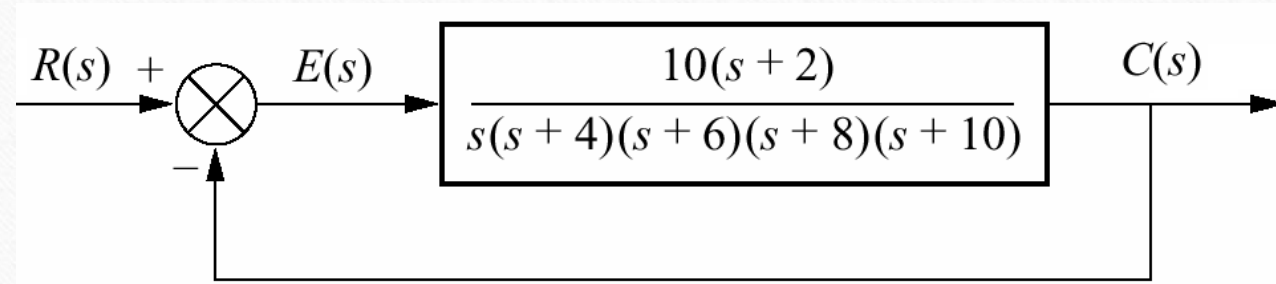


Week 11 – DCSE

7th Semester

In Class Exercise

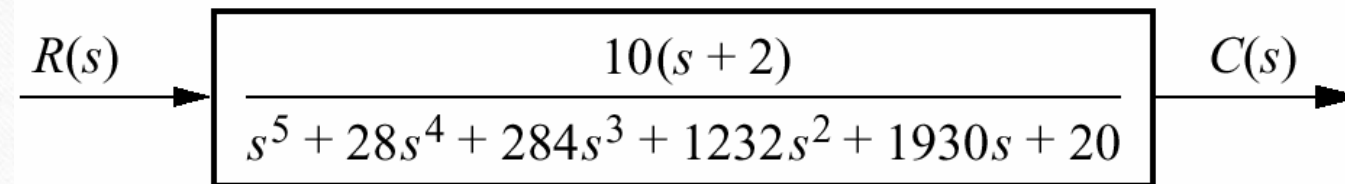
Find the transfer function or $C(s)/R(s)$ – and is it a stable transfer function



$$T(s) = \frac{G(s)}{1 + G(s)}$$

$$T(s) = \frac{10(s+2)}{s(s+4)(s+6)(s+8)(s+10) + 10(s+2)}$$

$$T(s) = \frac{10(s+2)}{s^5 + 28s^4 + 284s^3 + 1232s^2 + 1930s + 20}$$



Stability Checking techniques

If you recall, there are 3 main techniques of checking stability

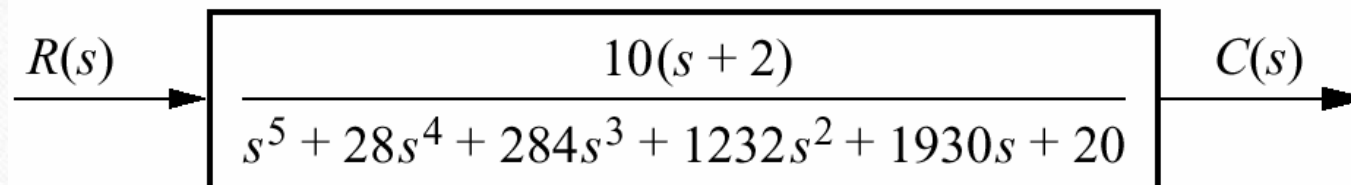
1 – Poles of transfer function (all poles must be negative for stable tf)

2 – Eigen values of matrix A in State-space Domain

(all eigenvalues must be negative for stable ss model)

3 – Step Response (should be bounded response for stability)

Who will compute poles of 5th Order Transfer function by hand?



Routh Hurwitz Stability Criteria

- In today's lecture, we will study a technique to check stability of higher order systems
- If you **CAN** compute poles of high order systems, that's perfect
- For example

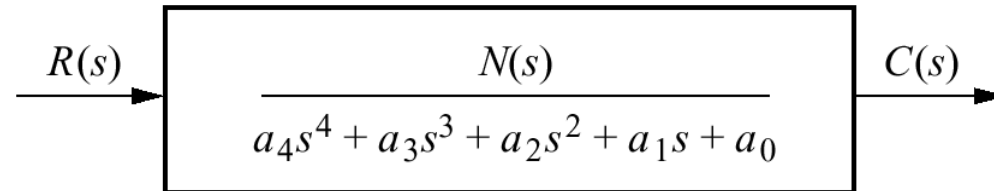
$$\frac{10(s + 2)}{s(s + 4)(s + 6)(s + 8)(s + 10)}$$

Routh Hurwitz Stability Criteria

- But if you can **NOT** easily compute poles, then what to do?
- How to check stability of high order systems?
- Routh Hurwitz is a technique to check stability of high order systems
 - (Remember: Its is a technique to check stability – **NOT** to compute poles)
- Let us study this technique in next slide

Steps of Routh Hurwitz Stability Criteria

Step 1: Compute *transfer function*



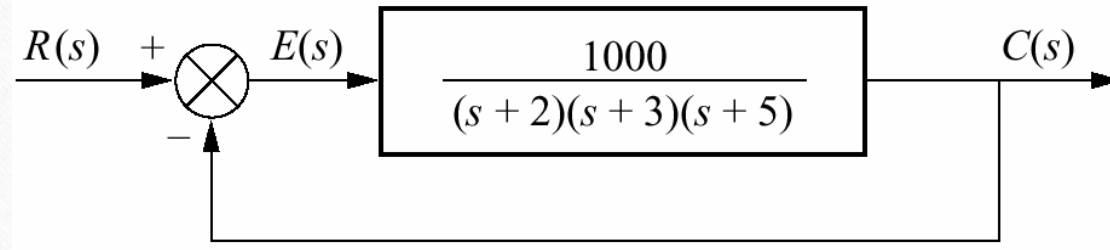
Step 2: Develop Routh-Hurwitz Table

$$P(s) = a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$b_1 = \frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3}$	$b_2 = \frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$	$b_3 = \frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3}$
s^1	$c_1 = \frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}$	$c_2 = \frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$c_3 = \frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s	$d_1 = \frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1}$	$d_2 = \frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$d_3 = \frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

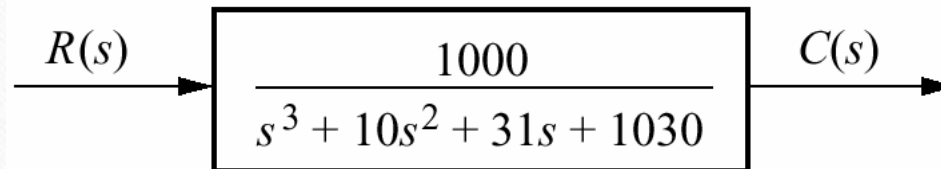
Step 3: Interpretation: The number of RHP poles = The number of SIGN CHANGES of COL 1

Example: Determine whether the system is stable or not.



Step 1: Find the transfer function

$$T(s) = \frac{G(s)}{1 + G(s)}$$



Step 2: Develop Routh Table

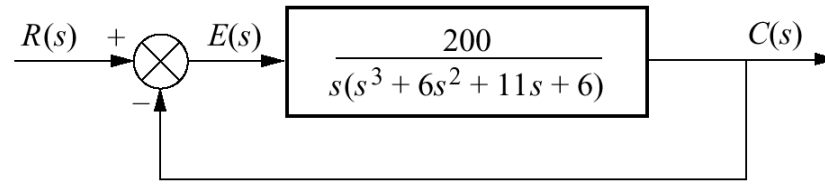
$$P(s) = s^3 + 10s^2 + 31s + 1030$$

s^3	1	31	0
s^2	10	1030	0
s^1	$b_1 = \frac{-\begin{vmatrix} 1 & 31 \\ 10 & 1030 \end{vmatrix}}{1} = \mathbf{-72}$	$b_2 = \frac{-\begin{vmatrix} 1 & 0 \\ 10 & 0 \end{vmatrix}}{1} = \mathbf{0}$	$b_3 = \frac{-\begin{vmatrix} 1 & 0 \\ 10 & 0 \end{vmatrix}}{1} = \mathbf{0}$
s^0	$c_1 = \frac{-\begin{vmatrix} 1 & 1030 \\ -72 & 0 \end{vmatrix}}{-72} = \mathbf{103}$	$c_2 = \frac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = \mathbf{0}$	$c_3 = \frac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = \mathbf{0}$

Step 3: Interpret Routh Table

TWO sign changes: Thus **2 RHP Poles**

Therefore, system is **UNSTABLE**

Example 2:

$$T(s) = \frac{200}{s^4 + 6s^3 + 11s^2 + 6s + 200}$$

s^4	1	11	200
s^3	6	1	0
s^2	10	1	0
s^1	-19	0	0
s^0	20	0	0

2 sign changes: 2 RHP (UNSTABLE)

Poles: 2 LHP and 2 LHP

Example 3: Zero only in the first column

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

s^5	1	3	5
s^4	2	6	3
s^3	$0 \quad \varepsilon$	$7/2$	0
s^2	$\frac{6\varepsilon - 7}{\varepsilon}$	3	0
s^1	$\frac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$	0	0
s^0	3	0	0

Assume ε is small POSITIVE (follow the previous signage) :

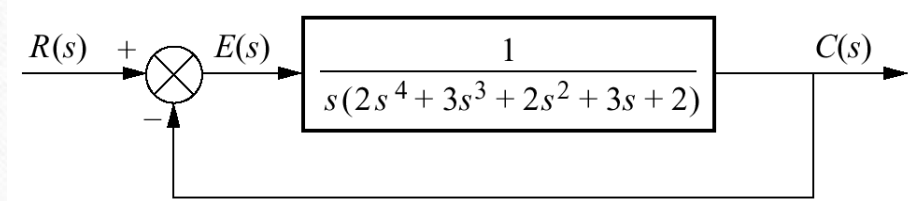
TWO sign changes at first column

Poles: 2 RHP, 3 LHP

System is unstable

Example 4:

$$T(s) = \frac{1}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1}$$



s^5	2	2	2
s^4	3	3	1
s^3	$0 \quad \varepsilon$	$4/3$	0
s^2	$\frac{3\varepsilon - 4}{\varepsilon}$	1	0
s^1	$\frac{12\varepsilon - 16 - 3\varepsilon^2}{9\varepsilon - 12}$	0	0
s^0	1	0	0

$$T(s) = \frac{1}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1}$$

Assume ε is small POSITIVE : TWO sign changes
Poles: 2 RHP, 3 LHP

Example 5: Entire Row is Zero

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

s^5	1	6	8
s^4	7 1	42 6	56 8
s^3	0 4 1	0 12 3	0 0 0
s^2	3	8	0
s^1	1/3	0	0
s^0	8	0	0

Polynomial above the row of zeros

$$P(s) = s^4 + 6s^2 + 8$$

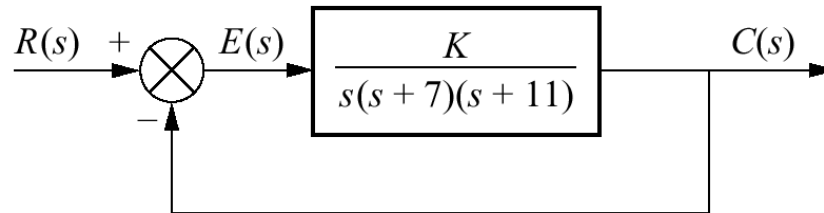
$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

Used to replace row of zeros

1 on LHP + 4 on jw axis = 5 poles

Example 6: Stability Design via Routh-Hurwitz

Find range of K that will cause the system to be stable / unstable / marginally stable for the given diagram.



Step 1: Find transfer function:

$$T(s) = \frac{K}{s^3 + 18s^2 + 77s + K}$$

Step 2: Develop Routh-Hurwitz table

s^3	1	77
s^2	18	K
s^1	$\frac{1386 - K}{18}$	0
s^0	K	0

Step 3: Analyze table

STABLE: $0 < K < 1386$

UNSTABLE: $K > 1386$

MARGINALLY STABLE: $K = 1386$

Example 7: Stability in State-Space

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

$$\det(\lambda\mathbf{I} - \mathbf{A}) = 0$$

Values of the system's poles are equal to the eigenvalues of matrix A

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [1 \ 0 \ 0] \mathbf{x}$$

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s-0 & -3 & -1 \\ -2 & s-8 & -1 \\ 10 & 5 & s+2 \end{vmatrix}$$

$$|s\mathbf{I} - \mathbf{A}| = s^3 - 6s^2 - 7s - 52$$

s^3	1	-7
s^2	-3	-26
s^1	-1	0
s^0	-26	0

1 sign change: 1 RHP

Poles: 1 RHP, 2 LHP

\therefore System is UNSTABLE