

EDUC 231D

Advanced Quantitative Methods: Multilevel Analysis
Winter 2025

Multisite and Cluster Randomized Design

Lecture 10 Presentation Slides

February 11, 2025

Today's Topics

- Reading discussion
- Sample size requirements for multilevel designs

Reading Discussion (in small groups)



Accelerating early math learning with research-based personalized learning games: A cluster randomized controlled trial

- Describe the general study design?
 - What is the primary outcome measure and at what level/unit was the outcome collected?
 - What is the level/unit of the treatment assignment?
- “Although the intraclass correlation coefficient (ICC) was 0.10, indicating a low variation across classrooms, we used HLM to control for the variability that may exist across schools (ICC = .03).” (Thai et al., 2022, p. 39)
 - What are the authors trying to say with this sentence? Does it make sense to you, and do you think the sentence achieves the intended purpose? Why or why not?

Accelerating early math learning with research-based personalized learning games: A cluster randomized controlled trial

- Define the following terms in the impact model on page 39:
 - π_{0jk} :
 - π_{1jk} :
 - β_{00k} :
 - β_{01k} :

Accelerating early math learning with research-based personalized learning games: A cluster randomized controlled trial

- Ignoring level 3 (school level), write-out a 2-level version of the model that has the following features:
 - *PreMath* is centered at level 1 so that π_{0j} represents the mean outcome score for students in class j
 - A class-mean version of *PreMath* is included at level 2 to adjust for the between-class relationship between pre- and post-test scores
 - There's a cross-level interaction to test whether the treatment effect differs based on a student's *PreMath* score

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- “... students who used *My Math Academy* in fall 2017 outperformed their control group peers on selected *TEMA-3* items by 5.71% at posttest (effect size = .23).” (Thai et al., 2022, p. 44)
 - Something in this sentence isn’t stated correctly. What is it?
- What do you think are the main strengths of the study? The main limitations?
- If you were to conduct a new study of *My Math Academy* (or a similar intervention), in what ways would you change the study design? Why?

Sample Size Requirements for Multilevel Designs

Statistical Power and the Minimum Detectable Effect Size (MDES)

- When planning a randomized study, it is often helpful (and sometimes required) to get a sense of how large of a sample size you need to detect a meaningful effect
- Requires a “power analysis” that is (usually) based on a null hypothesis test of the average treatment effect

Commonly set at 0.05

	Probability to reject H_0	Probability to not reject H_0
If H_0 is true	α	$1 - \alpha$
If H_1 is true	$1 - \beta$ (power)	β

Commonly set at 0.80

Statistical Power and the Minimum Detectable Effect Size (MDES)

- Minimum detectible effect size (MDES) = the smallest effect (in standard deviation units) for which you can reject the null hypothesis
- Relationship between MDES and sample size for an individual random assignment design:

$$MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{1 - R^2}{p(1 - p)N}}$$

Proportion of variance explained by covariates

Total sample size

Proportion of units in treatment group

Hypothesis test multiplier given α , β , and degrees of freedom
M approximately 2.81 for a traditional two-tailed t-test with $\alpha = 0.05$ and $\beta = 0.20$

Statistical Power and the Minimum Detectable Effect Size (MDES)

- For independent random assignment design, MDES depends on:
 - N : total sample size
 - p : proportion of units assigned to treatment (50/50 provides most power)
 - R^2 : proportion of variance explained by covariates

$$MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{1 - R^2}{p(1 - p)N}}$$

Statistical Power and the Minimum Detectable Effect Size (MDES)

- For multisite randomized design where there's a site-level random effect for the treatment effect, the MDES formula is a little more complicated:

The diagram illustrates the MDES formula for a multisite randomized design. The formula is presented as $MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{\rho\omega(1 - R_{2t}^2)}{J} + \frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}}$. Annotations with yellow lines point to specific parts of the formula: 'Proportion of between-site variance' points to ρ ; 'Amount of between-site treatment effect variance' points to ω ; 'Proportion of between-site treatment effect variance explained by covariates' points to R_{2t}^2 ; 'Level 2 error variance' points to the first fraction $\frac{\rho\omega(1 - R_{2t}^2)}{J}$; and 'Level 1 error variance' points to the second fraction $\frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}$.

$$MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{\rho\omega(1 - R_{2t}^2)}{J} + \frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}}$$

Annotations:

- Proportion of between-site variance (ρ)
- Amount of between-site treatment effect variance (ω)
- Proportion of between-site treatment effect variance explained by covariates (R_{2t}^2)
- Level 2 error variance ($\frac{\rho\omega(1 - R_{2t}^2)}{J}$)
- Level 1 error variance ($\frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}$)

Statistical Power and the Minimum Detectable Effect Size (MDES)

- For two-level cluster randomized design, the MDES formula is similar:

Proportion of between-site variance

Proportion of between-site variance explained by covariates

$$MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{\rho(1 - R_2^2)}{J} + \frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}}$$

Statistical Power and the Minimum Detectable Effect Size (MDES)

- For two-level cluster randomized design, MDES depends on:
 - n : average number of level-1 units per group
 - J : total number of groups at level-2
 - p : proportion of groups assigned to treatment (50/50 provides most power)
 - ρ : ICC
 - R_1^2 : proportion of level-1 variance explained by covariates
 - R_2^2 : proportion of level-2 variance explained by covariates

$$MDES = M_{(\alpha, \beta, df)} \sqrt{\frac{\rho(1 - R_2^2)}{J} + \frac{(1 - \rho)(1 - R_1^2)}{p(1 - p)Jn}}$$

Statistical Power and the Minimum Detectible Effect Size (MDES)

- Website with a lot more about power analysis:

<https://www.causalevaluation.org/power-analysis.html>

- Online tool for power analysis:

<https://powerupr.shinyapps.io/index/>

Small group work



- Use the online PowerUp! tool to explore how the MDES of a multisite randomized design and a cluster randomized design changes based on different values for the following parameters:
 - J :
 - ρ :
 - R_1^2 :
 - R_2^2 :

Small group work



- Use the online PowerUp! tool to determine the number of sites (J) needed for a MDES = 0.20 for a 2-level multisite design with random treatment effects given the following conditions:
 - A two-tailed t-test with $\alpha = 0.05$; $\beta = 0.20$
 - $n = 25$; $p = 0.50$; $\rho = 0.20$; $R_1^2 = 0.30$; $g_2 = 2$; $\omega = 0.25$; $R_{2t}^2 = 0$
- Use the online PowerUp! tool to determine the number of sites (J) needed for a MDES = 0.20 for a 2-level cluster design given the following conditions:
 - A two-tailed t-test with $\alpha = 0.05$; $\beta = 0.20$
 - $n = 25$; $p = 0.50$; $\rho = 0.20$; $R_1^2 = 0.30$; $g_2 = 2$; $R_2^2 = 0.50$