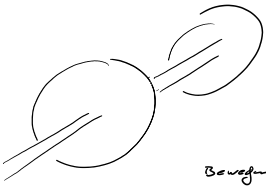
1.4 Gekoppelte harm. Schwingung



Bewegungsgleichung:

$$① \quad J\ddot{\varphi}_1 + b^*\dot{\varphi}_1 + D^*\varphi_1 + D^{**}(\varphi_1 - \varphi_2) = \hat{M}_E \sin(\omega_e t) \tag{1.22a}$$

$$② \quad J\ddot{\varphi}_2 + b^*\dot{\varphi}_2 + D^*\varphi_2 + D^{**}(\varphi_2 - \varphi_1) = 0 \tag{1.22b}$$

AB: $\varphi_1(0) = \varphi_{10}$ $\quad \varphi_2(0) = \varphi_{20}$
$\dot{\varphi}_1(0) = \dot{\varphi}_{10}$ $\quad \dot{\varphi}_2(t) = \dot{\varphi}_{20}$ $\Big\}$ (1.23)

1.4.1 freie Schwingung

$$\ddot{\varphi}_1 + \frac{b^*}{J}\dot{\varphi}_1 + \frac{D^*}{J}\varphi_1 + \frac{D^{**}}{J}(\varphi_1 - \varphi_2) = 0 \tag{1.24a}$$

$$\ddot{\varphi}_2 + \frac{b^*}{J}\dot{\varphi}_2 + \frac{D^*}{J}\varphi_2 + \frac{D^{**}}{J}(\varphi_2 - \varphi_1) = 0 \tag{1.24b}$$

Ges.: φ_1, φ_2 => neue Koordinaten u_a, u_b

(1.24a) + (1.24b) :

$$\frac{d^2}{dt^2}\underbrace{(\varphi_1 + \varphi_2)}_{u_a} + \frac{b^*}{J}\frac{d}{dt}\underbrace{(\varphi_1 + \varphi_2)}_{u_a} + \underbrace{\frac{D^*}{J}}_{\omega_a^2}\underbrace{(\varphi_1 + \varphi_2)}_{u_a} = 0 \tag{1.25}$$

(1.24a) − (1.24b) :

$$\frac{d^2}{dt^2}\underbrace{(\varphi_1 - \varphi_2)}_{u_b} + \underbrace{\frac{b^*}{J}}_{2\delta}\frac{d}{dt}\underbrace{(\varphi_1 - \varphi_2)}_{u_b} + \frac{D^*}{J}\underbrace{(\varphi_1 - \varphi_2)}_{u_b} + \frac{D^{**}}{J}\, 2\underbrace{(\varphi_1 - \varphi_2)}_{u_b} = 0 \tag{1.26}$$

ω_b^2

$$\Rightarrow \ddot{u}_a + 2\delta\dot{u}_a + \omega_a^2 u_a = 0 \tag{1.25a}$$

$$\Rightarrow \ddot{u}_b + 2\delta\dot{u}_b + \omega_b^2 u_b = 0 \tag{1.25b}$$

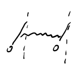
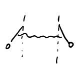
Lösung wie (1.5):

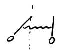
für $\delta = 0$ $\quad \dot{u}_{a0} = \dot{u}_{b0} = 0$

$$u_a(t) = u_{a0}\cos(\omega_a t) \;;\quad u_b(t) = u_{b0}\cos(\omega_b t) \tag{1.27a, 1.27b}$$

$$\boxed{\Rightarrow \varphi_1(t) = \frac{1}{2}(u_a + u_b) = \frac{1}{2}(\varphi_{10} + \varphi_{20})\cos(\omega_a t) + \frac{1}{2}(\varphi_{10} - \varphi_{20})\cos(\omega_b t)} \tag{1.28a}$$

$$\boxed{\varphi_2(t) = \frac{1}{2}(u_a - u_b) = \frac{1}{2}(\varphi_{10} + \varphi_{20})\cos(\omega_a t) - \frac{1}{2}(\varphi_{10} - \varphi_{20})\cos(\omega_b t)} \tag{1.28b}$$

- Für $\varphi_{10} = \varphi_{20}$  (1.28ab): $\varphi_1(t) = \varphi_2(t) = \varphi_{10}\cos(\omega_a t)$
- Für $\varphi_{10} = -\varphi_{20}$  (1.28ab): $\varphi_1(t) = \varphi_{10}\cos(\omega_b t)$ $\quad \varphi_2(t) = -\varphi_{10}\cos(\omega_b t)$
- Für sonst werden beide Normalschwingungen angeregt.

  z.B.: $\varphi_{10} \neq 0$ $\quad \varphi_{20} = 0$ 

$$\Rightarrow \varphi_1(t) = \frac{1}{2}\varphi_{10}\left[\cos(\omega_a t) + \cos(\omega_b t)\right]$$

$$= \varphi_{10}\left[\cos\Big(\underbrace{\frac{\omega_b-\omega_a}{2}}_{\frac{\Delta\omega}{2}}t\Big)\cdot\cos\Big(\underbrace{\frac{\omega_a+\omega_b}{2}}_{\bar\omega}t\Big)\right] = \varphi_{10}\cos\left(\frac{\Delta\omega}{2}t\right)\cos(\bar\omega t) \qquad (1.30a)$$

$$\varphi_2(t) = \frac{1}{2}\varphi_{10}\left[\cos(\omega_a t) - \cos(\omega_b t)\right] = \varphi_{10}\sin\left(\frac{\Delta\omega}{2}t\right)\sin(\bar\omega t) \qquad (1.30b)$$

„Schwebung“

Momentane Schwingungsenergie für $b^*=0$

$(1.24a)\,\dot\varphi_1 + (1.24b)\,\dot\varphi_2 \;\hat{=}\;$ Leistung

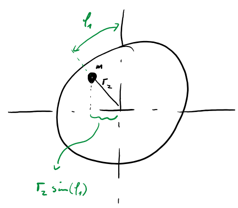
$$J\ddot\varphi_1\dot\varphi_1 + J\ddot\varphi_2\dot\varphi_2 + D^*\varphi_1\dot\varphi_1 + D^*\varphi_2\dot\varphi_2 + D^{**}\dot\varphi_1(\varphi_1-\varphi_2) + D^{**}\dot\varphi_2(\varphi_2-\varphi_1) = 0$$

$$\text{''} \qquad \text{''} \qquad + (D^*+D^{**})(\varphi_1\dot\varphi_1 + \varphi_2\dot\varphi_2) - D^{**}(\varphi_2\dot\varphi_1 + \varphi_1\dot\varphi_2) = 0$$

$$\Rightarrow \frac{d}{dt}\underbrace{\left[\frac{1}{2}J\dot\varphi_1^2 + \frac{1}{2}J\dot\varphi_2^2\right]}_{\frac{1}{2}J(\dot\varphi_1^2+\dot\varphi_2^2)} + \frac{d}{dt}\underbrace{\left[\frac{1}{2}D^*\varphi_1^2 + \frac{1}{2}D^*\varphi_2^2\right]}_{\frac{1}{2}D^*(\varphi_1^2+\varphi_2^2)} + \frac{d}{dt}\underbrace{\left[\frac{1}{2}D^{**}\varphi_1^2 + \frac{1}{2}D^{**}\varphi_2^2 - D^{**}\varphi_1\varphi_2\right]}_{\frac{1}{2}D^{**}(\varphi_1-\varphi_2)^2} = 0$$

$$\underset{\text{Integration}}{\Rightarrow} \quad \underbrace{\frac{1}{2}J(\dot\varphi_1^2+\dot\varphi_2^2)}_{E_{kin}} + \underbrace{\frac{1}{2}D^*(\varphi_1^2+\varphi_2^2) + \frac{1}{2}D^{**}(\varphi_1-\varphi_2)^2}_{E_{pot}} = E_0 = \text{const.}$$

Was, wenn ein Rad eine extra Masse bekommt?



$$\ddot\varphi_1 + \frac{b^*}{J}\dot\varphi_1 + \frac{D^*}{J}(\varphi_1-\varphi_2) - \frac{mg\,r_2\sin(\varphi_1)}{J} = 0$$

$\Rightarrow$ Zusatzterm zu E_{pot} von oben.