The ESS of K MCMC samples from the distribution $p(X)$ for estimating the expectation $\mathrm{E}_p[f(X)]$ is

$$\mathrm{ESS}_{p,f}(\{X\}_1^K) = \frac{K}{1 + 2\sum_{j=1}^{K-1}(1 - \frac{j}{K})\rho_j^f}$$

where ρ_k^f gives the autocorrelation of q (the MCMC distribution) for estimating f at lag k:

$$\rho_k^f \equiv \frac{\mathrm{E}_q[(f(X^t) - \mathrm{E}_p[f(X)])(f(X^{t-k}) - \mathrm{E}_p[f(X)])]}{\mathrm{Var}_p[f(x)]}$$

The autocorrelations ρ_k^f can be estimated from the MCMC samples:

$$\hat{\rho}_k^f = \frac{1}{\sigma_f^2(K-k)} \sum_{j=k+1}^{K} (f(X^j) - \mu_f)(f(X^{j-k}) - \mu_f)$$

where $\mu_f = \mathrm{E}_p[f(X)]$ and $\sigma_f^2 = \mathrm{Var}_p[f(X)]$.

For Problem II.1, the true distribution p can be sampled from directly.

For Problem II.2, it is not possible to sample directly from p. In this case, use HMC with 25 Leapfrog steps as the baseline for p, and compare other methods against it. Use $\hat{\mu}_f$ and $\hat{\sigma}_f^2$ from the MCMC samples in the ESS calculations for II.2.