Date: 2/5/2023 S.S Day: M T W T F S

Systems :-

$$\xrightarrow{\text{i/p}} \boxed{\text{System}} \xrightarrow{\text{o/p}}$$

$x(t)$ $h(t)$ $y(t)$

$$y(t) = x(t) * h(t)$$

$$y(t) = 0.8\, x(t)$$

$$x(t) \longrightarrow y(t) = x(t - t_d)$$

$$y(t) = x(t)$$

$y(0) = x(0)$ Present i/p

$$y(t) = x(t-1)$$

$$t = 0$$

$y(0) = x(-1)$ Previous

$$y[n] = x[n-1]$$

$$y(t) = x(t+1)$$

$$y(0) = x(1)$$

Memory less System

★ Static & Dynamic Systems:

$y(t) = x(t)$

$y(t) = x(t) + x(t-1)$

★ $y(t) = 2x(t)$

$t = 0$

$y(0) = 2 . x(0)$

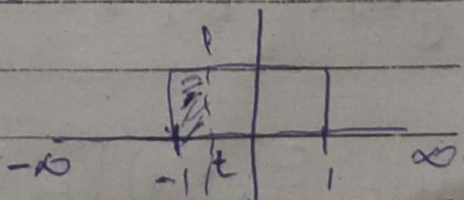
★ $y(t) = x(2t)$

$y(0) = x(0)$

$y(1) = x(2)$] → Dynamic

★ $y(t) = x(-t)$ → Dynamic

★ $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$ →



★ $y(t) = x(\sin t)$

$y(0) = x(0)$

$y(\pi) = x(0)$

If there is a time scaling
time shifting
or integration.
then system is dynamic.

$y(t) = Re[x(t)]$

$x(t) = a + ib$
$x^*(t) = a - ib$

$x(t) + x^*(t) = 2a$

$$\boxed{a = \frac{x(t) + x^*(t)}{2}}$$

Now $y(t) = Re[x(t)] = a = \frac{x(t) + x^*(t)}{2}$

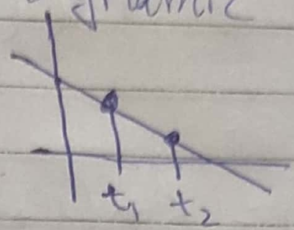
$y(t) = E[x(t)]$

$y(t) = E[x(t)] = \frac{x(t) + x(-t)}{2}$

Date: $K=3$, $K<2$, $K=1$ Day: M T W T F S

(9) $y(t) = x(t^3)$ —— Dynamic

(10) $y(t) = \frac{d}{dt} x(t) \longrightarrow$ Dynamic



★ Causal & Non Causal System

↳ does not depend upon future values.

(i) $y(t) = x(t) \longrightarrow$ Causal

$y(t) = x(t) + x(t+1) \longrightarrow$ Non Causal

$y(t) = x(t+1) \longrightarrow$ Anti - Causal

Causal :- Past / Present

Non Causal :- Past / Present + Future

Anti Causal :- Future

(i) $y(t) = x(t+2)$

(ii) $y(t) = x(t) + x(t-1) + x(t+1)$

(1) $y(t) = x(3t)$ Non-Causal

(2) $y(t) = \begin{cases} x(3t) & t < 0 \\ x(t-1) & t \geq 0 \end{cases}$ → Causal System

(3) $y(t) = \sin(t+1) \cdot x(t-1)$

Causal

$y(0) = \sin(1) \cdot x(-1)$

(4) $y(t) = x(e^t)$

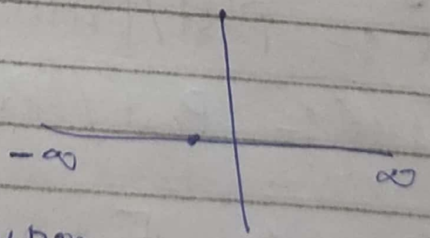
Non-Causal

$y(0) = x(e^0) = x($

---

(5) $y(t) = \int_{\infty}^{t} x(\tau)\,d\tau$

Causal



$\cancel{y(t)}$ = depends upon present and past values.

(6) $y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau$

Non-Causal

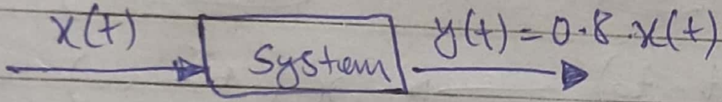
$x(-\infty)$ $x(t+1)$

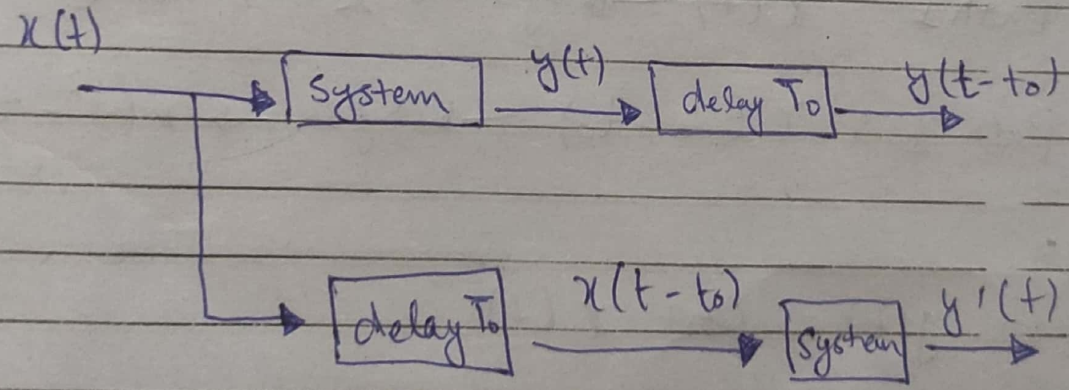
⑦ $y(t) = \int_{-\infty}^{t} x(3\tau)\, d\tau$

Non-Causal

⑧ $y(t) = \frac{d}{dt} \cdot x(t)$

Time Invariant and Time Variant Systems:-





$y'(t) = y(t-t_0)$ TIV

$y'(t) \neq y(t-t_0)$ TV

Date: Day:

① $y(t) = x(\cos(t))$

Path I: $x(t) \to \text{System} \to x(\cos t) \to x[\cos(t-t_0)]$

Path II: $x(t) \to x(t-t_0) \to \text{System} \to x[\cos(t) - t_0]$

② $y(t) = x(t^2)$

Path I = $x(t^2) \to x((t-t_0)^2)$

Path II = $\cancel{x((t-t_0))} \to \cancel{x(t^2-t_0)}$

$x(t-t_0) \to x(t^2-t_0)$

Path I: $x(t) \to \text{Sys} \to x(t^2) \to x[(t-t_0)^2]$

Path II: $x(t-t_0) \to \text{Sys} \to x(t^2-t_0)$

③ $y(t) = \cos t \cdot x(t)$

Path I: $x(t) \to \text{Syst} \to \cos t \cdot x(t) \to \cos(t-t_0) \cdot x(t-t_0)$

Path II: $x(t) \to \cancel{\cos(t-t_0) \cdot x(t-t_0)}$ $\cos(t) \cdot x(t-t_0)$

$x(t-t_0) \to$

Date: 9/5/2023 Day: M T W T F S

$$x(t) \xrightarrow{} \boxed{\text{System}} \xrightarrow{} y(t)$$

$h(t)$

$X(s)$ $Y(s)$

$x(t) * h(t)$ → convolution

then $X(s) \cdot H(s) = Y(s)$ → simple multiplication.

$x(t)$ → System → Delay → $y(t)$

$x(t)$ → Delay → System → $y'(t)$

if $y(t) = y'(t)$ Time Invariant

$y(t) = x(t^3)$

$x(t)$ → System → $y(t) = x(t^3)$ → delay

$(x(t-t_0)^3)$

x

If the power operator exist in input then system will be time invarient

---

$e^{t} \cdot x(t)$ — Sys — $\boxed{e^{-t}}\, x(t)$ → $e^{-(t-t_0)} \cdot x(t-t_0)$

$e^{-t} \cdot x(t-t_0)$ → Sys → $e^{-t} \cdot x(t-t_0)$

a co-efficient then, TV

Date: Whenever there is Day: M T W T F S

$y(t) = 2t + x(t) \longrightarrow Sys \rightarrow 2t + x(t)$

$2(t-t_0) + x(t-t_0)$

$2t + x(t-t_0) \longrightarrow Sys \longrightarrow 2t + x(t-t_0)$

If any addition or subtraction other than the input ~~and output~~ signal the system is said to be $\boxed{TV}$

If any added or subtracted term is constant at ~~all~~ ~~the~~ instance of time then the system is TIV

If additive or subtractive term depends upon t, then TV

If there is a time scaling, TIV

$x(t-1) + x(t+1) \longrightarrow x(t-t_0-1) + x(t-t_0+1)$

$x(t-t_0-1) + x(t-t_0+1) \longrightarrow x(t-t_0-1) + x(t-t_0+1)$

Date:

Day: M T W T F S

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

$$x(\tau) \xrightarrow{Sys} \int_{-\infty}^{t} x(\tau)\,dt \xrightarrow{delay} \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$$

$$x(\tau) \xrightarrow{delay} x(\tau - t_0) \xrightarrow{Sys} \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$$

$$y(t) = \frac{d}{dt} x(t)$$

$$Sys \rightarrow \int_{-\infty}^{t} x(3\tau)\,d\tau \longrightarrow \int_{-\infty}^{t} x(3(\tau - t_0))\,d\tau$$

$$x(\tau - t_0) \longrightarrow \int_{-\infty}^{t} x(3(\tau - t_0))\,d\tau$$

$$\int_{-\infty}^{t} x(3\tau - t_0)\,d\tau$$

TV

Integration of time scaled

Date: Day: M T W T F S

★ $y(t) = \int_{-\infty}^{t} \cos\tilde{\tau} \cdot x(\tilde{\tau})\, d\tilde{\tau}$

~~y(t)~~ $x(t) \longrightarrow \text{sys} \xrightarrow{\text{delay}} \int_{-\infty}^{t} \cos(\tilde{\tau} - t_0)\, x(\tilde{\tau} - t_0)\, d\tilde{\tau}$

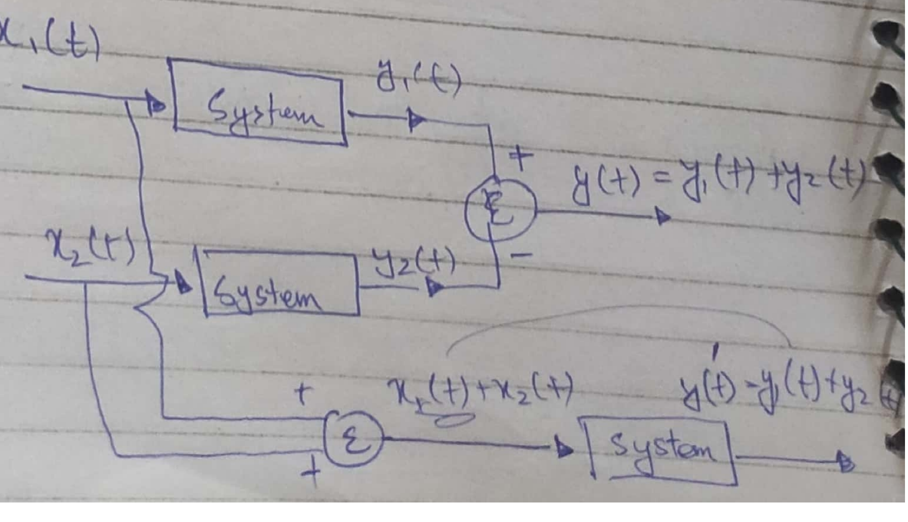
Linear & Non-Linear Systems :-

→ which follows the law of superposition are linear.

(i) Law of additivity — LOA

(ii) Law of homogenity — LOH

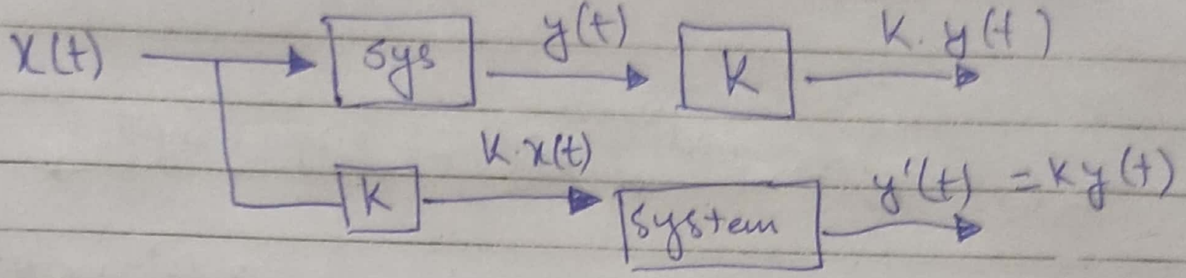
(i) LOA :-



Date: Day: M T W T F S

if $y'(t) = y(t)$ LOA

$y'(t) \neq y(t)$ LOA ✗



if $y'(t) = K \cdot y(t)$ LOH

$\neq K y(t)$ LOH ✗

$y(t) = x(\sin t)$

1st Path

$x_1(t) \xrightarrow{Sys} x_1(\sin t)$

$x_2(t) \xrightarrow{Sys} x_2(\sin t)$ } $x_1(\sin t) + x_2(\sin t)$

2nd Path:

$x_1(t) + x_2(t) \longrightarrow Sys \longrightarrow x_1(\sin t) + x_2(\sin t)$

$K \cdot x(\sin t)$

$K \cdot x(t) \longrightarrow K \cdot x(\sin t)$

$L(KU) = K.L(U)$

Date: Day: M T W T F S

$y(t) = x(t^2)$

$x_1(t) \xrightarrow{sys} x_1(t^2)$

$x_2(t) \xrightarrow{sys} x_2(t^2)$

$\rightarrow x_1(t^2) + x_2(t^2)$

$x_1(t) + x_2(t) \xrightarrow{add} x_1(t^2) + x_2(t^2) \xrightarrow{syst}$

$x_1(t^2) + x_2(t^2)$

$K.x(t^2)$

$K.x(t) \longrightarrow K.x(t^2)$

---

* $y(t) = x(\log t)$

$x_1(t) \xrightarrow{sys} x_1(\log t)$

$x_2(t) \xrightarrow{sys} x_2(\log t)$

$\rightarrow x_1(\log t) + x_2(\log t)$

$x_1(t) + x_2(t) \xrightarrow{sys} x_1(\log t) + x_2(\log t)$

$K.x(\log t)$

$K.x(t) \Rightarrow K\,x(\log t)$

Scanned with CamScanner

Date: Day: M T W T F S

★ $y(t) = \log[x(t)]$

$\log[x_1(t)] + \log[x_2(t)]$

$\cancel{\log[x_1(t)] + \log[x_2(t)]} \longrightarrow \cancel{\log[x_1(t)}$

$x_1(t) + x_2(t) \longrightarrow \log[x_1(t) + x_2(t)]$

$y(t) = x^2(t)$

$x_1^2(t)$

$x_2^2(t)$ — $x_1^2(t) + x_2^2(t)$

$x_1(t) + x_2(t) \longrightarrow [x_1(t) + x_2(t)]^2$

★ $y(t) = \sin t \cdot x(t)$

$\sin t \cdot x_1(t)$

$\sin t \; x_2(t)$ — $\sin t\,(x_1(t) + x_2(t))$

Date: Day: M T W T F S

$$x_1(t) + x_2(t) \xrightarrow{sys} \sin t \cdot x_1(t) + \sin t \cdot x_2(t)$$

$$\cancel{\sin t [x_1(t) + x_2(t)]}$$

If any addition / subtra other than the input signals, the sys is said to be non-linear system.

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$\int_{-\infty}^{t} x_1(\tau)\, d\tau + \int_{-\infty}^{t} x_2(\tau)\, d\tau$$

$$\cancel{x_1(\tau) + x_2(\tau)} = \int_{-\infty}^{t} (x_1(\tau)\, d\tau + x_2(\tau)\, d\tau)$$

Linear

$$y(t) = E(x(t)) \qquad E[x(t)] = \frac{x(t) + x(-t)}{2}$$

$$\left.\begin{aligned} E[x_1(t)] &= \frac{x_1(t) + x_1(-t)}{2} \\ E[x_2(t)] &= \frac{x_2(t) + x_2(-t)}{2} \end{aligned}\right] \longrightarrow x_1(t) + x_1(-t)$$

Date: Day: M T W T F S

$y(t) = Re[x(t)]$ NLinear

$x(t) = a(t) + i\,b(t)$

$$a = \frac{x(t) + x^*(t)}{2}$$

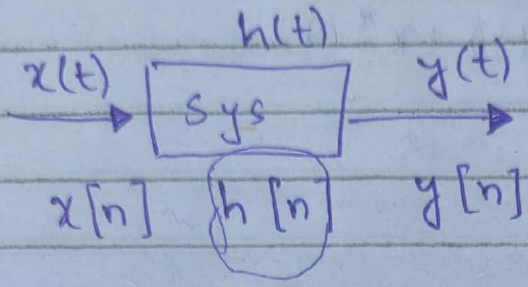
$$\frac{x_1(t) + x_1^*(t)}{2} + \frac{x_2(t) + x_2^*(t)}{2}$$

$$\frac{x_1(t) + x_1^*(t) + x_2(t) + x_2^*(t)}{2}$$

$$x_1(t) + x_2(t) \xrightarrow{\text{System}} \frac{[x_1(t) + x_2(t)] + [x_1(t) + x_2(t)]^*}{2}$$

Linear Time Invariant Systems

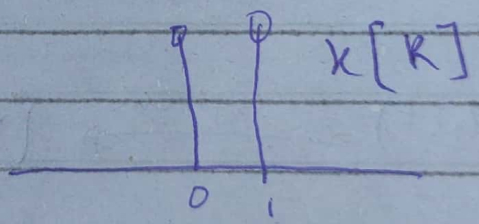
★ LTI got a fundamental role

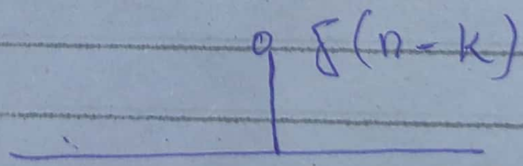


If $x(t) = \delta(t)$
then response of sys will be <u>unit impulse response</u>.

$$\delta(t)\ h(t) \longrightarrow y(t) = h(t)$$

★ $x[n]$ $\delta[n]$



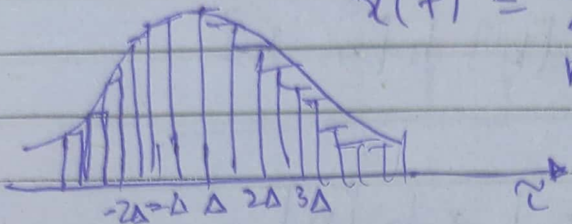


$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot\delta(n-k]$$

$$x[0]\cdot\delta[n] + x[1]\delta[n-1]$$

★ Continuous time Signals :-

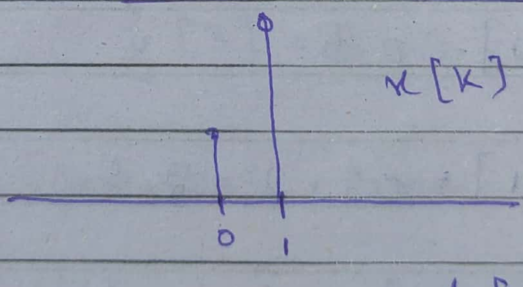
$$x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) \cdot \delta_\Delta(t - k\Delta)$$

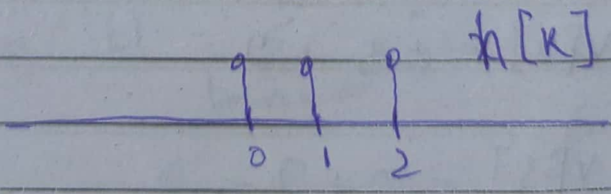


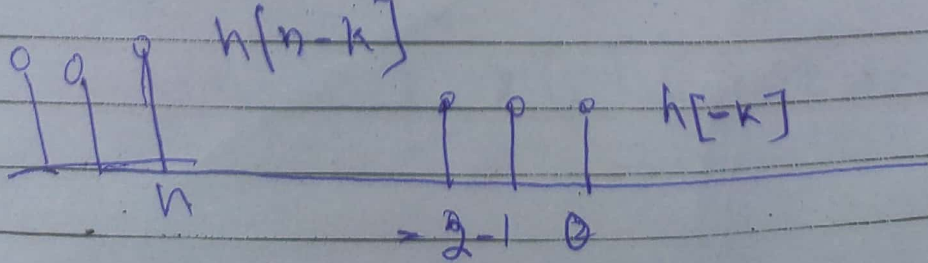
Let $k \cdot \Delta = \tilde{\tau}$

$$\lim_{k\Delta \to 0} \hat{x}(t) = \sum_{n=-\infty}^{\infty} x(\tilde{\tau}) \cdot \underbrace{\delta_\Delta(t - \tilde{\tau})}_{\text{height}} \cdot \underbrace{\Delta}_{\text{width}}$$

$$x(t) = \int_{-\infty}^{\infty} x(\tilde{\tau}) \cdot \delta_\Delta(t - \tilde{\tau})\, d\tilde{\tau}$$







Day: M T W T F S

$$y[n] = \sum_{k=-\infty}^{\infty} x[n] \cdot h_k[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$y[n] = x[n] * h[n]$$

$y[n] = 0.5(1) + \boxed{2(1)}$ → nullify

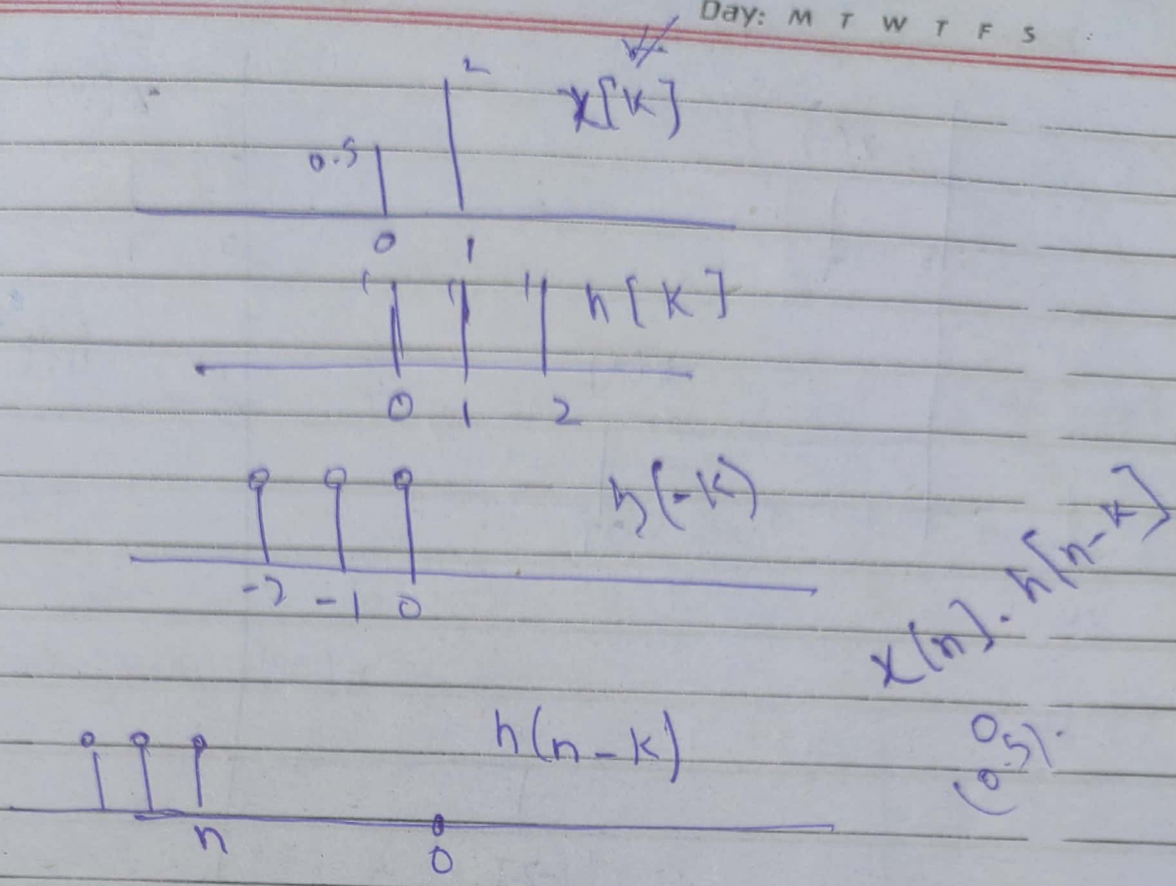
$y[0] = 0.5$

$y[1] = 0.5 + 2 = 2.5$

$y[2] = 0.5 + 2$

$y[3] = 0 + 2 = 2$

Date:
Day: M T W T F S



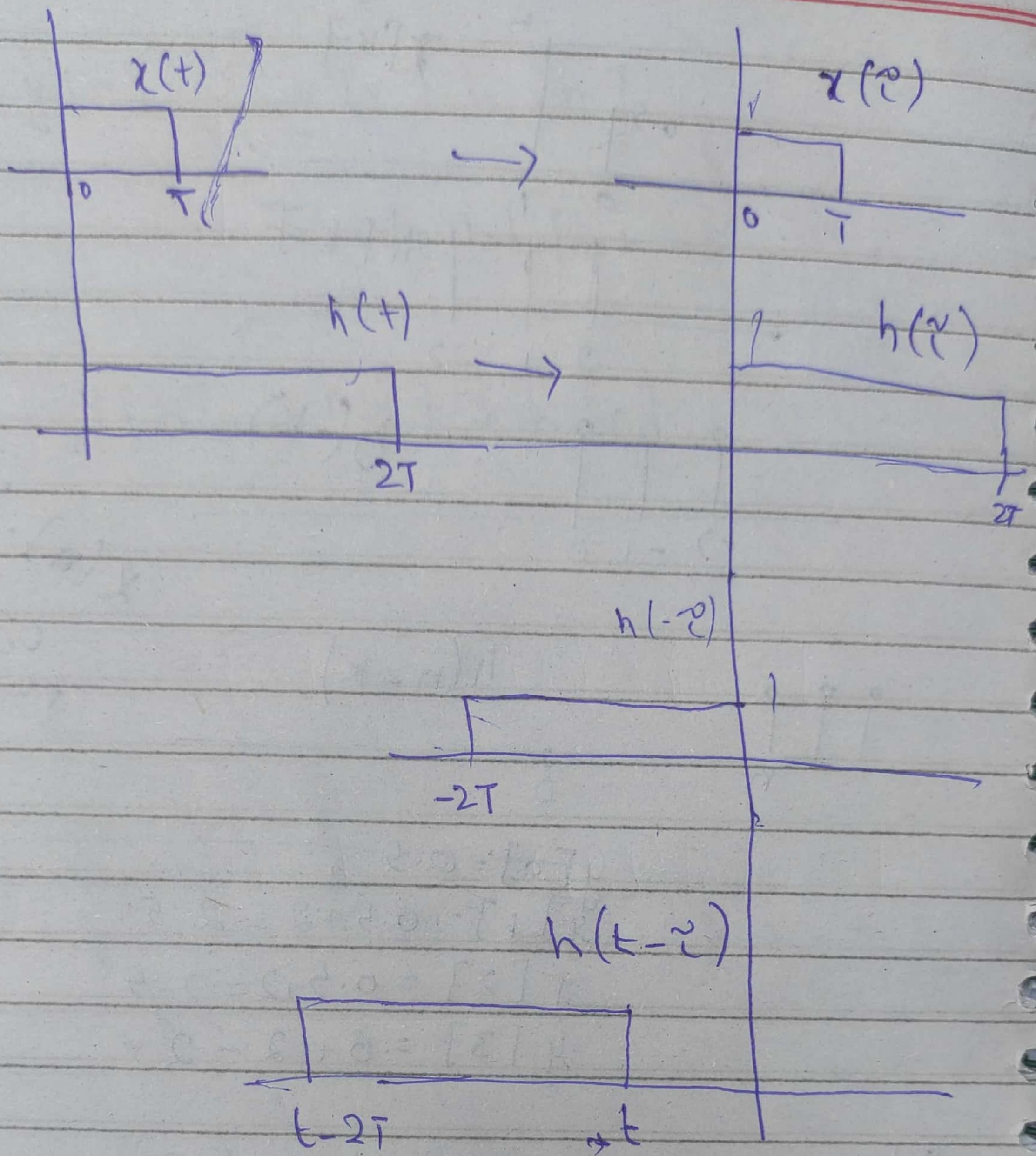
$y[0] = 0.5$
$y[1] = 0.5 + 2 = 2.5$
$y[2] = 0.5 + 2 = 2.5$
$y[3] = 0 + 2 = 2$

Now in continous time

$$y(t) = \lim_{\Delta \to 0} \hat{y}(t) = \lim_{\Delta \to 0} \sum_{n=-\infty}^{\infty} x[k\Delta).h(t-k\Delta)\Delta$$

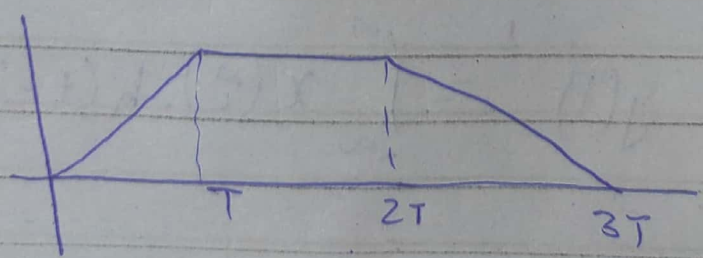
$$y(t) = \int_{-\infty}^{\infty} x(\tilde{\tau}).h(t-\tilde{\tau})d\tilde{\tau}$$

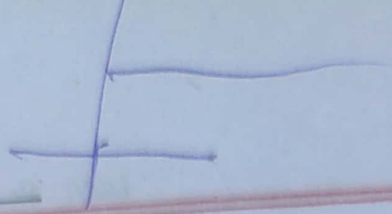
$$y(t) = x(t) * h(t)$$



$$x(t) * h(t) = \int_{-\infty}^{t} x(\tau).h(t-\tau)\,d\tau$$

$-\infty$ to 0



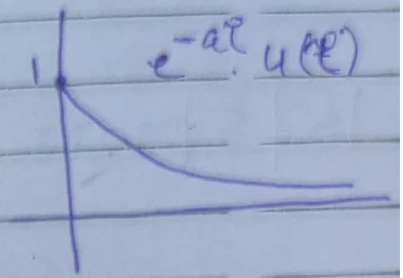


Date: Day: M T W T F S

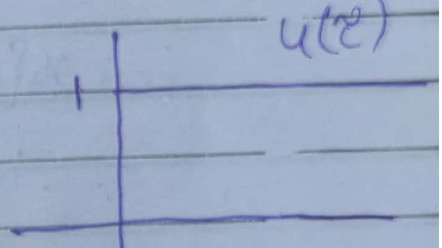
Prob

$$x(t) = e^{-at} . u(t) \quad , a>0$$
$$h(t) = u(t)$$

$$x(\tau) = e^{-a\tau} . u(\tau)$$

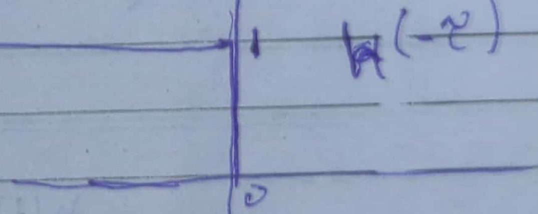


from $-\infty$ to 0, it is disjoint,

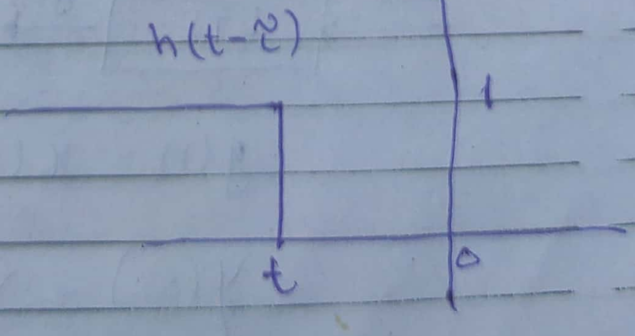


$$\text{So } y(t) = \int_0^t x(\tau) . h(t-\tau) d\tau$$

$$y(t) = x(t) * h(t)$$



$$y(t) = \int_0^t e^{-a\tau} d\tau$$



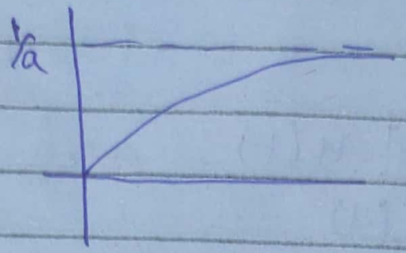
$$y(t) = \frac{e^{-a\tau}}{-a} \Big|_0^t$$

$$y(t) = \frac{e^{-at} - e^{0}}{-a}$$

$$y(t) = \frac{1}{a} (1 - e^{-at})$$

Scanned with CamScanner

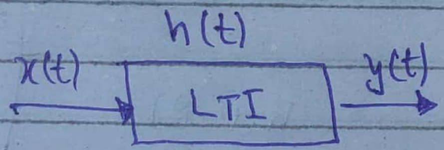
Date: Day: M T W T F S



16|5|2023

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$



$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$X(\omega) = \underbrace{|X(\omega)|}_{\text{Ampl.}} \cdot \underbrace{e^{j\theta_x(\omega)}}_{\text{Phase angle}}$$

$$Y(\omega) = |Y(\omega)| \cdot e^{j\theta_y(\omega)}$$

Date: Day: M T W T F S

$$H(\omega) = |H(\omega)| e^{j\theta_h(\omega)}$$

$$|Y(\omega)| \cdot e^{j\theta_y(\omega)} = |X(\omega)| \cdot e^{j\theta_x(\omega)} \cdot |H(\omega)| \cdot e^{j\theta_h(\omega)}$$

$$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$$

$$\theta_y(\omega) = \theta_x(\omega) + \theta_h(\omega)$$

$$|H(\omega)| = \frac{|Y(\omega)|}{|X(\omega)|}$$

$$y(t) = \underline{k \cdot x(t - t_d)}$$

$$|Y(\omega)| \cdot e^{j\theta_y(\omega)} = k \cdot |X(\omega)| \cdot e^{-j\omega t_d}$$

$$|H(\omega)| = k$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{Rj\omega C + 1}$$

$$\cancel{H(\omega) =}$$

Date: Day: M T W T F S

$$H(\omega) = \frac{1/j\omega RC}{1 + \frac{1}{j\omega RC}}$$

Let $\underline{RC = a}$

$$H(\omega) = \frac{1/j\omega a}{1 + 1/j\omega a} \times \frac{j\omega a}{j\omega a}$$

$$H(\omega) = \frac{1}{1 + j\omega a}$$ OR

$$H(\omega) = \frac{a}{a + j\omega}$$

$$H(\omega) = \frac{a}{a + j\omega} \times \frac{a - j\omega}{a - j\omega}$$

$$H(\omega) = \frac{a^2 - ja\omega}{a^2 + \omega^2}$$

$$|H(\omega)| = \sqrt{\left(\frac{a^2}{a^2+\omega^2}\right)^2 + \left(\frac{a\omega}{a^2+\omega^2}\right)^2}$$

$$\text{"} = \sqrt{\frac{a^4 + a^2\omega^2}{(a^2+\omega^2)^2}}$$

$$H(\omega) = \sqrt{\frac{a^2(a^2+\omega^2)}{(a^2+\omega^2)^2}}$$

$$= a\sqrt{\frac{\cancel{a^2+\omega^2}}{(a^2+\omega^2)^{\cancel{2}}}} = \boxed{\frac{a}{\sqrt{a^2+\omega^2}}}$$

$$\theta_h(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

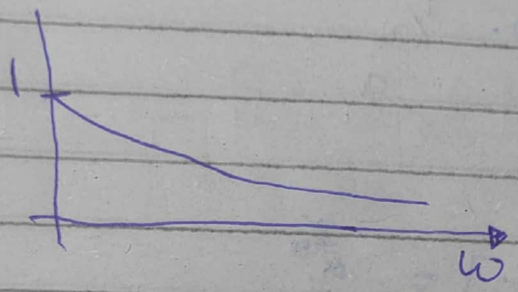
$1+x^2$

Date: ______

Day: M T W T F S

$$|H(\omega)| = \frac{a}{\sqrt{a^2+\omega^2}}$$

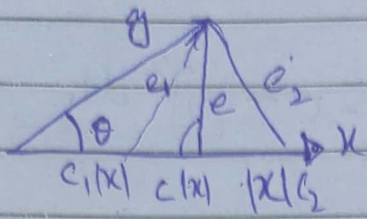
$$\boxed{a = 1/RC}$$



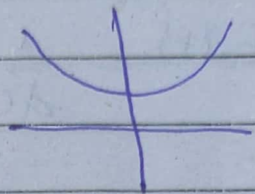
Date: 22/3/2023 Day: M T W T F S

Vectors & Signals :-

Signals are vectors.



$$g = c_1|x| + e_1$$



$$|g| \simeq c|x|$$

$$\cos\theta = \frac{c \cdot |x|}{|g|}$$

$$c \cdot |x| = |g| \cos\theta$$

$$c \cdot |x| \cdot |x| = |g| \cdot |x| \cdot \cos\theta$$

$$c = \frac{g \cdot x}{|x|^2}$$

$$g(t) \simeq c \cdot x(t)$$

$$e(t) = g(t) - c \cdot x(t)$$

$$g(t) = c \cdot x(t) + e(t)$$

Date: Day: M T W T F S

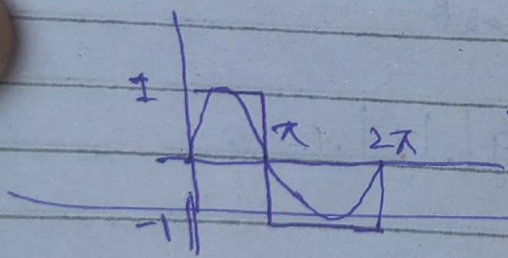
Strength of signal in time domain will be represented in energy.

$$E_e(t) = \int_{t_1}^{t_2} e^2(t)\, dt$$

$$E_e(t) = \int_{t_1}^{t_2} [g(t) - c.x(t)]^2\, dt$$

$$\frac{d}{dc} E_e(t) = \frac{d}{dc}\left[\cancel{\int_{t_1}^{t_2} g^2(t) dt}^{\,0} + c^2 \int_{t_1}^{t_2} x^2(t)\, dt - 2c\int_{t_1}^{t_2} g(t).x(t) dt\right] = 0$$

$$c = \frac{\int_{t_1}^{t_2} g(t).x(t)\, dt}{\int_{t_1}^{t_2} x^2(t)\, dt}$$



$x(t) . \sin t$

$$E_x = \int_0^{2\pi} \left(\frac{1 - \cos 2t}{2}\right) dt$$

$$= \frac{1}{2}\left[\int_0^{2\pi} 1\, dt - \int_0^{2\pi} \cos 2t\, dt\right]$$

$$= \frac{1}{2}\left[t\Big|_0^{2\pi} - 0\right] = \frac{1}{2}[2\pi - 0]$$

$$= \pi$$

Scanned with CamScanner

$$C = \frac{\int_{t_1}^{t_2} g(t) . x(t)\, dt}{\int_{t_1}^{t_2} x^2(t)\, dt} = 4/\lambda$$

When $C = 0$, the signals are said to be orthogonal.

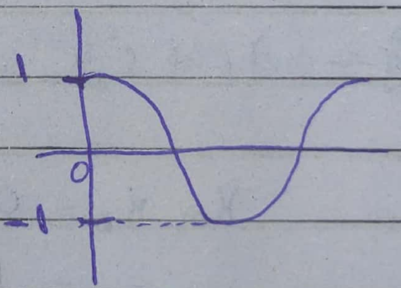
Correlation :- $\rightarrow$ direct multiplication.

$C_n = \cos\theta$

$g.x = |g|\,|x| . \cos\theta$

$$C_n = \cos\theta = \frac{g.x}{|g|.|x|}$$

$-1 \leq C_n \leq 1$

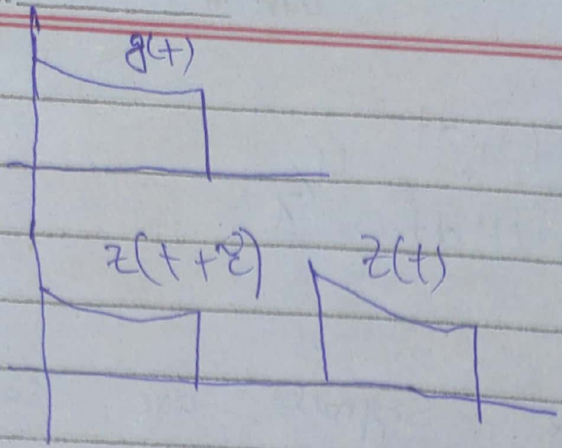


$$C_n = \frac{\int_{t_1}^{t_2} g(t) . x(t)\, dt}{\sqrt{E_g . E_x}}$$

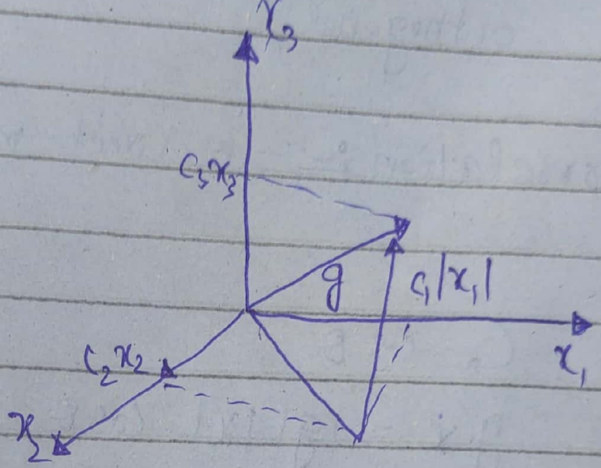
$= \psi_{gx}(t) = \int_{-\infty}^{\infty} g(t) . x(t+\tau)\, dt$

Date:

Day: M T W T F S



$$\psi_{gz}(t) = \int_{-\infty}^{\infty} g(t) \cdot z(t+\tau)\, dt$$



$$g \cong c_1 x_1 + c_2 x_2$$

$$e = g - [c_1 x_1 + c_2 x_2]$$

$$g = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$x_m \cdot x_n = 0 \quad m \neq n$$
$$= 1 \quad m = n$$

$$g(t) = c_1 \cdot x_1(t) + c_2 x_2(t) + c_3 \cdot x_3(t) - \cdots + x_N(t)$$

$$g(t) = \sum_{n=1}^{\infty} c_n \cdot x_n(t)$$

Date:

Day: M T W T F S

Trignometric Fourier Series.

$1, \cos\omega_0 t, \cos^2\omega_0 t, \ldots, \cos n\omega_0 t, \sin\omega_0 t,$
$\sin 2\omega_0 t \ldots \sin n\omega_0 t$

$$\int_{T_0} \cos m\omega_0 t \cdot \cos n\omega_0 t \, dt = 0 \quad n \neq m$$
$$= T_0/2 \quad m = n$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right]$$

$$g(t) = \sum_{n=0}^{\infty} \left[a_n \cancel{\cos n\omega_0 t} + b_n \cdot \sin n\omega_0 t \right]$$

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = C_n \cdot \cos(n\omega_0 t + \theta_n)$$

$$= C_n \cdot \cos n\omega_0 t \cdot \cos\theta_n - C_n \cdot \sin n\omega_0 t \sin\theta_n$$

Iff

$$a_n = C_n \cdot \cos\theta_n, \quad b_n = -C_n \cdot \sin\theta_n$$

$$a_n^2 + b_n^2 = C_n^2 \cdot \cos^2\theta_n + C_n^2 \cdot \sin^2\theta_n$$

$$\sqrt{a_n^2 + b_n^2} = \sqrt{C_n^2}$$

$$\boxed{C_n = \sqrt{a_n^2 + b_n^2}}$$

Scanned with CamScanner

Date:

Day: M T W T F S

$$\frac{-C_n \cdot \sin\theta_n}{C_n \cdot \cos\theta_n} = \frac{b_n}{a_n}$$

$$\tan\theta_n = \frac{-b_n}{a_n}$$

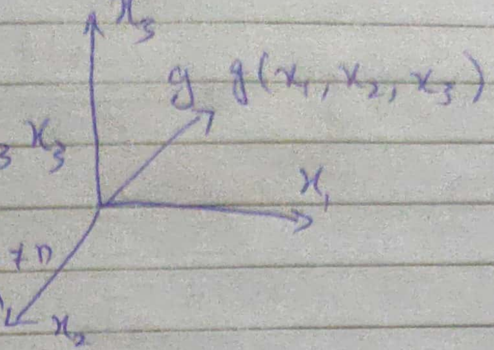
$$\boxed{\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)}$$

$\therefore$ Compact form of trig Fourier Series.

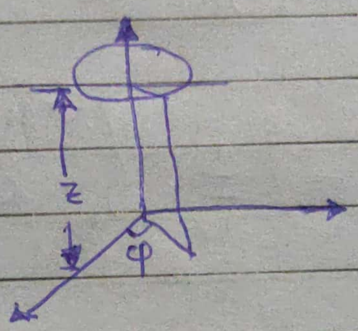
$$g(t) = \sum_{n=0}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$$

Date: 29/5/2023 Day: M T W T F S

$$g = c_1 x_1 + c_2 x_2 + c_3 x_3$$



$$x_m \cdot x_n = 0 \quad m \neq n$$
$$= 1 \quad m = n$$



$$x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots a_n \cos n\omega_0 t$$
$$b_1 \sin\omega_0 t + b_2 \cos 2\omega_0 t + \dots b_n \sin n\omega_0 t$$

$$x(t) = a_0 + \sum_{n=-\infty}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right]$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot \cos(n\omega_0 t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$c = \underline{g \cdot x}$$

Date: Day: M T W T F S

$$a_n = \frac{\int_{t_1}^{t_1+T_0} g(t) . \cos n\omega_0 t \, dt}{\int_{t_1}^{t_1+T_0} \cos^2 n\omega_0 t \, dt}$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) . \sin n\omega_0 t \, dt$$

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) . dt$$

Exponenential form: of fourier Series

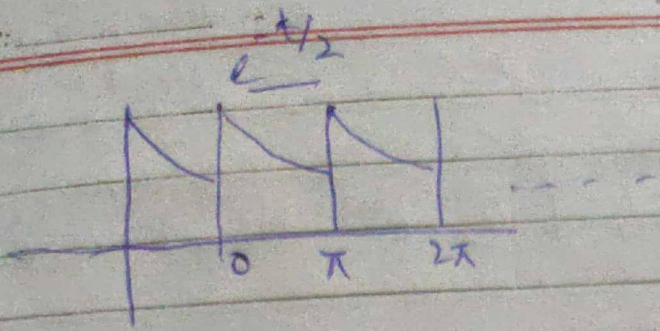
$$C_n . \cos(n\omega_0 t + \theta_n) = C_n \left[\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(\ldots)}}{2} \right.$$

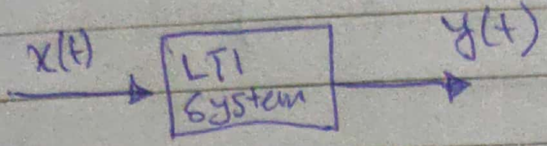
$$'' = \frac{C_n e^{j\theta_n}}{2} . e^{jn\omega_0 t} + \frac{C_n . e^{-j\theta_n}}{2} . e^{-jn\omega_0 t}$$

Magnitude

$$'' = D_n e^{jn\omega_0 t} + D_n e^{-jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n . e^{jn\omega_0 t}$$





$$y(t) = x(t) * h(t)$$

$$y(t) = h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) . x(t-\tau) dt$$

Let $x(t) = e^{j\omega_0 t}$

$$x(t-\tau) = e^{j\omega_0 (t-\tau)}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) . e^{j\omega(t-\tau)} d\tau$$

$$y(t) = e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} h(\tau) . e^{-j\omega\tau} d\tau}_{H(j\omega)}$$

s

$$y(t) = e^{j\omega t} . H(s)$$

Response to the Sum is the sum of responses.

Date: Day: M T W T F S

$x(t) = a_1 . e^{j\omega_0 t} + a_2 . e^{j2\omega_0 t} + a_3 e^{j3\omega_0 t}$

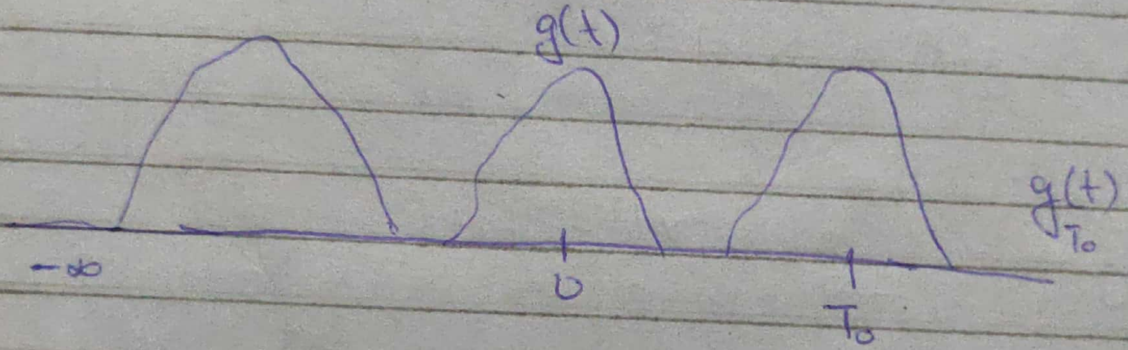
$x(t)$ Passed through LTI system.

~~$x(t)$~~ $y(t) = a_1 e^{s_1 t} . H(s_1) + a_2 . e^{s_2 t} H(s_2) + a_3 . e^{s_3 t} . H(s_3)$

If n components

$$y(t) = \sum_{n=0}^{\infty} a_n . e^{s_n t} H(s_n)$$

Fourier Series for Aperiodic Signals:-

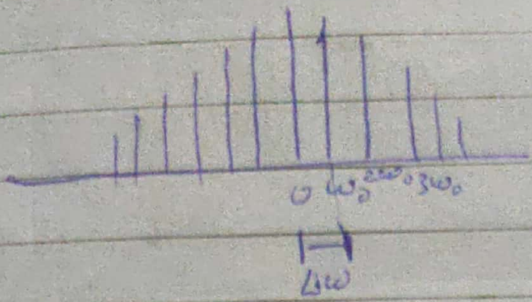


If $T_0 \to \infty$ then

$$\lim_{T_0 \to \infty} g_{T_0}(t) = g(t)$$

$$D_n = \frac{1}{T} \int_0^{T_0} g(t) . e^{-jn\omega_0 t} dt$$

$$D_n = \frac{G(n\omega_0)}{T_0}$$



$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{G(n\omega_0)}{T_0} e^{-jn\omega_0 t}$$

$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{G(n\,\Delta\omega)}{T_0} \cdot e^{-jn\Delta\omega t}$$

If $T_0 \to \infty$

$\Delta\omega \to 0$

$$\lim_{T_0\to\infty} g_{T_0}(t) = g(t) = \lim_{\Delta\omega\to 0} \sum_{n=-\infty}^{\infty} \frac{G(n\cdot\Delta\omega)}{T_0} \cdot e^{+jn\Delta\omega t}$$

$$T_0 = \frac{2\pi}{\omega_0}$$

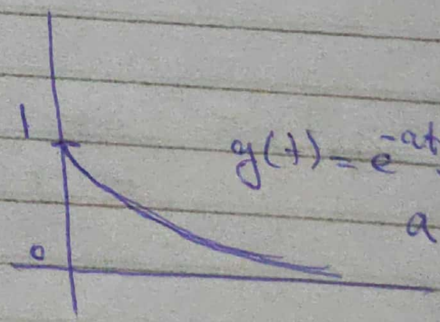
$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \cdot e^{+j\omega t}\, d\omega$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} \cdot dt$$

Fourier Transform of the signal.

$$g(t) \longrightarrow \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} \longrightarrow G(\omega) \longrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} \longrightarrow g(t)$$

$g(t)$



$g(t) = e^{-at} \cdot u(t)$

$a > 0$

Dirichlet Condition

A Value of $g(t)$ must be finite.

$$G(\omega) = \int_{-\infty}^{\infty} \cdot e^{-at} \cdot u(t) \cdot e^{-j\omega t} dt$$

$$G(\omega) = \int_{0}^{\infty} \cdot e^{-at} \cdot e^{-j\omega t} dt$$

Date:

$$G(\omega) = \int_0^\infty e^{-(a+j\omega)t}$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^\infty$$

$$= \frac{e^{-\infty} - e^{-0}}{-(a+j\omega)} = \frac{0-1}{-(a+j\omega)}$$

$$G(\omega) = \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega} = \frac{a-j\omega}{a^2+\omega^2}$$

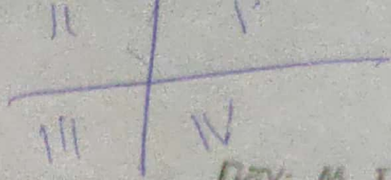
$$G(\omega) = |G(\omega)| \cdot e^{j\theta_g(\omega)}$$

$$|G(\omega)| = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}}$$

$$= \sqrt{\frac{\cancel{a^2+\omega^2}}{(a^2+\omega^2)^{\cancel{2}}}}$$

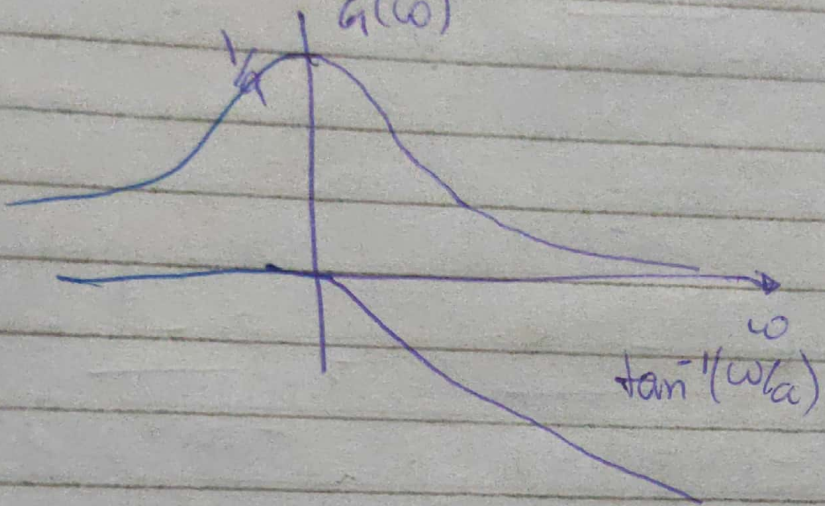
$$= \sqrt{\frac{\cancel{a^2+\omega^2}\ 1}{a^2+\omega^2}} = \frac{1}{\sqrt{a^2+\omega^2}}$$

Date:



Day: M T W T F S

$$\theta_g(\omega) = -\tan^{-1}(\omega/a)$$



$$|G(-\omega)| = |G(\omega)| \quad \text{even}$$

$$\theta_g(-\omega) = -\theta_g(\omega) \quad \text{odd}$$

Scanned with CamScanner

Date: 30/5/2023 Day: M T W T F S

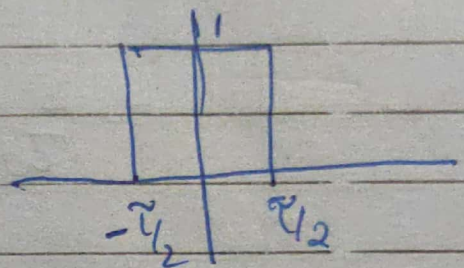
Linearity:-

$$a_1 g_1(t) \Longleftrightarrow a_1 G_1(\omega)$$
$$a_2 g_2(t) \Longleftrightarrow a_2 \cdot G_2(\omega)$$

$$a_1 g_1(t) + a_2 g_2(t) \Longleftrightarrow a_1 G_1(\omega) + a_2 G_2(\omega)$$

★ Unit Gate function:

$$\text{rect}(t/\tau)$$



$$\text{Sinc } x = \frac{\sin\theta}{\theta}$$

$$G(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) \cdot e^{-j\omega t} \, dt$$

$$\text{"} = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j\omega t} \, dt$$

$$\text{"} = \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{-\tau/2}^{\tau/2}$$

$$\text{"} = \frac{e^{-j\omega \tau/2} - e^{j\omega\tau/2}}{-j\omega}$$

Date:

Day: M T W T F S

$$G(\omega) = \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega} \times \frac{2}{2}$$

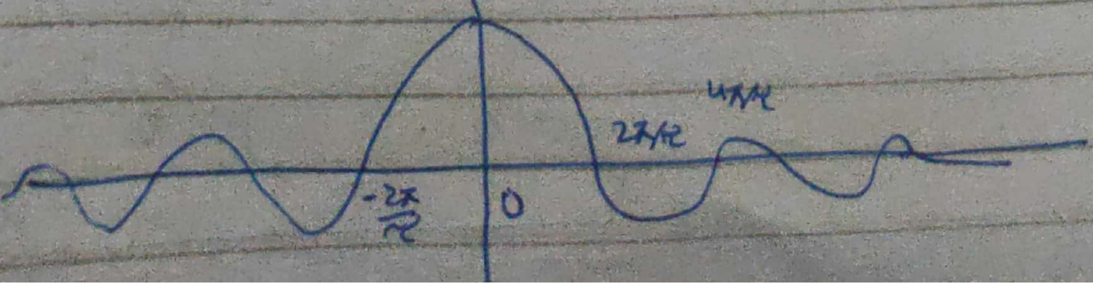
$$G(\omega) = \frac{2}{\omega} \cdot \left[\frac{1}{2j} \left(e^{j\omega\tau/2} - e^{-j\omega\tau/2} \right) \right]$$

$$= \frac{2}{\omega} \sin(\omega\tau/2)$$

$$= \frac{\sin(\omega\tau/2)}{\omega/2} \times \frac{\tau}{\tau}$$

$$= \tau \cdot \frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

$$= \tau \cdot \mathrm{Sinc}\left(\frac{\omega\tau}{2}\right)$$



Date: Day: M T W T F S

$$\frac{\omega\tau}{2} = \pm n\pi$$

$$\omega = \pm\frac{2\pi}{\tau}$$

$$rect\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau . Sinc\left(\frac{\omega\tau}{2}\right)$$

1 MBPs

$\tau = 1 \mu Sec$

2 MBps

$\tau = 0.5 \mu Sec$

Keep the width of rect function small in order to get more bandwidth.

$$g(t) = \delta(t)$$

$$G(\omega) = \int_{-\infty}^{\infty} \delta(t) . e^{-j\omega t} dt = e^{0}$$

$$= \underline{1}$$

Date:

Day: M T W T F S

$$G(\omega) = \delta(\omega - \omega_0)$$

$$g(t) = ?$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} \, dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) \cdot e^{j\omega t} \, dt$$

$$g(t) = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$2\pi \times \frac{1}{2\pi} \cdot e^{j\omega_0 t} \Longleftrightarrow \delta(\omega - \omega_0) \times 2\pi$$

$$\boxed{\begin{aligned} e^{j\omega_0 t} &\Longleftrightarrow \delta(\omega - \omega_0) \cdot 2\pi \\ e^{-j\omega_0 t} &\Longleftrightarrow 2\pi \cdot \delta(\omega + \omega_0) \end{aligned}}$$

$$g(t) = \cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

G(~~[illegible]~~

$$= \int_{-\infty}^{\infty} \frac{e^{j\omega_0 t}}{2} \cdot e^{-j\omega t} \quad \int_{-\infty}^{\infty} \frac{e^{-j(\omega-\omega_0)t}}{2} \, dt$$

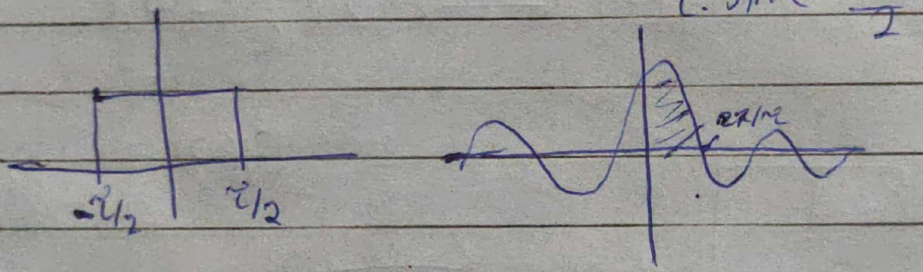
using above results.

$$G(\omega) = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

5/6/2023

$$g(t) \longleftrightarrow G(\omega)$$

$$F\{g(t)\} = G(\omega)$$



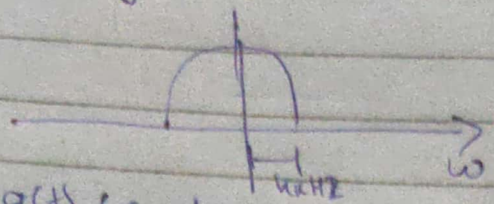
$$= \int_{-\infty}^{\infty} \delta(t) \, e^{-j\omega t} \, dt = 1$$

$$\delta(t) \longleftrightarrow 1$$

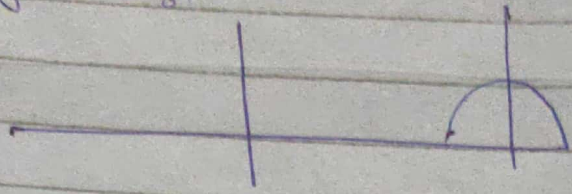
$$F^{-1}\left[\delta(\omega-\omega_0)\right] = \frac{1}{2\pi} \int \delta(\omega-\omega_0) \cdot e^{j\omega t} \, d\omega$$

Date: Day: M T W T F S

$g(t) \longleftrightarrow G(\omega)$



$g(t) \cdot \cos \omega_0 t$



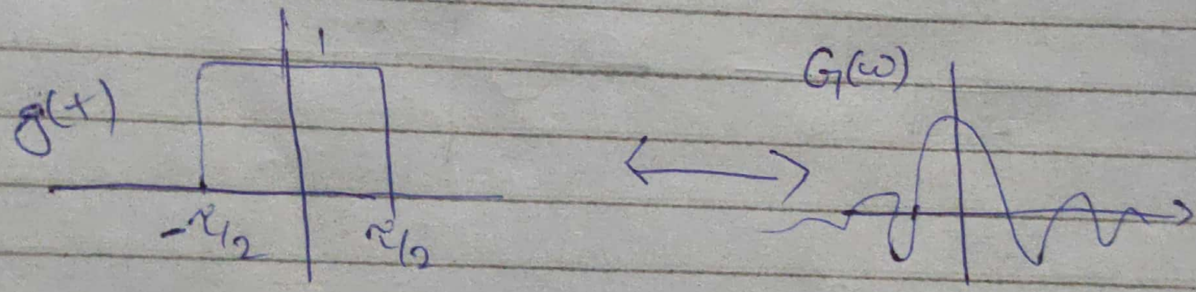
Props of F.T:-

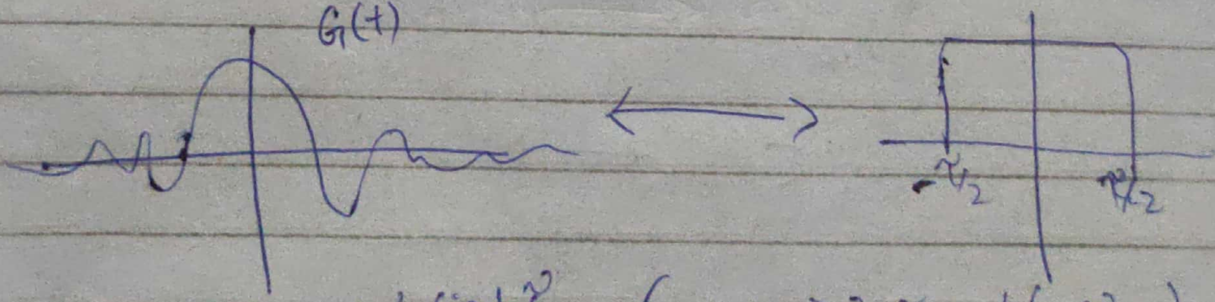
* Symmetry

$g(t) \longleftrightarrow G(\omega)$

$G(t) \longleftrightarrow 2\pi g(-\omega)$

e.g





$\tau \cdot \sin \frac{t\tau}{2} \longleftrightarrow 2\pi \, rect(-\omega/\tau)$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} \, d\omega$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} \, dt$$

Scanned with CamScanner

Date: Day: M T W T F S

✱ Scaling

$$g(t) \longleftrightarrow G(\omega)$$
$$g(at) \longleftrightarrow ?$$

$$F\{g(at)\} = \int_{-\infty}^{\infty} g(at) \cdot e^{-j\omega t} \, dt$$

Let $at = x$

$$t = x/a \Rightarrow dt = \frac{dx}{a}$$

$$\therefore F\{g(at)\} = \int_{-\infty}^{\infty} g(x) \cdot e^{-j\omega \cdot \frac{x}{a}} \cdot \frac{dx}{a}$$

From $G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} \, dt$

$\cancel{F\{g(at)\}} =$

$$\frac{1}{a} G\left(\frac{\omega}{a}\right)$$

Compress in time domain, expand in frequency domain.

$$\therefore g(at) \longleftrightarrow \frac{1}{a} G(\omega/a)$$

Date: Day: M T W T F S

★ Shifting property

$$g(t-t_d) \longleftrightarrow ?$$

$$\int_{-\infty}^{\infty} g(t-t_d) . e^{-j\omega t} dt$$

Let $t - t_d = x$

$t = x + t_d$

$dx = dt$

$$\int_{-\infty}^{\infty} g(x) . e^{-j\omega(x+t_d)} dx$$

$$e^{-j\omega t_d} \int_{-\infty}^{\infty} g(x) . e^{-j\omega x} dx = \boxed{G(\omega). e^{-j\omega . t_d}}$$

Now.

$$g(t-t_d) \longrightarrow G(\omega) . e^{-j\omega . t_d}$$

★ Convolution:

$$F\left[g_1(t) * g_2(t)\right] = \int_{-\infty}^{\infty} \left[g_1(t) * g_2(t)\right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} (g_1(\tau) g_2(t-\tau) d\tau) . \right] e^{-j\omega t} dt$$

Date: Day: M T W T F S

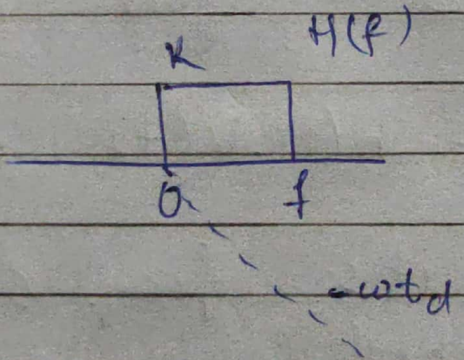
$$\int_{-\infty}^{\infty} g_1(\tilde{\tau}) \left[\int_{-\infty}^{\infty} g_2(t-\tilde{\tau}) \cdot e^{-j\omega t} \, dt \right] d\tilde{\tau}$$

$$\int_{-\infty}^{\infty} g_1(\tilde{\tau}) \cdot G_2(\omega) \, e^{-j\omega\tilde{\tau}} \, d\tilde{\tau}$$

$$G_2(\omega) \left[\int_{-\infty}^{\infty} g_1(\tilde{\tau}) \cdot e^{-j\omega\tilde{\tau}} \, d\tilde{\tau} \right]$$

$$G_1(\omega) \cdot G_2(\omega)$$

Ideal Filter :-



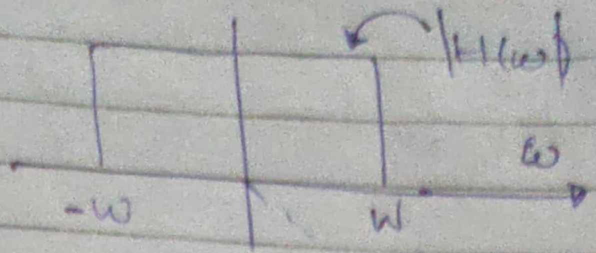
$$y(t) = g(t - t_d)$$

$$Y(\omega) = G(\omega) \cdot e^{-j\omega t_d}$$

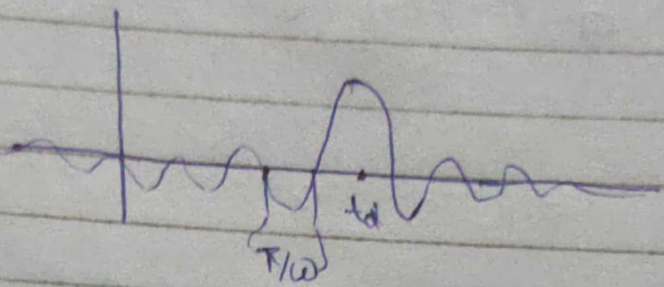
$$H(\omega) = 1 \cdot e^{-j\omega t_d}$$

$$\boxed{H(\omega) = \text{rect}\, \frac{\omega}{2W} \, e^{-j\omega t_d}}$$ → Ideal Filter ka proper response

Date: Day: M T W T F S

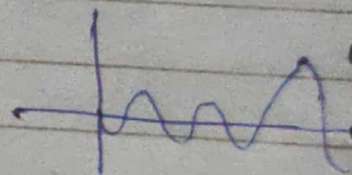


$h(t) = \frac{\omega}{\pi} \cdot \operatorname{Sinc} \omega (t - t_d)$

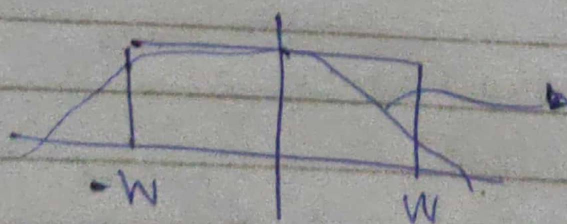


↳ is not realizable as it is a non-causal signal.

So $\hat{h}(t) = h(t) \cdot u(t)$



But now it's freq. domain is also disturbed.



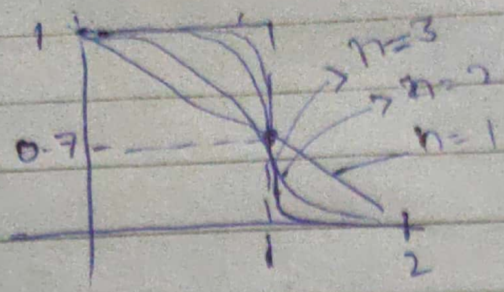
freq. selective filter.

Butterworth Filter

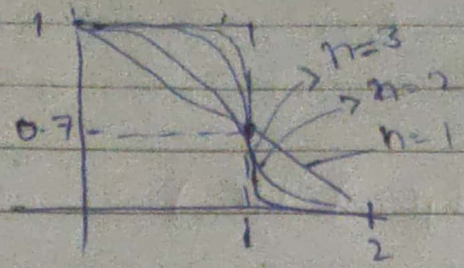
$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi B}\right)^{2n}}}$$

Day: M T W T F S

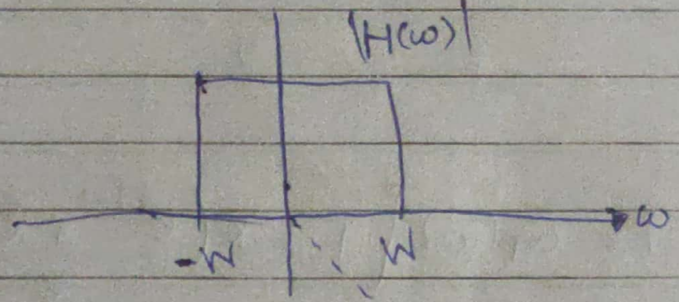
Date:



Date: Day: M T W T F S

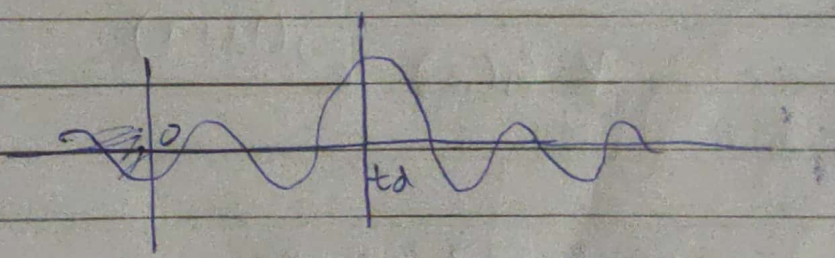


Date 6|6|2023



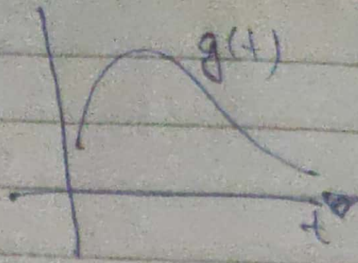
$\theta_n(\omega) = -\omega . t_d$

$rect\left(\frac{\omega}{2W}\right) . e^{-j\omega t_d}$



$$H(\omega) = \begin{cases} (1 + K . \cos T\omega)\, e^{-j\omega t_d} & |\omega| < 2\pi B \\ 0 & |\omega| > 2\pi B \end{cases}$$

$\theta_n(\omega) = -\omega . t_d$



$y(t) = ?$

$Y(\omega) = G(\omega) . H(\omega)$

$Y(\omega) = G(\omega) \left[(1 + k \cos T\omega) e^{-j\omega t_d} \right]$

$$\boxed{\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

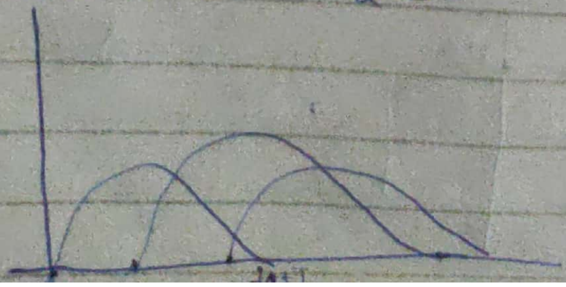
$$Y(\omega) = G(\omega) \left[1 + k \left(\frac{e^{jT\omega} + e^{-jT\omega}}{2} \right) \right] e^{-j\omega t_d}$$

$$Y(\omega) = G(\omega) . e^{-j\omega t_d} + \frac{k}{2} G(\omega) . e^{-j(t_d - T)\omega} + \frac{k}{2} G(\omega) . e^{-j(t_d + T)}$$

$g(t) \longleftrightarrow G(\omega)$

$g(t - t_d) \longleftrightarrow G(\omega) . e^{-j\omega t_d}$

$$y(t) = g(t - t_d) + \frac{k}{2} g[t - (t_d - T)] + \frac{k}{2} g[t - (t_d + T)]$$



Date: Day: M T W T F S

Fourier Transform for DTS:-

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jn\omega}$$

$$x[n] = a^n \cdot u[n]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^n \cdot u[n] \cdot e^{-j\omega n}$$

$$= \sum_{0}^{\infty} a^n \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left[(a e^{-j\omega})^n \right] \longrightarrow \text{Power Series}$$

$$\boxed{\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}}$$

$$= \frac{1}{1 - a e^{-j\omega}}$$

$$= \frac{1}{1 - a[\cos\omega - j\sin\omega]}$$

$$= \frac{1}{1 - a\cos\omega + a j \sin\omega}$$

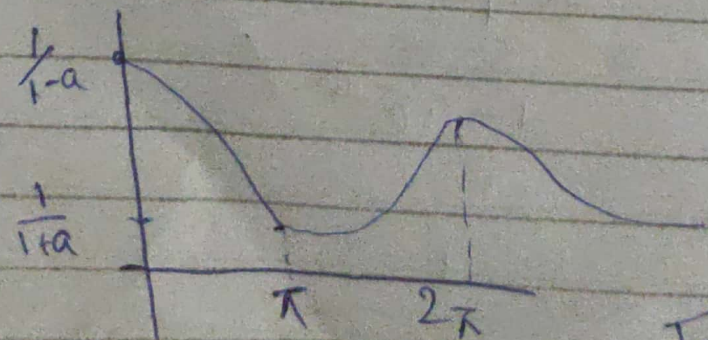
$$= \frac{1}{(1 - a\cos\omega) + j(a\sin\omega)} \times \frac{(1 - a\cos\omega) - j(a\sin\omega)}{(1 - a\cos\omega) - j(a\sin\omega)}$$

$$= \frac{1 - a\cos\omega - j a\sin\omega}{(1 - a\cos\omega)^2 + a^2\sin^2\omega}$$

$$= \frac{1 - a\cos\omega - j a\sin\omega}{1 + a^2\cos^2\omega - 2a\cos\omega + a^2\sin^2\omega}$$

$$= \frac{1 - a\cos\omega - j a\sin\omega}{1 - 2a\cos\omega + a^2}$$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1 - a\cos\omega)^2 + a^2\sin^2\omega}}$$



Find Phase Angle:

$$\tan^{-1}\left(\frac{-a\sin\omega}{1 - a\cos\omega}\right)$$

SS 6/6/2023 ①

Examples in chapter ⑤

$$x[n] = \sum_{n=-\infty}^{\infty} a^{|n|} \cdot e^{-j\omega n}$$

$$x\{n] = \sum_{n=0}^{\infty} a^{n} \cdot e^{-j\omega n} + \sum_{n=-\infty}^{-1} \cdot a^{-n} \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (a e^{-j\omega})^{n} + \sum_{n=-\cancel{\infty}}^{\cancel{\infty}1} (a e^{j\omega})^{m} \quad m = -n$$

$$= \frac{1}{1 - a e^{-j\omega}} + \frac{a e^{j\omega}}{1 - a e^{-j\omega}}$$

Scanned with CamScanner