A robust stamp detection framework on degraded documents

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ABSTRACT

Detecting documents with a certain stamp instance is an effective and reliable way to retrieve documents associated with a specific source. However, this unique problem has essentially remained unaddressed. In this paper, we present a novel stamp detection framework based on parameter estimation of connected edge features. Using robust basic-shape detectors, the approach is effective for stamps with analytically shaped contours, when only limited samples are available. For elliptic/circular stamps, it efficiently exploits the orientation information from pairs of edge points to determine its center position and area, without computing all the five parameters of an ellipse. In our approach, we considered the set of unique characteristics of stamp patterns. Specifically, we introduced effective algorithms to address the problem that stamps often spatially overlay their background contents. These give our approach significant advantages in detection accuracy and computation complexity over traditional Hough transform method in locating candidate ellipse regions. Experimental results on real degraded documents demonstrated the robustness of this retrieval approach on large document database, which consists of both printed text and handwritten notes.

Keywords: stamp/seal detection, ellipse detection, Hough transform, document analysis, pattern retrieval

1. INTRODUCTION

Document stamps, also known as seals in many Oriental cultures, serve different purposes in different settings today. In business environments, they are often used to provide supplemental information (date received/approved, etc). On official documents, they have served over centuries as the authoritative means of demonstrating the authenticity of those documents. This enables us to reliably identify the document sources by detecting instances of their official stamps.

This paper presents a novel stamp detection approach, which has been recently implemented and evaluated. The objective of the project is to retrieve documents associated with a specific source by detecting its official stamps from degraded document images. The system is able to process volumes of scanned binary images with dimensions in the order of 2000 by 2500 pixels in 2-3 seconds average processing time on each image on a Pentium IV PC. The recall-precision graph of our approach on stamp detection is shown in Figure 7.

To our best knowledge, this is the pioneering work in document recognition and retrieval that specifically addresses the problem of stamp detection and retrieval in general, although the task of seal imprint identification on bank checks, envelops, and transaction receipts have emerged from mid-1980s [1-3], based on the different assumption that the stamp/seal regions have either been detected or spatially confined within a fixed bounded area.

2. PROBLEM FORMULATION

A key challenge to stamp detection is to provide a concrete formulation of the problem. Stamps possess a unique set of characteristics that make it difficult to apply common logo detection techniques directly, although they can loosely fit into the category of logos. First, stamps are generally unstable and somehow unpredictable patterns in documents due to imperfect ink condition, surface contact, and noise. Second, they have much lower spatial density compared to logos. Even with well preserved closed contour, stamps appear as weaker regions in a full spectrum of background contents, which may consist of text, images, and even watermarks, as shown in Figure 1. Outlier and occlusion are typical. Last but not least, a stamp instance may land on any position within the source, which requires detection to be carried out on the entire document.

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In our approach, we treat stamps as regions with analytically shaped contours within noisy documents and use strongly connected edges as feature points. From our experiments on heavily degraded documents, it consistently demonstrates that, if extracted carefully, connected edges are a robust feature against noise and other document degradation.


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ЛИЦЕНЗИЯ

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Figure 1: A sample document and its edge pattern, illustrating the diverse background in a stamp region.

Robust analytical-shape detectors in our system are the critical component to achieve high accuracy at reasonable speed. As majority of the retrieved stamp patterns in the project are either elliptic or circular in shape, we will focus on elliptic stamp detection in the following discussions.

In fact, using ellipse detector to handle both elliptic and circular objects has significant practical implications. First, it allows a unified and more general detection approach, in which a circular object is simply a special case. Analytically this may seem redundant for detecting circular objects, as a complete parameterization of an ellipse would require five parameters, namely, the center of the ellipse in the x and y coordinates (x_o, y_o), semimajor axis a, semiminor axis b, and the orientation of the major axis θ. At the same time, three parameters are sufficient to uniquely determine a circle. However, due to processing artifacts introduced incrementally at each stage, especially when stamping direction and surface are not perfectly normal to each other, stamps from a rounded seal appear as ellipses with a small value of eccentricity e (where $e = 0$ is the case of circle). Second, stamps typically exhibit multiple closely spaced concentric traces at their boundaries inherited from the source seal pattern. Parameter estimation using the ellipse model gives natural error allowance even if a set of feature points is chosen out of separate closely spaced concentric traces, which may occur often unintentionally when linking connected edges. We can potentially leverage this multi-concentric-trace pattern of stamps to improve detection accuracy under the ellipse model. In section 4, we will present an efficient ellipse detection method based on the Hough transform scheme, with the exceptional capability that enables us to find the center position (x_o, y_o) and the area of an ellipse, without completely determining all the five parameters.

It is interesting to note that ellipses are important elements in many natural objects in documents, including logos with elliptic contours, human photos and curved signatures. We are currently working on more generalized approach on retrieving and classifying regions of interest automatically based on this work.

3. PROPOSED STAMP DETECTION FRAMEWORK

In this section, we present our proposed stamp detection framework as shown in Figure 2 and highlight the critical considerations on each building block. As our detection approach relies on the connected edge pattern of stamp contour, it is essential that processing at each stage must well preserve edge features.

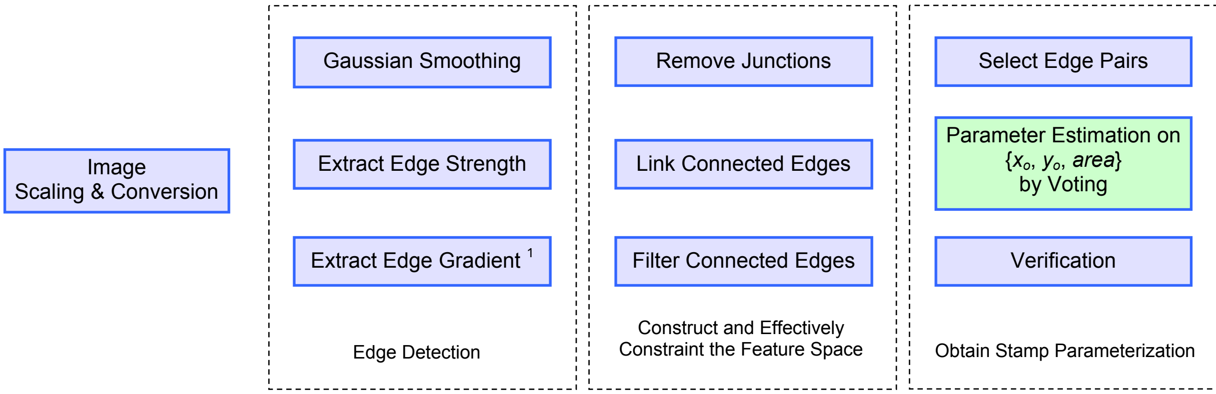

Figure 2: The proposed stamp detection framework.

3.1 Image scaling & conversion

As illustrated in Figure 1, the edge image obtained directly from the original image contains considerable amount of fine details, which makes the processing at later stages rather inefficient, especially during parameter estimation. As edge features are able to preserve reasonably well across scale change, a scaled-down image proves sufficient for detection purpose. A number of factors in image rescaling have considerable impact on the overall detection precision. The image rescaling factor should reasonably reflect the contour width of retrieved stamp patterns. For example, if the contour is typically between 8 to 16 pixels, downscaling image beyond a factor of 8 will naturally eliminate all edge features of a stamp. The quality of image rescaling function is crucial. In the context of stamp detection, a good image resizing function should serve as a FIR high-pass filter that is able to preserve sharpened and crisp edges. In our implementation, we use the Lanczos filter, which is essentially a windowed *sinc* function.

3.2 Edge detection

The purpose of edge detection is to extract the strength and orientation of edge points from the rescaled image, which are used in estimating center position and area of the stamp. Ideally, a good edge detection method should meet the following performance criteria so that we can compute a set of ellipse parameters accurately at later stage:

- **Good detection**. The detection algorithm should be sensitive in marking real edge points, while maintaining a low probability of falsely marking non-edge points. This requirement is essential for getting complete and meaningful connected edges around stamp contour, especially if part of the contour is occluded or appears weak due to imperfect ink condition.
- **Good localization**. The marked edge points should be as close as possible to the location of the true edges.
- **Only one response to a single edge**. Specifically, when there are multiple responses to the same edge, only one of them is marked as edge point. This criterion is critical for legitimate grouping of connected edges as it greatly reduces the possibility of getting excessively connected edges that do not strictly reflect spatial proximities between objects in the source image.

This set of criteria is almost identical to the assumptions from which Canny derived the optimal edge detector [4]. For these reasons, we use the Canny edge detector, which convolves the input image with the first derivative of the two-dimensional Gaussian function, as illustrated by the first pillar in Figure 2. [1]

3.3 Construct and effectively constraint the feature space

A major concern to any unconstraint searching algorithm running on entire images is the computing efficiency. By obtaining legitimate connected edges and estimating their parameters through an efficient voting scheme within each connect component, we effectively constraint the search space. This greatly reduces the computation required. Furthermore, this approach improves the accuracy in parameter estimation as it naturally suppresses unrelated feature points in the set, which is a major issue for any Hough transform scheme based on mapping of set of feature points.

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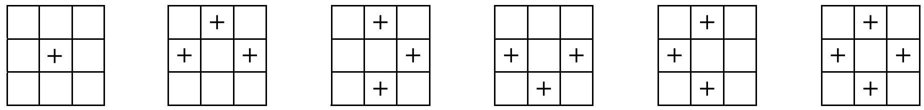
[1] For grayscale images, the edge gradient is given by $\nabla f(x,y) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y}$, where $f(x,y)$ is the grayscale intensity function at edge points. The gradient direction of ∇f is the orientation in which the directional derivative $|\nabla f|$ has the maximum value.



(a) (b) (c) (d) (e) (f)

Figure 3: Identification and removal of junction points in connected edge components.

There exist different approaches for grouping connected components in literature. We consider two feature points to be connected if they are immediate neighbors to each other. In Figure 3(a), if another feature point appears at one of the eight neighbors to the marked pixel, it is connected to the center feature point and subsequently will be added to the same connected component. When linking the connected edges, we also consistently update the key parameters of the connected component, including the dimensions of its bounding box, run length at each row. There features are very effective criteria in filtering out unwanted segments within connected components, as shown in Figure (4).

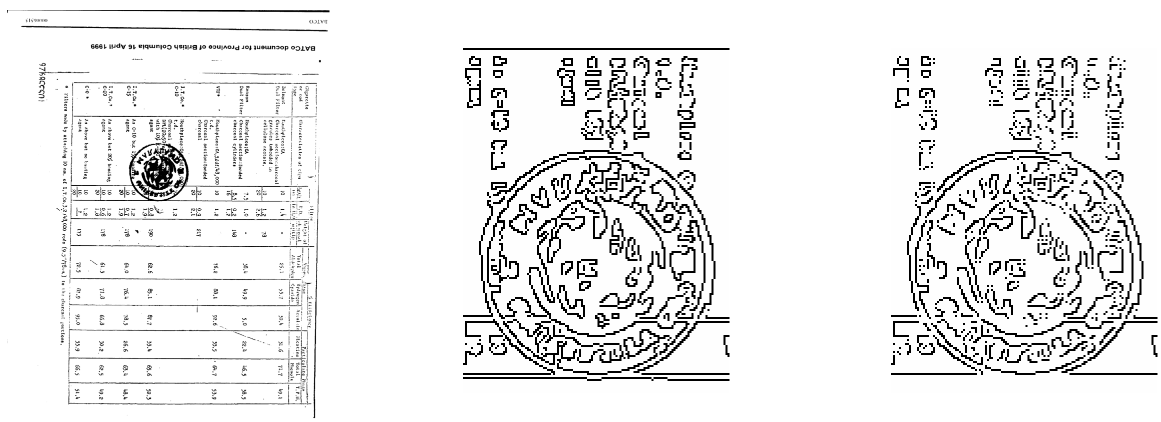


Figure 4: Filtering edge segments and removing junction points effectively suppress unwanted feature points.

This simple definition of connected component gives good result if the stamp instance does not spatially overlay with the content in the background. However, stamps commonly appear amid text, signatures, tables and other objects. In these cases, this method presented above generates bigger connected edges that spans beyond the interested stamp region, introducing additional unwanted feature points into the search space. We introduce the following algorithm to solve this problem. If any of the patterns in Figure 3(b)-(f) appears within a 3 by 3 neighborhood, we consider the center as a junction, irregardless whether it is an edge point. This template-based removal comes from the observation that on a legitimate connected edge component around stamp contour, neighboring edge points always form a trace with smooth variation in orientation. The abrupt orientation change as shown in Figure 3(b)-(f) is the result of local disturbances, which happens at the junctions where a stamp spatially overlaps with its background content, or sometime simply due to imperfect ink condition. This algorithm effectively breaks long connected edges precisely at the junction points, thus effectively ensuring with a high probability, that any two connected edge points are truly from the same object. Although this method generally results in larger number of shorter connected edges, our experiments demonstrate that piecewise segments are sufficient for detection purpose in most cases.

4. ELLIPSE DETECTION METHOD

The core of the proposed stamp detection framework is an efficient and robust ellipse detection approach that is able to compute a set of accurate ellipse parameters from connected edge pattern against document degradations. Ellipse detection has been an active research area in pattern recognition over the last two decades. Traditional ellipse detection algorithms can be broadly divided into two groups. Methods in the first group, including Hough transform (HT) and its variants [5-8], RANSAC [9] and fuzzy logic [10-11], map sets of points from a feature space to the parameter space through a voting/clustering scheme. These methods are typically robust against outlier and occlusion, but are computationally expensive due to the nature of the five-dimensional parameterization. The other group, which includes least-square fitting [12] and genetic algorithm [13], detects ellipse instances by optimizing certain objective functions. These approaches generally require pre-processing such as segmentation and grouping and works better for clean data.

From the application perspective, however, it is sufficient in most cases to know the center position and area of the ellipse. If other unknown parameters of an ellipse are needed, we can always run a robust traditional ellipse detection method within this estimated detected region, and would be able to obtain its complete set of parameters at much lower computing cost with higher accuracy.

This is the motivation for us to find a new ellipse detection approach that adopts the brutal-force voting scheme from the Hough transform paradigm and at the same time, does not require mapping the feature points into parameter space in high dimensions. In this section, we present a single-stage method which detects the center position and area of ellipse instances.

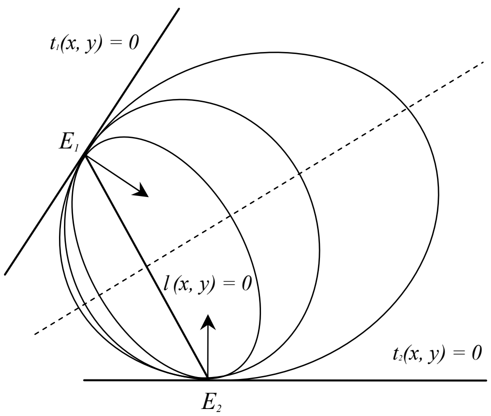


Figure 5: Family of ellipsis inferred by a pair of edge points.

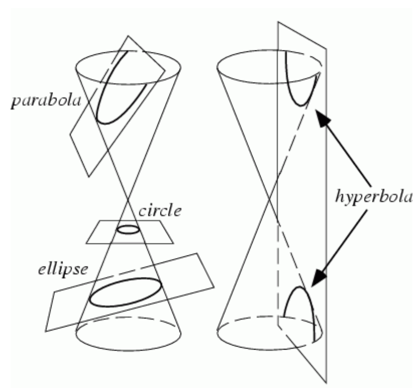


Figure 6: Geometric interpretation based on conic sections.

The key to this approach is to make use of the orientation information from pairs of edges to obtain the general second-order expression for the family of elliptic curves that may possibly pass both edge points. For a pair of edge points E_1 at (x_1, y_1) and E_2 at (x_2, y_2), let the slopes of their associated gradient directions be q_1/p_1 and q_2/p_2, which have been extracted from edge detection. As shown in Figure 5, the two tangent lines at E_1 and E_2 are normal to their edge gradients and can be written as (1) and (2), respectively.

$$t_1(x, y) \equiv p_1(x - x_1) + q_1(y - y_1) = 0 \tag{1}$$

$$t_2(x, y) \equiv p_2(x - x_2) + q_2(y - y_2) = 0 \tag{2}$$

The straight line $l(x, y)$ that passes E_1 and E_2 can be simply written as

$$l(x, y) \equiv \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_1 & 1 \end{vmatrix} = 0 \tag{3}$$

Notice that functions $t_1(x, y)$, $t_2(x, y)$ and $l(x, y)$ are all first-order linear functions in x and y. The whole family of second-order curves that are tangent to $t_1(x, y)$ and $t_2(x, y)$ can be expressed in the canonical form in (4).

$$\begin{aligned} C(x, y) &\equiv l^2(x, y) - \lambda t_1(x, y) t_2(x, y) = 0 \\ &= ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \end{aligned} \tag{4}$$

where parameters a, b, c, f, g, h can all be mapped as first-order linear functions in λ.

$$
\begin{aligned}
a(\lambda) &= (y_1 - y_2)^2 - \lambda p_1 p_2 \\
b(\lambda) &= (x_2 - x_1)^2 - \lambda q_1 q_2 \\
c(\lambda) &= (x_1 y_2 - x_2 y_1)^2 - \lambda (p_1 x_1 + q_1 y_1)(p_2 x_2 + q_2 y_2) \\
f(\lambda) &= (x_1 y_2 - x_2 y_1)(x_2 - x_1) + \frac{\lambda}{2}[q_1(p_2 x_2 + q_2 y_2) + q_2(p_1 x_1 + q_1 y_1)] \\
g(\lambda) &= (x_1 y_2 - x_2 y_1)(y_1 - y_2) + \frac{\lambda}{2}[p_1(p_2 x_2 + q_2 y_2) + p_2(p_1 x_1 + q_1 y_1)] \\
h(\lambda) &= (y_1 - y_2)(x_2 - x_1) - \frac{\lambda}{2}(p_1 q_2 + p_2 q_1)
\end{aligned}
\tag{5}
$$

As the value of λ varies within $0 < \lambda < \infty$, this quadratic function has a nice geometric interpretation that it represents the intersections of the fixed plane determined by $t_1(x, y)$ and $t_2(x, y)$ with one or two nappes of a double cone, as the cone rotates and maintains tangential to both $t_1(x, y)$ and $t_2(x, y)$. As illustrated in Figure 6, when λ gradually increases from 0, the corresponding intersection changes from ellipse to parabola at a specific value $\lambda = \lambda_0$, and further to hyperbola when $\lambda > \lambda_0$. The value of λ_0 was derived by Maxwell [14].

$$
\lambda_0 = \frac{4[p_1(x_2 - x_1) + q_1(y_2 - y_1)][p_2(x_1 - x_2) + q_1(y_1 - y_2)]}{(p_1 q_2 - p_2 q_1)^2} \tag{6}
$$

In fact, for a pair of edge points, any value of λ that satisfies $0 < \lambda < \lambda_0$ corresponds to a uniquely deterministic ellipse which has its center along the dotted line shown in Figure 6. This center position and the area of this ellipse was derived in [15] for a fixed value of λ.

$$
x_o(\lambda) = \frac{h(\lambda) f(\lambda) - b(\lambda) g(\lambda)}{a(\lambda) b(\lambda) - h^2(\lambda)}, \; y_o(\lambda) = \frac{h(\lambda) g(\lambda) - a(\lambda) f(\lambda)}{a(\lambda) b(\lambda) - h^2(\lambda)} \tag{7}
$$

$$
area(\lambda) = \frac{\pi \, |d(\lambda)|}{\sqrt{a(\lambda) b(\lambda) - h^2(\lambda)}}, \text{ where } d(\lambda) = \frac{a(\lambda) f^2(\lambda) + b(\lambda) g^2(\lambda) - 2 f(\lambda) g(\lambda) h(\lambda)}{a(\lambda) b(\lambda) - h^2(\lambda)} - c(\lambda) \tag{8}
$$

Based on this ellipse parameterization, we can construct a Hough transform with the three-dimensional parameter space in (x_0, y_0, area). For any two edges on the same connected edge component (i.e. for a given set of parameters (x_1, y_1), (x_2, y_2), (p_1, q_1), (p_2, q_2)), we exponentially decrease the value of λ from the calculated upper bound of λ_0 in (6) and add a vote to the corresponding three-dimensional accumulator, given by (7) and (8). This ellipse detection approach allows a prior information from stamp samples (such as minimum area, maximum area, maximum eccentricity) to be effectively incorporated so that only legitimate votes are accounted.

The proposed ellipse detection method is orders of magnitude faster than the traditional Hough transforms. Suppose there is a total number E of detected edge pixels used as feature points. Let D be the number of distinct values for each parameter. As the classic Hough transform attempts to obtain the five parameters of an ellipse by mapping a single edge point to the parameter space each time, its running time is in order of $O(ED^3)$, even if orientation information is exploited [16]. Our approach uses pairs of edge points. This gives us four equations, two in the form of ellipse equations evaluated at the two edge points and another two involving the first-order derivatives at the edge points which can be readily obtained at the stage of edge detection. Therefore, the running time of our approach is loosely upper-bounded by $O(E^2 D)$, as the degree of freedom in this problem reduces to one. By linking connected edges and removing junction points, as we discussed in section 3.3, we effectively ensure that each edge pair is taken from a truely connected component of bounded length L, further reducing the E^2 term to EL. The computation of our proposed stamp detection approach is of order $O(ED)$, significantly lower than $O(ED^3)$ required in the classic Hough transform.

5. EXPERIMENTS

The implemented stamp detection system delivered promising performance using the University of Maryland Arabic document databases, which include a balanced collection of degraded printed documents and handwritten notes in dimensions of 2000 by 2500 pixels at 200 by 200 DPI. We groundtruthed two independent databases of binary images for evaluation, which consist of 436 and 193 images from different unidentified sources respectively. Detailed information on these two database are summarized in Table 1.

In all tests, we only provided the following a priori knowledge as input to the system, which account for stamp characteristics that can be either reasonably assumed or practically obtainable from limited stamp samples: (a) rough estimates of the minimum and maximum areas of the retrieved stamp, (b) an eccentricity bound of 0.94 (i.e. $a \leq 3b$), which represents the range of eccentricities for normal stamp patterns.

Table 1: Summary of the two test databases of real degraded binary images.

Test Databases	Total Images	Images with The Retrieved Stamp	Image Quality	Stamp Quality
Database 1	436	92	Mediate – Poor	Mediate – Poor
Database 2	193	68	Good – Poor	Mediate – Poor

On each image, top stamp candidates in the three-dimensional parameter space (x_0, y_0, area) are ranked by their scores, which are calculated from their weighted sum of accumulated votes. Strong stamp patterns typically correspond to peaks in the parameter space that are an order of magnitude larger than any other closely ranked top candidates. Once a stamp candidate emerges, we obtain its confidence value by taking the ratio of its scores with the sum of scores from itself and the immediate next top 10 non-stamp candidates. This normalization ensures that multiple stamp instances can detected within the same document.

Retrieved stamps from all images are ranked by their confidence values and the detected regions are saved as sub-images for verification. When calculating the mean average precision, we declare the retrieved stamp relevant if the detected region is correct. Figure 7 shows the overall recall-precision trend on the testing data by unconstraint search on entire documents using a priori information described above.

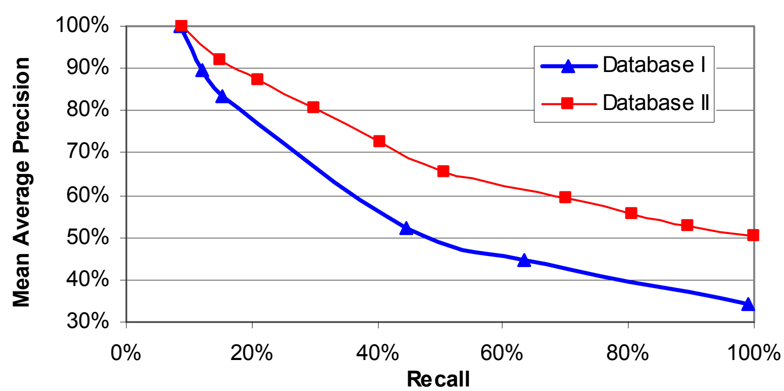


Figure 7: Evaluation of stamp retrievals on groundtruthed document databases.

We can interpret the difference in retrieval performance with reference to the percentage of the total number of relevant within the queried documents in the two databases. From our observation, the diversity of the objects within documents and the quality of retrieved stamp also play major roles in overall retrieval performance.

The system has also gone through production test on databases with more than 5000 documents. The average processing time on each image is 2-3 seconds on a Pentium IV PC. Due to the sensitivity of the data, we demonstrate our approach in Figure 8 using scanned documents with the University of Maryland stamp.

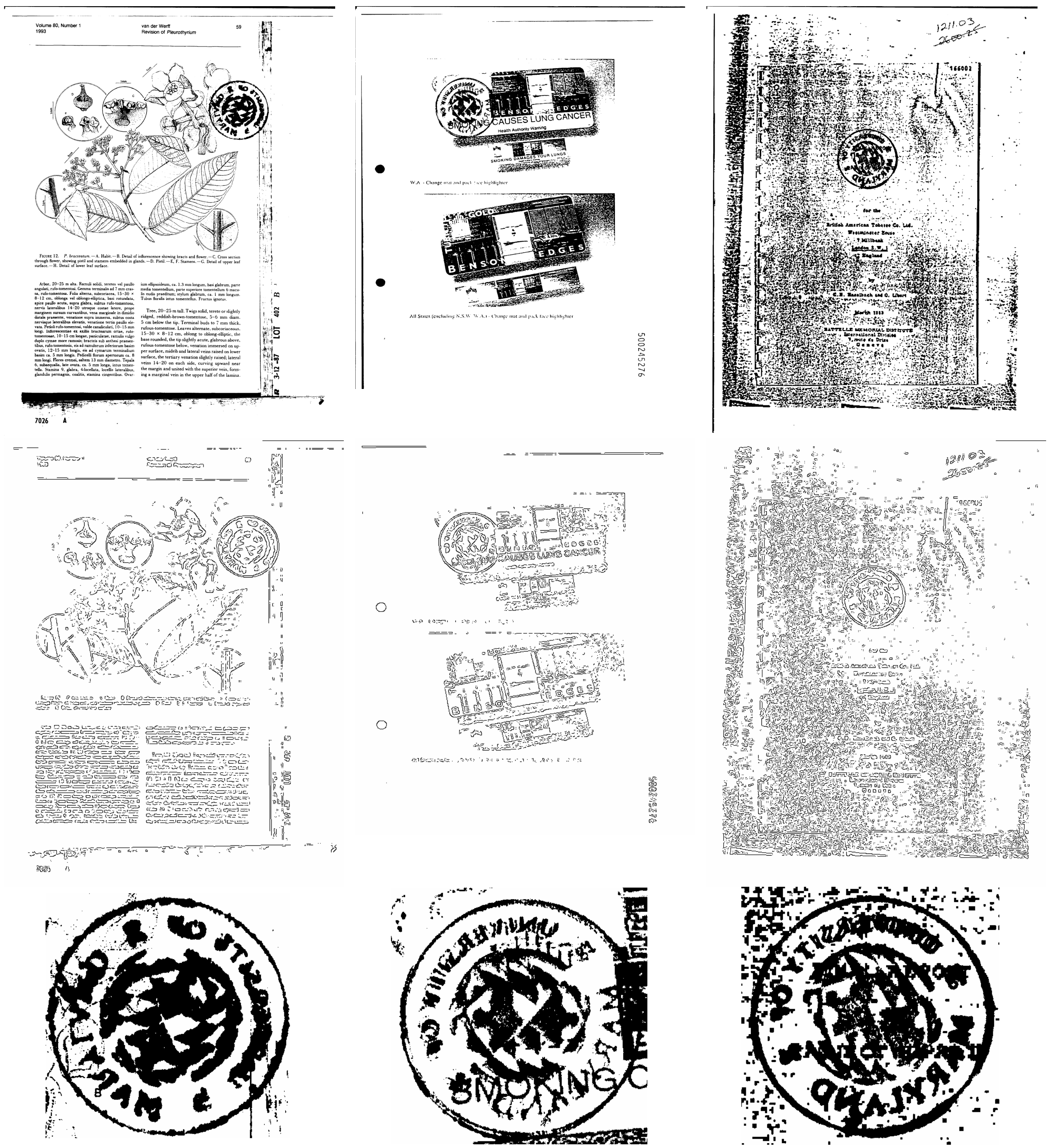


Figure 8: A few detected stamps, shown with their processed edge images and retrieved stamp regions.

6. CONCLUSION

We present a novel stamp detection approach in this paper, which treats stamps as regions with analytically shaped contours. Using robust basic shape detectors as building blocks, the system can effectively retrieve documents with specific stamps using limited information from available samples. When linking connected edges, we introduce an effective template-based junction removal technique to specifically address the problem that stamps commonly overlay with a diverse spectrum of background contents. Based on recognition of strongly connected edge feature points, our method is tested to be robust on printed and handwritten documents with diverse layouts, and noise. We analyzed the running time of our approach for elliptic/circular stamps in comparison with classic Hough transform methods. Our approach delivers exceptional performance on volumes of real degraded documents. This demonstrates that intelligent retrieval and classification of documents from different sources is viable by recognizing native objects within documents. As future work, we are working towards a more general approach on retrieving and classifying regions of interest in documents, including stamps, logos and photos.

REFERENCES

1. K. Ueda, Y. Nakamura, "Automatic verification of seal impression patterns,' in *Proc. 7th. Int. Conf. on Pattern Recognition*, pp. 1019-1021, 1984.
2. T. Fan, W. Tsai, "Automatic Chinese seal identification", *Comput. Vision Graphics Image Process.*, vol. 25, pp. 311-330, 1984.
3. Y. Chen, "Automatic identification for a Chinese seal image," *Pattern Recognition*, vol. 29, no. 11, pp. 1807-1820, 1996.
4. J. Canny, "A computational approach to edge detection," *IEEE Trans. Pattern Anal. Machine Intell.,* vol. 8, no. 6, pp. 679-697, 1986.
5. P.V.C. Hough, *Method and means for recognizing complex patterns*, U.S. Patent 3069654, 1962.
6. S. Tsuji, F. Matsumoto, "Detection of ellipses by a modified Hough transformation," *IEEE Trans. Comput.*, vol. 27, no. 8, pp.777–781, 1978.
7. H. Li, M.A. Lavin, R. J. Le Master, "Fast Hough transform: a hierachical approach," *J. Comput. Vision Graphics Image Process.*, vol. 36, pp.139-161, 1986.
8. R. A. McLaughlin, "Randomized Hough transform: improved ellipse detection with comparison," *Pattern Recognition Lett.*, vol. 19, no. 3-4, pp.299-305, 1998.
9. Y. C. Cheng, S. Lee, "A new method for quadratic curve detection using K-RANSAC with acceleration techniques," *Pattern Recognition*, vol. 28, no. 5, pp. 663-682, 1995.
10. R. Krishnapuram, O. Nasraoui, H. Frigui, "The Fuzzy C spherical shells algorithms: a new approach," *IEEE Trans. Neural Networks,* vol. 3, no. 5, pp. 663-671, 1992.
11. R. N. Dave, "Generalized fuzzy C-shells clustering and detection of circular and elliptical boundaries," *Pattern Recognition*, vol. 25, no.7, pp. 713-721, 1992.
12. A. Fitzgibbon, M. Pilu, R. B. Fisher, "Direct least square 2tting of ellipses," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 2l, no. 5, pp. 477-480, 1999.
13. P. Yin, "A new circle/ellipse detector using genetic algorithms," in *Proc. IPPR on CVGIP Taipei*, pp. 362-368, 1998.
14. E. A. Maxwell, *The Methods of Plane Projective Geometry Based on the Use of General Homogeneous Coordinates*. Cambridge, England: Cambridge Univ. Press, 1946.
15. N. Bennett, R. Burridge, N. Saito, "A method to detect and characterize ellipses using the Hough transform," *IEEE Trans. Pattern Anal. Machine Intell.,* vol. 21, no. 7, pp. 652-657, 1999.
16. D. H. Ballard, "Generalizing the Hough transform to detect arbitrary shapes," *Pattern Recognition*, vol. 13, no. 2, pp. 111-122, 1981.