Model Selection

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Introduction

Load the ISLR package and check the the Hitters data.

```
library(ISLR)
data(Hitters)
?Hitters
str(Hitters)
  'data.frame':
                    322 obs. of 20 variables:
##
   $ AtBat
                     293 315 479 496 321 594 185 298 323 401 ...
               : int
   $ Hits
               : int 66 81 130 141 87 169 37 73 81 92 ...
               : int 1 7 18 20 10 4 1 0 6 17 ...
##
   $ HmRun
##
   $ Runs
               : int
                     30 24 66 65 39 74 23 24 26 49 ...
## $ RBI
               : int 29 38 72 78 42 51 8 24 32 66 ...
## $ Walks
              : int 14 39 76 37 30 35 21 7 8 65 ...
## $ Years
              : int
                     1 14 3 11 2 11 2 3 2 13 ...
## $ CAtBat
                     293 3449 1624 5628 396 4408 214 509 341 5206 ...
              : int
## $ CHits
              : int 66 835 457 1575 101 1133 42 108 86 1332 ...
## $ CHmRun
             : int 1 69 63 225 12 19 1 0 6 253 ...
## $ CRuns
                     30 321 224 828 48 501 30 41 32 784 ...
               : int
## $ CRBI
               : int 29 414 266 838 46 336 9 37 34 890 ...
## $ CWalks
             : int 14 375 263 354 33 194 24 12 8 866 ...
             : Factor w/ 2 levels "A", "N": 1 2 1 2 2 1 2 1 2 1 ...
## $ League
## $ Division : Factor w/ 2 levels "E", "W": 1 2 2 1 1 2 1 2 2 1 ...
## $ PutOuts : int 446 632 880 200 805 282 76 121 143 0 ...
## $ Assists : int 33 43 82 11 40 421 127 283 290 0 ...
## $ Errors
             : int
                     20 10 14 3 4 25 7 9 19 0 ...
               : num NA 475 480 500 91.5 750 70 100 75 1100 ...
##
   $ Salary
   $ NewLeague: Factor w/ 2 levels "A","N": 1 2 1 2 2 1 1 1 2 1
Are there any missing values?
NA index <- is.na(Hitters)
length(Hitters[NA_index])
## [1] 59
There are 59 missing values here, so before we proceed we will remove them:
Hitters <- na.omit(Hitters)</pre>
NA_index <- is.na(Hitters)</pre>
length(Hitters[NA_index])
```

Best Subset regression

[1] 0

We will now use the package leaps to evaluate all the best-subset models. It considers all possible variable combinations for each possible model size. The * in each row of the model output below signifies the chosen variable.

```
library(leaps)
```

```
subset_full <- regsubsets(Salary ~ ., data = Hitters)</pre>
summary(subset_full)
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters)
## 19 Variables (and intercept)
##
              Forced in Forced out
## AtBat
                  FALSE
                              FALSE
## Hits
                  FALSE
                              FALSE
## HmRun
                  FALSE
                              FALSE
## Runs
                  FALSE
                              FALSE
## RBI
                  FALSE
                              FALSE
## Walks
                  FALSE
                              FALSE
## Years
                   FALSE
                              FALSE
## CAtBat
                  FALSE
                              FALSE
## CHits
                   FALSE
                              FALSE
## CHmRun
                   FALSE
                              FALSE
## CRuns
                   FALSE
                              FALSE
## CRBI
                   FALSE
                              FALSE
## CWalks
                   FALSE
                              FALSE
## LeagueN
                  FALSE
                              FALSE
## DivisionW
                   FALSE
                              FALSE
## PutOuts
                   FALSE
                              FALSE
## Assists
                   FALSE
                              FALSE
## Errors
                   FALSE
                              FALSE
## NewLeagueN
                   FALSE
                              FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
            AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
##
     (1)""
                   11 11
                               11 11
                                   11 11
                                                     11 11
                                                            11 11
## 1
                               11 11
                                    11 11 11 11
## 2 (1)""
                   11 🕌 11
## 3 (1)
## 4
     (1)
            11 11
## 5
     (1)"*"
     (1)"*"
     (1)
            11 11
## 7
                                              11 11
                               11 11
                                   11 11 11 *11
                                                     .. ..
                                                                   "*"
## 8
      (1)
            "*"
                                                                          11 * 11
##
            CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
            "*"
                                 11 11
                                            11 11
                                                     11 11
     (1)
     (1)"*"
## 2
     (1)
            "*"
                         11 11
                                  11 11
                                            "*"
                                  "*"
                                            "*"
## 4 (1) "*"
                         11 11
                                  "*"
                                            "*"
## 5
     (1)
                                  "*"
## 6
      (1)
            "*"
                         11 11
                                            11 * 11
     (1)""
                         11 11
                                  "*"
                                            "*"
                                                     11 11
                                                             11 11
                                                                     11 11
## 7
                                  "*"
                                            "*"
     (1)""
```

Notice above, the default best-subsets up to size 8. Lets increase that to 19, which is all the variables, create summary statistics on the model and view their names. Calling names on the full_summary gives us the output categories it contains.

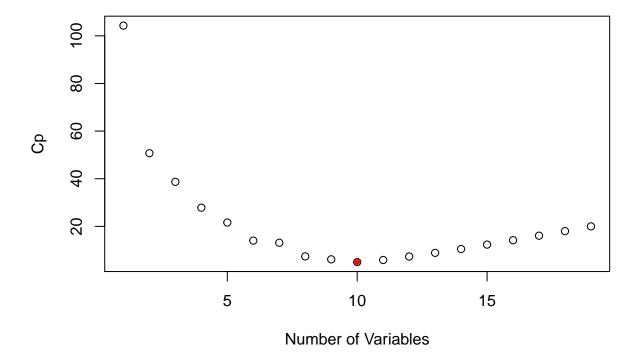
```
subset_full <- regsubsets(Salary ~ ., data = Hitters, nvmax = 19)
full_summary <- summary(subset_full)</pre>
```

```
names(full_summary)
```

```
## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"
```

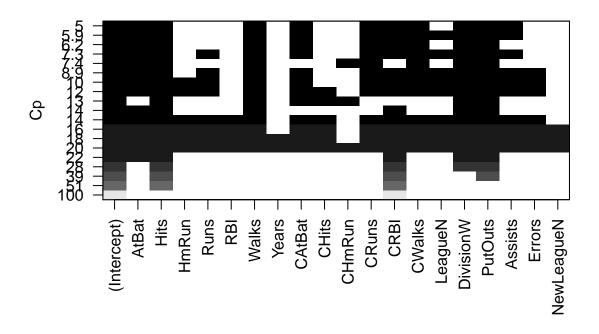
So lets plot the Cp, or the estimated prediction error, for each variable. As we are looking for the Min, we can use the which.min function and color it red.

```
plot(full_summary$cp, xlab = "Number of Variables", ylab = "Cp")
points(which.min(full_summary$cp), full_summary$cp[which.min(full_summary$cp)],
    pch = 20, col = "red")
```



There is a plot method designed specifically for the **regsubsets** object which is displayed below. It also plots the **Cp** statistic but each variable. Areas that are colored black indicate the variable is present in the model at the corresponding **Cp** level, while white areas communicate an absence of the variable.

```
plot(subset_full, scale = "Cp")
```



coef(subset_full, 10)

```
(Intercept)
                                                                    \mathtt{CAtBat}
##
                         AtBat
                                         Hits
                                                      Walks
##
    162.5354420
                    -2.1686501
                                   6.9180175
                                                  5.7732246
                                                               -0.1300798
##
           CRuns
                          CRBI
                                       CWalks
                                                  DivisionW
                                                                   PutOuts
##
      1.4082490
                     0.7743122
                                  -0.8308264 -112.3800575
                                                                0.2973726
##
        Assists
##
      0.2831680
```

Forward Stepwise Selection

Here we use the regsubsets function but specify the method="forward" option:

```
forward_step <- regsubsets(Salary ~ ., data=Hitters, nvmax=19, method="forward")
summary(forward_step)</pre>
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables (and intercept)
##
              Forced in Forced out
## AtBat
                  FALSE
                             FALSE
## Hits
                  FALSE
                             FALSE
## HmRun
                  FALSE
                             FALSE
## Runs
                  FALSE
                             FALSE
## RBI
                  FALSE
                             FALSE
## Walks
                  FALSE
                             FALSE
```

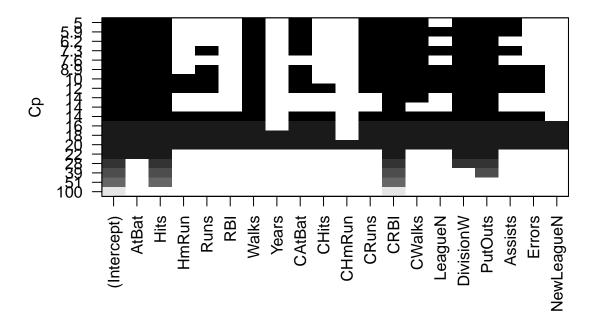
```
## CAtBat
                       FALSE
                                     FALSE
                                     FALSE
## CHits
                       FALSE
## CHmRun
                       FALSE
                                     FALSE
## CRuns
                       FALSE
                                     FALSE
## CRBI
                       FALSE
                                     FALSE
## CWalks
                       FALSE
                                     FALSE
## LeagueN
                                     FALSE
                       FALSE
## DivisionW
                       FALSE
                                     FALSE
## PutOuts
                       FALSE
                                     FALSE
## Assists
                       FALSE
                                     FALSE
## Errors
                       FALSE
                                     FALSE
## NewLeagueN
                       FALSE
                                     FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##
                 AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1
                        11 11
                               11 11
                                      11 11
                                             \Pi=\Pi=\Pi=\Pi
                                                          11 11
                                                                           11 11
                                                                                  11 11
                                                                                           11 11
       (1)
                                      11 11
                                             11 11 11
                                                                                           11 11
                11 11
                        "*"
##
      (1)
                                                  11
                                                                                           11 11
##
   3
       (1)
                 11 11
                        "*"
## 4
       (1)
                                      11 11
                                                                           11
                                                                             11
## 5
       ( 1
            )
                 "*"
                                                                           11 11
## 6
       (1)
                 "*"
                               11 11
                                      11 11
                                                                                           11 11
## 7
       (1)
                 "*"
                        "*"
                               11 11
                                       11 11
                                                                           11 11
                                                                                   11 11
## 8
       (1
            )
                 "*"
                                                                                            "*"
                "*"
                        "*"
                                                                                           "*"
## 9
       (1)
                                                                           11 11
## 10
         (1)
                "*"
                                      11 11
                                                          11 11
                                                                  "*"
                                                                                   11 11
                                                                                           "*"
                                                                           11 11
                                                                                           "*"
##
   11
         ( 1
             )
                "*"
                        "*"
                                                                  "*"
##
   12
         (1
              )
                               11 11
                                      "*"
                                             اايداا اا اا
                                                          .. ..
                                                                  "*"
                                                                           11 11
                                                                                   11 11
                                                                                           "*"
                "*"
                        "*"
                                       "*"
                                                                  "*"
                                                                                           "*"
##
   13
         (1
              )
                                                                           11 11
                                      "*"
                                             11 11
                                                          11 11
                                                                  "*"
                                                                                   11 11
                                                                                           "*"
## 14
         (1
                 "*"
                                       "*"
                                                                           "*"
                                                                                            "*"
                        "*"
                               " * "
                                                                  "*"
## 15
         (
           1
              )
##
   16
         (1
              )
                 "*"
                        "*"
                               "*"
                                       "*"
                                             "*"
                                                          11 11
                                                                  "*"
                                                                           "*"
                                                                                   11 11
                                                                                           "*"
                                      "*"
         (1
                "*"
                        11 🕌 11
                               الياا
                                             11 4 11
                                                                  11 🕌 11
                                                                           11 🕌 11
                                                                                           "*"
##
   17
             )
##
                "*"
                        "*"
                                      "*"
                                             "*" "*"
                                                                                   11 11
                                                                                           "*"
         (1)
   18
                                             "*" "*"
                                                                                           "*"
                        "*"
                               "*"
                                      "*"
                                                          "*"
                                                                           "*"
                                                                                   "*"
                                                                  "*"
##
   19
         (
##
                CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
                                11 11
                                          11 11
                                                       11 11
                                                                            11 11
                                                                                     11 11
## 1
       (1)
                "*"
                       11 11
## 2
       (1)
                 "*"
                                          11 11
                                                       11 11
                                                                                       11
## 3
                                          11 11
                                                       11 * 11
       (1
            )
                                                       "*"
## 4
                 "*"
                                          "*"
       ( 1
            )
                                11 11
                                                       "*"
## 5
       (1
                 "*"
                                          "*"
            )
                 "*"
                                          "*"
                                                        "*"
## 6
       ( 1
            )
##
       (1
                 "*"
                                          "*"
                                                       "*"
                                                                    11
## 8
                 "*"
                                          "*"
                                                       "*"
       ( 1
            )
                                11 11
                                                       "*"
                                                                                       "
## 9
       (1)
                 "*"
                                          "*"
                       "*"
                                          "*"
                                                        "*"
                                                                  "*"
         (1)
                 "*"
## 10
                                                                            11 11
              )
                       "*"
                                "*"
                                          "*"
                                                       "*"
                                                                  "*"
                                                                                     11 11
##
   11
         (
           1
                                          "*"
                                                       "*"
##
   12
         (1
                "*"
                       "*"
                                "*"
                                                                  "*"
              )
                "*"
                                          "*"
                                                       "*"
                                                                  "*"
                                                                                     11 11
##
   13
         (1)
                                          "*"
                                                       "*"
                 "*"
                       11 🕌 11
                                اليواا
                                                                  11 🕌 11
                                                                            11 🕌 11
##
   14
         (
           1
              )
                                "*"
                                          "*"
                                                       "*"
                                                                  "*"
                                                                            "*"
                                                                                     11 11
##
   15
         (1
              )
                                                       "*"
                                                                                     11 11
                       11 * 11
                                11 * 11
                                          11 * 11
                                                                  11 * 11
                                                                            11 * 11
## 16
         (1
              )
                                "*"
                                          "*"
                                                       "*"
                                                                  "*"
                                                                            "*"
                                                                                     "*"
## 17
         (1)
         (1)
                "*"
                       11 * 11
                                11 * 11
                                          11 * 11
                                                       11 * 11
                                                                  11 * 11
                                                                            "*"
                                                                                     "*"
## 18
```

Years

FALSE

FALSE

```
## 19 ( 1 ) "*" "*" "*" "*" "*" "*"
plot(forward_step, scale = "Cp")
```



Model Selection Using a Validation Set

Lets make a training and validation set, so that we can choose a good subset model. We will do it using a slightly different approach from what was done in the the book.

```
dim(Hitters)
## [1] 263 20
set.seed(1)
train <- sample(seq(263), 180, replace = FALSE)

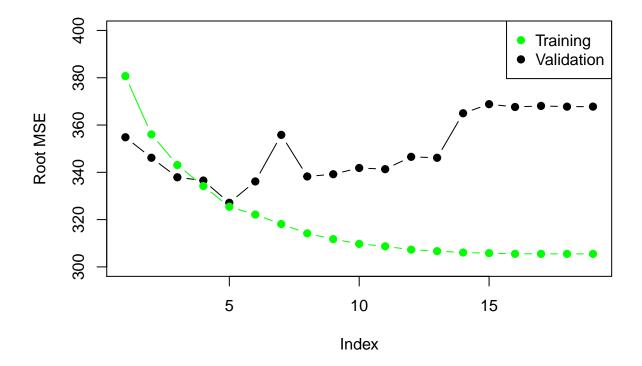
forward_step <- regsubsets(Salary ~ ., data = Hitters[train, ], nvmax = 19,
    method = "forward")</pre>
```

Now we will make predictions on the observations not used for training. We know there are 19 models, so we set up some vectors to record the errors. We have to do a bit of work here, because there is no predict method for regsubsets.

```
val.errors <- rep(NA, 19)
x.test <- model.matrix(Salary ~ ., data = Hitters[-train, ])
for (i in 1:19) {</pre>
```

```
coefi <- coef(forward_step, id = i)
   pred <- x.test[, names(coefi)] %*% coefi
   val.errors[i] <- mean((Hitters$Salary[-train] - pred)^2)
}

plot(sqrt(val.errors), ylab = "Root MSE", ylim = c(300, 400), pch = 19, type = "b")
points(sqrt(forward_step$rss[-1]/180), col = "green", pch = 19, type = "b")
legend("topright", legend = c("Training", "Validation"), col = c("green", "black"),
   pch = 19)</pre>
```



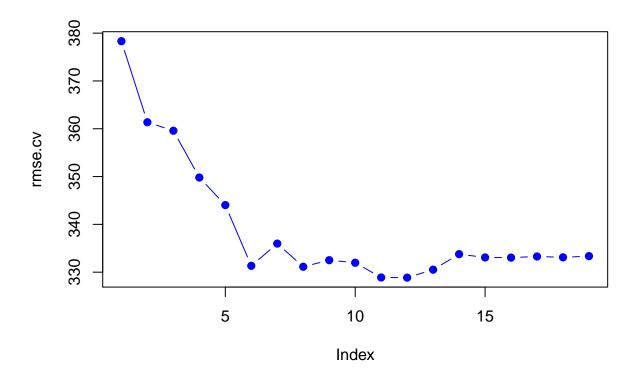
As we expect, the training error goes down monotonically as the model gets bigger, but not so for the validation error.

This was a little tedious - not having a predict method for regsubsets. So we will write a generic function for it.

Model Selection by Cross-Validation

We will do 10-fold cross-validation. Its really easy!

```
set.seed(11)
folds <- sample(rep(1:10, length = nrow(Hitters)))</pre>
folds
##
        3 1 4 4 7 7 3 5 5 2 5
                                      2
                                        8
                                           3 3 3 9
                                                      2 9 8 10
                                                                5
    [1]
   [24] 5 5 5 5 10 10 4
                          4 7
                                6
                                   7
                                      7
                                        7
                                           3
                                              4 8 3
                                                      6 8 10 4
##
##
   [47] 9
           3 4 9 8 7 10 6 10
                                3
                                   6 9 4
                                           2
                                              8
                                                 2
                                                   5
                                                      6 10 7 2
##
   [70]
        1
           3 6
                2
                   5
                      8 1
                           1
                             2
                                8
                                   1 10
                                         1
                                           2
                                              3 6
                                                   6
                                                      5
                                                         8
                                                           8 10
## [93] 6 1 7 4 8 3 7 8 7
                                1 10
                                      1
                                         6
                                           2 9 10
                                                      7
                                                         7
                                                           4
                                                             7
                                                                 4 10
                                                   1
## [116] 3 6 10 6 6
                      9 8 10 6
                                7
                                   9
                                      6
                                        7
                                           1 10
                                                 2
                                                   2
## [139] 2 9 4 10 5 3 7
                          7 10 10
                                   9
                                      3 3
                                           7
                                              3
                                                   4
                                                      6
                                                         6 10 4
                                                1
        1
## [162]
           3 6 8 10
                      8 5 4
                             5
                                6
                                   2
                                      9 10
                                           3
                                              7
                                                 7
                                                   6
                                                      6
## [185] 4 4 8 2 3 5 9 9 10
                                2
                                   1
                                      3 9
                                           6 7
                                                 3 1 9 4 10 10
## [208] 8
           2 5 9 8 10 5 8 2 4 1 4 4 5 5 2 1 9 5 2 9 9 5
## [231] 3
           2 1
                9 1 7 2 5 8 1
                                   1 7 6 6 4 5 10 5 7 4 8 6 9
## [254] 1
           2 5 7 1 3 1 3 1
table(folds)
## folds
## 1 2 3 4 5 6 7 8 9 10
## 27 27 27 26 26 26 26 26 26 26
cv.errors <- matrix(NA, 10, 19)</pre>
for (k in 1:10) {
   best.fit <- regsubsets(Salary ~ ., data = Hitters[folds != k, ], nvmax = 19,
       method = "forward")
   for (i in 1:19) {
       pred <- predict(best.fit, Hitters[folds == k, ], id = i)</pre>
       cv.errors[k, i] <- mean((Hitters$Salary[folds == k] - pred)^2)</pre>
   }
}
rmse.cv <- sqrt(apply(cv.errors, 2, mean))</pre>
plot(rmse.cv, col = "blue", pch = 19, type = "b")
```



Ridge Regression and the Lasso

We will use the package glmnet, which does not use the model formula language, so we will set up an x and y.

```
library(glmnet)

## Loading required package: Matrix

## Loading required package: foreach

## Loaded glmnet 2.0-10

?glmnet

## starting httpd help server ...

## done

x <- model.matrix(Salary ~ .-1, data = Hitters)

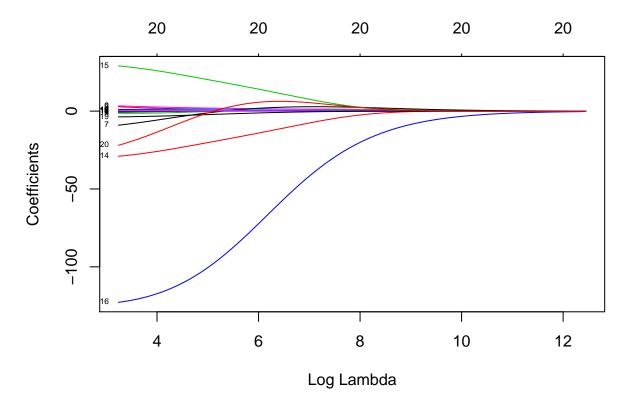
y <- Hitters$Salary</pre>
```

First we will fit a ridge-regression model. Use glmnet with alpha = 0. Remember from the lectures, ridge regression penalizes by the sum squares of the coefficients. It takes the usual linear regression Residual Sum of Squares (RSS), and has been modified by adding a penalty placed on the coefficients.

$$RSS + \lambda \sum_{j=1}^{p} \beta_j^2$$

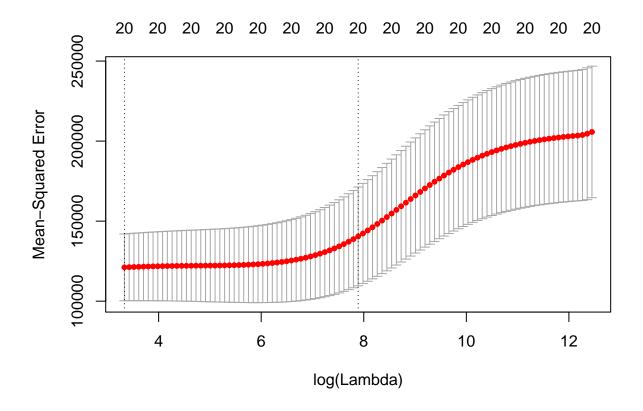
As λ increases, the coefficients shrink to zero. The following plot illustrates this relationship well. When $\lambda = 0$, you have the coefficients of linear regression, with their parameters resting on the y-axis where x = 0.

```
ridge_model <- glmnet(x, y, alpha = 0)
plot(ridge_model, xvar = "lambda", label = TRUE)</pre>
```



here is also a cv.glmnet function which will do the cross-validation for us and has a plot method.

```
cv_ridge_model <- cv.glmnet(x, y, alpha = 0)
plot(cv_ridge_model)</pre>
```



str(cv_ridge_model)

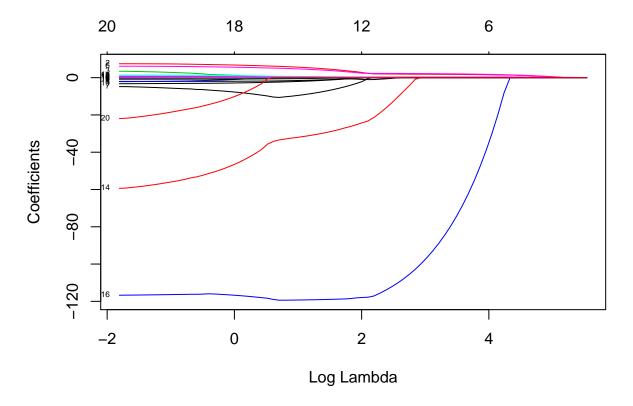
```
## List of 10
               : num [1:99] 255282 232604 211940 193112 175956 ...
   $ lambda
   $ cvm
               : num [1:99] 205730 204596 203812 203439 203183 ...
               : num [1:99] 41078 41192 40895 40748 40722 ...
   $ cvsd
               : num [1:99] 246809 245788 244708 244187 243905 ...
##
   $ cvup
               : num [1:99] 164652 163404 162917 162691 162461 ...
##
   $ cvlo
##
               : Named int [1:99] 20 20 20 20 20 20 20 20 20 ...
   $ nzero
     ..- attr(*, "names")= chr [1:99] "s0" "s1" "s2" "s3" ...
##
               : Named chr "Mean-Squared Error"
##
   $ name
    ..- attr(*, "names")= chr "mse"
##
    $ glmnet.fit:List of 12
                  : Named num [1:100] 536 528 527 526 525 ...
##
     ..$ a0
     ....- attr(*, "names")= chr [1:100] "s0" "s1" "s2" "s3" ...
##
##
     ..$ beta
                 :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
##
     .. .. ..@ i
                      : int [1:2000] 0 1 2 3 4 5 6 7 8 9 ...
##
     .. .. ..@ р
                       : int [1:101] 0 20 40 60 80 100 120 140 160 180 ...
     .. .. ..@ Dim
                       : int [1:2] 20 100
##
     .. .. ..@ Dimnames:List of 2
##
     .....$ : chr [1:20] "AtBat" "Hits" "HmRun" "Runs" ...
##
     .....$: chr [1:100] "s0" "s1" "s2" "s3" ...
##
##
     .. .. ..@ x
                       : num [1:2000] 1.22e-36 4.43e-36 1.78e-35 7.49e-36 7.91e-36 ...
##
     .. .. ..@ factors : list()
               : int [1:100] 20 20 20 20 20 20 20 20 20 20 ...
##
     ..$ df
                 : int [1:2] 20 100
     ..$ dim
##
```

```
: num [1:100] 255282 232604 211940 193112 175956 ...
##
##
     ..$ dev.ratio: num [1:100] 6.19e-36 1.16e-02 1.27e-02 1.39e-02 1.53e-02 ...
                  : num 53319113
##
     ..$ nulldev
     ..$ npasses
##
                  : int 701
##
     ..$ jerr
                  : int 0
     ..$ offset
                  : logi FALSE
##
##
     ..$ call
                  : language glmnet(x = x, y = y, alpha = 0)
                   : int 263
##
     ..$ nobs
##
     ..- attr(*, "class")= chr [1:2] "elnet" "glmnet"
    $ lambda.min: num 28
##
    $ lambda.1se: num 2674
    - attr(*, "class")= chr "cv.glmnet"
```

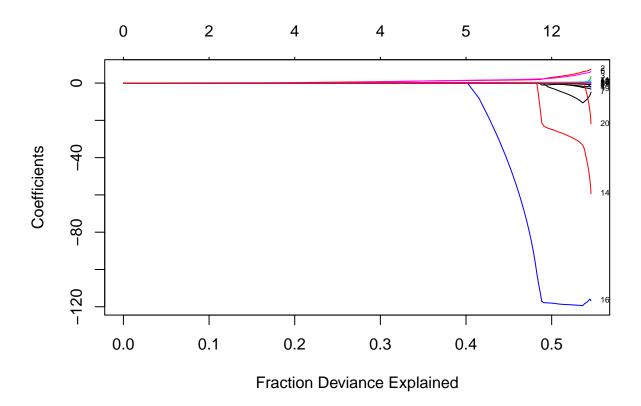
Now we fit a lasso model, calling glmnet but using the default alpha=1. This time, instead of penalizing the sum of squares of the coefficients, we penalize their absolute values instead. This actually restricts some coefficients to be exactly zero, which makes them effectively NULL. Your variable selection has now been performed for you in a much more efficient manner than the subset and step-wise methods.

$$RSS + \lambda \sum_{j=1}^{p} |\beta_j|$$

```
lasso_model <- glmnet(x, y, alpha = 1)
plot(lasso_model, xvar = "lambda", label=TRUE)</pre>
```

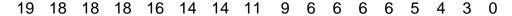


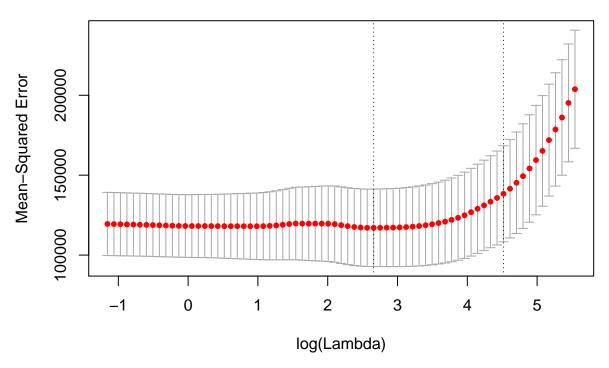
```
plot(lasso_model, xvar = "dev", label=TRUE)
```



Lets use Cross-Validation for the Lasso.

```
cv.lasso <- cv.glmnet(x, y, alpha = 1)
plot(cv.lasso)</pre>
```





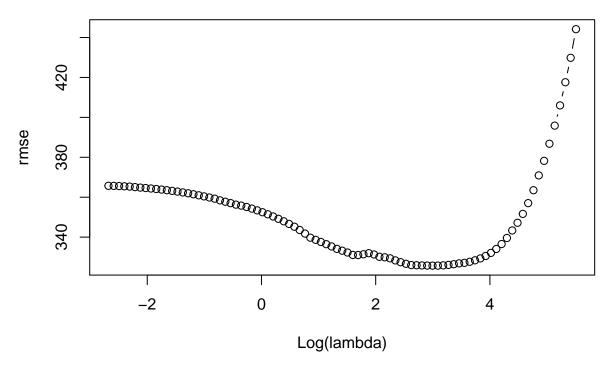
coef(cv.lasso)

```
## 21 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept) 193.74263858
## AtBat
## Hits
                 1.21471320
## HmRun
## Runs
## RBI
## Walks
                 1.28957902
## Years
## CAtBat
## CHits
## CHmRun
                 0.12923755
## CRuns
## CRBI
                 0.31515925
## CWalks
## LeagueA
## LeagueN
## DivisionW
## PutOuts
                 0.02533305
## Assists
## Errors
## NewLeagueN
```

Suppose we want to use our earlier train/validation set to select the lambda for the lasso.

```
lasso_train <- glmnet(x[train,],y[train])</pre>
lasso_train
## Call: glmnet(x = x[train, ], y = y[train])
##
##
               %Dev
                       Lambda
         Df
         0 0.00000 246.40000
    [1,]
##
    [2,] 1 0.05013 224.50000
         1 0.09175 204.60000
    [3,]
##
    [4,]
         2 0.13840 186.40000
    [5,]
          2 0.18000 169.80000
    [6,]
          3 0.21570 154.80000
##
##
    [7,]
          3 0.24710 141.00000
    [8,]
##
          3 0.27320 128.50000
    [9,]
          4 0.30010 117.10000
## [10,]
          4 0.32360 106.70000
## [11,]
          4 0.34310
                     97.19000
## [12,]
          4 0.35920
                     88.56000
## [13,]
          5 0.37360
                     80.69000
## [14,]
          5 0.38900
                     73.52000
## [15,]
          5 0.40190
                     66.99000
## [16,]
          5 0.41260
                     61.04000
## [17,]
          5 0.42140
                     55.62000
## [18,]
          5 0.42880
                     50.67000
## [19,]
          5 0.43490
                     46.17000
## [20,]
          5 0.43990
                     42.07000
## [21,]
          5 0.44410
                     38.33000
## [22,]
          5 0.44760
                     34.93000
## [23,]
          6 0.45140
                     31.83000
## [24,]
          7 0.45480
                     29.00000
## [25,]
          7 0.45770
                     26.42000
## [26,]
          7 0.46010
                     24.07000
## [27,]
          8 0.46220
                     21.94000
## [28,]
          8 0.46380
                     19.99000
## [29,]
          8 0.46520
                     18.21000
## [30,]
          8 0.46630
                     16.59000
## [31,]
          8 0.46730
                     15.12000
## [32,]
          8 0.46810
                     13.78000
## [33,]
          9 0.47110
                     12.55000
## [34,]
          9 0.47380
                     11.44000
## [35,]
         9 0.47620
                     10.42000
## [36,] 10 0.48050
                      9.49500
## [37,] 9 0.48450
                      8.65200
## [38,] 10 0.48770
                      7.88300
## [39,] 10 0.49360
                      7.18300
## [40,] 11 0.49890
                      6.54500
## [41,] 12 0.50450
                      5.96300
## [42,] 12 0.51010
                      5.43400
                      4.95100
## [43,] 13 0.51470
## [44,] 13 0.51850
                      4.51100
## [45,] 13 0.52170
                      4.11000
## [46,] 14 0.52440
                      3.74500
```

```
## [47,] 14 0.52670
                      3.41200
## [48,] 15 0.52870
                      3.10900
## [49,] 15 0.53030
                      2.83300
## [50,] 15 0.53160
                      2.58100
## [51,] 16 0.53280
                      2.35200
## [52,] 17 0.53420
                      2.14300
## [53,] 18 0.53580
                      1.95300
## [54,] 18 0.53760
                      1.77900
## [55,] 18 0.53890
                      1.62100
## [56,] 18 0.54000
                       1.47700
## [57,] 18 0.54090
                      1.34600
## [58,] 18 0.54160
                       1.22600
## [59,] 18 0.54220
                      1.11700
## [60,] 18 0.54280
                      1.01800
## [61,] 18 0.54320
                      0.92770
## [62,] 18 0.54360
                      0.84530
## [63,] 18 0.54380
                      0.77020
## [64,] 19 0.54410
                      0.70180
## [65,] 19 0.54430
                      0.63940
## [66,] 19 0.54450
                      0.58260
## [67,] 19 0.54470
                      0.53090
## [68,] 19 0.54490
                      0.48370
## [69,] 20 0.54510
                      0.44070
## [70,] 20 0.54520
                      0.40160
## [71,] 20 0.54530
                      0.36590
## [72,] 20 0.54540
                      0.33340
## [73,] 20 0.54550
                      0.30380
## [74,] 20 0.54560
                      0.27680
## [75,] 20 0.54570
                      0.25220
## [76,] 20 0.54570
                      0.22980
## [77,] 20 0.54580
                      0.20940
## [78,] 20 0.54580
                      0.19080
## [79,] 20 0.54590
                      0.17380
## [80,] 20 0.54590
                      0.15840
## [81,] 20 0.54590
                      0.14430
## [82,] 20 0.54590
                      0.13150
## [83,] 20 0.54600
                      0.11980
## [84,] 19 0.54600
                      0.10920
## [85,] 19 0.54600
                      0.09948
## [86,] 19 0.54600
                      0.09064
                      0.08259
## [87,] 19 0.54600
## [88,] 20 0.54600
                       0.07525
## [89,] 20 0.54600
                      0.06856
pred <- predict(lasso_train, x[-train, ])</pre>
dim(pred)
## [1] 83 89
rmse <- sqrt(apply((y[-train]-pred)^2, 2, mean))</pre>
plot(log(lasso_train$lambda), rmse, type = "b", xlab = "Log(lambda)")
```



```
lambda_best <- lasso_train$lambda[order(rmse)[1]]</pre>
lambda_best
## [1] 19.98706
coef(lasso_train, s = lambda_best)
## 21 x 1 sparse Matrix of class "dgCMatrix"
## (Intercept) 107.9416686
## AtBat
## Hits
                  0.1591252
## HmRun
## Runs
## RBI
                  1.7340039
## Walks
                  3.4657091
## Years
## CAtBat
## CHits
## CHmRun
## CRuns
                  0.5386855
## CRBI
## CWalks
## LeagueA
                -30.0493021
## LeagueN
## DivisionW
               -113.8317016
```

PutOuts 0.2915409

Assists .
Errors .

NewLeagueN 2.0367518