UCLA School of Education & Information Studies
Department of Education

EDUC 231D
Advanced Quantitative Methods: Multilevel Analysis
Winter 2025

Three-Level Models

Lecture 13 Presentation Slides

February 20, 2025

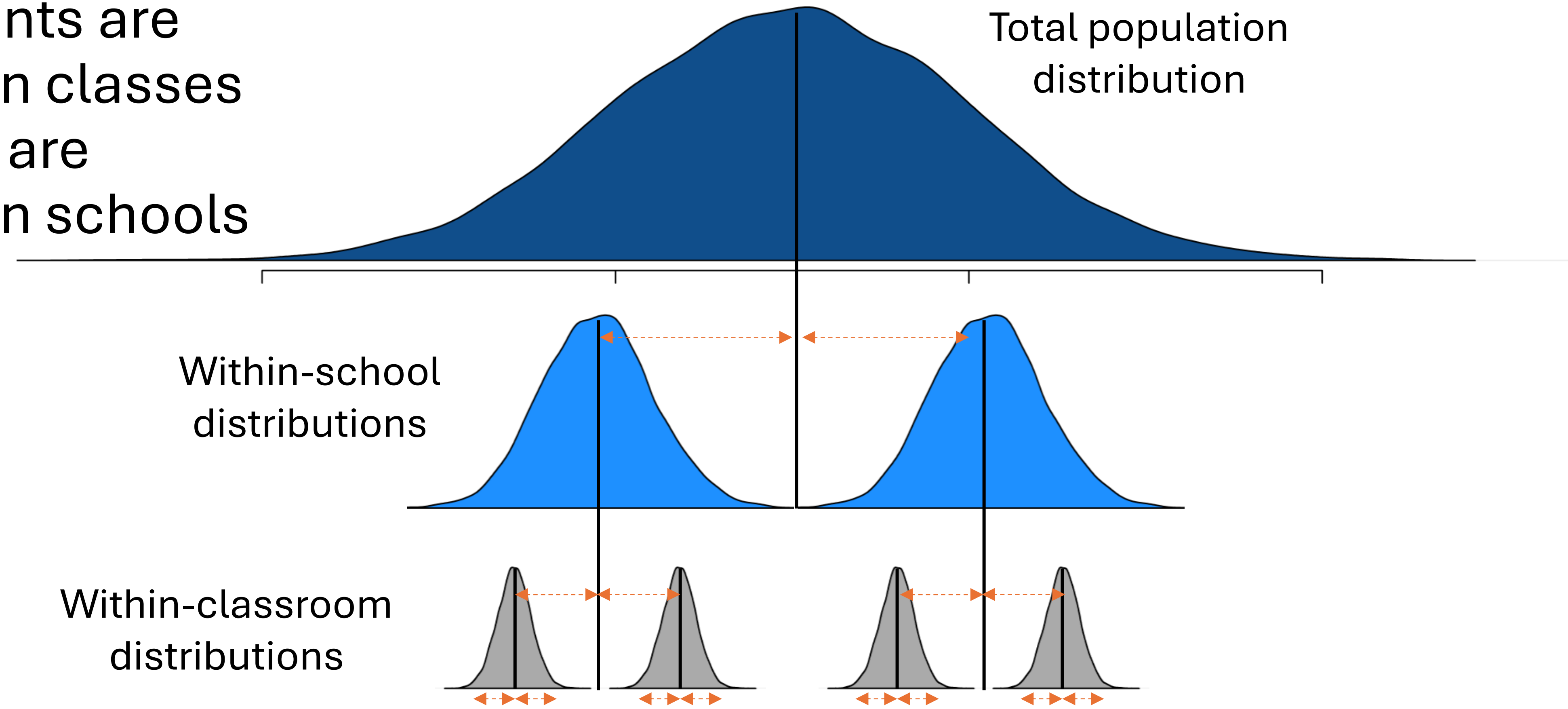
Today's Topics

- The three-level model specification
- Application to longitudinal analysis
- Application to randomized designs
- Reading discussion

The Three-Level Model Specification

3-Level Model

- Consider the example where students are nested within classes and classes are nested within schools



Unconditional 3-Level Model

- Consider the example where students are nested within classes that are nested within schools
- Level 1 (student-level):

Outcome for student *i* in class *j* in school *k*

The within-class variance component

$$Y_{ijk} = \pi_{0jk} + e_{ijk}, \qquad e_{ijk} \sim N(0, \sigma^2)$$

True mean outcome for class *j* in school *k*

Deviation of student *i*'s outcome from the class *j* mean

Unconditional 3-Level Model

- Level 2 (class-level):

True mean outcome for class *j* in school *k*

The between-class (within-school) variance component

$$\pi_{0jk} = \beta_{00k} + r_{0jk}, \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$

True mean outcome for school *k*

Deviation of class *j*'s outcome from the school *k* mean

Unconditional 3-Level Model

- Level 3 (school-level):

True mean outcome for school *k*

The between-school variance component

$$\beta_{00k} = \gamma_{000} + u_{00k}, \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$

True grand mean for the population of schools

Deviation of school *k*'s outcome from the grand mean

Unconditional 3-Level Model

- Level 1 (student-level): $Y_{ijk} = \pi_{0jk} + e_{ijk}\,, e_{ijk} \sim N(0, \sigma^2)$
- Level 2 (class-level): $\pi_{0jk} = \beta_{00k} + r_{0jk}\,,\ r_{0jk} \sim N(0, \tau_{\pi_0})$
- Level 3 (school-level): $\beta_{00k} = \gamma_{000} + u_{00k}\,,\ u_{00k} \sim N(0, \tau_{\beta_{00}})$
- Combined model: $Y_{ijk} = \gamma_{000} + u_{00k} + r_{0jk} + e_{ijk}$

Model with Level-1 Covariate

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(X_{ijk}) + e_{ijk}\ , \qquad e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + r_{0jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$
$$\pi_{1jk} = \beta_{10k} + r_{1jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_1})$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + u_{00k}\ , \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$
$$\beta_{10k} = \gamma_{100} + u_{10k}\ , \qquad u_{10k} \sim N(0, \tau_{\beta_{10}})$$

Model with Level-1 & Level 2 Covariates

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(X_{ijk}) + e_{ijk}\ , \qquad e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}(W_{jk}) + r_{0jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$
$$\pi_{1jk} = \beta_{10k} + \beta_{11k}(W_{jk}) + r_{1jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_1})$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + u_{00k}\ , \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$
$$\beta_{01k} = \gamma_{010} + u_{01k}\ , \qquad u_{01k} \sim N(0, \tau_{\beta_{01}})$$
$$\beta_{10k} = \gamma_{100} + u_{10k}\ , \qquad u_{10k} \sim N(0, \tau_{\beta_{10}})$$
$$\beta_{11k} = \gamma_{110} + u_{11k}\ , \qquad u_{11k} \sim N(0, \tau_{\beta_{11}})$$

Model with Level-1 & Level 2 & Level 3 Covariates

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(X_{ijk}) + e_{ijk}\ , \qquad e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}(W_{jk}) + r_{0jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$
$$\pi_{1jk} = \beta_{10k} + \beta_{11k}(W_{jk}) + r_{1jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_1})$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + \gamma_{001}(Z_k) + u_{00k}\ , \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$
$$\beta_{01k} = \gamma_{010} + \gamma_{011}(Z_k) + u_{01k}\ , \qquad u_{01k} \sim N(0, \tau_{\beta_{01}})$$
$$\beta_{10k} = \gamma_{100} + \gamma_{101}(Z_k) + u_{10k}\ , \qquad u_{10k} \sim N(0, \tau_{\beta_{10}})$$
$$\beta_{11k} = \gamma_{110} + \gamma_{111}(Z_k) + u_{11k}\ , \qquad u_{11k} \sim N(0, \tau_{\beta_{11}})$$

Small group discussion

- In groups of 3-4, take 10 minutes to write out the equation for the combined model

  - Level 1 (student-level):
  $$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(X_{ijk}) + e_{ijk}\ , \qquad e_{ijk} \sim N(0, \sigma^2)$$
  - Level 2 (class-level):
  $$\pi_{0jk} = \beta_{00k} + \beta_{01k}(W_{jk}) + r_{0jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$
  $$\pi_{1jk} = \beta_{10k} + \beta_{11k}(W_{jk}) + r_{1jk}\ , \qquad r_{0jk} \sim N(0, \tau_{\pi_1})$$
  - Level 3 (school-level):
  $$\beta_{00k} = \gamma_{000} + \gamma_{001}(Z_k) + u_{00k}\ , \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$
  $$\beta_{01k} = \gamma_{010} + \gamma_{011}(Z_k) + u_{01k}\ , \qquad u_{01k} \sim N(0, \tau_{\beta_{01}})$$
  $$\beta_{10k} = \gamma_{100} + \gamma_{101}(Z_k) + u_{10k}\ , \qquad u_{10k} \sim N(0, \tau_{\beta_{10}})$$
  $$\beta_{11k} = \gamma_{110} + \gamma_{111}(Z_k) + u_{11k}\ , \qquad u_{11k} \sim N(0, \tau_{\beta_{11}})$$

Small group discussion

- Combined model:

$$Y_{ijk} = \gamma_{000} + r_{0jk} + u_{00k} + e_{ijk}$$

$$+ (\gamma_{100} + r_{1jk} + u_{10k})(X_{ijk}) + (\gamma_{010} + u_{01k})(W_{jk}) + (\gamma_{001})(Z_k)$$

$$+ (\gamma_{110} + u_{11k})(W_{jk})(X_{ijk}) + (\gamma_{101})(Z_k)(X_{ijk}) + (\gamma_{011})(Z_k)(W_{jk})$$

$$+(\gamma_{111})(Z_k)(W_{jk})(X_{ijk})$$

Application to Longitudinal Analysis

Longitudinal Analysis: LSAY Example

- Longitudinal Survey of American Youth (LSAY) followed students from 7th grade (1987) through 12th grade (1992)
  - Level 1: 6 observations over time (math test scores)
  - Level 2: 1,762 students
  - Level 3: 50 schools

- Let's reexamine the rate of change in math scores during secondary school
- And test whether there's a differential growth rate by student sex

Longitudinal Analysis: LSAY Example

- Unconditional linear growth model

- Level 1 (observations): $Y_{ijk} = \pi_{0jk} + \pi_{1jk}(YEAR_{ijk}) + e_{ijk}$

- Level 2 (students):
$$\pi_{0jk} = \beta_{00k} + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k} + r_{1jk}$$

- Level 3 (schools):
$$\beta_{00k} = \gamma_{000} + u_{00k}$$
$$\beta_{10k} = \gamma_{100} + u_{10k}$$

Longitudinal Analysis: LSAY Example

- Unconditional linear growth model

Level 2 random effects:
tells R that students are nested within school and we want the (within-school) student-level random effects for the intercept and YEAR slope

```
m1 <- lmer(MTHSCORE ~ 1 + YEAR
           + (1 + YEAR | SCHOOLID:CASENUM)
           + (1 + YEAR | SCHOOLID), data = lsayx)
```

Level 3 random effects:
tells R that we want school-level random effects for the intercept and YEAR slope

Longitudinal Analysis: LSAY Example

- Unconditional linear growth model

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: MTHSCORE ~ 1 + YEAR + (1 + YEAR | SCHOOLID:CASENUM) + (1 + YEAR | SCHOOLID)
   Data: lsayx

REML criterion at convergence: 68165.3

Scaled residuals: 
    Min      1Q  Median      3Q     Max 
-5.0643 -0.5013  0.0057  0.5330  3.8069 

Random effects:
 Groups           Name        Variance Std.Dev. Corr
 SCHOOLID:CASENUM (Intercept) 72.2117  8.4977       
                  YEAR         2.2996  1.5164   0.34
 SCHOOLID         (Intercept) 19.4068  4.4053       
                  YEAR         0.3197  0.5655   0.47
 Residual                     17.0300  4.1267       
Number of obs: 10572, groups:  SCHOOLID:CASENUM, 1762; SCHOOLID, 50

Fixed effects:
            Estimate Std. Error       df t value Pr(>|t|)    
(Intercept) 52.31576    0.66799 47.32745   78.32   <2e-16 ***
YEAR         3.44401    0.09269 46.80726   37.16   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Longitudinal Analysis: LSAY Example

- Estimate differential math growth based on student sex
  - Level 1 (observations): $Y_{ijk} = \pi_{0jk} + \pi_{1jk}(YEAR_{ijk}) + e_{ijk}$
  - Level 2 (students): $$\pi_{0jk} = \beta_{00k} + \beta_{01k}(FEM_{jk} - \overline{FEM}_{.k}) + r_{0jk}$$ $$\pi_{1jk} = \beta_{10k} + \beta_{11k}(FEM_{jk} - \overline{FEM}_{.k}) + r_{1jk}$$
  - Level 3 (schools): $$\beta_{00k} = \gamma_{000} + u_{00k}$$ $$\beta_{01k} = \gamma_{010} + u_{01k}$$ $$\beta_{10k} = \gamma_{100} + u_{10k}$$ $$\beta_{11k} = \gamma_{110} + u_{11k}$$

Coded 1 for female students
and 0 for male students

Longitudinal Analysis: LSAY Example

- Estimate differential math growth based on student sex

```
m2 <- lmer(MTHSCORE ~ 1 + YEAR + FEM.gpc + FEM.gpc:YEAR
     + (1 + YEAR | SCHOOLID:CASENUM)
     + (1 + YEAR + FEM.gpc + FEM.gpc:YEAR | SCHOOLID),
     data = lsayx)
```

We now have 4 random effect terms at Level 3

Longitudinal Analysis: LSAY Example

- Estimate differential math growth based on student sex
  - Note: a likelihood ratio test suggests there is not statistically significant between-school variation in the differential growth rate by sex

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: MTHSCORE ~ 1 + YEAR + FEM.gpc + FEM.gpc:YEAR + (1 + YEAR | SCHOOLID:CASENUM)
+ (1 + YEAR + FEM.gpc + FEM.gpc:YEAR | SCHOOLID)
   Data: lsayx

REML criterion at convergence: 68137.3

Scaled residuals:
    Min      1Q  Median      3Q     Max
-5.0981 -0.5016  0.0030  0.5371  3.7915

Random effects:
 Groups           Name         Variance Std.Dev. Corr
 SCHOOLID:CASENUM (Intercept)  71.57640 8.4603
                  YEAR          2.26826 1.5061    0.35
 SCHOOLID         (Intercept)  19.41631 4.4064
                  YEAR          0.32226 0.5677     0.46
                  FEM.gpc       1.26897 1.1265    -0.57  0.31
                  YEAR:FEM.gpc  0.05828 0.2414    -0.24 -0.97 -0.49
 Residual                      17.02972 4.1267
Number of obs: 10572, groups:  SCHOOLID:CASENUM, 1762; SCHOOLID, 50

Fixed effects:
              Estimate Std. Error        df t value Pr(>|t|)
(Intercept)   52.30288    0.66778  47.27931  78.323  < 2e-16 ***
YEAR           3.44140    0.09280  46.92354  37.083  < 2e-16 ***
FEM.gpc        1.31467    0.46661  45.44171   2.817  0.00714 **
YEAR:FEM.gpc  -0.26656    0.09432 134.45537  -2.826  0.00543 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Longitudinal Analysis: LSAY Example

- Included a Level 3 covariate in the model (use of ability grouping)
  - Level 1 (observations): $Y_{ijk} = \pi_{0jk} + \pi_{1jk}(YEAR_{ijk}) + e_{ijk}$
  - Level 2 (students): $\pi_{0jk} = \beta_{00k} + \beta_{01k}(FEM_{jk} - \overline{FEM}_{.k}) + r_{0jk}$
    $\pi_{1jk} = \beta_{10k} + \beta_{11k}(FEM_{jk} - \overline{FEM}_{.k}) + r_{1jk}$
  - Level 3 (schools): $\beta_{00k} = \gamma_{000} + \gamma_{001}ABGROUP_k + u_{00k}$
    $\beta_{01k} = \gamma_{010} + \gamma_{011}ABGROUP_k + u_{01k}$
    $\beta_{10k} = \gamma_{100} + \gamma_{101}ABGROUP_k + u_{10k}$
    $\beta_{11k} = \gamma_{110} + \gamma_{111}ABGROUP_k$

Longitudinal Analysis: LSAY Example

- Included a Level 3 covariate in the model (use of ability grouping)

```
m3 <- lmer(MTHSCORE ~ 1 + YEAR
     + FEM.gpc + FEM.gpc:YEAR + (1 + YEAR | SCHOOLID:CASENUM)
     + ABGROUP + ABGROUP:YEAR + ABGROUP:FEM.gpc
     + ABGROUP:FEM.gpc:YEAR
     + (1 + YEAR + FEM.gpc | SCHOOLID),
     data = lsayx)
```

Longitudinal Analysis: LSAY Example

- Included a Level 3 covariate in the model

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: MTHSCORE ~ 1 + YEAR + FEM.gpc + FEM.gpc:YEAR + (1 + YEAR | SCHOOLID:CASENUM)
 + ABGROUP + ABGROUP:YEAR + ABGROUP:FEM.gpc + ABGROUP:FEM.gpc:YEAR
 + (1 + YEAR + FEM.gpc | SCHOOLID)
   Data: lsayx

REML criterion at convergence: 68139.4

Scaled residuals:
    Min      1Q  Median      3Q     Max
-5.1117 -0.5010  0.0001  0.5345  3.7872

Random effects:
 Groups           Name        Variance Std.Dev. Corr
 SCHOOLID:CASENUM (Intercept) 71.5751  8.4602
                  YEAR         2.2848  1.5116   0.35
 SCHOOLID         (Intercept) 19.8683  4.4574
                  YEAR         0.3273  0.5721    0.47
                  FEM.gpc      1.6034  1.2663   -0.47  0.40
 Residual                     17.0301  4.1268
Number of obs: 10572, groups:  SCHOOLID:CASENUM, 1762; SCHOOLID, 50

Fixed effects:
                       Estimate Std. Error         df t value Pr(>|t|)
(Intercept)            52.48475    1.55881   52.01201  33.670   <2e-16 ***
YEAR                    3.42790    0.22175   55.79213  15.459   <2e-16 ***
FEM.gpc                 1.64761    1.26872   73.94265   1.299    0.198
ABGROUP                -0.22069    1.72911   50.62814  -0.128    0.899
YEAR:FEM.gpc           -0.15085    0.23972 1710.12597  -0.629    0.529
YEAR:ABGROUP            0.02182    0.24453   53.42598   0.089    0.929
FEM.gpc:ABGROUP        -0.37450    1.36817   67.70381  -0.274    0.785
YEAR:FEM.gpc:ABGROUP   -0.14182    0.25759 1710.12597  -0.551    0.582
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Application to Randomized Designs

Multisite Individual Randomized Design

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \boxed{\pi_{1jk} Trt_{ijk}} + e_{ijk}\,,\;\; e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + r_{0jk}\,,\;\; r_{0jk} \sim N\left(0, \tau_{\pi_0}\right)$$

$$\pi_{1jk} = \beta_{10k} + r_{1jk}\,,\;\; r_{1jk} \sim N(0, \tau_{\pi_1})$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + u_{00k}\,,\;\; u_{00k} \sim N\left(0, \tau_{\beta_{00}}\right)$$

$$\beta_{10k} = \gamma_{100} + u_{10k}\,,\;\; u_{10k} \sim N(0, \tau_{\beta_{10}})$$

Treatment assignment at Level 1: Can estimate variation in treatment effect at Level 2 and Level 3

Multisite Cluster Randomized Design

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + e_{ijk}\,,\;\; e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + \boxed{\beta_{01k} Trt_{jk}} + r_{0jk}\,,\;\; r_{0jk} \sim N\left(0, \tau_{\pi_0}\right)$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + u_{00k}\,,\;\; u_{00k} \sim N\left(0, \tau_{\beta_{00}}\right)$$

$$\beta_{01k} = \gamma_{010} + u_{01k}\,,\;\; u_{01k} \sim N\left(0, \tau_{\beta_{01}}\right)$$

Treatment assignment at Level 2: Can estimate variation in treatment effect at Level 3

Cluster Randomized Design

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + e_{ijk}\ ,\ \ e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + r_{0jk}\ ,\ \ r_{0jk} \sim N\left(0, \tau_{\pi_0}\right)$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + \boxed{\gamma_{001} Trt_k} + u_{00k}\ ,\ \ u_{00k} \sim N\left(0, \tau_{\beta_{00}}\right)$$

Treatment assignment at Level 3

Multisite Individual Randomized Design: Project STAR Example

- Project STAR is a classic experimental study of class size reduction conducted in the mid-1980s
  - Grade 1 students in Tennessee were randomly assigned to a small class with 13 – 17 students or a regular class with 22 – 25 students
  - Level 1: 6,377 students
  - Level 2: 334 classes
  - Level 3: 75 schools

- Let's reexamine the data to get the grand-mean treatment effect and see how much the treatment effect varies across schools

Multisite Individual Randomized Design: Project STAR Example

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}\left(Trt_{ijk} - \overline{Trt}_{..k}\right) + e_{ijk}\,,\ \ e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + r_{0jk}\,,\ \ r_{0jk} \sim N\left(0, \tau_{\pi_0}\right)$$

$$\pi_{1jk} = \beta_{10k}$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + u_{00k}\,,\ \ u_{00k} \sim N\left(0, \tau_{\beta_{00}}\right)$$

$$\beta_{10k} = \gamma_{100} + u_{10k}\,,\ \ u_{10k} \sim N\left(0, \tau_{\beta_{10}}\right)$$

Multisite Individual Randomized Design: Project STAR Example

- Unconditional treatment effect model

Level 2 random effects:
tells R that teachers are nested within school and we want the (within-school) teacher-level random effects for the intercept

```
m1 <- lmer(zscorem ~ 1 + trt.gpc
           + (1 | g1schid:g1tchid)
           + (1 + trt.gpc | g1schid), data = starx)
```

Level 3 random effects:
tells R that we want school-level random effects for the intercept and treatment effect

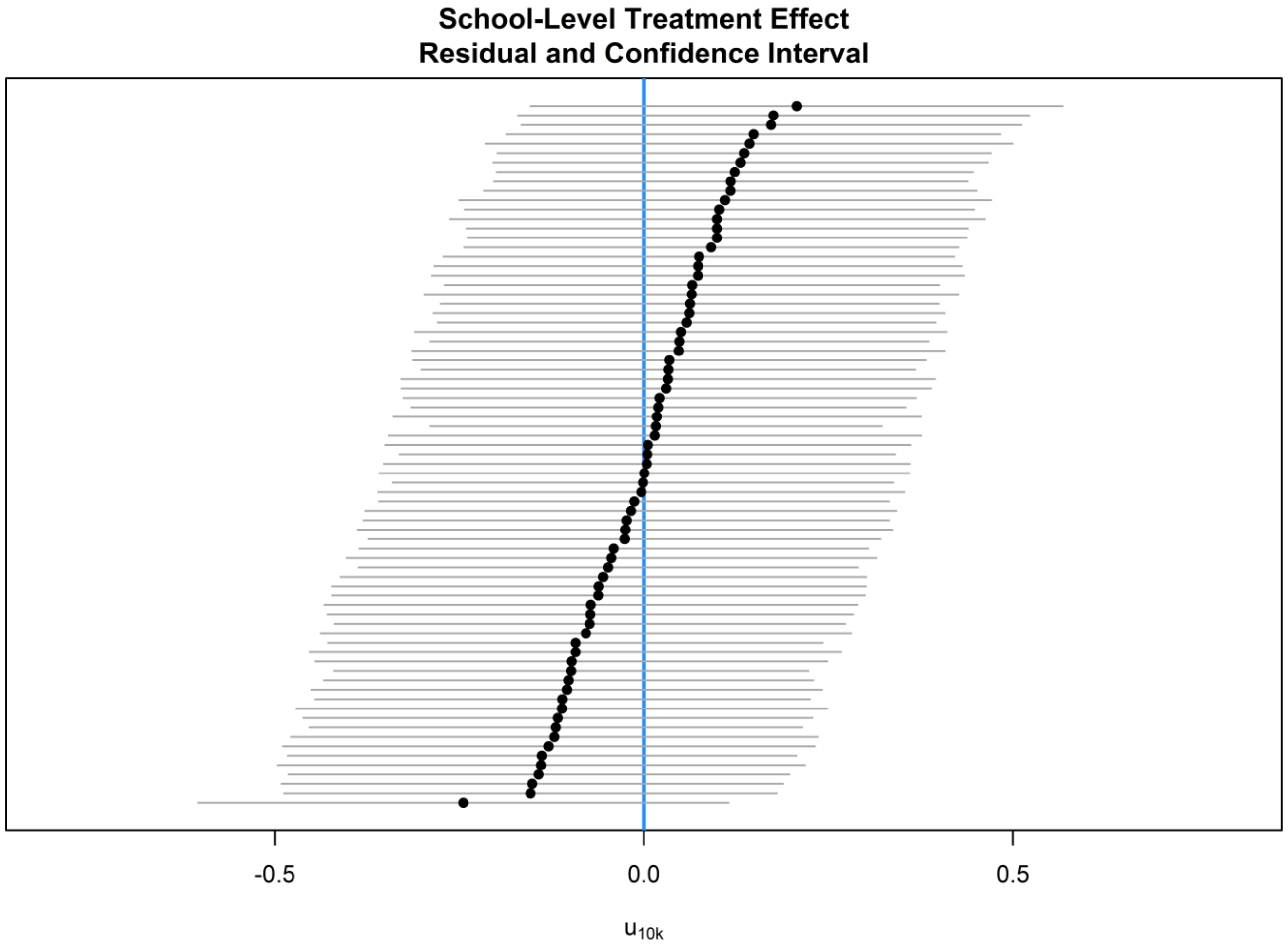
Multisite Individual Randomized Design: Project STAR Example

- Unconditional treatment effect model
  - Note: a likelihood ratio test suggests there is not statistically significant between-school variation in the treatment effect

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: zscorem ~ 1 + trt.gpc + (1 | g1schid:g1tchid) + (1 + trt.gpc |      g1schid)
   Data: starx

REML criterion at convergence: 16279.5

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.2828 -0.6435 -0.0350  0.6290  3.7734

Random effects:
 Groups          Name        Variance Std.Dev. Corr
 g1schid:g1tchid (Intercept) 0.1005   0.3170
 g1schid         (Intercept) 0.1994   0.4465
                 trt.gpc     0.0408   0.2020   0.08
 Residual                    0.6827   0.8263
Number of obs: 6377, groups:  g1schid:g1tchid, 334; g1schid, 75

Fixed effects:
             Estimate Std. Error        df t value Pr(>|t|)
(Intercept)  0.001715   0.055761 74.006323   0.031    0.976
trt.gpc      0.286747   0.049697 65.900915   5.770 2.31e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Multisite Individual Randomized Design: Project STAR Example

- Unconditional treatment effect model
  - Note: a likelihood ratio test suggests there is not statistically significant between-school variation in the treatment effect



Multisite Individual Randomized Design: Project STAR Example

- Test for differential treatment effect between inner-city schools and other schools in the state (cross-level interaction)
- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + \gamma_{001} Inner_k + u_{00k}\,,\;\; u_{00k} \sim N\left(0, \tau_{\beta_{00}}\right)$$
$$\beta_{10k} = \gamma_{100} + \gamma_{101} Inner_k + u_{10k}\,,\;\; u_{10k} \sim N\left(0, \tau_{\beta_{10}}\right)$$

```
m2 <- lmer(zscorem ~ 1 + trt.gpc
           + (1 | g1schid:g1tchid)
           + schinner + schinner:trt.gpc
           + (1 + trt.gpc | g1schid), data = starx)
```

Multisite Individual Randomized Design: Project STAR Example

- Test for differential treatment effect between inner-city schools and other schools in the state (cross-level interaction)

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: zscorem ~ 1 + trt.gpc + (1 | g1schid:g1tchid) + schinner + schinner:trt.gpc
                        + (1 + trt.gpc | g1schid)
   Data: starx

REML criterion at convergence: 16261.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.2771 -0.6458 -0.0302  0.6278  3.7604

Random effects:
 Groups          Name        Variance Std.Dev. Corr
 g1schid:g1tchid (Intercept) 0.10126  0.3182
 g1schid         (Intercept) 0.13868  0.3724
                 trt.gpc     0.04232  0.2057   0.11
 Residual                    0.68273  0.8263
Number of obs: 6377, groups:  g1schid:g1tchid, 334; g1schid, 75

Fixed effects:
                  Estimate Std. Error        df t value Pr(>|t|)
(Intercept)       0.124485   0.053648 71.232187   2.320   0.0232 *
trt.gpc           0.285631   0.056104 66.322465   5.091 3.16e-06 ***
schinner         -0.611865   0.119809 70.608144  -5.107 2.66e-06 ***
trt.gpc:schinner  0.003871   0.123823 61.164459   0.031   0.9752
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Reading Discussion (in small groups)

The Impacts of Reading Recovery at Scale: Results From the 4-Year i3 External Evaluation

- Describe the general study design
  - What is the primary outcome measure and at what level/unit was the outcome collected?
  - What is the level/unit of the treatment assignment?
  - What other levels/groupings are part of the study design?

- Translate equation 1 (p. 324) into the 3-level model notation we've been using in class

- Can you define each parameter in the model?

The Impacts of Reading Recovery at Scale: Results From the 4-Year i3 External Evaluation

- Map the parameters in the 3-level model notation you just created to the model results presented in Table 8 (p. 329)

- The article reports results for “exploratory analyses of ELL and rural [school] subgroup impacts” (p. 324; results discussed on p. 329). But the article does not explain how the subgroup effects were estimated. How do you think the subgroup effects were estimated for this study?

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- "A final limitation of this study is it's inability to explain substantial variation in program effect that were observed across schools" (p. 331).
  - What results indicate there's substantial variation in effect across schools?
  - What factors do you think might contribute to this variation?
- What do you think are the main strengths of the study? The main limitations?
- If you were to conduct a new study of *Reading Recovery* (or a similar intervention), in what ways would you change the study design? Why?

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- Translate equation 1 (p. 324) into the 3-level model notation we've been using in class

Level 1: Students

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}Pretest_{ijk} + \pi_{2jk}Trt_{ijk} + \pi_{3jk}Year_{ijk} + \pi_{4jk}\left(Trt_{ijk} \times Year_{ijk}\right) + e_{ijk}$$

Level 2: Matched Pairs (within schools)

$$\begin{aligned} \pi_{0jk} &= \beta_{00k} + r_{0jk} \\ \pi_{1jk} &= \beta_{10k} \\ \pi_{2jk} &= \beta_{20k} \\ \pi_{3jk} &= \beta_{30k} \\ \pi_{4jk} &= \beta_{40k} \end{aligned}$$

Level 3: Schools

$$\begin{aligned} \beta_{00k} &= \gamma_{000} + u_{00k} \\ \beta_{10k} &= \gamma_{100} \\ \beta_{20k} &= \gamma_{200} + u_{20k} \\ \beta_{30k} &= \gamma_{300} \\ \beta_{40k} &= \gamma_{400} \end{aligned}$$