UCLA School of Education & Information Studies
Department of Education

EDUC 231D
Advanced Quantitative Methods: Multilevel Analysis
Winter 2025

Application of Multilevel Models

Lecture 7 Presentation Slides

January 28, 2025

Today's Topics

- Use of multilevel models to study organizational effects (introduction)
- Use of centering to study contextual effects

Use of Multilevel Models to Study Organizational Effects

What do we mean by “organizational effects”?

- How do organizations affect the individuals within them?
- Could focus on the effects of specific organizational characteristics or practices
  - Effect of charter school status on student learning
  - Effect of a workplace onboarding program on employee satisfaction
  - Effect of hospital nursing workload on patient health outcomes
- Could focus on the effects of organizational “climate” or “context”
  - Effect of school safety on teacher retention
  - Effect of neighborhood public transit access on resident employment
  - Effect of doctor “bedside manner” on patient trust

What do we mean by “organizational effects”?

- Could focus on the relative effectiveness of specific organizational units
  - School effects or teacher effects
  - Which hospitals are the “best”?
  - Which cities are the “healthiest”?
- Could focus on how disparities in outcomes or opportunities within organizations differs across organizations
  - Effect of school counselor workload on college eligibility disparity between low and high family income students
  - Effect of company size on employee gender salary gap

What do we mean by "organizational effects"?

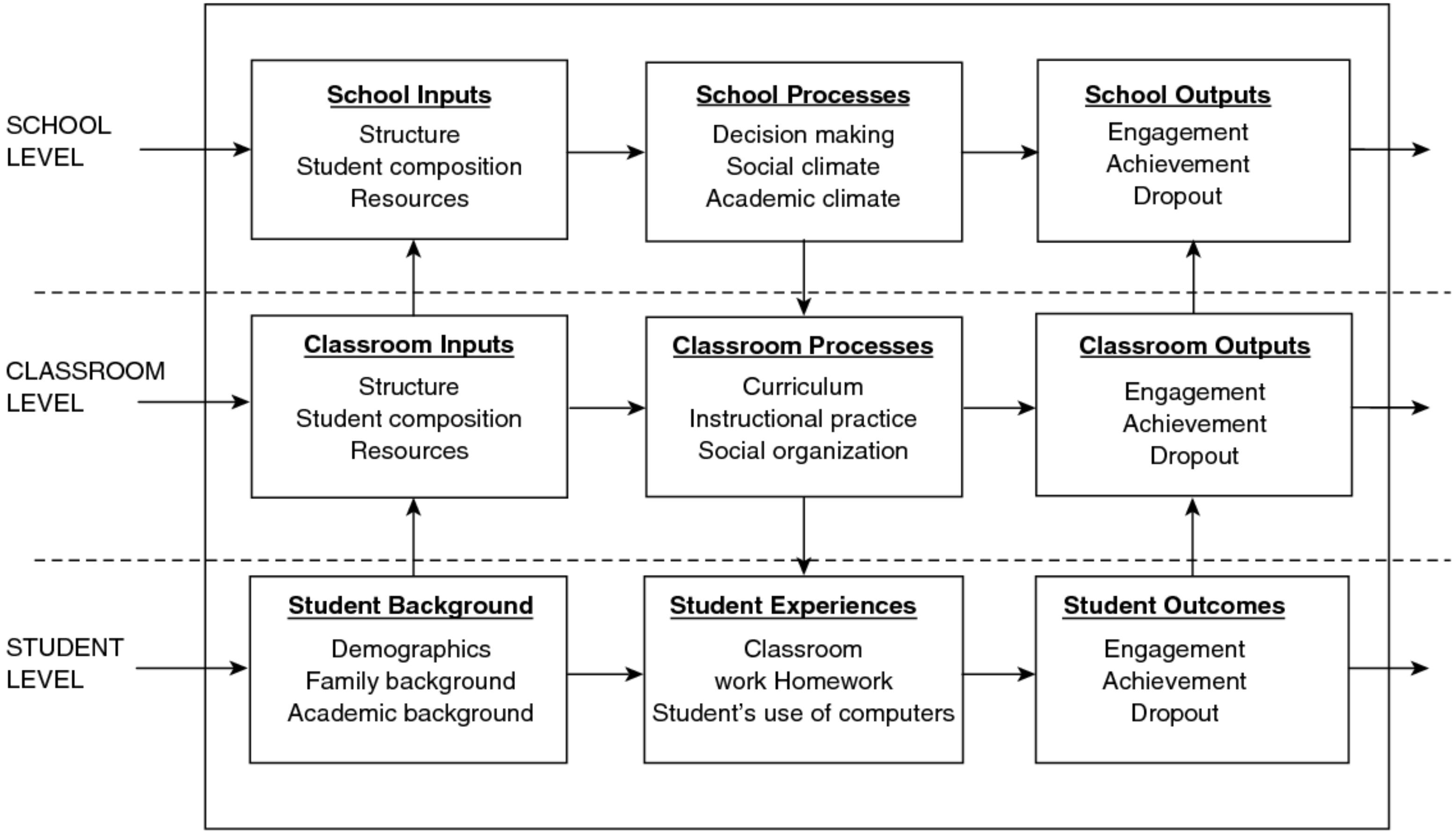

Image source: Rumberger, R. W., & Palardy, G. J. (2004). Multilevel models for school effectiveness research. In *The SAGE Handbook of Quantitative Methodology for the Social Sciences* (pp. 236-259). SAGE Publications, Inc., https://doi.org/10.4135/9781412986311

Generic model specifications

- (Level 1) Person-level model

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + r_{ij}\,, \qquad r_{ij} \sim N(0, \sigma^2)$$

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}\,, \qquad u_{0j} \sim N(0, \tau_{00})$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j}\,, \qquad u_{1j} \sim N(0, \tau_{11})$$

Generic model specifications

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}, \qquad u_{0j} \sim N(0, \tau_{00})$$

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True mean outcome value in organization *j*, conditional on *X* = Effect of organizational characteristic *W* + Unique effect of organization *j*, conditional on *W*

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True distributive effect of *X* in organization *j* = Effect of organizational characteristic *W* + Unique effect of organization *j*, conditional on *W*

Issues with research on organizational effects

- Research questions are frequently posed as causal questions but are mostly observational/descriptive in nature
- Aggregation bias can occur when measures of seemingly the same thing have different meanings at different levels
- Misestimation of standard errors if dependencies are not properly taken into account
- Failure to account for heterogeneity in relationships across organizations can limit the utility of the research

The ECLS-K reading example

- Let’s go back to the ECLS-K grade 1 reading achievement example from previous classes
- But now think about it from the lens of an organizational effects study where we want to investigate whether a student’s reading performance in grade 1 is influenced by their school’s organizational sector (private vs. public)

The ECLS-K reading example

- Three modeling approaches:
  - Model 1: student-level analysis

$$\bar{Y}_{.j} = \beta_0 + \beta_1 Sector_j + e_j$$

  - Model 2: school-level analysis

$$\bar{Y}_{.j} = \gamma_0 + \gamma_1 Sector_j + u_j$$

  - Model 3: multilevel analysis

$$Y_{ij} = \beta_{0j} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} Sector_j + u_{0j}$$

The ECLS-K reading example

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The ECLS-K reading example

- Three modeling approaches:
  - Model 1: student-level analysis
    $\bar{Y}_{.j} = \beta_0 + \beta_1 Sector_j + e_j$
  - Model 2: school-level analysis
    $\bar{Y}_{.j} = \gamma_0 + \gamma_1 Sector_j + u_j$
  - Model 3: multilevel analysis
    $$Y_{ij} = \beta_{0j} + r_{ij}$$
    $$\beta_{0j} = \gamma_{00} + \gamma_{01} Sector_j + u_{0j}$$

	Model 1		**Model 2**		**Model 3**	
	Est.	SE	Est.	SE	Est.	SE
Intercept	94.847	0.175	94.586	0.318	94.657	0.317
sector	6.553	0.566	6.513	1.036	6.512	1.030
Percentage of variance accounted for by sector						
	1.2%		5.1%		6.7%	

Biased because of level 1 dependency (i.e., clustering)

Biased because of unreliability of observed group means

Use of Centering to Study Organizational Effects

What are we trying to estimate?

- If we're studying organizational effects, we might be interested in the following types of questions:
  - What is the organizational effect, independent of any person-level effect?
  - How much between-organization variation exists?
  - What is the compositional/contextual effect of an organizational characteristic?

- Centering level-1 predictors affects which question the model can address

What are we trying to estimate?

- What is the organizational effect, independent of any person-level effect?
  - We want to estimate level-2 effects while adjusting for level-1 covariates
  - Grand-mean centering is appropriate

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(X_{ij} - \bar{X}_{..}\right) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

What are we trying to estimate?

- How much between-organization variation exists?
  - We want to estimate the variance of level-1 coefficients
  - Group-mean centering is appropriate

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}, \qquad u_{0j} \sim N(0, \tau_{00})$$

$$\beta_{1j} = \gamma_{10}$$

What are we trying to estimate?

- What is the compositional/contextual effect of an organizational characteristic?
- Compositional (or contextual) effects exist when there's a difference between the effect of a person-level characteristic and the organizational-level aggregate of that characteristic
  - We want to disentangle the level-1 and level-2 effects
  - Group-mean or grand-mean centering can work

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

Interpreting parameters under different types of centering

- (Level 1) Person-level model, <u>no centering</u>

How do you interpret the circled parameters?

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij}\,, \qquad r_{ij} \sim N(0, \sigma^2)$$

Intercept for group j, controlling for X; or Expected Y value in group j for a level-1 unit with X = 0

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01} \bar{X}_{.j} + u_{0j}\,, \qquad u_{0j} \sim N(0, \tau_{00})$$

Relationship between X and Y, controlling for the group-mean of X

$$\beta_{1j} = \gamma_{10}$$

Relationship between group j mean of X and the intercept for group j, controlling for X at level-1; or level-2 relationship between mean of X and Y, controlling for X at level-1

Interpreting parameters under different types of centering

How do you interpret the circled parameters?

- (Level 1) Person-level model, <u>grand-mean centering</u>

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{..}) + r_{ij}\,, \qquad r_{ij} \sim N(0, \sigma^2)$$

Intercept for group j, controlling for X; or Expected Y value in group j for an average level-1 unit in the sample

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}\,, \qquad u_{0j} \sim N(0, \tau_{00})$$

Relationship between X and Y, controlling for the group-mean of X

$$\beta_{1j} = \gamma_{10}$$

Relationship between group j mean of X and the intercept for group j, controlling for X at level-1; or level-2 relationship between mean of X and Y, controlling for X at level-1

Interpreting parameters under different types of centering

How do you interpret the circled parameters?

- (Level 1) Person-level model, <u>group-mean centering</u>

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{.j}) + r_{ij}\,, \qquad r_{ij} \sim N(0, \sigma^2)$$

Expected Y value in group j for an average level-1 unit in group j; or Mean Y for group j

- (Level 2) Organization-level model

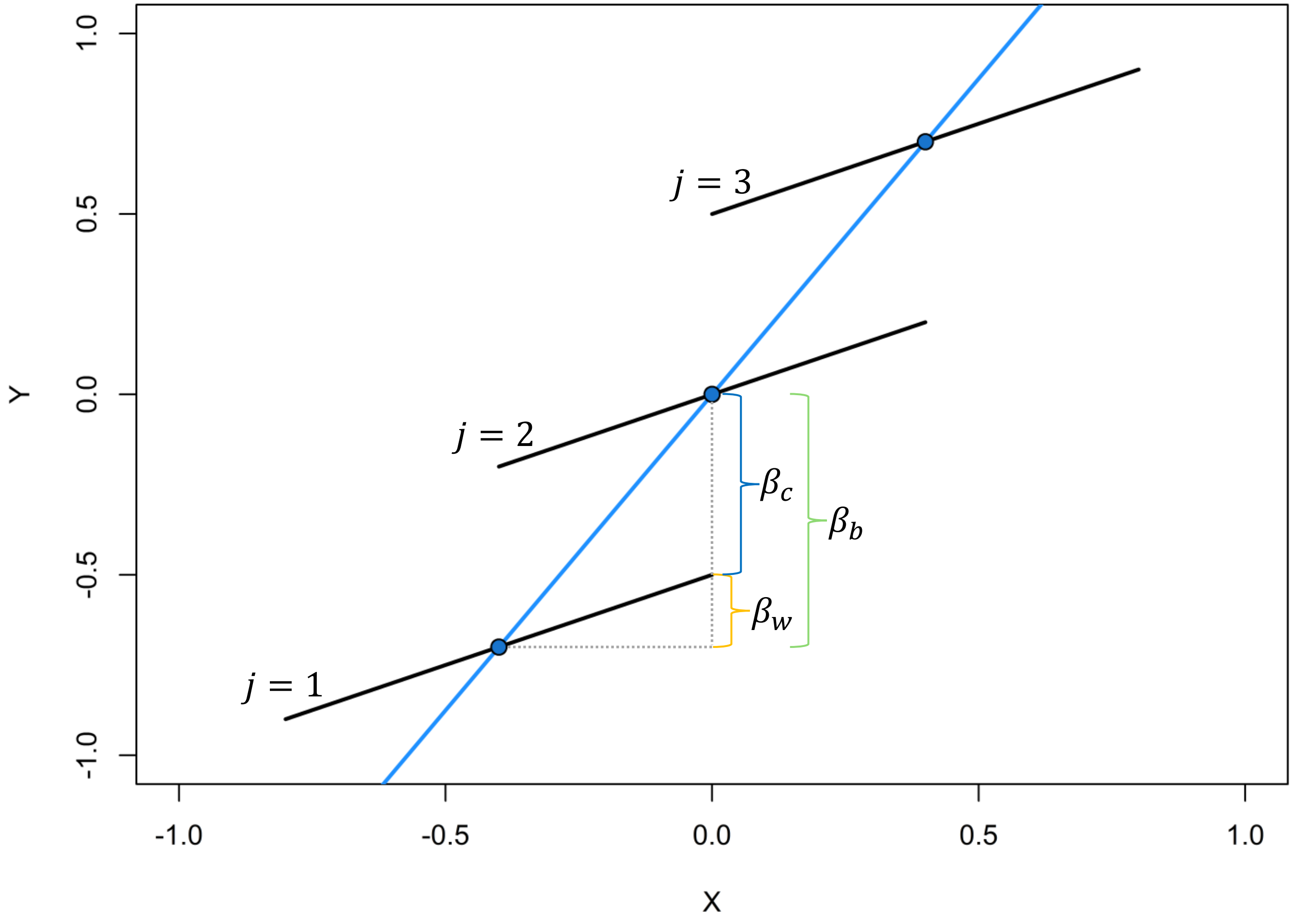
$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}\,, \qquad u_{0j} \sim N(0, \tau_{00})$$

Level-2 relationship between group j mean of X and the Mean Y value for group j

Within-group relationship between X and Y

$$\beta_{1j} = \gamma_{10}$$

The contextual effect with group-mean centering



- (Level 1) Person-level model

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + r_{ij}$$

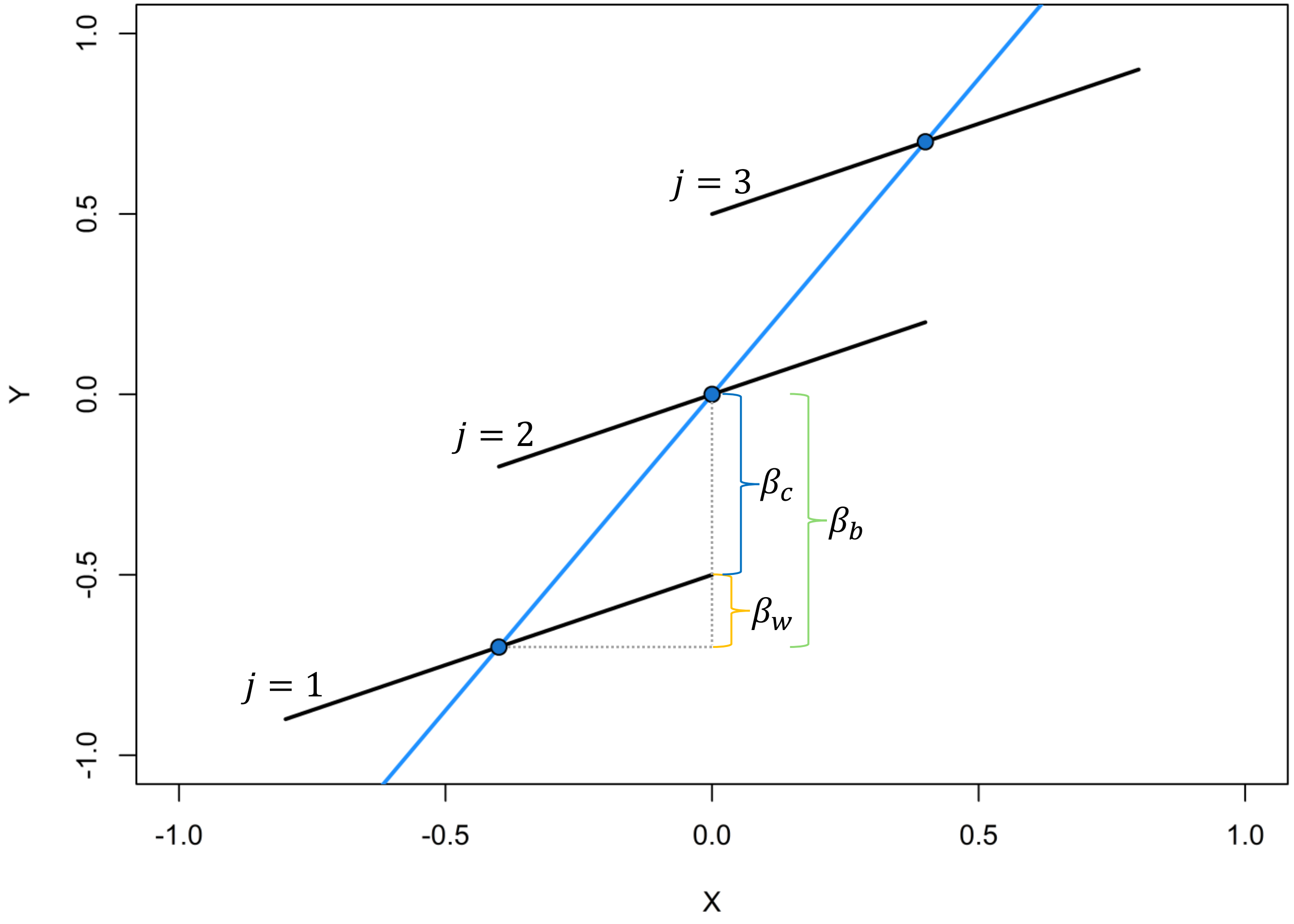
- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

- Contextual effect

$$\beta_b = \gamma_{01}$$
$$\beta_w = \gamma_{10}$$
$$\beta_c = \gamma_{01} - \gamma_{10}$$

The contextual effect with grand-mean centering



- (Level 1) Person-level model

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{..}) + r_{ij}$$

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

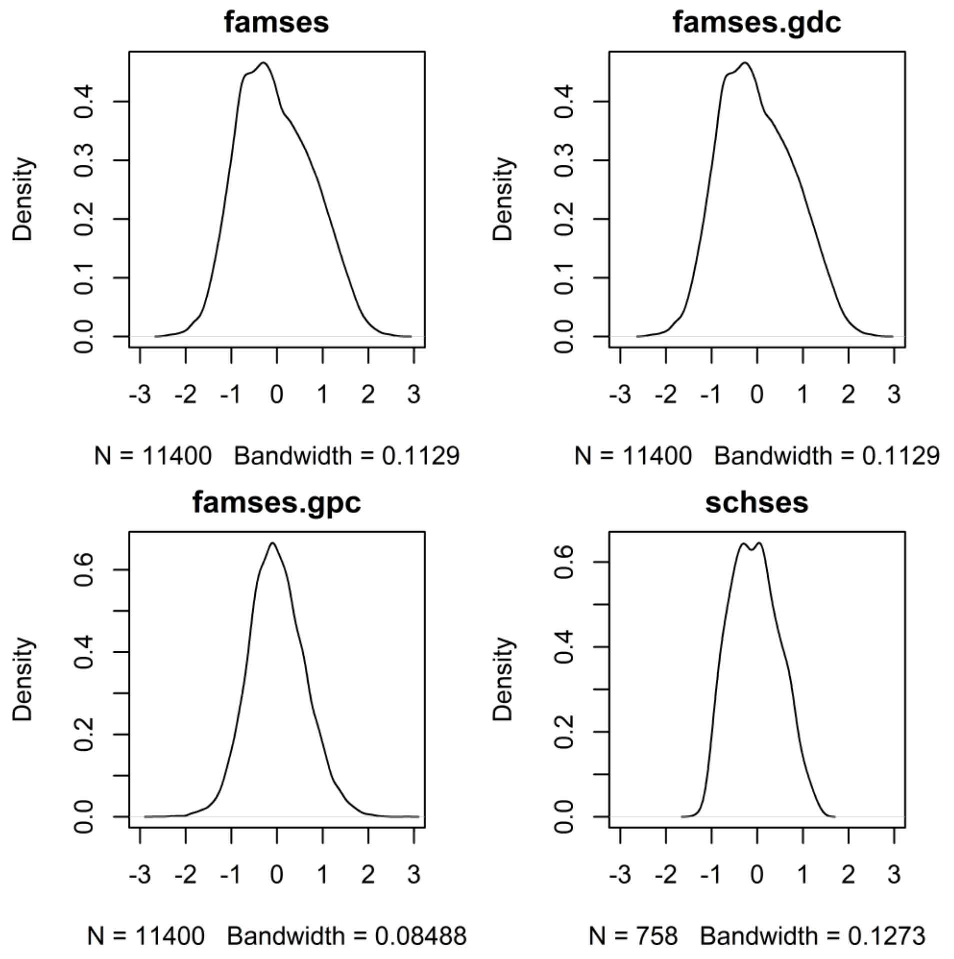
- Contextual effect

$$\beta_b = \gamma_{01} + \gamma_{10}$$
$$\beta_w = \gamma_{10}$$
$$\beta_c = \gamma_{01}$$

Contextual effects illustration

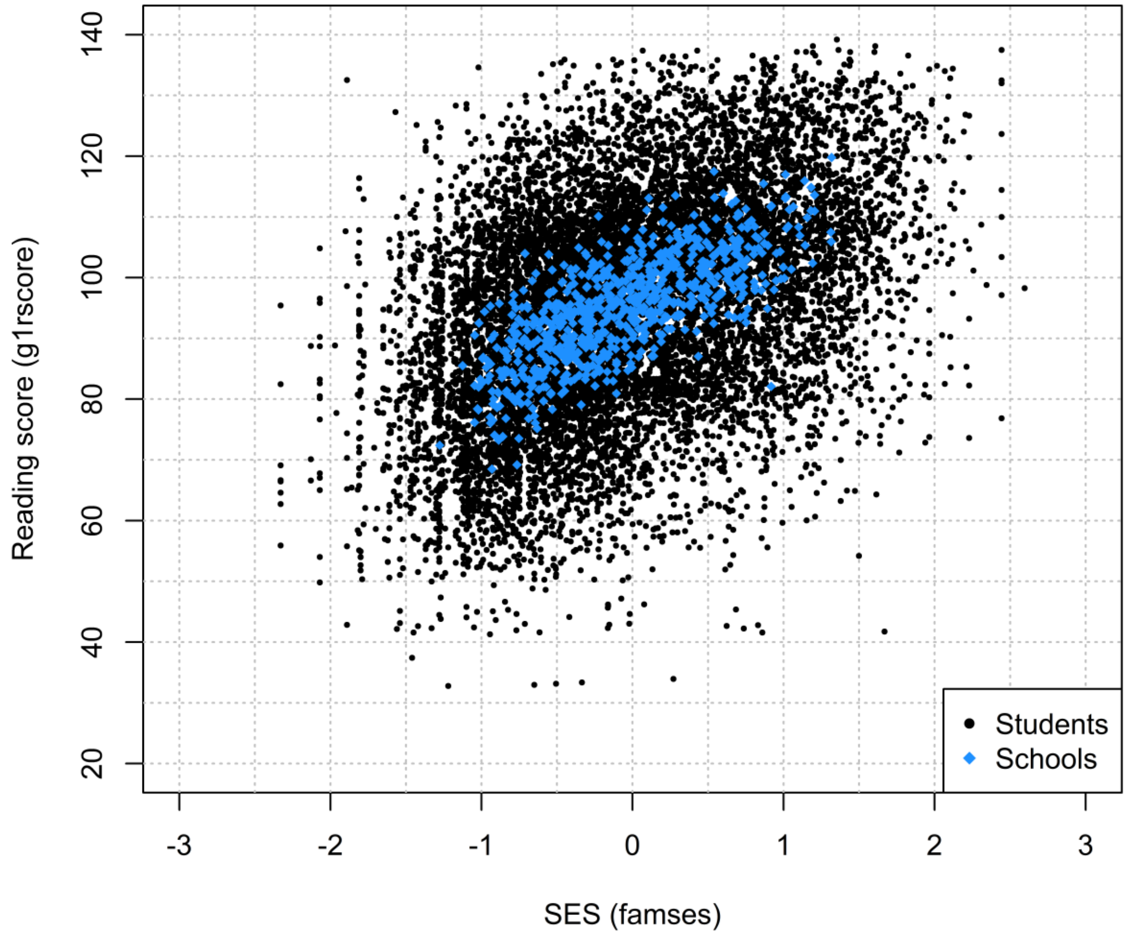
- Back to the ECLS-K data
- What is the relationship between SES and Grade 1 reading achievement?

variable	mean	sd	p0	p25	p50	p75	p100
g1rscore	95.40	17.72	32.77	82.54	96.69	107.77	139.21
famses	-0.03	0.81	-2.33	-0.65	-0.10	0.56	2.60
famses.gpc	0.00	0.61	-2.64	-0.42	-0.03	0.40	2.84
famses.gdc	0.00	0.81	-2.30	-0.62	-0.07	0.59	2.62
schses	-0.03	0.53	-1.28	-0.44	-0.03	0.37	1.32



Contextual effects illustration

- What is the relationship between SES and Grade 1 reading achievement?



Contextual effects illustration

- Grand-mean centering (M2)

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{..}) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

- Contextual effect

$$\beta_c = \gamma_{01} = 4.28$$
$$\beta_w = \gamma_{10} = 7.35$$
$$\boxed{\beta_b = \gamma_{01} + \gamma_{10}}$$
$$= 11.64$$

	(M2) Grand-mean		**(M3) Group-mean**	
Coef.	Est.	SE	Est.	SE
γ_{00}	95.52	0.21	95.73	0.21
γ_{10}	7.35	0.24	7.35	0.24
γ_{01}	4.28	0.46	11.64	0.39

- Group-mean centering (M3)

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{.j}) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

- Contextual effect

$$\beta_b = \gamma_{01} = 11.64$$
$$\beta_w = \gamma_{10} = 7.35$$
$$\boxed{\beta_c = \gamma_{01} - \gamma_{10}} = 4.28$$