ISLR 3: Linear Regression

Justin M Shea

Contents

Load the following packages.

```r
library(MASS)
library(ISLR)
```

Simple linear regression

Check out the `Boston` data

```r
?Boston
```

```r
names(Boston)
```

```
##  [1] "crim"    "zn"      "indus"   "chas"    "nox"     "rm"      "age"
##  [8] "dis"     "rad"     "tax"     "ptratio" "black"   "lstat"   "medv"
```

```r
class(Boston)
```

```
## [1] "data.frame"
```

```r
dim(Boston)
```

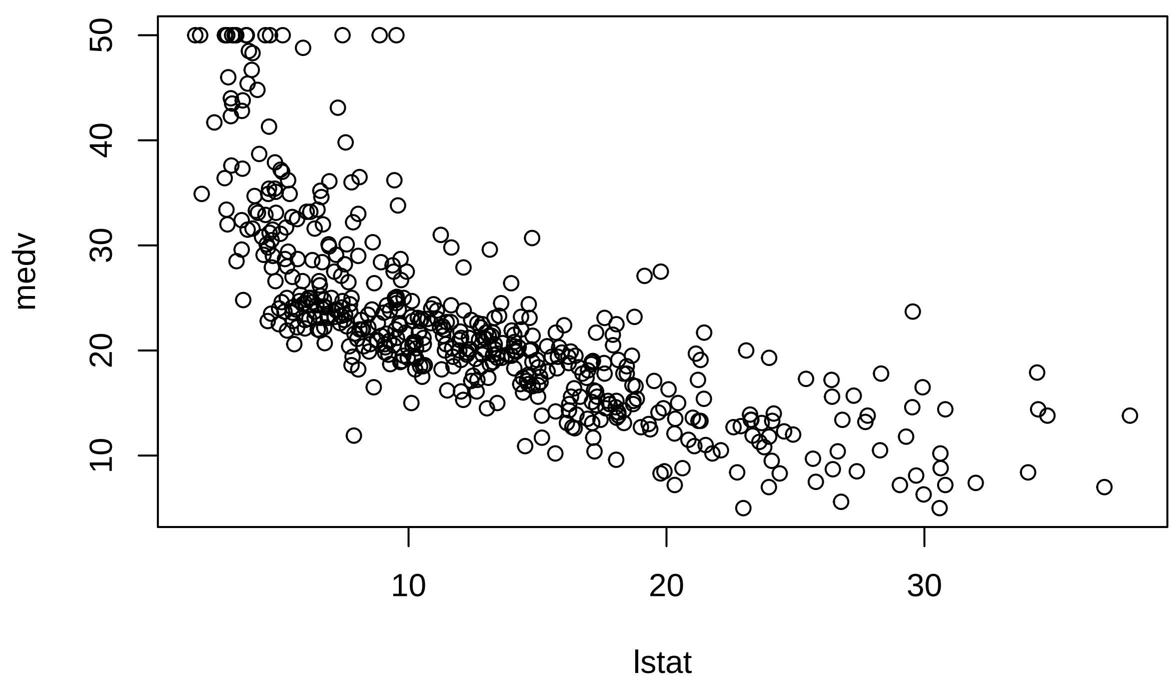
```
## [1] 506  14
```

```r
summary(Boston)
```

```
##       crim                zn             indus            chas
##  Min.   : 0.00632   Min.   :  0.00   Min.   : 0.46   Min.   :0.00000
##  1st Qu.: 0.08204   1st Qu.:  0.00   1st Qu.: 5.19   1st Qu.:0.00000
##  Median : 0.25651   Median :  0.00   Median : 9.69   Median :0.00000
##  Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917
##  3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000
##  Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000
##       nox               rm             age              dis
##  Min.   :0.3850   Min.   :3.561   Min.   :  2.90   Min.   : 1.130
##  1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100
##  Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207
##  Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795
##  3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188
##  Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127
##       rad              tax           ptratio          black
##  Min.   : 1.000   Min.   :187.0   Min.   :12.60   Min.   :  0.32
##  1st Qu.: 4.000   1st Qu.:279.0   1st Qu.:17.40   1st Qu.:375.38
##  Median : 5.000   Median :330.0   Median :19.05   Median :391.44
##  Mean   : 9.549   Mean   :408.2   Mean   :18.46   Mean   :356.67
##  3rd Qu.:24.000   3rd Qu.:666.0   3rd Qu.:20.20   3rd Qu.:396.23
##  Max.   :24.000   Max.   :711.0   Max.   :22.00   Max.   :396.90
##      lstat            medv
##  Min.   : 1.73   Min.   : 5.00
##  1st Qu.: 6.95   1st Qu.:17.02
##  Median :11.36   Median :21.20
##  Mean   :12.65   Mean   :22.53
##  3rd Qu.:16.95   3rd Qu.:25.00
##  Max.   :37.97   Max.   :50.00
```

Lets plot the `Boston` data

```r
plot(medv ~ lstat, data = Boston)
```



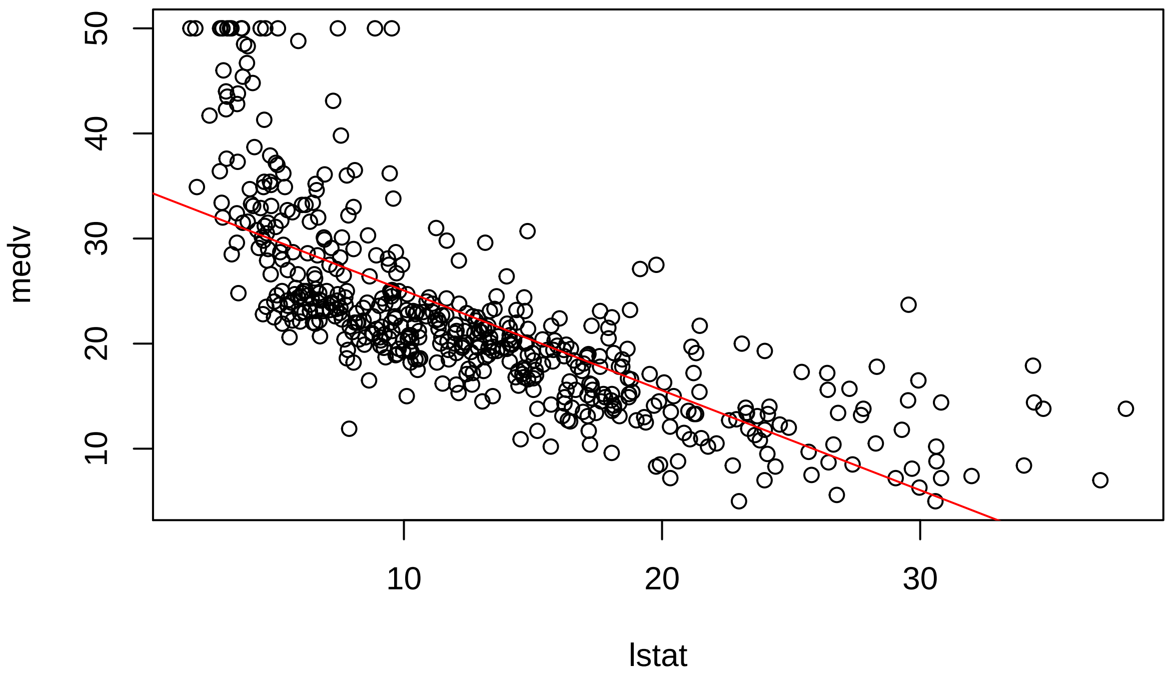
Run a linear model (`lm`) on it and print the results

```r
Boston_lm <- lm(medv ~ lstat, data = Boston)
Boston_lm
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Coefficients:
## (Intercept)        lstat
##       34.55        -0.95
```

Lets plot the linear model against a scatter plot of `medv` and `lstat`.

```r
plot(medv ~ lstat, data = Boston)
abline(Boston_lm, col="red")
```



We can check the confidence intervals of our models parameters using the `confint` function.

```
confint(Boston_lm)
```

```
##                 2.5 %     97.5 %
## (Intercept) 33.448457 35.6592247
## lstat       -1.026148 -0.8739505
```

In addition, the `predict` function is useful in making some predictions with the `Bonston_lm` model we created.

```
predict(Boston_lm, data.frame(lstat=c(5,10,15)), interval="confidence")
```

```
##        fit      lwr      upr
## 1 29.80359 29.00741 30.59978
## 2 25.05335 24.47413 25.63256
## 3 20.30310 19.73159 20.87461
```

Multiple linear regression

Regress `lstat` and `age` against `medv` of the `Boston` data set and print the `summary` diagnostics.

```
Boston_lm2 <- lm(medv ~ lstat + age, data = Boston)
summary(Boston_lm2)
```

```
##
## Call:
## lm(formula = medv ~ lstat + age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
## lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
## age          0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

Use the `.` notation to regress all variables in the `Boston` data against `medv` and print the `summary` diagnostics.

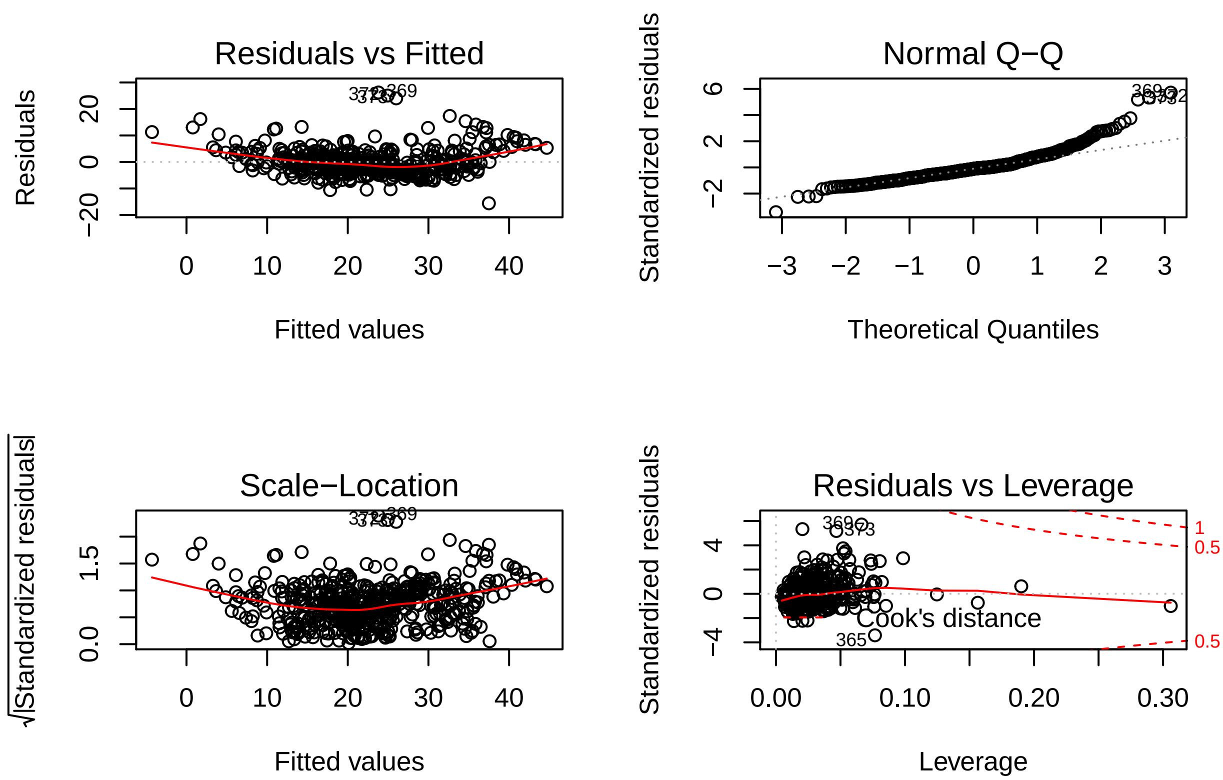
```
Boston_lm3 <- lm(medv ~., data = Boston)
summary(Boston_lm3)
```

```
##
## Call:
## lm(formula = medv ~ ., data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.595  -2.730  -0.518   1.777  26.199
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
## crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
## zn           4.642e-02  1.373e-02   3.382 0.000778 ***
## indus        2.056e-02  6.150e-02   0.334 0.738288
## chas         2.687e+00  8.616e-01   3.118 0.001925 **
## nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
## rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
## age          6.922e-04  1.321e-02   0.052 0.958229
## dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
## rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
## tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
## ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
## black        9.312e-03  2.686e-03   3.467 0.000573 ***
## lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.745 on 492 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7338
## F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

Lets plot the model

```r
par(mfrow=c(2,2))
plot(Boston_lm3)
```



Use the . notation again to select all variables, but this time subtract `age` and `indus` before regressing against `medv`. Print the `summary` diagnostics.

```r
Boston_lm4 <- update(Boston_lm3,~.-age-indus)
summary(Boston_lm4)
```

```
##
## Call:
## lm(formula = medv ~ crim + zn + chas + nox + rm + dis + rad +
##     tax + ptratio + black + lstat, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.5984  -2.7386  -0.5046   1.7273  26.2373
##
## Coefficients:
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.341145   5.067492   7.171 2.73e-12 ***
## crim         -0.108413   0.032779  -3.307 0.001010 **
## zn            0.045845   0.013523   3.390 0.000754 ***
## chas          2.718716   0.854240   3.183 0.001551 **
## nox         -17.376023   3.535243  -4.915 1.21e-06 ***
## rm            3.801579   0.406316   9.356  < 2e-16 ***
## dis          -1.492711   0.185731  -8.037 6.84e-15 ***
## rad           0.299608   0.063402   4.726 3.00e-06 ***
## tax          -0.011778   0.003372  -3.493 0.000521 ***
## ptratio      -0.946525   0.129066  -7.334 9.24e-13 ***
## black         0.009291   0.002674   3.475 0.000557 ***
## lstat        -0.522553   0.047424 -11.019  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.736 on 494 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7348
## F-statistic: 128.2 on 11 and 494 DF,  p-value: < 2.2e-16
```

Nonlinear terms and Interactions

Multiply `lstat` by `age` and regress against `medv`:

```
Boston_mult <- lm(medv ~ lstat * age, data = Boston)
Boston_mult
```

```
##
## Call:
## lm(formula = medv ~ lstat * age, data = Boston)
##
## Coefficients:
## (Intercept)        lstat          age    lstat:age
##  36.0885359   -1.3921168   -0.0007209    0.0041560
```

Create a quadratic interaction using the `I` function with `lstate`:

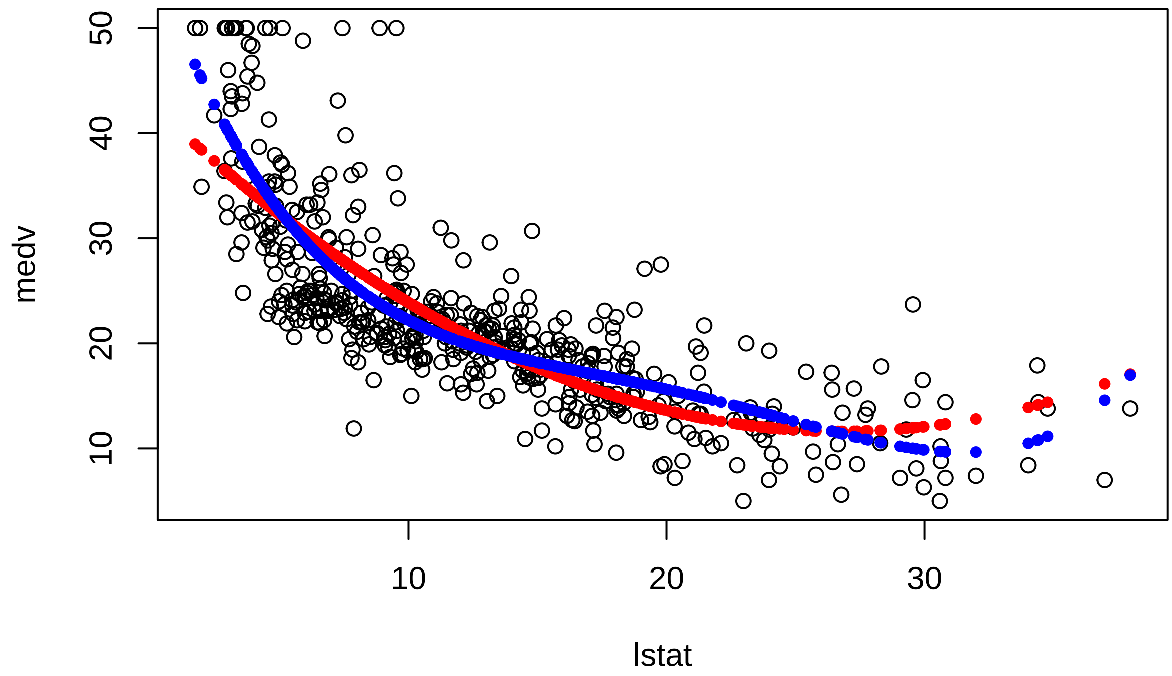
```
Boston_Interaction <- lm(medv ~ lstat + I(lstat^2), data = Boston)
```

Create a 4^{th} order polynomial wrapping `lstat` in the `poly` function, defining the degree arguments a `4`.

```
Boston_poly <- lm(medv ~ poly(lstat, degree=4), data = Boston)
```

Plot the results of the both the `Boston_Interaction` and `Boston_poly` models.

```
attach(Boston)
par(mfrow=c(1,1))
plot(medv ~ lstat, data = Boston)
points(lstat, fitted(Boston_Interaction), col="red", pch=20)
points(lstat, fitted(Boston_poly), col="blue", pch=20)
```



Finally, lets look at the coeffients of both:

```
Boston_Interaction
```

```
##
## Call:
## lm(formula = medv ~ lstat + I(lstat^2), data = Boston)
##
## Coefficients:
## (Intercept)        lstat   I(lstat^2)
##    42.86201     -2.33282      0.04355
```

```
Boston_poly
```

```
##
## Call:
## lm(formula = medv ~ poly(lstat, degree = 4), data = Boston)
##
## Coefficients:
##              (Intercept)  poly(lstat, degree = 4)1
##                    22.53                   -152.46
## poly(lstat, degree = 4)2  poly(lstat, degree = 4)3
##                    64.23                    -27.05
## poly(lstat, degree = 4)4
##                    25.45
```

Qualitative predictors

For this section, use the `Carseats` data. Lets explore:

```r
?Carseats
```

```r
names(Carseats)
```

```
##  [1] "Sales"       "CompPrice"   "Income"      "Advertising" "Population"
##  [6] "Price"       "ShelveLoc"   "Age"         "Education"   "Urban"
## [11] "US"
```

```r
summary(Carseats)
```

```
##      Sales          CompPrice       Income        Advertising
##  Min.   : 0.000   Min.   : 77   Min.   : 21.00   Min.   : 0.000
##  1st Qu.: 5.390   1st Qu.:115   1st Qu.: 42.75   1st Qu.: 0.000
##  Median : 7.490   Median :125   Median : 69.00   Median : 5.000
##  Mean   : 7.496   Mean   :125   Mean   : 68.66   Mean   : 6.635
##  3rd Qu.: 9.320   3rd Qu.:135   3rd Qu.: 91.00   3rd Qu.:12.000
##  Max.   :16.270   Max.   :175   Max.   :120.00   Max.   :29.000
##    Population        Price        ShelveLoc        Age
##  Min.   : 10.0   Min.   : 24.0   Bad   : 96   Min.   :25.00
##  1st Qu.:139.0   1st Qu.:100.0   Good  : 85   1st Qu.:39.75
##  Median :272.0   Median :117.0   Medium:219   Median :54.50
##  Mean   :264.8   Mean   :115.8                Mean   :53.32
##  3rd Qu.:398.5   3rd Qu.:131.0                3rd Qu.:66.00
##  Max.   :509.0   Max.   :191.0                Max.   :80.00
##    Education    Urban       US
##  Min.   :10.0   No :118   No :142
##  1st Qu.:12.0   Yes:282   Yes:258
##  Median :14.0
##  Mean   :13.9
##  3rd Qu.:16.0
##  Max.   :18.0
```

Run a model regressing all variables as well as two new interactive variables resulting form combining `Income:Adversting` and `Age:Price`.

```r
Carseats_lm <- lm(Sales ~. + Income:Advertising + Age:Price, data = Carseats)
summary(Carseats_lm)
```

```
##
## Call:
## lm(formula = Sales ~ . + Income:Advertising + Age:Price, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9208 -0.7503  0.0177  0.6754  3.3413
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)         6.5755654  1.0087470   6.519 2.22e-10 ***
## CompPrice           0.0929371  0.0041183  22.567  < 2e-16 ***
## Income              0.0108940  0.0026044   4.183 3.57e-05 ***
## Advertising         0.0702462  0.0226091   3.107 0.002030 **
## Population          0.0001592  0.0003679   0.433 0.665330
```

```
## Price               -0.1008064  0.0074399 -13.549  < 2e-16 ***
## ShelveLocGood        4.8486762  0.1528378  31.724  < 2e-16 ***
## ShelveLocMedium      1.9532620  0.1257682  15.531  < 2e-16 ***
## Age                 -0.0579466  0.0159506  -3.633 0.000318 ***
## Education           -0.0208525  0.0196131  -1.063 0.288361
## UrbanYes             0.1401597  0.1124019   1.247 0.213171
## USYes               -0.1575571  0.1489234  -1.058 0.290729
## Income:Advertising   0.0007510  0.0002784   2.698 0.007290 **
## Price:Age            0.0001068  0.0001333   0.801 0.423812
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.011 on 386 degrees of freedom
## Multiple R-squared:  0.8761, Adjusted R-squared:  0.8719
## F-statistic:   210 on 13 and 386 DF,  p-value: < 2.2e-16
```

Call the `contrasts` function on the `ShelveLoc` variable to display a table of levels corresponding to the quality of the shelving location for the car seats at each site.

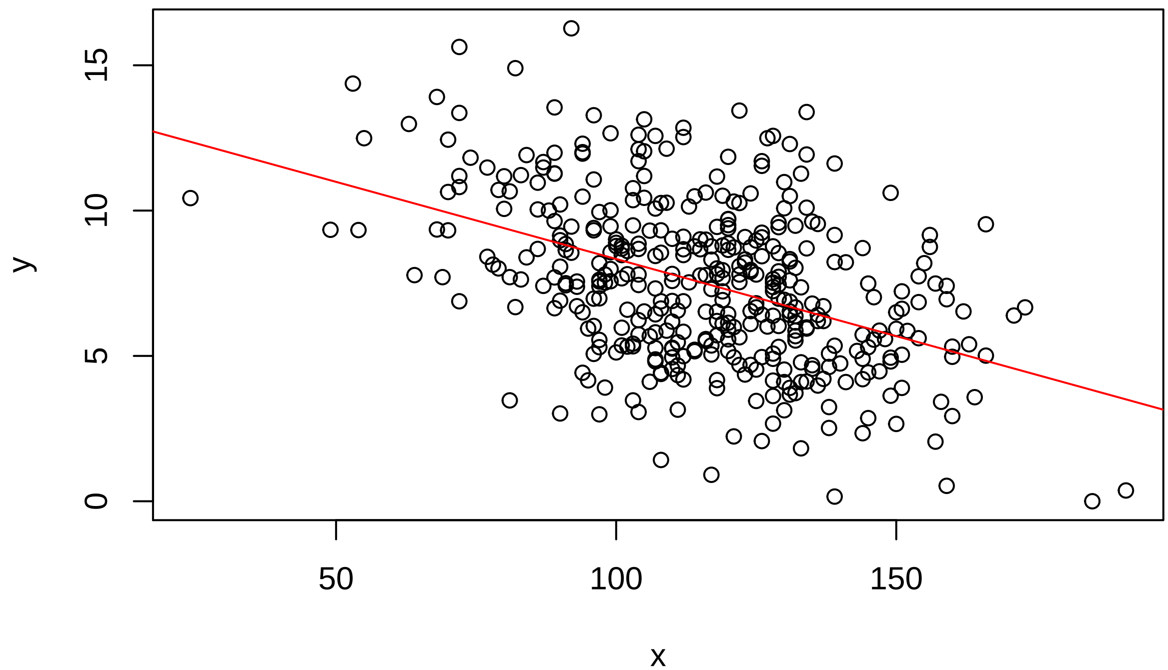
```
contrasts(Carseats$ShelveLoc)
```

```
##        Good Medium
## Bad       0      0
## Good      1      0
## Medium    0      1
```

Brief section writing R functions

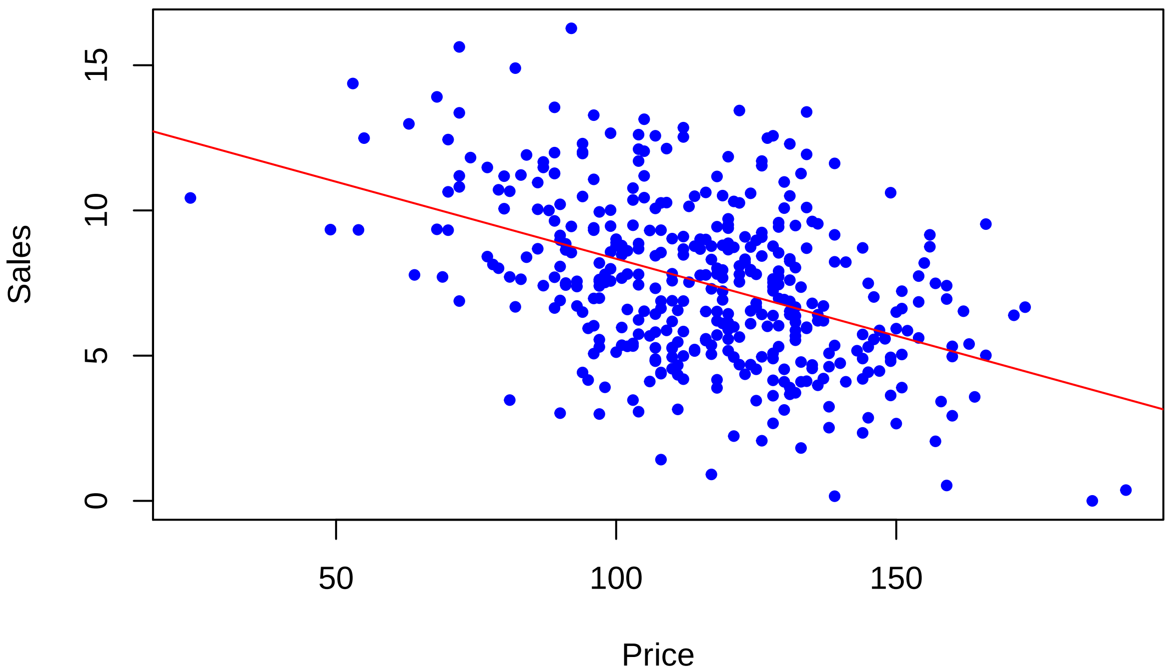
Function creating a plot which displays linear model regressin line.

```
regplot <- function(x, y){
          fit <- lm(y ~ x)
          plot(x, y)
          abline(fit, col="red")
}

attach(Carseats)
regplot(Price, Sales)
```



This time, add the ... argument to the function, which allowing one to pass arguements to functions within the function.

```
regplot <- function(x, y, ...){
          fit <- lm(y ~ x)
          plot(x, y, ...)
          abline(fit, col="red")
}

regplot(Price, Sales, xlab="Price", ylab="Sales", col="blue", pch=20)
```


Sales
Price
0
5
10
15
50
100
150