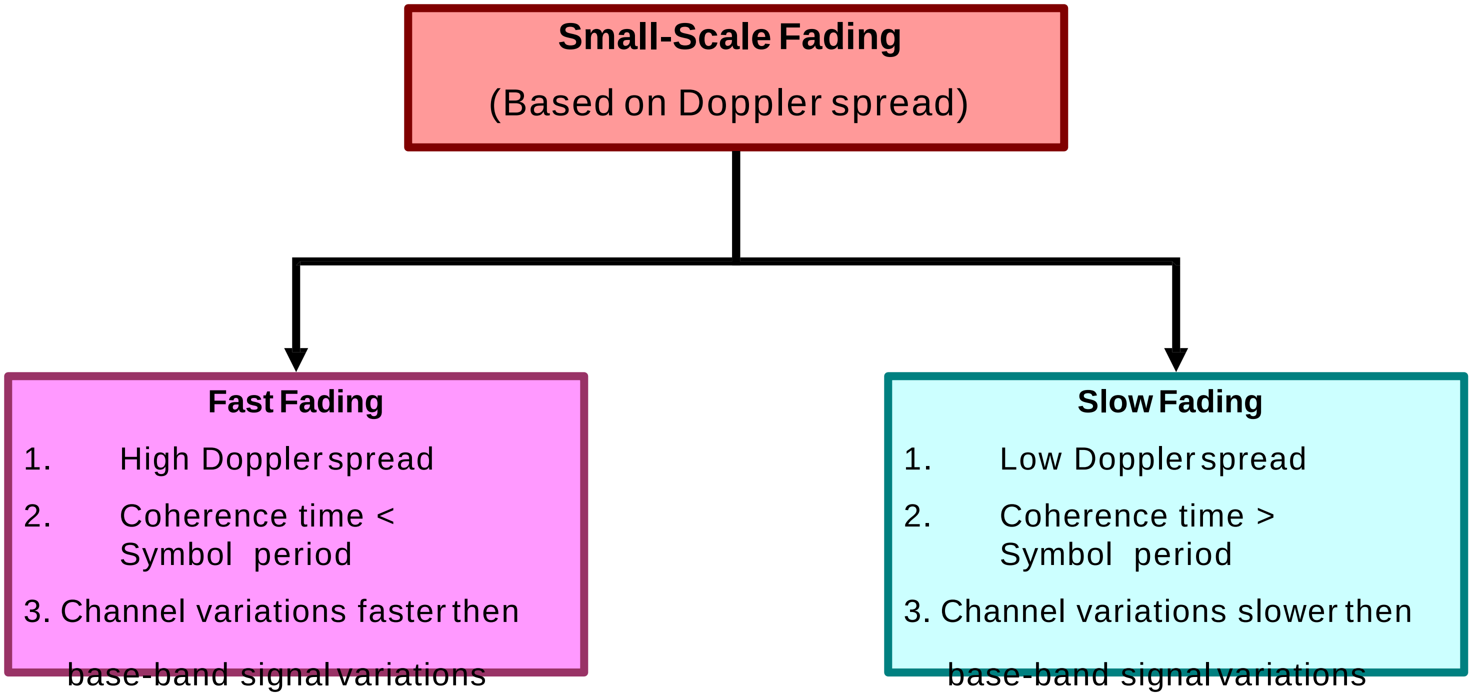
Slow Fading

1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Small-Scale Fading
Delay Spread



Small-Scale Fading
Time Variations

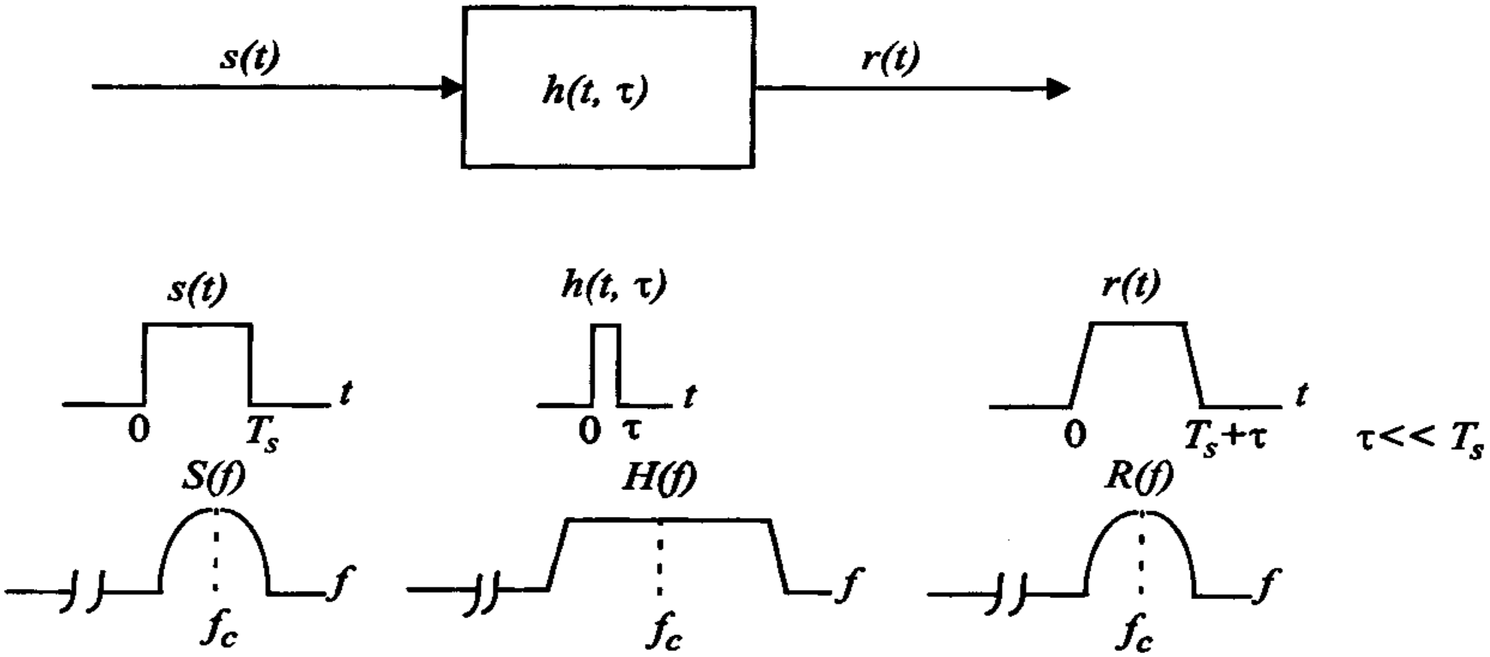


Flat Fading 1

- If the mobile radio channel has a constant gain and linear phase over a bandwidth ***greater*** than the bandwidth of the transmitted signal - the received signal will undergo ***flat fading***

- Please, observe that the fading is flat (or frequency selective) depending on the signal bandwidth relative to the channel coherence bandwidth.

Flat Fading 2

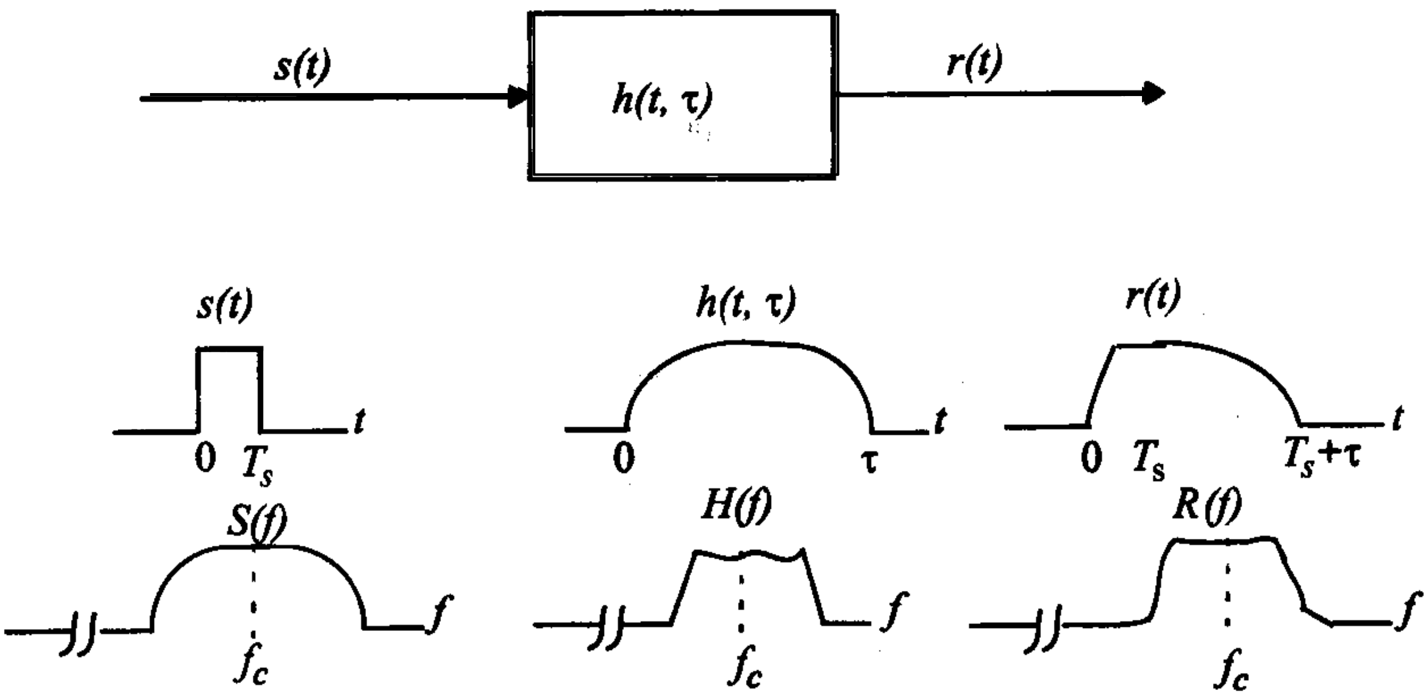


- $BS << B_C$ & $T_S >> \sigma_\tau$

Frequency Selective Fading 1

- If the mobile radio channel as a constant gain and linear phase over a coherence bandwidth, ***smaller*** than the bandwidth of the transmitted signal - the received signal will undergo *frequency selective fading*
- Again, the signal bandwidth is wider then the channel coherence bandwidth, causing one or more areas of attenuation of the signal within the signal bandwidth

Frequency Selective Fading 2



- $BS > B_C$ & $T_S < \sigma_\tau$

Fast Fading

- The channel impulse response changes rapidly within the symbol duration - coherence time < symbol period
- $T_S > T_c$ and $B_S < B_D$
- Channel specifies as a fast or slow fading channel does not specify whether the channel is flat fading or frequency selective fading

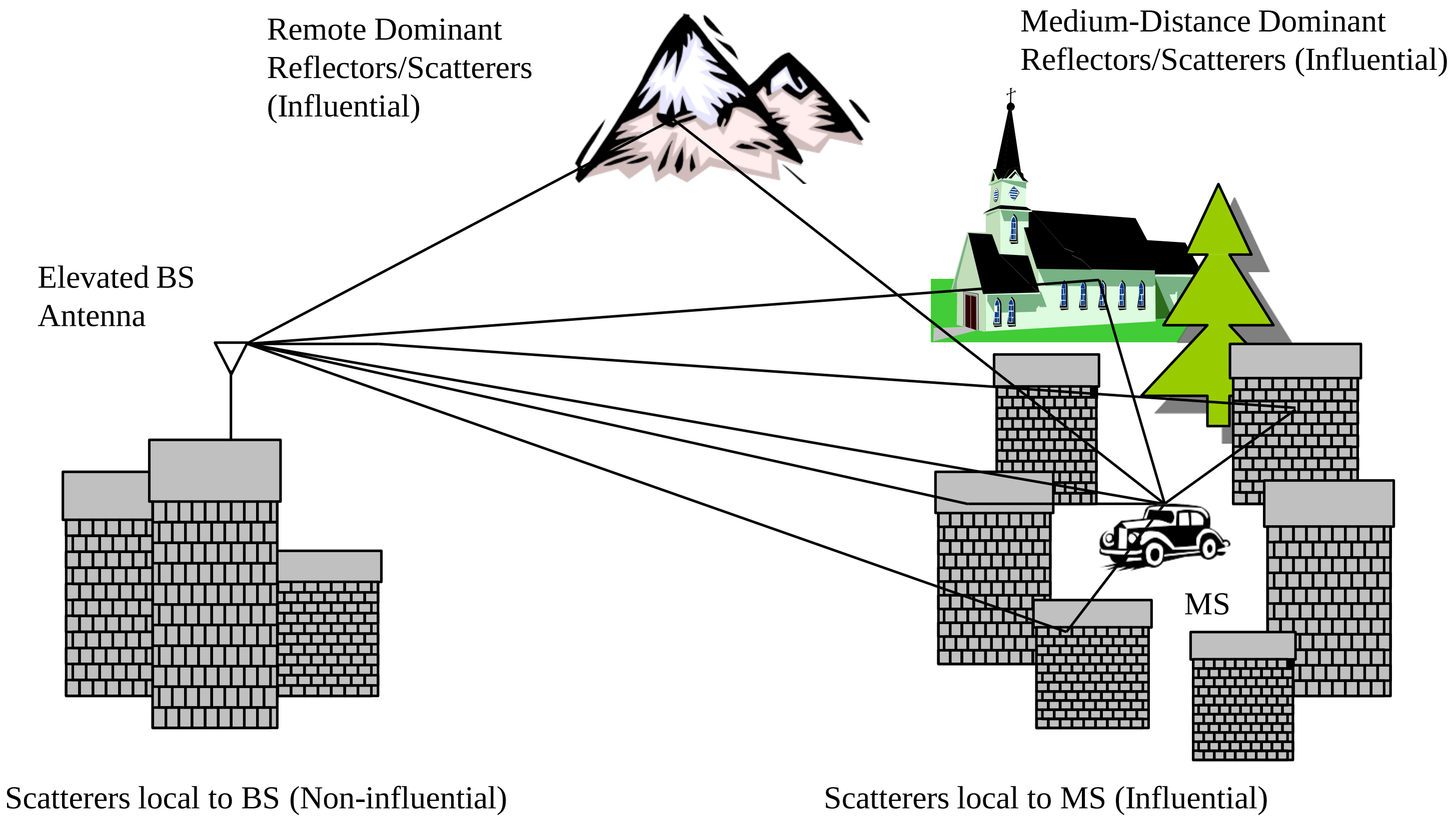
Slow Fading

- The channel impulse response changes at a rate much slower than the transmitted base-band signal.
- Doppler spread is much less than the bandwidth of the base-band signal
- $T_S << T_c$ and $B_S >> B_D$
- Velocity of the MS and the base-band signaling determines whether a signal undergoes fast or slow fading

Summary

- Fast and slow fading deal with the relationship between the time rate of change in the channel and the transmitted signal, NOT with propagation path loss models

Typical Cellular Mobile Environment



Fading

- Fading: The interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times
- Multipaths: Above mentioned versions of the transmitted signal

Fading (Continued)

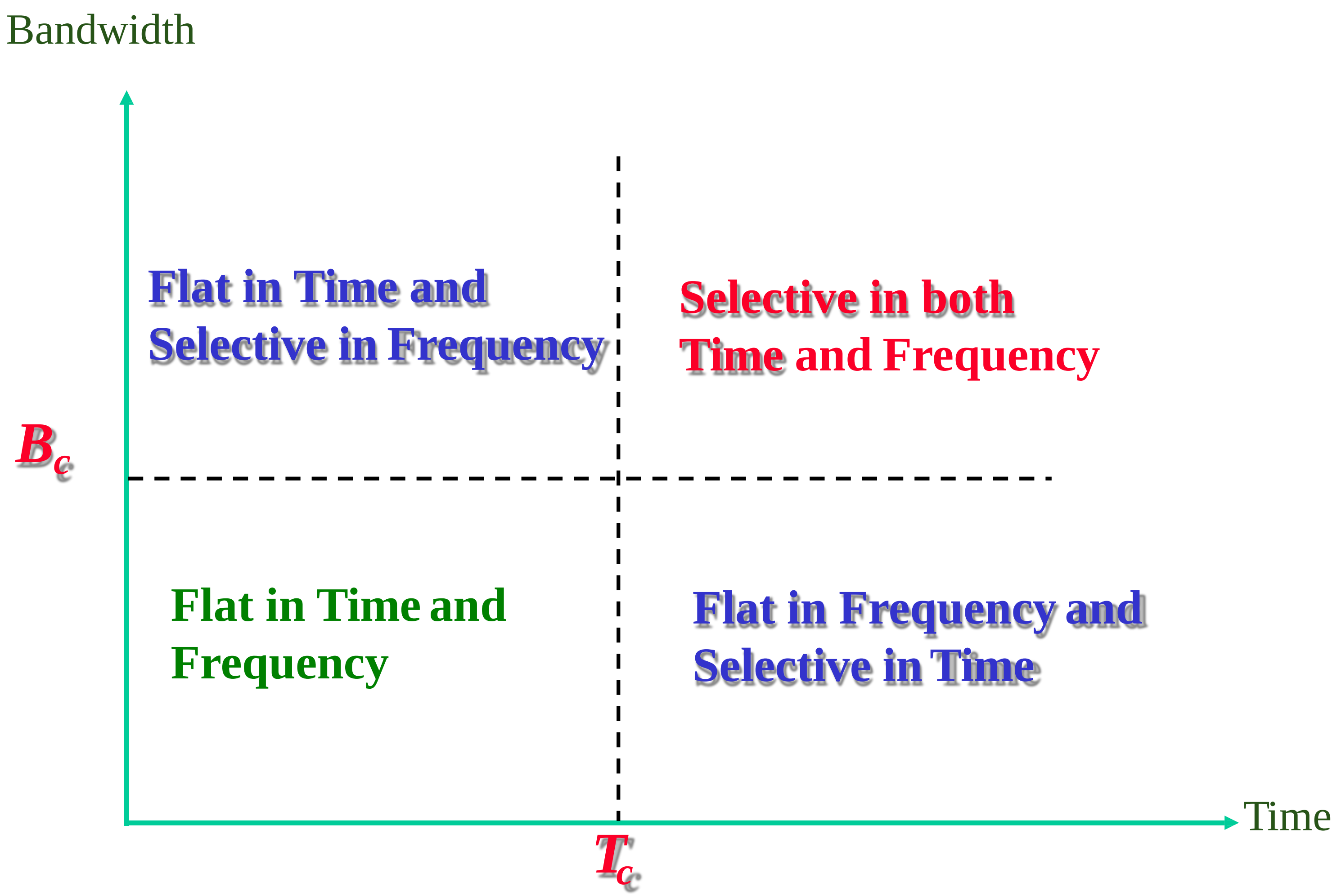
Delay Spread ←→ Coherence Bandwidth

Frequency separation at which two frequency components of Tx signal undergo independent attenuations

Doppler Spread ←→ Coherence Time

Time separation at which two time components of Tx signal undergo independent attenuations

Fading (Continued)



Fading (Continued)

Fast and Slow Fading

If the channel response changes within a symbol interval, then the channel is regarded FAST FADING

Otherwise

the channel is regarded as SLOW FADING

Fast Fading

When?
The channel impulse response changes rapidly within the symbol period of the transmitted signal.

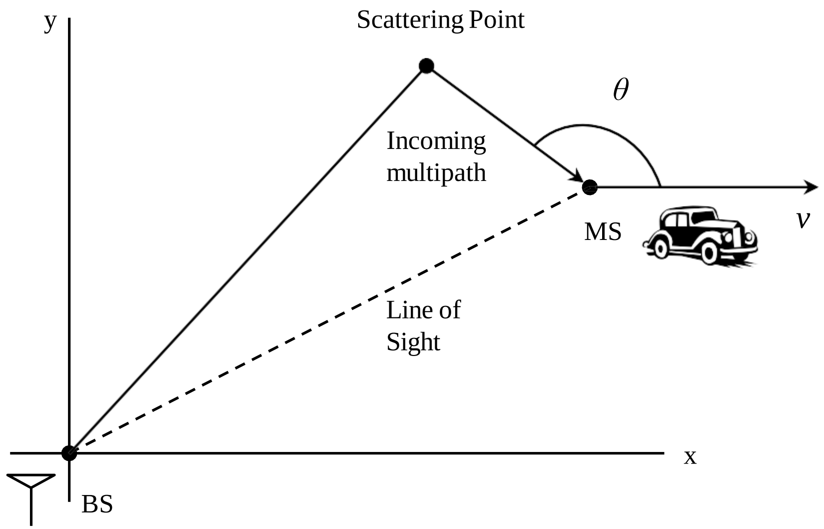
What?
The Doppler Spread causes frequency dispersion which leads to signal distortion.

Doppler Spread

The Doppler effect (in addition to the fading effect) renders the received pulse to be time-varying

The State Transitions are determined from the dynamics of the fading channel (Fading Correlation Function or The Doppler Spectrum)

Doppler Spread (Continued)



f: carrier frequency

c: speed of light

v: mobile speed

θ: Angle of motion with incoming multipath

Doppler Spread (Continued)

$$f_d = \frac{f\, v \cos\theta}{c}$$

f: carrier frequency

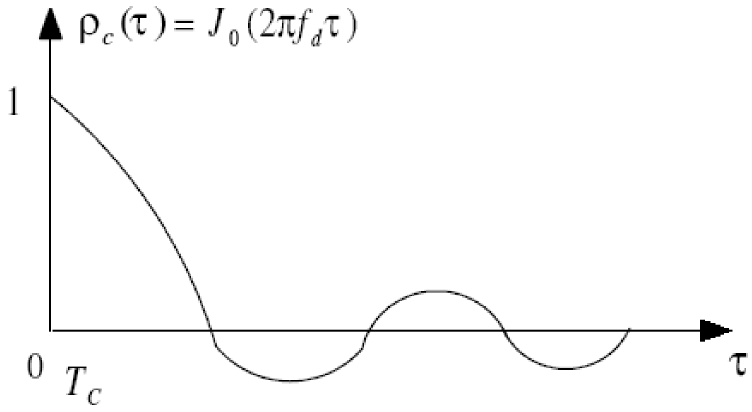
c: speed of light

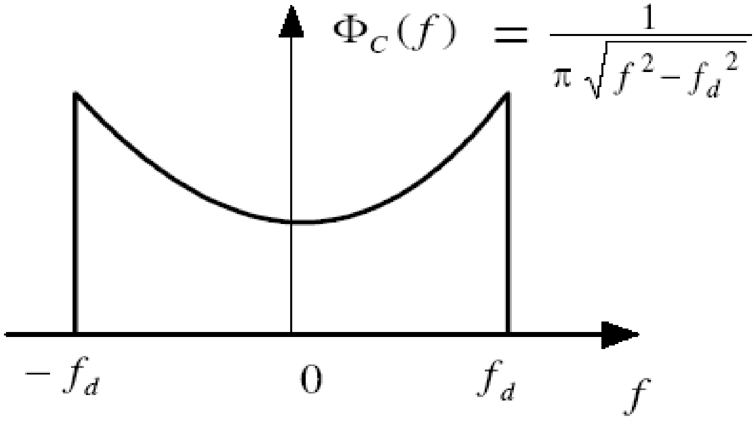
v: mobile speed

θ: Angle of motion with incoming multipath

Doppler Spread (Continued)

For the land mobile fading spectrum,

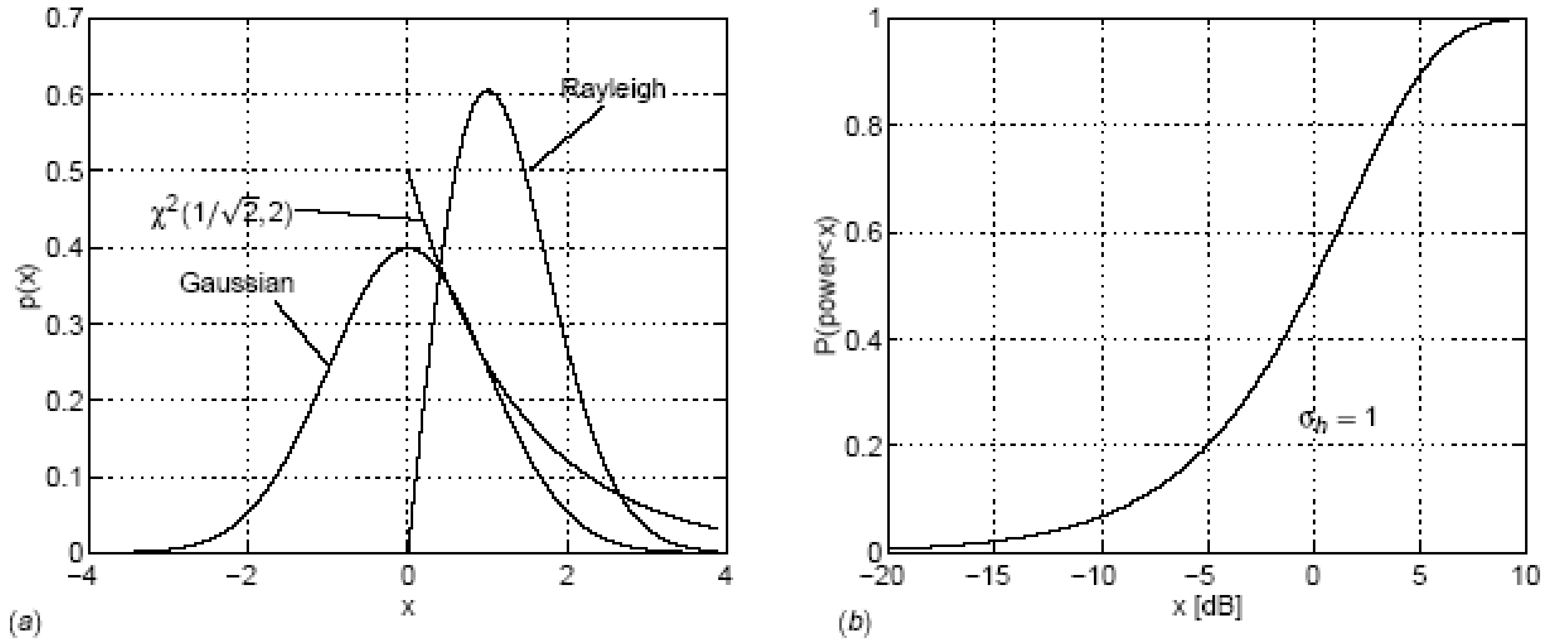




The Auto-Correlation Function Doppler Fading Spectrum

Doppler Spread (Continued)

- ***h*** **is the channel impulse response**
- ***h*** **has a complex normal distribution with zero mean**
- **|*h*| is Raleigh distributed**
- **Phase *φ* is uniformly distributed between 0 and 2*π***
- $|h|^2$ **is *Chi-square* distributed**



Fading in Brief

Large Doppler Spread

⇓

Time-Selective Fading

Large Delay Spread

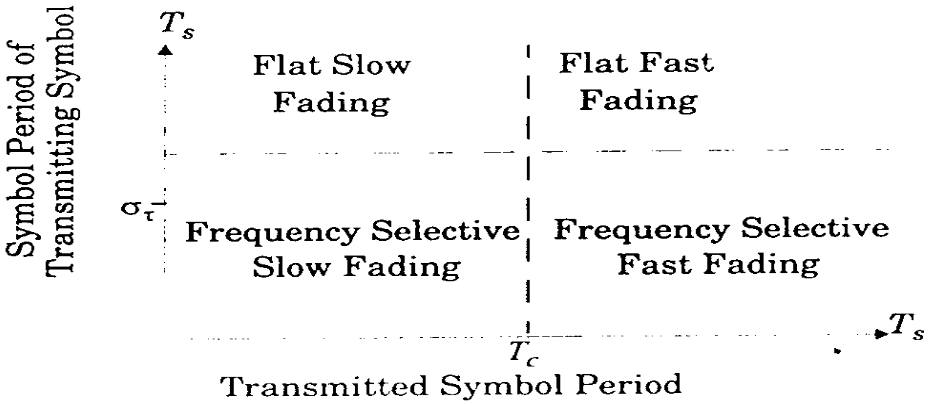
⇓

Frequency-Selective Fading

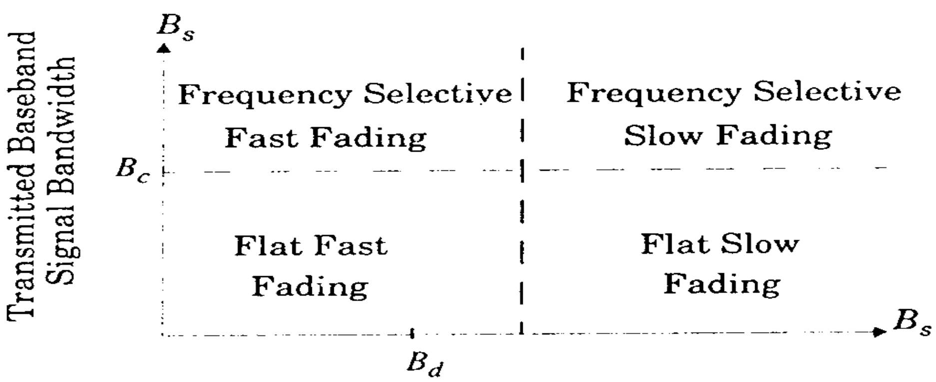
Large Angle Spread

⇓

Space-Selective Fading


T_s
Symbol Period of Transmitting Symbol
Flat Slow Fading
Flat Fast Fading
σ_τ
Frequency Selective Slow Fading
Frequency Selective Fast Fading
T_c
T_s
Transmitted Symbol Period

(a)


B_s
Transmitted Baseband Signal Bandwidth
Frequency Selective Fast Fading
Frequency Selective Slow Fading
B_c
Flat Fast Fading
Flat Slow Fading
B_d
B_s
Transmitted Baseband Signal Bandwidth

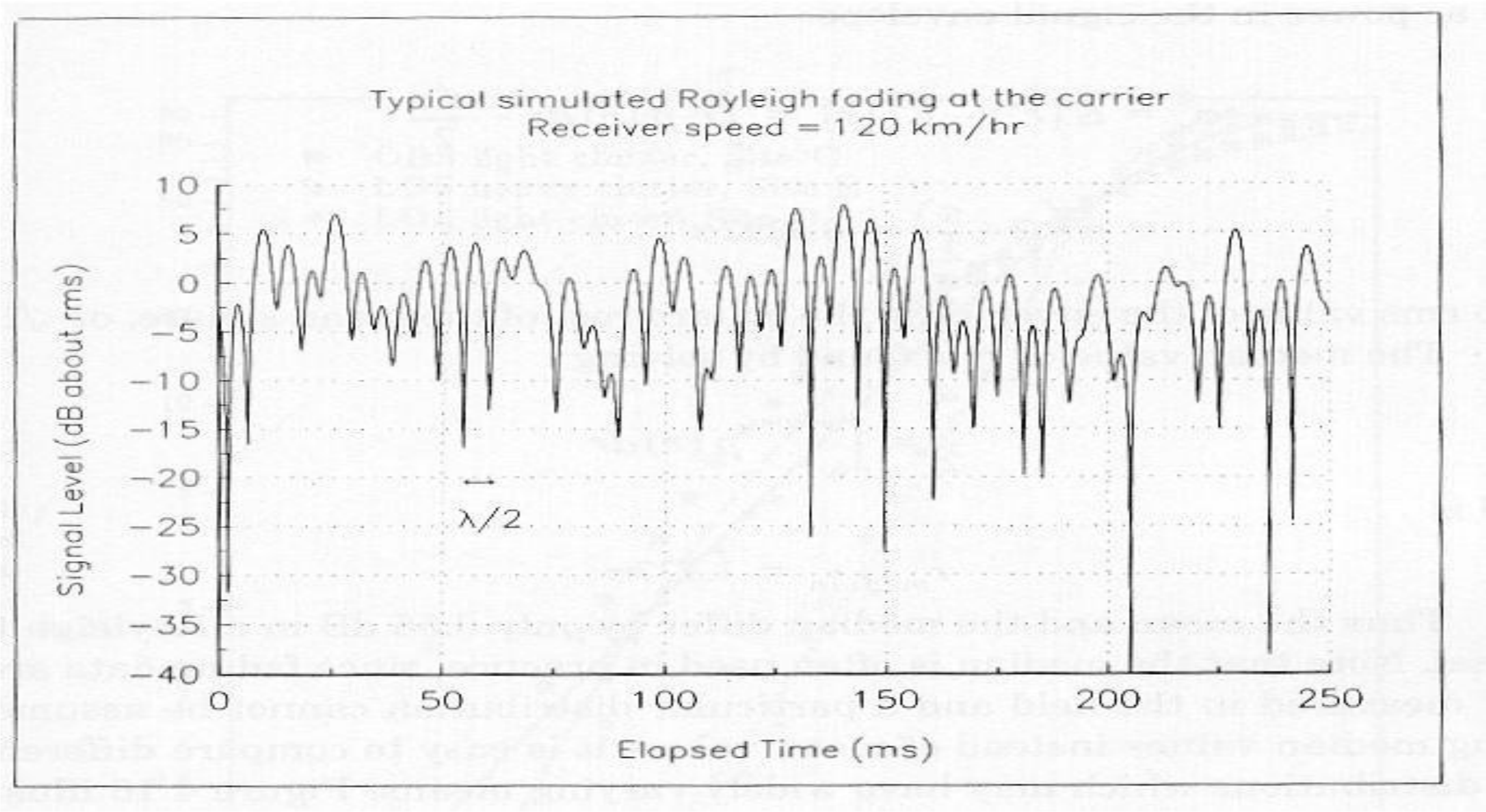
(b)

Rayleigh Fading 1

- The received envelope (amplitude) of a flat fading signalis described as a Rayleigh distribution
  - Square root sum ***r***, of two quadrature Gaussian noise signals $\boldsymbol{x_I}$ and $\boldsymbol{y_Q}$ has a Rayleigh distribution (Papoulis65)

$$r = \sqrt{x_I^2 + y_Q^2} \qquad p(r) = \left\{ \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2} \right); \ (0 \le r \le \infty) \right.$$

Rayleigh Fading 2



Rayleigh Fading PDF

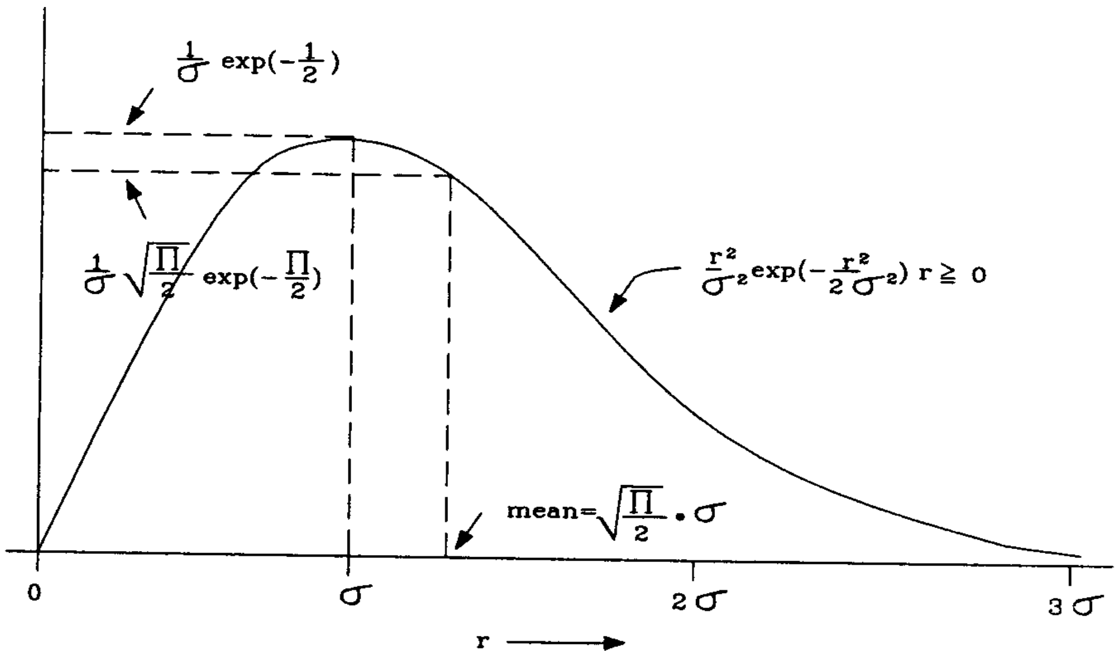


Figure 1.2: Rayleigh PDF.

Rayleigh Fading 3

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{r^2}{2\sigma^2} \right) \end{cases} \qquad (0 \le r \le \infty)$$

- σ - rms value of the received voltage signal before envelope detection
- σ^2 - time average power before envelope detection
- The probability that the received signal envelope does not exceed R is given by:

$$P(R) = \Pr(r \le R) = \int_0^R p(r)\,dr = 1 - \exp\left(-\frac{R^2}{2\sigma^2} \right)$$

Rayleigh Fading 4

- The median value of *r* is found by solving

$$\frac{1}{2} = \int_0^{r_{median}} p(r)dr$$

$$r_{median} = 1.77\sigma$$

- Mean and median differ by only 0.55dB

Ricean Fading 1

- When there is a dominant stationary signal component
- At the output of an envelope detector - adding a DC component ot the random multi-path

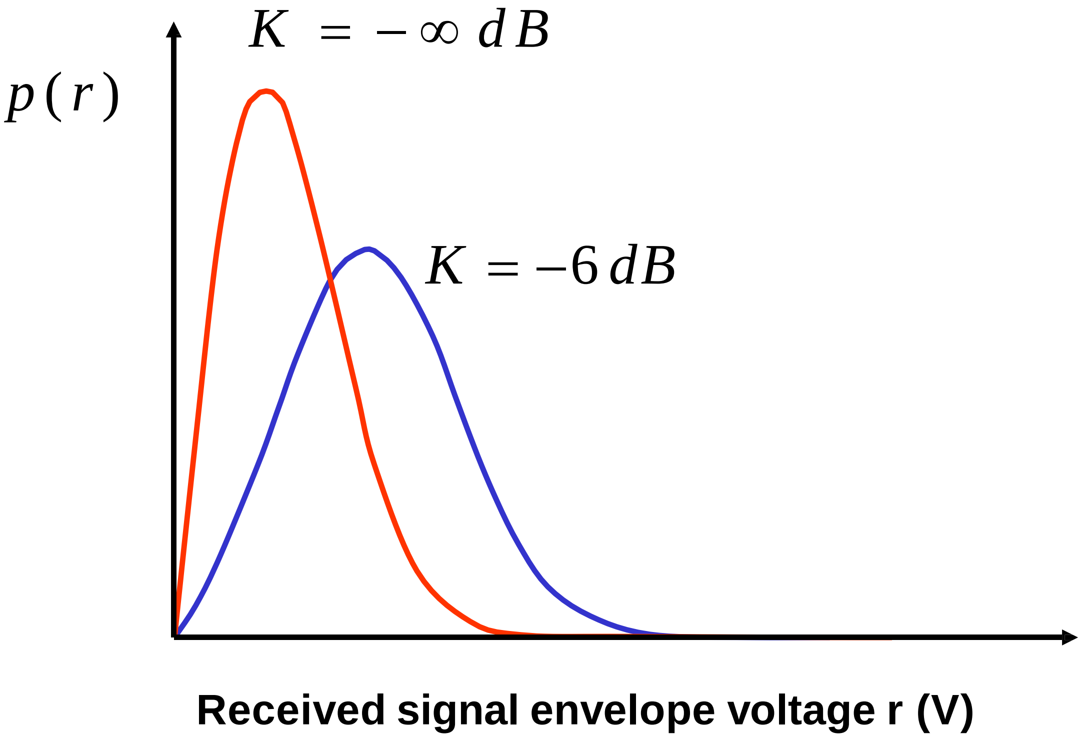
$$p(r) = \frac{r}{\sigma^e} e^{-\frac{(r^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right); \qquad for \quad (A \geq 0, r \geq 0)$$

Ricean Fading 2

- A - peak amplitude of the dominant signal
- $I_0()$ - modified Bessel function of the first kind and zero order
- Described in terms of a Ricean factor, K

$$K(dB) = 10\log \frac{A^2}{2\sigma^2}(dB)$$

Ricean PDF



Clarks Model for Flat Fading 1

- Statistical Characteristics of the EM fields of the received signal at the MS are obtained from scattering
- Assumes
  - Fixed transmitter & vertically polarized antenna
  - Fields incident on the mobile antenna comprises of N waves in azimuth plane with arbitrary carrier phases and azimuth angels of arrival
  - equal average signal amplitude

Clarks Model for Flat Fading 2

- The model shows that the random received signal envelope *r* has a Rayleigh distribution and is given by:

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2} \right); \qquad 0 < r \leq \infty$$

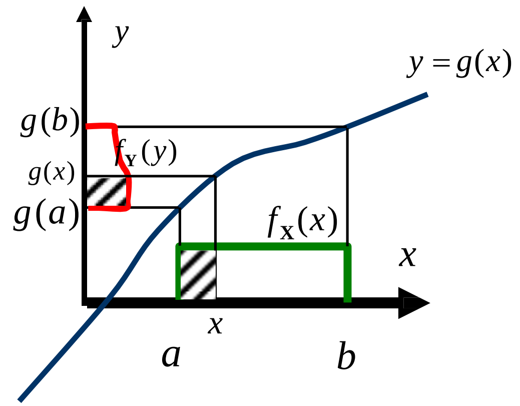
Effect of Doppler Spread

- It can be shown that if the angle of the received signals, α_i is uniformly distributed that the Doppler frequency has a random cosine distribution.
- Then the Doppler power spectral density *S(f)* can be computed by equating the incident received power in an angle $d\alpha$ with Doppler power *S(f)df*
  - *df* is found by differentiating the Doppler term $f_m cos\alpha$ wrt α.

Function of One Random Variable

x - random variable

$$\int_a^b f_X(x)\, dx = 1$$



y =g(x); function of x

y - random variable

$$\int_{g(a)}^{g(b)} f_Y(y)\, dy = 1 \qquad \int_{g(a)}^{g(b)} f_Y(y)\, dy = 1$$

$$f_Y(y)\, dy = f_X(x)\, dx$$

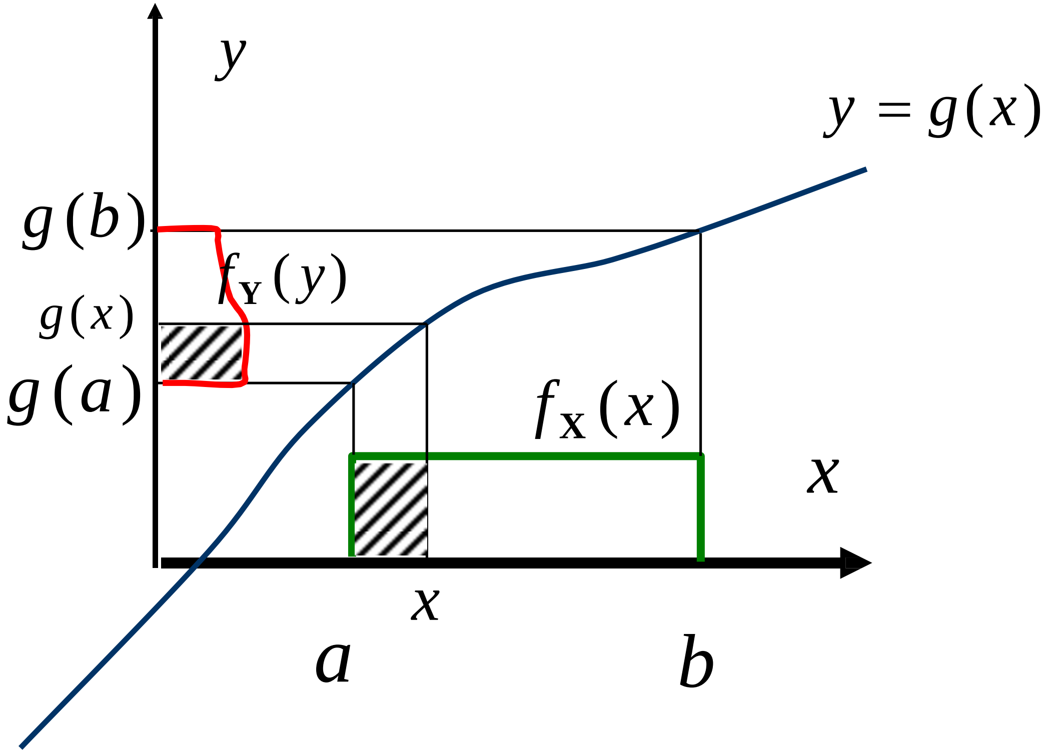
Function of One Random Variable

x - random variable

$$\int_a^b f_{\mathbf{X}}(x)\ dx = 1$$

y =g(x); function of x

y - random variable



$$f_{\mathbf{Y}}(y)\, dy = f_{\mathbf{X}}(x)\, dx$$

Function of One Random Variable

y - random variable

$$y\Big|_{g(a)}^{g(b)}$$

$$\int_{g(a)}^{g(b)} f_{\mathbf{Y}}(\ y)\ dy = 1$$

y = g(x); function of x

$$\int_{g(a)}^{g(b)} f_{\mathbf{Y}}(\ y)\ dy = 1$$

y = g(x); substitution in

$$f_{\mathbf{Y}}(y) = \ f_{\mathbf{Y}}(g(x)); \quad dy = g'(x)\,dx; \qquad y\Big|_{g(a)}^{g(b)} \Rightarrow \ x\Big|_a^b$$

$$\int_a^b f_{\mathbf{Y}}(g(x))g'(x)\,dx \ = \int_a^b \ f_{\mathbf{X}}\ (x)dx = 1$$

$$\int_a^{g(x)} \ f_{\mathbf{Y}}(g(x))g'(x)\ dx = \int_a^x f_{\mathbf{X}}\ (x)dx$$

Function of One Random Variable

$$\int_a^{g(x)} f_{\mathbf{Y}}(g(x))g'(x)\,dx = \int_a^{x} f_{\mathbf{X}}(x)dx$$

$$f_{\mathbf{Y}}(g(x))g'(x) = f_{\mathbf{X}}(x)$$

$$f_{\mathbf{Y}}(y)g'(x) = f_{\mathbf{X}}(x)$$

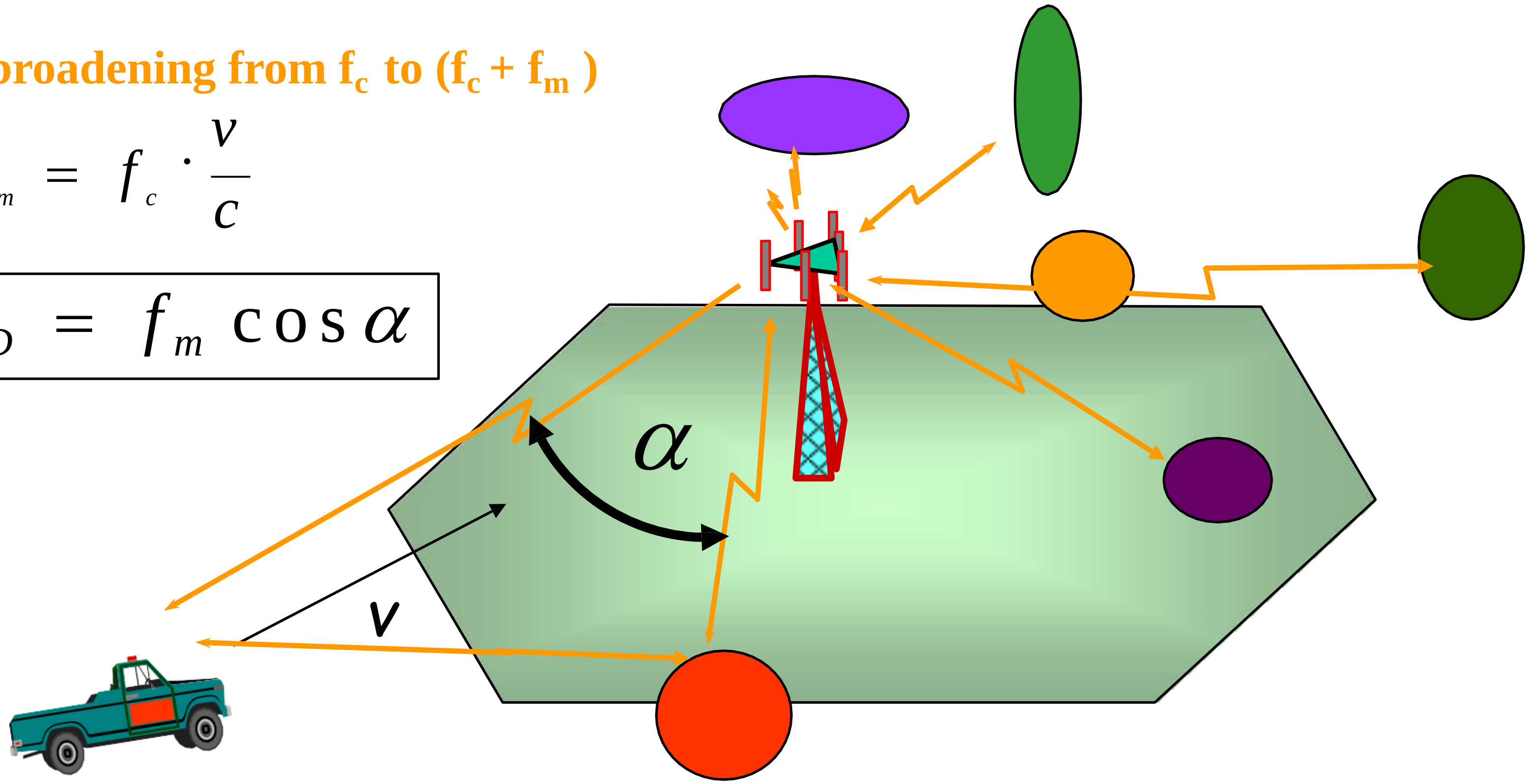
$$\boxed{f_{\mathbf{Y}}(y) = \frac{f_{\mathbf{X}}(x)}{|g'(x)|}}$$

Doppler Shift

- f_c broadening from f_c to $(f_c + f_m)$

$$f_m = f_c \cdot \frac{v}{c}$$

$$\boxed{f_D = f_m \cos \alpha}$$



Effect of Doppler Spread

$f = f_m \cos\alpha$ α - uniformly distributed (0,2π)

$$S_{\mathbf{f}}(f) = \frac{S_{\boldsymbol{\alpha}}(\alpha)}{\left|\left(f_m \cos\alpha\right)'\right|}$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha}$$

$$S_{\mathbf{f}}(f) = \frac{1}{2\pi f_m \sin\alpha}$$

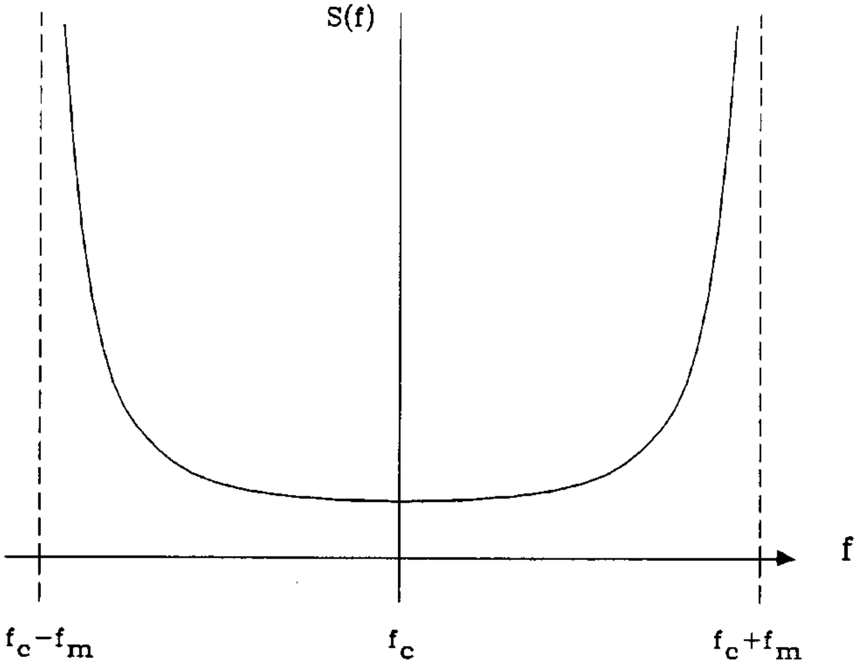
$$\cos\alpha = \frac{f}{f_m}$$

$$S_{\mathbf{f}}(f) = \frac{1}{2\pi f_m \sqrt{1-\frac{f^2}{f_m^2}}}$$

Doppler Spectrum

- the incident received power at the MS depends on the power gain of the antenna and the polarization used

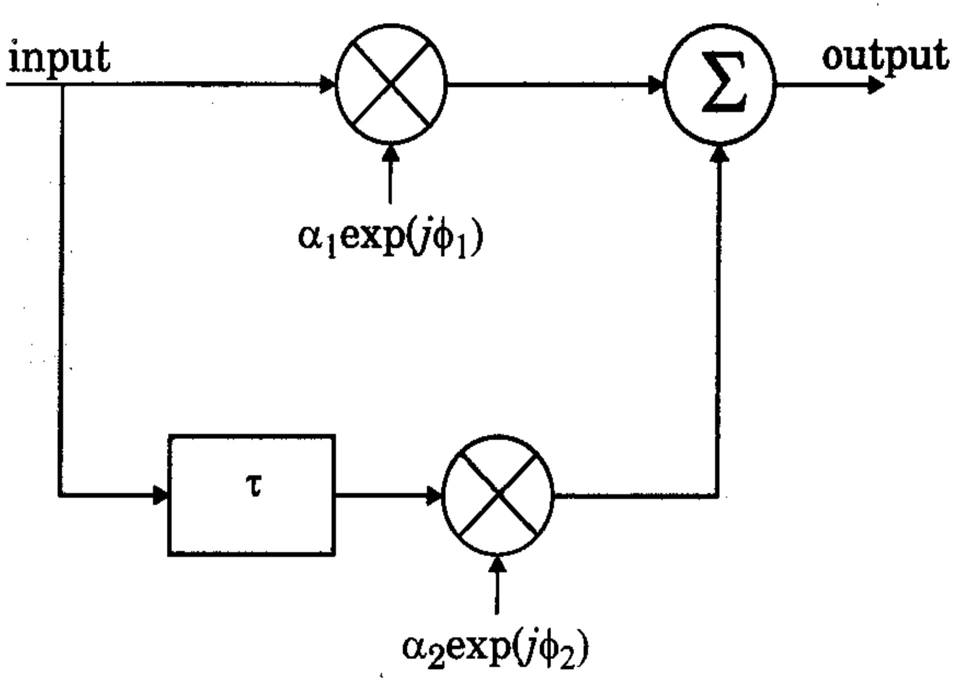
$$S(f) = \frac{A}{\sqrt{1-(f/f_m)^2}}$$



Two-ray Rayleigh Fading Model

- Clarke's model for flat fading
- It is necessary to model multi-path delay spread as well
- Commonly used model is the two-ray model

Two-ray Rayleigh Fading Model



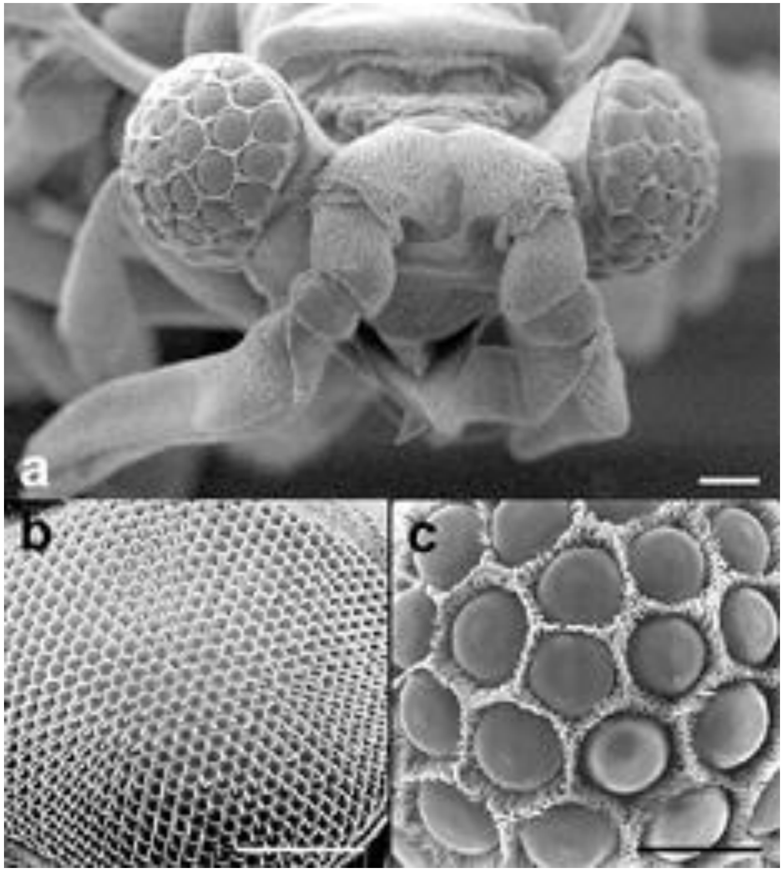
Two-ray Rayleigh Fading Model

- The impulse response of the model

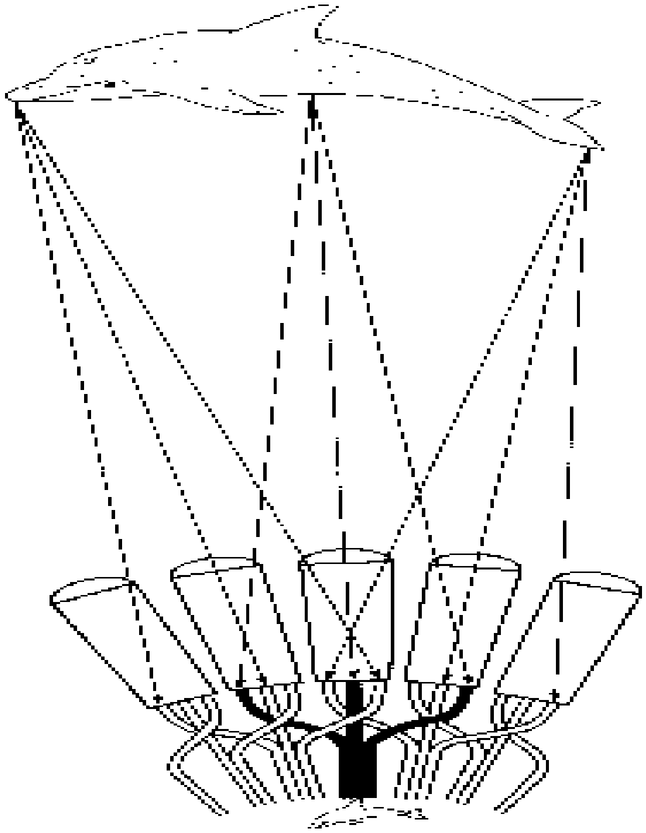
$$h_b = \alpha_1 \exp(j\phi_1)\delta(t) + \alpha_1 \exp(j\phi_2)\delta(t-\tau)$$

- $\alpha_{.1}$ and $\alpha_{.2}$ are independent and Rayleigh distributed
- ϕ_1 and ϕ_1 are independent and uniformly distributed over [0,2π]
- τ - time delay between the two rays
- By varying τ it is possible to create a wide range of frequency selective fading effects

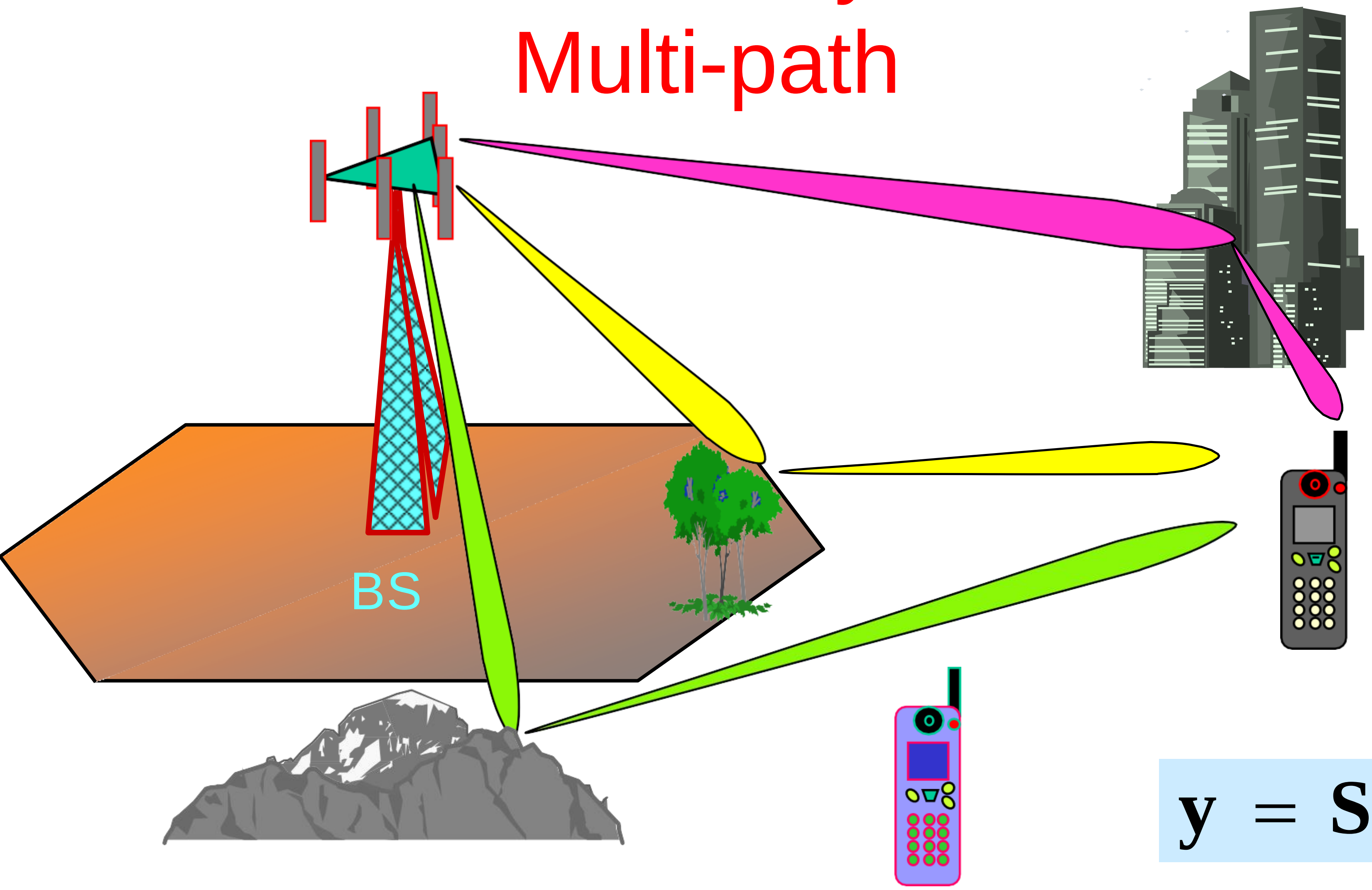
Beyond Current Engineering Practice



Antenna Arrays are Electromagnetic Eyes



Smart Antenna System with Multi-path



$$\mathbf{y} = \mathbf{S}\,\mathbf{x} + \mathbf{n}$$

Multi-user System Model

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{n}$$

$$E\{\mathbf{n}\mathbf{n}^H\} = \sigma^2 \mathbf{I}$$

- **y** - received signal(N X 1)dimensional vector
- **S** - signature matrix (N X K) dimensional matrix
- **x** - transmitted symbols (K X 1) dimensional vector
- **n** - Gaussian noise (N X 1) dimensional vector
- N - Number of antenna elements
- K - Number of **Users**

Multi-user System Information Capacity

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{n}$$

$$C = \log_2 \left| I + \frac{\mathbf{S} V_x \mathbf{S}^H}{\sigma_n^2} \right|$$

$$\mathbf{V}_{\mathbf{x}} = E\left\{ \mathbf{x}\mathbf{x}^H \right\} = P\mathbf{I} - \text{Signal symbol power}$$

$$\sigma_{\mathbf{n}}^2 - \text{Noise } \mathbf{n} \text{ (variance) power}$$

$$\boxed{C = \log_2 \left(1 + \frac{P}{\sigma_n^2} \right)}$$

Multi-Path Fading Channel

Multiuser Spatial Filter

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{n}$$

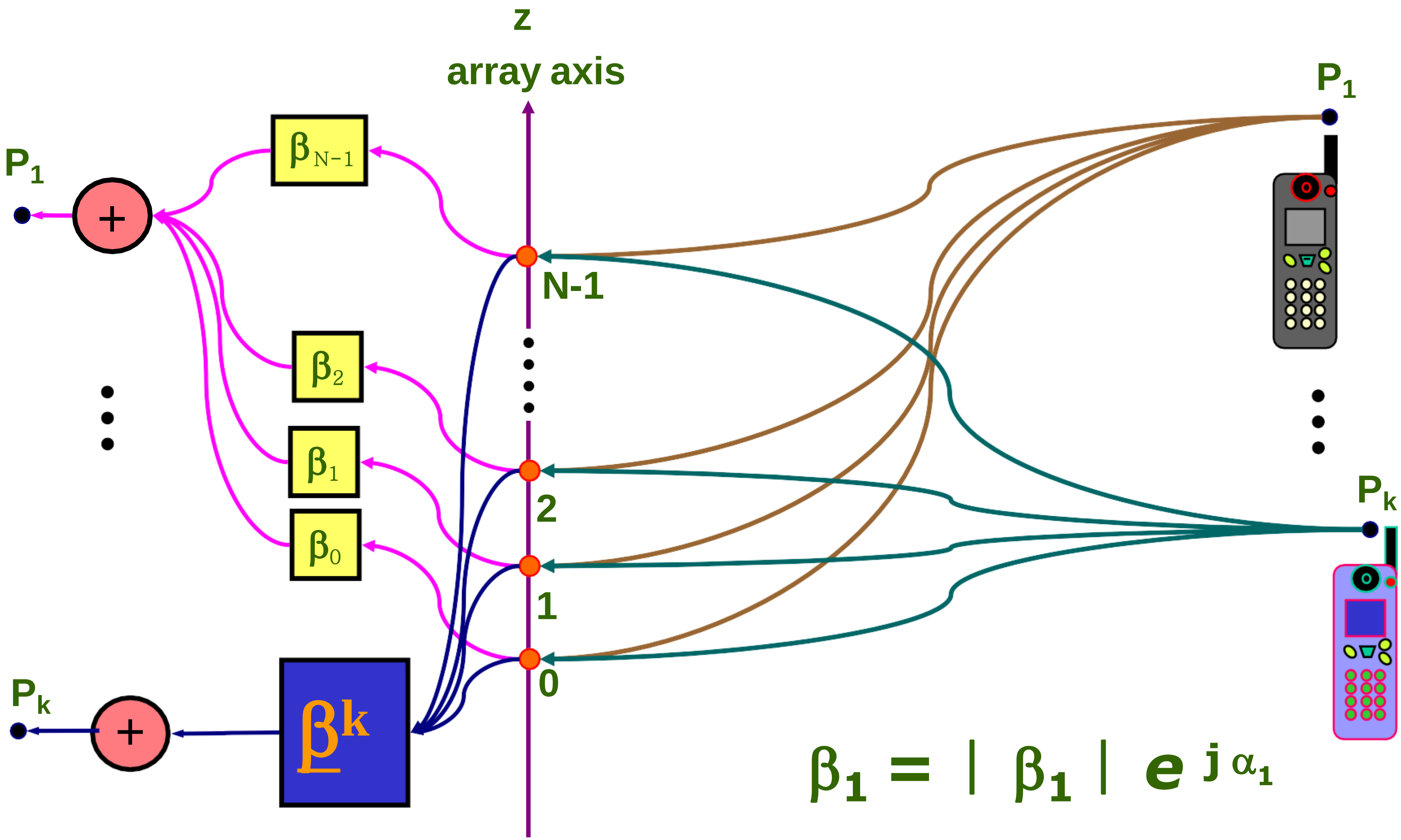
$$\boldsymbol{\beta} = (\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H$$

Moore-Penrose Pseudo Inverse

Optimum Spatial Filter

$$\begin{aligned}\hat{\mathbf{x}} &= (\mathbf{S}^H \mathbf{S})^{-1}\mathbf{S}^H \cdot \mathbf{y} \\ &= (\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H \cdot \mathbf{S}\mathbf{x} + \boxed{(\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H}\mathbf{n} \\ &= \mathbf{x} + 0_{MAI} + \tilde{\mathbf{n}}\end{aligned}$$

Array of N Elements



$$\beta_1 = |\beta_1| e^{j\alpha_1}$$

Capacity of2G/3G vs Achievable Capacity

$$C = 0.1\log_2\left(1+\frac{1}{0.1}\right) = 0.3\,bits/Hz/s$$

2G/3G

$$C = 30\log_2\left(1+\frac{1}{0.001}\right) = 300\,bits/Hz/s$$

3G+

300 / 0.3= 1000

Capacity improvement factor