# Solution 12

## 4.4

06. a)

$$\begin{aligned}17 &= 8 \cdot 2 + 1\\ 2 &= 2 \cdot 1\\ 1 &= 17 - 8 \cdot 2\end{aligned}$$

So, $\overline{a} = -8 \pmod{17} = 9$

b)

$$\begin{aligned}89 &= 2 \cdot 34 + 21\\ 34 &= 1 \cdot 21 + 13\\ 21 &= 1 \cdot 13 + 8\\ 13 &= 1 \cdot 8 + 5\\ 8 &= 1 \cdot 5 + 3\\ 5 &= 1 \cdot 3 + 2\\ 3 &= 1 \cdot 2 + 1\end{aligned}$$

$$\begin{aligned}1 &= 3 - 2\\ &= 3 - (5 - 3) = 2 \cdot 3 - 5\\ &= 2 \cdot (8 - 5) - 5 = 2 \cdot 8 - 3 \cdot 5\\ &= 2 \cdot 8 - 3 \cdot (13 - 8) = 5 \cdot 8 - 3 \cdot 13\\ &= 5 \cdot (21 - 13) - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13\\ &= 5 \cdot 21 - 8 \cdot (34 - 21) = 13 \cdot 21 - 8 \cdot 34\\ &= 13 \cdot (89 - 2 \cdot 34) - 8 \cdot 34 = 13 \cdot 89 - 34 \cdot 34\end{aligned}$$

So, $\overline{a} = -34 \pmod{89} = 55$

c)

$$
\begin{aligned}
233 &= 1 \cdot 144 + 89 \\
144 &= 1 \cdot 89 + 55 \\
89 &= 1 \cdot 55 + 34 \\
55 &= 1 \cdot 34 + 21 \\
34 &= 1 \cdot 21 + 13 \\
21 &= 1 \cdot 13 + 8 \\
13 &= 1 \cdot 8 + 5 \\
8 &= 1 \cdot 5 + 3 \\
5 &= 1 \cdot 3 + 2 \\
3 &= 1 \cdot 2 + 1
\end{aligned}
$$

$$
\begin{aligned}
1 &= 3 - 2 \\
&= 3 - (5 - 3) = 2 \cdot 3 - 5 \\
&= 2 \cdot (8 - 5) - 5 = 2 \cdot 8 - 3 \cdot 5 \\
&= 2 \cdot 8 - 3 \cdot (13 - 8) = 5 \cdot 8 - 3 \cdot 13 \\
&= 5 \cdot (21 - 13) - 3 \cdot 13 = 5 \cdot 21 - 8 \cdot 13 \\
&= 5 \cdot 21 - 8 \cdot (34 - 21) = 13 \cdot 21 - 8 \cdot 34 \\
&= 13 \cdot (55 - 34) - 8 \cdot 34 = 13 \cdot 55 - 21 \cdot 34 \\
&= 13 \cdot 55 - 21 \cdot (89 - 55) = 34 \cdot 55 - 21 \cdot 89 \\
&= 34 \cdot (144 - 89) - 21 \cdot 89 = 34 \cdot 144 - 55 \cdot 89 \\
&= 34 \cdot 144 - 55 \cdot (233 - 144) = 89 \cdot 144 - 55 \cdot 233
\end{aligned}
$$

So, $\overline{a} = 89 \pmod{233} = 89$

d)

$$1001 = 5 \cdot 200 + 1$$

$$1 = 1 \cdot 1001 - 5 \cdot 200$$

So, $\overline{a} = -5 \pmod{1001} = 996$

12. a) From 06.($b$), 55 is an inverse of 34 modulo 89.
$x \equiv 77 \cdot 55 \pmod{89} \equiv 52 \pmod{89}$
Check: $34 \cdot 52 = 1768 \equiv 77 \pmod{89}$

b) From 06.($c$), 89 is an inverse of 144 modulo 233.
$x \equiv 4 \cdot 89 \pmod{233} \equiv 123 \pmod{89}$
Check: $144 \cdot 123 = 17712 \equiv 4 \pmod{233}$

c) From 06.($b$), 996 is an inverse of 200 modulo 1001.
$x \equiv 13 \cdot 996 \pmod{1001} \equiv 936 \pmod{89}$
Check: $200 \cdot 936 = 187200 \equiv 13 \pmod{1001}$

# 4.5

02. a) $104578690 \pmod{101} = 58$

b) $432222187 \pmod{101} = 60$

c) $372201919 \pmod{101} = 52$

c) $501338753 \pmod{101} = 3$

06.

$$\begin{aligned} x_0 &= 3 \\ x_1 &= (4 \cdot 3 + 1) \mod 7 = 6 \\ x_2 &= (4 \cdot 6 + 1) \mod 7 = 4 \\ x_3 &= (4 \cdot 4 + 1) \mod 7 = 3 = x_0 \end{aligned}$$

So, the sequence is $3, 6, 4, 3, 6, 4, 3, 6, 4 \ldots$

# 4.6

02. STOP POLLUTION $\rightarrow 18, 19, 14, 15, 15, 14, 11, 11, 20, 19, 8, 14, 13$

a) $22, 23, 18, 19, 19, 18, 15, 15, 24, 23, 12, 18, 17 \rightarrow$ WXST TSPPYXMSR

b) $13, 14, 9, 10, 10, 9, 6, 6, 15, 14, 3, 9, 8 \rightarrow$ NOJK KJGGPODJI

c) $16, 7, 0, 17, 17, 0, 1, 1, 24, 7, 2, 0, 9 \rightarrow$ QHAR RABBYHCAJ

# 5.1

32. If $n = 0$, $3|0$
Suppose $3|(n^3 + 2n)$, then
$(n+1)^3 + 2(n+1) = (n^3 + 2n) + 3(n^2 + n + 1) \rightarrow 3|(n+1)^3 + 2(n+1)$
So,the statement is true.

44. If $n = 1$, the statement is true.

If $n = 2$£¬ $(A_1 - B) \cap (A_2 - B) = (A_1 \cap \overline{B}) \cap (A_2 \cap \overline{B}) = (A_1 \cap A_2) - B$ Suppose $(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) = (A_1 \cap A_2 \cap \cdots \cap A_n) - B$, then

$(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_{n+1} - B) = ((A_1 \cap A_2 \cap \cdots \cap A_n) - B) \cap (A_{n+1} - B)$

$= (A_1 \cap A_2 \cap \cdots \cap A_n) - B$

54. If $n = 1$, $1|2$, then the statement is true. Suppose $n = k$, the statement is true and the set is $A_k$, then

$n = k + 1$, we must choose $k$ integers form $A_{k+1}$ to become $A_k$, and $A_k \subseteq A_{k+1}$

According to suppose, $A_k$ is true. So $A_{k+1}$ is true.

# 5.2

04. a) $18 = 1 \cdot 4 + 2 \cdot 7 \to P(18) = true$

$19 = 3 \cdot 4 + 1 \cdot 7 \to P(19) = true$

$20 = 5 \cdot 4 \to P(20) = true$

$21 = 3 \cdot 7 \to P(21) = true$

b) Using 4-cent and 7-cent stamps, we can form $j$ cents postage for all $j$ with $18 \leqslant j \leqslant k$ , $k \geqslant 21$.

c) Using 4-cent and 7-cent stamps, we can form $k + 1$ cents.

d) If $P(k - 3) = true$, then $P(k) = true$, for all $k \geqslant 21$.

d) By the principle of strong induction, the statement is true for every integer $n \geqslant 18$.

26. a) $0, 2, 4, 6, 8, 10, \ldots$

b) $0, 3, 6, 9, 12, 15, \ldots$

c) All nonnegative integers $n$.

d) 0 and all nonnegative integers $n >= 2$