Control Systems - 7th Semester

Lecture 4

Topics of last week

In last weeks, we studied about obtaining state-space models from differential equations

Then we studied about converting state-space model to transfer functions using formula

We also studied on converting transfer function to state-space model using canonical forms

Model - Recalling concepts

A model is representation or abstraction of reality/system

Who invent model? We, human beings, invent model based on our knowledge

This means the more knowledge a person has, the better he/she can write a model

What is mathematical model?

- A set of equations (linear or differential) that describes the relationship between input and output of a system

Types of Model

There are three types of mathematical models

- Black Box
- Grey Box
- White Box

Introduction to Transient Analysis

Sometimes we cannot write white box models for the systems

Either we do NOT know what is inside the system or either the system is too complex to verify the components

Perhaps sometimes the components are not easily identifiable and their configuration or layout is not readable

So another way to obtain model of a system is to apply a test input signal and obtain the output signal

Standard Input Signals

Though there are many possible combinations of input signals, the following are famous or popular input signals

- ❑ Impulse Signal
- ❑ Step Signal
- ❑ Ramp Signal
- ❑ Parabolic Signal

Impulse Signal

The impulse signal imitate the sudden shock characteristics of a signal

$$\boldsymbol{\delta(t) = \begin{cases} A, & if\ t = 0 \\ 0, & if\ t \neq 0 \end{cases}}$$

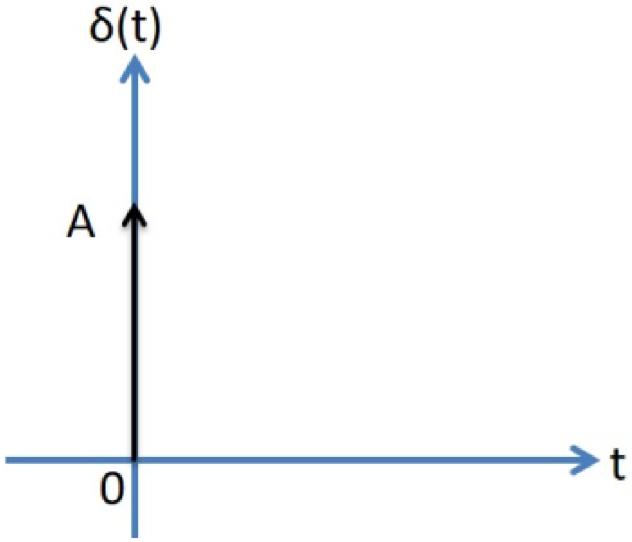

Figure: Impulse Signal

If A = 1, it is called unit impulse signal

Step Signal

The step signal is used to imitate the sudden change of a signal

$$u(t) = \begin{cases} A, & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}$$

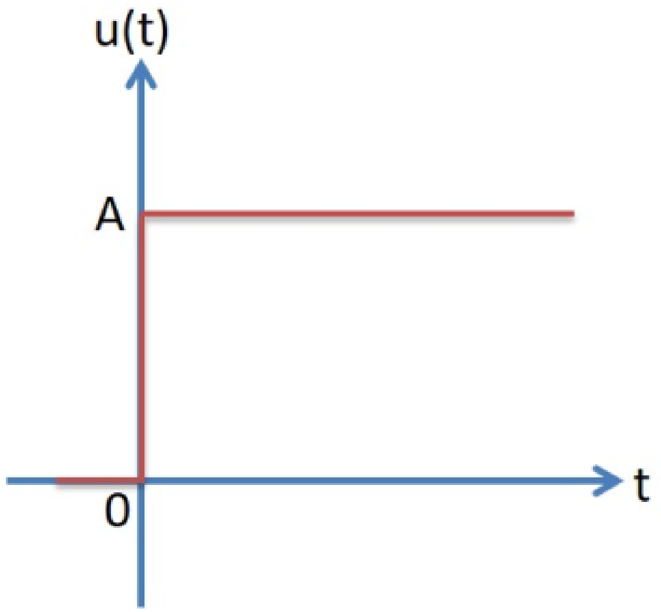

Figure: Step Signal

If A = 1, the step signal is called unit step signal

Ramp Signal

The ramp signal is used to imitate the constant velocity characteristic of a signal

$$\boldsymbol{r(t) = \begin{cases} At, & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}}$$

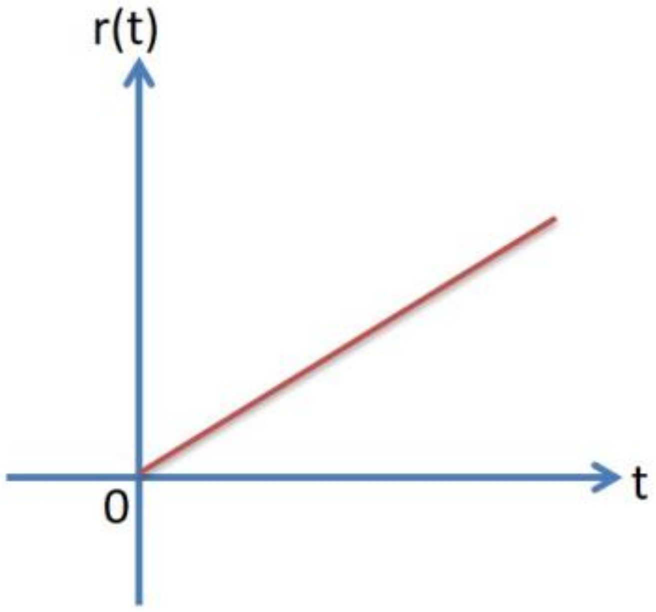

Figure: Ramp Signal

If A = 1, the ramp signal is called unit ramp signal

Parabolic Signal

The parabolic signal is used to imitate the constant acceleration characteristic of a signal

$$\boldsymbol{p(t) = \begin{cases} \dfrac{At^2}{2}, & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}}$$

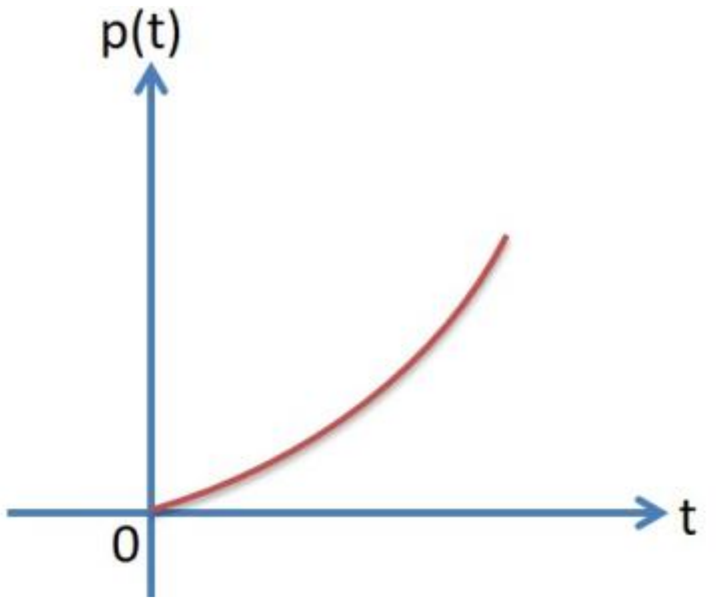

Figure: Parabolic Signal

If A = 1, the parabolic signal is called unit parabolic signal

Step Response

Impulse response is the best response because transfer function is defined as impulse response of a system

However, in practical life generating impulse signal is not easy

We use unit step signal as test input signal and then analyze the output of the system

First Order System

A first order system has a single pole (irrespective of number of zeros)

Many systems are of first order. Examples include

- velocity of a car on road
- an RLC circuit with only one capacitor and no inductor
- an RLC circuit with only one inductor and no capacitor
- level control in a tank
- speed control of dc motor in hard disk system
- time taken by queries in database management system, e.g.

  SELECT * from STUDENT

  where ATTENDANCE PERCENTAGE > 75;
- energy consumed by an IoT device

Step Response of First Order System

A general first order system without zeros can be written as follows:

$$\boldsymbol{G(s) = \frac{b}{s+a}}$$

Let ***C(s)*** be the output of a system having transfer function ***G(s)*** (expressed above). If the input to ***G(s)*** is a unit step, then the output can be expressed as follows:

$$Output\ Signal = Input\ Signal\ \times Transfer\ function$$

We can further write the following:

$$\boldsymbol{C(s) = Unit\ step\ signal\ \times G(s)}$$

$$\boldsymbol{C(s) = \frac{1}{s} \times \frac{b}{s+a} = \frac{b}{(s)(s+a)}}$$

Step Response of First Order System

A first order system without zeros can be written as follows:

$$\boldsymbol{G(s) = \frac{b}{s+a}}$$

The term $\boldsymbol{a}$ is important time. The inverse of $\boldsymbol{a}$ is called time constant i.e.

$$\boldsymbol{\tau = \frac{1}{a}}$$

The gain ***K*** is also called as dc-gain or steady-state gain of a system

$$\boldsymbol{K = \frac{b}{a}}$$

Sometimes in numerator, we have a gain term. If we can identify the gain and time-constant from step response of a system, then we can obtain its transfer function also

Step Response of First Order System

For example compute $\boldsymbol{\tau}$ and gain $\boldsymbol{K}$ of the following system:

$$\boldsymbol{G(s) = \frac{3}{s+2}}$$

Here $\boldsymbol{\tau} = \frac{\mathbf{1}}{\mathbf{2}} = \mathbf{0.5}$

gain $K = \frac{\mathbf{3}}{\mathbf{2}} = \mathbf{1.5}$

The value of gain $\boldsymbol{K}$ indicates the final steady-state value of the step response

Step Response of First Order System

In order to compute transfer function from a plot, we need to define a few more terminologies

Rise Time: $\boldsymbol{T_r}$, time taken to reach **90%** or **0.9** of final value from **10%** or **0.1**

Mathematically:

$$\boldsymbol{T_r = \frac{2.2}{a}}$$

Settling Time: $\boldsymbol{T_s}$, time taken to stay within **2%** of its final value (or reach **98%** of final value).

Mathematically:

$$\boldsymbol{T_s = \frac{4}{a}}$$

Step Response of First Order System

Can you compute the transfer functions?

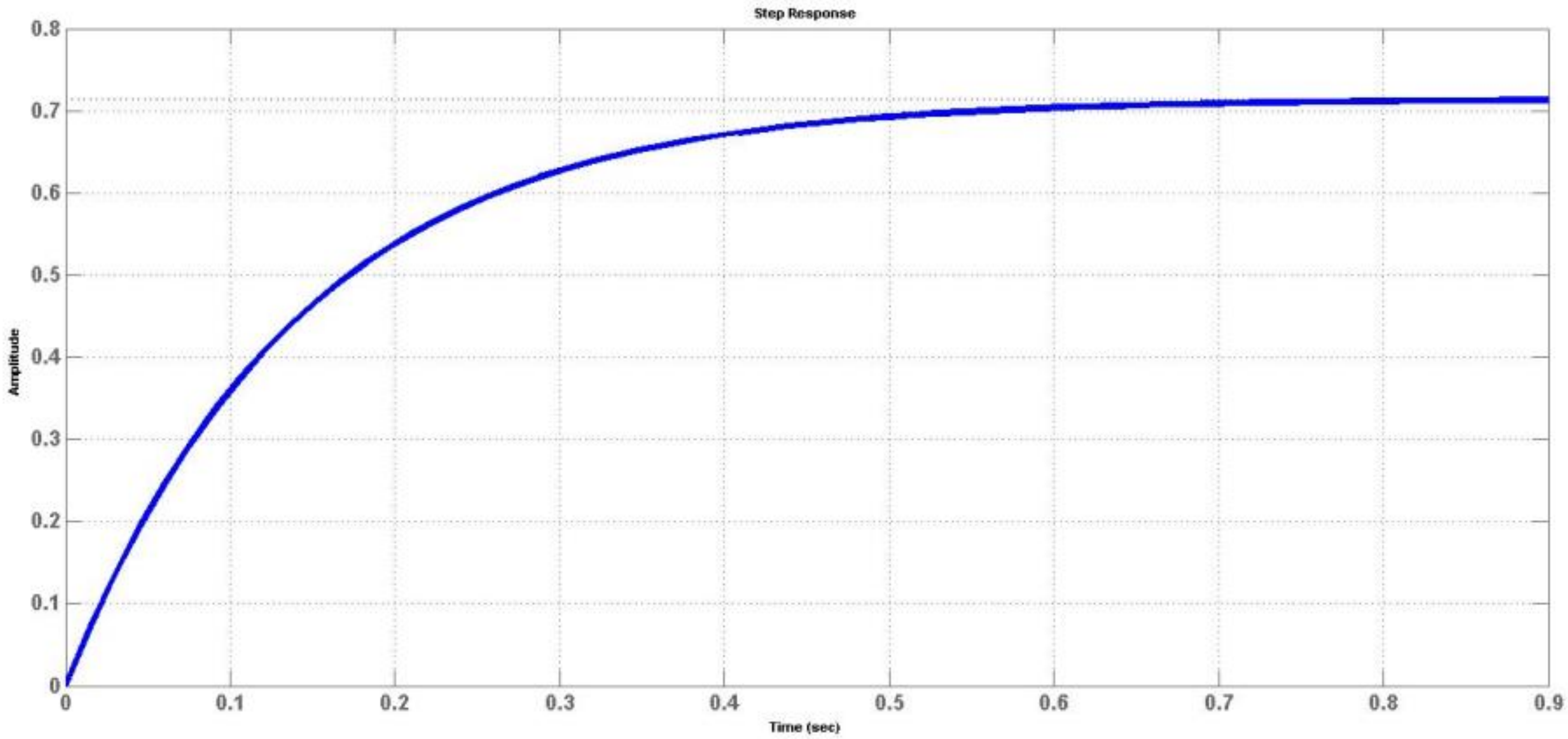

Figure: Step Response of a transfer function

Step Response of First Order System

Time constant: Time to reach **63%** of final value. Compute the transfer function from the previous plot

Solution:

Final value = steady-state value = gain, ***K*** = **0.72**

63% of final value is **0.63 × 0.72** = **0.4464**

Time taken to reach **0.45** value is **0.15** seconds

And a = 6.67

The value of b is 4.802

Step Response of First Order System

The final transfer function is

$$G(s) = \frac{4.802}{s + 6.67}$$

The previous step-response was obtained for the following actual transfer function:

$$G(s) = \frac{5}{s + 7}$$

MATLAB code for obtaining step response

```
num = [5] ;

den = [1 7] ;

step(num, den)
```

Step Response of First Order System

Effects of decreasing time constant

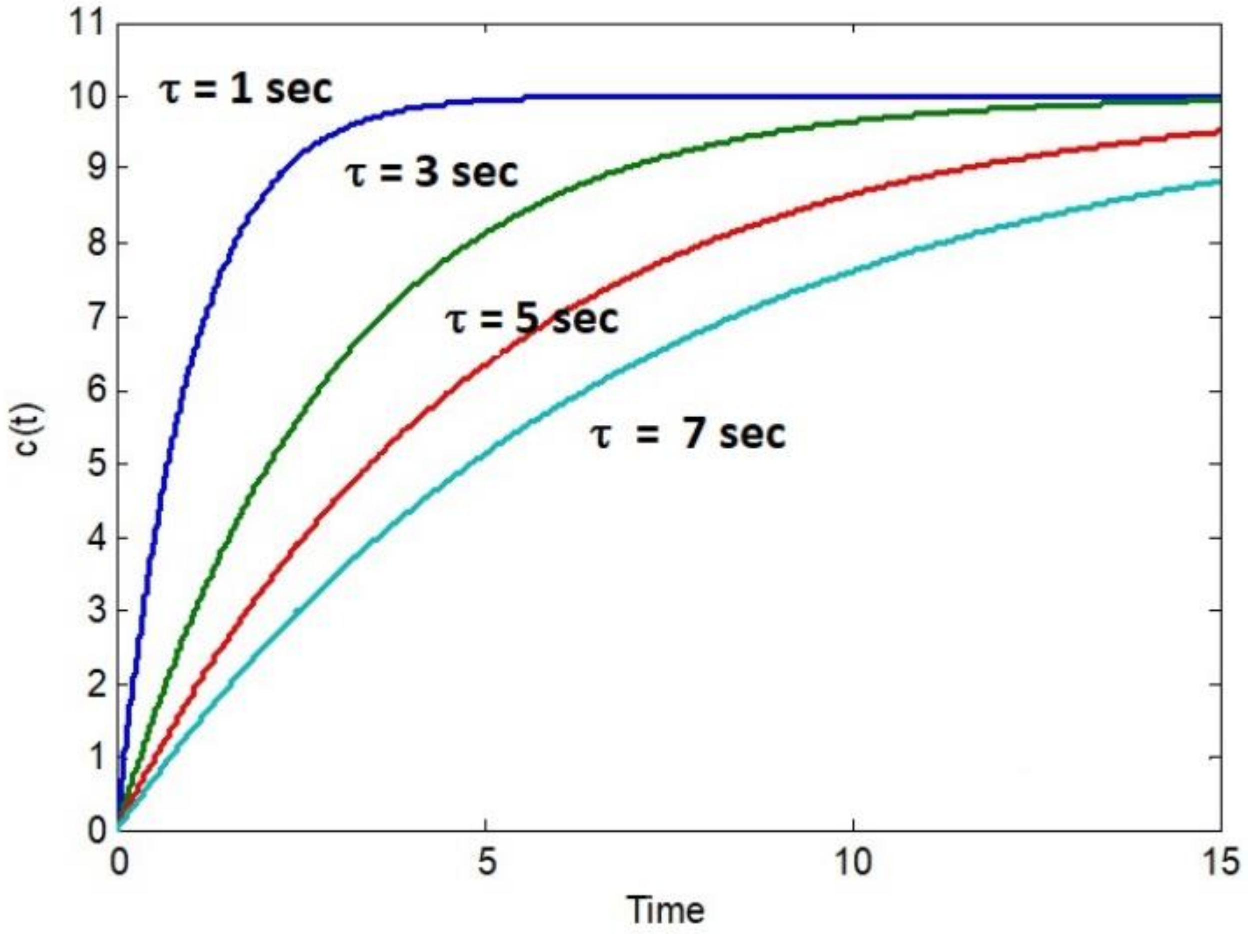

Figure: Effects of decreasing time constants of first order transfer function

Step Response of First Order System

Effects of increasing gains (remember its ***K*** not the term ***b***)

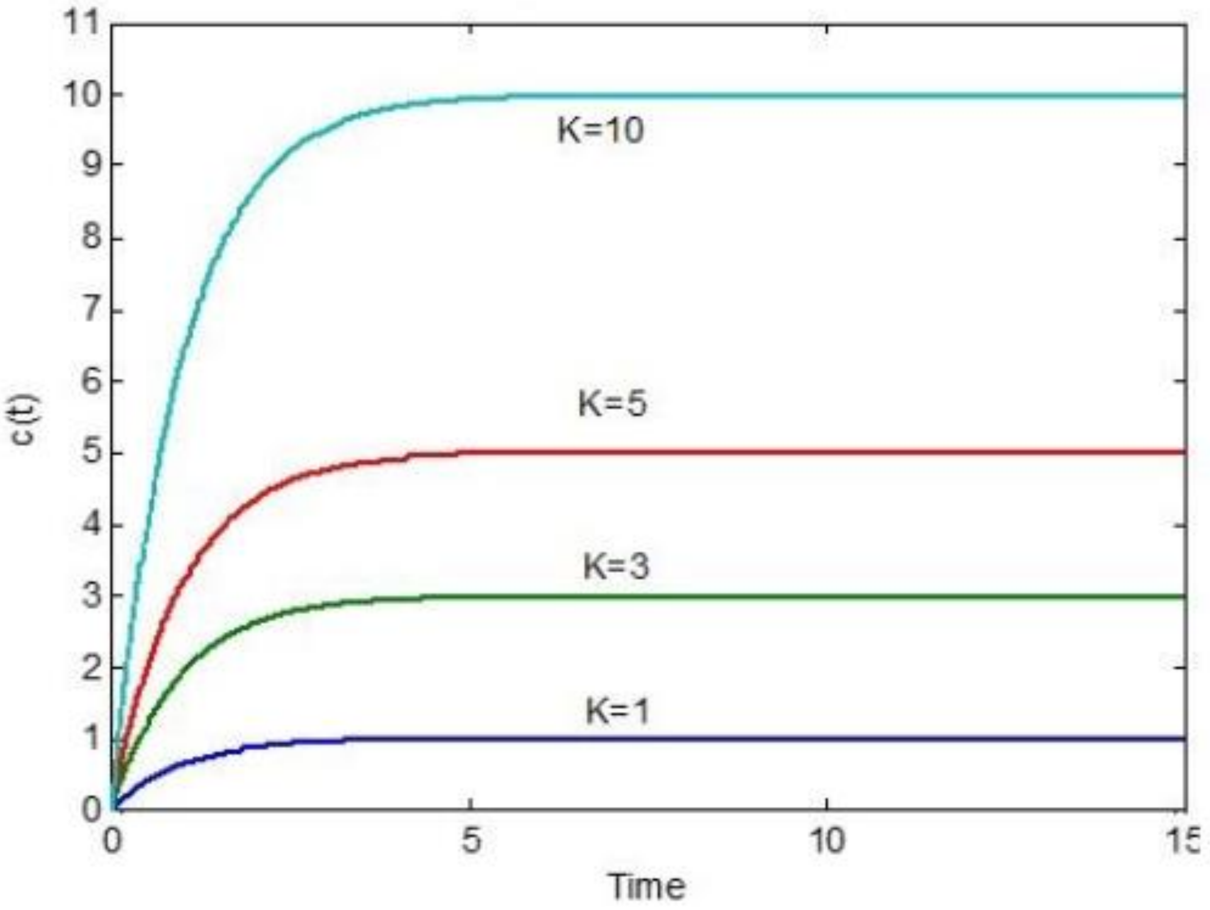

Figure: Effects of increasing gains of first order transfer function

First Order Systems Summary

In first order system, we only have 2 parameters: dc gain and time-constant

Varying these two parameters only change the speed or amplitude of step response

Which parameter changes the speed of first order transfer function?

Which parameter changes the amplitude of first order transfer function?