CITS4403 Computational Modelling Test
Sample Answers
Semester 2, 2024

26 August 2024

Full Name:

Student Number:

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1. **Complex Systems:**

   (a) What is the phenomenon of emergence in complex system? Explain your answer with an example. **[3 marks]**

   Sample Answer:

> Interaction between components generate novel information and exhibit non-trivial collective structures and behaviors at larger scales (1.5 marks); E.g. an ant does not have much of will or plan, but an colony can construct complex structures (1.5 marks).

2. **Graphs**

   (a) What's the difference between the complete graph and the regular graph? Explain your answer with examples. **[3 marks]**

   Sample Answer:

> Complete graph: a graph where every node is connected to every other node. Regular graph: a graph that each node has the same number of neighbours or every node has the same degree (1.5 marks); The example can be any accurate complete graph and regular graph drawings or reasonable example descriptions e.g. using social networks (1.5 marks).

   (b) Given an undirected graph G and a starting node, describe the steps that a Python program would take to check whether the graph is connected.

   [5 marks]

```python
def reachable_nodes(G, start):
    seen = set()
    stack = [start]
    while stack:
        node = stack.pop()
        if node not in seen:
            seen.add(node)
            stack.extend(G.neighbors(node))
    return seen


def is_connected(G):
    start = next(iter(G))
    reachable = reachable_nodes(G, start)
    return len(reachable) == len(G)
```

Sample Answer:

> Correctly describing the starting empty set and stack list of start node ①; While the stack list is non-empty, pop out the last item of stack list ①; If the node is not seen, add this node to the seen set ①; identify all the neighbours of this node and add them to the stack ①; Once the while loop ends, check if the number of elements in the seen set equals to number of nodes in the graph ①.

(c) Given an Erdös Rényi Graph $G(n, p)$, discuss how the values of the two parameters influence the graph connectivity. **[4 mark]**

Sample Answer:

> In an ER graph, the probability that the graph is connected is very low when p is small and nearly 1 when p is large. Between these two regimes, there is a rapid transition at a particular value of p, which we call the critical value ①; The graph is unlikely to be connected is p is smller than the critical value and more likely to be connected if p is larger than the critical value ①; As n increases, the critical value gets smaller, the the transition gets more abrupt ②.

(d) An alternative definition of Erdös Rényi graph, $G(n, m)$, is characterized by two parameters: the number of nodes, n, and the number of edges, m. Describe the process to construct Erdös Rényi graph with these two parameters. **[4 marks]**

```python
def m_pairs(nodes, m):
    pairs = list(all_pairs(nodes))
    return random.sample(pairs, m)

def make_m_graph(n, m):
    G = nx.Graph()
    nodes = range(n)
    G.add_nodes_from(nodes)
    G.add_edges_from(m_pairs(nodes, m))
    return G
```

Sample Answer:

> Randomly select m edges among all the possible edges in a graph of n nodes ②
> Add n nodes and randomly selected m edges to the empty graph ②

(e) Clustering coefficient quantifies the likelihood that two nodes that are connected to the same node are also connected to each other. Describe the steps to compute the local clustering coefficient for node u of Graph G. **[5 marks]**

Sample Answer:

> Finding neighbours of node u ①; Ensuring sufficient neighbours for triangle to form ①; Determining the possible edges: $k(k-1)/2$ ①; Counting the edges that actually exist ①; Returning proportion of edges that exist ①

3. Modelling Social Networks

(a) Given a social network dataset of 3000 nodes and 66000 edges, to construct a Watts-Strogatz graph by using the code below, n is the number of nodes, what is the value for k? Explain your answer. **[4 marks]**

```
import networkx as nx
ws = nx.watts_strogatz_graph(n, k, 0.05, seed=15)
```

Sample Answer:

> k is the estimated degree of node (number of neighbours of each node in ring lattice), we can take the value as the average degree of node of the graph. Average degree of node is the sum of degree divided by the number of nodes. Number of node is 3000, the sum of degree of node is $2 * 66000$. $k = 2 * 66000/3000$. ②;

k is 44 ②
Note that only showing the working process but not giving $k = 44$ will not get full marks.

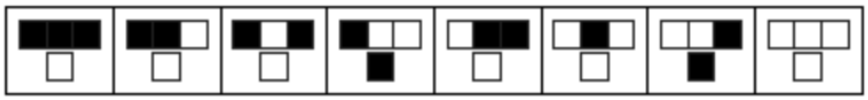
(b) Describe the process to verify if a network is scale-free. **[5 marks]**

Sample Answer:

We are looking for a heavy tail distribution such as a power law ①. To do this we first determine node degree distribution ①, plot the PMF of degree on log-log scale ① and seek a linear scaling region ①, particularly for large degree ①.

4. **1-D Cellular Automata**

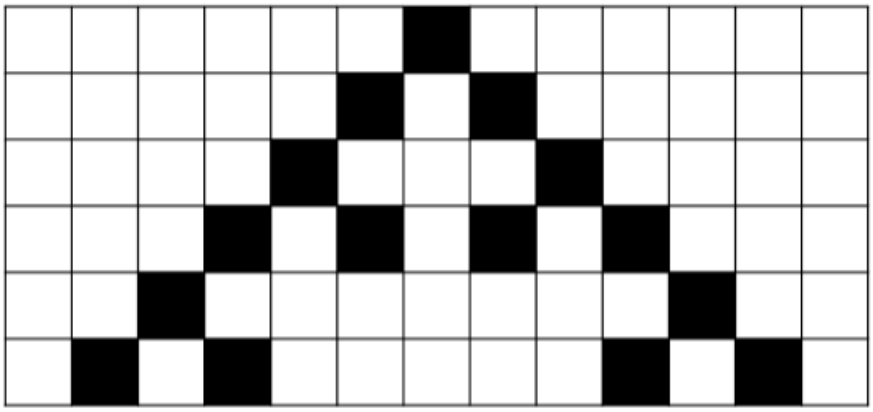
(a) Wolfram's experiments using 3-cell neighborhoods for elementary cellular automata have 256 different rules. What is the name of the rule below? Explain your answer. **[4 marks]**



Sample Answer:

$2^4 + 2 = 18$
Note that 2 marks are given for correctly providing the name of Rule: 18; and 2 marks are given for the correct working process.
If you have written $2^4 + 2$, but didn't provide the correct result of 18, then only 2 marks are given.

(b) Based on the rule in the previous question, complete the next 5 generations with the initial configuration below. **[4 marks]**



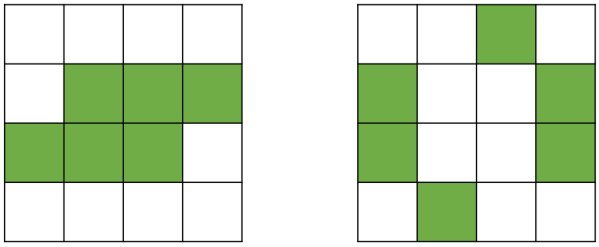
No partial marks for this question. You get 4 marks only when your drawing is completely correct.

5. **Game of Life**

(a) What is the oscillator in Game of Life? Explain your answer with an example. **[3 marks]**

Sample Answer:

A pattern that repeats itself after a fixed number of generations (known as its period). ②



A toad is a period-2 oscillator. ②

2 marks for the definition of oscillator and 2 marks for the example. The 2 marks are not given if your example doesn't show the feature of oscillator. Valid examples could be the drawings of the correct oscillator pattern that matches the definition of oscillator and follows the correct GoL rules.

(b) A variation of Game of Life (GoL) has the same rules as GoL, except for one change: a dead cell with 6 neighbours comes to life. When implementing this variation of

GoL in Python using cross correlation on 2-dimensional arrays, we have a kernel and a lookup table to update the cell's states. Complete the last row of the code below to implement the rules for this variation of GoL. Provide explanations to support your answers. **[6 marks]**

```
kernel = np.array([[1, 1, 1],
                   [1,10, 1],
                   [1, 1, 1]])
table = np.zeros(20, dtype=np.uint8)
table[[6, 12, 13]] = 1
```

③

Sample Explanation:

The cell's state is either 0 or 1 in GoL. When using cross correlation on 2-dimensional arrays with the given kernel, the output array will contain values range from 0 to 18. Based on rules of GoL, if a cell is alive, its state is 1; it will stay alive if it has 2 or 3 neighbours whose states are 1, and dies otherwise. Thus, for an alive cell, it will keep alive if outcome of 2D cross correlation is 12 or 13. Based on the rules of this variation of GoL, a dead cell will come back to life with 6 neighbours. Thus, for a dead cell whose state value is 0, its state will change to 1 if the 2D cross correlation outcome is 6. *table* is used as the rules to set the value of item with index number 6, 12 and 13 to 1. It will then be used to update the cell's state. ③

Note that for students who wrote table[[3,6,12,13]] = 1, full marks are also given if the explanations are reasonable.

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