Q
is for
QUANTUM

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For Xavier, Aby, Lydia, Jesse and Caleb

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Part III: Q-REALITY

Realism and physics

Science works, believe (in) it or not. Fortunately for phone companies the laws and scientific principles underpinning how your cellphone works are the same whether built by a person in China or one in the USA. Although you likely take this for granted, why should the laws of physics behave the same for different people in different countries?

One way to explain this universality is to posit that there is a deeper "underlying reality" upon which physics is based. More specifically we could presume that: (i) there exists a physical world that is external to us, and (ii) that we are not particularly important to what is going on in this external world—the stuff within this universe existed before we came along and will continue to exist once any or all of us are gone.

The extent to which we can draw "ultimately true" conclusions about this underlying reality from our scientific theories is debatable, tuned as the theories are to solving pragmatic problems for human-scale creatures. Still, one might think it unarguable that "there really is something there," and the fact that the human endeavor of science succeeds equivalently across variations in time, location and culture of the practitioner is somehow due to this.

The preceding paragraphs summarize a view known as (naive) scientific realism, of which there are multiple subtle variations. There are many, many people who would disagree with some, or all, of this view, both scientists and non-scientists. If you are interested in alternatives then start with the Wikipedia article on philosophical realism, and follow your nose until your brain hurts.

I am not qualified to enter a debate on such topics. My goal in this final part of the book is to quantitatively expose to you the extent to which the somewhat weird phenomena introduced in Parts I and II, and our physical laws that describe them in terms of misty states, yield new insights and conundrums on such questions, irrespective of your own philosophical preferences. Feel free to ignore any armchair philosophy that has crept in (I have tried hard to avoid it). I also don't want to get tied up in knots being overly cautious about language. I hope you are both willing and able to apply your own filters without rejecting the overall message, because the quantitative and technical things that you are now able to understand about our physical theories will sharpen your understanding of your own viewpoints; as well as, I hope, rule out some things you thought were obviously true.

Physical properties

If we at least agree that there are other people than ourselves (and frankly, if solipsism is true and you are creations of my mind, then you would all be better looking and made of chocolate), and if we also agree those people are experiencing the world in a similar manner to ourselves (again, you will find people who make a big deal of the fact that this is unprovable), then the consistency of our conversations about what we are individually experiencing ("it's hot," "that book is heavy") leads us to implicitly or explicitly form the useful notion that "things have physical properties."

We all appreciate it can be a blurry line between properties we know are subjective ("the banana smells nice") and those our common agreement indicates are objective ("the banana is yellow"). But it is a natural concept that there are at least some objective physical properties, of some kinds of material things, which are out there in the universe and independent of our subjective experiences. Similarly, it's a natural concept that some of those properties are more fundamental than others. Think of the incredible diversity of textures, smells, colors, and tastes of everything that we personally experience; all this originates from less than a hundred different building blocks we call atoms. Lego can't even make a model Millennium Falcon without double that number of different building blocks.

It is only a slight idealization to say that all theories of physics begin by stating the types of objects that exist, which is done by listing their fundamental properties. The ways objects evolve and interact are the basis of physical laws—statements about how the "values" of the fundamental physical properties change and affect each other. For example, if "mass" and "position" and "speed" are fundamental properties of a rock, then "a two-ton rock is located directly above your head and falling at a hundred miles per hour" is a statement about the initial values of those properties, from which you can deduce the exciting future consequences.

Originally our physical laws were based on properties we have personal experience with—we know rocks have mass, speed, position, color, texture, and so on. As time went on, we found that there are properties whose value cannot be experienced directly by a human quite so easily—energy for example. Typically, the properties are all interdependent—you can deduce the speed of the rock from its mass and energy and current position, but equivalently you can deduce the energy of the rock from its mass and speed and current position. Making good choices about the properties to take as fundamental is crucial to doing good physics—the theory of seismic wave propagation should not begin with considerations of the locations of individual grains of sand on the beach.

Correspondence between mathematics and physical properties

Our physical theories cover incredible scales, from the tiniest particles to the whole universe. The properties of the things at the smallest scales—the subatomic particles—include properties like mass, charge, position, energy, spin, etc. These would seem to be the "absolutely most fundamental" properties of the stuff that makes up the world. But the set of "most fundamental" physical properties keeps undergoing revision as we deepen our understanding, and this changes the terms in which we couch our smallest-scale theories. Often things that seemed absolutely fundamental turn out to be built up from more fundamental things, and so previously absolutely fundamental properties are now seen as derivative.

We infer new properties—and even the existence of new things—that we cannot directly experience, either by realizing that a good explanation of some phenomenon requires them, or because the mathematical consistency of some physical laws requires them. In either case the laws of physics are ultimately mathematical, and so for every object and every associated physical property "out there" in the world there is some kind of mathematical counterpart in our physical theory. The converse, however, is not true. Our theories can contain mathematical objects that we do not believe necessarily have a direct counterpart in the physical world. A subtle example, relevant to what comes later, occurs when you're about to flip a coin. "50% probability" is a mathematical object you assign to the coin's landing on "heads." It is not a property of the coin, it is not something that affects the coin's trajectory through the air, but it's manifestly useful to you predicting the outcome of a future observation. It is a state of your knowledge or information. While it may seem that such a subjective thing surely cannot be an integral part of a physical law, in fact the laws of thermodynamics have just such an "ignorance-quantifying" mathematical object indispensably hardwired in, known as entropy. Without entropy in our physical laws we would not be able to understand how the engine of your (future?) motorcycle works.

Sometimes, and this is where our story becomes interesting, explaining what we observe can only be done using some particular mathematical object, but we are unsure if that object is or is not in correspondence with some physical property. The misty states are the example we will examine in detail, because our whole notion of "what is real" and "what is really going on" changes dramatically according to whether or not they are a physical property of the world.

In order to appreciate the full ramifications of what we are going to talk about, I want to emphasize that if our current physical understanding of the universe based on misty states is correct, then everything you have learned about them applies equally well to all physical properties of everything that makes up the universe. The black and white balls (via which I introduced you to misty states) are typically composed from many particles, and their color is a derivative property. Yet their color can be in a misty state. To date we do not know of any physical property, fundamental or derivative, that in principle cannot be put into a misty state, and it would be mysterious if such a non-misterious property existed.

To refine some concepts and obtain some useful language we first consider a case about which there is no controversy, namely the distinction between states that are in your mind and the real states of the world when you flip a coin.

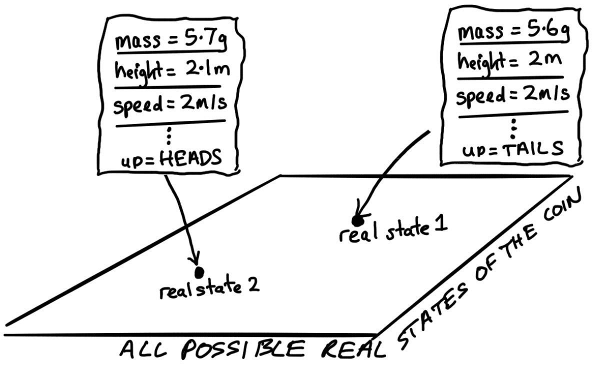
A deeper description of the rocky state of a coin

You might assign a coin you have just flipped the rocky state:

This is a more diagrammatic representation of the "50% probability of heads" you assign to the coin. It would be implausible to suggest that what happens to the coin when you toss it in the air is a result of the coin thinking to itself: "Now I should fly over there and land this way because I can sense that the weird-looking human over there thinks I play fair and is thus assigning me that particular rocky state."

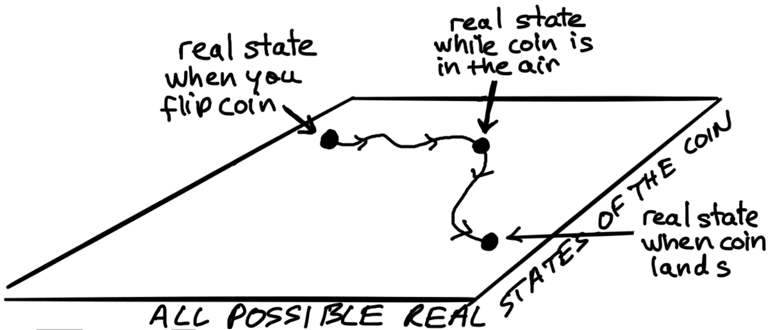
A rocky state of the coin is therefore unarguably a state of your mind: a state of your knowledge or your information. It is a tool that lets you summarize your best predictions as to what you will subsequently observe. The connection between the rocky state in your mind, and the dynamics of the coin, is indirect. The coin responds to the "actually real" physical circumstances—the same ones that you believe are relevant but realize you are ignorant about, causing you to assign fair odds in the first place.

The real state of the coin—as opposed to its rocky state—consists of physical properties, like its mass, shape, color, height above the floor, speed and rotation through the air, and so on. We can make a simple diagram that captures all the possible real states of a coin:

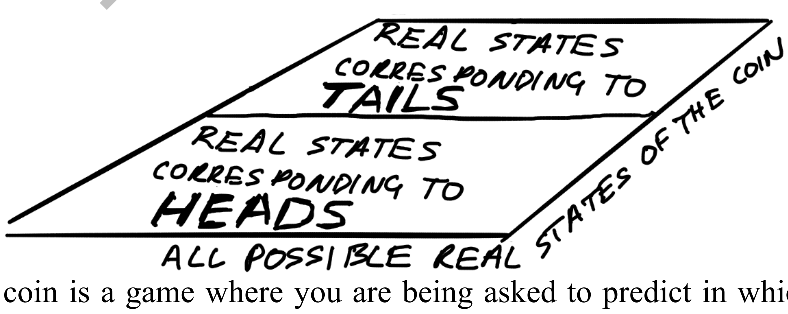


The square plane in the diagram represents every possible real state of the coin—there are an infinite number of possibilities. I have labelled two of them. You can imagine a piece of paper at each point which lists all the relevant physical properties and their values corresponding to that particular real state—the mass, height, speed and so on. One of the real states of the coin is whether the upwards face of the coin is heads or tails. (While it is spinning in the air there are fractions of a second for which the coin is "on its side," but to keep things simple I will ignore those.) Although I have represented the set of all real states by a square plane, this is just a schematic diagram. As more properties of the coin are included on each piece of paper that defines a real state physicists prefer to use higher-dimensional shapes to represent the set of all real states. This is because they like the representation to have the feature that two nearby points in the set of real states list physical properties with values very similar to each other. My drawing skills are not up to that, so you should not take the particular square plane I have drawn too literally.

Some of the properties of the coin—its mass, for example—stay the same when you flip it. Other properties change—for example, its height above the ground; its speed; which side faces upwards. In terms of the diagram of the real states of the coin, the whole procedure maps out a line through the set of real states:



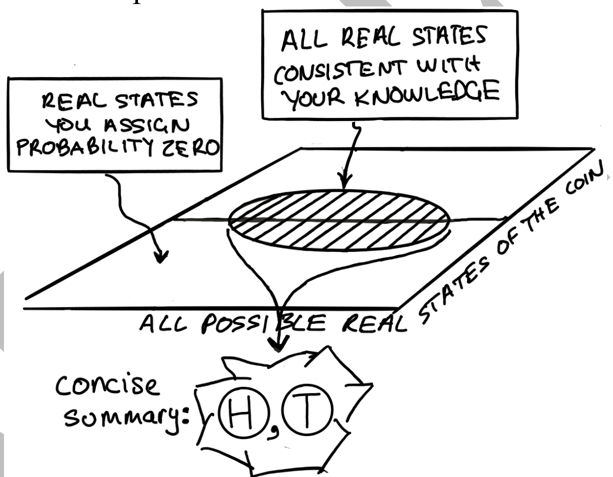
The set of all possible real states can be divided in two, according to whether the real state corresponds to the coin having heads facing upwards or tails facing upwards:



Flipping a coin is a game where you are being asked to predict in which of these two regions the final real state of the coin (i.e. after it has landed) will end up. If you knew the actual real state of the coin at the instant you flipped it, as well as all the physical laws governing how it flies through the air, then you would know whether it was going to land heads or tails. In practice, you do not know the initial real state perfectly, nor do you have the capability of calculating the trajectory of the coin fast enough, and so you cannot predict in which of the two regions the final real state of the coin will be.

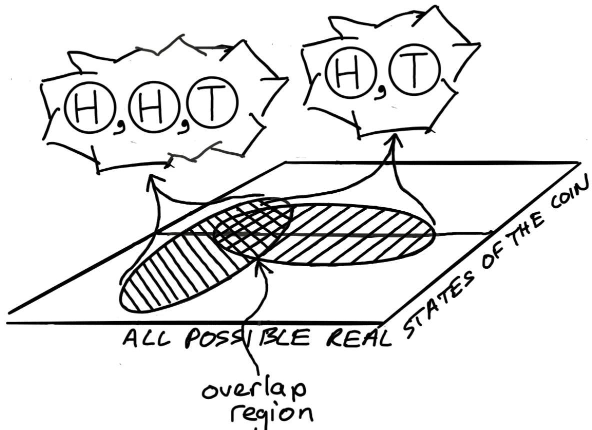
However, at any instant of time, you do know some things about the real state of the coin. You know its height in the air is not a hundred meters; you know its mass is not a kilogram. All of those ridiculous options exist within the set of all possible real states depicted above. You do not, therefore, assign equal likelihood to all of the possible real states—that is not why you say the coin is fair. The reason you assign the coin the rocky state of a "fair flip," is that you are saying: "Amongst all the potential real states that are consistent with what I know about the coin once it has landed, half of them are in the region corresponding to heads, and half are in the region corresponding to tails." For simplicity, let's assume that—amongst all the real states consistent with what you know about the coin—you assign an equal likelihood to the coin "actually being" in any particular one of them once it has landed. To all the rest, you assign probability zero.

This state of affairs, then, is the more precise meaning of the rocky state—your state of knowledge. We can depict it:



In the diagram, half of the real states consistent with your knowledge lie in the region where the coin lands heads, and half lie in the region where it lands tails. This is the reason you use the rocky state <H,T> corresponding to a fair coin as your "state of knowledge" about the coin.

The final lesson we can learn from the coin requires us to consider a situation where, for whatever reason, you think the coin is biased—say more likely to land heads. For the sake of sticking with a simple diagrammatic language, imagine the special case where you believe heads to be twice as likely as tails (i.e., the probability of heads is 2/3 and tails is 1/3). We could represent this new state, as well as the old one, in terms of both real and rocky states as follows:



Because the rocky states are unquestionably representing our knowledge, it is fine to avoid the "squaring" rule that the misty states forced upon us in Part II—so, for this new rocky state, the two heads and one tail means heads is twice as likely as tails.

In the diagram, you can see a feature that will be essential to understand for much of what follows, so if you've started to drift off it's time to refocus. The feature is that there are some real states that are common to both of the rocky states. That is, there is a nontrivial overlap between the real states consistent with the two different (rocky) states of your knowledge.

This is not particularly strange in and of itself. To give a completely different example where a real state of the world might be consistent with two different states of knowledge, consider someone draws a random playing card and tells one person the card is red and another that the card is an ace. The two people have completely different states of knowledge about the card, but the "real states" Ace of Hearts and Ace of Diamonds are in the overlap region.

The most important point about such overlap regions is as follows. Imagine someone comes along and is powerful enough to determine the real state of the coin; that is, they find out exactly all the physical properties of the coin, including whether heads or tails faces upwards. If it so happens that the real state is in the overlap region, then they cannot tell for sure which rocky state you are assigning to the coin. The rocky state is in your head, and no matter how much they know about the physical properties of the coin, they cannot know what is your state of knowledge about the coin. This is true even if you narrowed it down in advance for them to just the two rocky states depicted above.

More subtly, the very fact that two different rocky states can correspond to the same real state indicates that the rocky states themselves are not a physical property of the coin. Remember, the real states are like a sheet of paper listing all physical properties of the coin and their particular values for a coin that happens to be in that real state. So, by definition, if a rocky state was a physical property then we would be able to look at the real state (read the sheet of paper) and know what the "value of the rocky state" was. Since there is more than one rocky state associated with real states in the overlap region, the value of the rocky state cannot be "on the paper" as it were—a rocky state is not a physical property; it is not (part of) the "real physical state" of a coin.

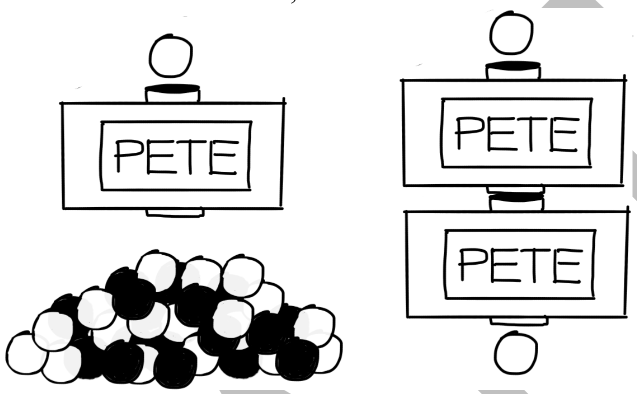
This has the following consequence: if, for some reason, we were unsure about whether rocky states were a physical property of a coin or not, then a proof that multiple different rocky states could correspond to the same real state would prove to us that they were not. Similar reasoning will form a crucial part of one of the arguments that misty states are not a physical property.

First, however, we delve into the simplest arguments that misty states are, in one way or another, a physical property of a ball.

Revisiting our first conundrum

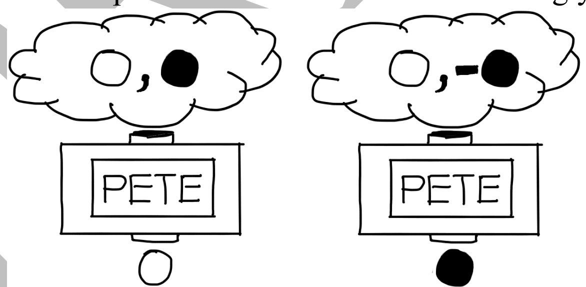
When I introduced you to misty states I emphasized that the one thing every physicist can agree on is that they are, at a minimum, a practical tool for calculation. They work incredibly well—much of modern technology (transistors/computers, lasers/the internet) is built upon unbelievably precise calculations that misty states enable us to make about concrete physical stuff, that we frantically push together in order to bring you a better cellphone each year.

Let us revisit the first conundrum we encountered with misty states, namely that the output of a single PETE box is random, but of two stacked PETE boxes is not.



If the misty state [W,B] that we use to describe the ball emerging from the first PETE box (in the case where we don't observe its color) is an "actual real thing" or, alternatively, "an actual physical property of a ball," then it is plausible that the second PETE box can "see" that the ball is in a misty state—and, in fact, which particular misty state it is in.

Perhaps, unlike ourselves, PETE boxes do not destroy the mist by looking at it? If so, then the second PETE box can distinguish whether the mist is [W,B] or [W, −B], and evolve the color of the output ball to white or black accordingly:



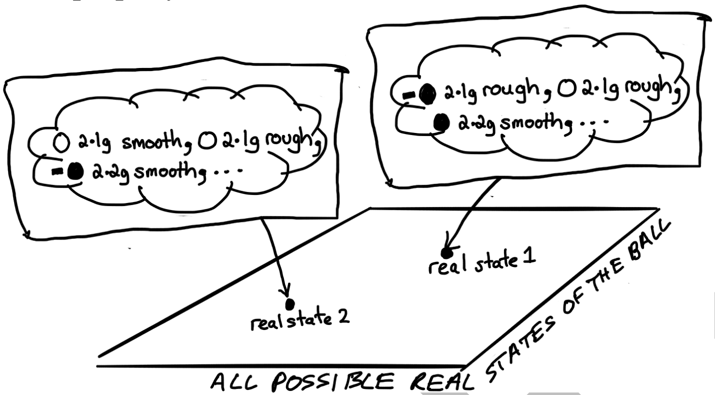
Thus, by taking the misty states to be real physical things—that is, actual physical properties of the balls—we immediately obtain a potential explanation of (some of) the weirdness of the PETE boxes. It seems like such a simple and natural explanation of the PETE boxes experiment that you may be wondering: why is there any controversy at all?

Two variations on a misty state "being real"

The only requirement for the explanation in the preceding section to work is that the PETE boxes be able to "see," one way or another, exactly what the misty state of the ball is. We would like to say this sort of explanation relies on the misty state "being real," or "being a physical property" such that the PETE box can respond to (or interact with) the mist.

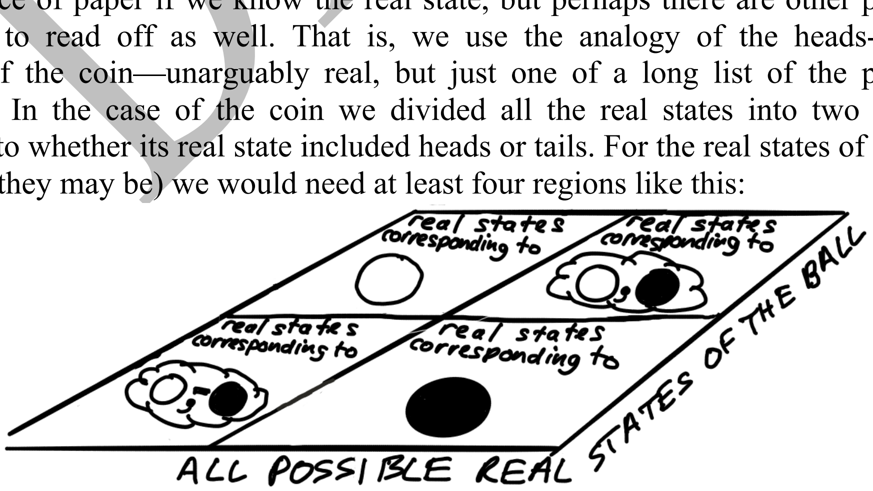
At this point there is considerable potential for confusion, some of which you even overhear amongst physicists who get paid to argue about such things. The confusion arises when we are not clear about whether we are considering the mist to be the full and only real state itself, or whether we take it to be just one, amongst many, physical properties. Both options are, in fact, tenable, so let me describe each.

As mentioned, although technically difficult, there is no reason to believe we cannot make a superposition of any given physical properties of a ball. There is nothing special about the color. This suggests the option of just using a mist as the one and only physical property of the ball. In this view the mist is the full real state:



In the figure we see two different mists of a single ball, and the mists contain not only the ball colors but also their mass, perhaps whether their surface is rough or smooth and potentially much more. The details are unimportant for our considerations, but for completeness let me say that that the manner in which a mist is constructed to incorporate multiple different physical properties has many precise rules, and is not quite as arbitrary as the above figure implies. Also I should point out that under this view the real state of more than one ball is just a suitably enlarged mist.

The second option is that we allow for the real states to include the misty state as one of the physical properties, but to potentially include a whole bunch of other things that we do not (yet) know about as well. Under this more conservative view we do not commit to knowing everything about the underlying real states of the world per se. We just say that whatever the real states of the world are, they unambiguously let us (well, the PETE boxes) determine what the misty state is—we can read the misty state off the piece of paper if we know the real state, but perhaps there are other physical properties to read off as well. That is, we use the analogy of the heads-or-tails property of the coin—unarguably real, but just one of a long list of the physical properties. In the case of the coin we divided all the real states into two regions according to whether its real state included heads or tails. For the real states of the ball (whatever they may be) we would need at least four regions like this:



This is not sufficient, however, because which real states correspond to [W,W,B]? Or [−W,B,B]? Or.... Well, you get the picture: we would need to divide the set of real states up into an infinite number of distinct regions, one for each misty state. Note that if you know which point in the plane describes the ball, you know which region it is in, and so you know which misty state describes it.

Both of the options discussed above—one real state per misty state, or many real states per misty state—have the feature that if someone knows the real state of the ball then they also know the misty state. In particular, for both these options there is nothing like the overlap region discussed above for a rocky state. For this reason I will lump them together as options in which we "take the mist as a real physical property," or more simply just that "the mist is real". We just mean that at any point in the set of real states, you can read on the piece of paper uniquely what the appropriate misty state is.

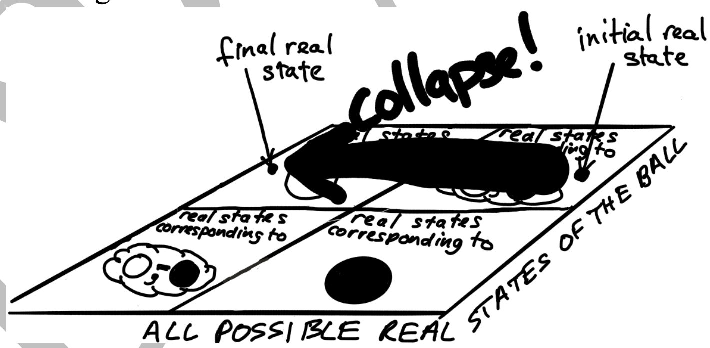
Both options also provide a natural explanation of the two-PETE-boxes conundrum: The ball begins in a real state where it is definitely white; it evolves (via the action of the PETE box) to a real state where it is definitely [W,B]; it then evolves via a second PETE box back to a real state where it is white again; and so on.

Although the details are beyond this book, the main thing to know is that this evolution follows a nice and precise wandering trajectory through the real states (much like the trajectory of a flipped coin through its own series of real states depicted earlier). We can calculate very precisely how one misty state changes into another and that one into yet another as time progresses. In fact this nice type of meandering through the mist occurs for any of the boxes we have encountered, as well as many others we have not. It is all very calm and pleasant.

Until, that is, a human (such as me and I hope you) leaps in and observes the ball.

If misty states are real, should we collapse in confusion?

When we observe the ball, we only ever see it black or white. This means, for example, if the ball is actually in a real state corresponding to a mist such as [W,B], it must immediately "jump" into a different real state for which the ball is definitely black or white, according to what was observed:



It is difficult to convey how repulsive most physicists find such "jumping." In fact physicists usually use the term "collapse" which sounds more negative. Both words, when they correspond to familiar processes in our everyday lives (such as a building collapsing or a frog jumping), refer to something which is sudden, perhaps very fast, but which is still continuous. By *continuous*, I mean the building's fall or the frog's leap might initially appear to be completely instantaneous, but if you filmed it and then slowed the film right down, you would be able to see all the intermediate stages between the beginning and the end of the jump/collapse process.

We saw a continuous type of evolution through the real states of a coin when I sketched how the real state changes from the time you flip it until the time it lands. If our present laws of physics are correct, however, once you make an observation the misty states necessarily collapse absolutely instantaneously and completely discontinuously. There is no time lag, and there are no intermediate states. The laws essentially demand this for self-consistency, so it is not even a case of us being able to hope that “maybe it just happens so fast we haven't been able to see it in slow-motion yet.”

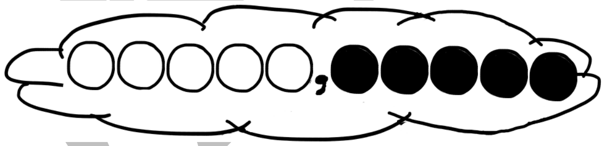
There is no precedent for something like this in even the most sudden physical processes we ever encounter in other areas of our experience—or, for that matter, in all non-misty physics.

To make matters worse, the laws demand that the collapse only ever happens when an observer, such as you or I, looks at the color of the ball. As we considered in Part I, we might employ a wide variety of different techniques to try to observe the ball’s color in different ways, some more gentle than others. The collapse process doesn't care about the method we use to interact with the ball, it just happens as soon as that method is strong enough to tell us the ball’s color, and it doesn't happen at all otherwise.

Does it then matter in this situation whether or not we actually observe the color? What if we use some kind of unintelligent observing device to probe the ball between the boxes, but we do not look at the color recorded on the device? What we have done is create a really large misty state. The mist representing the ball is now entangled with the mist representing the stuff making up the device. It is this entanglement which causes the “disturbance” to the ball which is responsible for the fact it no longer always falls from the second PETE box the same color it entered the first. However, the actual collapse, the laws claim, only happens when you, the observer, look at this large misty state, either indirectly by reading the device or more directly just looking at the ball. Then the ball mist collapses into “black” or “white” and the observation-device mist collapses into a “black” or “white” reading.

Why should observers be so important? Why should the widely different physical mechanisms they use to interact with the ball make no difference to the final real state the ball ends up in, or how that process occurs?

And if you thought all that was bad enough, consider what happens when we extend our considerations to misty states of two or more balls. Consider the five-ball entangled (generalized Bella) misty state:



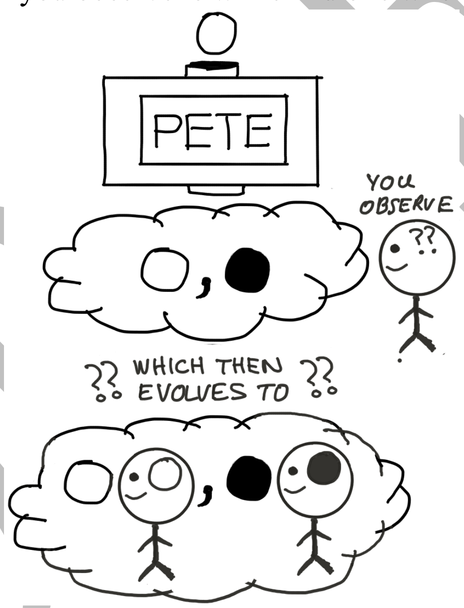
If all misty states are real, then in some large set of all the real states of five balls, there sits this particular state. What happens when we observe one of the balls? At that point we collapse all of the balls to either the state where they are all black, or the state where they are all white—that is, to a completely different real state. None of the balls are left in a misty state, because if one of them is white they all are. That is more disconcerting than the single ball collapse, because the balls in this state can all be in completely different spatial locations.

This means that (again, only if the misty states are real) it is possible in one spatial location to instantaneously change the real state of the world, in another location, arbitrarily far away. Imagine a giant misty state of many balls, spread throughout the universe. Is it really plausible that one person on one little planet orbiting an insignificant star is instantaneously changing the real state of a part of the universe arbitrarily far away?

Faced with all this ugliness surrounding collapse, proponents of the notion that misty states are real have for the most part tried to modify our current physical laws by either (i) trying to find a more physically appealing model of collapse (one where it is proper dynamical process and doesn't need observers); or (ii) trying to find a way to use the misty states as real states, but to completely remove collapse from the picture.

Option (i) is difficult to make compatible with both current and near-future high precision experiments, as well as consistent with other aspects of physical laws (such as not being able to send messages faster than light), which we hold quite dearly. But there are some models which work (at least for the ball type of experiments we have considered; making them work for all experiments we can presently do is more tricky), and which soon will be experimentally ruled in or out.

Option (ii) is more subtle—it typically involves thinking about the absolutely giant misty state that makes up everything in the universe including the observers of the balls, and denying that this giant misty state ever collapses. The challenge then is to extract an explanation of how the very small pieces of that giant mist that comprise you and me experience a world where we can talk about little mists of one and two balls, little mists that give random outcomes when we observe them; a world where the assumption of collapse works so well. (This is often called "the measurement problem.") The most studied option along these lines has been to assume that the giant mist actually describes many different universes, and when you observe the misty state of a single ball there are actually two copies of you created, one that lives in a universe where the ball you observe is white and one where it is black:

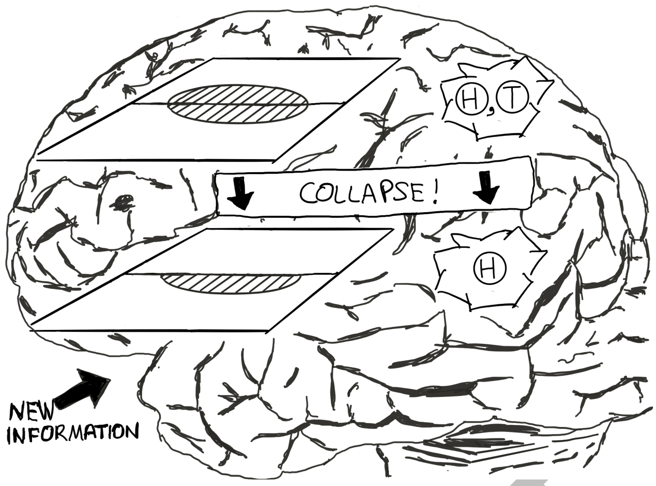


It is a dramatically different view of all of physical reality.

A completely different option is to discard the idea that misty states are real, and we now turn to understanding this possibility.

Currency collapse, mental collapse

Imagine you have prepared a coin and flipped it, and you believe that its real state is equally likely to be in the heads region as the tails region of real coin states, as depicted previously. You describe the coin with the appropriate rocky state. Now, before you look at it yourself, a friend you trust tells you "Hey, that coin is heads." At that point you immediately and instantaneously "collapse" your knowledge about the coin. Here is a to-scale diagram of you undergoing mental collapse:



Acquiring information has caused you to change your mind about both the appropriate rocky state and the appropriate distribution over the set of real variables of the coin, because now you know for sure that the coin shows heads.

Such "collapse via updating one's knowledge" is clearly not a physical process as far as the coin is concerned—something changed in your mind, but the coin doesn't care what happens in your mind. It is also clearly something that happens instantaneously; you do not slowly and continuously change the state you assign to the coin. The collapse will occur whenever you gain the appropriate information, and it doesn't matter what physical mechanism you use to acquire that information—if it can provide the requisite information, you collapse; if not, you don't. These are all features shared by the collapse of a misty state, but here they are not at all strange.

These similarities motivate the question: Is it plausible that misty states are also features of our knowledge, rather than real states of the world?

However, there is a very simple dissimilarity between coin collapse and misty state collapse. Collapse of the misty state is accompanied by some tangible disturbance to the experiment, because the output of the second PETE box depends on whether or not you observe the ball after the first PETE box (and cause the collapse).

Because observing the ball involves interacting with it somehow (shining light off it, smelling it, licking it—whatever) it is not ridiculous to conclude: "Collapse happens in my head, which is where misty states live. There is, however, some new fundamental principle of physics which ensures that to probe a system strongly enough to be able to collapse my misty state of knowledge, I must use concrete physical interactions that cause a random disturbance to the real states of that system, whatever they may be." The nice thing about this proposal is that it is completely non-committal about what the "actual real states" of a ball are. I gave in Part I a silly version of such an explanation when I imagined the balls could have hidden stickers on them which PETE boxes manipulate.

However, the proponent of "misty states = real states" counters with: "Why bother thinking about these hidden real states at all? Just let the real state be the misty state and that is the physical thing disturbed by observation."

This in turn stimulates the counterpoint—even if you accept the weird behavior of such a supposedly real physical state when it is disturbed, why can a PETE box see the real states without disturbing them, but we cannot see them at all? What is the principle which decides which devices do or do not cause such disturbance?

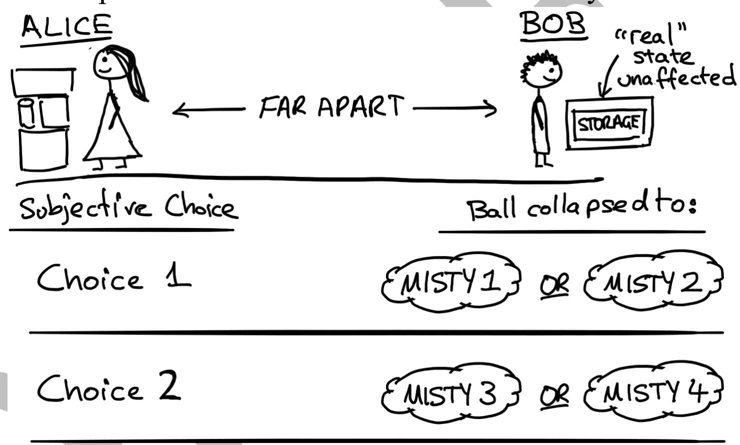
Well, comes the rejoinder, *something* enters the top of the second PETE box, since something falls out the bottom, and when we finally observe that something, it always looks like a colored ball. If the physical property of that something is not that it is "really in the misty state" while it is in transit, then why can't you tell me what actually is the real state of that thing about which the mist represents only your knowledge?

Into such back and forth charges no less a person than Albert Einstein, with a brilliant argument (what more do you expect?) that most people understand incompletely. You, I hope, will not be one of them.

Einstein throws himself in completely

Einstein presented an argument to prove that, even if we have no idea what the real states of a ball are, there exist real states in the overlap region of at least two misty states—that is, real states for which there is not a unique corresponding misty state. If a misty state was a physical property of a ball then we would be able to look at the real state and know what the "value of the mist" was (by definition of the real states). Einstein claims to prove there is an overlap between the real states corresponding to two or more misty states, and thus he concludes the "value of the mist" cannot be "on the paper"—it is not a physical property, it is not part of the "real physical state" of a ball.

Einstein's argument is built upon a demonstration that a subjective choice by one person (a.k.a. Alice), arbitrarily far away from a ball held in a storage box by another person (Bob), can collapse Bob's ball to different sets of misty states:



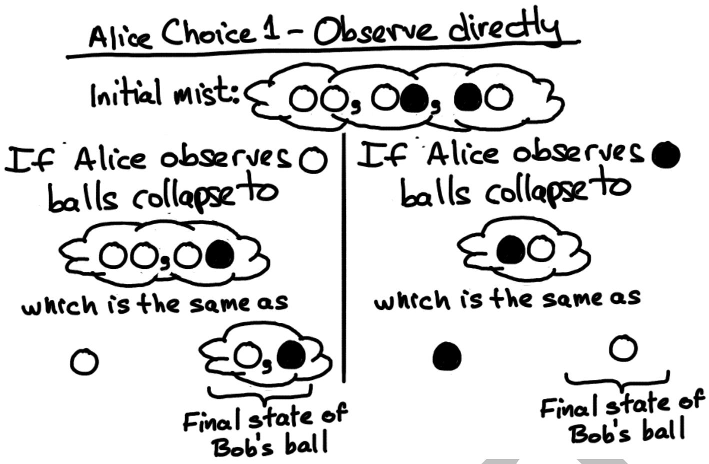
In particular, one of Alice's choices leads Bob's ball to end up in either misty state 1 or 2, while a different choice causes it to end up in either misty state 3 or 4; and all four misty states are different from each other. (Saying they are different means an experiment can be performed for which each misty state predicts a different set of probabilities for concrete observations, not just that they look different as a diagram.)

Alice's measurement, Einstein's argument goes, does not affect Bob's real state. Since she does change the misty states it can be in (at her whim), any given real state must correspond to at least two distinct misty states. More precisely, assuming that (i) a real state of Bob's ball exists, and (ii) the real state of Bob's ball cannot depend on what Alice does arbitrarily far away, Einstein concludes: there are many different misty states corresponding to a single real state of the ball.

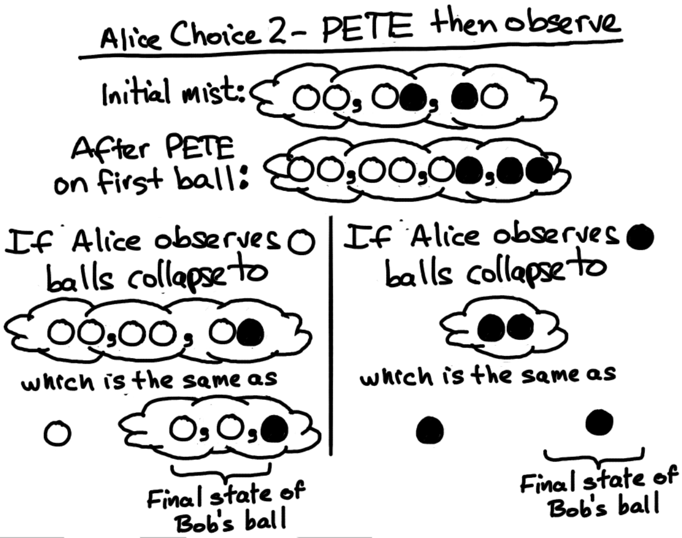
For a concrete example, imagine we start with two balls prepared in the entangled mist [WW,WB,BW] that we encountered in Part II. We give one ball to Alice and one to Bob (Einstein called them A and B; he wasn't a very imaginative guy), who then move far apart from each other.

Alice now chooses between two different experiments to perform. Much like when she was winning your gold, she can choose either to observe her ball directly, or to first pass it through a PETE box before observing it.

In the case where she does nothing first, just observes her ball, she collapses both balls like this:



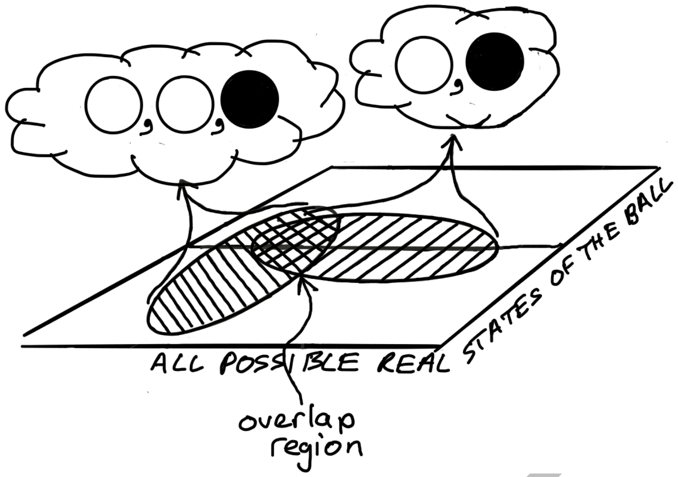
Her other choice is to first pass her ball through a PETE box and then observe it. In doing so she collapses the state of Bob's ball like this:



Alice's free choice of measurement—her choice to use the PETE box or not—made far away from Bob, can collapse his ball to either being in one of the states [W,B] or W, or to being in one of the pair of states [W,W,B] or B.

For our particular example Einstein would say that since some of the time Alice collapses Bob's ball to [W,W,B], it must be the case that some of the time the real state of Bob's ball corresponds to [W,W,B], but it must also correspond to one (or both) of either W or [W,B]—the state it would have collapsed to had she done the other measurement. For Einstein's argument it doesn't particularly matter which of these two possibilities [W,W,B] overlaps with.

In terms of our schematic diagrams, Einstein's conclusion that at least some misty states correspond to overlapping sets of real states, perhaps something like this:



That is enough to prove the misty states are not themselves real.

It is possible to include arbitrarily more misty states in the argument by letting Alice do many more choices of measurements. The final conclusion is that for any given real state of Bob's ball, there are actually arbitrarily many misty states.

Einstein called all this “incompleteness” of the misty state description. He uses the term *incompleteness* (I surmise) because the real state was, by definition, meant to capture all and everything about the physical properties of the ball, and the misty states clearly do not do that if there exist real states in an overlap region of two misty states. But the confusing thing is that we (and he) could also claim the misty states are incomplete if they happened to be just one amongst many physical properties. That is, the “many real states per misty state” option, discussed previously as a more conservative claim about reality of the mist, is also a view in which the misty states are incomplete. Einstein gave other arguments for this very different type of incompleteness, but I’m going to skip them—if I teach you everything there is to know about this topic then you won’t be able to make a career as a philosopher, should you so wish, because everything will be clear to you. If you accept the argument of Einstein’s just given then whether there are additional underlying physical properties comprising the real states, or whether the mist is the only thing that is real, is moot—the misty states cannot be real at all.

Note in passing that Einstein does not give a sausage (his words) about what Bob does or doesn't measure, or what he can or cannot infer about the properties of his ball. And neither should you—it is irrelevant to this particular argument.

The existence of real states in the overlap region of two misty states would provide a nice explanation of the following: while there is an experiment for which the probabilities of the various outcomes are all different according to whether we drop into it a ball prepared in W or [W,B] or [W,W,B], that experiment never lets determine *for sure* which particular misty state the ball originated in. (This is obvious if we just observe the color, since seeing the ball is white we cannot for sure distinguish the three options, but it actually remains true no matter what experiment we do). If misty states are real, this is somewhat strange—why are we prevented from knowing something that nature knows? However, if we accept Einstein’s conclusion, then the reason we cannot always for sure determine whether a ball was prepared in W or [W,B] or [W,W,B] is because sometimes the real state of the ball itself is ambiguous about the matter—that is, in an overlap region compatible with all three of these. By hypothesis, what we mean by the real state is anything and everything that can affect the outcome of an experiment. Thus if the real state itself does not uniquely determine the misty state, neither can any experiment we do. Our inability to distinguish different misty states with certainty is then no more surprising than the fact we cannot always tell for sure, just by seeing a coin with heads facing upwards, whether the person who flipped that coin assigned it the rocky state <H,T> or the rocky state <H,H,T>.

Questioning Einstein's two assumptions

It is natural (and common) to question Einstein's first assumption—the existence of real states. As mentioned at the start of Part III, it would be difficult to question "macroscopic" reality (the stuff we observe directly, and all agree occurs). But we might question the existence of "microscopic reality." This viewpoint calls into question the process whereby we assume (or infer) the existence of microscopic things built from ever-smaller things. In particular, if we question the existence of microscopic reality, we question the assumption that the macroscopic physical properties are built from—or are manifestations of—microscopic ones. The assumption of microscopic reality has served science well, but given the difficulties we have in finding a narrative to explain what the misty states are in terms of well-behaved, microscopic, unobserved real states, perhaps we should abandon the notion that there is any connection between them? (That is, if the real states have meaning or "exist" at all.)

Proponents of this view see the misty states as an inferential tool for predicting the outcomes of future experiments (and how our observations will affect such). In that sense, they propose, the misty states are states of knowledge, but not states of knowledge that tell us anything at all about an underlying reality that is the cause of what we see.

Einstein's second assumption (which he called the "separability hypothesis") was that the real state of Bob's ball cannot depend on what Alice chooses to do with her ball when they are far away from each other. At the time Einstein made his argument, it seemed completely obvious to him (and, in fact, to the people with whom he was arguing) that there could be no question of some kind of mechanical disturbance to (the real state of) Bob's ball based on Alice's measurement choice.

As we now know, based on Part II, "some kind of mechanical disturbance" does have to be contemplated seriously as a possible explanation for the nonlocal correlations. Moreover, it would have to have all of the physically anathematic features that caused Einstein and his contemporaries to dismiss such a thing out of hand. (Einstein's argument preceded the discovery that measurements on entangled misty states can generate nonlocal correlations, as discussed in Part II. Similar to the method the psychics used to get your gold, Einstein's argument requires measurements on an entangled pair of balls. However; he did not discover the type of strong nonlocality presented in Part II, where the data appears in the outcomes of experiments, and does not rely on any interpretation of mathematical objects, mists, or philosophical prejudices.)

With one of Einstein's core assumptions thrown into doubt in such a concrete manner, we cannot rely on his argument to tell us whether every real state of a system uniquely determines a misty state or not. Even worse, in a moment, I will explain an argument built on very, very similar (but arguably less questionable) assumptions than Einstein's; one that reaches the complete opposite conclusion.

Why no faster-than-light communication?

Before we turn to the argument that reaches the opposite conclusion to Einstein's, let me briefly explain why it is that we cannot send messages instantaneously by making measurements on one of an entangled pair of balls. I often hear from people confused about this.

Imagine that Alice, in the experimental setup we've been discussing, wants to communicate something to Bob—let's say either "ATTACK" or "RETREAT." What does she have control over to use as a transmitter? Well, she has one choice: she can observe her ball directly (and collapse Bob's ball to W or [W,B] according to whether she sees her ball is black or white respectively), or she can send it through a PETE box and then observe it (collapsing Bob's ball to B or [W,W,B] according to whether she sees her ball is black or white respectively). Let's say she and Bob have planned that if she observes her ball directly, that means "ATTACK," and if she runs it through the PETE box first, she's sending a "RETREAT" signal.

What does this mean for Bob? A small calculation shows if she observes her ball directly, she will collapse Bob's ball to [W,B] twice as often as to W. That is, the probability of collapsing to [W,B] is 2/3, while the probability of collapsing to W is 1/3. Alternatively, if she wishes to communicate "RETREAT" (and runs her ball through a PETE box first), we find that the probability of collapsing Bob's ball to [W,W,B] is five times as likely as collapsing to B. That is, the probability of collapsing to [W,W,B] is 5/6, while the probability of collapsing to B is 1/6.

The key point is that although Alice can choose which pair to collapse Bob's ball to, she cannot control which member of the pair it ends up in. The probabilities cannot be changed from what the misty states predict.

Bob needs to determine which pair she has collapsed his ball to. Imagine he doesn't do anything to his ball, he just observes its color. When Alice is communicating ATTACK, the probability he sees a white ball is 1 if she collapsed it to W, and 1/2 if she collapsed it to [W,B]. His overall probability of seeing a white ball is 2/3.

When Alice is communicating RETREAT the probability he sees his ball is white is 0 if she collapsed his ball to B, and 4/5 if she collapsed his ball to [W,W,B] (remember the squaring rule). Since there is a 5/6 chance of collapsing to [W,W,B] the total probability he sees a white ball is the product of 4/5 times 5/6—which is 2/3.

This is identical to his probability if she was communicating the complete opposite message. He has gained no information when he sees his ball is white.

Even if Bob first sends his ball through some complicated array of boxes along with other balls this conclusion remains true. All he sees is that it is equally likely Alice is telling him to attack versus retreat, and so he gains no information at all.

All this is mainly strange if you consider the mist to be real—if you follow Einstein and deny that the real state of Bob's ball is changing at all then you would not expect them to be able to communicate in this manner.

But then how would you explain how the psychics won your gold?

Pooh-Bear creates complete confusion

Any argument contradicting a conclusion of Einstein's should be made by a figure of equivalent stature, although they will, of course, necessarily be of inequivalent intelligence. By consensus of his friends, Winnie-the-Pooh "has no brain." It seems appropriate to attribute these characteristics to an advocate of this counterargument, and so I will call it the Pooh-Bear argument.

Pooh adopts Einstein's first assumption, that there is such a thing as a real state of a ball. Einstein's second assumption was that the real state of Bob's ball cannot depend on what Alice (located arbitrarily far away) does to her ball. The weakness of Einstein's argument, premised as it is on his second assumption, is that it requires the two balls to be entangled. This in turn requires them to, by some means or another, have already come into contact with each other prior to being separated by Alice and Bob. As such they are definitely not necessarily completely independent (entanglement is, in fact, the misty-state manifestation of this). Perhaps when they come into contact they create a magical hole in spacetime that can stretch through extra dimensions and this lets them instantaneously coordinate their actions? Any ridiculous possibility must be considered, and the fact they have had to have some kind of causal connection already to get entangled means it is hard to rule any such things out. The fact the balls have already had this connection also makes the pretty natural expectation that they have real states (physical properties) “all of their own” a lot more questionable.

Pooh replaces Einstein's separability assumption by a separability assumption that is simple enough for even a brainless creature to understand. His assumption is: if two balls have never come into contact (for example, perhaps they were originally created at far corners of the universe), then they have physical properties/real states of their own which are independent from each other, and if you never ever bring the balls together then you can always treat them as separate physical things, completely ignoring the existence of the other one if you like.

Pooh's separability assumption is not a mechanical one like Einstein's. It is an assumption that we can always isolate the things we are examining to a sufficient extent, and treat them as separate physical entities. Without the ability to do independent things we would not actually be able to make scientific progress, because the process of science is built on “independent verification.” The thought that we cannot reason and infer things based on an experiment here, because of what might be going on right now on the other side of the universe (with which we have not interacted in billions of years), reaches a new level of absurdity, and makes one wonder how we could investigate nature at all. (Although *Nature* has been known to be both absurd and capricious when it comes to such questions.)

Without the ability to consider a rock on Pluto as having its own properties independent of the properties of a rock here on Earth, it's very hard to see how we could begin to organize our thoughts about the world. Never mind tie our shoelaces.

To set the scene for the Pooh-Bear argument, we turn to:

UNIMAGINABLE CONVERSATION BETWEEN POOH-BEAR AND EINSTEIN

POOH: We should think about what we will have for lunch, which is always nearby in both time and space thankfully.

EINSTEIN: Time, space? Boring. Although your clock is not working, which has started me thinking about... hang on a second, didn't you just have elevenses?!?

POOH: (*Smiling*) Oh yes, I did just have a small something. Perhaps that is why my Very Little Brain is so ready to explain. Hey, that rhymes. My “brain” can “explain,” hum tiddely pom...

Pooh begins to hum

EINSTEIN: Ah, Mr. Pooh,...

POOH: Oh sorry, yes, well imagine there are two types of packed lunch. The first type either contains hunny or it contains condensed milk.

EINSTEIN: Honey?

POOH: Yes, hunny. Just one of the two, of course. I wouldn't be allowed to have both. (*Looks wistful, pats his oversize tummy*) I will call that packed lunch the "sweet" option for lunch, because…

EINSTEIN: (*Interrupting*) I think I know why.

POOH: Sorry, I do tend to be a little slow of thought, and perhaps I over-explain. I heard you have a Very Clever Brain. Anyway, the other lunch option contains either hunny or a banana. I'm going to call that the "healthy" option, because...

EINSTEIN: (*Impatiently*) Yes, Mr. Pooh...

POOH: Sorry, there I go again. Now, Christopher Robin is going to pack me either a sweet lunch or a healthy lunch. And Eeyore is, completely independently, going to pack you either a sweet lunch or a healthy lunch, choosing from the same three foods. And then we are going to visit Owl.

EINSTEIN: Good, I have always wished to meet that knowledgeable creature.

POOH: There are four possible lunch combinations we could be carrying: sweet-sweet, sweet-healthy, healthy-sweet and....and....

Pooh looks a bit anxious

EINSTEIN: healthy-healthy?

POOH: (*Smiling*) Yes, exactly, thank you. The plan is to set Owl a challenge. He must tell us, for absolute sure, a lunch combination that was definitely *not* prepared.

EINSTEIN: Why can't he tell us one that *was* prepared?

POOH: I don't really know, this is the only way at the moment that I can make the whole argument work.

EINSTEIN: That is fine. So Owl must definitively rule out one of the four options. I presume he is allowed to look inside the two lunchboxes?

POOH: Oh yes, he can look inside if he wants. Now the question is, will Owl definitely be able to rule out one of the lunch combinations?

EINSTEIN: Well, let's just consider all the possibilities. If Owl sees a banana in my lunchbox and a condensed milk in your lunchbox, he knows the combination prepared was—listing mine first and yours second—definitely "healthy-sweet." So he can say, "The combination prepared was definitely not sweet-sweet." Or, for that matter, he can say it was definitely not healthy-healthy or not sweet-healthy.

POOH: Exactly. In that case there are three different answers he could give, because he knows for sure what was prepared, so he can say for sure what was not prepared.

EINSTEIN: Yes, that's a pretty easy case. The same goes for if he sees bananas in both lunchboxes, or condensed milk in both, or condensed milk in mine and a banana in yours. He knows for sure what was prepared, so he has lots of options to give an answer about what was not prepared.

POOH: What if he sees hunny in your lunchbox and a condensed milk in mine?

EINSTEIN: Hmm. That is more tricky, because honey is both healthy and sweet.

POOH: (*Beaming*) I know, isn't hunny wonderful?

EINSTEIN: So, he knows from the condensed milk in yours that your lunchbox was prepared sweet. But he can't be sure about mine because of the honey.

POOH: Remember the rule was, Owl needs to only tell us one of the combinations that was *not* prepared. He doesn't actually need to know which combination *was* prepared.

EINSTEIN: Aha—I see. Since he knows from the condensed milk that your lunchbox was prepared sweet, he can answer that healthy-healthy was definitely not prepared. Or he can answer that sweet-healthy definitely was not prepared. Because he sees you have the condensed milk, he knows any combination involving your lunchbox being healthy was definitely not prepared.

POOH: Yes, there are only two safe options for him to answer now, but he can still meet the challenge easily.

EINSTEIN: The same will be true whenever one of us has honey and the other one does not. In all those cases Owl will be able to give a satisfactory answer because he will know for sure from the lunchbox without honey how it was prepared.

POOH: You really do think so much faster than me, Mr. Einstein. I have to be honest, that when I was first thinking about this I had to sit and write out all of the options. There were quite a few of them, and I got so absorbed I missed having a little something at eleven o'clock that day. (*Looks sad at the memory*)

EINSTEIN: I can also see now that there is no way that Owl can be certain of winning this challenge. If we played it many times he must eventually be stumped. Because, what can he say if he finds honey in both of our lunchboxes?

POOH: Exactly, Mr. Einstein, you have seen the problem! I think hunny in both lunchboxes would be a Very Good Thing to find, especially if you are willing to share (*glances anxiously at Einstein*). But when that happens Owl will be stumped. He cannot say for sure one of the combinations that was not prepared.

EINSTEIN: I'm not able to see how all this might be relevant to my argument about misty states...

POOH: Well your claim is that just as the two different “states of lunchbox knowledge”—sweet versus healthy—have a “real state” of hunny in common, the misty states have at least some real states in common.

EINSTEIN: Yes, assuming there are such things as real states, and assuming what someone does in one location can't change a real state somewhere far away, it is simple to see that there must be real states common to more than one misty state. I don't know why I've had to spell it out so many times to those...

POOH: Careful Mr. Einstein, children read these books. Well, let’s imagine we redo the challenge with Owl, but this time Christopher Robin will prepare me a STORAGE lunchbox that contains one of the infamous black or white balls. And he will either prepare it as a white ball W, or he will prepare it in the misty state [W,B]. Eeyore is independently going to do the same thing for you—prepare you a STORAGE lunchbox that contains a ball either in W or in [W,B]. You should think of W like a “sweet” lunchbox preparation and [W,B] like a “healthy” one.

EINSTEIN: I see. My claim is that W and [W,B] are not themselves the real states because by my argument there are at least some real states common to both of them. Those common real states are like the honey. Even if Owl is so wise and knowledgeable that he can see real states, whatever they may be, at least some of the time both balls will be in a honey-like real state. This makes it impossible for Owl to always say one combination that was *not* prepared.

POOH: Yes, yes—now we are there. Because here is the amazing thing. Owl always manages to win this challenge!

EINSTEIN: What? Are you sure?

POOH: Yes, we did it many, many times. In fact it doesn't matter which pair of misty states Christopher Robin and Eeyore pick from, Owl can always meet the challenge. This is why I claim that every real state is associated with one and only one misty state.

EINSTEIN: Remarkable. Things are really not the Things that they seem to be.

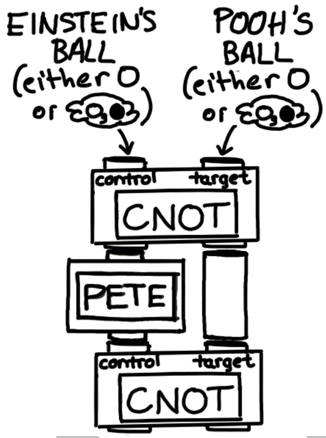
POOH: Oh I am so glad you appreciate that. When you are a Bear of Very Little Brain, and you Think of Things, you find sometimes that a Thing which seemed very

Thingish inside you is quite different when it gets out into the open and has other people looking at it.

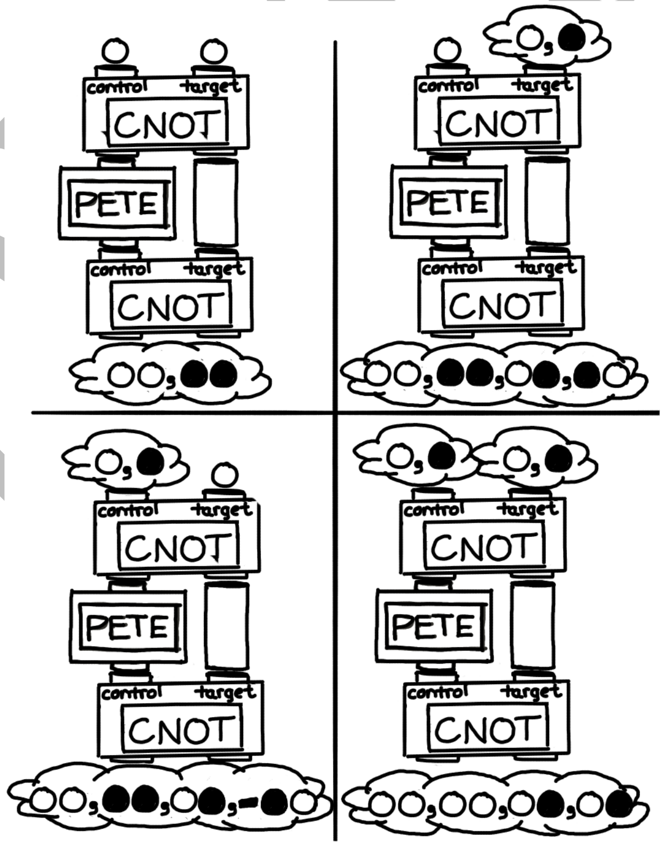
How can Owl do it?

Owl needs to take the two balls from Pooh and Einstein—about which he only knows that they could be in any one of the four misty states WW, W[W,B], [W,B]W, or [W,B][W,B]—and rule out one of the four combinations.

The schematic of the procedure Owl devised (with a little help from Rabbit, Tigger and Roo) is to take Pooh's and Einstein's balls (without looking at them) and drop them through these boxes:



Working out what happens for the four different possible input misty states is old hat to you by now:



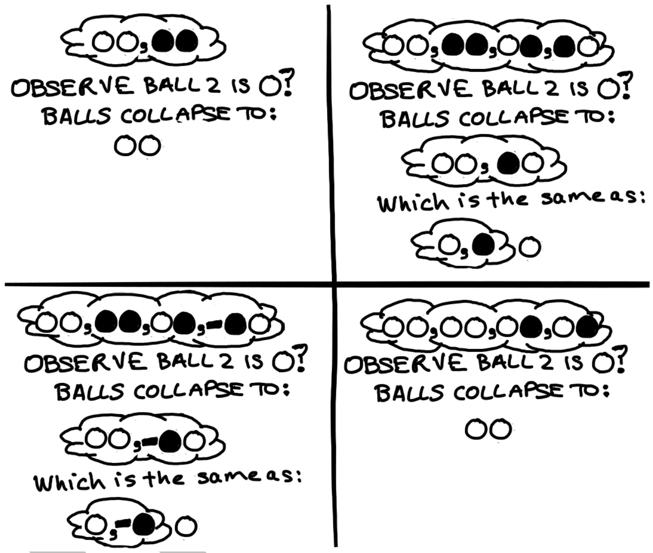
Holding Einstein's ball in storage, the next stage of Owl's procedure is to look at the color of Pooh's ball. If he observes Pooh's ball is black he then observes

Einstein's ball directly, but if he sees it is white he first drops Einstein's ball through a PETE box and then observes it.

Ball 2 is black: If Owl sees Pooh's ball is black, then he releases Einstein's ball from storage and directly observes it. Looking at the four possible output mists in the above figure, we see that the only configurations appearing which have Pooh's ball black are BB and WB. We further note that the WB configuration could have resulted from three of the possible combinations that Einstein and Pooh might have presented Owl with. It does not, however, appear in the output mist when both Pooh's and Einstein's balls started out white (top left corner figure). So, if Owl sees that Pooh's ball is black and then sees that Einstein's ball is white, he can safely announce "it is not the case that both balls were initially prepared white."

Similarly, the BB configuration does not appear in the output mist when the two balls Owl drops in are [W,B][W,B] (bottom right corner figure). Therefore if he sees both balls are black he announces "it is not the case that both balls were initially prepared in the misty state [W,B]."

Ball 2 is white: If Owl sees Pooh's ball is white after the CNOT-PETE-CNOT series, he collapses the two balls according to these rules:



Let's say the balls were W[W,B] at the very start, when Eeyore and Christopher Robin separately packed them. After passing the three boxes this has evolved to the situation in the top right corner figure of this diagram. Therefore, Owl seeing Pooh's ball is white means Einstein's ball is now in the misty state [W,B]. If Owl now passes Einstein's ball through a PETE box, he will definitely observe it to be white.

If, on the other hand, the balls were [W,B]W at the very start (bottom left corner figure), then, after Owl observes Pooh's ball is white, Einstein's ball is in the misty state [W,−B]. If Owl now passes Einstein's ball through a PETE box, he will definitely observe it to be black.

Thus, if Owl sees Einstein's ball is black he safely announces "it is not the case that Einstein's ball was initially W and Pooh's ball initially [W,B]." If he sees Einstein's ball is white he announces "it is not the case that Einstein's ball was initially [W,B] and Pooh's ball initially W."

This is quite a complicated procedure to verify works (and after all that, Pooh and Einstein have to go back to Eeyore and Christopher Robin to check if Owl was right). It has the feature that Owl does not even have to look at the mysterious, apparently invisible real state of the balls, he just passes them through some simple boxes and observes their color. We don't know what goes on inside the boxes—perhaps they can look at real physical states, but Owl does not need to (although he probably would claim that he can).

Recapping, rexplaining

The basic question we are addressing is: although we have no clue what the underlying real physical properties of a ball in a misty state might be, can we deduce something about the relationship between misty states and the presumed real states? Einstein gave an argument that the relationship must be "many-to-one," that is, there are many different misty states corresponding to a single real state. His method was to show that one person's arbitrary choice in handling a ball in their possession can cause a remote ball (arbitrarily far away from the first one) to end up in one of many different misty states. Assuming distant subjective choices should not change what is "really going on" with a ball, his conclusion follows. Einstein's argument requires the two balls in question initially be entangled, it makes a strong presumption of locality and (in light of the nonlocal correlations generated by measurements on entangled states discussed in Part II) it can be considered suspect.

As with Einstein, Pooh's goal is deduce something about the relationship between misty states and real states of a single ball. Pooh-Bear's argument also used two balls. For both Einstein's and Pooh's arguments two balls are necessary because it is known that observations on just a single ball cannot be used to distinguish between the possibilities of whether the misty states overlap on the real states (whatever they may be) or not.

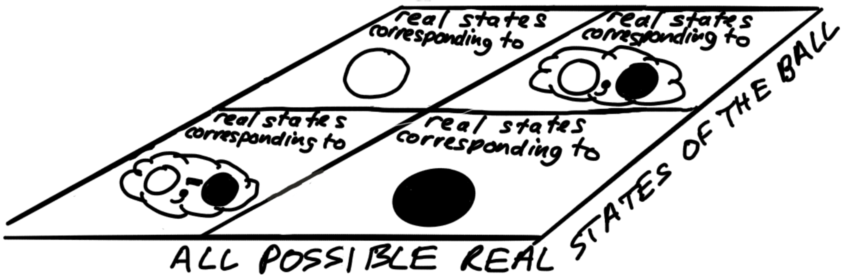
Pooh's argument explicitly relied on the balls not being initially entangled; in fact, it relies on them having been prepared independently (by Christopher Robin and Eeyore), and this assumption of preparation independence replaces Einstein's "obvious" locality assumption that an arbitrarily distant choice in handling one ball cannot change the real state of another ball.

Before his argument dealing with two balls Pooh first considered the example of a single lunchbox that can be prepared in two different ways, "sweet" or "healthy," such that there is a real state (honey) common to both possible preparations. If you happen to open a single lunchbox like this and see honey, there is no way to tell which of the two preparations was used. A similar situation arises with balls in misty states—in general two different misty states of a single ball cannot be distinguished with certainty. But with the balls, unlike the lunchboxes, we cannot see the proposed real states. As such it is unclear if the inability to distinguish them is because there are overlapping real states, that is, real states in common to both, just as Einstein argued there must be. Alternatively, it could just have to do with some fundamental restriction on the kinds of boxes we can build and observations we can make.

What Pooh argued was that the presence of honey—a real state in common to both possible preparations of a single lunchbox—also made impossible a different type of game you can play with two lunchboxes. This is the slightly strange game where Owl has to say which of the four possible combinations of two lunchboxes was not prepared.

Putting balls inside the lunchboxes using different preparations of misty states is a game changer (and less palatable). Unlike the case of preparation of sweet versus healthy lunches, if a ball is placed in each lunchbox prepared in one of two different misty states, then there does exist a way for Owl to win the game. This was shown explicitly for the case where the ball was prepared either W or [W,B], but something similar can be done for any pair of possible misty state preparations.

Pooh's conclusion is that there cannot be real states compatible with two different misty states, so that there are completely distinct regions of real states for any misty state, much like this diagram from earlier:



That is, Pooh's argument (should you accept its premises) proves the misty states *are* actually real in the sense described near the beginning of this part of the book.

Final thoughts

There are a many, many arguments that I have not covered, both for and against thinking of the mist as real. I have tried to explain, and get you interested in, the challenges on both sides—not to indoctrinate you with my own ever-fungible opinions.

I personally live in cognitive dissonance: on a day-to-day basis I talk about the physical properties of the photons (from which I am currently trying to build the mystical computer discussed in Part I) as if they are as tangible as any of the physical properties of the human-scale objects in the room around me. They are not. I suspect I should treat the misty states as states of knowledge, but to be understood within a more general framework of theories of inference than our present theories find comfortable.

The extreme view along these lines is to say we can only use the misty states to infer about potential happenings within the world that we may, by our personal choices, bring to being with our observations. That we should not attempt to connect those happenings to... well to stuff actually going on. While this extreme view may seem strange to us now, there was a time that the gods were attributed responsibility for the events of the world. Thousands of years ago the radical view (best captured by Lucretius in *De Rerum Natura*) was to propose that actually the things important to humans were just flotsam of the motion of microscopic constituents. This perspective eventually won out, and has served science incredibly well—it is difficult (for me) to let go of.

But because all attempts to provide an underpinning narrative for what is "really going on" when we do experiments in the mist—nonlocality being my personal bugbear—I have no confidence in the "correctness" of any of the physical properties our theories are premised upon. It seems crazy that I should either abandon the idea there is stuff doing things out there that doesn't care about me, or believe my experiences of the world can depend on things happening in a galaxy far, far away.

In the end I guess I cannot escape my naive realistic belief that there is stuff there, and it has physical properties of some form independent of my concerns. I am willing to contemplate the possibility that perhaps all the physical properties I implicitly consider fundamental are merely artifacts of the evolution of human perceptions, their presumed importance an anthropocentric mistake. Confronted by nonlocal correlations, I sometimes even wonder if space and time—important properties for monkeys who need to find food and mates—might be no more relevant to the true underlying reality of the universe than the pleasant smell of a banana.

Summary of Part III

* Although we have only discussed the color of a ball as a property, any physical property can be "put into" a misty state.
* The relationship of the mathematical object of a misty state to "real physical properties" is contentious.
* One option is to take the mist itself as a real physical property. This results either in some weird mechanical properties of the mist (if we accept collapse) or problems to do with explaining our own experiences within a single universe.
* Another option is to take the mist as a state of knowledge. Within this option there are two possibilities. The misty states might be states of knowledge (like rocky states) where the macroscopic-scale events they represent arise from underlying real states. Or the misty states (not at all like rocky states) might be a new type of inferential object, which does not need a connection to underlying "real states."
* If there are real underlying states (i.e. properties) of physical objects, then, Einstein argued, there are multiple misty states corresponding to a single real state. His argument assumed locality. This assumption is particularly questionable as he also needed entangled states, which we now know can produce nonlocal correlations.
* Without using entangled misty states, but also with a different locality/independence assumption, Pooh-Bear gave an argument for the opposite conclusion to Einstein—namely that real states correspond to unique misty states.

Epilogue

Quantum. It is a word I read hundreds of times a week in scientific papers. But here I was in an airport pharmacy, buying underarm deodorant, reading that magical word through the mental fog of travel fatigue. My armpits definitely need to be "quantum dry," so I purchased it. I long ago gave up being irritated by stupid use of the q-word in everything from "quantum auto-body repair" to "quantum financial services." As a word that actually originated in describing the very smallest quantities of stuff, calling your product a "quantum leap" makes about as much sense as advertising it as "miniscule progress you definitely won't notice but which you should pay more for."

Although this has been a book about quantum theory—the most important theory of modern physics—I have avoided using the word "quantum" per se. Primarily this is because I want to avoid what is now extremely loaded jargon about which a ridiculous number of misconceptions abound. Many of these misconceptions are due to poor use of language describing much more precise mathematics and/or experimental facts. My goal here has been to teach those facts and avoid the jargon. There is also the danger of thinking that quantum theory is difficult to sort out because we cannot "see" the microscopic things to which we normally apply it. But in fact you can see with your naked eye the fluorescence from a single atom. We could therefore build boxes very much as I have presented, where instead of black and white balls we would see something like a small dot of red or green light bouncing off (or emitted by) an atom. We have no indications whatsoever (and in fact many to the contrary) that quantum theory will fail to be the correct description of the world up to human scales. The difficulty of building PETE boxes for large-scale objects appears to be only one of engineering. If somehow quantum theory fails to be the correct theory at larger scales that would be a remarkable discovery.

The "misty states" are just "quantum states," and the phenomena of nonlocality and entanglement and superposition would often be called "quantum nonlocality" and "quantum entanglement," and "quantum superposition." I have taken "the laws of quantum physics" to be the axioms of the theory as typically taught to an undergraduate physicist (often associated with what is called a "Copenhagen Interpretation" although they are basically just the rules taken at face value, and not really an interpretation of those rules per se).

As Part I hopefully convinced you, we are entering an age of amazing new technologies based on these counterintuitive phenomena. Actually there are many things we already use every day that rely on quantum phenomena. The internet is powered by lasers, which require quantum physics. The GPS system relies on atomic clocks, which require quantum physics. Although remarkable and important, these, and many other, "old quantum technologies" are basically only making use of "quantum superposition"— the phenomenon which can be exhibited by a single ball in a misty state of black and white. The challenge is to build larger, entangled, misty states with which we can do even more valuable things. It is a challenge I am having fun taking on, and perhaps you can join in one day too.

It is also fun to try and come up with "natural" explanations of quantum weirdness, but hopefully you now realize that this is an extremely challenging task, and should be approached with both care and precision.

History, Context and Further Reading

I intend to put some material on www.qisforquantum.org, both to help understand misty states better, as well as to try and connect the formalism of the mist to regular quantum theory for students who go on to university level study.

Although the misty balls do not easily describe every possible part of quantum theory, they are an extremely powerful subset, universal for quantum computing, and so in principle can simulate and calculate everything in the full theory. This means there are many other parts of quantum theory I did not try to cover that can be readily learned in this diagrammatic form—the uncertainty principle, teleportation, secret key distribution, all the main quantum algorithms, superdense coding and so on.

Part I

Let me relate some other jargon, which you will hear as you go about your physics reading, to things you now know from this book. “Wave-particle duality” is just one manifestation of the phenomenon that a ball can sometimes be “definitely white” and sometimes “definitely in a misty state of black and white.” A ball that is definitely black or white is like a particle. A ball in a misty state is “wavy” because, just as a peak in a water wave can interfere with a trough to yield no wave at all, a ball configuration with a negative-sign label can cancel with one with a positive sign label. The Wikipedia articles *Wave–particle_duality* and *Interference_(wave_propagation)* are a good start if you want to understand a bit more. However, once we are considering two or more balls, there are very many other ways in which misty states are not at all like water waves (or any other regular physical waves), so the usefulness of this wave/particle way of thinking is debatable.

“Quantum coherent” means (in the terms we’ve been using) “misty”; “relative phase” or “quantum phase” refers to the negative-sign labels. The weirdness of quantum superposition is often prosaically illustrated in terms of Schroedinger’s cat being in a (misty state) superposition of dead and alive. Schroedinger actually used the cat to illustrate the even weirder phenomenon of quantum entanglement, discussed in Part II.

The remarkable (to me) fact that I can go so far in building up proper quantum theory for you using only basic arithmetic is due to the proof by Shih (arxiv.org/abs/quant-ph/0205115) that the Toffoli plus Hadamard gates are universal for quantum computing, which means in fact they could be used to accurately calculate any quantum phenomena. The Toffoli gate is just our CCNOT box, the Hadamard gate is the infamous PETE box. (The infamous Pete person is at peteshadbolt.co.uk). The CSWAP box can be used in place of the CCNOT, and its universality for regular computation was discovered by Fredkin. The Wikipedia articles *Toffoli_gate*, *Quantum_gate#Hadamard_gate* and *Fredkin_gate* have more information.

If you are interested in the interplay between our processes of logical reasoning and computation, then it is worth reading the introduction to Turing’s famous papers which formalized them, along with one of Andrew Hodges’ many books or essays about Turing at www.turing.org.uk/publications/. If you want to go the next step to understanding more quantum information theory try Scott Aaronson’s Quantum Computing Since Democritus (www.scottaaronson.com/democritus/).

The robbing-a-bank example I used to illustrate the power of multi-ball misty state (quantum) computation is called the Deutsch-Jozsa algorithm; I stole the idea of using a bank robbery to explain this algorithm from Naomi Nickerson.

Part II

The remarkable discovery that quantum theory allows for the generation of nonlocal correlations is due to Bell, and is typically called Bell's Theorem. Coincidentally, his version makes use of the "Bella" misty state. The particular game with the psychics that I set up exhibits a special case of a version of quantum nonlocality due to Hardy (doi.org/10.1103/PhysRevLett.71.1665). Hardy's paper is difficult to understand; I provide the link for completeness only. The Wikipedia article on Bell's theorem isn't great at the time of writing, and the internet is full of incomprehensible, wrong, or overly technical stuff about quantum nonlocality. Good luck. However Bell's original papers, some of which are not overly technical, are still a beautiful read. They are collected in the book *Speakable and Unspeakable in Quantum Mechanics*.

It is more common when discussing nonlocal quantum correlations to consider a game due to Clauser, Horne, Shimony, and Holt that the psychics win if they give the same color answers (BB or WW) if either one, or both, of them are told tails, and opposite color answers (BW or WB) when both are told tails. There are no other rules per se. Any local strategy can only win this game 75% of the time (known as the CHSH inequality), but with the psychics sharing balls prepared in the Bella mist it can be won over 85% of the time. I may add a fuller discussion to the webpage for the book if enough people are interested in it.

I have only touched on the options people consider seriously for avoiding unpleasant consequences or conclusions of nonlocal correlations. An option that treats the mist as a physically real medium (with all the requisite weird properties necessary) is Bohm-de Broglie theory, Wikipedia has a decent exposition at *De_Broglie–Bohm_theory*.

If you start reading around this area there is a lot of confusion. For instance, people often conflate or equate the Einstein-Podolsky-Rosen paradox with Bell's theorem. EPR's paradox is about incompleteness, addressed in Part III, but it is more poorly explained than Einstein did on his own in other places. In fact all aspects of the EPR paradox can be reproduced in a local theory (as pointed out originally by Bell) and so it is distinct from Bell's considerations. Because the EPR paper itself mashed up incompleteness with Heisenberg's uncertainty principle, in Part III, I have chosen to use the much clearer argument for incompleteness Einstein gave in a letter (EA 47-22 at the Einstein archives) to Schroedinger.

Part III

More precise distinctions between types of explanations for the quantum state are elucidated by Harrigan and Spekkens at arxiv.org/abs/0706.2661. To read more on models with dynamical collapse, find the Stanford Encyclopedia of Philosophy's (plato.stanford.edu) article *qm-collapse*. To read more on many-universe (non-collapse) explanations try their article *qm-manyworlds* or Wallace's book, *The Emergent Multiverse*. The multiverse/many-worlds view is an example of taking the mist to be real, and taking it to be the only real state (the "first option" discussed in the text). An example where the mist is taken as real but there are other physical properties in the real states as well (the "second option") is Bohm-de Broglie theory mentioned above.

If you want to read excerpts from the letter EA 47-22 to Schroedinger where Einstein made his argument most clearly (along with some musings on how Einstein or Schroedinger could have discovered Bell's theorem) then try arxiv.org/abs/1411.4387. Gilder's lovely book *The Age of Entanglement* seems to get the conceptual history mostly right, as do any of the papers by Howard (www3.nd.edu/~dhoward1/) upon which it is partly based.

Einstein also sometimes gave an argument along the lines that since a misty state of [W,B] sometimes looks like the "observably real" state W and sometimes like the "observably real" state B when we look at the ball, there is clearly more than one real state for a misty (i.e. quantum) state. This is an argument for the "second option" for taking the mist to be real, discussed in the text, namely that many different real states corresponding to a single misty state. It is a different form of "incompleteness", and confusion with Bell's theorem and the EPR paradox and Einstein's letter to Schroedinger is rife.

Such a type of incompleteness would not alleviate us from all the "unphysical" properties of the real states when we take the mist to be real, for example that a measurement on one ball of the mist [WWWW,BBBB] in one location can change the purported real state of the world at arbitrarily distant locations. In the text I have focused on the stronger argument Einstein gave as to why misty and real states were not in "one to one correspondence", because it is very often misunderstood, and, for reasons discussed in the Harrigan and Spekkens paper mentioned above, would have been much stronger in terms of ruling out various "realist interpretations" of quantum theory, had it not been found suspect by Bell's discovery of nonlocality. Reading any of the writings of Fine, such as *The Shaky Game*, is a way of getting into this somewhat confusing topic.

In modern papers you will often read about "ontic states" (what I called real states) and the distinction between "the mist (quantum state) is real" versus "the mist (quantum state) is just knowledge/information about the real state" referred to as "psi-ontic" and "psi-epistemic" respectively.

Approaches to (and arguments for) understanding quantum states as epistemic can be found in Ballentine's textbook *Quantum Mechanics: A Modern Development*, in Spekkens' classic paper arxiv.org/abs/quant-ph/0401052, in Brukner and Zeilinger's discussion arxiv.org/abs/quant-ph/0212084, and in any of many papers on QBism by Fuchs and collaborators, for example arxiv.org/abs/1412.4211. QBism is an example of an approach that denies a connection between real states/physical properties and how we use the mist to explain our experiences and interactions with the world. There are also many great links and references in the article *qt-issues* at the Stanford Encyclopedia of Philosophy.

The "Pooh-Bear" argument is from arxiv.org/abs/1111.3328, but Leifer's blog post *can-the-quantum-state-be-interpreted-statistically* is a good place to start if you want a clearer explanation. The very last sentence Pooh says to Einstein is direct quotation from *Winnie-the-Pooh* by A. A. Milne. The rest I made up.

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