UCLA School of Education & Information Studies Department of Education

EDUC 231D
Advanced Quantitative Methods: Multilevel Analysis
Winter 2025

Looking Back and Looking Forward

Lecture 16 Presentation Slides

March 11, 2025

Today's Topics

- Looking back on what we've covered
- Looking forward to more advanced topics we didn't cover

Looking Back

We started with simple school-specific OLS models to look at relationships within schools

School 5006	Estimate	Standard Error	t value	Pr(>|t|)	
(Intercept)	489.648	11.626	42.116	0.0000	***
homesesc	25.294	10.181	2.484	0.0192	*

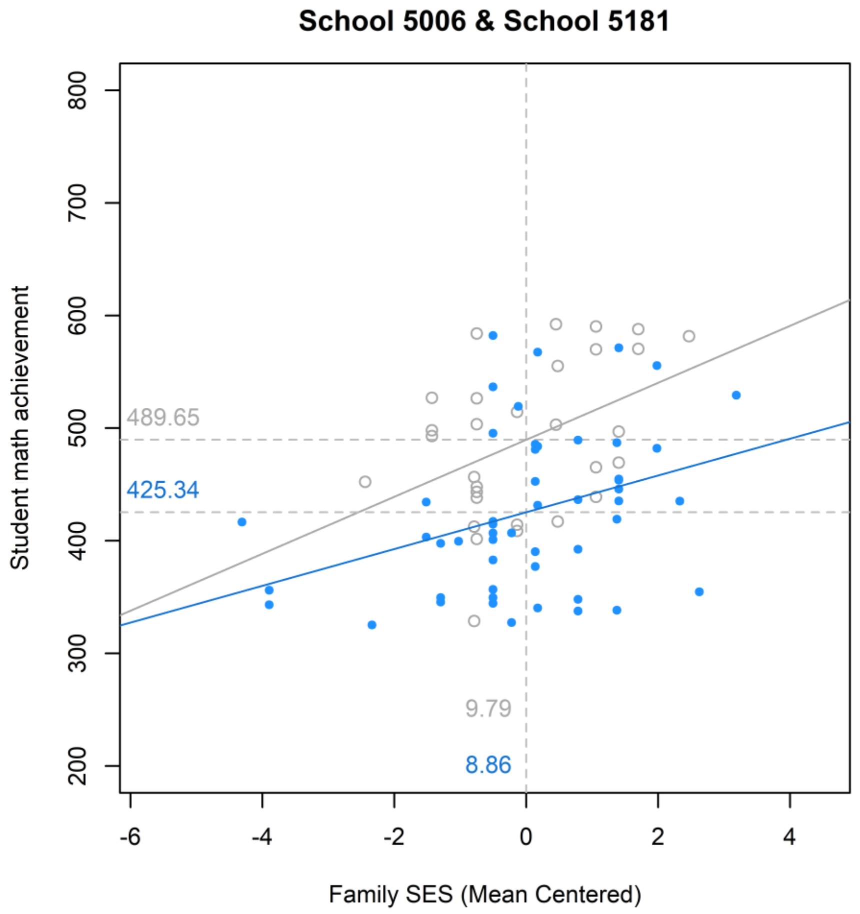
*Signif. codes: 0 <= '***' < 0.001 < '**' < 0.01 < '*' < 0.05*

Residual standard error: 63.68 on 28 degrees of freedom
Multiple R-squared: 0.1806, Adjusted R-squared: 0.1513
F-statistic: 6.172 on 28 and 1 DF, p-value: 0.0192

School 5181	Estimate	Standard Error	t value	Pr(>|t|)	
(Intercept)	425.338	9.324	45.619	0.0000	***
homesesc	16.333	6.150	2.656	0.0106	*

*Signif. codes: 0 <= '***' < 0.001 < '**' < 0.01 < '*' < 0.05*

Residual standard error: 66.58 on 49 degrees of freedom
Multiple R-squared: 0.1258, Adjusted R-squared: 0.108
F-statistic: 7.053 on 49 and 1 DF, p-value: 0.0106



We learned how to do something similar with random intercept and slope multilevel models

- Level-1 (within-school) model:

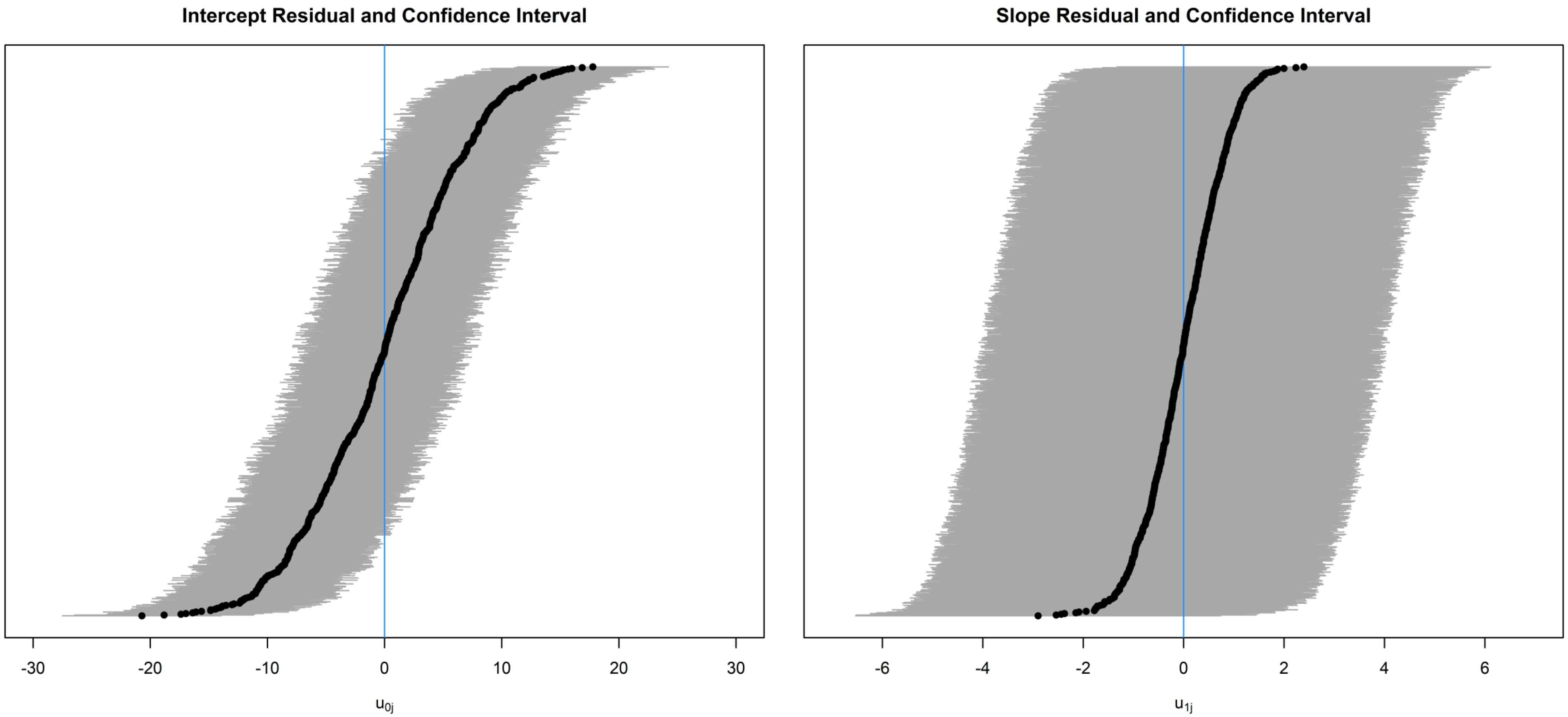
$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(SES_{ij} - \overline{SES}_{.j}\right) + r_{ij}\,, \qquad r_{ij} \sim N(0, \sigma^2)$$

- Level-2 (between-school) model:

$$\beta_{0j} = \gamma_{00} + u_{0j}\,, \qquad u_{0j} \sim N(0, \tau_{00})$$
$$\beta_{1j} = \gamma_{10} + u_{1j}\,, \qquad u_{1j} \sim N(0, \tau_{11})$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim MVN(\mathbf{0}, \mathbf{T}), \qquad \mathbf{T} = \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}$$

Where we can more formally examine variation across schools



And we can examine what organizational factors “explain” that variation

- Level-1 (within-school) model:

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(SES_{ij} - \overline{SES}_{.j}\right) + r_{ij}\ , \qquad r_{ij} \sim N(0, \sigma^2)$$

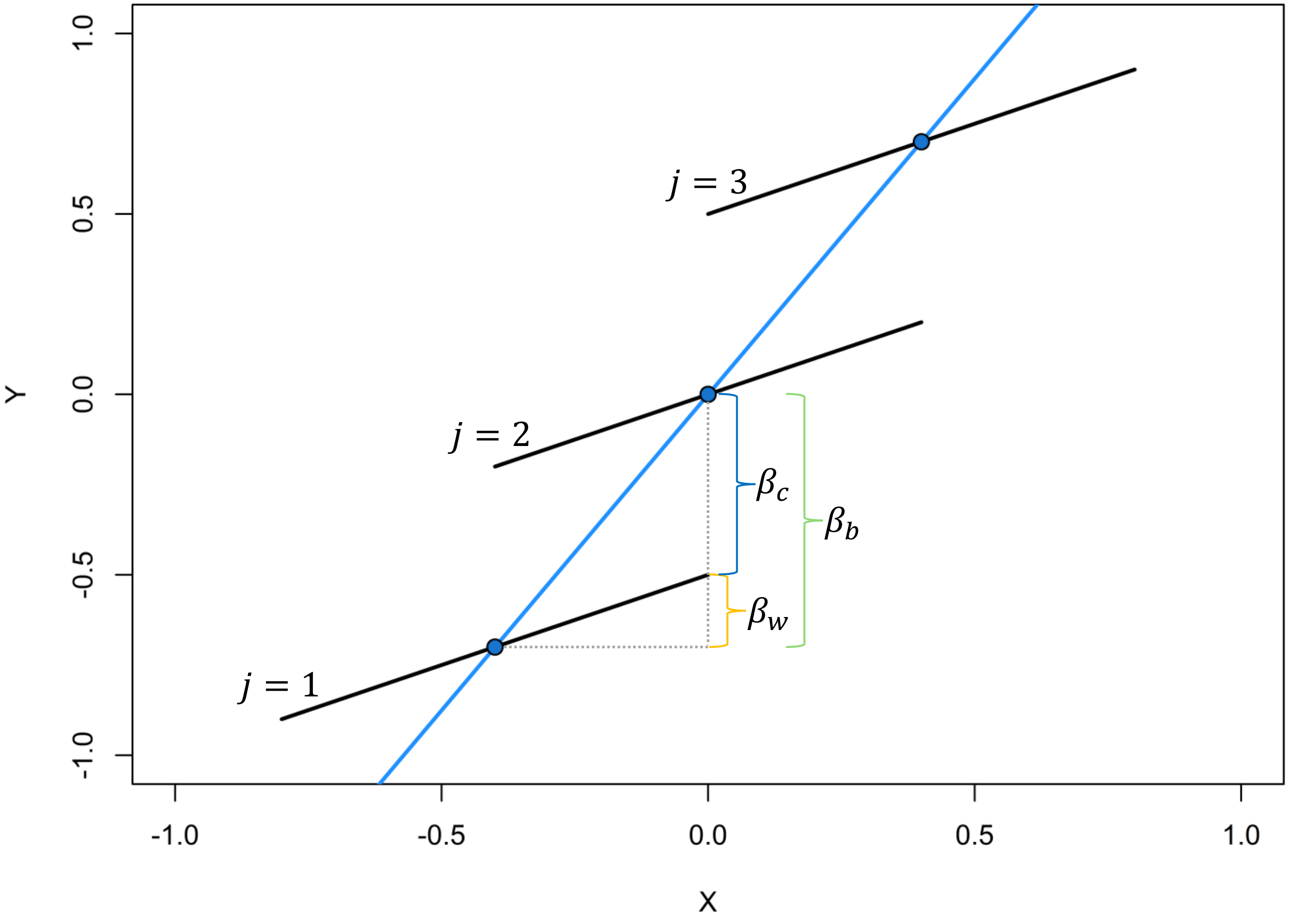
- Level-2 (between-school) model:

$$\beta_{0j} = \gamma_{00} + \gamma_{01} sector_j + u_{0j}\ , \qquad u_{0j} \sim N(0, \tau_{00})$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} sector_j + u_{1j}\ , \qquad u_{1j} \sim N(0, \tau_{11})$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim MVN(\mathbf{0}, \mathbf{T}), \qquad \mathbf{T} = \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}$$

Note: *sector* = 1 for private schools and 0 for public schools

This includes decomposing the effect of person-level factors into the “within group” effect and contextual effect



- (Level 1) Person-level model

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{.j}) + r_{ij}$$

- (Level 2) Organization-level model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{X}_{.j} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

- Contextual effect

$$\beta_b = \gamma_{01}$$
$$\beta_w = \gamma_{10}$$
$$\beta_c = \gamma_{01} - \gamma_{10}$$

We applied multilevel models to multisite randomized studies

- TM example: What's the average effect of TM on geometry readiness?

$$Y_{ij} = \beta_{0j} + \beta_{1j}\left(Trt_{ij} - \overline{Trt}_{.j}\right) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}, \qquad u_{0j} \sim N(0, \tau_{00})$$

$$\beta_{1j} = \gamma_{10} + u_{1j}, \qquad u_{1j} \sim N(0, \tau_{11})$$

```
m1 <- lmer(gebtot ~ trt.gpc + (1 + trt.gpc | site),
          data = tm2
summary(m1)
```

And cluster randomized studies

- MMA example: What's the average effect of MMA on math performance?

$$Y_{ij} = \beta_{0j} + \beta_{1j}(Pretest_i - \overline{Pretest}_{..}) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\left(Trt_j - \overline{Trt}_{..}\right) + \gamma_{02}\left(\overline{Pretest}_{.j} - \overline{Pretest}_{..}\right) + u_{0j},$$

$$u_{0j} \sim N(0, \tau_{00})$$

$$\beta_{1j} = \gamma_{10}$$

We applied multilevel models to longitudinal analyses

- (Level 1) Within-person model

$$Y_{ti} = \pi_{0i} + \pi_{1i}(Time_{ti}) + e_{ti} \; ,$$

$$e_{ti} \sim N(0, \sigma^2 \;)$$

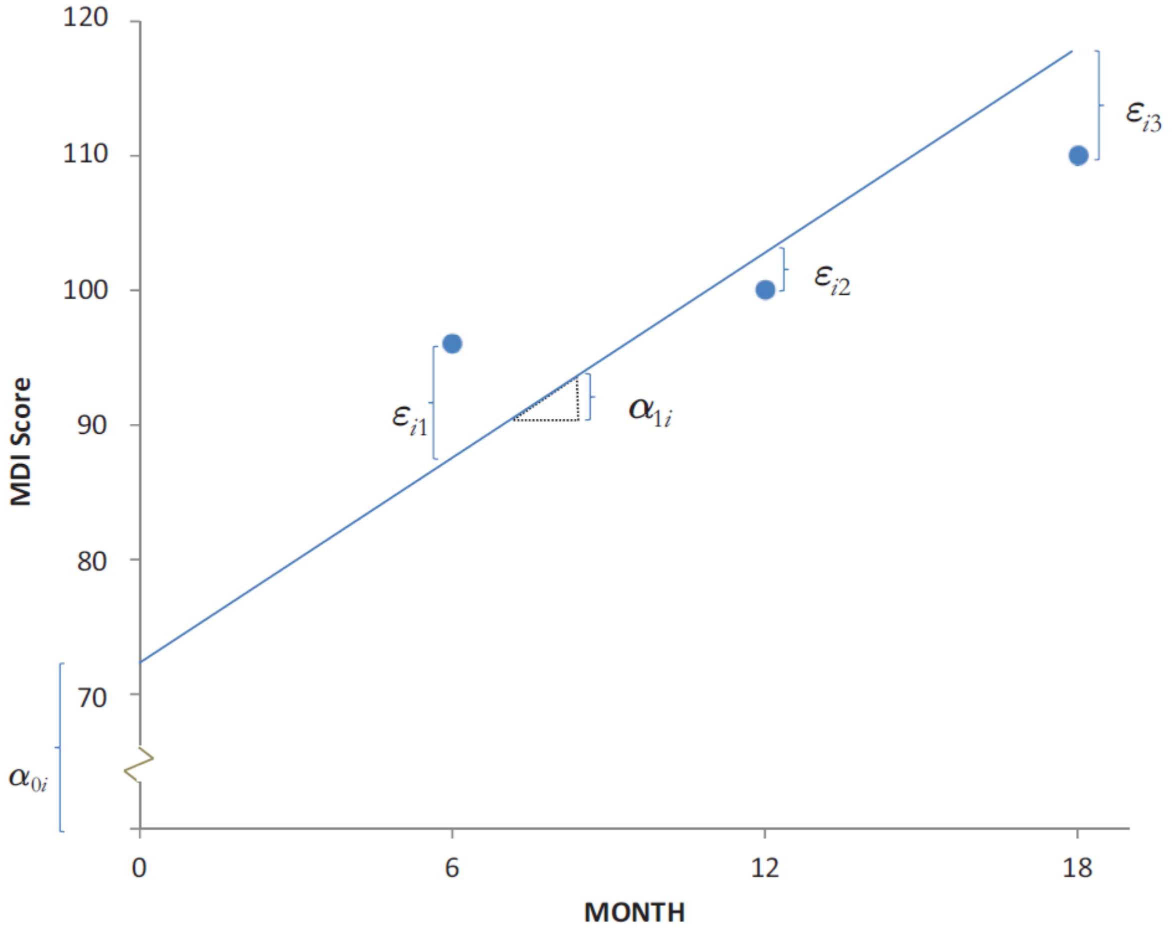


Image source: Gee, K. A. (2014). Multilevel Growth Modeling: An Introductory Approach to Analyzing Longitudinal Data for Evaluators. *American Journal of Evaluation*, *35*(4), 543–561. https://doi.org/10.1177/1098214014523823

We applied multilevel models to longitudinal analyses

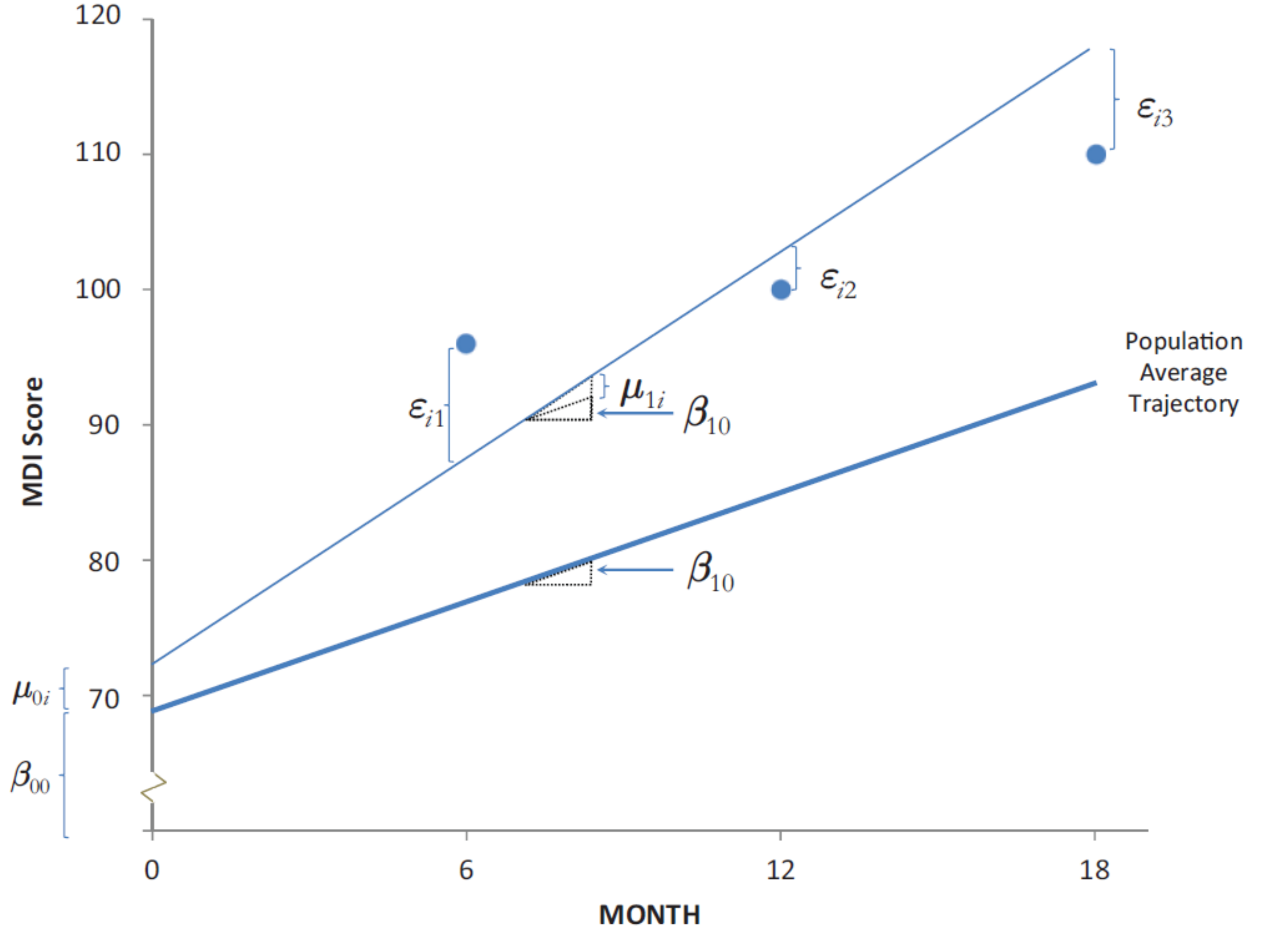
- (Level 2) Between-person model

$$\pi_{0i} = \beta_{00} + r_{0i}$$

$$\pi_{1i} = \beta_{10} + r_{1i}$$

$$\begin{pmatrix} r_{0i} \\ r_{1i} \end{pmatrix} \sim MVN(\mathbf{0}, \mathbf{T})$$

$$\mathbf{T} = \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}$$



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And worked through how to extend what we've learned to a three-level model

- Level 1 (student-level):

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(X_{ijk}) + e_{ijk}\,, \qquad e_{ijk} \sim N(0, \sigma^2)$$

- Level 2 (class-level):

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}(W_{jk}) + r_{0jk}\,, \qquad r_{0jk} \sim N(0, \tau_{\pi_0})$$
$$\pi_{1jk} = \beta_{10k} + \beta_{11k}(W_{jk}) + r_{1jk}\,, \qquad r_{1jk} \sim N(0, \tau_{\pi_1})$$

- Level 3 (school-level):

$$\beta_{00k} = \gamma_{000} + \gamma_{001}(Z_k) + u_{00k}\,, \qquad u_{00k} \sim N(0, \tau_{\beta_{00}})$$
$$\beta_{01k} = \gamma_{010} + \gamma_{011}(Z_k) + u_{01k}\,, \qquad u_{01k} \sim N(0, \tau_{\beta_{01}})$$
$$\beta_{10k} = \gamma_{100} + \gamma_{101}(Z_k) + u_{10k}\,, \qquad u_{10k} \sim N(0, \tau_{\beta_{10}})$$
$$\beta_{11k} = \gamma_{110} + \gamma_{111}(Z_k) + u_{11k}\,, \qquad u_{11k} \sim N(0, \tau_{\beta_{11}})$$

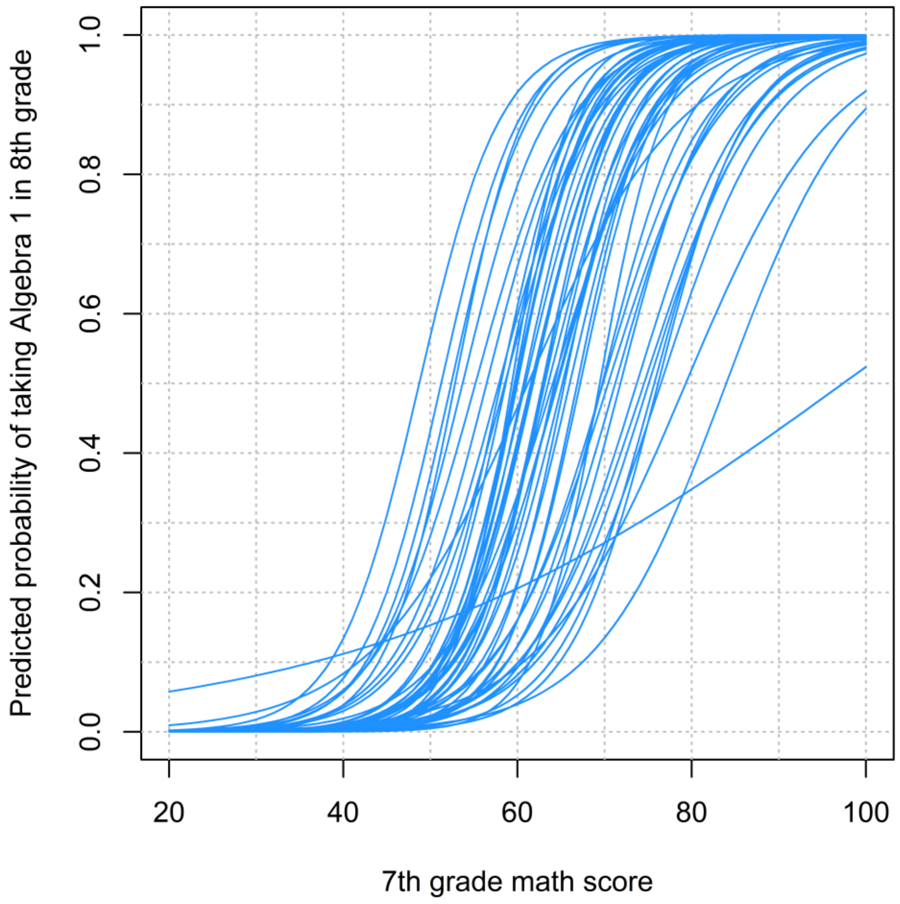
And a multilevel logistic model

- Level 1 (student-level):

$$logit(p_{ij}) = \beta_{0j} + \beta_{1j}(X_{ij} - \bar{X}_{.j})$$

- Level 2 (school-level):

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$



We discussed how multilevel models can support critical quantitative analysis

- Multilevel models are an explicit and efficient way to address the role of contextual and structural factors
- Multilevel models facilitate exploration of heterogeneity
- Multilevel models facilitate exploration of intersectionality, particularly the intersection between individual characteristics and social context

Looking Forward

We were not able to cover more advanced topics

- Using survey/sampling weights with multilevel models
  - Rabe-Hesketh, S., & Skrondal, A. (2006). Multilevel Modelling of Complex Survey Data. *Journal of the Royal Statistical Society Series A: Statistics in Society*, *169*(4), 805–827. https://doi.org/10.1111/j.1467-985X.2006.00426.x
- Missing data and multiple imputation for multilevel data
  - Enders, C. K., Du, H., & Keller, B. T. (2020). A model-based imputation procedure for multilevel regression models with random coefficients, interaction effects, and nonlinear terms. *Psychological Methods*, *25*(1), 88–112. https://doi.org/10.1037/met0000228

We were not able to cover more advanced topics

- Cross-classified models
  - Beretvas, S. N. (2010). Cross-Classified and Multiple-Membership Models. In *Handbook of Advanced Multilevel Analysis*. Routledge.
  - Zaccarin, S., & Rivellini, G. (2002). Multilevel analysis in social research: An application of a cross-classified model. *Statistical Methods and Applications*, *11*(1), 95–108. https://doi.org/10.1007/BF02511448
- Meta-analysis (variance known models)
  - Pastor, D. A., & Lazowski, R. A. (2018). On the Multilevel Nature of Meta-Analysis: A Tutorial, Comparison of Software Programs, and Discussion of Analytic Choices. *Multivariate Behavioral Research*, *53*(1), 74–89. https://doi.org/10.1080/00273171.2017.1365684
  - Goldstein, H., Yang, M., Omar, R., Turner, R., & Thompson, S. (2000). Meta-analysis using multilevel models with an application to the study of class size effects. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, *49*(3), 399–412. https://doi.org/10.1111/1467-9876.00200

We were not able to cover more advanced topics

- Network analysis

  - Grunspan, D. Z., Wiggins, B. L., & Goodreau, S. M. (2014). Understanding Classrooms through Social Network Analysis: A Primer for Social Network Analysis in Education Research. *CBE—Life Sciences Education*, *13*(2), 167–178. https://doi.org/10.1187/cbe.13-08-0162
  - Sweet, T. M. (2016). Social Network Methods for the Educational and Psychological Sciences. *Educational Psychologist*, *51*(3–4), 381–394. https://doi.org/10.1080/00461520.2016.1208093
  - Vacca, R., Stacciarini, J.-M. R., & Tranmer, M. (2022). Cross-classified Multilevel Models for Personal Networks: Detecting and Accounting for Overlapping Actors. *Sociological Methods & Research*, *51*(3), 1128–1163. https://doi.org/10.1177/0049124119882450

We were not able to cover more advanced topics

- Multilevel latent variable modeling
  - Muthén, B., & Asparouhov, T. (2010). Beyond Multilevel Regression Modeling: Multilevel Analysis in a General Latent Variable Framework. In *Handbook of Advanced Multilevel Analysis*. Routledge.
  - Kaplan, D., Kim, J.-S., & Kim, S.-Y. (2009). Multilevel Latent Variable Modeling: Current Research and Recent Developments. In *The SAGE Handbook of Quantitative Methods in Psychology* (pp. 592–612). SAGE Publications Ltd. https://doi.org/10.4135/9780857020994
- Bayesian multilevel models
  - Seltzer, M. H., Wong, W. H., & Bryk, A. S. (1996). Bayesian Analysis in Applications of Hierarchical Models: Issues and Methods. *Journal of Educational and Behavioral Statistics*, *21*(2), 131–167. https://doi.org/10.3102/10769986021002131