Solution 6

(1)

Step	Reason
1. $\exists x F(x)$	Premise #2
2. $\exists x F(x) \rightarrow \forall y((F(y) \vee G(y)) \rightarrow R(y))$	Premise #1
3. $\forall y((F(y) \vee G(y)) \rightarrow R(y))$	Modus ponens using 1,2
4. $F(c)$	1.EI
5. $F(c) \vee G(c)$	Addition of 4.
6. $(F(c) \vee G(c)) \rightarrow R(c)$	3.UI
7. $R(c)$	Modus ponens using 5,6
8. $\exists x R(x)$	7.EG

(2)

Step	Reason
1. $\forall x(F(x) \vee G(x))$	Premise #1
2. $F(c) \vee G(c)$	1.UI
3. $\forall x(\neg R(x) \vee \neg G(x))$	Premise #2
4. $\neg R(c) \vee \neg G(c)$	3.UI
5. $R(c) \rightarrow \neg G(c)$	Equive of 4.
6. $\forall x R(x)$	Premise #3
7. $R(c)$	6.UI
8. $\neg G(c)$	Modus ponens using 6,7
9. $\neg G(c) \rightarrow F(c)$	Equive of 2.
10. $F(c)$	Modus ponens using 8,9
11. $\exists x F(x)$	10.EG

(3) P(x):Patient x D(x):Doctor x L(x):Liar x T(x,y):x trust y

$\exists x(P(x) \wedge \forall y D(y) \rightarrow T(x,y)), \quad \forall x \forall y((P(x) \wedge L(y)) \rightarrow \neg T(x,y))$

$\Rightarrow \forall x(D(x) \rightarrow \neg L(x))$

Step	Reason
1. $\exists x(P(x) \wedge \forall y D(y) \rightarrow T(x,y))$	Premise #1
2. $\exists x \forall y(P(x) \wedge (\neg D(y) \vee T(x,y)))$	Equive of 1.
3. $P(c) \wedge (\neg D(a) \vee T(c,a))$	2.EI UI
4. $P(c)$	Simplification of 3
5. $\neg D(y) \vee T(c,y)$	Simplification of 3
6. $\forall x \forall y((P(x) \wedge L(y)) \rightarrow \neg T(x,y))$	Premise #2
7. $\neg P(x) \vee \neg L(y) \vee \neg T(x,y)$	Equive of 6.
8. $\neg L(y) \vee \neg D(y)$	7.UI and Resolution using 4,5,7
9. $D(y) \rightarrow \neg L(y)$	Equive of 8.
10. $\forall x(D(x) \rightarrow \neg L(x))$	9.UG

1.6

04. a) Rule of simplification
    b) Rule of disjunctive syllogism
    c) Rule of modus ponens
    d) Rule of addition
    e) Rule of hypothetical syllogism

06. r: It rains f: It's foggy
p:The sailing race will be held
q:The life saving demonstration will go on
s:The trophy will be awarded
$\neg r \vee \neg f \rightarrow (p \wedge q),\ p \rightarrow s,\ \neg s \Rightarrow r$

Step	Reason
1. $\neg s$	Premise #3
2. $p \rightarrow s$	Premise #2
3. $\neg p$	Modus tollen using 1,2
4. $\neg p \vee \neg q$	Addition of 3.
5. $\neg r \vee \neg f \rightarrow (p \wedge q)$	Premise #1
6. $(\neg p \vee \neg q) \rightarrow (r \wedge f)$	Contrapositive of 5.
7. $r \wedge f$	Modus ponens using 4,6
8. r	Simplification of 7.

12.

Step	Reason
1. q	Premise #5
2. $q \to (u \wedge t)$	Premise #2
3. $u \wedge t$	Modus ponens using 1,2
4. u	Simplification of 3.
5. $u \to p$	Premise #3
6. p	Modus ponens using 4,5
7. t	Simplification of 3.
8. $p \wedge t$	Conjunction using 6,7
9. $(p \wedge t) \to (r \vee s)$	Premise #1
10. $r \vee s$	Modus ponens using 8,9
11. $\neg s$	Premise #4
12. r	Disjunctive syllogism using 10,11

16. a) Correct. Rule of modus tollens.

    b) Not correct.

    c) Not Correct.

    d) Correct. Rule of modus ponens.

20. a) Invalid. If $a < 0$, then a^2 is a positive real number, but a is a negative real number.

    b) Valid. Rule of modus ponens.

24.

Step 3. Rule of simplification is used to conjunction.

Step 5. Rule of simplification is used to conjunction.