

Solution 5

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1.5

10. a) $\forall x F(x, Fred)$
b) $\forall x F(Evelyn, x)$
c) $\forall x \exists y F(x, y)$
d) $\neg \exists x \forall y F(x, y)$
e) $\exists x \forall y F(x, y)$
f) $\neg \exists x (F(x, Fred) \wedge F(x, Jerry))$
g) $\exists x \exists y (F(Nancy, x) \wedge F(Nancy, y) \wedge \forall z F(Nancy, z) \rightarrow (z = x \vee z = y) \wedge x \neq y)$
h) $\exists x (\forall y F(y, x) \wedge \forall y (\forall z F(z, y) \rightarrow y = x))$
i) $\neg \exists x F(x, x)$
j) $\exists x \exists y (F(x, y) \wedge x \neq y \wedge \forall z ((F(x, z) \wedge x \neq z) \rightarrow z = y))$
28. a) *true* ($y = x^2$)
b) *false* ($x < 0$)
c) *true* ($x \equiv 0$)
d) *false*
e) *true* ($y = \frac{1}{x}$)
f) *false* (*only* $y = \frac{1}{x}$)
g) *true* ($y = 1 - x$)
h) *false* ($x + 2y = 2 \wedge x + 2y = 2.5$)
i) *false* (*only* $(2 - x = 2x - 1 \rightarrow x = 1)$)
j) *true* ($z = (x + y)/2$)
34. a) $x = 0 \quad y = 1 \quad z = 1$
b) $x = y = 0 \quad z = 1$
46. a) *false* let $y = \frac{\sqrt{x}}{3}$ then $x = 9y^2 > y^2$
b) *true* let $x < 0 \quad \forall y (y^2 \geq 0)$

c) *true* let $x < 0 \ \forall y(y^2 \geq 0)$

50. a) $\exists x(P(x) \vee Q(x) \vee A)$

b) $\exists x \exists y \neg(P(x) \vee Q(y))$

c) $\forall x \exists y(\neg P(x) \vee Q(y))$

(1.)

Step	Reason
1. $p \rightarrow (q \rightarrow r)$	Premise #1
2. p	Premise #2
3. $q \rightarrow r$	Modus ponens using 1,2
4. q	Premise #3
5. r	Modus ponens using 3,4
6. $r \vee s$	Addition of 5

(2.)

Step	Reason
1. $\neg(q \wedge r)$	Premise #2
2. $\neg q \vee \neg r$	De Morgan's laws of 1
3. r	Premise #3
4. $\neg q$	Disjunctive syllogism using 2,3
5. $p \rightarrow q$	Premise #1
6. $\neg p$	Modus tollens using 4,5