EDUC 231D: Homework 3

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Scenario

The Director of Math Instruction at a large school district is looking for a supplemental math program to improve pre-algebra learning among middle school students. They heard about a program called Transition Mathematics (TM) and a study that showed it has the potential to improve student learning. However, they're concerned that random assignment was not used in all of the sites and that the reported positive effect might be due to selection bias. They'd like you to reanalyze the data to see if the positive effects hold up when controlling for assignment type at the site level and pretest scores at the student level.

To conduct this investigation, the director provided you with the TM data file and a description of the file. The file was uploaded to our BruinLearn site and is named hw3_tm_data.RDS.

Use this data file to answer the following questions for the director. Submit your responses as a PDF file by **12PM on February 18**. The file name for the PDF you submit should use the following naming convention:

HW3_[LastName]_[FirstName].pdf

Set Up

To get started, you need to load some R packages and the data file.

> **Note**
>
> If you have not already installed these packages, you will first have to install them before loading the libraries.

```
# clear the R environment just in case there are things loaded that we don't want
# (start with a clean slate)
rm(list=ls())

# load packages
library("tidyverse") # optional package useful for data processing
library("skimr") # optional package useful for summarizing data file contents
library("flextable") # optional package useful for creating tables
library("lme4") # the primary package we'll use for estimating multilevel models
library("lmerTest") # package to view p-values for estimates

# load data file: make sure the file path matches where you have the file saved
hw <- readRDS("/Users/aishuhan/Desktop/EDUC 231D Multilevel Analysis/Assignments/HW3/hw3_tm_data.R
```

```r
# set a working directory for where you can save files
setwd("/Users/aishuhan/Desktop/EDUC 231D Multilevel Analysis/Assignments/HW3/")
```

Description of the Data File

The data file includes data on 549 students from 19 sites. The following variables are included in the file:

- *site* = unique ID for each site
- *sitempre* = mean pretest score for site j
- *sitempre.gdc* = version of sitempre that is grand-mean centered
- *sitempost* = mean posttest score for site j
- *rand* = indicator for whether the site used random assignment (1) or not (0) to determine which class in the site got TM
- *pretest* = pretest score for student i
- *pretest.gpc* = version of pretest that is group-mean centered
- *posttest* = posttest score for student i
- *trt* = indicator for whether the student was in the treatment (1) or comparison (0) class
- *trt.gpc* = version of trt that is group-mean centered

```
skim(hw) # get descriptive statistics
```


Table 1: Data summary

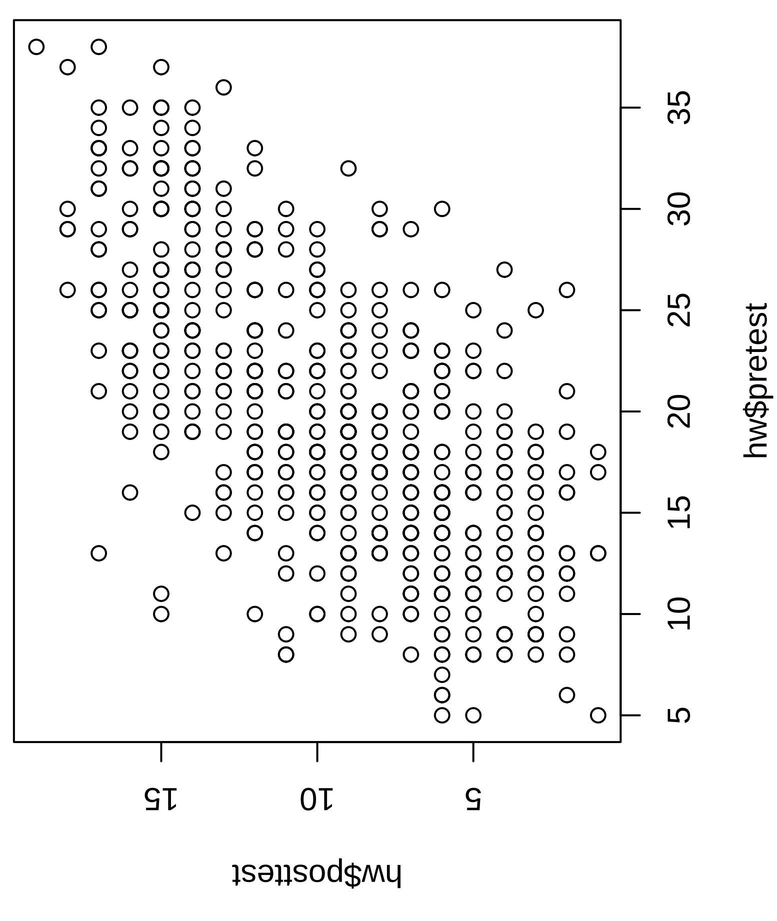
Name	hw
Number of rows	549
Number of columns	10
_____	
Column type frequency:	
numeric	10
_____	
Group variables	None

Variable type: numeric

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
site	0	1	9.98	5.71	1.00	5.00	10.00	15.00	20.00	
sitempre	0	1	19.55	4.25	11.79	16.06	20.06	22.44	28.00	
sitempre.gdc	0	1	0.27	4.25	-7.48	-3.22	0.78	3.16	8.72	
sitempost	0	1	9.39	2.86	5.15	6.98	8.70	11.45	14.42	

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
rand	0	1	0.51	0.50	0.00	0.00	1.00	1.00	1.00	
pretest	0	1	19.55	6.81	5.00	14.00	19.00	24.00	38.00	
pretest.gpc	0	1	0.00	5.32	-14.14	-3.59	-0.55	3.31	15.30	
posttest	0	1	9.39	4.22	1.00	6.00	9.00	13.00	19.00	
trt	0	1	0.50	0.50	0.00	0.00	0.00	1.00	1.00	
trt.gpc	0	1	0.00	0.49	-0.65	-0.48	-0.28	0.48	0.72	

```
# a quick look at the relationship between pretest and posttest math scores
plot(hw$pretest, hw$posttest)
```


hw$pretest
hw$posttest
5
10
15
20
25
30
35

Question 1

Start by fitting the following model to estimate the treatment effect without controlling for anything else (Model 1):

$Y_{ij} = \beta_{0j} + \beta_{1j}(trt - \bar{trt}_{.j}) + r_{ij}$, $r_{ij} \sim N(0, \sigma^2)$

$\beta_{0j} = \gamma_{00} + u_{0j}$, $u_{0j} \sim N(0, \tau_{00})$

$\beta_{1j} = \gamma_{10} + u_{1j}$, $u_{1j} \sim N(0, \tau_{11})$

> Show your work
>
> As part of your response to this question, include any R code and/or output you used to help answer the question.

1.A. Which parameter in this model represents the average math score for the population of sites in the study?

Answer: γ_{00} represents the grand mean for the population of sites.

1.B. Which parameter represents the average effect of TM for the population of sites in the study?

Answer: γ_{10} represents the average effect of TM across sites.

1.C. Based on the model results, what point estimate do you get for the average effect of TM for the population of sites in the study? Is this estimate statistically significant? Interpret the meaning of this result for the director.

Answer: The fixed effect estimate for `trt.gpc` the 1.344 (s.e. $= 0.428$, $p < 0.01$). On average across sites, students in the TM program score about 1.34 points higher on the posttest compared to the students in the control group (without this intervention). Although magnitude of the effect is relatively modest, it is statistically significant in improving students math performance.

```
#fit the model
model1 <- lmer(posttest ~ 1 + trt.gpc + (1 + trt.gpc | site), data = hw)
summary(model1)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: posttest ~ 1 + trt.gpc + (1 + trt.gpc | site)
   Data: hw

REML criterion at convergence: 2846.1

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.1314 -0.6874  0.0252  0.6597  3.0634

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 site     (Intercept) 7.941    2.818
          trt.gpc     2.115    1.454    -0.21
 Residual             9.100    3.017
```

```
Number of obs: 549, groups:  site, 19

Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)
(Intercept)   9.2241     0.6603 18.1048  13.970 3.87e-11 ***
trt.gpc       1.3440     0.4282 17.3130   3.139  0.00588 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
        (Intr)
trt.gpc -0.162
```

1.D. Does the average TM effect appear to vary across sites to a substantive or significant degree? What results led you to this conclusion?

Answer: The result shows a between-sites variance (τ_{11}) of 2.12, with a s.d. of 1.45 for the random slope associated with `trt.gpc`. This suggests that the effect of TM is not identical across all sites, some might have larger benefits and other might have smaller benefits. Other contextual predictors might influence the size of its effect.

Question 2

The director is worried that differences in a student's pretest score could affect the results. To address this concern, fit the following model that includes `pretest.gpc` at level 1 and `pretest.gdc` at level 2 (Model 2):

$$Y_{ij} = \beta_{0j} + \beta_{1j}(trt - \bar{trt}_{.j}) + \beta_{2j}(pretest - \bar{pretest}_{.j}) + r_{ij} \ , \ r_{ij} \sim N(0, \sigma^2)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(sitempre - \bar{sitempre}_{..}) + u_{0j} \ , \ u_{0j} \sim N(0, \tau_{00})$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \ , \ u_{1j} \sim N(0, \tau_{11})$$

$$\beta_{2j} = \gamma_{20}$$

> Show your work
>
> As part of your response to this question, include any R code and/or output you used to help answer the question.

2.A. Does including pretest score as a covariate in the level-1 and level-2 model change the meaning of β_{1j}? If so, in what way(s)?

Answer: Yes, it changed of the meaning of β_{1j}. In model1 without pretest covariate, β_{1j} represent the raw treatment effect of TM on posttest scores in each site. However, in the model2, β_{1j} now represents the treatment effect of TM on posttest scores while controlling for students' pretest scores (or for students who have an average pretest score within their site.

2.B. Does including pretest score as a covariate in the level-1 and level-2 model change the meaning of γ_{10} and τ_{11}? If so, in what way(s)?

Answer: Yes, they both changed. γ_{10} in model1 represent the average treatment effect across all sites without any conditions; in model2 it presents the conditional treatment effect, meaning the difference in

posttest scores between treatment and control groups after accounting for individual differences in pretest scores. τ_{11}, in model1 represents the variance of treatment effect across sites; in model2, τ_{11} now represents the residual variance of treatment effect above and beyond what can be explained by pretest differences.

2.C. How do the results for γ_{10} and τ_{11} based on this model compare with the results for γ_{10} and τ_{11} based on the first model?

Answer: The fixed treatment effect γ_{10} was estimated at 1.34 in model1 and 1.28 in model2. The slight change may suggest that the average effect of TM on posttest scores is not strongly confounded by pretest performance differences. τ_{11} changed from 2.12 in model1 to 2.55 in model2. It increased a little bit (wondering to know why). After adjusting for pretest differences, there remains some between-site differences in the TM effect.

```
#fit the model
model2 <- lmer(posttest ~ 1 + trt.gpc + pretest.gpc + sitempre.gdc + (1 + trt.gpc | site), data =
summary(model2)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: posttest ~ 1 + trt.gpc + pretest.gpc + sitempre.gdc + (1 + trt.gpc |
    site)
   Data: hw

REML criterion at convergence: 2674.4

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.98559 -0.65254  0.01965  0.64779  2.86049

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 site     (Intercept) 1.336    1.156
          trt.gpc     2.549    1.596    -0.23
 Residual             6.788    2.605
Number of obs: 549, groups:  site, 19

Fixed effects:
              Estimate Std. Error        df t value Pr(>|t|)
(Intercept)    9.21469    0.28918  16.64575  31.865 2.35e-16 ***
trt.gpc        1.28437    0.43419  17.65688   2.958  0.00854 **
pretest.gpc    0.27872    0.02111 516.62234  13.204  < 2e-16 ***
sitempre.gdc   0.61095    0.06919  16.03383   8.830 1.48e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) trt.gp prtst.
trt.gpc     -0.177
pretest.gpc  0.000 -0.013
sitempr.gdc -0.015 -0.004 -0.004
```

2.D. Based on the model results, what would you tell the director about the sensitivity of the TM effect estimate to controlling for the pretest score?

Answer: The magnitude of the treatment effect γ_{10} decreased slightly when pretest scores are added to the model (from 1.34 to 1.28). The small change indicates that the TM effect is not largely driven by differences in pretest scores. In both models, the treatment effect remains statistically significant, suggesting that the positive impact of TM on student math performance is robust and not driven by differences in students' baseline achievement.

Question 3

Now fit the following model to test whether the TM effect estimate differs between sites that used random assignment or not (Model 3):

$Y_{ij} = \beta_{0j} + \beta_{1j}(trt - \bar{trt}_{.j}) + \beta_{2j}(pretest - \bar{pretest}_{.j}) + r_{ij}\ ,\ r_{ij} \sim N(0, \sigma^2)$

$\beta_{0j} = \gamma_{00} + \gamma_{01}(sitempre - \bar{sitempre}_{..}) + u_{0j}\ ,\ u_{0j} \sim N(0, \tau_{00})$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(rand_j) + u_{1j}\ ,\ u_{1j} \sim N(0, \tau_{11})$

$\beta_{2j} = \gamma_{20}$

> Show your work
>
> As part of your response to this question, include any R code and/or output you used to help answer the question.

3.A. What do the parameters γ_{10}, γ_{11}, and τ_{11} represent? Explain them using language the director should be able to understand.

Answer: γ_{10} represents the average treatment effect of the TM program on students' posttest scores at sites that did not use random assignment, controlling for the pretest score. γ_{11} represents the difference in the TM effect between sites that used random assignment and those that did not, controlling for the pretest score. It tells us how much the TM effect changes if a site uses random assignment. τ_{11} represents the residual variance of the TM treatment effect across sites that remains after considering whether a site used random assignment or not, controlling for prettest scores.

3.B. Based on the model results, is there evidence that the TM effect systematically differs between sites that used random assignment and those that did not? Explain your conclusion using language the director should be able to understand. As part of your answer, comment on the direction and magnitude of the estimated difference and whether the point estimate is statistically significant. And report the expected average TM effect at sites that used random assignment and the expected average TM effect at sites that did not use random assignment.

Answer: At the nonrandom sites, the students receiving the TM program are expected to score about 1.66 points higher on the posttest compared to the control group, conditioning on the pretest score. For the sites that did use random assignment, the expected benefit of TM effect on posttest score is 0.95, (1.66-0.71), controlling for the pretest score. The difference in the TM effect between random and nonrandom sites is estimated at –0.71 points (the TM effect is 0.71 points lower at random sites than at nonrandom sites). However, this difference is not statistically significant (p = 0.53), so we couldn't say there is strong evidence that the TM effect systematically differs based on the random assignment.

```
model3 <- lmer(posttest ~ 1 + trt.gpc + pretest.gpc + sitempre.gdc +
               + trt.gpc:rand + ( 1 + trt.gpc | site), data = hw)
summary(model3)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: posttest ~ 1 + trt.gpc + pretest.gpc + sitempre.gdc + +trt.gpc:rand +
    (1 + trt.gpc | site)
   Data: hw

REML criterion at convergence: 2672.2

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.96737 -0.63874  0.01663  0.65066  2.91441

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 site     (Intercept) 1.332    1.154
          trt.gpc     2.651    1.628    -0.27
 Residual             6.790    2.606
Number of obs: 549, groups:  site, 19

Fixed effects:
              Estimate Std. Error        df t value Pr(>|t|)
(Intercept)    9.21394    0.28883  16.67565  31.901  < 2e-16 ***
trt.gpc        1.65707    0.63134  16.67085   2.625    0.018 *
pretest.gpc    0.27896    0.02112 515.96260  13.210  < 2e-16 ***
sitempre.gdc   0.61203    0.06862  15.92493   8.919 1.37e-07 ***
trt.gpc:rand  -0.70761    0.86210  16.46586  -0.821    0.423
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) trt.gp prtst. stmpr.
trt.gpc     -0.150
pretest.gpc  0.000  0.004
sitempr.gdc -0.015  0.026 -0.004
trt.gpc:rnd  0.001 -0.717 -0.019 -0.042
```