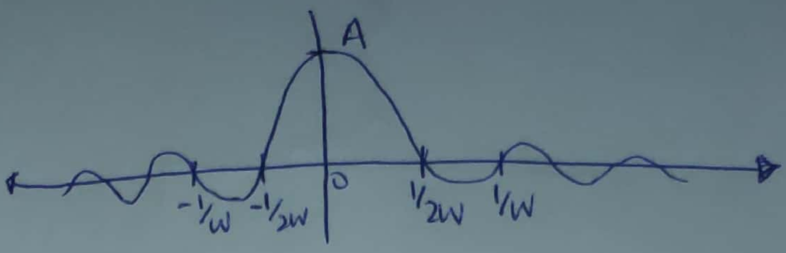
QUESTION #1

Ansr



The given function in time domain is:

$$x(t) = A \cdot \text{Sinc}(2\pi W t)$$

Taking Fourier Transform of $x(t)$.

$$X(\omega) = \mathcal{F}\left[A \cdot \text{Sinc}(2\pi W t)\right] \quad \text{— (i)}$$

We know that

$$\frac{W}{\pi} \text{sinc}(Wt) \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$$

Replacing 'W' by '$2\pi W$', we get

$$\frac{2\cancel{\pi} W}{\cancel{\pi}} \text{sinc}(2\pi W t) \longleftrightarrow \text{rect}\left(\frac{\omega}{4\pi W}\right)$$

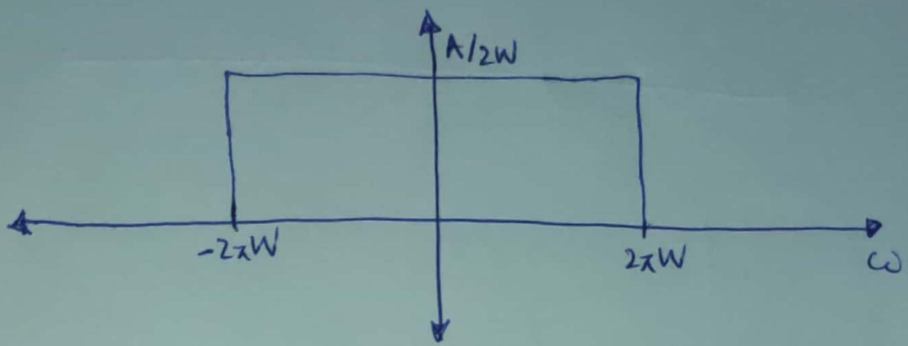
$$\text{sinc}(2\pi W t) \longleftrightarrow \frac{1}{2W} \cdot \text{rect}\left(\frac{\omega}{4\pi W}\right)$$

eq(i) → $X(\omega) = \cancel{A \cdot 2W}$

$$A \cdot \frac{1}{2W} \cdot \text{rect}\left(\frac{\omega}{4\pi W}\right)$$

$$\boxed{X(\omega) = \frac{A}{2W} \cdot \text{rect}\left(\frac{\omega}{4\pi W}\right)}$$

Spectrum of given function



QUESTION #2

Ans:-

$$x(t) = \cos(2\pi f_o t)$$

Using Euler's Identity. and $\omega_o = 2\pi f_o$

$$x(t) = \cos(\omega_o t)$$

$$x(t) = \frac{1}{2}\left[e^{-j\omega_o t} + e^{j\omega_o t}\right]$$

Taking Fourier Transform of $x(t)$.

$$X(\omega) = \frac{1}{2}\left[\mathcal{F}\left[e^{-j\omega_o t}\right] + \mathcal{F}\left[e^{j\omega_o t}\right]\right] \quad \text{(i)}$$

Now ~~X~~ $\mathcal{F}\left[e^{-j\omega_o t}\right] = 2\pi\delta(\omega+\omega_o)$ and

$$\mathcal{F}\left[e^{j\omega_o t}\right] = 2\pi \cdot \delta(\omega-\omega_o)$$

eq(i) $\Rightarrow$ $X(\omega) = \frac{1}{2}\left[2\pi \cdot \delta(\omega+\omega_o) + 2\pi \cdot \delta(\omega-\omega_o)\right]$

$$X(\omega) = \frac{\cancel{2}\pi}{\cancel{2}}\left[\delta(\omega-\omega_o) + \delta(\omega+\omega_o)\right]$$

$$\boxed{X(\omega) = \pi\left[\delta(\omega-\omega_o) + \delta(\omega+\omega_o)\right]}$$

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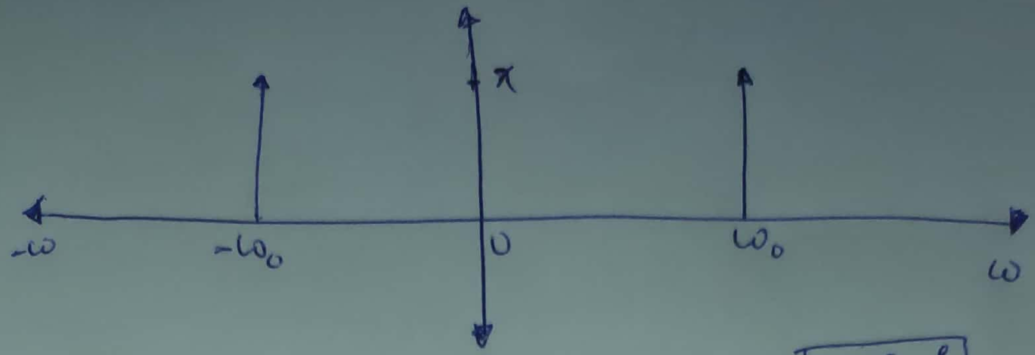

where $\boxed{\omega = 2\pi f}$

QUESTION #3

Ans:-

$$x(t)\cdot\cos(\omega_c t) = x(t)\left[\frac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right]$$

$$\quad // \quad = \frac{x(t)\cdot e^{j\omega_c t}}{2} + \frac{x(t)\cdot e^{-j\omega_c t}}{2}$$

Taking Fourier Transform of both sides

$$F\left[x(t)\cdot\cos(\omega_c t)\right] = F\left[\frac{x(t)\cdot e^{j\omega_c t}}{2}\right] + F\left[\frac{x(t)\cdot e^{-j\omega_c t}}{2}\right] \quad (i)$$

Now $x(t)\cdot e^{j\omega_c t} \Longleftrightarrow X(\omega-\omega_c)$ [Time Delay Property]

and $x(t)\cdot e^{-j\omega_c t} \Longleftrightarrow X(\omega+\omega_c)$

$\therefore$ eq (i) $\Rightarrow F\left[x(t)\cdot\cos(\omega_c t)\right] = \frac{X(\omega-\omega_c)}{2} + \frac{X(\omega+\omega_c)}{2}$

Soo we can say that

$$x(t)\cdot\cos(\omega_c t) \Longleftrightarrow \frac{X(\omega-\omega_c)}{2} + \frac{X(\omega+\omega_c)}{2}$$

Question #5

Ans:- Total power is given by:

$$P_t = P_c + P_s \quad \text{— (i)}$$

Here $P_t \rightarrow$ Total Power

$P_c \rightarrow$ Carrier Power

$P_s \rightarrow$ Signal Power

The power of carrier is given as:

$$\boxed{P_c = A^2/2}$$

The power of signal is given as:

$$P_s = \frac{\overline{m^2(t)}}{2}$$

Now for a sinusoid,

$$\overline{m(t)} = \frac{V_m}{\sqrt{2}}$$

$$\therefore P_s = \frac{V_m^2}{2} \cdot \frac{1}{2}$$

Put all values in (i)

$$P_t = \frac{A^2}{2} + \frac{1}{2} \cdot \frac{V_m^2}{2}$$

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2} \cdot \frac{V_m^2}{A^2}\right] = \frac{A^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{A}\right)^2\right] \quad \text{— (ii)}$$

The modulation index, ma is given as:

$$\boxed{m_a = \frac{V_m}{A}}$$

put in (ii)

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2} . m_a^2\right]$$

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$