Date: 27/2/2023 Day: M T W T F S

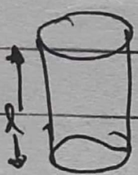
$$y(t) = x(t) + h(t)$$

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow{y(t)}$$

* CT & DT

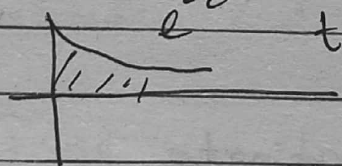
$x(t)$ $x[n]$

$x(t)$ size $\longrightarrow$ Energy / Power



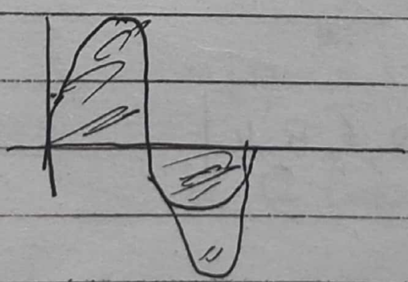
$$V = \int_0^l \pi r^2 \, dl$$

* Signals may be energy signals / Powe

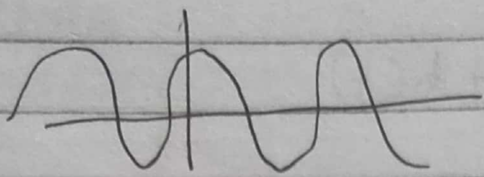


e^{-2t} $t \geq 0$

$$E_x = \int_0^\infty \left(e^{-2t}\right)^2 dt$$



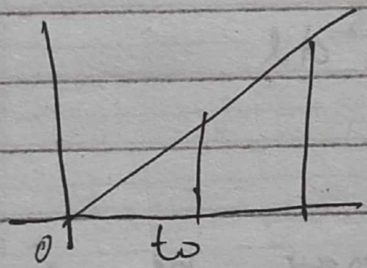
$$E_x = \int_0^\infty x^2(t) \, dt$$



$\sin t$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$x(t) = t$

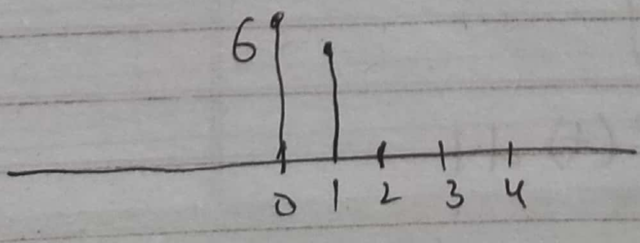


$t \geq 0$

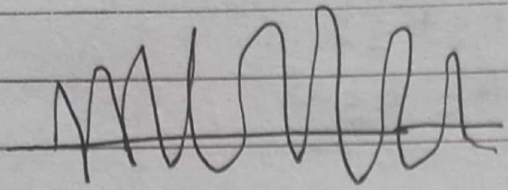
Not an energy, neither power signal.

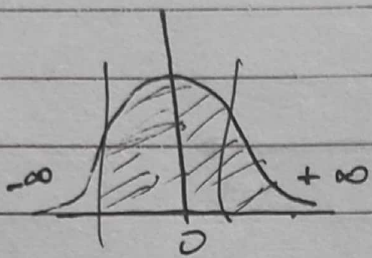
☆ Determinate & Indeterminate

$x[n] = [-1, 8, 0, 6, 5, 4]$



★ NOISE (indeterminate)





Gaussian Curve

Additive Wide Gaussian Noise

AWGN

• Now $g(t) = x(t) + h(t) + n(t)$

★ $x(t)$ Operations on signal.

Scaling:-

(i) Amplitude scaling

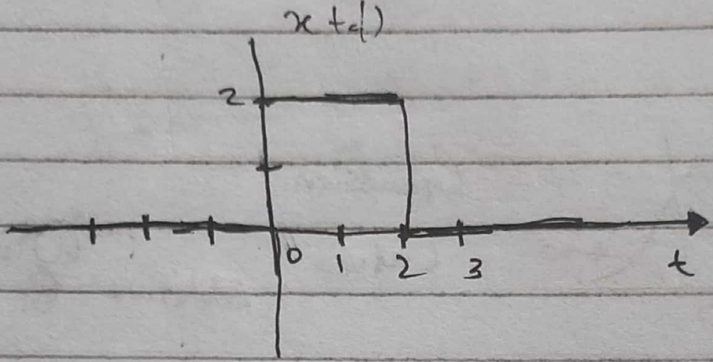
(ii) Time "

$x(t) \xrightarrow{A.S} \alpha . x(t)$

$\alpha \in (-\infty, 1) \cup (1, \infty)$

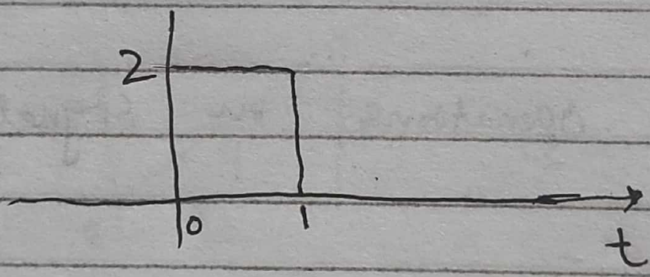
Lets suppose $\alpha = +Ve$

$$x(t) = \begin{cases} 0 & -\infty \leq t \leq 0 \\ 2 & 0 \leq t \leq 2 \\ 0 & 2 \leq t \leq \infty \end{cases}$$



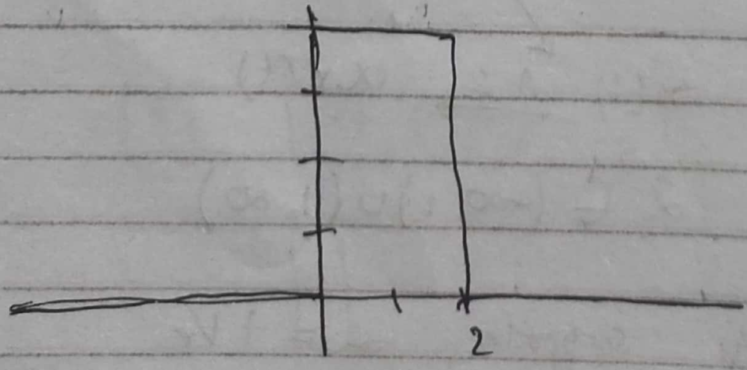
$x(2t)$

divide t values by 2

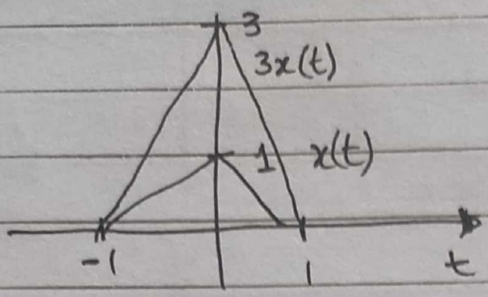


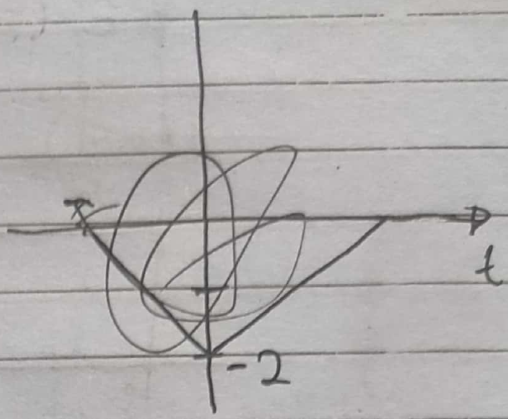
$2x(t)$

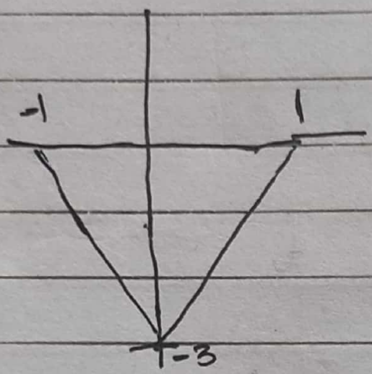
Multiply 2 with amplitude.

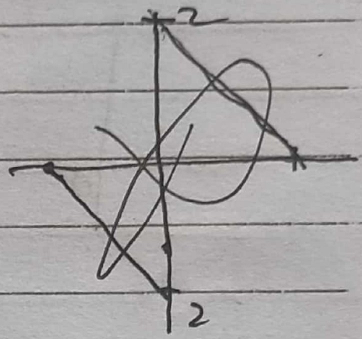


Scanned with CamScanner









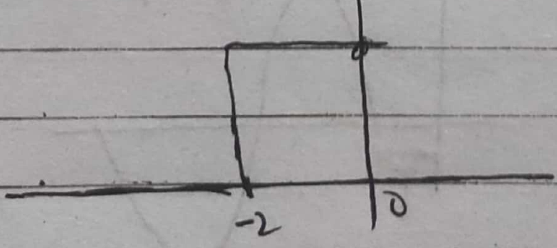
★ Time Scaling

$x(\beta t)$

$\beta \in (-\infty, -1) \cup (1, \infty)$

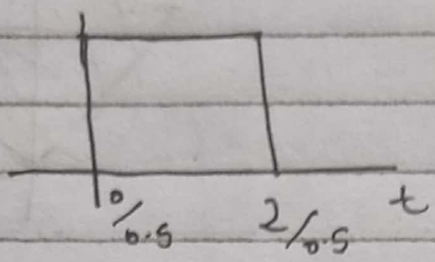
$x(2t)$

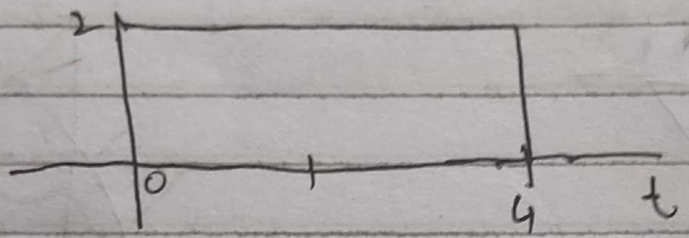
Time reversal $x(t) \xrightarrow{T.R} x(-t)$



$\beta \in (0, 1)$

$x(0.5t)$





★ $x(t) \xrightarrow{\text{Time Shifting}} x(t + K)$

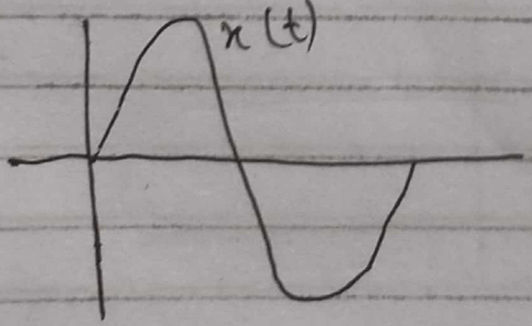
$K \in (-\infty, -1) \cup (1, \infty)$

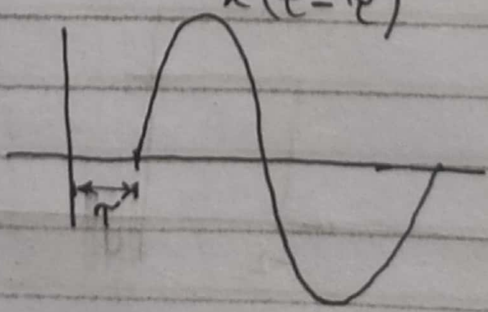
Case I : $K = +ve$

$K = 3$

$x(t+3)$

If +ve ⟶ advanced , -ve ⟶ delayed





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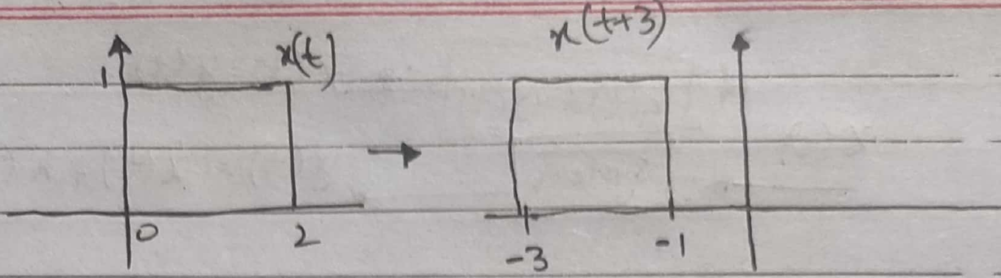
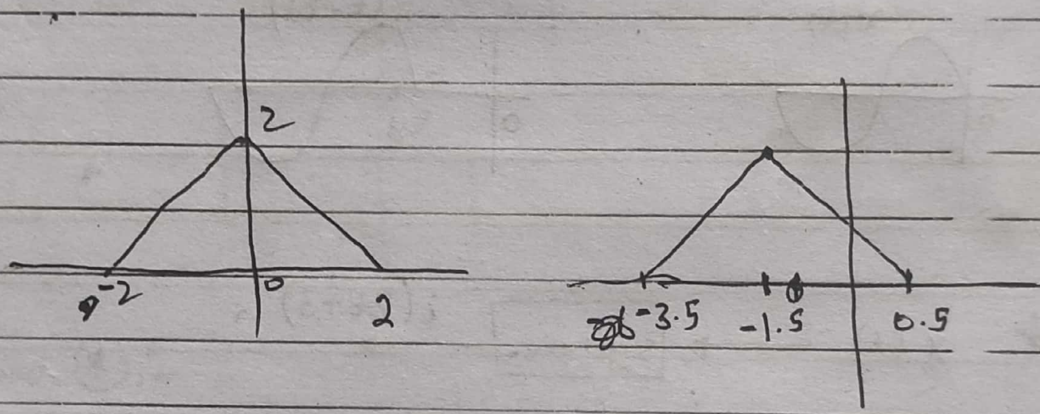


$t+3=0$
$t=-3$

$t+3=2$
$t=-1$



$t+1.5=-2$
$t=-2-1.5$
$t=-3.5$

$t+1.5=2$
$t=2-1.5$
$t=0.5$

$t+1.5=0$
$t=-1.5$

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if $\boxed{h(t)=\delta(t)} \rightarrow x(t)=y(t)$

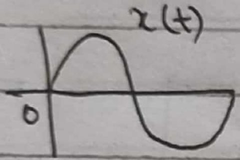
$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = x(t) + h(t) + n(t)$

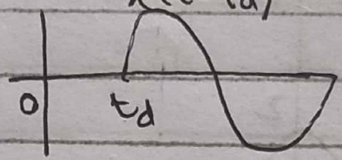
$x(t) \xrightarrow{A.S} \alpha\, x(t)$

$x(t) \xrightarrow{T.\,scaling} x(\beta t)$

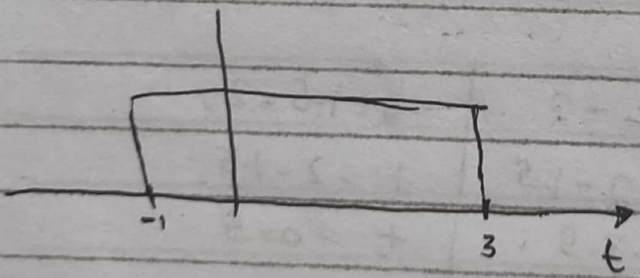
$x(t) \xrightarrow{T.R} x(-t)$

$x(t) \xrightarrow{T.\,Shifting} x(t+k)$



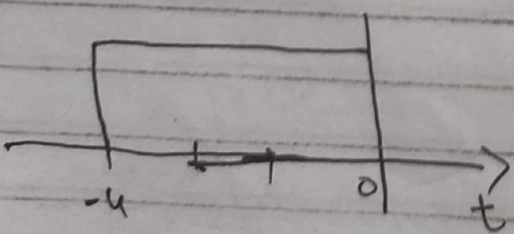


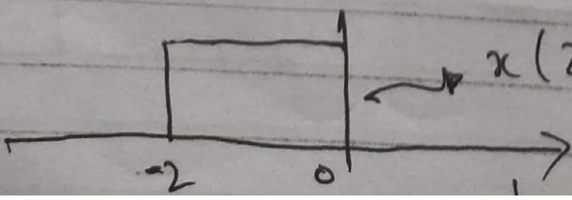
★ $x(t) \longrightarrow \boxed{\text{System}} \longrightarrow x(2t+3)$



M①:-

$x(t+3)$

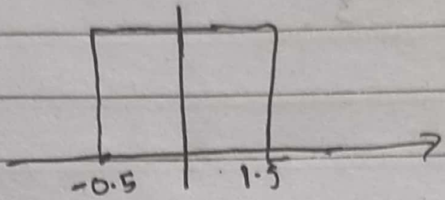




$\rightarrow x(2t+3)$

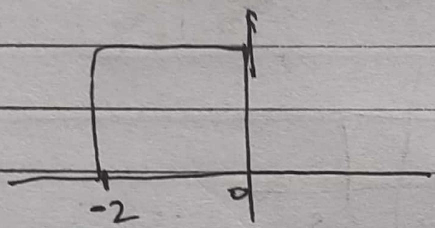
Method ②:-

$x(2t)$



$x\big(2(t+1.5)\big)$

We must shift with respect to t rather than 2t.

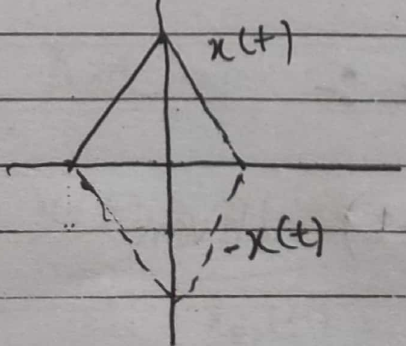


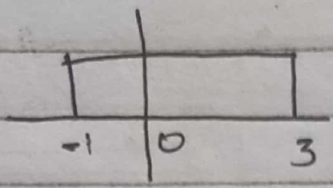
Method ③:-

$2t+3 = -1$	$2t+3 = \cancel{0}\ 3$
$2t = -4$	$2t = 0$
$\boxed{t = -2}$	$\boxed{t = 0}$

Amplitude Reversal:-

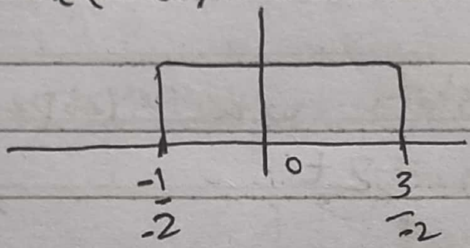
$$x(t) \xrightarrow{A.R} -x(t)$$

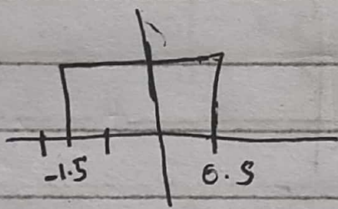




$x(-2t+3) =$

$x(-2t)$

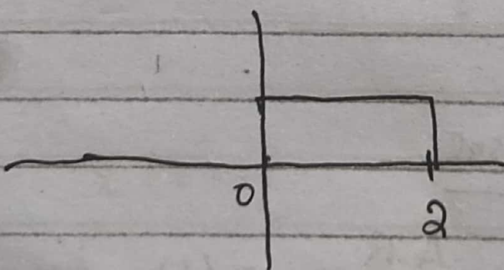




$x(-2t+3)$

$x(-2(t-1.5))$

?



Even & Odd Signals :-

$x(t) = x(-t)$ Even

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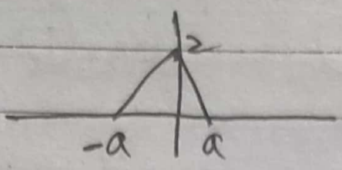
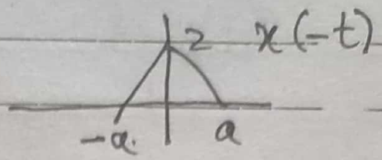
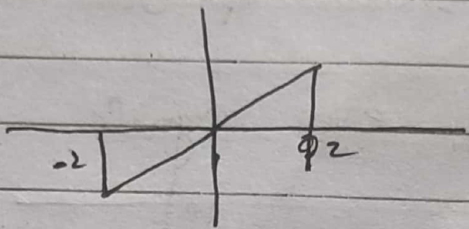
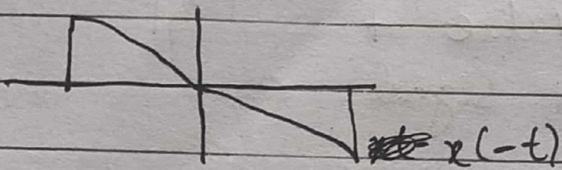


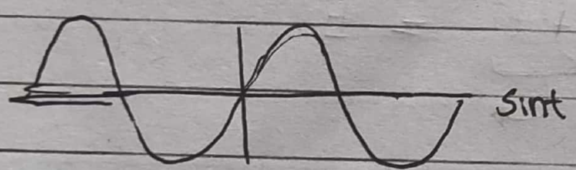


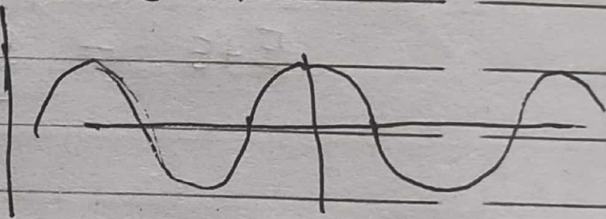
Odd Signals:-

$$x(t) = -x(-t)$$









$$x(t) = x_e(t) + x_o(t) \quad \text{—(i)}$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{—(ii)}$$

Add (i) and (ii)

$$x(t) + x(-t) = 2 \cdot x_e(t)$$

$$\boxed{x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]}$$

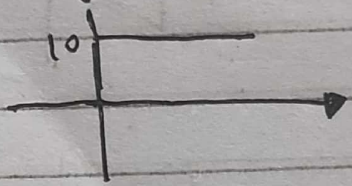
Subtract eq(ii) from (i)

$$x(t) - x(-t) = 2x_o(t)$$

$$\boxed{x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]}$$

Properties of Even & Odd Ftions:-

1. dc value is even signals



2. If dc value + Even signal

$$E + E = \underbrace{10 + t^2} = E$$

3. $E \times E = E$
4. Odd x odd = Even
5. $E \times O$ = Odd
6. $\frac{1}{E} = E$
7. $\frac{1}{Odd}$ = Odd
8. $\frac{d}{dt} E$ = Odd
9. $\frac{d}{dt} O$ = Even

(10) $\int E \, dt = \text{odd}$

(11) $\int \text{Odd} \, dt = \text{Even}$

(12) dc value + Odd = 0

$$x(t) = \underset{E}{\cos t} + \sin t + \underset{E}{\cos t \cdot \sin t}$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$\quad " = \frac{1}{2}\left[\cos t + \sin t + \cos t \cdot \sin t + \cos(-t) + \sin(-t) + \cos(-t) \cdot \sin(-t)\right]$$

$$\quad " = \frac{1}{2}\left[\cos t + \cancel{\sin t} + \cancel{\cos t \cdot \sin t} + \cos t - \cancel{\sin t} - \cancel{\cos t \cdot \sin t}\right]$$

$$\quad " = \frac{1}{2}\left[2 \cdot \cos t\right] = \boxed{\cos t}$$

$$x_o(t) = \left[\cos t + \sin t + \cos t \cdot \sin t - \left(\cos(-t) + \sin(-t) + \cos(-t) \cdot \sin(-t)\right)\right] \frac{1}{2}$$

$$x_o(t) = \frac{1}{2}\left[\cos t + \sin t + \cos t \cdot \sin t - \left(\cos t - \sin t - \cos(t) \cdot \sin(t)\right)\right]$$

$$x_o(t) = \frac{1}{2}\left[\cancel{\cos t} + \sin t + \cos t.\sin t - \cancel{\cos t} + \sin t + \cos t.\sin t\right]$$

$$x_o(t) = \frac{1}{2}\left[2.\sin t + 2\cos t.\sin t\right]$$

$$x_o(t) = \sin t + \cos t.\sin t.$$

$$x(t) = t^2.\sin t - \frac{t^3}{\sin^2 t} + t^3\cos t - \frac{\cos^2 t}{t^2} + \frac{t^5}{\sin^5 t}$$

$$x_1(t) = t^2.\sin t = E \times O = O$$

$$x_2(t) = \frac{t^3}{\sin^2 t} = \frac{O}{O \times O} = O \times \frac{1}{E}$$

$$x_2(t) = O \times E = O$$

$$x_3(t) = t^3.\cos t$$

$$x_3(t) = O \times E = \text{Odd}$$

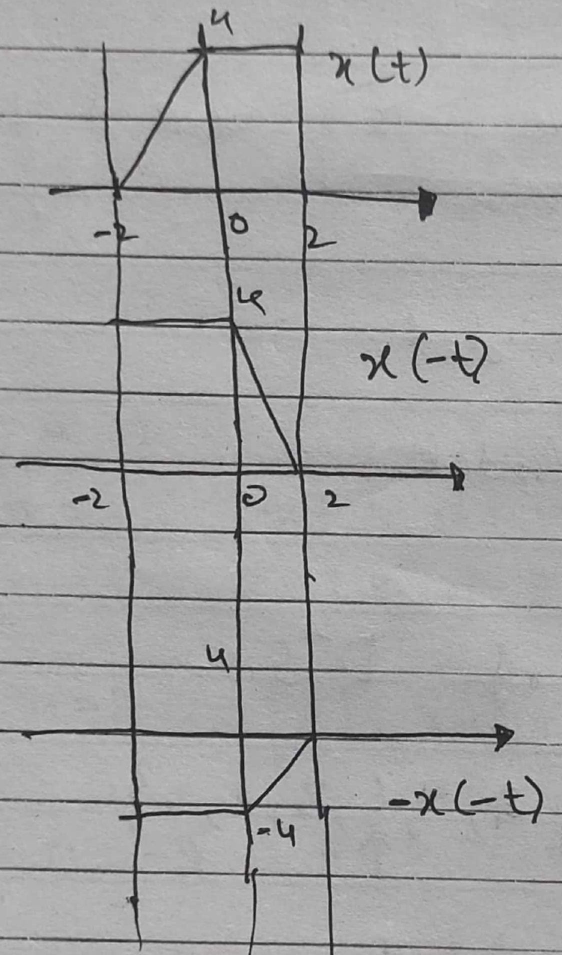
$$x_4(t) = \frac{\cos^2 t}{t^2} = E \times \frac{1}{E} = E \times E = \text{Even}$$

$$x_5(t) = \frac{t^5}{\sin^5 t} = E \times \frac{1}{O \times O \times O \times \underbrace{O \times O}} = E \times \frac{1}{O \times O \times O \times E}$$

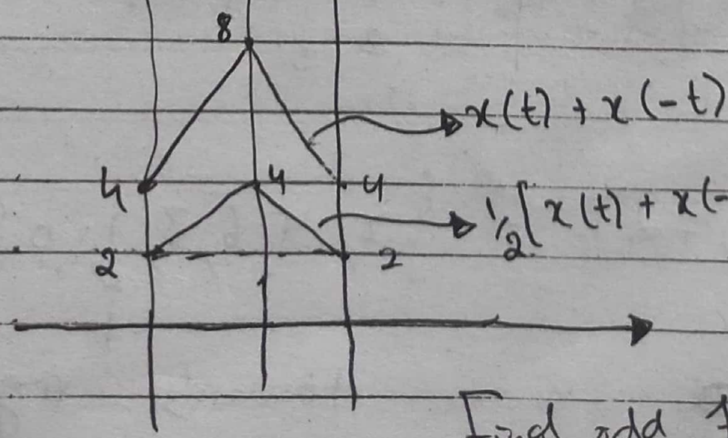
$$= E \times \frac{1}{O \times O \times O} = E \times \frac{1}{O \times E} = E \times \frac{1}{O}$$

$= \text{Odd}$

★



$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$



Find odd ftion too as an assignment

$$x(t) = x(t+nT)$$

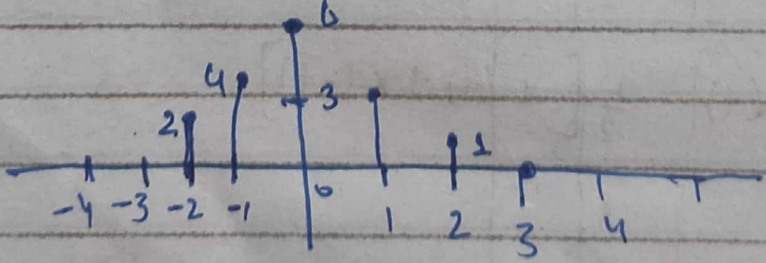
$$x[n]$$

$$\int_{-\infty}^{\infty} x(t)\,dt < \infty$$

- ☆ Gaussian Curve is used to represent noise.
- ☆ Others are
  - Relaigh Model
  - Ricean Model.

☆ Time Shifting of DTS :-

$$x[n] = \{2, 4, \underset{\uparrow}{6}, 3, 1, 0\}$$



$$x\{n+2\} = \{2, 4, 6, 3, \underset{\uparrow}{1}, 0\}$$

Arrow will move towards right.

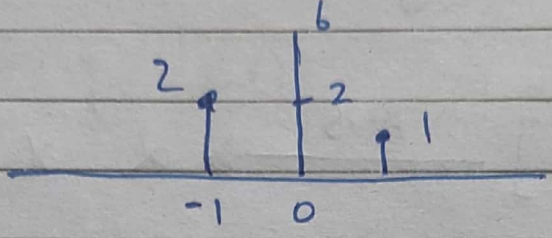
★ Time Compression of DTS:

$$x[2n]$$

$n = 0 \quad x[2 \times 0] = x[0] = 6$

$n = 1 \quad x[2 \times 1] = x[2] = 1$

$n = -1 \quad x[2 \times -1] = x[-2] = 2$

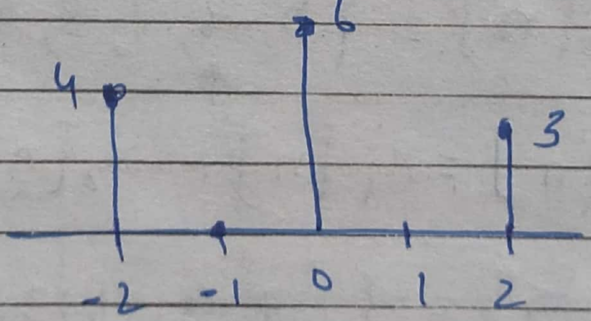


$n = 0 \quad x[\frac{1}{2} \times 0] = x[0] = 6$

$n = 1 \quad x[\frac{1}{2} \times 1] = x[\frac{1}{2}] = 0$

$n = 2 \quad x[\frac{1}{2} \times 2] = x[1] = 3$

$n = -2 \quad x[\frac{1}{2} \times -2] = x[-1] = 4$



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★ Time Reversal of DTS :-

$$x\{-n\} = \{0, 1, 3, \underset{\uparrow}{6}, 4, 2\}$$

★ Amplitude Scaling of DTS :-

$$x[n] \xrightarrow{A.S} \alpha . x[n]$$

$$x[n] = \{2, 4, \underset{\uparrow}{6}, 3, 1, 0\}$$

★ Addition of DTS :

$$x_1[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5, 6\}$$

$$x_2[n] = \{5, \underset{\uparrow}{6}, 7, 8, 9\}$$

$$x[n] = x_1[n] + x_2[n]$$

$$x[n] = \{1, 7, 9, 11, 13, 15\}$$

$x_1[n] = \{$

$$\{1, 2, \underset{\uparrow}{3}, 4, 5, 6\}$$

$$\{0, 5, \underset{\uparrow}{6}, 7, 8, 9\}$$

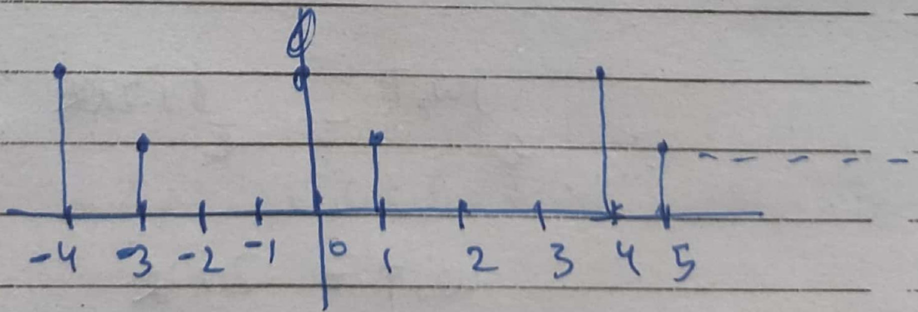
$x[n] = \{0, 10, 18, 28, 40, 54\}$ multiplied

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

★ Periodic & Aperiodic Signals:-



$N = 4$

what if its is finite ?

$$x[n] = x[n \pm kN]$$

$$x(t) = A_0 e^{j\omega_0 t}$$

$$x(t \pm nT_0) = A_0 e^{j\omega_0 (t \pm nT_0)}$$

$$x(t+T) = A_0 e^{j\omega_0 (t+T)}$$

$$x(t) = x(t+T)$$

$$\cancel{A_0}\,\cancel{e^{j\omega_0 t}} = \cancel{A_0}\,\cancel{e^{j\omega_0 t}}\, e^{j\omega_0 T}$$

$$1 = e^{j\omega_0 T}$$

$$\cancel{\ln(1) = \ln(e^{j\omega_0 T})}$$

$$\cancel{0 = j\omega_0} T$$

Generally, $e^{j2\pi k} = 1$

$$\therefore\ e^{j\omega_0 T} = e^{j \cdot 2\pi k}$$

$$\omega_0 T = 2\pi k$$

$$\boxed{T = \frac{2\pi}{\omega_0}}$$

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$\omega_0 \rightarrow$ fundamental frequency.

$$x(t) = \sin^2 4\pi t$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$x(t) = \frac{1}{2} - \frac{\cos 8\pi t}{2}$$

ω_0

$$A \cos(\omega_0 t + \phi)$$

Now $t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$ Seconds

$$f_0 = 4\ Hz$$

$x(t) = \sin 6\pi t + \cos 5\pi t$ (Composite ftion

$$T_0 = ?$$

$x_1(t) = \sin 6\pi t$, $x_2(t) = \cos 5\pi t$

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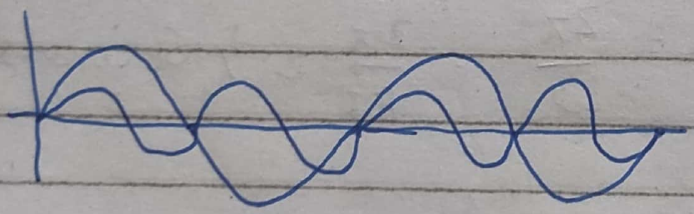
$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$ $\quad T_2 = \frac{2\pi}{5\pi} = 2/5$

$T_0 = \frac{T_1}{T_2} = LCM(T_1, T_2)$

$T_0 = \frac{LCM(1, 2)}{HCF(3, 5)}$ $\quad \frac{\text{Numerators}}{\text{Denominators}}$

$T_0 = \frac{2}{1} = 2$ sec

$f_0 = 0.5$ sec



Verify in MATLAB

$\boxed{x(t) = \sin 2\pi t + \cos 3\pi t}$ $\quad t_1 = \frac{2\pi}{2\pi} = 1$

$t_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

★ $x_1(t) = \sin 2\pi t$ $\quad T_0 = 1$ sec

$f_0 = 1$ Hz

★ Effect of time shifting on FTP

$$x_2(t) = x_1(t+2)$$

$$x_2(t) = \sin(2\pi(t+2))$$

$$x_2(t) = \sin(2\pi t + 4\pi)$$

$$\omega_0$$

No change of fundamental frequency

$$\therefore \quad T_0 = \frac{2\pi}{2\pi} = 1$$

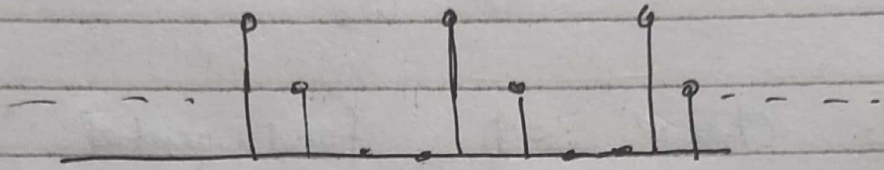
$$T_0 = \frac{LCM(1,2)}{HCF(1,3)} = \frac{2}{1}$$

$T_0 = 2\,sec$

★ Periodic Signals

$$x(t) = x(t \pm T)$$

$$x[n] = x[n \pm KN]$$



★ COMPOSITE DT SIGNAL :-

$$x[n] = x_1[n] + x_2[n]$$

Fundamental Period

$$T_1 , T_2$$

$$T_0 = \frac{T_1}{T_2} = R$$

$$N_1 , N_2$$

$$N = \frac{N_1}{N_2} = \frac{int}{int} = R$$

★ Discrete time exponential signals may be periodic / Aperiodic.

$$x[n] = A_0 e^{j\omega_0 n}$$

$$\updownarrow$$

$$x[n \pm kn] = A_0 \cdot e^{j\omega_0 (n \pm kn)}$$

$$A_0 e^{j\omega_0 n} = A_0 . e^{j\omega_0 (n \pm kN)}$$

$$\cancel{A_0} . \cancel{e^{j\omega_0 n}} = \cancel{A_0} . \cancel{e^{j\omega_0 n}} . e^{j\omega_0 N}$$

$$1 = e^{j\omega_0 N}$$

$$e^{j2\pi k} = 1$$

$$\therefore \quad 2\pi k = \omega_0 N$$

$$\frac{2\pi}{\omega_0} = \frac{N}{K} \rightarrow \text{Must be rational in order to be periodic.}$$

Example :-

$$x[n] = e^{j2n}$$

$$N = \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{\pi} \longrightarrow \text{is irrational}$$

It is Aperiodic.

$x[n] = \cos \frac{3\pi}{4} n$

$\frac{N}{K} = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{2\cancel{\pi}}{3\cancel{\pi}} \times 4 = \frac{8}{3}$

$\frac{N}{K} = \frac{8}{3} \Rightarrow N = \frac{8}{3} \times K$

For $K = 3$, $\boxed{N_0 = 8}$

$K = 6$, $N = 16$

$x[n] = \cos \frac{3\pi}{4} + \sin \frac{5\pi}{4} n$

$\frac{N_1}{K} = \frac{8}{3}$

$N_1 = \frac{8}{3} \times K$

$\boxed{N_1 = 8}$

$\frac{N_2}{K} = \frac{2\cancel{\pi}}{5\cancel{\pi}} \times 4$

$\frac{N_2}{K} = \frac{8}{5}$

$N_2 = \frac{8}{5} \times K$

$\boxed{N_2 = 8}$

$N_0 = LCM(8, 8)$
$= 8$

★ Discrete - Time Energy Signal.

which have finite energy.

$|E| < \infty$

$$E_x = \sum_{n=0}^{\infty} |x[n]|^2$$

$$x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$$

$$E_x = (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2$$

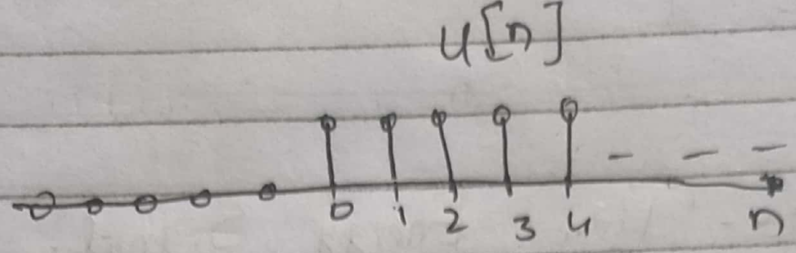
$$E_x = 1 + 4 + 9 + 16 + 25$$

$$E_x = 55$$

$$x[n] = \left(\tfrac{1}{3}\right)^n \cdot u[n]$$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$E_x = \sum_{n=0}^{\infty} |x[n]|^2$$



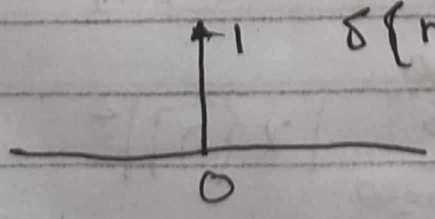
$$E_x = 1 + \left(\tfrac{1}{9}\right)^{1} + \left(\tfrac{1}{9}\right)^{2} + \left(\tfrac{1}{9}\right)^{3} + \left(\tfrac{1}{9}\right)^{6} \cdots$$

$$S_\infty = \frac{a_0}{1-r} = \frac{1}{1-\left(\frac{1}{9}\right)} = \frac{1}{\frac{9-1}{9}} = \frac{9}{8}$$

$$\cancel{E_x = 1 + \tfrac{1}{9}(1) +}$$

② $x[n] = \delta[n]$ → Unit Impulse Signal → have the prop. to extract sample.

Exist only at $x=0$



$E_x = 1$

★ Is there any effect of the time reversal, time shifting, amplitude reversal and time expansion on energy of the signal?

$$x[n] = [1, \underset{\uparrow}{2}, 3, 4, 5]$$

$$x[n+2] = [1, 2, 3, \underset{\uparrow}{4}, 5]$$

★ DISCRETE TIME POWER SIGNALS:-

$x(t)$ P for continous time.

if (Periodic)

$$P = \frac{1}{T} \int_T |x(t)|^2 dt$$

else

$$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 dt$$

★ $x[n]$ $\xrightarrow{\text{APeriodic}}$ $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x[n]|^2$

$$P = \frac{1}{N} \sum_{-N}^{N} |x[n]|^2$$

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★ Conjugate Symmetric Signals:-

$a+ib \qquad a-ib$

$$x(t) = a(t) + jb(t)$$

For conjugate symmetric signals:

$$\boxed{x(t) = x^*(-t)}$$

$$x(-t) = a(-t) + jb(-t)$$

$$x^*(-t) = a(-t) - jb(-t)$$

$$a(t) = a(-t) \longrightarrow \text{Even}$$

$$b(t) = -b(-t) \longrightarrow \text{Odd}$$

① $x(t) = \underbrace{t^2}_{\text{Even}} + j\underbrace{\sin t}_{\text{Odd}}$

② $x(t) = \sin t^2 + jt^3$

★ Conjugate Anti-Symmetric Signals:

$$x(t) = -x^*(-t)$$

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$$x(t) = a(t) + j.b(t)$$

$$x(-t) = a(-t) + jb(-t)$$

$$x^*(-t) = a(-t) - j.b(-t)$$

$$-x^*(-t) = -a(-t) + j.b(-t)$$

Odd function

$a(t) = -a(-t)$, $b(t) = b(-t)$ → Even

① $x(t) = \underbrace{2.\sin t}_{\text{odd}} + j\underbrace{t^5}_{\text{odd}}$

② $x(t) = \sin^3 t + j\,t^2$ ⟹ Anti-Symmetric

$$x(t) = x_{CS}(t) + x_{CAS}(t)$$ ⟶ Note it.

$$x_{CS}(t) = \frac{x(t) + x^*(-t)}{2}$$

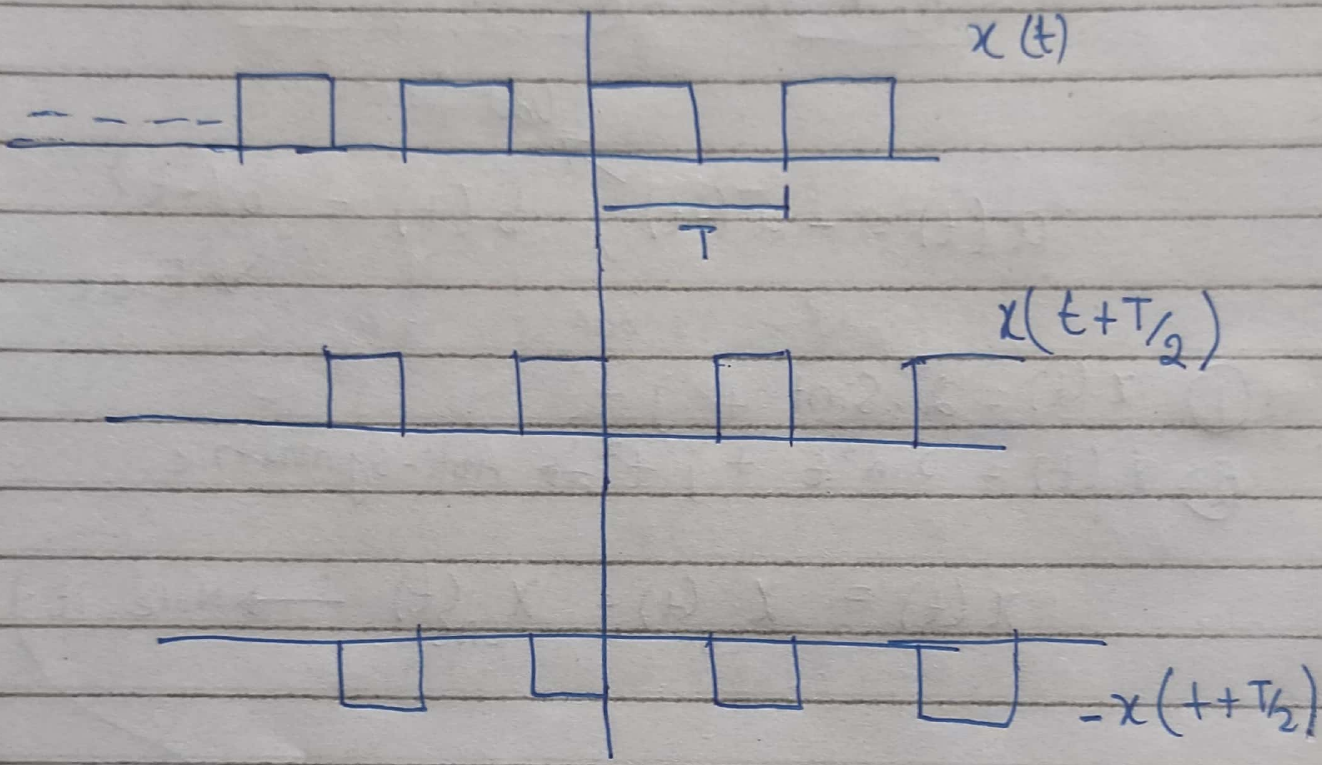
$$x_{CAS}(t) = \frac{x(t) - x^*(-t)}{2}$$

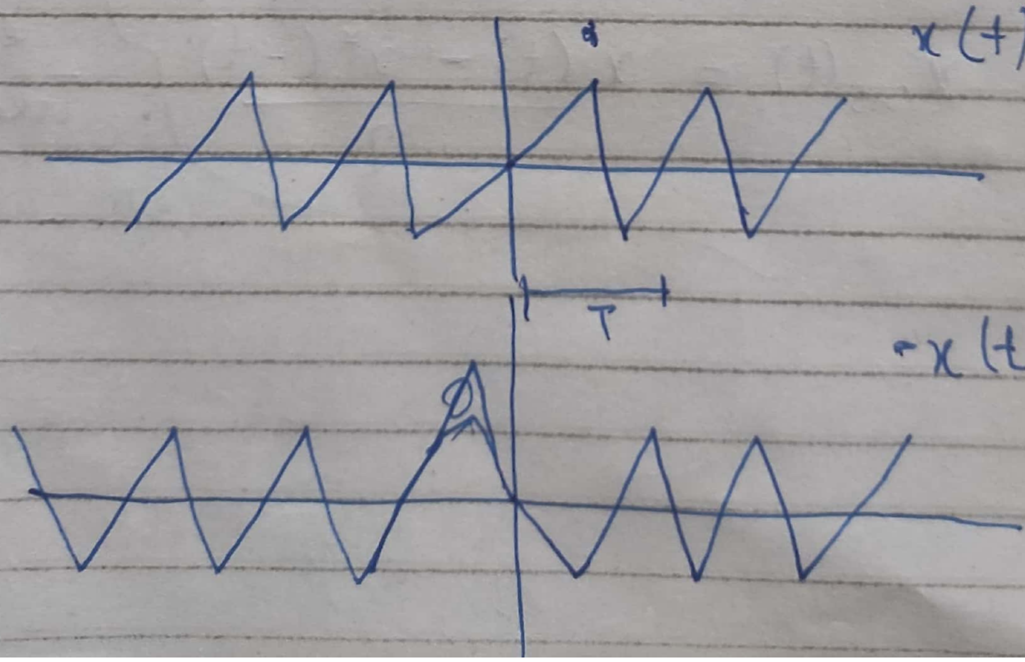
Take it as an assignment

★ Half Wave Symmetric Signals:

$$x(t) = -x(t \pm T/2)$$

→ only applicable to periodic signals.





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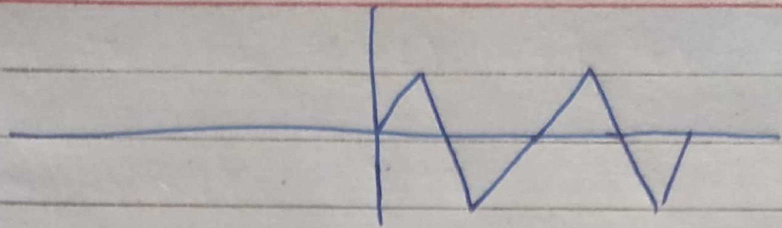
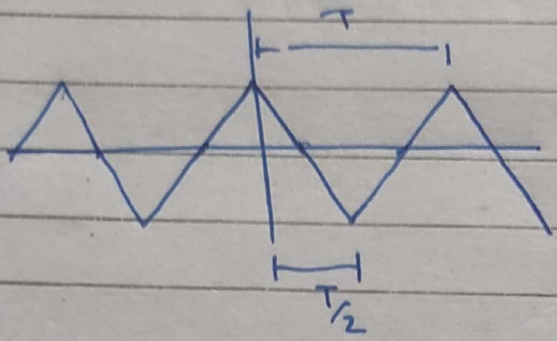
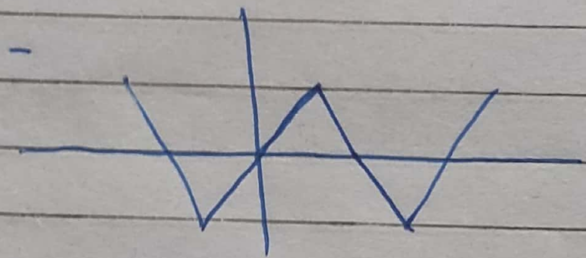
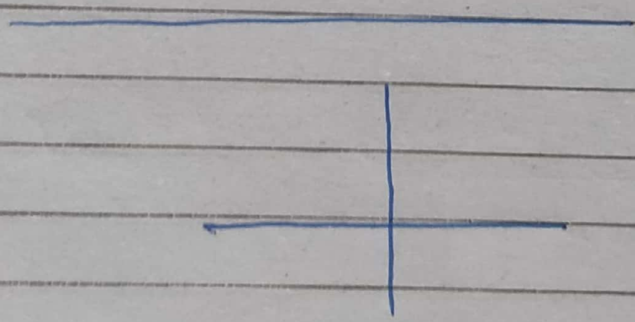






★ Area of continous time signal;

$$A = \int_{-\infty}^{\infty} x(t)\, dt$$



★ Energy & Power of CTS:-

$$P(t) = V(t) \cdot i(t)$$

$$P(t) = \frac{V^2(t)}{R} = i^2(t) R$$

Normalized 'R', $R = 1\,\Omega$

$$P(t) = V^2(t) = i^2(t) \text{ Watts}$$

$$P = \frac{W}{T} = E = P/T$$

$$E = P \times T$$

$$x_1(t) = e^{-at} u(t), \quad a > 0$$

$$E = \int_{-\infty}^{\infty} |x_1(t)|^2 dt$$

$$E = \int_0^{\infty} e^{-2at}$$

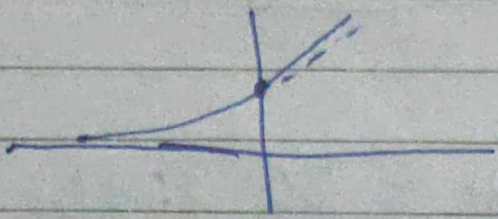
$$= \left. \frac{e^{-2at}}{-2a} \right|_0^{\infty} \quad \frac{1}{-2a}$$

$$= \frac{1}{-2a} \left[e^{-\infty} - e^0 \right] = 1/2a$$

$x_2(t) = x_1(-t)$

$x_2(t) = e^{at} \cdot u(-t)$

$u(t)$ is limiting this signal.



$$E_{x_2} = \int_{-\infty}^{0} \left| e^{at} \cdot u(-t) \right|^2 dt$$

$$E_{x_2} = \int_{-\infty}^{0} e^{2at} \, dt$$

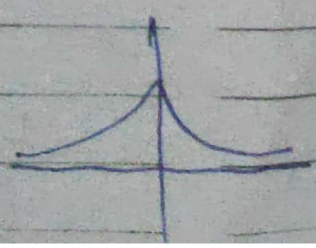
$$'' = \left. \frac{e^{2at}}{2a} \right|_{-\infty}^{0}$$

$$'' = \frac{1}{2a} \left(e^{0} - e^{-\infty} \right)$$

$$'' = \frac{1}{2}$$

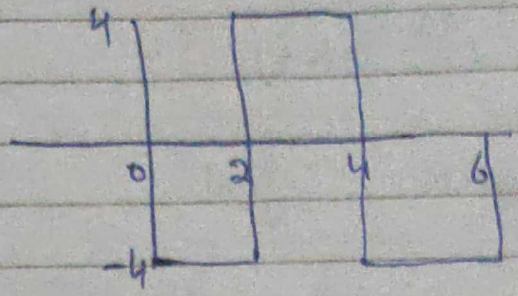
(3) =

$x_3(t) = x_1(t) + x_2(t)$

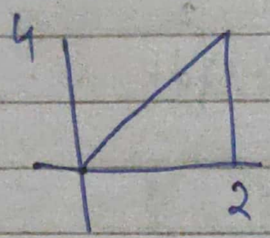


$E_3 = \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}$

Energy:



$$E = \int_0^2 16\,dt = 32 \text{ joules}$$



$y = mx + c$

$m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$

$y = 2x$

$$E = \int_0^2 |x_5(t)|^2\,dt$$

$$E = \int_0^2 |2t|^2\,dt$$

$$E = \int_0^2 4t^2\,dt$$

$$E = \left.\frac{4t^3}{3}\right|_0^2$$

$$E = \frac{4(2)^3}{3} - 0 = \frac{32}{3}$$

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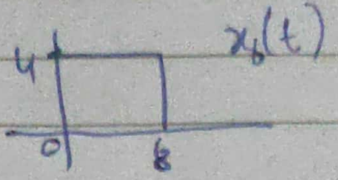
$x_6(t) =$ same
$x_6(t-2) =$ same



$x_7(t) \longrightarrow E = 4$

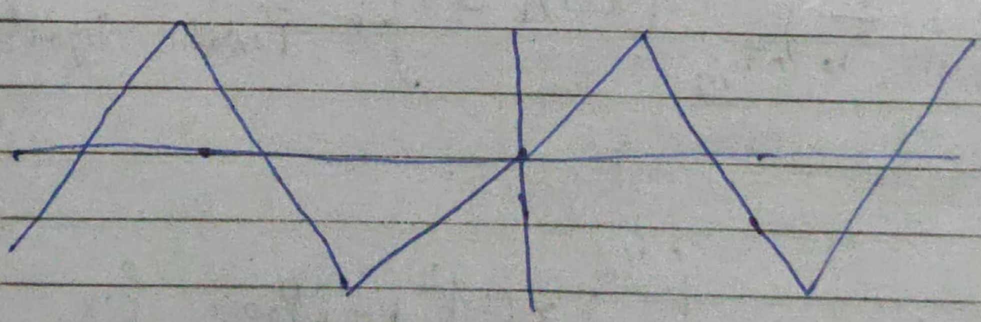
$y(t) = 2j\, x_7(2t-1)$

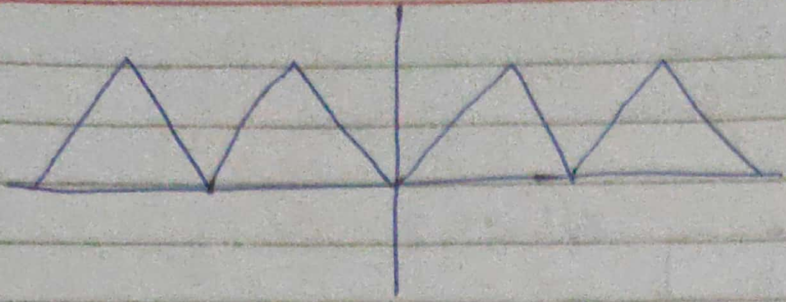
$x_7(t-1) \longrightarrow E = 4$

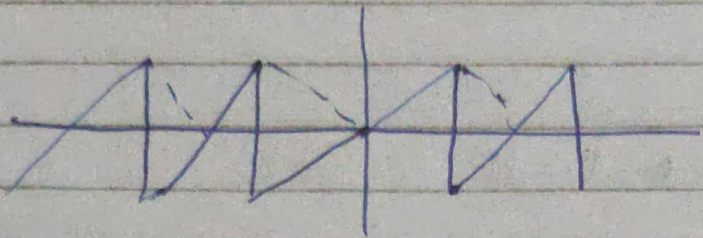
$x_7(2t-1) \longrightarrow E = 2$

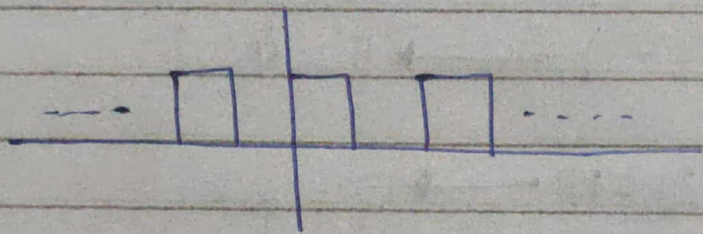
H.W

Power Signals :-









$$Ave = \frac{\text{Total Area}}{\text{Total time}}$$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt$$ → Power for Periodic Signal

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$ → Power for Non-periodic signal.

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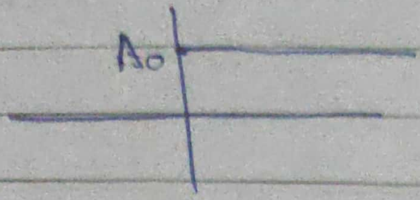
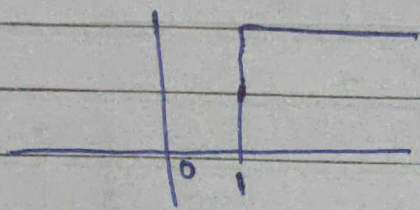


$$\lim_{T \to \infty} \frac{1}{T}\int_{0}^{T/2} A_0^2 \, dt = \frac{A_0^2}{2}$$



$A_0 u(t-1)$ will have same power.

$x_1(t) = A_0 \sin\omega_0 t$

$P = ?$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A_0^2 \sin^2\omega_0 t \, dt$$

$$P = \frac{A_0^2}{T_0}\int_{-T_0/2}^{T_0/2} \left[\frac{1 - \cos 2\omega_0 t}{2}\right] dt$$

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$$P = \frac{A_0^2}{T \times 2}$$

$$P = \frac{A_0^2}{2}$$

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$P < \infty$ Power Signal

$P = 0$ Energy "

$P = \infty$ Neither.

$$x(t) = x_1(t) + x_2(t)$$

$$P_x = P_{x_1} + P_{x_2}$$

Valid only if $x_1(t)$ and $x_2(t)$ are mutually independant signals/orthogonal signals

$$x(t) = A_0 \cdot \cos \omega_0 t \Rightarrow A_0^2/2$$

$$= A_0 \sin(\omega_0 t + \pi/2) \Rightarrow A_0^2/2$$

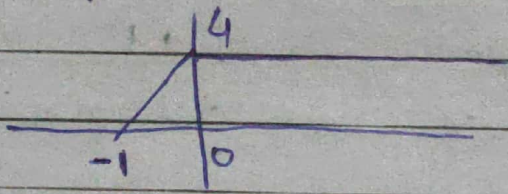
RMS

$$\sqrt{P} = \sqrt{\frac{A_0^2}{2}} \Rightarrow \sqrt{P} = \frac{A_0}{\sqrt{2}}$$

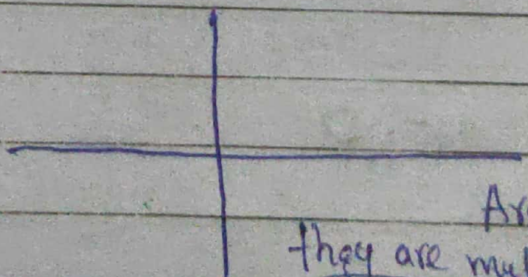
$$x(t) = 4j\, x_1(t+2)$$

$$P_{av} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |4j \cdot x_1(t+2)|^2\, dt$$

$$P_{av} =$$



$$\frac{A_0^2}{2}$$



Are orthogonal b/c they are multiples of each other

$$x(t) = 5\cos(10t + \phi) + 10\sin(5t + \phi)$$

$$P_x = \frac{5^2}{2} + \frac{10^2}{2} = \frac{25+100}{2} \quad \frac{5^2}{2} + \frac{10^2}{2} = 62.5 \text{ Watts}$$

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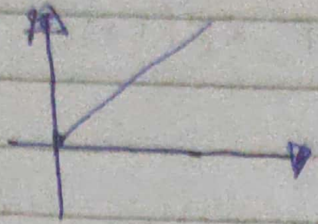
If not orthogonal then make $C.\sin(\alpha+\beta)$

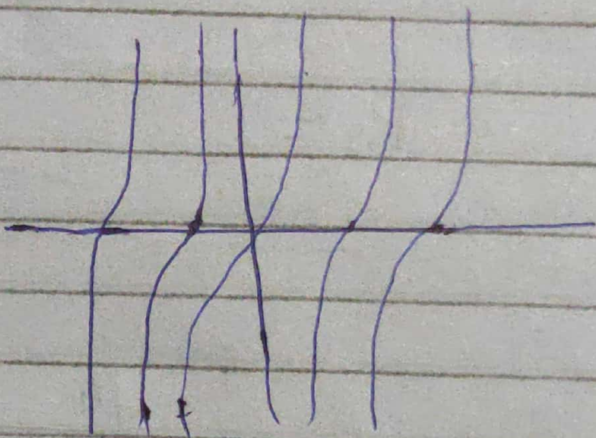
$x(t) = u(t).t$ Not Energy Nor Power

NENP



$x(t) = \tan t$

$x(t) = \frac{\sin t}{\cos t}$



This is a periodic signal but we cant determine its power.

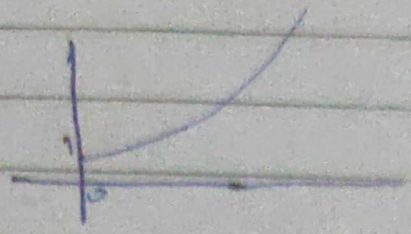
NENP

$x(t) = t^2$ NENP

$x(t) = \frac{1}{t}$

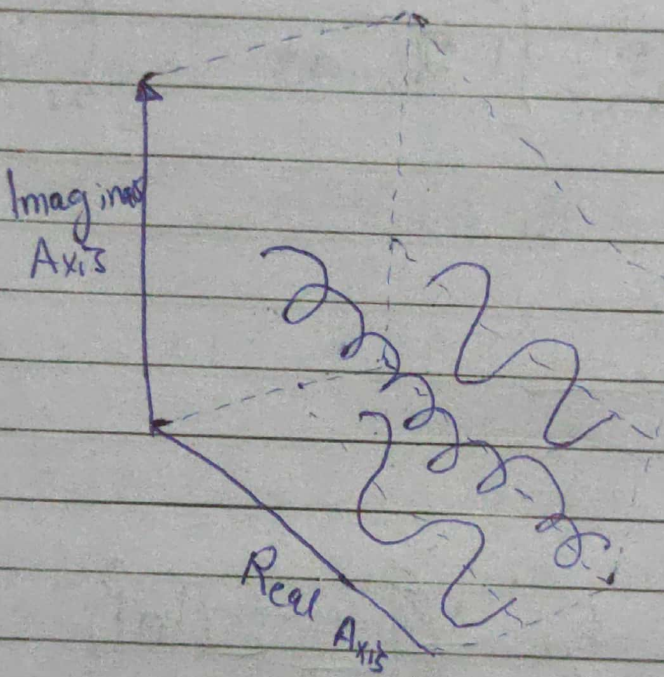
NENP

$x(t) = e^{t} \cdot u(t)$ NENP



$$x(t) = e^{j(2t + \pi/4)}$$

$$= \cos(2t + \pi/4) + j\sin(2t + \pi/4)$$



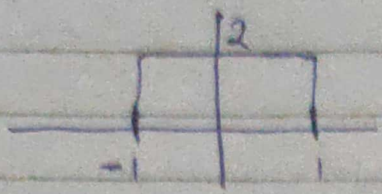
$$e^{jx} = \cos x + j\sin x$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \left| e^{j(2t+\pi/4)} \right|^2 dt$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} 1\, dt = \frac{1}{T_0} \cdot T_0 = \underline{1}$$

Date:

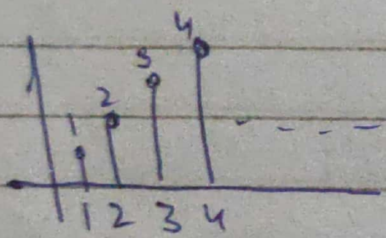
$x(t) = 2 \cdot rect(\underline{t/2}) \rightarrow$ Duration



$E_x = 4$

$E_x = \int_{-1}^{1} (2 \cdot rect(t/2))$

$E_x = \int_{-1}^{1} (2)^2 \, dt = \int_{-1}^{1} 4 \, dt$



$P = \lim_{N \to \infty} \frac{1}{N} \sum_{i=\infty}^{\infty} (|a|)^2$

$P = \lim_{N \to \infty} \frac{1}{(2N+1)} \cdot \sum_{n=-N}^{N} |x[n]|^2$

$P = \lim_{N \to \infty} \frac{1}{2N+1} \cdot \sum_{n=0}^{N} n^2$

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$$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\lim_{N \to \infty} \frac{1}{\cancel{2N+1}} \cdot \frac{N(N+1)\cancel{(2N+1)}}{6} = \infty$$

Date 3/4/2023

ORTHOGONAL SIGNALS:-

$$\int x_1(t) \cdot x_2(t) \, dt = 0$$

$$x(t) = x_1(t) + x_2(t)$$

$P_x = P_{x_1} + P_{x_2} \longrightarrow$ Valid only if x_1 and x_2 are orthogonal signals.

★ PROPERTIES OF ORTHOGONAL SIGNALS:

(i) $x(t) = \sin(m\omega_0 t + \varphi_1) + \sin(n \cdot \omega_0 t + \varphi_2)$

$$m \neq n$$

then $\int_0^T x_1(t) \, x_2(t) \, dt = 0$

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★ (ii) $x(t) = \sin(m\omega_0 t + \varphi_1) + \cos(n\omega_0 t + \varphi)$

$$m = n, \quad \varphi_1 = \varphi_2$$

(iii) dc value $+ \sin(\omega_0 t + \varphi)$

$$\int_0^T \text{dc value} \sin(\omega t + \varphi)\, dt$$

(iv) $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)\, dt = 0$

★ $y(t) = 2\sin(3\omega_0 t + 45°) + 4\sin(4\omega_0 t + 35°)$

$$P_y = \frac{2^2}{2} + \frac{4^2}{2} = 2 + 8 = 10 \text{ watts}$$

★ $x(t) = 2\cos(2\omega_0 t + 45°) + 3\sin(2\omega_0 t + 45°)$

★ $x(t) = 2 + 4\sin(3\omega_0 t + 35°)$

★ $x(t) = 2\sin 3t + 3\cos(3t + \pi/3)$

$$= 2\sin 3t + 3\sin\left(3t + \frac{\pi}{3} + \pi/2\right)$$

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$$A_1 \sin(\alpha + \varphi_1) + A_2 \sin(\alpha + \varphi_2) = A_0 \sin(\alpha + \varphi)$$

$$A_1 \sin\alpha \cdot \cos\varphi_1 + A_1 \cos\alpha \sin\varphi_1 + A_2 \sin\alpha \cdot \cos\varphi_2 +$$

$$A_2 \cdot \cos\alpha \cdot \sin\varphi_2 = A_0 \cdot \sin\alpha \cdot \cos\varphi + A_0 \cos\alpha \cdot \sin\varphi$$

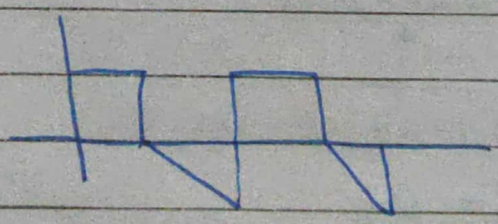
$$(A_1 \cos\varphi_1 + A_2 \cdot \cos\varphi_2) \sin\alpha + (A_1 \sin\varphi_1 + A_2 \cdot \sin\varphi_2) \cos\alpha$$

$$A_1 \cos\varphi_1 + A_2 \cos\varphi_2 = A_0 \cdot \cos\varphi \quad \text{— (i)}$$

$$A_1 \sin\varphi_1 + A_2 \cdot \sin\varphi_2 = A_0 \cdot \sin\varphi \quad \text{— (ii)}$$

Square and add

$$A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\varphi_1 - \varphi_2)}$$



☆ UNIT IMPULSE SIGNAL :—

$$\delta(t) = \begin{cases} \infty & \text{at } t = 0 \\ 0 & \text{at } t \neq 0 \end{cases}$$

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(i) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

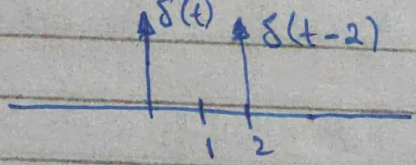
(ii) $\int_{-\infty}^{\infty} A_0 \,\delta(t)\, dt = A_0$

(iii) $\int_{-\infty}^{t} \delta(t)\, dt = u(t)$

(iv) $\delta(t) = \delta(-t)$

(V) NENP b/c $P = \infty$, $E = \infty$

(vi) $\delta(t) \xrightarrow{T.S} \delta(t-2)$



vii)

$\delta(t) \xrightarrow{T.S} \delta(at)$

$\frac{1}{|a|} \cdot \delta(t)$

$\int_{-\infty}^{\infty} f(t) \cdot \delta(at)\, dt$

$\tau = at$

$d\tau = a \cdot dt$

$dt = \frac{d\tau}{a}$

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$$\frac{1}{|a|}\int_{-\infty}^{\infty} f(\tau/a)\,\delta(\tau)\,d\tau$$

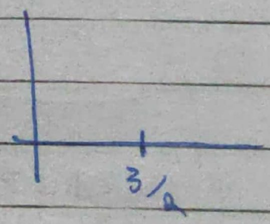
$$\frac{1}{|a|}\cdot f(0)$$

$$f(0) = \int_{-\infty}^{\infty} f(t).\,\delta(t)\,dt$$

$$\boxed{\delta(at) = \frac{1}{|a|}\cdot\delta(t)}$$

★ $\delta(-2t+3)$

$$\delta(-2(t-1.5))$$



$$\therefore \frac{1}{2}\cdot\delta(t-1.5)$$

(Viii) Multiplication

$$x(t).\,\delta(t-t_1)$$

$$\rightarrow x(t_1)$$

$$y(t) = 2t^2.\,\delta(t-3)$$

$$2(3)^2 = 18$$

$2(t-1)$

1

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$\int_{-\infty}^{\infty} \cos \pi t \cdot \delta(t-2)\, dt$

$\cos 2\pi$

$e^{-t} \delta(2t-2)$

$\frac{e^{-1}}{|2|} = \frac{1}{2e}$

$\int_{-\infty}^{\infty} e^{-2t} \delta(-2t+3)\, dt$

$\delta(-2(t-1.5))$

$\frac{1}{2} e^{-2(1.5)} = \int_{-\infty}^{\infty} \frac{e^{-3}}{2}$

$\cos 2\pi + 3 + \frac{1}{2} \sin \frac{\pi}{2} = \frac{e^{-3}}{2}$

$1 + 3 + \frac{1}{2}$

$4 + \frac{1}{2} = 4.5$

★ Signum (t) = 1   t > 0

0   t = 0

−1   t < 0

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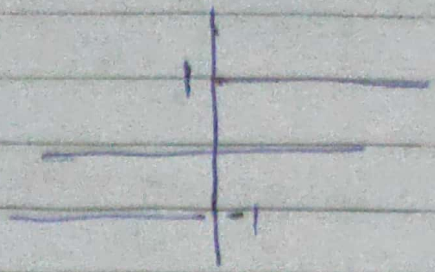
17 is a sign func.



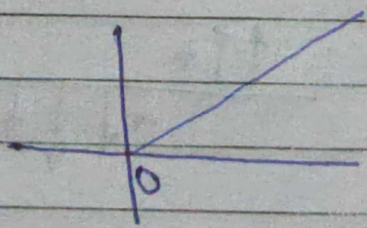
$$\frac{1 + \text{Sign}(t)}{2} = u(t)$$

$$1 + \text{Sign}(t) = 2.\, u(t)$$

$$\frac{d\, \text{Sign}(t)}{dt} = 2 \frac{d\, u(t)}{dt}$$

$$\frac{d.\, \text{Sign}(t)}{dt} = 2\, \text{Sign}(t)$$

Ramp Function



$r(t) = t.\, u(t)$

$$r(t) = \begin{cases} t.\, u(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

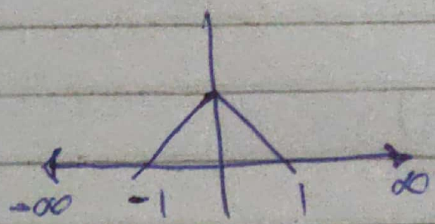
$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

$$r(t) = \int_0^t 1 \, d\tau = \tau \Big|_0^t = t\, u(t)$$

By integrating $u(t)$, we get $r(t)$.

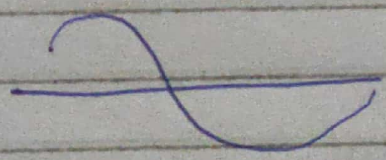
★ Unit Triangle Function

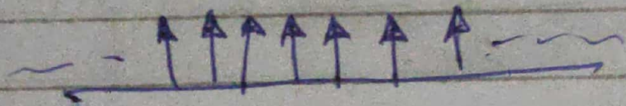
$$tri(t) = t + 1 \qquad -1 \le t \le 0$$
$$= -t + 1 \qquad 0 \le t \le 1$$



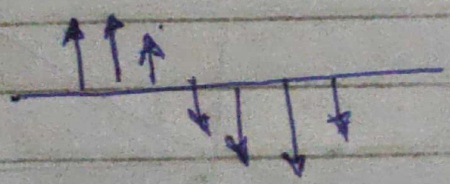
~~Property~~ Applications of Unit Impulse ftion.

★ ① Sampling / Sifting





$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\, x(t)$$



★ Sampling theorem: or

Nyquist Criteria

e.g $B = 0 - 4KHz$

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$f_s = 4K \times 2 = 8000$ Samples/sec

(2) Quantization:-

(3) Encoding.

4×8000 Samples/sec

32kbps

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