Solution 9

2.3

08. a) $\lfloor 1.1 \rfloor = 1$

    b) $\lceil 1.1 \rceil = 2$

    c) $\lfloor -0.1 \rfloor = -1$

    d) $\lceil -0.1 \rceil = 0$

    e) $\lceil 2.99 \rceil = 3$

    f) $\lceil -2.99 \rceil = -2$

    g) $\left\lfloor \frac{1}{2} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor = \left\lfloor \frac{1}{2} + 1 \right\rfloor = 1$

    h) $\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{1}{2} \right\rceil = \left\lceil 1 + \frac{1}{2} \right\rceil = 2$

20. a) $f(x) = x + 1$

    b) $f(x) = \left\lceil \frac{x}{2} \right\rceil$

    c)

$$f(x) = \begin{cases} x+1 & , x \ is \ odd \\ x-1 & , x \ is \ even \end{cases}$$

    d) $f(x) = 1$

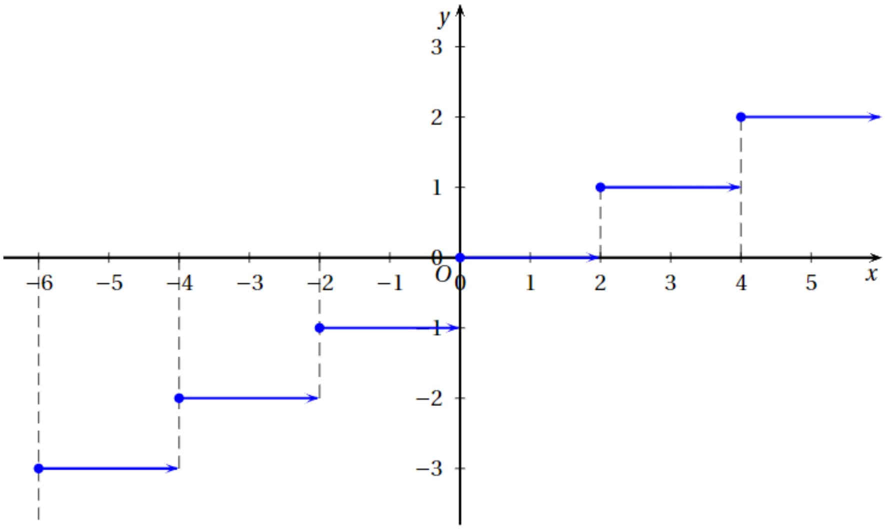
24. First.
    If $f(x)$ is strictly increasing, then $f(a) < f(b)$ when $a < b$.
    Because $g(x) = \frac{1}{f(x)}$, therefore $g(a) = \frac{1}{f(a)} > \frac{1}{f(b)} = g(b)$.
    So, $g(x)$ is strictly decreasing.

    Second.
    If $g(x)$ is strictly decreasing, then $g(a) > g(b)$ when $a < b$.
    Because $f(x) = \frac{1}{g(x)}$, therefore $f(a) = \frac{1}{g(a)} < \frac{1}{g(b)} = f(b)$.
    So, $f(x)$ is strictly increasing.

33. a) Let $x, y \in A$, and $x \neq y$.
    Because g is one-to-one, then $g(x), g(y) \in B$,and $g(x) \neq g(y)$.
    Because f is one-to-one, then $f \circ g(x) = f(g(x)) \neq f(g(y)) = f \circ g(y)$.
    So, $f \circ g(x)$ is also one-to-one.

    b) Let $\forall y \in C$.
    Because f is onto, then $\exists x \in B, f(x) = y$.
    Because g is onto, then $\exists a \in A, g(a) = x$.
    Therefore, $y = f(g(a)) = (f \circ g)(a)$.
    So, $f \circ g(x)$ is also onto.

66.



2.4

16. a) $a_n = -a_{n-1} = (-1)^2 a_{n-2} = \cdots = (-1)^n a_0 = (-1)^n \cdot 5$

b)

$$\begin{aligned} a_n &= a_{n-1} + 3 \\ &= a_{n-2} + 3 \times 2 \\ &= a_{n-3} + 3 \times 3 \\ &\;\;\vdots \\ &= a_0 + 3 \cdot n \\ &= 3n + 1 \end{aligned}$$

c)

$$\begin{aligned} a_n &= a_{n-1} - n \\ &= a_{n-2} - [n + (n-1)] \\ &= a_{n-3} - [n + (n-1) + (n-2)] \\ &\;\;\vdots \\ &= a_0 - [n + (n-1) + \cdots + 2 + 1] \\ &= 4 - \frac{n(n+1)}{2} \end{aligned}$$

d)

$$
\begin{aligned}
a_n &= 2a_{n-1} - 3 \\
&= 4a_{n-2} - 3 - 6 \\
&= 8a_{n-3} - 3 - 6 - 12 \\
&\ \ \vdots \\
&= 2^n a_0 - 3(1 + 2 + 4 + \cdots + 2^{n-1}) \\
&= -2^n - 3(2^n - 1) \\
&= -2^{n+2} + 3
\end{aligned}
$$

e)

$$
\begin{aligned}
a_n &= (n+1)a_{n-1} \\
&= (n+1)na_{n-2} \\
&= (n+1)n(n-1)a_{n-3} \\
&\ \ \vdots \\
&= (n+1)n(n-1)\cdots[n-(n-2)]a_0 \\
&= (n+1)n(n-1)\cdots 2 \cdot a_0 \\
&= 2(n+1)!
\end{aligned}
$$

f)

$$
\begin{aligned}
a_n &= 2na_{n-1} \\
&= 2^2 \cdot n(n-1)a_{n-2} \\
&= 2^3 \cdot n(n-1)(n-2)a_{n-3} \\
&\ \ \vdots \\
&= 2^n \cdot n(n-1)(n-2)\cdots[n-(n-1)]a_0 \\
&= 2^n \cdot n(n-1)(n-2)\cdots 1 \cdot 3 \\
&= 3 \cdot 2^n n!
\end{aligned}
$$

g)

$$
\begin{aligned}
a_n &= n - 1 - a_{n-1} \\
&= (n-1) - (n-2) + a_{n-2} \\
&= (n-1) - (n-2) + (n-3) - a_{n-3} \\
&\ \ \vdots \\
&= (n-1) - (n-2) + \cdots + (-1)^{n-1}(n-n) + (-1)^n a_0 \\
&= \frac{2n - 1 + (-1)^n}{4} + (-1)^n \cdot 7
\end{aligned}
$$

26. a) $f(n) = n^2 + 2$. The next three terms are $123, 146, 171$.

b) $f(n) = 4n + 3$. The next three terms are $47, 51, 55$.

c) $f(n)$ is the binary number of n. The next three terms are $1100, 1101, 1110$.

d) The first two number are 1 and 2 and each subsequent value is the sum of the previous two values. And first subsequent's length is 1 and the next subsequent's length is increase 2. The next three terms are $8, 8, 8$.

e) $f(n) = 3^n - 1$. The next three terms are $59048, 177146, 531440$.

f) $f(n) = (2n-1)!!$. The next three terms are $654729075, 13749310575, 316234143225$.

g) The first is one 1, then two 0, three 1, four 0... The next three terms are $0, 0, 0$.

h) $f(n) = f(n-1)^2, \quad f(1) = 2$. The next three terms are 18446744073709551616 $340282366920938463463374607431768211456$, $115792089237316195423570985008687907853269984665640564039457584007913129639936$.

30. a) $\sum_{j \in S} j = 1 + 3 + 5 + 7 = 16$

b) $\sum_{j \in S} j^2 = 1 + 9 + 25 + 49 = 84$

c) $\sum_{j \in S} \frac{1}{j} = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{176}{105}$

d) $\sum_{j \in S} 1 = 1 + 1 + 1 + 1 = 4$

34.

a) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i-j) = (1-1)+(1-2)+(1-3)+(2-1)+(2-2)+(2-3)+(3-1)+(3-2)+(3-3 = 3$

b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (i-j) = (0+0)+(0+2)+(0+4)+(3+0)+(3+2)+(3+4)+(6+0)+(6+2)+(6+4)+(9+0)+(9+2)+(9+4) = 78$

c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j = 0+1+2+0+1+2+0+1+2 = 9$

d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3 = (0+0+0+0)+(0+1+8+27)+(0+4+32+108) = 180$

2.5

02. a) The set is countably infinite. $n \leftrightarrow (n+10)$

b) The set is countably infinite. $n \leftrightarrow -(2n-1)$

c) The set is finite. $\{-999999, -999998, \ldots, -1, 0, 1, \ldots, 999998, 999999\}$

d) The set is uncountable.

e) The set is countably infinite. $1 \leftrightarrow (2,1), 2 \leftrightarrow (3,1), 3 \leftrightarrow (2,2), 4 \leftrightarrow (3,2) \ldots$

f) The set is countably infinite. $1 \leftrightarrow 0, 2 \leftrightarrow 10, 3 \leftrightarrow -10, 4 \leftrightarrow 20 \quad n \leftrightarrow (-1)^n \left\lfloor \frac{n}{2} \right\rfloor \cdot 10$

10. a) A and B are all the set of real numbers;
then $A - B = \varnothing$, is finite.

b) A is the set of real numbers; B is the set of real numbers but not consist the integers;
then $A - B = \mathbf{Z}$, is countably infinite.

c) A is the set of real numbers; B is the set of positive real numbers;
then $A - B$ is the set of negative real numbers, is uncountable.

2.6

04. a)

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

b)

$$AB = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$$

c)

$$AB = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$$

20. a)

$$A^{-1} = \begin{bmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

b)

$$A^3 = A^2 A = \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$$

c)

$$(A^{-1})^3 = (A^{-1})^2 A^{-1} = \begin{bmatrix} \frac{11}{25} & -\frac{4}{25} \\ -\frac{2}{25} & \frac{3}{25} \end{bmatrix} \begin{bmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} -\frac{37}{125} & \frac{18}{125} \\ \frac{9}{125} & -\frac{1}{125} \end{bmatrix}$$

d)

$$(A^3)^{-1} = \begin{bmatrix} -\frac{37}{125} & \frac{18}{125} \\ \frac{9}{125} & -\frac{1}{125} \end{bmatrix} = (A^{-1})^3$$

28.

$$AB = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

32. a)

$$\begin{aligned}(A \vee B) \vee C &= [(a_{ij} \vee b_{ij}) \vee c_{ij}] \\ &= [a_{ij} \vee (b_{ij} \vee c_{ij})] \\ &= A \vee (B \vee C)\end{aligned}$$

b)

$$\begin{aligned}(A \wedge B) \wedge C &= [(a_{ij} \wedge b_{ij}) \wedge c_{ij}] \\ &= [a_{ij} \wedge (b_{ij} \wedge c_{ij})] \\ &= A \wedge (B \wedge C)\end{aligned}$$