University of Engineering and Technology, Peshawar

Engineering Economics

CSE-305

(Chapter 03a)

Principles of Money-Time Relationship

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Agenda

- Understanding Money-Time Relationship
- Considering Return on Capital

- Simple Interest
- Compound Interest
- Cash Flow Diagram
- Fundamental Law of Engineering Economics
- Market Value
- Concept of Economic Equivalence
- Single Payment Methods
- Multiple Payment Methods
- Uneven Payment Series
- Evaluating Market Value

Capital

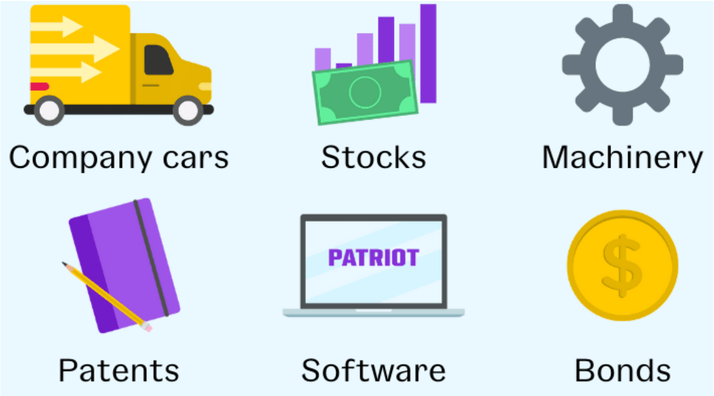
- Wealth in the form of money or property that is capable of earning more wealth

Equity Capital

- Money or wealth owned by individuals that are invested in a business or project or venture in the hope of profit

Debt/ Borrowed Capital

- Money or wealth borrowed from others that are invested in a business or project or venture in the hope of profit



Why consider Return on Capital?

Why not consider investing:

- If the project or venture is successful, the return to the owner of capital can be substantially more than the interest received by the lender of the capital; however, the owners could lose some or all of their money invested while the lenders still could receive all the interest owed plus repayment of the money borrowed by the firm.

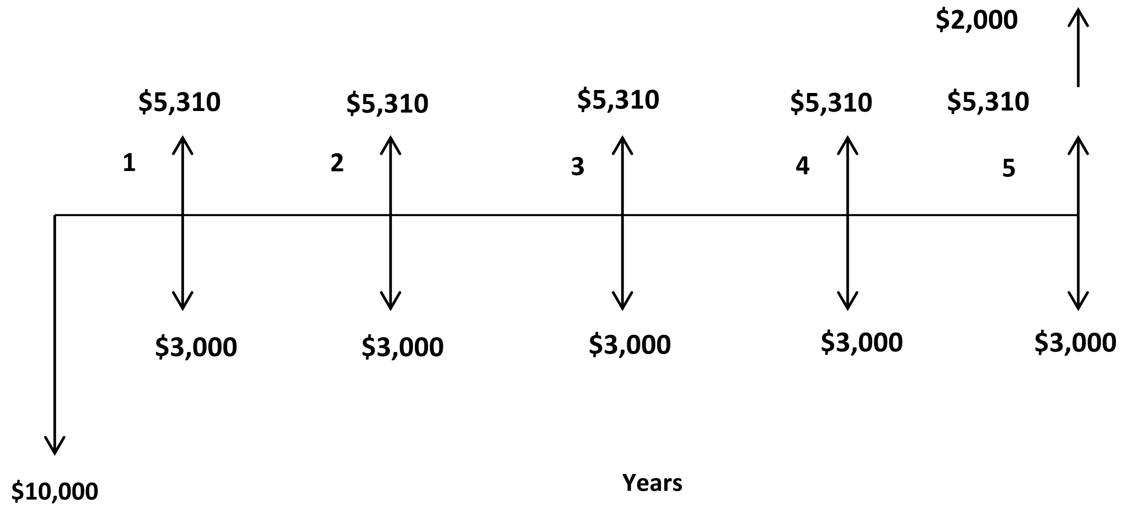
How to payback?

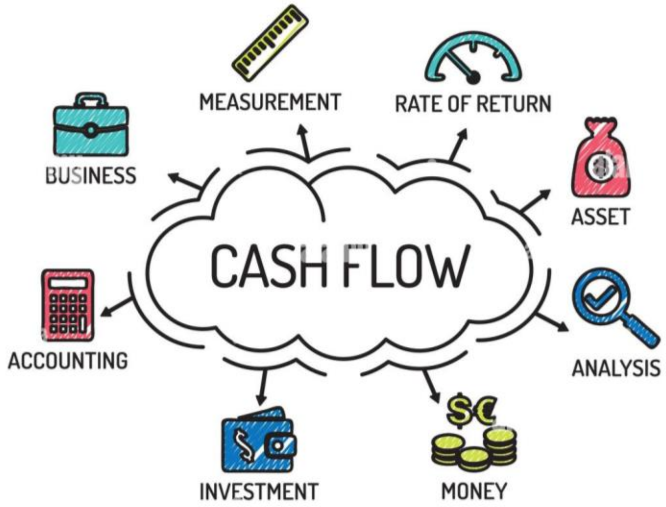
- Pay the lenders of capital for forgoing its use during the time the capital is being used. And risk the investor takes in permitting another person to use his or her capital
- **Opportunity cost:** by not investing in a possibly better alternative.

Cash Flow Diagrams

- **Horizontal Axis:** Time Scale
- **Cash Flows:** Flow of cash through the process
- **Point of view:** Lender or Borrower

Example: An investment of $10,000 can be made that will produce uniform annual revenue of $5,310 for 5 years and then have a positive salvage value of $2,000 at the end of year 5. Annual expenses will be $3,000 at the end of each year for operating and maintaining the project. Draw a cash flow diagram for the 5-year life of the project.





Simple Interest

- The practice of charging an interest rate only to an initial sum (principal amount).
- When the total interest earned or charged is directly proportional to the initial amount of loan (principal), the interest rate and the number of interest periods for which the principal is committed

$$\text{Simple Interest Formula} = P \times (1 + r \times t)$$

- Cumulative interest owned is a linear function of time until the interest is paid

Simple Interest: Example

- ***P*** **= Principal amount**
- ***i*** **= Interest rate**
- ***N*** **= Number of interest periods**
- **Example:**
  - ***P*** **= $1,000**
  - ***i*** **= 8%**
  - ***N*** **= 3 years**

Simple Interest Rate Formula = Principal Amount x Rate of Interest x Time Period

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$80	$1,160
3	$1,160	$80	$1,240

Simple Interest: Example

$$F = P + (iP)N$$

where

P = Principal amount

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

$$\begin{aligned} F &= \$1,000 + (0.08)(\$1,000)(3) \\ &= \$1,240 \end{aligned}$$

Compound Interest

- The practice of charging an interest rate to an ***initial sum*** and ***to any previously accumulated interest*** that has not been withdrawn
- Whenever the interest rate for any interest period is based on the ***remaining principal amount*** plus any ***accumulated interest charges*** up to the beginning of that period
- ***Compounding*** is essentially a calculation of interest on the previously earned interest

P = Principal amount

i = Interest rate

N = Number of interest periods

Example:

P = $1,000

i = 8%

N = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$86.40	$1,166.40
3	$1,166.40	$93.31	$1,259.71

Compound Interest

Commonly known as single payment compound amount: also denoted as

F= P(F/P, i%, N) ***"Check at end of the book"***

"The greatest mathematical discovery of all time,"

Albert Einstein

Some Fundamental Laws

$$F = m a$$

$$V = i R$$

$$E = m c^2$$

The Fundamental Law of Engineering Economy

$$F = P(1+i)^N$$

Compounding Process

Compound Interest Formula

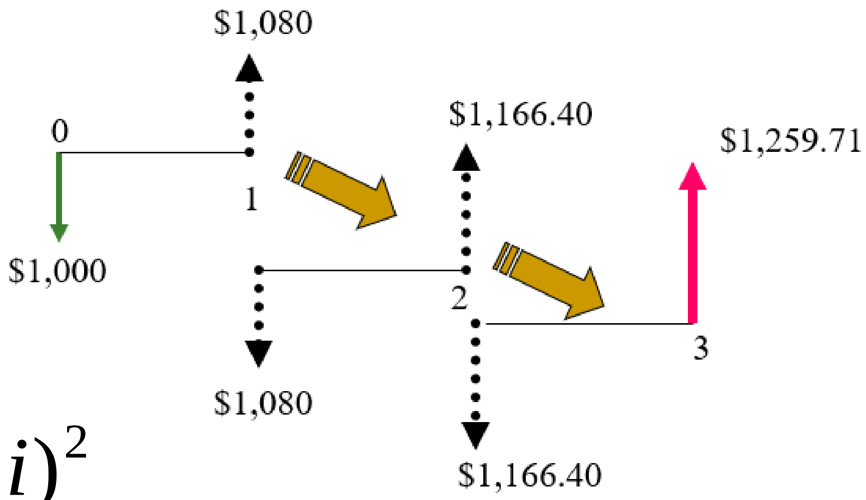
$$n = 0 : P$$

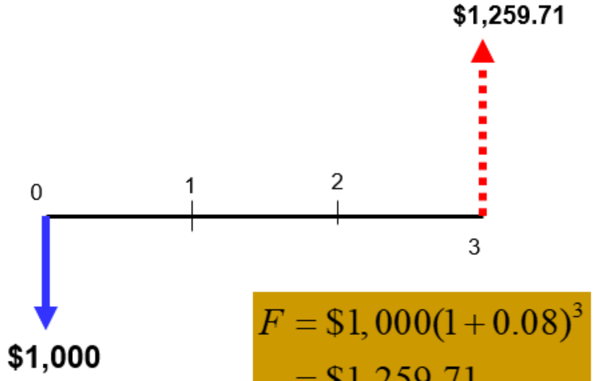
$$n = 1 : F_1 = P(1+i)$$

$$n = 2 : F_2 = F_1(1+i) = P(1+i)^2$$

$$\vdots$$

$$n = N : \boxed{F = P(1+i)^N}$$

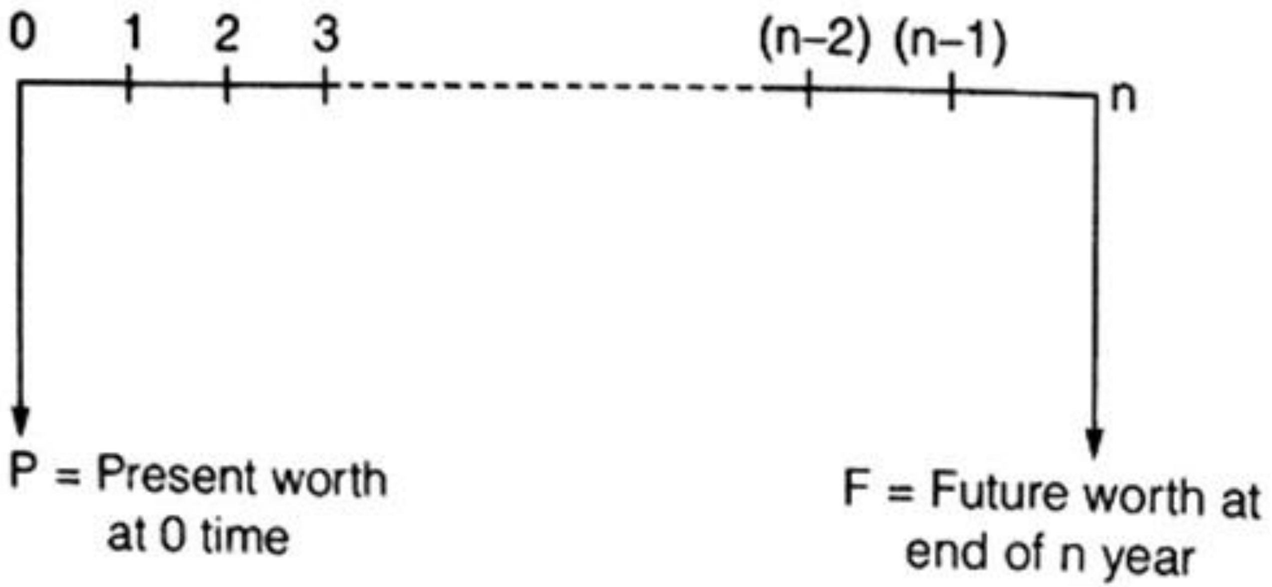


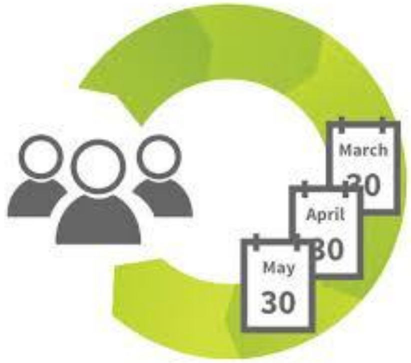


$$F = \$1,000(1+0.08)^3$$
$$= \$1,259.71$$

Compounding Process

Compound Interest formula relating Present and Future Equivalent Values of Single Cash Flow





Single Cash Flow Formula: Growth Factor

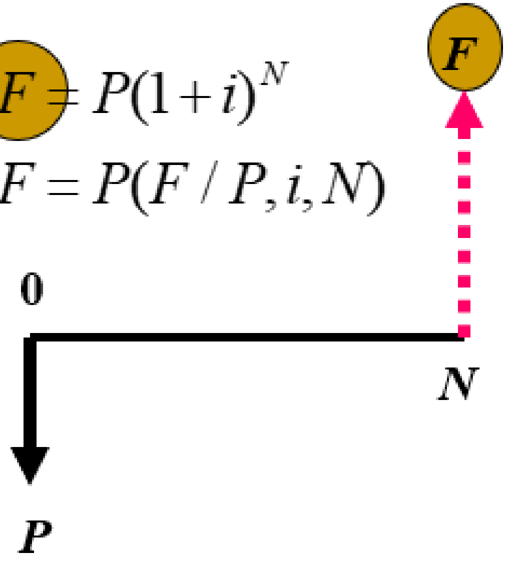
Single payment compound amount factor (growth factor)

Given:

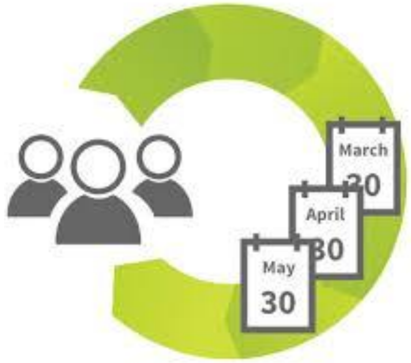
$$i = 10\%$$
$$N = 8 \text{ years}$$
$$P = \$2{,}000$$

Find: **F**

$$F = P(1+i)^N$$
$$F = P(F/P, i, N)$$



$$F = \$2{,}000(1+0.10)^8$$
$$= \$2{,}000(F/P, 10\%, 8)$$
$$= \$4{,}287.18$$



Single Cash Flow Formula: Growth Factor

If you had \$2,000 now and invested it at 10%, how much would it be worth in 8 years?

Given:

$$P = \$2,000$$
$$i = 10\%$$
$$N = 8 \text{ years}$$

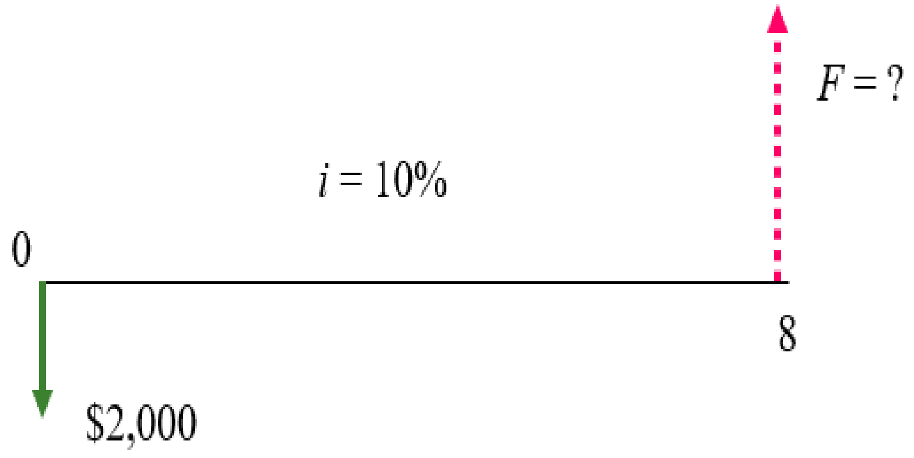
Find: F

$$F = \$2,000(1+0.10)^{8}$$
$$= \$2,000(F/P,10\%,8)$$
$$= \$4,287.18$$

EXCEL command:

=FV(10%,8,0,2000,0)

=\$4,287.20



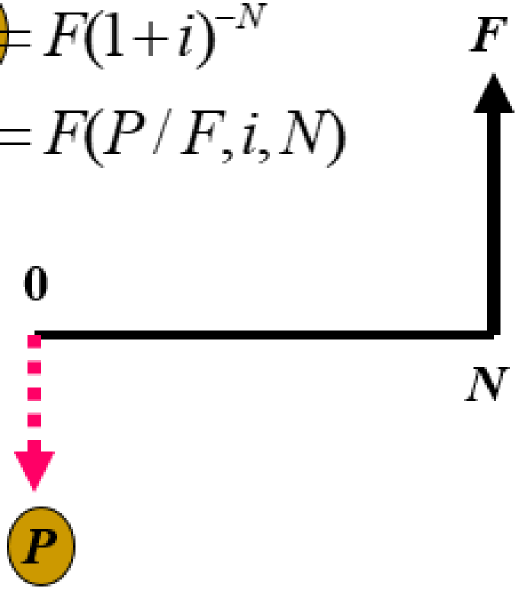
Single Cash Flow Formula: Discount Factor

If you have accumulated $1,000 now on an amount that you invested 5 years back at 12%, how much was it worth 5 years back?

Given:

$$i = 12\%$$
$$N = 5 \text{ years}$$
$$F = \$1{,}000$$

$$P = F(1+i)^{-N}$$
$$P = F(P/F, i, N)$$



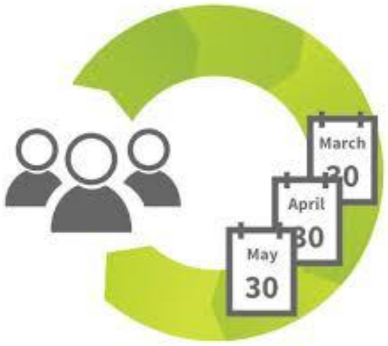
Find:

$$\begin{aligned} P &= \$1{,}000(1+0.12)^{-5} \\ &= \$1{,}000(P/F, 12\%, 5) \\ &= \$567.40 \end{aligned}$$

Which investment plan?

End of Year	Receipts	Payments	
		Plan 1	Plan 2
Year 0	$20,000.00	$200.00	$200.00
Year 1		5,141.85	0
Year 2		5,141.85	0
Year 3		5,141.85	0
Year 4		5,141.85	0
Year 5		5,141.85	30,772.48

The amount of loan = $20,000, origination fee = $200, interest rate = 9% APR (annual percentage rate)



Compound Interest: Market Value

Example: Assume that the company's stock worth \$ 100.36 will continue to appreciate at an annual rate of 24.58% for the next 27 years.

$$F = \$100.36(1+0.2458)^{27}$$
$$= \$37.902 \text{ trillions}$$

Example: Suppose that you borrow \$8,000 now, with the promise to repay the loan principal plus accumulated interest in 4 years at i=10% per year. How much would you owe at the end of 4 years?

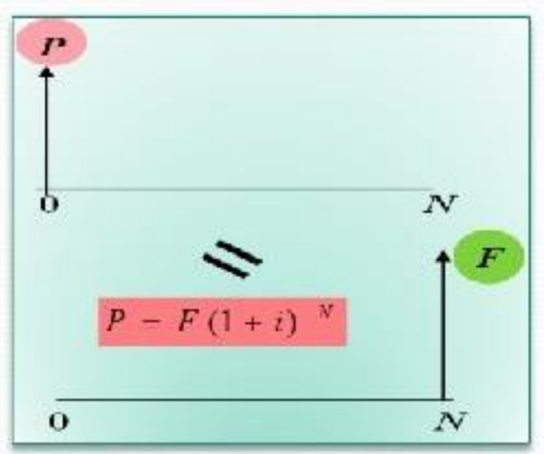
Year	Amount Owed at Start of Year	Interest Owed at for Each Year	Amount Owed at End of Year	Total End of year payment
1	P =\$8,000	iP =\$800	$P(1+i) = \$8,800$	0
2	$P(1+i) = \$8,800$	$iP(1+i) = \$880$	$P(1+i)^2 = \$9,680$	0
3	$P(1+i)^2 = \$9,680$	$iP(1+i)^2 = \$968$	$P(1+i)^3 = \$10,648$	0
4	$P(1+i)^3 = \$10,648$	$iP(1+i)^3 = \$1,065$	$P(1+i)^4 = \$11,713$	F =\$11,713

Concept of Equivalence

- How can alternatives for providing the same service or accomplishing the same function be compared when interest is involved over extended periods of time?

- Thus, we should consider the comparison of alternative options or proposals, by reducing them to ***an equivalent basis*** that is dependent on some factors

  - What do we mean by "economic equivalence?"
  - Why do we need to establish an economic equivalence?
  - How do we establish an economic equivalence?



Economic Equivalence

Economic equivalence exists between cash flows that have the **same economic effect** and could therefore be traded for one another.

Even though the amounts and timing of the cash flows may differ, the **appropriate interest rate** makes them equal.

Equivalence Basis

Mechanics of interest and notion of economic equivalence:

- **The interest rate**
- **The amounts of money involved**
- **The timing of monetary receipts and/or disbursements**
- **The way the interest or profit on invested capital is repaid**
- **The initial capital recovered**

Economic Equivalence: Example

We have borrowed $ 8000 and agreed to repay in 4 years, at an annual interest rate of 10%. There are several ways in which this principal amount and the interest on it can be repaid.

Four plans for repayment of $8,000 in 4 years with interest at 10%. Here equivalence means that all four plans are equally desirable to the borrower. (Check the book for detailed values)

Plan1

At end of each year pay $2,000 principal plus interest due.

Plan2

Pay interest due at end of each year and principal at end of 4 years.

Plan3

Pay in 4 equal end-of-year payments.

Plan4

Pay principal and interest in one payment at end of 4 years.

Economic Equivalence: Example

Which plan is better?

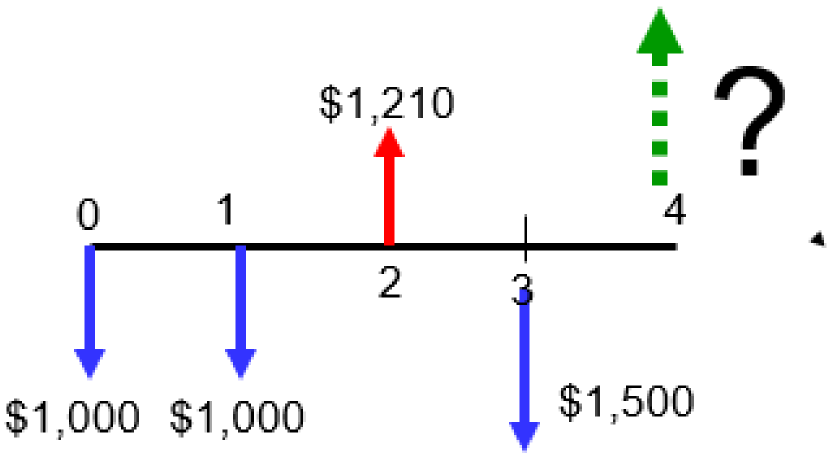
- When total dollar-years are calculated for each plan and divided into total interest paid over the 4 years (the sum of column 3), the ratio is found to be constant:

Plan	Area Under Curve(Dollar-years)	Total Interest paid	Ratio of Total Interest to Dollar-Years
1	$20,000	$2,000	0.10
2	32,000	3,200	0.10
3	20,960	2,096	0.10
4	37,130	3,713	0.10

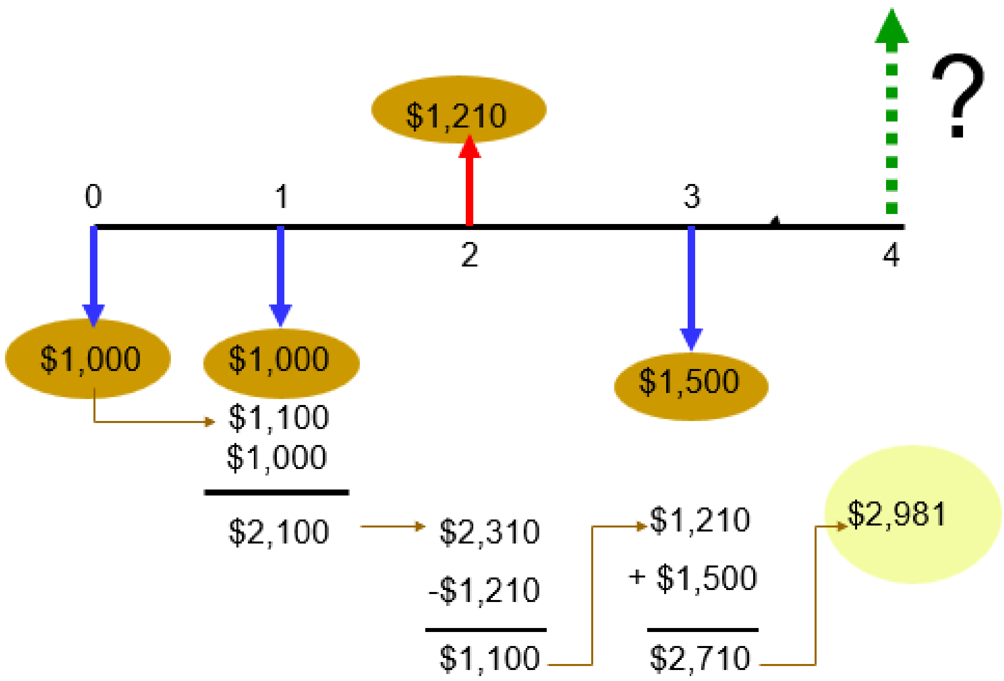
Compound Interest: Uneven Payments

Example:

- **Consider the following sequence of deposits and withdrawals over a period of 4 years. If you earn 10% interest, what would be the balance at the**



Compound Interest: Uneven Payments



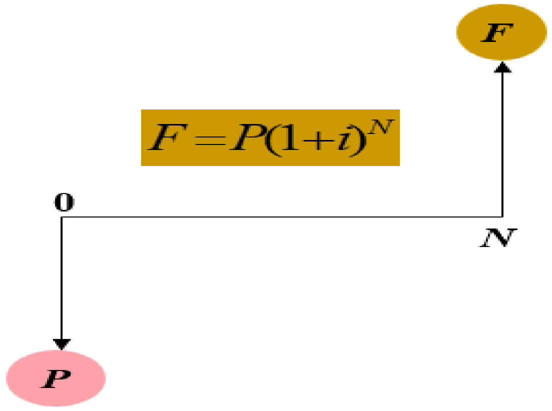
End of Period	Beginning balance	Deposit made	Withdraw	Ending balance
n = 0	0	$1,000	0	$1,000
n = 1	$1,000(1 + 0.10) =$1,100	$1,000	0	$2,100
n = 2	$2,100(1 + 0.10) =$2,310	0	$1,210	$1,100
n = 3	$1,100(1 + 0.10) =$1,210	$1,500	0	$2,710
n = 4	$2,710(1 + 0.10) =$2,981	0	0	$2,981

Compound Interest: Equivalence

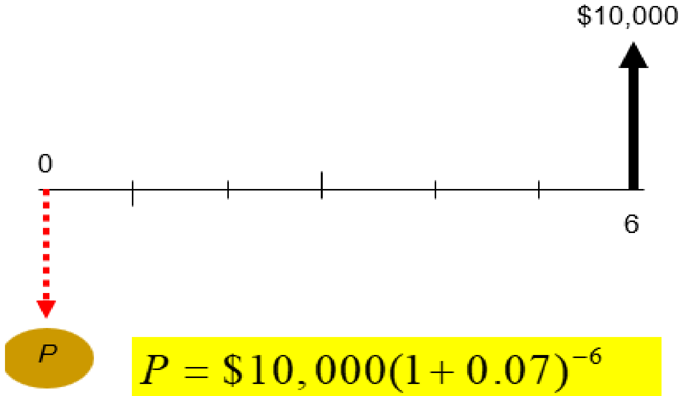
Equivalence from Personal Financing Point of View:

If you deposit ***P*** dollars today for ***N*** periods at ***i,*** you will have ***F*** dollars at the end of period *N*.

$$P \equiv F$$



Example: You want to set aside *a lump sum* amount today in a savings account that earns ***7%*** annual interest to meet a future expense in the amount of ***$10,000*** to be incurred in 6 years. How much do you need to deposit today?

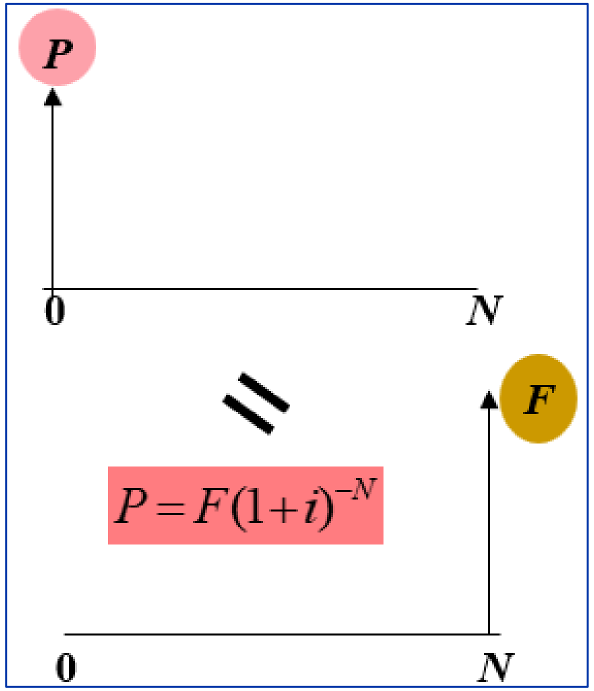


$$\begin{aligned} P &= \$10,000(1+0.07)^{-6} \\ &= \$10,000(P/F, 7\%, 6) \\ &= \$6,663 \end{aligned}$$

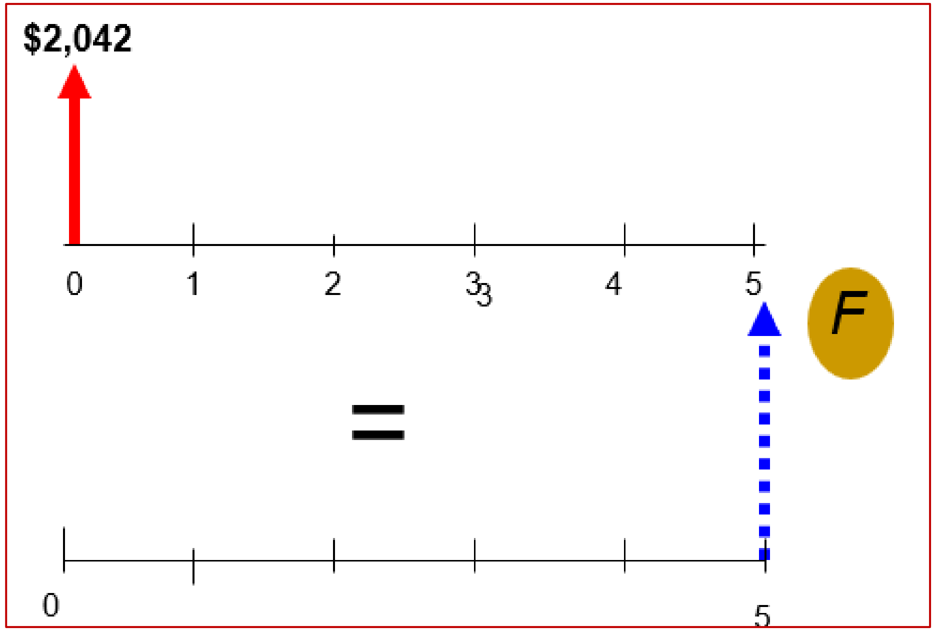
Compound Interest: Equivalence

Alternate Way of Defining Equivalence:

F dollars at the end of period *N* is ***equal*** to a single sum *P* dollars now, if your earning power is measured in terms of interest rate *i*.



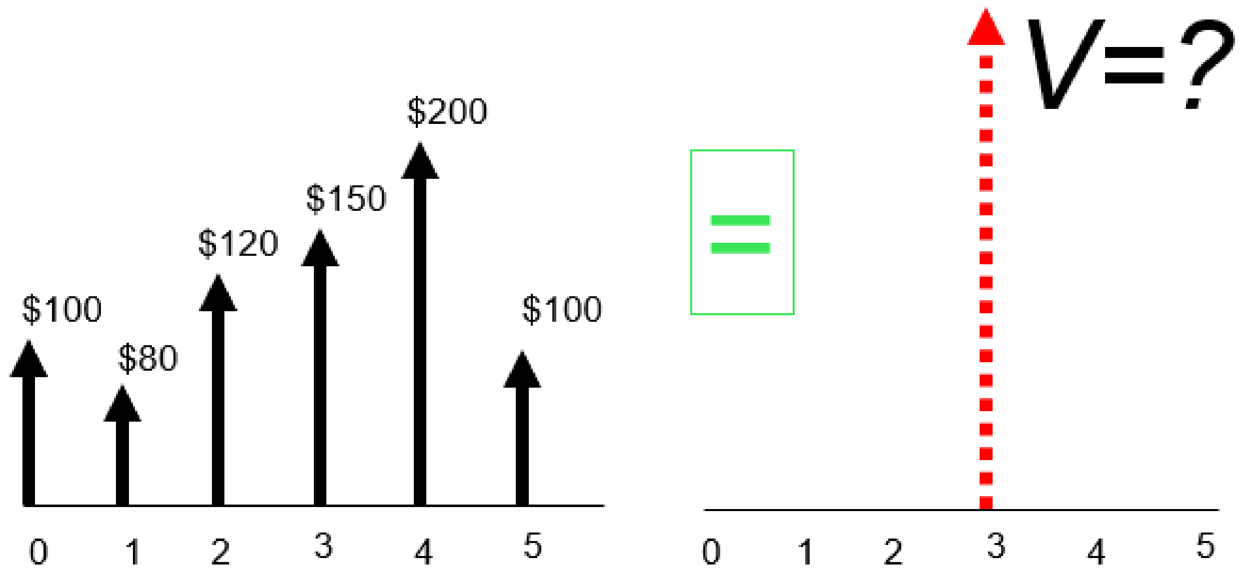
Example: At 8% interest, what is the equivalent worth of \$2,042 now 5 years from now? If you deposit \$2,042 today in a savings account that pays 8% interest annually. How much would you have at the end of 5 years?



$$F = \$2,042(1+0.08)^5$$
$$= \$3,000$$

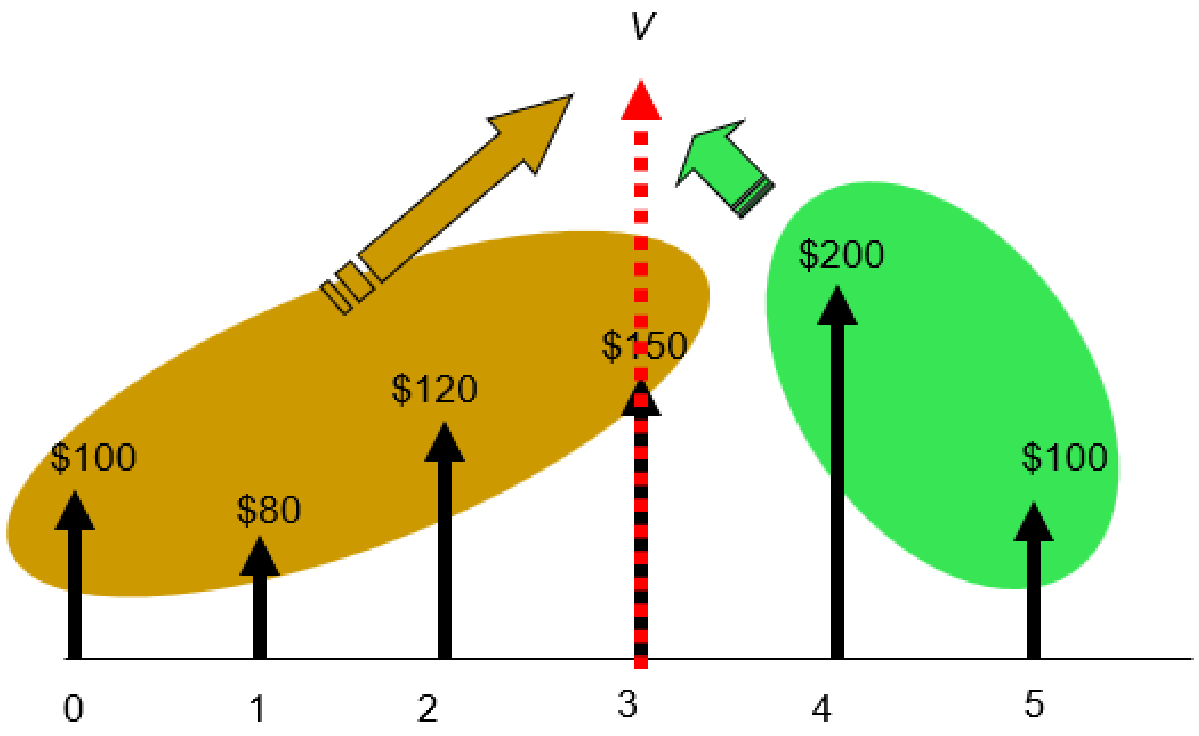
Uneven Payments: Example

Compute the equivalent lump-sum amount at n = 3 at 10% annual interest.



Uneven Payments: Example

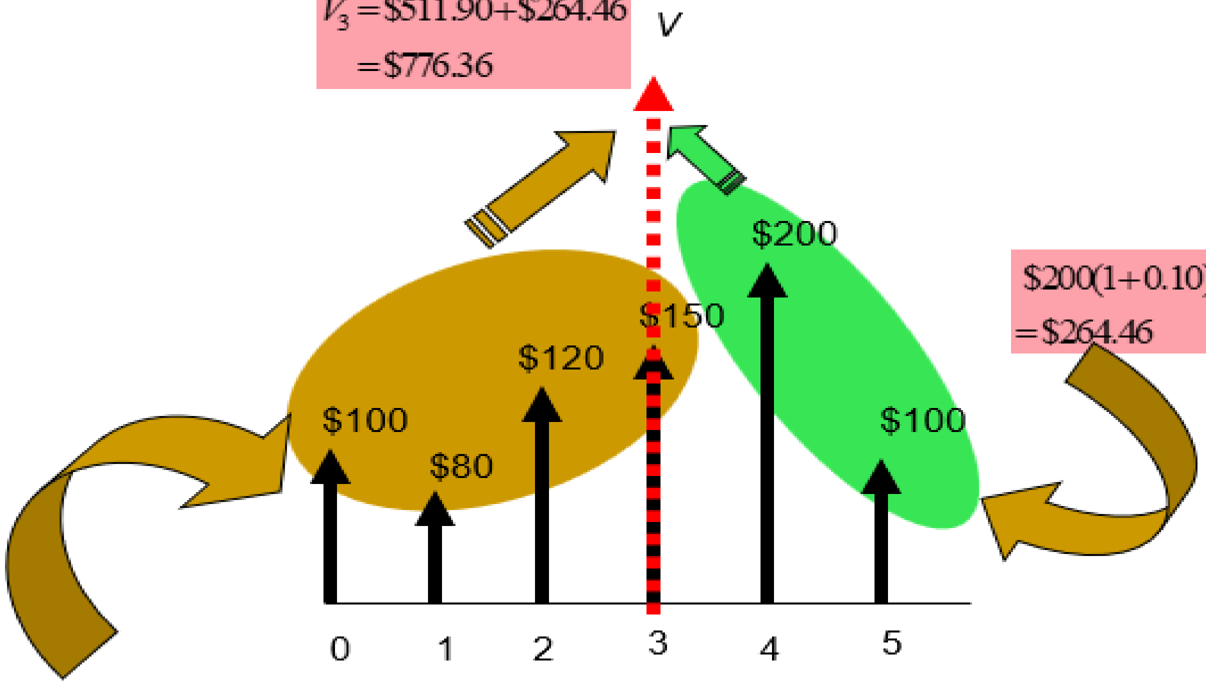
Compute the equivalent lump-sum amount at n = 3 at 10% annual interest.



Uneven Payments: Example

Compute the equivalent lump-sum amount at n = 3 at 10% annual interest.

$$V_3 = \$511.90 + \$264.46 = \$776.36$$

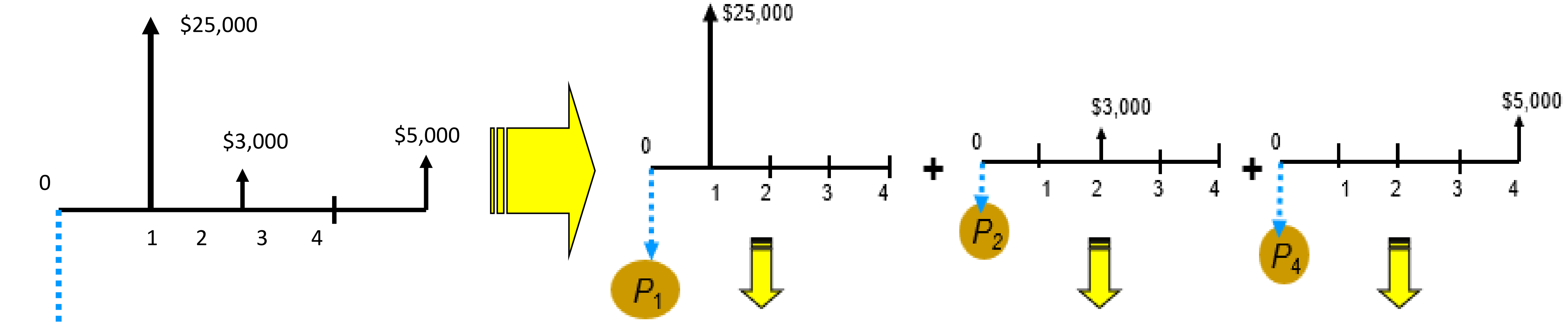


$$\$200(1+0.10)^{-1} + \$100(1+0.10)^{-2} = \$264.46$$

$$100(1+0.10)^{3} + \$80(1+0.10)^{2} + \$120(1+0.10) + \$150 = \$511.90$$

Multiple Payments: Example

How much do you need to deposit today (P) to withdraw $25,000 at n =1, $3,000 at n = 2, and $5,000 at n =4, if your account earns 10% annual interest?



$$P_1 = \$25{,}000(P/F, 10\%, 1) = \$22{,}727$$

$$P_2 = \$3{,}000(P/F, 10\%, 2) = \$2{,}479$$

$$P_4 = \$5{,}000(P/F, 10\%, 4) = \$3{,}415$$

$$P = P_1 + P_2 + P_3 = \$28{,}622$$

Multiple Payments: Example

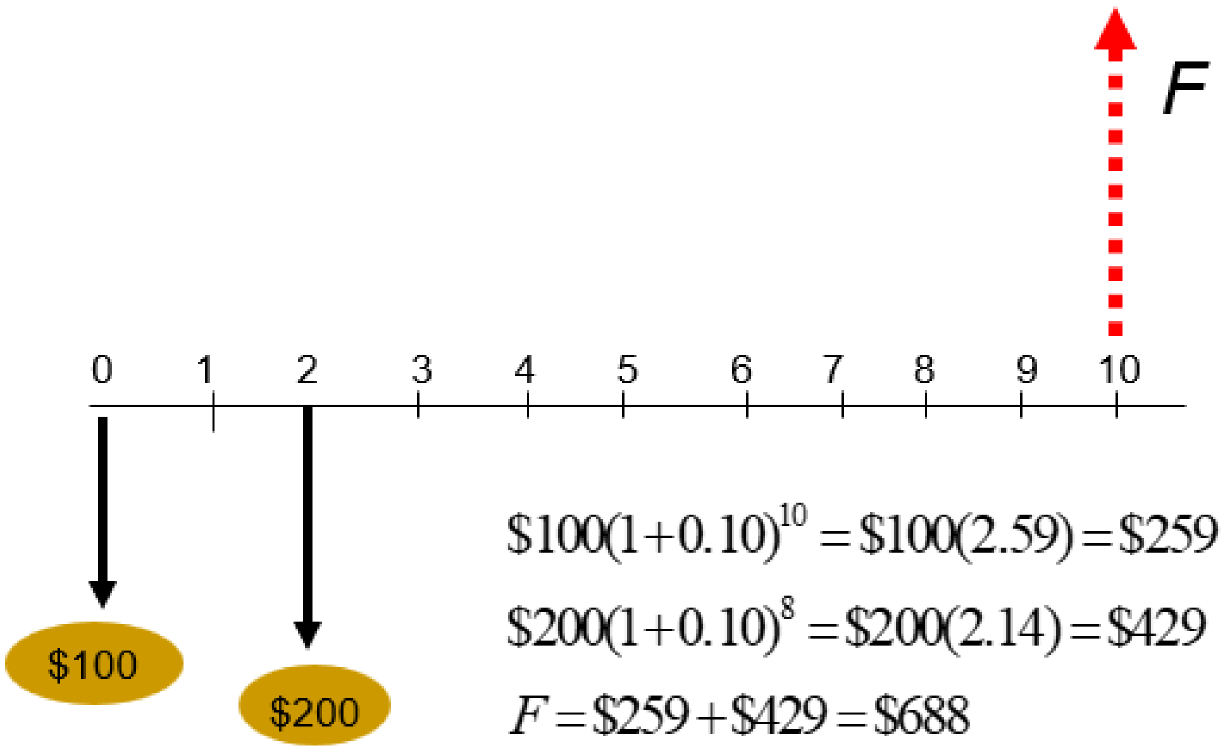
How much do you need to deposit today (P) to withdraw $25,000 at n =1, $3,000 at n = 2, and $5,000 at n =4, if your account earns 10% annual interest?

	0	1	2	3	4
Beginning Balance	0	28,622	6,484.20	4,132.62	4,545.88
Interest Earned (10%)	0	2,862	648.42	413.26	454.59
Payment	+28,622	-25,000	-3,000	0	-5,000
Ending Balance	$28,622	6,484.20	4,132.62	4,545.88	0.47

Rounding error
It should be “0.”

Multiple Payments: Example

If you deposit \$100 now (n = 0) and \$200 two years from now (n = 2) in a savings account that pays 10% interest, how much would you have at the end of year 10?



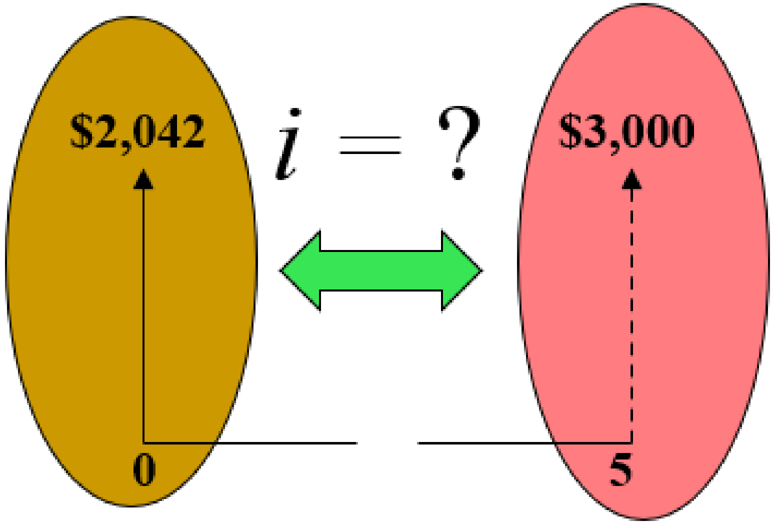
$$\$100(1+0.10)^{10} = \$100(2.59) = \$259$$

$$\$200(1+0.10)^{8} = \$200(2.14) = \$429$$

$$F = \$259 + \$429 = \$688$$

Economic Equivalence: Example

At what interest rate would these two amounts be equivalent?



Various dollar amounts that will be ***economically equivalent*** to \$3,000 in 5 years, given an interest rate of 8%.

$$P = \frac{\$3,000}{(1+0.08)^5} = \$2,042$$

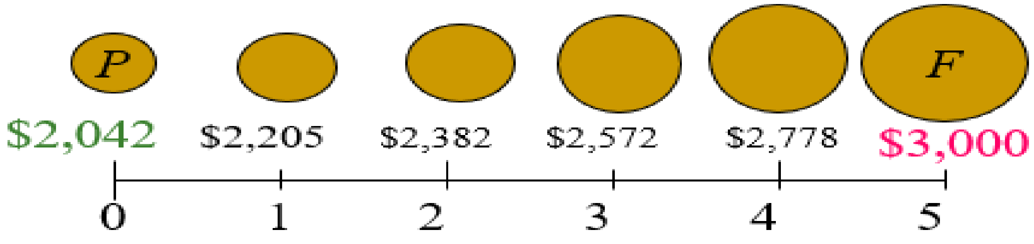
Solution:

- **Step 1**: Determine the base period, say, year 5.
- **Step 2**: Identify the interest rate to use.
- **Step 3**: Calculate equivalence value.

$$i = 6\%, F = \$2,042(1+0.06)^5 = \$2,733$$

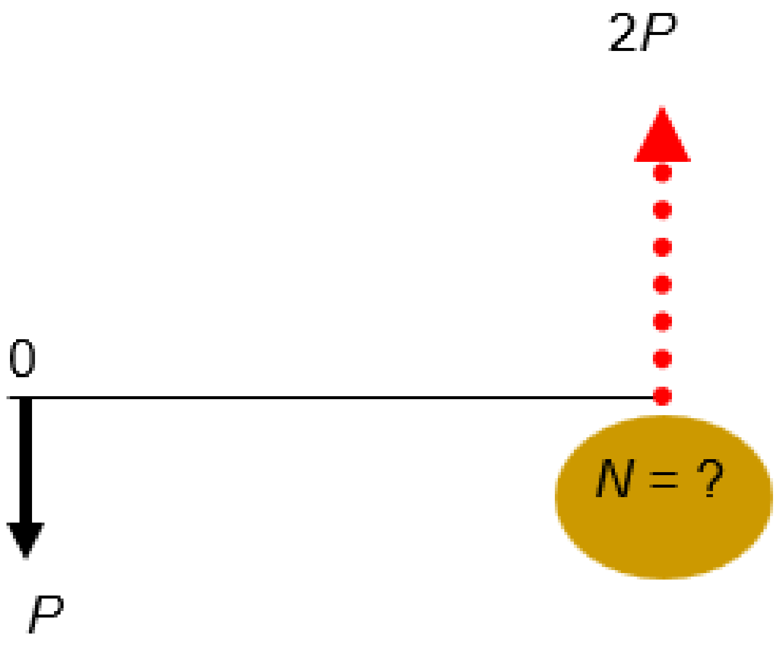
$$i = 8\%, F = \$2,042(1+0.08)^5 = \$3,000$$

$$i = 10\%, F = \$2,042(1+0.10)^5 = \$3,289$$



Economic Equivalence: Example

How many years would it take an investment to double at 10% annual interest?



$$F = 2P = P(1+0.10)^N$$

$$2 = 1.1^N$$

$$\log 2 = N \log 1.1$$

$$N = \frac{\log 2}{\log 1.1}$$

$$= 7.27 \text{ years}$$

Economic Equivalence: Example

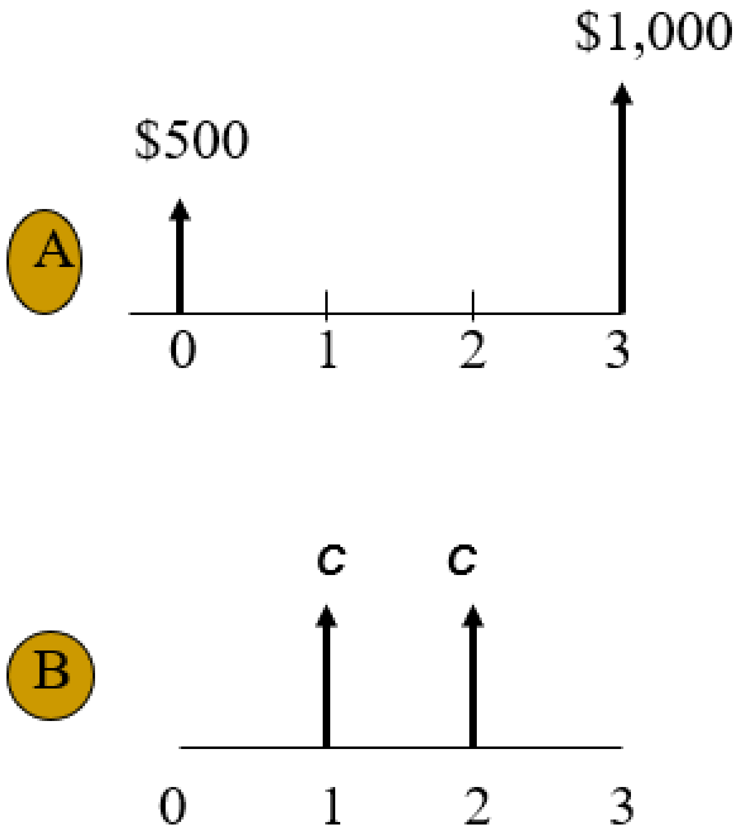
Given: i = 10%
Find C that makes the two cash flow streams to be indifferent.

Approach:

Step 1: Select the base period to use, say $n = 2$.

Step 2: Find the equivalent lump sum value at $n = 2$ for both A and B.

Step 3: Equate both equivalent values and solve for unknown *C*.



Economic Equivalence: Example

Approach:

For A:

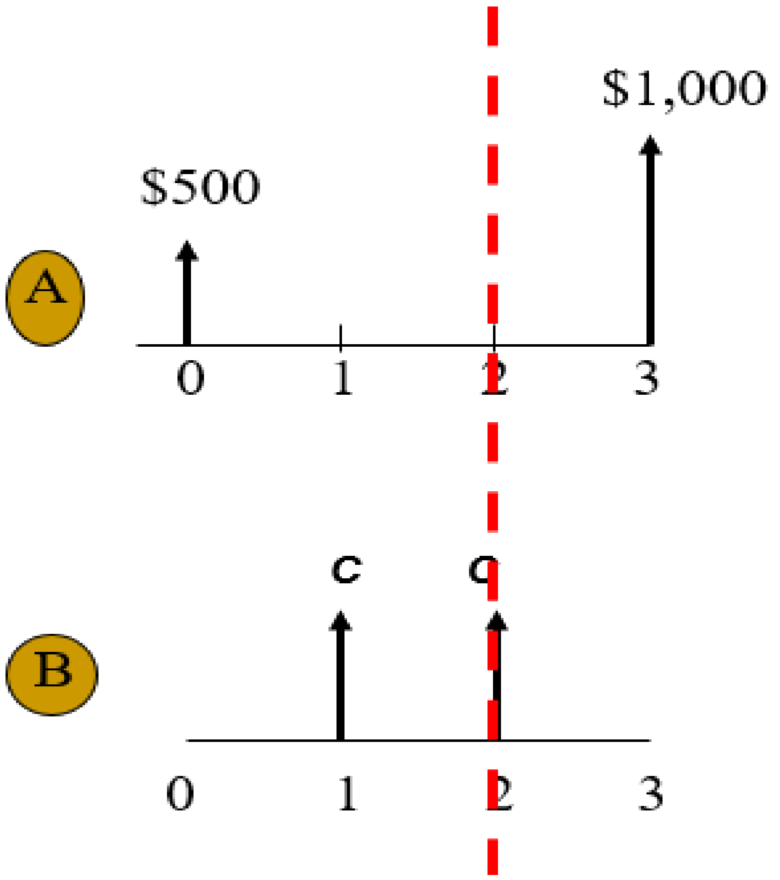
$$V_2 = \$500(1+0.10)^2 + \$1,000(1+0.10)^{-1}$$
$$= \$1,514.09$$

For B:

$$V_2 = C(1+0.10) + C$$
$$= 2.1C$$

Finding C:

$$2.1C = \$1,514.09$$
$$C = \$721$$



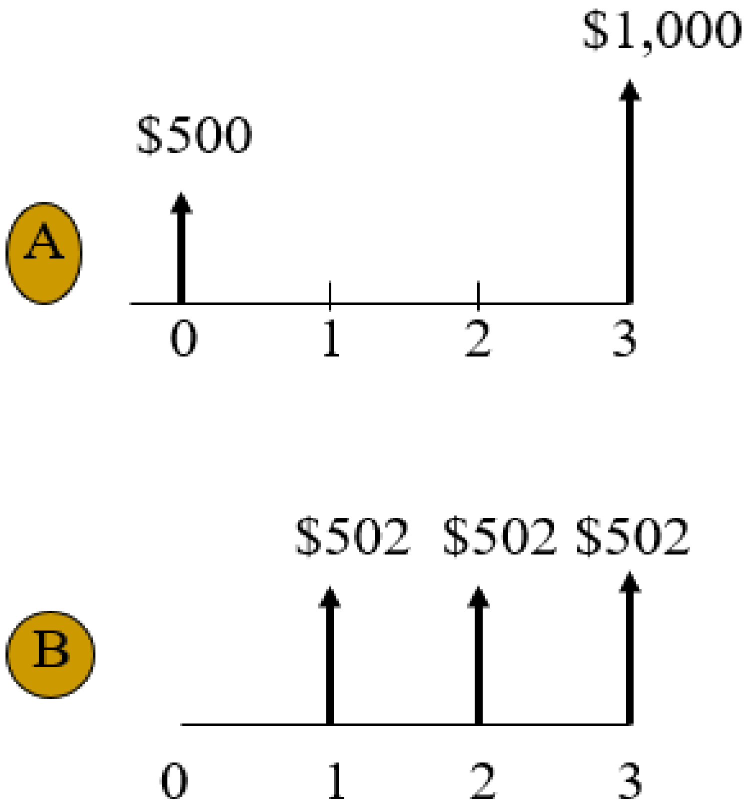
Economic Equivalence: Example

At what interest rate would it be indifferent between the two cash flows?

Approach:

Step 1: Select the base period to compute the equivalent value (say, n = 3)

Step 2: Find the net worth of each at n = 3.



Economic Equivalence: Example

Establish equivalence at n=3

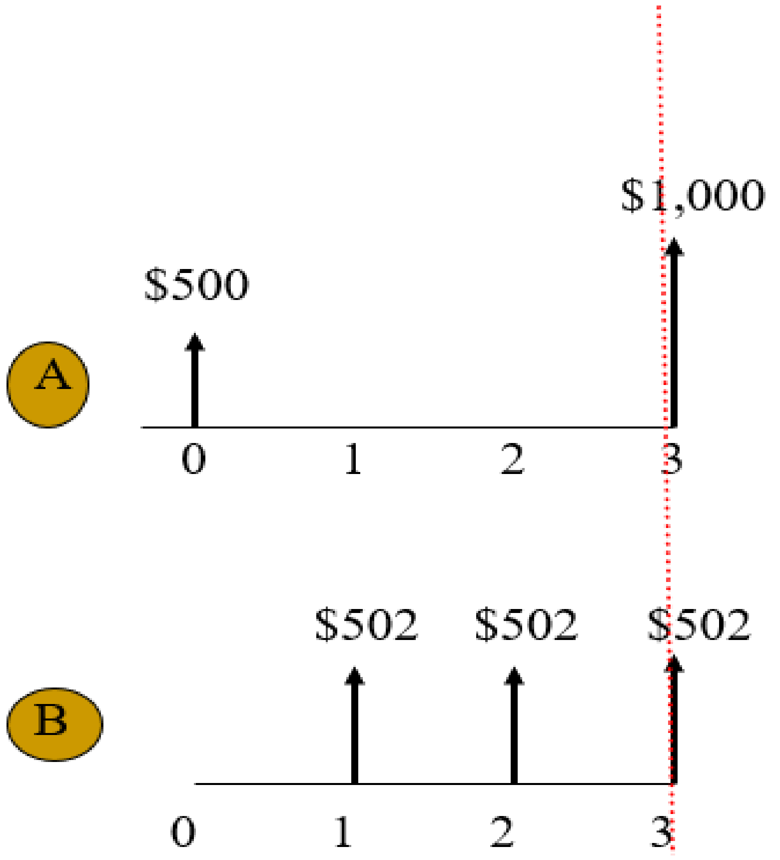
$$\text{Option A}: F_3 = \$500(1+i)^3 + \$1{,}000$$

$$\text{Option B}: F_3 = \$502(1+i)^2 + \$502(1+i) + \$502$$

Find the solution by trial and error, say i = 8%

$$\text{Option A}: F_3 = \$500(1.08)^3 + \$1{,}000 = \$1{,}630$$

$$\text{Option B}: F_3 = \$502(1.08)^2 + \$502(1.08) + \$502 = \$1{,}630$$



Summary

- **Understanding Money-Time Relationship**
- **Considering Return on Capital**

- Simple Interest
- Compound Interest
- Cash Flow Diagram
- Fundamental Law of Engineering Economics
- Market Value
- Concept of Economic Equivalence
- Single Payment Methods
- Multiple Payment Methods
- Uneven Payment Series
- Evaluating Market Value