Solution 8

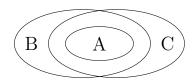
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2.1

08. $B \subseteq A$ $C \subseteq A$ $C \subseteq D$

- 12. a) $\emptyset \in \{\emptyset\}$ true
 - b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ true
 - c) $\{\emptyset\} \in \{\emptyset\}$ false
 - d) $\{\emptyset\} \in \{\{\emptyset\}\}\$ true
 - e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}\$ true
 - f) $\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}\$ true
 - g) $\{\{\emptyset\}\}\subset\{\{\emptyset\},\{\emptyset\}\}\}$ false



18.

- 24. Contrapositive: if $A \neq B$, then A and B are two sets with not same power set. let P(X) means the power set of X.
 - $\therefore A \subseteq P(A) \text{ and } B \subseteq P(B)$
 - $\therefore P(A) \neq P(B)$

So, A = B, if A and B are two sets with the same power set.

- 34. a) $\{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$
 - b) $\{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$
 - c) $\{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y)$ $(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$
 - d) $\{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y), \}$
- 48. a) $\{1, 2, 3, 4, \dots\}$
 - b) ∅
 - c) $\{\ldots, -3, -2, -1, 2, 3, 4, \ldots\}$

04. Find $A \subseteq B$

a)
$$A \cup B = B = \{a, b, c, d, e, f, g, h\}$$

b)
$$A \cap B = A = \{a, b, c, d, e\}$$

c)
$$A - B = \emptyset$$

d)
$$B - A = \{f, g, h\}$$

21. a) Proof is as follows.

A	B	A - B	\overline{B}	$A \cap \overline{B}$
0	0	0	1	0
0	1	0	0	0
1	0	1	1	1
1	1	0	0	0

b) Proof is as follows.

A	B	$A \cap B$	\overline{B}	$A \cap \overline{B}$	$(A \cap B) \cup (A \cap \overline{B})$
0	0	0	1	0	0
0	1	0	0	0	0
1	0	0	1	1	1
1	1	1	0	0	1

32. a) No. $A = \{1\}, B = \{1, 2\}, C = \{1, 2, 3\}$ $A \cup C = B \cup C = C$ but $A \neq B$

b) No.
$$A = \emptyset, B = \{1\}, C = \{2, 3\}$$
 $A \cap C = B \cap C = \emptyset$ but $A \neq B$

c) Yes.

Suppose $A \neq B$, let $x_0 \in A, x_0 \notin B$

$$A \cup C = B \cup C$$
 $x_0 \in C$

$$\therefore A \cap C = B \cap C \text{ and } x_0 \in A \text{ and } x_0 \in C$$

$$\therefore x_0 \in A \cap C \Rightarrow x_0 \in B \cap C \Rightarrow x_0 \in B$$

So, the suppose is not right, and the original proposition is right.

54. Find $A_1 \subseteq A_2 \subseteq A_3 \dots$

a)
$$\bigcup_{i=1}^{n} A_i = A_n = \{\dots, -2, -1, 0, 1, \dots, n\}$$

b)
$$\bigcap_{i=1}^{n} A_i = A_1 = \{\dots, -2, -1, 0, 1\}$$

58. a) 00 1110 0000

b) 10 1001 0001

- c) 01 1100 1110
- 62. $(A B) \cup (B A)$ $(x \in A \text{ but } x \notin B) \text{ or } (x \in B \text{ but } x \notin A)$