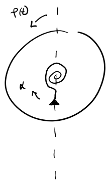
Simaufgabe Theorie part 1, yay!

Donnerstag, 6. Juni 2024 11:43

Kap. 1.2 Erzw. harm. Schwingung

Pohlsch. Rad:



Externes Drehmoment:

$M_e = \hat{M}_e \sin(\omega_e t)$ (Gl. 1.9)

$\alpha_e(t) = \hat{\alpha}_e \sin(\omega_e t)$

DGL: $\ddot{\varphi} + \underbrace{\frac{b^*}{J}}_{2\delta} \dot{\varphi} + \underbrace{\frac{D^*}{J}}_{\omega_0^2} \varphi = \frac{\hat{M}_e \sin(\omega_e t)}{J}$ (Gl 1.10)

Trägh.mom.

Lin, inhomo, 2. Ordnung → Allgem. Lsg = Lsg. homogen + Spez. Lsg. inhomo

$\rightarrow \varphi(t) = \hat{\varphi}_e(\omega_e) \sin[\omega_e t - \xi(\omega_e)]$ (1.11)

mit $\hat{\varphi}_e(\omega_e) = \frac{\hat{M}}{J} \frac{1}{\sqrt{(\omega_0^2-\omega_e^2)^2 + (2\delta\omega_e)^2}}$ (1.11a)

$\xi(\omega_e) = \arctan\left(\frac{2\delta\omega_e}{\omega_0^2-\omega_e^2}\right)$ (1.11b)

a) <u>Statischer Grenzfall</u> $\omega_e \rightarrow 0$, $\omega_e \ll \omega_0$

1.11a wird zu: $\hat{\varphi}_e(0) = \frac{\hat{M}_e}{J} \frac{1}{\omega_0^2} = \frac{\hat{M}_e}{D^*} = \hat{\alpha}_e$ (1.12)

$\omega_e \approx 0$

1.11b wird zu: $\xi(0) = 0$

$\cos(\xi_e) = \frac{1}{\sqrt{1+\tan^2(\xi_e)}} = \frac{1}{\sqrt{1+\left(\frac{2\delta\omega_e}{\omega_0^2-\omega_e^2}\right)^2}} = \frac{\omega_0^2-\omega_e^2}{\sqrt{(\omega_0^2-\omega_e^2)^2+(2\delta\omega_e)^2}}$

1.11 zerlegen: $\varphi(t) = \hat{\varphi}_e(\omega_e)[\sin(\omega_e t)\cos(\xi_e) - \cos(\omega_e t)\sin(\xi_e)]$

$\sin(\xi_e) = \frac{\tan(\xi_e)}{1+\tan^2(\xi_e)} = \dots = \frac{2\delta\omega_e}{\cancel{\omega_0^2-\omega_e^2}} \cdot \frac{\cancel{\omega_0^2-\omega_e^2}}{\sqrt{(\omega_0^2-\omega_e^2)+(2\delta\omega_e)^2}} = \frac{2\delta\omega_e}{\sqrt{(\omega_0^2-\omega_e^2)^2+(2\delta\omega_e)^2}}$

$\Rightarrow \varphi(t) = \frac{\hat{M}_e}{J}\left[\underbrace{\frac{\omega_0^2-\omega_e^2}{\sqrt{\dots}} \sin(\omega_e t)}_{\text{In Phase mit } M_e} - \underbrace{\frac{2\delta\omega_e}{\sqrt{\dots}} \cos(\omega_e t)}_{90°/\frac{\pi}{2} \text{ phasenverschoben zu } M_e}\right]$

↪ Energietransport

1.12 wird zu: $\frac{\hat{M}_e}{J} = \hat{\alpha}_e \omega_0^2$

$\Rightarrow \ddot{\varphi} + \frac{b^*}{J}\dot{\varphi} + \omega_0^2 \varphi = \omega_0^2 \underbrace{\alpha_e \sin(\omega_e t)}_{\alpha_e(t)}$

$\Rightarrow \ddot{\varphi} + \frac{b^*}{J}\dot{\varphi} + \omega_0^2(\varphi - \alpha_e(t))$

b) Hochfrequenter Grenzfall $\omega_e \rightarrow \infty$, $\omega_e \gg \omega_0$

1.11a wird zu: $\hat{\varphi}_e(\infty) \longrightarrow 0$

1.11b wird zu: $\xi(\infty) \rightarrow \pi$ $\tan(\xi) = \frac{1}{\infty} \Rightarrow \xi = 0, \pi, 2\pi, \dots$

c) Resonanzfall $\omega_e \approx \omega_0 \approx \omega_R$

$(1.11a) \Rightarrow \hat{\varphi}_e(\omega_0) = \underbrace{\omega_0^2 \hat{\alpha}}_{\frac{\hat{M}_e}{J}} \frac{1}{2\delta\omega_0}$

Ges: Maximum $\hat{\varphi}_e(\omega_R) \rightarrow \partial_{\omega_R} \hat{\varphi}_e(\omega_R) = 0$ mit $\omega_R = \sqrt{\omega_0^2 - 2\delta^2}$ (1.14)

$\hat{\varphi}(\omega_R) = \omega_0^2 \hat{\alpha} \left(2\delta\sqrt{\omega_0^2 - \delta^2}\right)$ (1.14a)

Amplitude: $\hat{\varphi}(\omega_e) = \frac{\hat{M}_e}{J} \frac{1}{\underbrace{\sqrt{(\omega_0^2-\omega_e^2)^2 + (2\delta\omega_e)^2}}_{f(\omega_e)}}$ $\Rightarrow \hat{\varphi}(\omega_e)$ max wenn $f(\omega_e)$ min

$\Rightarrow \frac{d}{d\omega_e} f(\omega_e) \overset{!}{=} 0$

$\rightarrow \hat{\varphi}(\omega_e) = \omega_0^2 \hat{\alpha} \frac{1}{2\delta\sqrt{\omega_0^2-\delta^2}}$

Phasenverschiebung $\xi(\omega_e \approx \omega_0) = \arctan\left(\frac{1}{0}\right) = \frac{\pi}{2}$

Was ist hier passiert? 1.11a $\omega_e^2 \approx \omega_0^2 - 2\delta^2$ in $f(\omega_e)$ eingesetzt.

1.3 Leistungstransfer vom Erreger zum Oszisystem

Leistungsbilanz vor 1.8: $\underbrace{\frac{d}{dt}\left[\frac{1}{2} J \dot{\varphi}^2(t)\right]}_{E_{kin}\ (1.15)} + \underbrace{\frac{d}{dt}\left[\frac{1}{2} D^* \varphi^2(t)\right]}_{E_{pot}\ (1.16)} = \underbrace{-b^* \dot{\varphi}^2(t)}_{P_{verlust}} + \underbrace{\dot{\varphi}(t)\, \hat{M}_e \sin(\omega_e t)}_{P_{in}\ (1.17)}$

$P_{in} \uparrow (1.10) \cdot \dot{\varphi} \cdot J$

$E_{kin} + E_{pot} = E_{ges} = \text{const.}$ nur für ungedämpft

oder wenn Reibung vorhanden und $P_{in} = P_{verlust}$

$P_{in} = \underbrace{\dot{\varphi}(t)}\, \hat{M}_e \sin(\omega_e t)$

↳ (1.11) ableiten

Ziel: P_{in} in Abh. von ω_e beschreiben.

$P_{in}(\omega_e)$

$$\dot{\varphi}(t) = \omega_e \hat{\varphi}_e(\omega_e) \cos\left(\omega_e t - \xi(\omega_e)\right) = \omega_e \hat{\varphi}_e(\omega_e) \left[\cos(\omega_e t)\cos(\xi_e) + \sin(\omega_e t)\sin(\xi_e)\right]$$

$$\Rightarrow P_{in}(\omega_e) = \omega_e \hat{\varphi}_e \hat{M}_e \left[\underbrace{\sin(\omega_e t)\cos(\omega_e t)\cos(\xi_e)} + \underbrace{\sin^2(\omega_e t)}\sin(\xi_e)\right]$$

zeitliches Mittel von P_{in}:

$$\langle P_{in} \rangle = \omega_e \hat{\varphi}_e \hat{M}_e \left[\ 0 + \frac{1}{2}\sin(\xi_e)\right]$$

$$\boxed{= \frac{1}{2} \omega_e \hat{\varphi}_e \hat{M}_e \sin(\xi_e)}$$

$$\sin(\xi_e) = \frac{\tan(\xi_e)}{\sqrt{1 + \tan^2(\xi_e)}}$$

$$\tan(\xi_e) = \frac{2\delta\omega_e}{\omega_0^2 - \omega_e^2}$$

$\hat{\varphi}_e$ nach (1.11a): $\frac{\hat{M}_e}{J} \frac{1}{\sqrt{(\omega_0^2 - \omega_e^2) + (2\delta\omega_e)^2}}$

$$\Rightarrow \langle P_{in} \rangle = \frac{1}{2}\omega_e \frac{\hat{M}_e}{J} \frac{1}{\sqrt{(\omega_0^2 - \omega_e^2) + (2\delta\omega_e)^2}}\, \hat{M}_e \frac{2\delta\omega_e}{(\omega_0^2 - \omega_e^2)\sqrt{1 + \left(\frac{2\delta\omega_e}{\omega_0^2 - \omega_e^2}\right)^2}}$$

Reibung/Dämpfung

$$= \ldots = \frac{1}{2}\omega_e^2 \hat{M}_e^2 \frac{1}{J} \frac{\delta\omega_e^2}{(\omega_0^2 - \omega_e^2)^2 + (2\delta\omega_e)^2} \qquad (1.20)$$

Fazit:
- Leistungstransfer nur für $\delta \neq 0$
- $\langle P_{in} \rangle$ max für $\omega_e = \omega_0$
- falls $\omega_e - \omega_0 \ll \omega_0 \ \hat{=}\ \omega_e \approx \omega_R \approx \omega_0$

$$(\omega_0^2 - \omega_e^2)^2 = \left[(\omega_0 + \omega_e)(\omega_0 - \omega_e)\right]^2 \approx \left[2\omega_e(\omega_0 - \omega_e\right.$$

Näherung

↳ in (1.2

$\Big]^2$

$0)$

$$\langle P_{in} \rangle \approx \frac{\hat{\mu}_e^2}{J} \frac{\delta \omega_e^2}{4\omega_e^2(\omega_0-\omega_e)^2 + 4\delta^2\omega_e^2}$$

$$\approx \frac{\hat{\mu}_e^2}{4J\delta} \underbrace{\frac{\delta^2}{(\omega_0-\omega_e)^2 + \delta^2}}_{\text{Lorentzkurve}}$$