

# Control Systems

2 Oct

SERIAL: JB13 TI, TRANS

CLR RI

MOV A, SBUFF

MOV P0, A

RETI

TRANS:

CLR TI

RETI

3 main parts of course.

→ System process information.

→ Amplifier example

→ Robot → electromechanical system

→ outputs update according to input.

→ efficiency

↳ noise

↳ Stability

↳ oscillation

↳ speed

→ effusion pump

input unit      step  
unit output

- washing machine → washing
- Automobiles → Tesla Car
- PLC → Logic Controller
- Adding more data in table  
use  $y = mx + c$

$$y = 2.5x - 3$$

→ Mathematically, see the world.  
→ In control systems, we see the mathematical form of system.

→ Cyber-Physical World.

Digital World

When we integrate physical world with digital world

→ Solar Panelling → Smart Metering.

→ Traditionally → One way

04/10/2024

→ Model → A representation, map based on knowledge  
    ↳ Examples

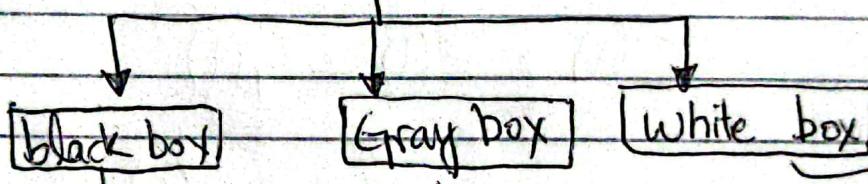
→ Mathematical Model → set of equations ( $y=2x$ )

→ Meteorologist makes weather forecast model  
for predicting weather.

→ Two types of models

    ↳ Stochastic

    ↳ Deterministic



↓  
Atomic bomb

↓  
little knowledge  
about internal

No idea of internal dynamics

↓  
all info  
about internal  
structure

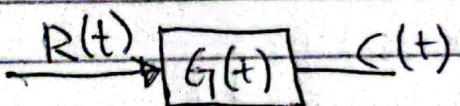
→ Difference b/w step ftn  $w(t)$  and  
unit step ftn

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

$$= F(s)$$

Convolution in time domain is multiplication  
in frequency domain.

$$T.F = \frac{\text{L.T of O/P}}{\text{L.T of I/P}}$$



$$C(t) = R(t) * G(t)$$

$$C(s) = R(s) \cdot G(s)$$

$$\frac{C(s)}{R(s)} = G(s)$$

$$2 \left\{ \frac{d^2 y(t)}{dt^2} \right\} = s^2 \cdot Y(s)$$

$$2 \left\{ \frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2 y(t) \right\} = \left\{ x(t) \right\}$$

$$s^2 y(s) + 3s y(s) + 2y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} \left[ s^2 + 3s + 2 \right] = \frac{1}{s^2 + 3s + 2}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + 3s + 2}$$

QG

Jx

- A sys is → stable, unstable or marginally  
When poles are  
on j axis
- For discrete-time system is stable  
on basis of unit circle.
- For STABILITY FIND THE POLE ONLY

9 Oct 2024

$$\mathcal{L}\{h(t)\} = H(s)$$

## → Recap

- State-Space Model discuss internal dynamics as well not just i/p's and o/p's
- Contains extra variables (state-space <sup>imp</sup> variable)

SS variables      SS Model

- State Space variables completely describe the behaviour of a system.

→  $x \rightarrow$  one SS var

$$\rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{or} \quad 3 \text{ SS vars}$$

→  $\boxed{x(t)}$  → State Space var

- We have two options
  - (i) State option
  - (ii) Output

$$\dot{x} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

D B mostly zero.

→ Example 1

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = [2 \ 3] u(t)$$

→ Ex 2

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \text{[Redacted]} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\dot{x}_1 = x_1 + 2x_2$$

$$\dot{x}_2 = 3x_1 + 4x_2$$

$$\ddot{y} + 4y + 3y = 2u$$

Order 2 means 2 ~~not~~ SS vars

Sol:

$$x_1 = y, \quad x_2 = \frac{dy}{dt} = \ddot{y}$$

$$\dot{x}_1 = \dot{y}, \quad \dot{x}_2 = \frac{d^2y}{dt^2} = \ddot{y}$$

$$\boxed{\dot{x}_1 = \dot{y} = x_2}$$

$$\dot{x}_2 = \ddot{y} =$$

$$\ddot{y} = -4\dot{y} - 3y + 2u$$

$$\boxed{\dot{x}_2 = -4x_2 - 3x_1 + 2u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$y = x_1 + 0x_2$$

$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Now we come to transfer function.

- For transfer function, ~~to~~ convert it to differential equation.
- Take inverse Laplace transform.

$$\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

$$C(s) [s^3 + 9s^2 + 26s + 24] = 24.R(s) \quad \frac{dx}{dt} = x(t)$$

$$\frac{d^2x}{dt^2} - s^2 x(s)$$

$$\ddot{C} + 9\dot{C} + 26C - 24x$$

$$\begin{aligned}x_1 &= c \\ \dot{x}_2 &= \ddot{c} \\ \ddot{x}_3 &= \dddot{c}\end{aligned}$$

$$\begin{aligned}\dot{x}_1 &= c \\ \ddot{x}_2 &= \ddot{c} \\ \ddot{x}_3 &= \dddot{c}\end{aligned}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}$$

$$c = x_1$$

$$C = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$