Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results

Control Systems Project

Ali Asghar
21PWCSE2059
Section C DCSE
UET Peshawar.

January 8, 2025

Introduction to Project

Perform the following for Problem 22 at Page 148:

a. Consider the state-space of Problem 22, Page 148 of Norman Nise Book Edition 5.
b. Check the stability of the system using all the methods that you know.
c. Compute the controllability and observability for the system. If the system is unstable, design a suitable controller for it.
d. Simulate the system using the controller that you design and show all the responses.
e. Design a PID Controller and show the response of the system using PID Controller. Compare the results obtained in part d and e.
f. Compute the steady state errors before and after designing controller.

Introduction to Project

22. In the past, Type-1 diabetes patients had to inject themselves with insulin three to four times a day. New delayed-action insulin analogues such as insulin Glargine require a single daily dose. A similar procedure to the one described in the Pharmaceutical Drug Absorption case study of this chapter is used to find a model for the concentration-time evolution of plasma for insulin Glargine. For a specific patient, state-space model matrices are given by (*Tarín, 2007*)

$$\mathbf{A} = \begin{bmatrix} -0.435 & 0.209 & 0.02 \\ 0.268 & -0.394 & 0 \\ 0.227 & 0 & -0.02 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} 0.0003 & 0 & 0 \end{bmatrix}; \quad \mathbf{D} = 0$$

where the state vector is given by

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The state variables are

x_1 = insulin amount in plasma compartment
x_2 = insulin amount in liver compartment
x_3 = insulin amount in interstitial (in body tissue) compartment

The system's input is u = external insulin flow. The system's output is y = plasma insulin concentration.

State-space Representation of the System

The state-space representation of the system can be written as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.435 & 0.209 & 0.02 \\ 0.268 & -0.394 & 0 \\ 0.227 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (1)$$

$$y = \begin{bmatrix} 0.003 & 0 & 0 \end{bmatrix} x \quad (2)$$

Stability Analysis of the System

Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results


```
Sys1 =

   0.0003 s^2 + 0.0001242 s + 2.364e-06
  --------------------------------------
  s^3 + 0.849 s^2 + 0.1274 s + 0.0005188
```

Transfer Function

The eigen values of the system are:

$$\boldsymbol{\lambda_1 = -0.6560, \lambda_2 = -0.1889, \lambda_3 = -0.0042} \tag{3}$$

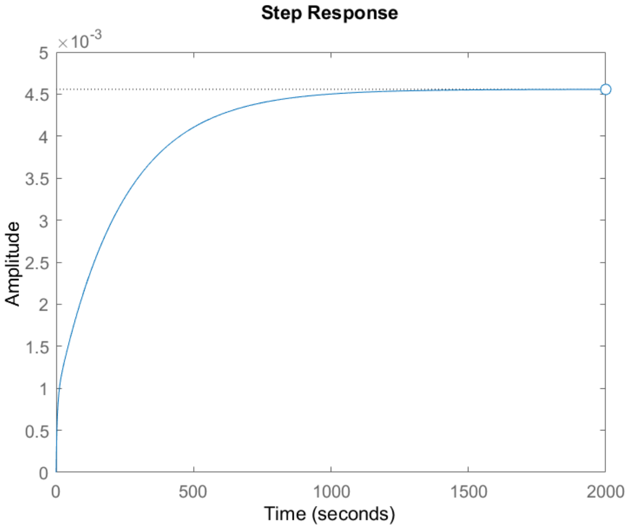
The poles of the system are:

$$\boldsymbol{p_1 = -0.6560, p_2 = -0.1889, p_3 = -0.0042} \tag{4}$$

Stability Analysis of the System

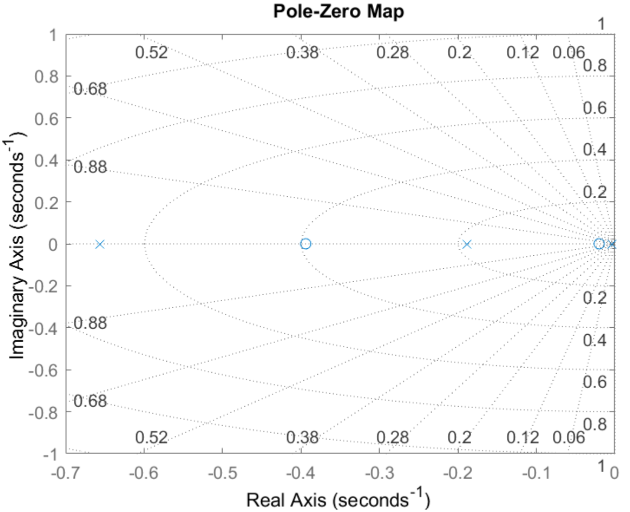
Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results

The step response of the system is:



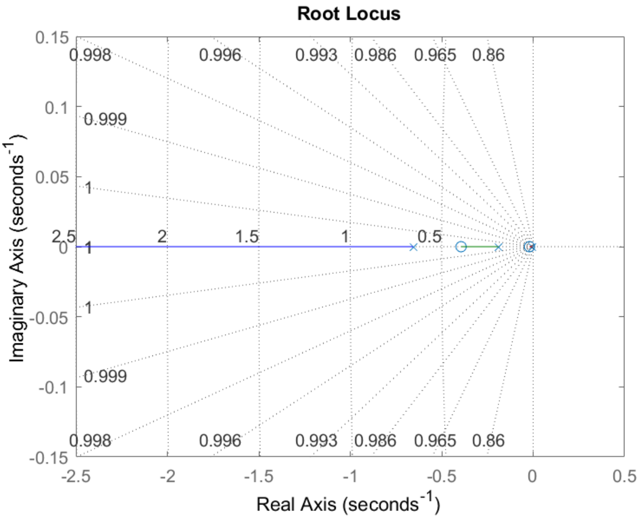
Stability Analysis of the System

The Pole-Zero Map of the system is:



Stability Analysis of the System

The Root Locus of the system is:

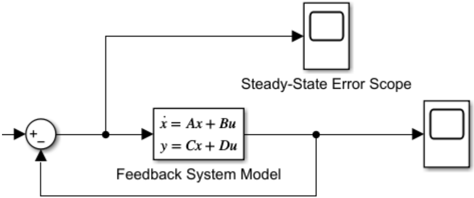


Controllability and Observability Analysis

No Need for Controllability and Observability Analysis as the system is stable already.

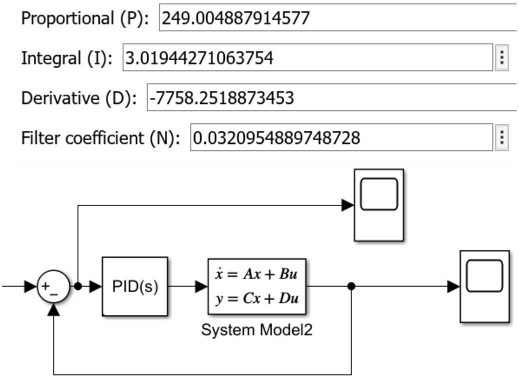
Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results

Controller Design



Steady State Error Computation

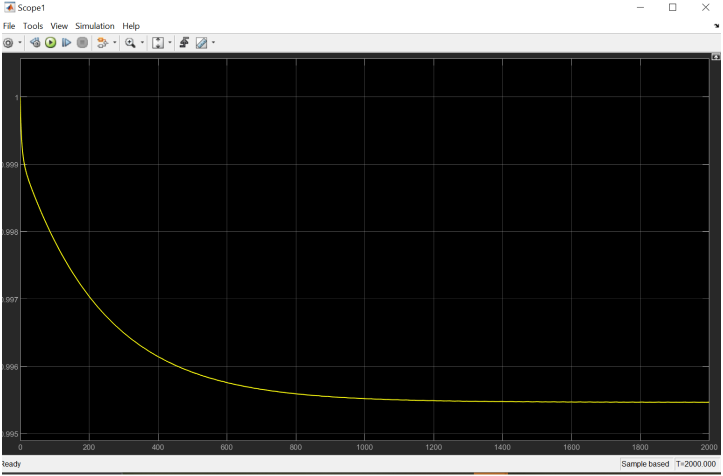
Controller Design



PID Controller Design

Simulink Results

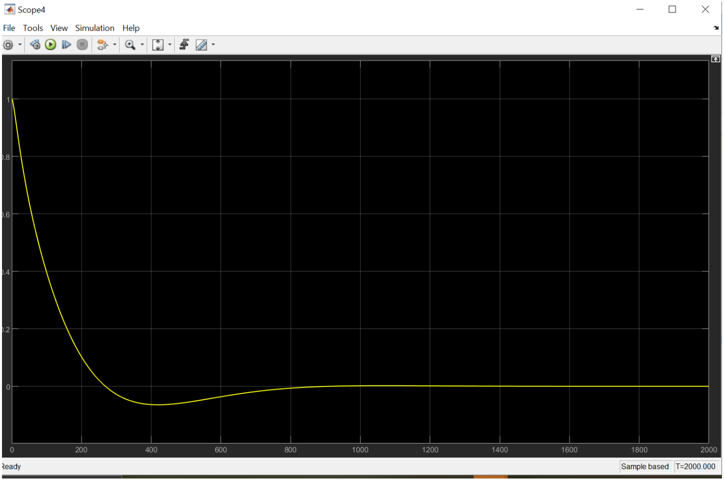
Steady-State Error before PID $\approx \mathbf{0.9955}$



Plot of SSE Before PID.

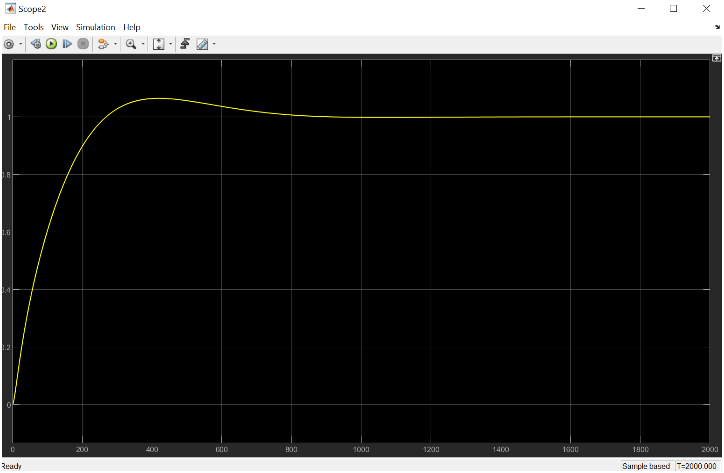
Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results

Steady-State Error After PID $\approx \mathbf{0}$



Plot of SSE after PID

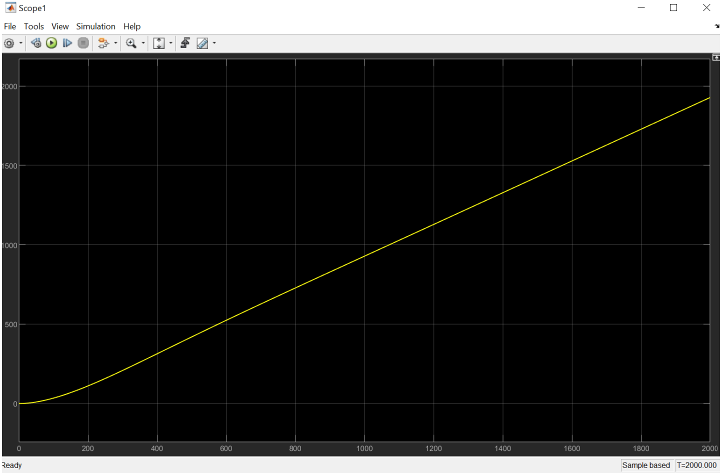
Simulink Results



Plot of Step Response in Simulink

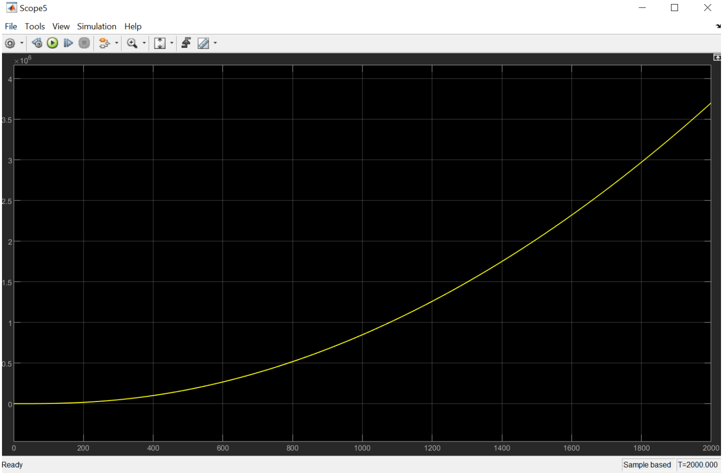
Simulink Results

Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results



Plot of Ramp Response in Simulink After PID

Simulink Results



Plot of Parabolic Response in Simulink After PID

MATLAB Results

Before PID

```
     RiseTime: 493.7075
TransientTime: 878.9076
 SettlingTime: 878.9076
  SettlingMin: 0.0041
  SettlingMax: 0.0045
    Overshoot: 0
   Undershoot: 0
         Peak: 0.0045
     PeakTime: 2.2123e+03

Steady-State Error (Step Input): 0.99592
```

After PID

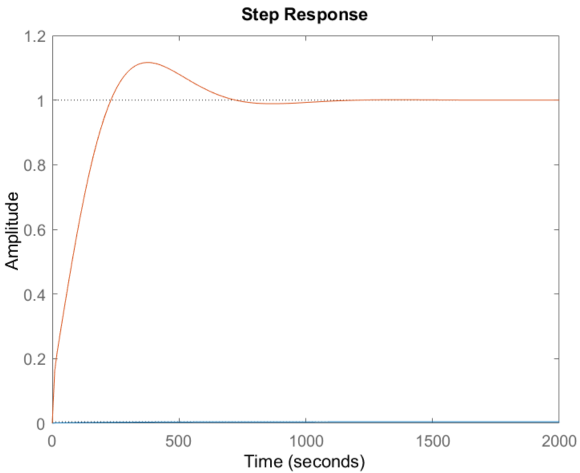
```
     RiseTime: 181.0784
TransientTime: 642.7400
 SettlingTime: 642.7400
  SettlingMin: 0.9166
  SettlingMax: 1.1164
    Overshoot: 11.6443
   Undershoot: 0
         Peak: 1.1164
     PeakTime: 376.8098

Steady-State Error (Step Input): 0.083
```

Comparison of Step Response before and after PID Controller

MATLAB Results

Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results



Plot of Step Response in MATLAB

Introduction
Solution
Stability Analysis
Controllability and Observability
Controller Design
Controller Design
Results

MATLAB Results