

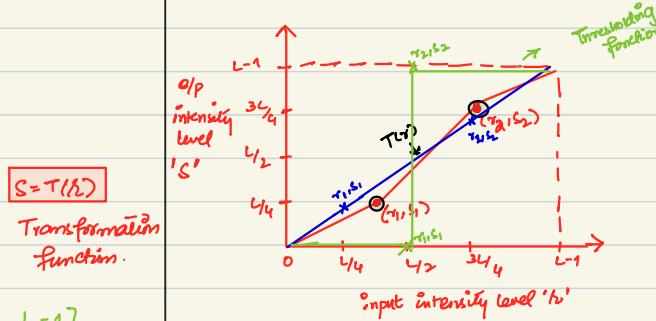
### Piece-wise Linear Transformation Functions-

This transformation function is complementary to the other functions we discussed previously. This function has both advantages & disadvantages. It is less complex but it requires more user inputs.

Three different applications:

1. Contrast Stretching.
2. Intensity level slicing.
3. Bit plane slicing.

1. Contrast Stretching:- A requisite image sometimes of very low quality because restriction of the dynamic range of the acquisition system we have used or may be due to poor illumination lighting conditions or may be due to wrong fixing of the aperture. So, those low contrast images should be enhanced.



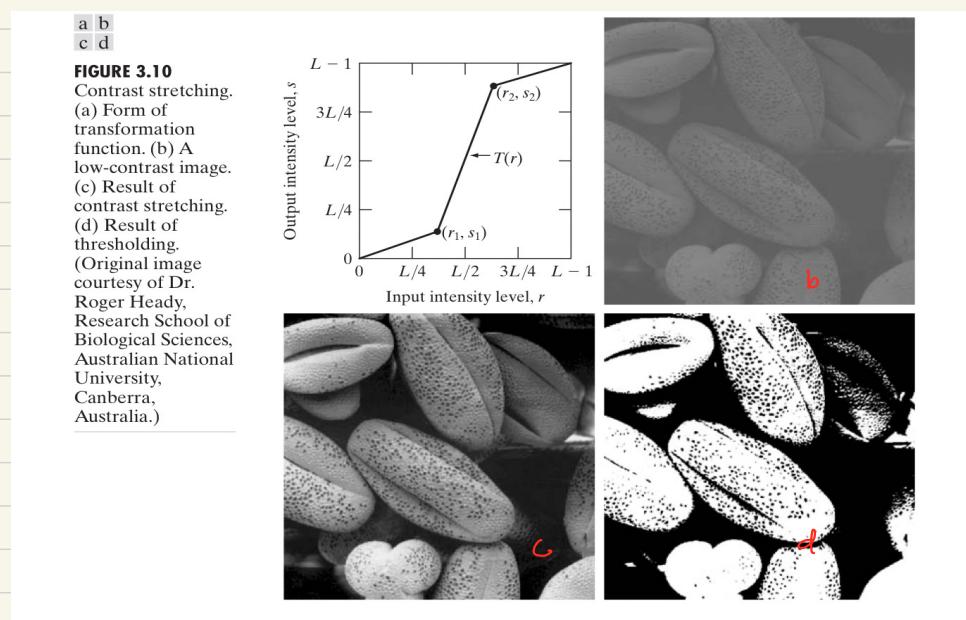
CASE I: If  $r_1 = r_2 \& s_1 = s_2$

Identical function, no intensity change.

CASE II: if  $r_1 = r_2 \& s_1 = 0, s_2 = L-1$

Hard Thresholding function.

CASE III: If  $r_1 = l_{\min} \& r_2 = l_{\max}, s_1 = 0, s_2 = L-1$   
contrast stretching

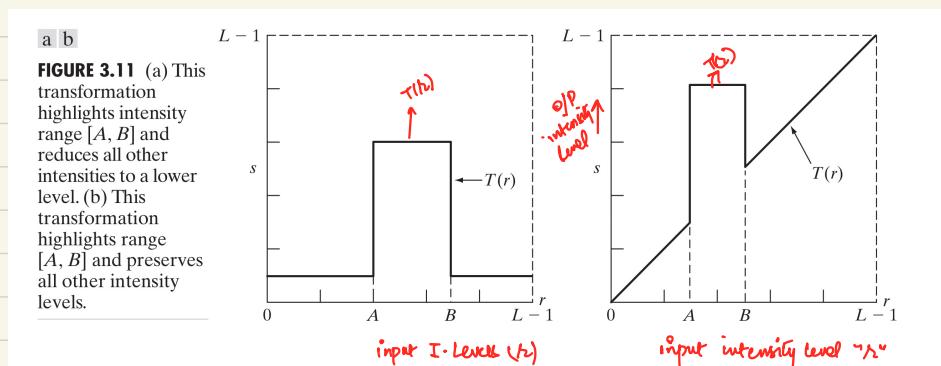


Intensity level slicing: ① used to highlight a particular range of intensity.

- ④ used in medical applications. (x-rays) - Enhance a part of

X-rays where the flaws are present. ⑤ Highlight the range of intensity levels which is of interest. ☐ Also useful in satellite images.

\* can be done using two approaches:



- Both these approaches are useful in the case(s) where we want to extract some object of interest.



**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

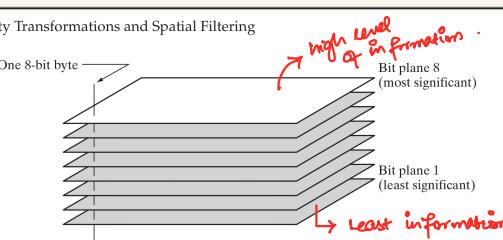
(3) Bit-plane slicing: ① Highlight the entire image appearance which is specific to that particular bit plane.

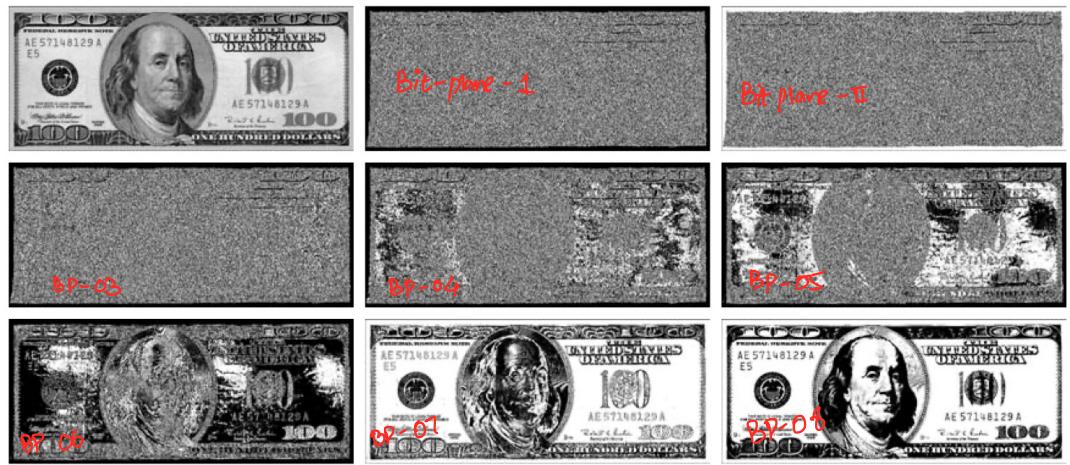
e.g. 8-bit image:  $2^8 = 256$

$$L = [0, 1, \textcircled{2}, 3, 4, 5, \dots, 255]$$

A horizontal line representing memory. Above it, the binary number **00000010** is written. The first bit, **0**, is circled in red and has an arrow pointing to it from the label **MSB** below. The last bit, **0**, is circled in red and has an arrow pointing to it from the label **LSB** below.

**FIGURE 3.13**  
Bit-plane representation of an 8-bit image.





**FIGURE 3.14** (a) An 8-bit gray-scale image of size  $500 \times 1192$  pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

- Used in compression techniques. We can avoid lower order bit plane and exploit the higher order bit plane for the reconstruction of the image.



### Histogram Processing :

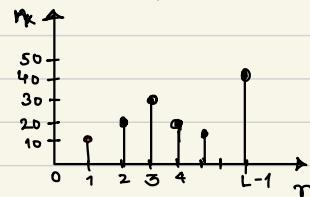
- Basis of various spatial domain processing technique
- Used to improve the quality of the image
- Used in image compression, enhancement and segmentation.

- $[0, L-1]$ , Histogram is nothing but the frequency of occurrence of each gray level in the image

- Graphically represented as two types of plots:  
I : x-axis : gray level & y-axis : no. of pixels in the image in each gray level.

$f(x_k) = n_k$

$x_k = k^{\text{th}}$  intensity level  
 $n_k = \text{no. of pixel values in the image with the intensity level.}$



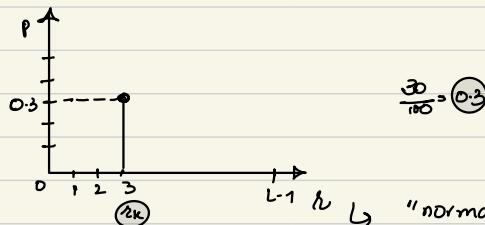
II:  $x$ -axis  $\rightarrow$  gray level &  $y$ -axis  $\rightarrow$  Probability of occurrence of pixels in each gray level

$$P(k) = \frac{n_k}{MN}$$

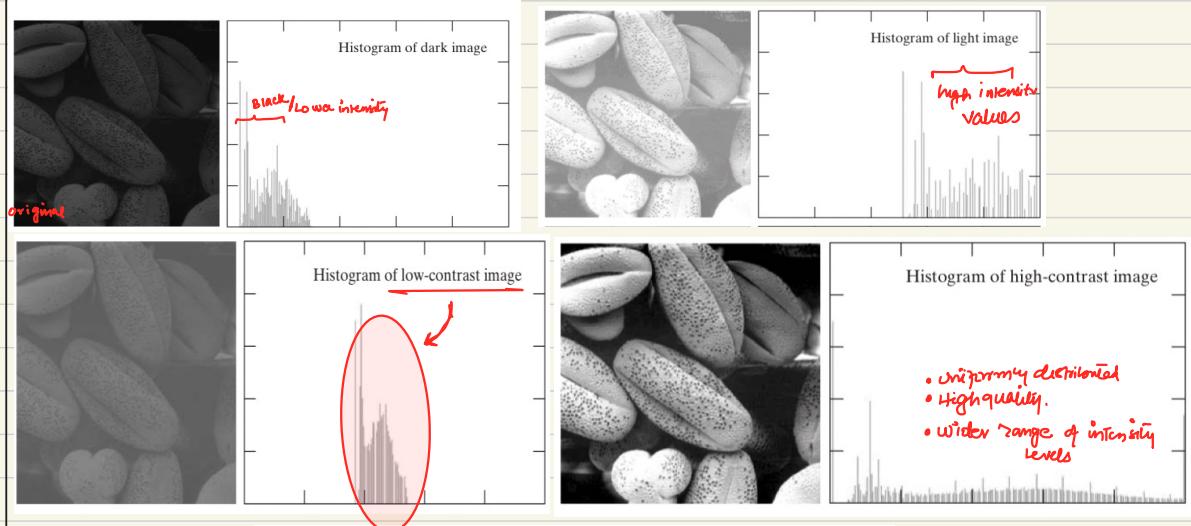
$n_k$  =  $k^{\text{th}}$  intensity level

$n_k$  = no. of pixels in the image with intensity level ( $k$ )

$MN$  = Total no. of pixels in an image.

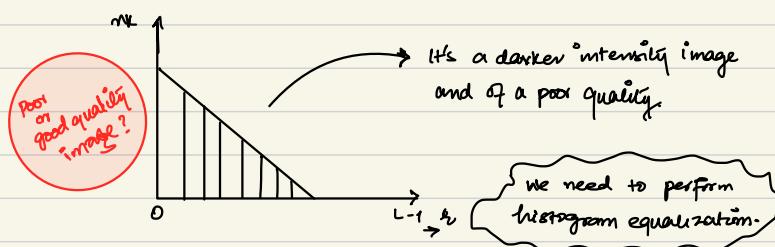


"normalized histogram"

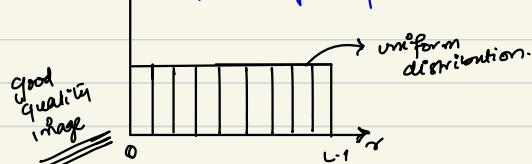


### Histogram Equalizations-

To obtain the flat-top histogram, we can use histogram equalization. It is a technique to improve the quality of the image. It helps to obtain the uniformly distributed histogram over the entire intensity levels from 0 to  $L-1$ .



After Histogram equalization.



$$S = T(r) \rightarrow ①$$

To transform the intensity levels of the input image to the new intensity levels we usually used the transformation functions e.g. using eq ①. The mapping of "r" to "s" depends on "T". This "T" can be anything like summation etc.

To perform histogram equalizations there are two important conditions:

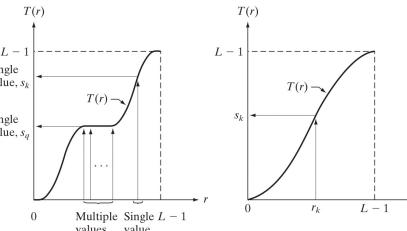
(i)  $T(r)$  must be a single value & monotonically increasing function in nature for the interval  $0 \leq r \leq L-1$ .

(ii)  $0 \leq T(r) \leq L-1$  for  $0 \leq r \leq L-1$ .

②

cumulative results

**FIGURE 3.17**  
(a) Monotonically increasing function, showing how multiple values can map to a single value.  
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



Recall that a function  $T(r)$  is monotonically increasing if  $T(r_2) \geq T(r_1)$  for  $r_2 > r_1$ .  $T(r)$  is a strictly monotonically increasing function if  $T(r_2) > T(r_1)$  for  $r_2 > r_1$ . Similar definitions apply to monotonically decreasing functions.

An image with a continuous intensity values shown in Fig 3.18a (probably a low contrast image) - a poor quality image requires to flatten the histogram producing a good quality image.

$P_r(r)$  is a PDF of "r"  $\rightarrow$  input image

$P_s(s)$  is a PDF of "s"  $\rightarrow$  output (transformed) image

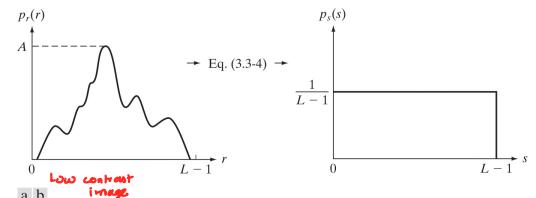
Using basic of Probability theory

$$P_s(s) = P_r(r) \left| \frac{dr}{ds} \right| \rightarrow \text{eq ②}$$

Let "T" be a particular transformation

$$T(r) = (L-1) \int_0^r P_r(w) dw \rightarrow ③$$

where "w" is a dummy variable for integration.



**FIGURE 3.18** (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels,  $r$ . The resulting intensities,  $s$ , have a uniform PDF, independently of the form of the PDF of the  $r$ 's.

Considering eq ②  $S = T(r)$

Differentiating w.r.t "r" on both sides

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$\frac{ds}{dr} = \frac{d}{dr} \left[ (L-1) \int_0^r p_r(\omega) d\omega \right]$$

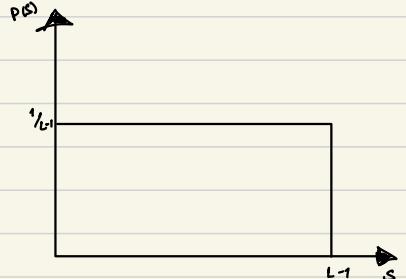
$$= (L-1) \frac{d}{dr} \int_0^r p_r(\omega) d\omega$$

From Leibnitz rule

$$\frac{ds}{dr} = (L-1) p_r(r) \rightarrow ④$$

$$\text{Substitute } ④ \text{ in eq ③ } P_S(S) = P_r(r) \left| \frac{1}{(L-1) p_r(r)} \right|$$

$$P_S(S) = \frac{1}{L-1} \quad ⑤ \quad 0 \leq S \leq L-1$$



- Suppose a continuous valued image "u" considered having

$$p_r(r) = \begin{cases} \frac{2\pi}{(L-1)^2}, & 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

$$P_S(S) = P_r(r) \left| \frac{ds}{dr} \right|$$

Transform function:  $S = T(r)$

$$\text{Let } T(r) = (L-1) \int_0^r p_r(\omega) d\omega$$

$$= (L-1) \int_0^r \frac{2\omega}{(L-1)^2} d\omega \Rightarrow \frac{2(L-1)}{(L-1)^2} \int_0^r \omega d\omega$$

$$T(r) = \frac{2r}{L-1} \left( \frac{r^2}{2} \right) \Rightarrow T(r) = \frac{r^3}{L-1}$$

Differentiating w.r.t "r" on both sides

$$\frac{ds}{dr} = \frac{d}{dr} [T(r)]$$

$$= \frac{d}{dr} \left( \frac{r^3}{L-1} \right) \Rightarrow \frac{ds}{dr} = \frac{3r^2}{L-1} \quad ⑥$$

Substitute eq ⑥ in eq ③ we get

$$P_S(S) = \frac{2\pi}{(L-1)^2} \left| \frac{(L-1)}{2r} \right|$$

$$P_S(S) = \frac{1}{L-1} \rightarrow ⑦$$

The given signal is converted to the good quality image by applying a histogram equalization.  
 $P_S(S) = \frac{1}{L-1}$  produces a uniform histogram for a range  $0 \leq S \leq L-1$

For Discrete Values :

$$P_Y(r_k) = \frac{n_k}{MN} \rightarrow ①$$

Where  $r_k \rightarrow k^{\text{th}}$  intensity value,  $n_k \rightarrow \text{no. of pixels with the intensity value } r_k$ , and  $MN \rightarrow \text{Total no. of pixels in an image}$

$$S_k = T(r_k) \rightarrow ②$$

Transformation function.

$$\text{Let } T(r_k) = (L-1) \sum_{j=0}^k P_Y(r_j) \rightarrow ③$$

Substitute ① in ③ we get

$$T_Y(r_k) = (L-1) \sum_{j=0}^k \frac{n_j}{(MN)}$$

$$S = T(r_k) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j \rightarrow ④$$

↳ Histogram equalization or "histogram linearization".

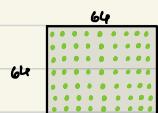


Histogram Equalization:

Example : Perform histogram equalization on a 3-bit image ( $L=8$ ) of size  $64 \times 64$  pixels. The intensity distribution of the image is given below:

Given levels ( $r_k$ )	0	1	2	3	4	5	6	7
No of pixels ( $n_k$ )	790	1023	850	656	329	245	122	81

= 4096



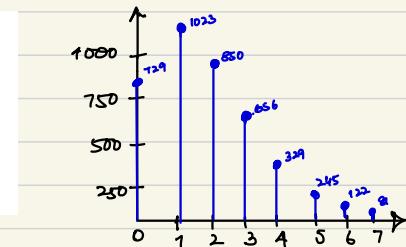
$$L = 2^3 = 8 \quad [0, L-1] \text{ i.e., } [0 \dots 7]$$

$$M = 64, N = 64$$

$$MN = 64 \times 64 = 4096 \text{ pixels}$$

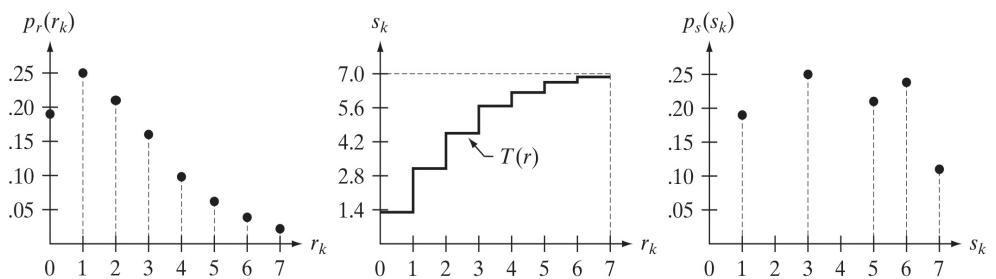
TABLE 3.1  
Intensity distribution and histogram values for a 3-bit,  $64 \times 64$  digital image.

$r_k$	$n_k$	$p_Y(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



For discrete intensity values

$$S_k = T(r_k) = (L-1) \sum_{j=0}^k P_Y(r_j)$$



a b c

**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

$$\text{For } k=0, s_0 = T(r_0) = 7 \sum_{j=0}^0 P_r(r_j) = 7 \cdot P_r(r_0) = 7 \cdot 0.19 = 1.33$$

$$\text{For } k=1, s_1 = T(r_1) = 7 \sum_{j=0}^1 P_r(r_j) = 7(P_r(r_0) + P_r(r_1)) = 7(0.19 + 0.25) = 1.33 + 1.75 = 3.08$$

$$\text{For } k=2, s_2 = T(r_2) = 7 \sum_{j=0}^2 P_r(r_j) = 7P_r(r_0) + 7P_r(r_1) + 7P_r(r_2) = 1.33 + 1.75 + 1.47 = 4.55$$

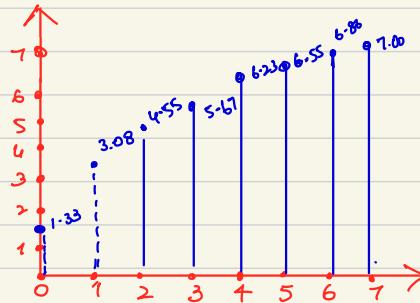
$$\text{For } k=3, s_3 = T(r_3) = 7 \sum_{j=0}^3 P_r(r_j) = 7P_r(r_0) + 7P_r(r_1) + 7P_r(r_2) + 7P_r(r_3) = 4.55 + 7 \cdot 0.16 = 5.67$$

$$\text{For } k=4, s_4 = T(r_4) = 7 \sum_{j=0}^4 P_r(r_j) = 5.67 + 7 \cdot 0.08 = 6.23$$

$$\text{For } k=5, s_5 = T(r_5) = 7 \sum_{j=0}^5 P_r(r_j) = 6.23 + 7 \cdot 0.06 = 6.65$$

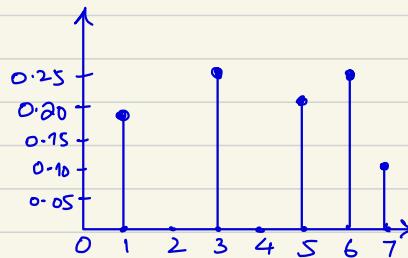
$$\text{For } k=6, s_6 = T(r_6) = 7 \sum_{j=0}^6 P_r(r_j) = 6.65 + 7 \cdot 0.03 = 6.86$$

$$\text{For } k=7, s_7 = T(r_7) = 7 \sum_{j=0}^7 P_r(r_j) = 6.86 + 7 \cdot 0.02 = 7.00$$



• Transformed function.

$r_K$	$n_K$	$P_K(r_K) = n_K/MN$	$S_K = T(r_K)$	round off	Histogram equalization $P_{Eq}(s_K)$
$r_0 = 0$	790	0.19	1.33	1	0.19
$r_1 = 1$	1023	0.25	3.08	3	0.25
$r_2 = 2$	850	0.21	4.55	5	0.21
$r_3 = 3$	656	0.16	5.67	6	$\frac{(656+329)}{4096} = 0.24$
$r_4 = 4$	329	0.08	6.23	6	
$r_5 = 5$	245	0.06	6.65	7	$\frac{(245+122+81)}{4096} = 0.10$
$r_6 = 6$	122	0.03	6.86	7	
$r_7 = 7$	81	0.02	7.00	7	



Perform the histogram equalization for the following image.

$$f(x, y) =$$

Input image.

1	2	1	1	1
2	5	3	5	2
2	5	5	5	2
2	5	3	5	2
1	1	1	2	1

$$\text{max value} = 5$$

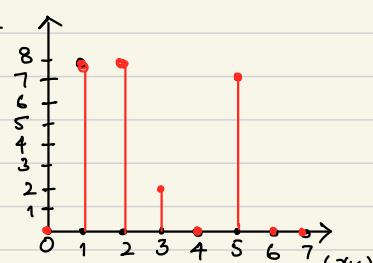
$$2^0 = 1, 2^1 = 2, 2^2 = 4$$

$$2^3 = 8$$

$$L = 8, L-1 = 7$$

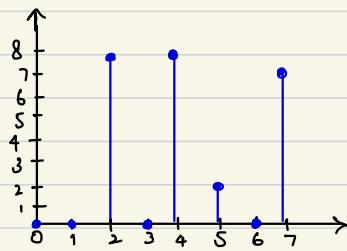
$$\begin{array}{ccccccccc} \text{Gray levels } (r_K) & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{No of pixels } (n_K) & 0 & 8 & 8 & 2 & 0 & 7 & 0 & 0 \end{array}$$

Gray Levels ( $r_K$ )	No of pixels ( $n_K$ )	$P_K(r_K) = n_K/M$ (PDF)	$S_K$ (CDF)	$S_K$	Histogram of Eq; level
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1.0	7	7
6	0	0	1.0	7	7
7	0	0	1.0	7	7
					$\sum n_K = 25$



Histogram of the input image

Gray Levels	0	2	4	5	7
No; of Pixels	0	8	8	2	7



Histogram of the  
equalized image

1	2	1	1	1
2	5	3	5	2
2	5	5	5	2
2	5	3	5	2
1	1	1	2	1

Output  
image

2	4	2	2	2
4	7	5	7	4
4	7	7	7	4
4	7	5	7	4
2	2	2	4	2