Solution 13

5.3

20. $max(a_1, a_2) = a_1$ if $a_1 \geqslant a_2$ and a_2 if $a_1 < a_2$
    $min(a_1, a_2) = a_2$ if $a_1 \geqslant a_2$ and a_1 if $a_1 < a_2$
    when $n \geqslant 2$,
    $max(a_1, a_2, \ldots, a_{n+1}) = max(max(a_1, a_2, \ldots, a_n), a_{n+1})$
    $min(a_1, a_2, \ldots, a_{n+1}) = min(min(a_1, a_2, \ldots, a_n), a_{n+1})$

34. a) $ones(0) = 0, ones(1) = 1$
       $ones(1s) = ones(s) + 1, ones(0s) = ones(s)$

    b) **Base step:**
       $ones(00) = ones(0) + ones(0)$
       $ones(01) = ones(0) + ones(1)$
       $ones(10) = ones(1) + ones(0)$
       $ones(11) = ones(1) + ones(1)$
       $ones(st) = ones(s) + ones(t)$ is true for base step.
       Recursive step:
       $ones(1s) = ones(s) + 1 = ones(1) + ones(s)$
       $ones(0s) = ones(s) + 0 = ones(0) + ones(s)$
       Hence, proved.

40. Suppose the set P is the set of all palindromes.
    Base step:
    λ is empty string, $\lambda \in P$
    x is a alpha, $x \in P$
    Recursive step:
    $xsx \in P$, if x is a alpha and $s \in P$
    Hence, proved.

50. a) $A(1, 0) = 0$

    b) $A(0, 1) = 2$

    c) $A(1, 1) = 2$

    d) $A(2, 2) = A(1, A(2, 1)) = A(1, 2) = A(0, A(1, 1)) = A(0, 2) = 4$

6.1

48. a) $6 \cdot 6 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 90720$

    b) $6 \cdot 5 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 50400$

c) From a) and b), exactly one of the bride is $90720 - 50400 = 40320$
So the answer is $40320 \cdot 2 = 80640$

52. a) 0000011111 1
b) 1111100000 1
c) $000001xxxx$ $(2^4 - 1) \cdot 2 = 30$
d) $x000001xxx$ $2^4 \cdot 2 = 32$
e) $xx000001xx$ $2^4 \cdot 2 = 32$
f) $xxx000001x$ $2^4 \cdot 2 = 32$
g) $xxxx000001$ $2^4 \cdot 2 = 32$
h) $xxxxx00000$ $(2^5 - 1) \cdot 2 = 62$

The answer is $1 + 1 + 30 + 32 + 32 + 32 + 32 + 62 = 222$

6.2

14. There are 25 results after moudle 5, that $(0,0), (0,1) \dots (4,3), (4,4)$. So the answer is $25 + 1 = 26$.

28. Suppose the friends are (a,b),(b,c),(c,d),(d,e),(e,a)
and the enemies are (a,c),(a,d),(b,d),(b,e),(c,e).
In this case, there are no three mutual friends or three mutual enemies.

48. Suppose the statement is wrong, so the ith box contains at most $n_i - 1$ objects.
Because $n_1 + n_2 + \cdots + n_t - t = (n_1 - 1) + (n_2 - 1) + \cdots + (n_t + 1)$, at this time,
$n_1 + n_2 + \cdots + n_t - t + 1 = (n_1 - 1) + (n_2 - 1) + \cdots + (n_t + 1) + 1$.
There must be a ith box contained n_i objects.
So the statement is true.

6.3

30. $C(40, 17) = 88732378800$

46. a) No ties. $P(4,4) = 24$
b) Two horses tie. $C(4,2) \cdot P(3,3) = 6 \cdot 6 = 36$
c) Three horses tie. $C(4,3) \cdot P(2,2) = 4 \cdot 2 = 8$
d) Four horses tie. $C(4,4) = 1$
e) Two groups tie. $C(4,2) = 6$

The answer is $24 + 36 + 8 + 1 + 6 = 75$