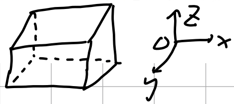
作业1. 声波方程的建立

如果媒质中存在体积流源，单位时间内流入单位体积里的质量为 $\rho_0 q(x,y,z,t)$，试导出有流源分布时的声波方程.



解. 对三维情况取一立方体微元

对于x.y.z方向分别讨论：

由x方向流入体积元质量为 $(\rho v_x)_x S_x$ 　其中 $S_x = dydz$

流出 ······ 为 $(\rho v_x)_{x+dx} S_x = \left[(\rho v_x)_x + \frac{\partial(\rho v_x)}{\partial x} dx\right] S_x$

y. z方向同理

已知单位时间注入体积元的流源质量为 $\rho_0 q(x,y,z,t)$

且质量单位时间的变化为 $\frac{\partial \rho}{\partial t} dx dy dz$

由质量守恒定律得

$$-\frac{\partial(\rho v_x)}{\partial x} dxdydz - \frac{\partial(\rho v_y)}{\partial y} dydxdz - \frac{\partial(\rho v_z)}{\partial z} dzdxdy + \rho_0 q = \frac{\partial \rho}{\partial t} dxdydz$$

$$\Rightarrow -\operatorname{div}(\rho v) + \rho_0 q = \frac{\partial \rho}{\partial t} \xrightarrow{\text{线性化}} -\operatorname{div}(\rho_0 v) + \rho_0 q = \frac{\partial \rho'}{\partial t} \quad ①$$

由运动方程：$\rho \frac{\partial v}{\partial t} = -\operatorname{grad} p$ 　即 $\rho_0(1+q)\frac{\partial v}{\partial t} = -\operatorname{grad}(p)$ ②

物态方程 $p = c_0^2 \rho'$ ③

对①求导：$-\operatorname{div}\left(\rho_0 \frac{\partial v}{\partial t}\right) + \rho_0 \frac{\partial q}{\partial t} = \frac{\partial^2 \rho'}{\partial t^2}$

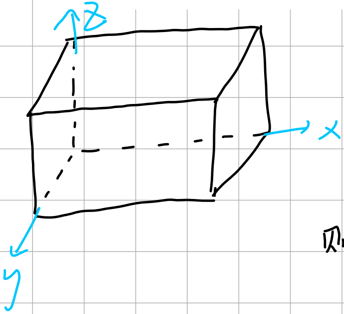
由③得 $\frac{\partial^2 \rho'}{\partial t^2} = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}$

由②得 $\rho_0 \operatorname{div}\left(\frac{\partial v}{\partial t}\right) = -\frac{1}{1+q} \nabla^2 p$

$\Rightarrow$ 整理后 $\frac{1}{1+q} \nabla^2 p + \rho_0 \frac{\partial q}{\partial t} = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}$

即 $\frac{1}{1+q} \nabla^2 p = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \rho_0 \frac{\partial q}{\partial t}$

如果媒质中有体力分布，设作用在单位体积媒质上的体力为 $F(x,y,z,t)$，试导出有体力分布时的声波方程。



对于x方向. $F_1 = P_x S_x = P_x dydz$

-x方向: $F_2 = P_{x+dx} S_x = P_{x+dx} dydz$

$P_x = P_0 + p$

$P_{x+dx} = P_0 + p + dp \quad dp = \frac{\partial p}{\partial x} dx$

则压强在x方向的合力是 $-\frac{\partial p}{\partial x} dx\, dydz$

同理. y方向 $-\frac{\partial p}{\partial y} dy\, dxdz$

z方向 $-\frac{\partial p}{\partial z} dz\, dxdy$.

而对于外力F.有x.y.z三方向的分量.

各方向的合力为 $-\frac{\partial p}{\partial x} dx\, dydz + \frac{\partial F}{\partial x} dx\, dydz$

注意. F是矢量. 本身有方向. 只需直接加上

则对于三维方程. 合力为 $-\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z}\right) dxdydz + \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\right) dxdydz$

由牛顿第二定律

$$\left(-\nabla p + \nabla F\right) dxdydz = \rho\, dxdydz \cdot \frac{\partial v}{\partial t}$$

$$\Rightarrow -\nabla(p-F) = \rho \frac{dv}{dt}$$

线性化, $-\nabla(p-F) = \rho_0 \frac{dv}{dt}$ ①

即 $-\nabla p + \nabla F = \rho_0 \frac{dv}{dt}$

又有
$$\begin{cases} -\mathrm{div}(\rho_0 v) = \frac{\partial \rho'}{\partial t} & ② \\ p = c_0^2 \rho' & ③ \end{cases}$$

① 求导 $-\nabla^2 p + \nabla^2 F = \rho_0\, \mathrm{div}\left(\frac{dv}{dt}\right)$

由②对t求导 $-\rho_0\, \mathrm{div}\left(\frac{dv}{dt}\right) = \frac{\partial^2 \rho'}{\partial t^2} \Rightarrow \rho_0\, \mathrm{div}\left(\frac{dv}{dt}\right) = -\frac{\partial^2 \rho'}{\partial t^2}$

由③对t求导 $\frac{\partial^2 p}{\partial t^2} = c_0^2 \frac{\partial^2 \rho'}{\partial t^2} \Rightarrow \frac{\partial^2 \rho'}{\partial t^2} = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}$

整理得 $-\nabla^2(p-F) = -\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}$

即 $\nabla^2(p-F) = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}$

作业2. 声波传播及隔声

3. 20 ℃ 时空气和水的特性阻抗分别为 $R_1 = 415 Pa \cdot s/m$ 及 $R_2 = 1.48\times10^6 Pa\cdot s/m$，计算平面声波由空气垂直入射于水面上时反射声压大小及声强透射系数。

(1)

$\rho_1 c_1 = R_1 = 415\ Pa\cdot s/m$

$\rho_2 c_2 = R_2 = 1.48\times10^6\ Pa\cdot s/m$

显然 $R_1 << R_2$ 是绝对硬边界

反射系数 $r_p = \frac{R_2 - R_1}{R_2 + R_1} \approx 1$ 即声压全反射. ⇒ 反射声压大小等于入射声压大小.

(2) 声强透射系数 $t_I = \frac{4R_1R_2}{(R_1+R_2)^2} = \frac{4\times415\times1.48\times10^6}{(415+1.48\times10^6)^2} \approx 1.12\times10^{-3}$

4. 声波由空气以 $\theta_i = 30^\circ$ 斜入射于水中，试问折射角为多大？θ_t 分界面上反射波声压于入射波声压之比为多少？r_p 平均声能量流透射系数为多少？

$c_1 = 344 m/s$ $c_2 = 1483 m/s$

1) 根据 Snell 折射定律 $\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_2}{k_1} = \frac{c_1}{c_2} \approx 0.23$

$\sin\theta_t = \frac{\sin\theta_i}{0.23} = \frac{0.5}{0.23} > 1$

则发生全反射. 不存在折射角 θ_t

2) $r_p = 1$

3) $t_W = 0$

5. 试求空气中厚为 1mm 的铁板对 200Hz 及 2000Hz 声波的声强透射系数 t_I(考虑垂直入射).

垂直入射 $t_I = \dfrac{4}{4\cos^2 k_2 D + (R_{12}+R_{21})^2 \sin^2 k_2 D}$

其中 $R_{12} = \dfrac{\rho_2 c_2}{\rho_1 c_1}$

$R_{21} = \dfrac{\rho_1 c_1}{\rho_2 c_2}$

$D = 1mm$

铁 $\rho = 7.7\times10^3\ kg/m^3$

$c = 4.35\times10^3\ m/s$

空气 $\rho_1 c_1 = 415\ N\cdot s/m^3$

① $f = 200Hz$

$k_2 = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c_2} = \dfrac{2\pi\times200}{4.35\times10^3} \approx 0.289$

$k_2 D = 0.289\times10^{-3} m = 2.89\times10^{-4} \ll 1$ $t_I \approx 1$

② $f = 2000Hz$

$k_2 = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c_1} = \dfrac{2\pi\times2000}{4.35\times10^3} \approx 2.89$

同理 $k_2 D = 2.89\times10^{-3} \ll 1$ $t_I \approx 1$

6. 空气中有一木质板壁，厚为 h，试问频率为 f 的声波的隔声量有多少？

$TL = 10\log\dfrac{1}{t_I}\ dB$

$t_I = \dfrac{4}{4\cos^2 k_2 h + (R_{12}+R_{21})^2 \sin^2 k_2 h}$

其中 $k_2 = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c_2}$ $R_{12} = \dfrac{\rho_2 c_2}{\rho_1 c_1}$ $R_{21} = \dfrac{\rho_1 c_1}{\rho_2 c_2}$

若满足质量律条件

则 $TL \approx -42 + 20\log f + 20\log M_2$ (dB)

设木质板密度为 ρ_2 则 $M_2 = \rho_2 h$

$\Rightarrow TL \approx -42 + 20\log f + 20\log(\rho_2 h)$ (dB)

7. 一骨导送话器的外壳用厚$1mm$的铁皮做成，试求这外壳对$1000Hz$气导声波的隔声量。

$$TL = 10\log\frac{1}{t_I}\ dB$$

$$t_I = \frac{4}{4\cos^2 k_2D + (R_{12}+R_{21})^2\sin^2 k_2D}$$

已知 $D = 1\times10^{-3}m$

$f = 1000Hz$

其中 $k_2 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c_2}$

$= \frac{2\pi\times1000}{3.7\times10^3}$

≈ 1.698

$R_{12} = \frac{\rho_2 c_2}{\rho_1 c_1}$

$R_{21} = \frac{\rho_1 c_1}{\rho_2 c_2}$

空气 $\rho_0 c_0 = 415\ N\cdot s/m^3$

铁 $\rho_2 = 7.7\times10^3\ kg/m^3$

$c_2 = 3.7\times10^3\ m/s$

$\Rightarrow R_{12} = \frac{7.7\times3.7\times10^6}{415} \approx 6.87\times10^4$

$R_{21} = \frac{415}{7.7\times3.7\times10^6} \approx 1.46$

$\Rightarrow k_2D = 1.698\times10^{-3} < 0.5$

$\Rightarrow \cos k_2D \approx 1 \quad \sin k_2D \approx k_2D$

$$TL \approx 10\log\left[1+\left(\frac{\omega M_2}{2\rho_1 c_1}\right)^2\right] dB$$

$$\approx 10\log\left[1+\left(\frac{6.28\times10^3\times7.7}{2\times415}\right)^2\right]$$

$$\approx 35.31dB$$

其中 $\omega = 2\pi f = 6.28\times10^3$

$M_2 = \rho_2\times D = 7.7\ kg/m^2$

作业3. 声管道作业

8. 有一声管在末端放一待测吸声材料，现用频率为 $500Hz$ 的平面声波，测得管中的驻波比 G 等于 10，并确定离材料表面 $0.25m$ 处出现第一个声压极小值. 试求该吸声材料的法向声阻抗率以及法向吸声系数.

$$\begin{cases}(-x)=(1+\sigma)\frac{\lambda}{4}\\ \lambda=\frac{c_0}{f}\end{cases}$$

$$\lambda=\frac{344}{500}=0.688m$$

$$0.25=(1+\sigma)\frac{0.688}{4}\Rightarrow\sigma\approx0.453$$

$$|r_p|=\left|\frac{1-G}{1+G}\right|=\left|\frac{1-10}{1+10}\right|=\frac{9}{11}$$

$$Z_s=\rho_0c_0\left[\frac{1+|r_p|e^{j\sigma\pi}}{1-|r_p|e^{j\sigma\pi}}\right]=415\times\left(\frac{1+\frac{9}{11}e^{j\cdot0.453\cdot\pi}}{1-\frac{9}{11}e^{j0.453\pi}}\right)$$

$$=96.104+j470.49$$

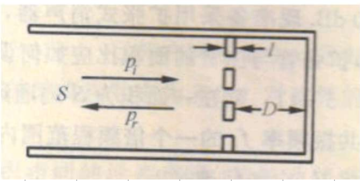
$Z_s=R_s+jX_s$ $\quad R_s=96.104$ $\quad X_s=470.49$

$$\alpha=\frac{4R_s\rho_0c_0}{(R_s+\rho_0c_0)^2+X_s^2}=\frac{4\times96.104\times415}{(96.104+415)^2+470.49^2}\approx0.331$$

也可以用 $\alpha=1-|r_p|^2$

9. 设在声管末端放一穿孔板共振吸声结构，如图所示，已知其共振频率为 500Hz，空腔深度 D=5cm，假设要求该吸声结构的吸声频带宽度为 2，试求该结构的声阻率比 x_s 以及在频率为 250，500，1 000Hz 时的吸声系数.

$$\left(z_1-z_2=\frac{1}{Q_R};\ Q_R=\frac{\lambda}{(1+x_s)2\pi D}\right)$$



$$\alpha_r=\frac{4x_s}{(1+x_s)^2}$$

$$\alpha=\frac{\alpha_r z^2}{z^2+[(z^2-1)Q_R]^2}$$

$$z=\frac{f}{f_r}$$

$Z_1 - Z_2 = 2 \Rightarrow \alpha_R = \frac{1}{2}$ $D = 5\times10^{-2} m$ $\lambda = \frac{c}{f}$

$X_s = \frac{c}{f \cdot 2\pi D \cdot \alpha_R} - 1$

其中 $f = f_r = 500Hz$

$X_s = \frac{344}{500\times2\times\pi\times5\times10^{-2}\times\frac{1}{2}} - 1 \approx 3.38$

$\alpha_r = \frac{4X_s}{(1+X_s)^2} \approx 0.7$

① $f = 250Hz$ $Z = \frac{f}{f_r} = 0.5$

$\alpha = \frac{0.7\times0.5^2}{0.5^2 + [(0.5^2-1)0.5]^2} \approx 0.448$

② $f = 500Hz$ 共振. $\alpha = \alpha_r = 0.7$

③ $f = 1000Hz$ $Z = \frac{f}{f_r} = 2$

$\alpha = \frac{0.7\times4}{4 + [(4-1)\times\frac{1}{2}]^2} \approx 0.448$

10. 设在一通风管道中传播着一频率为1000Hz 的声波，声压级为100dB . 现准备采用扩张式消声器, 把该声音消去 20 分贝, 试问扩张管的长度, 扩张管与主管的面积比应如何设计?

消声量公式 $TL = 10\lg[1 + \frac{1}{4}(S_{12} - S_{21})^2 \sin^2 kD]$ (dB)

已知 $TL = 20$

为实现消声量极大值

① $D = \frac{\lambda}{4} = \frac{c}{4f} = \frac{344}{4\times1000} \approx 0.086m$

此时消声量公式中 $kD = n\pi$ $\sin^2 kD = 1$

$TL_{max} = 10\lg[1 + \frac{1}{4}(S_{12} - S_{21})^2]$

$\Rightarrow 20 = 10\lg[1 + \frac{1}{4}(S_{12} - \frac{1}{S_{12}})^2]$

解得 $S_{12} \approx 19.95$

② 即 $\frac{S_2}{S_1} = 19.95$ 扩张管与主管面积比约为19.95.

求一元二次方程根
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{19.8997 \pm \sqrt{\cdots^2 + 4}}{2} \approx 19.95$

$x^2 - 19.8997x - 1 = 0$

$x^2 - 1 = 19.8997x$

$(x - \frac{1}{x})^2 = 396$

$x - \frac{1}{x} \approx 19.8997$

作业4 声波的辐射

11. 对于脉动球源，在满足 $kr_0 \ll 1$ 的情况下，如使球源半径比原来增加一倍，表面振速及频率仍保持不变，试问其辐射声压增加多少分贝？如果在 $kr_0 \gg 1$ 的情况下使球源半径比原来增加一倍，振速不变，频率也不变，试问声压增加多少分贝？

(1) $$p(r,t)=\frac{|A|}{r}e^{i(\omega t-kr+\theta)}$$

$kr_0 \ll 1$ 证明为低频条件.

$$|A_L| = \frac{\rho_0 c_0 k r_0^2}{\sqrt{(kr_0)^2+1}}u_0 \approx \rho_0 c_0 (kr_0) r_0 u_0 = \rho_0 c_0 k r_0^2 u_0$$

已知 u_0. k 不变.

则 $$|A_L|' = \rho_0 c_0 k (2r_0)^2 u_0 = 4\rho_0 c_0 k r_0^2 u_0$$

$$20\log|A_L|' - 20\log|A_L| = 20\log\frac{|A_L|'}{|A_L|} = 20\log 4 = 40\log 2 \approx 12\,dB$$

(2) $kr_0 \gg 1$ 是高频条件下.

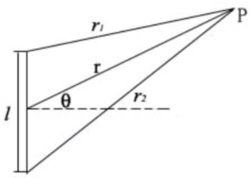
$$|A_H| = \rho_0 c_0 r_0 u_0$$

$$|A_H|' = \rho_0 c_0 \cdot 2r_0 \cdot u_0$$

$$20\log|A_H|' - 20\log|A_H| = 20\log 2 \approx 6.02\,dB$$

?

12. 求两个频率相同，源强相等，相位差 $\frac{\pi}{2}$ 的点声源相距为 l 时的远场辐射声压。



$p_1 = j\frac{k\rho_0 c_0 Q_0}{4\pi r_1} e^{j(\omega t - kr_1)}$ $\quad$ $p_2 = j\frac{k\rho_0 c_0 Q_0}{4\pi r_2} e^{j(\omega t - kr_2 + \frac{\pi}{2})}$

$$\Rightarrow p = p_1 + p_2 = j\frac{k\rho_0 c_0 Q_0}{4\pi r_1} e^{j(\omega t - kr_1)} + j\frac{k\rho_0 c_0 Q_0}{4\pi r_2} e^{j(\omega t - kr_2 + \frac{\pi}{2})}$$

$$\begin{cases} r_1 = r - \frac{l}{2}\sin\theta \\ r_2 = r + \frac{l}{2}\sin\theta \end{cases} \qquad \Delta = \frac{l}{2}\sin\theta \qquad \text{远场 } r \gg l$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$$\Rightarrow p \approx j\frac{k\rho_0 c_0 Q_0}{4\pi r} e^{j(\omega t - kr)} \left(e^{jk\Delta} + e^{-j(k\Delta - \frac{\pi}{2})}\right)$$

?

$$= j\frac{k\rho_0 c_0 Q_0}{4\pi r} e^{j(\omega t - kr)} \left[(\cos k\Delta - \sin k\Delta) - j(\cos k\Delta - \sin k\Delta)\right]$$

$$= j\frac{k\rho_0 c_0 Q_0}{4\pi r} e^{j(\omega t - kr)} \sqrt{2}(\cos k\Delta - \sin k\Delta) e^{-j\frac{\pi}{4}}$$

$$= j\frac{\sqrt{2}\, k\rho_0 c_0 Q_0}{4\pi r} e^{j(\omega t - kr - \frac{\pi}{4})} (\cos k\Delta - \sin k\Delta)$$

$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$

$\cos(\theta - \frac{\pi}{2}) = \sin\theta$

$\cos(\frac{\pi}{2} - \theta) = \sin\theta$

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\sin(\frac{\pi}{2} + \theta) = \cos\theta$

$e^{jk\Delta} = \cos k\Delta + j\sin k\Delta$

$e^{j(k\Delta - \frac{\pi}{2})}$

$= \cos(k\Delta - \frac{\pi}{2}) + j\sin(k\Delta - \frac{\pi}{2})$

$= \sin k\Delta - j\cos k\Delta$

作业5. 室内声场

13. 有一混响室已知空室时的混响时间为 T_{60}，平均吸声系数为 α_i，现在在某一壁面上铺上一层面积为 S'，平均吸声系数为 α'_i 的吸声材料，并测得该时室内的混响时间为 T'_{60}，试证明这层吸声材料的平均吸声系数可用下式求得

$$\alpha_i' = \frac{0.161V}{S_i'}\left(\frac{1}{T_{60}'} - \frac{1}{T_{60}}\right) + \alpha_i$$

根据赛宾公式

铺之前 $T_{60} = 0.161\frac{V}{S\alpha_i}$ $\quad\Rightarrow\begin{cases} S\alpha_i = \frac{0.161V}{T_{60}} \\ S\bar{\alpha}_i = \frac{0.161V}{T_{60}'} \end{cases}$

铺之后 $T_{60}' = 0.161\frac{V}{S\bar{\alpha}_i}$

$S\bar{\alpha}_i = (S - S')\alpha_i + S'\alpha_i'$

两式相减 $0.161V\left(\frac{1}{T_{60}} - \frac{1}{T_{60}'}\right) = S\alpha_i - S\bar{\alpha}_i$

$= S\alpha_i - S\alpha_i + S'\alpha_i + S'\alpha_i'$

$= S'(\alpha_i + \alpha_i')$

$$\Rightarrow 0.161V\left(\frac{1}{T_{60}} - \frac{1}{T_{60}'}\right) = S'(\alpha_i + \alpha_i')$$

则 $\alpha_i' = \frac{0.161V}{S'}\left(\frac{1}{T_{60}} - \frac{1}{T_{60}'}\right) - \alpha_i$

14. 有一$l_x \times l_y \times l_z = 10m \times 7m \times 4m$的长方体房间，已知室内的平均吸声系数$\bar{\alpha} = 0.2$，试求该房间的平均自由程，房间常数与混响时间（忽略空气吸收）。

①平均自由程 $\bar{l} = 4\frac{V}{S} = 4\frac{10\times7\times4}{2\times(10\times7+10\times4+7\times4)} = 4\frac{280}{2\times138}$
$\approx 4.06m$

②混响时间 $T_{60} \approx 0.161\frac{V}{-S\ln(1-\bar{\alpha})}$
$= 0.161\frac{280}{-2\times138\ln(1-0.2)} \approx 0.732s$

③房间常数 $R = \frac{S\bar{\alpha}}{1-\bar{\alpha}} = \frac{2\times138\times0.2}{1-0.2} = 69$

15. 有一体积为$l_x \times l_y \times l_z = 30m \times 15m \times 7m$的厅堂，要求它在空场时的混响时间为2s.

(1) 试求室内的平均吸声系数.

(2) 如果希望在该厅堂达到80dB的稳态混响声压级，试问要求声源辐射多少平均声功率(假设声源为无指向性的)?

(3) 假设厅堂中坐满400个观众，已知每个听众的吸声单位为$S\alpha_j = 0.5m^2$，问该时室内的混响时间变为多少?

(1) $T_{60} = \frac{0.161V}{S\bar{\alpha}}$ $\bar{\alpha} = \frac{0.161V}{T_{60}S} = \frac{0.161\times30\times15\times7}{2\times2\times[30\times15+30\times7+15\times7]}$
$= \frac{0.161\times3150}{2\times2\times765}$
≈ 0.166

{SPL 声压级.

~~(2)~~ 房间常数 $R = \frac{S\bar{\alpha}}{1-\bar{\alpha}} = \frac{2\times765\times0.166}{1-0.166} = \frac{253.98}{0.834} \approx 304.5$

则 $SPL = SWL - 10\log(\frac{4}{R}) = 80 - 10\log(\frac{4}{304.5}) \approx 98.82dB$

(3) $\bar{\alpha}' = \frac{\frac{0.161V}{T_{60}} + 400\times0.5}{S} = \frac{453.575}{2\times765} \approx 0.296$

$T_{60}' = 0.161\frac{V}{-S\ln(1-\bar{\alpha}')} = 0.161\frac{3150}{-2\times765\ln(1-0.296)} \approx 0.944s$

(2)' $10\log_{10}\bar{w} + \underset{26.18}{10\log 415} + 94 + \underset{-18.82}{10\log\left(\frac{4}{12}\right)} = 80$

$$10\log_{10}\bar{w} = -21.36$$

$$\log_{10}\bar{w} = -2.136$$

$$\bar{w} \approx 0.0073 \quad w$$

$w = 2\pi f$

$c = \lambda f$

$k = \frac{w}{c_0} = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$

4-1

	一维	三维
运动	$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$ ①	$\rho_0 \frac{\partial v}{\partial t} = -\mathrm{grad}\, p$ ①
连续	$-\rho_0 \frac{\partial v}{\partial x} = \frac{\partial \rho'}{\partial t}$ ②	$-\mathrm{div}(\rho_0 v) = \frac{\partial \rho'}{\partial t}$ ②
物态	$p = c_0^2 \rho'$ ③	$p = c_0^2 \rho'$ ③

1) 一维

① 对t求导 $\rho_0 \frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 p}{\partial x \partial t}$

② 对x求导 $-\rho_0 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 \rho'}{\partial t \partial x}$

③ 先对t再对x求导 $\frac{\partial^2 p}{\partial x \partial t} = c_0^2 \frac{\partial^2 \rho'}{\partial x \partial t}$

$\Rightarrow -\rho_0 \frac{\partial^2 v}{\partial t^2} = -c_0^2 \rho_0 \frac{\partial^2 v}{\partial x^2}$

$\Rightarrow \frac{\partial^2 v}{\partial t^2} = c_0^2 \frac{\partial^2 v}{\partial x^2}$

即 $\frac{\partial^2 v}{\partial x^2} = \frac{1}{c_0^2} \frac{\partial^2 v}{\partial t^2}$

2) 三维

① 对t求导 $\rho_0 \frac{\partial^2 v}{\partial t^2} = -\mathrm{grad} \frac{\partial p}{\partial t}$

③ 对t求导 $\frac{\partial p}{\partial t} = c_0^2 \frac{\partial \rho'}{\partial t}$

求梯度 $\mathrm{grad} \frac{\partial p}{\partial t} = c_0^2\, \mathrm{grad} \frac{\partial \rho'}{\partial t}$

$\Rightarrow \rho_0 \frac{\partial^2 v}{\partial t^2} = -c_0^2\, \mathrm{grad}[-\mathrm{div}(\rho_0 v)]$

$\Rightarrow \frac{1}{c_0^2} \frac{\partial^2 v}{\partial t^2} = \mathrm{grad}[\mathrm{div}(v)]$

4-4 $\rho_0 = \rho_0(x, y, z)$

x方向
- 流入质量 $(\rho v)_x S$
- 流入质量 $(\rho v)_{x+dx} S = \left[(\rho v)_x + \frac{\partial (\rho v)_x}{\partial x} dx\right] S$ $+ v \frac{\partial \rho}{\partial x} dx$

$S = dy\,dz$

质量变化量 $\frac{d\rho}{dt} dx\,dy\,dz$

$$\frac{\partial (\rho v)_x}{\partial x} dx(dy\,dz) + \frac{\partial (\rho v)_y}{\partial y} dy(dx\,dz) + \frac{\partial (\rho v)_z}{\partial z} dz(dx\,dy) = \frac{d\rho}{dt} dx\,dy\,dz$$

$$\Rightarrow \frac{\partial (\rho v)_x}{\partial x} + \frac{\partial (\rho v)_y}{\partial y} + \frac{\partial (\rho v)_z}{\partial z} = \frac{\partial \rho}{\partial t}$$ (连续性)

ρ_0和v都是位置的函数 $\left[\left(\frac{\partial (\rho v)}{\partial x} + v\frac{\partial \rho}{\partial x}\right) + \cdots\right]$

有 $\rho_0 \frac{\partial v}{\partial t} = -\mathrm{grad}\, p$ ①

$p = c_0^2 \rho'$ ③

若密度不均匀

$-[\mathrm{div}(\rho v) + v\,\mathrm{grad}\,\rho] = \frac{\partial \rho}{\partial t}$

↓线性化

$-[\mathrm{div}(\rho_0 v) + v\,\mathrm{grad}\,\rho_0] = \frac{\partial \rho'}{\partial t}$ ②

③对t求二阶导.

$\frac{\partial^2 p}{\partial t^2} = c_0^2 \frac{\partial^2 \rho'}{\partial t^2}$

②对t求导（只有v. ρ'是t的函数）

$-[\mathrm{div}(\rho_0 \frac{\partial v}{\partial t}) + \frac{\partial v}{\partial t}\mathrm{grad}\,\rho_0] = \frac{\partial^2 \rho'}{\partial t^2} = \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}$

①代入 $-[\mathrm{div}(-\mathrm{grad}\, p) + (-\frac{1}{\rho_0}\mathrm{grap}\,p\,\mathrm{grad}\,\rho_0)] = \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}$

$\Rightarrow \quad \nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\mathrm{grad}\,p\,\mathrm{grad}(\ln \rho_0)$ 正负号有问题

5-1 $|r_p| = \left|\frac{G-1}{G+1}\right| \rightarrow |r_p|$

$(-x) = (1+6)\frac{\lambda}{4} \rightarrow 6$

$Z_s = \frac{1+|r_p|e^{j6\pi}}{1-|r_p|e^{j6\pi}}\rho_0 c_0$

$\alpha = 1 - r_I = 1 - |r_p|^2 = \frac{(R_s-\rho_0 c_0)^2+X_s^2}{(R_s+\rho_0 c_0)^2+X_s^2}$

5-2 极小值 $2k(x+6\frac{\lambda}{4}) = \pm(2n+1)\pi$

极大值 $2k(x+6\frac{\lambda}{4}) = 2n\pi$ $\Big\}\ n=0,1,2\cdots$

$n=0$ $\begin{cases} 2k(x_1+6\frac{\lambda}{4}) = \pi \\ 2k(x_2+6\frac{\lambda}{4}) = 0 \end{cases} \Rightarrow \begin{array}{l} (-x_1) = \frac{\lambda}{4}(6-1) \\ (-x_2) = \frac{6\lambda}{4} \end{array}$

注意 $k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$

$\Rightarrow \frac{2\pi}{k} = \lambda \qquad \frac{\pi}{2k} = \frac{\lambda}{4}$

$\Rightarrow \Delta d = \frac{\lambda}{4}$

5-3

弹簧机械阻抗 $Z_m = j(\omega M_m - \frac{K_m}{\omega})$

↓对应声阻抗 $Z_a = \frac{Z_m}{S_1^2}$

↓ …… 率 $Z_s = Z_a \cdot S$

由于 $Z_s = \frac{1-r_p}{1+r_p}\rho_0 c_0$

$r_p = \frac{Z_s - \rho_0 c_0}{Z_s + \rho_0 c_0}$

5-4 $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{C_a M_a}}$

声容 $C_a = \frac{V}{\rho_0 c_0^2}$ $V = SD$

声质量 $M_a = \frac{\rho_0 l}{S_0}$

5-5. $\omega_r = 2\pi f_r$

声阻 $R_a = \frac{R_s}{S} = \frac{X_s \rho_0 c_0}{S}$

共振式 $Q_R = \frac{\omega_r M_a}{R'}$ $R' = R_a + \rho_0 c_0/S$

5-6 吸声频带宽度：$Z_1 - Z_2 = \frac{1}{Q_R}$

$$Q_R = \frac{\lambda}{(1+X_s)\,2\pi D} \qquad \lambda = \frac{c}{f} = \frac{2\pi}{k}$$

$$\omega = 2\pi f$$

$$\alpha_r = \frac{4X_s}{(1+X_s)^2}$$

$$\alpha = \frac{\alpha_r Z^2}{Z^2 + [(Z^2-1)Q_R]^2} \qquad Z = \frac{f}{f_r}$$

5-7 (1) p_i p_r v_i v_r

边界 $\begin{cases} x=l & v_i + v_r = 0 \\ x=D & v_i + v_r = u_a e^{j\omega t} \end{cases}$

(2) 声源力阻抗 Z_m

管口声阻抗 Z_{a0} ⇐ $Z_{a0} = \frac{\rho_0 c_0}{S} \frac{Z_{al} + j\frac{\rho_0 c_0}{S}\tan kl}{\frac{\rho_0 c_0}{S} + jZ_{al}\tan kl} \approx -j\frac{\rho_0 c_0}{S}\cot(kl)$

↓ 对应力阻抗 $Z_{a0}S^2$

总力阻抗 $Z_m + Z_{a0}S^2 = Z_w$

力振幅 F_a

↓ 声源振速 $u = \frac{F_a e^{j\omega t}}{Z_w}$

声源振幅 $u_a = \frac{u}{e^{j\omega t}}$

5-8 声功率透射系数

$$t_w = \frac{I_t S_2}{I_i S_1} \qquad \begin{cases} I_t = \frac{p_{ta}^2}{2\rho_2 c_2} S_2 \\ I_i = \frac{p_{ia}^2}{2\rho_1 c_1} S_1 \end{cases}$$

$$I = \frac{p_a^2}{2\rho c} = \frac{p_e^2}{\rho c}$$

5-9 扩张管式消声器的消声量公式 TL

$$TL = 10\log_{10}\frac{1}{t_I}$$

$$= 10\log_{10}\left[1 + \frac{1}{4}(S_{12} - S_{21})^2 \sin kD\right]^2$$

5-10 $kD=(2n-1)\frac{\pi}{2}$ $\quad k=\frac{2\pi}{\lambda}$

$\Rightarrow D=(2n-1)\frac{\lambda}{4}$ 消声量极大值

$TL_{max}=10\log_{10}[1+\frac{1}{4}(S_{12}-S_{21})^2]$ (dB)

$S_{12}=\frac{S_2}{S_1}$ $\quad S_{21}=\frac{S_1}{S_2}$

↓ 扩张管与主管的面积比

5-12 共振式消声器

$TL=10\log_{10}\frac{1}{t_I}=10\log_{10}[1+\frac{\beta^2 z^2}{(z^2-1)^2}]$ (dB)

$\beta=\frac{w_r V_b}{2C_0 S}$ $\quad z=\frac{f}{f_r}$

倍程：$z=2$ $\quad TL=20dB$

↓

$k=\frac{2\pi}{\lambda}$ $\quad w=2\pi f$ $\quad k=\frac{w}{c}=\frac{2\pi f}{c}$

$\beta'=\frac{w_r V_b}{2C_0 S}=\frac{\cancel{2}\pi f_r V_b}{\cancel{2}C_0 S}$

$V_b>\frac{\beta' C_0 S}{\pi f_r}$

5-12. $t_I=\frac{R_b^2+X_b^2}{(\frac{\rho_0 C_0}{2S}+R_b)^2+X_b^2}$ → 推导

$X_b=wM_b-\frac{1}{wC_b}$ $\quad M_b=\frac{\rho_0 l}{S_b}$

$C_b=\frac{V_b}{\rho C_0^2}$

由 $\chi_s=\frac{R_b S}{\rho_0 C_0}$ $\quad \beta=\frac{w_r V_b}{2C_0 S}$ $\quad f_r=\frac{1}{2\pi}\sqrt{\frac{1}{M_b C_b}}$

$TL=10\lg\frac{1}{t_I}=\cdots$

5-13. $\underbrace{\rho_b c_b}_{\text{声阻抗率}}=Z_s=S_b Z_b$ 声阻抗.

$\Rightarrow$ 质点速度 $v_b=\frac{p_b}{\rho_b C_0}=\frac{p_b}{S_b Z_b}$

$$t_I = \frac{R_b^2 + X_b^2}{\left(\frac{\rho_0 c_0}{2S} + R_b\right)^2 + X_b^2}$$

对于旁支管，末端封闭 $Z_{bl} = \infty$

⇒ 管口声阻抗 $Z_b = Z_{b0} = -j\frac{\rho_0 c_0}{S} ctg(kl)$

⇒ X_b . . R_b

代回 t_I

$TL = 10\lg\frac{1}{t_I}$

5-14 活塞特性 → $Z_{al}(a_1)$

阻抗连续 $= Z_{al}(a)$ ⟶ 阻抗转移 → $Z_{a0}(a)$

阻抗连续 $Z_{a0}(a_0)$

↓ 对应力阻抗 $Z_m' = Z_{a0} \cdot S_0^2$

+ 自身机械阻抗 $Z_m = j\left(\omega M_m - \frac{K_m}{\omega}\right)$

⇒ 总力阻抗 $Z_w = Z_m + Z_m'$

⇒ 表面振幅 $u = \frac{F}{Z_w} = \frac{F_a e^{j\omega t}}{Z_w}$

$u = u_a e^{j\omega t}$

⇒ 平均声辐射功率 $W = \frac{1}{2} Re(Z_m') u_a^2$

5.15

主管道 $x = l_1 + l_2 + l_3$ 声阻抗 $Z_{a3} = \frac{\rho_0 c_0}{S}$

阻抗连续 ↘ 2支管 $x = l_1 + l_2 + l_3$ 声阻抗 $= Z_{a3}$

↓ 声阻抗转移

2支管 $x = l_1 + l_2$ (管口) $Z_{a2} = \frac{\rho_0 c_0}{S_1} \frac{Z_{a3} + j\frac{\rho_0 c_0}{S_1}\tan(kl_3)}{\frac{\rho_0 c_0}{S_1} + jZ_{a3}\tan(kl_3)}$

已知 $l_3 = \frac{\lambda}{4}$ ⇒ $\tan kl_3 = \tan\frac{2\pi}{\lambda}\frac{\lambda}{4}$

$= \tan\frac{\pi}{2} = \infty$ 代入 $Z_{a2} = \frac{S\rho_0 c_0}{S_1}$

↓ 阻抗连续

主管 $x = l_1 + l_2$ Z_{a2}

↓ 转移 主管 $x = l_1$ $Z_{a1} = \frac{\rho_0 c_0}{S} \frac{Z_{a2} + j\frac{\rho_0 c_0}{S}\tan(kl_2)}{\frac{\rho_0 c_0}{S} + jZ_{a2}\tan(kl_2)} = \frac{\rho_0 c_0 S_1^2}{S^3}$

连续

1支管 $x=l_1$ Z_{a1}

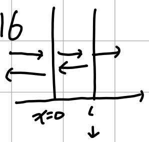
1支管 $x=0$ $Z_{a0} = \frac{\rho_0 c_0}{S_1} \frac{Z_{a1} + j\frac{\rho_0 c_0}{S}\tan(kl_1)}{\frac{\rho_0 c_0}{S_1} + jZ_{a1}\tan(kl_1)} = \frac{\rho_0 c_0 S_1^2}{S^3}$

$$\Rightarrow t_I = \frac{4R_a S\rho_0 c_0}{(R_a S+\rho_0 c_0)^2 + X_a^2 S^2}$$

$$TL = 10\lg\frac{1}{t_I} = 10\lg\left(\frac{S^2+S_1^2}{2SS_1}\right)^2$$ 若 $S=S_1$，消声量为0

声阻抗 Z_a → 声强透射系数 $t_I=\alpha$ → 消声量 TL

5-16



$x=l$ Z_{al}

l 中间层 $x=l$ 连续 Z_{al}

$x=0$ 转移 Z_{a0}

↓ $t_I=\alpha$ → TL

5-17

$S(x)=S_0 e^{\delta x}$ $r(x)=\sqrt{\frac{S_0}{\pi}}e^{\frac{\delta}{2}x}$

喉部半径 $a_0=2cm$

$S_0 = \pi a_0^2 = 4\pi\times10^{-4} m^2$

$\frac{S(1)}{S(0)} = 100 = e^{\delta}$ $\delta=\ln 100$ 蜿蜒指数

截止频率 $f_c = \frac{\delta c_0}{4\pi}$

5-18 (1) $R_{a0} = \frac{\rho_0 c_0}{S_0}\sqrt{1-\left(\frac{\delta}{2k}\right)^2}$

$\overline{W} = \frac{1}{2}(R_{a0}S_0^2)u_a^2$ u_a是声源速度振幅

位移振幅 $\xi = \frac{u_a}{\omega}$

(2) 活塞辐射阻 $R_r = \frac{\rho_0 c_0 k^2}{2\pi}(\pi a_0^2)$

$\overline{W} = \frac{1}{2}R_r u_a^2$

5-19 已知 $S(x)=S_0(1+\frac{x}{h})^2$

$S(x)=\pi r(x)^2$

得 r 随 x 变化规律 $r(x)$　$A(x)=\frac{1}{r(x)}$

$p(x)=A(x)e^{\pm j\gamma x}$

$\frac{r''}{r}=k^2-\gamma^2$

截止频率

$P(P_a)\to v(v_a)\to Z_a=\frac{p}{vS}\to R_a\xrightarrow{R_{a0}}\overline{W}=\frac{1}{2}(R_{a0}S_0^2)u_a^2$

看是否有平方根 / 满足物理意义的限制

5-20 $S(x)\to A(x)\sim r(x)$

5-21 $p=p_0e^{j\omega t}[e^{-\alpha x}e^{-jkx}+e^{\alpha x}e^{jkx}]$

波节
波腹

$\sinh(x)=\frac{e^x-e^{-x}}{2}$

$\cosh(x)=\frac{e^x+e^{-x}}{2}$

5-22 (1) $\alpha=\frac{1}{ac_0}\sqrt{\frac{\eta\omega}{2\rho_0}}$

a 管半径 2cm　　η 空气 $\frac{\eta}{\rho_0}=15.6\times10^{-6}m^2/s$

(c_0 声速 ρ_0 空气密度)　$\omega=2\pi f$　$f=100.500.1000$

(2) 单位长度压强差 $-\frac{\partial p}{\partial x}$

$l.\ \rho.\ a.\ \omega.\ \eta\ \rho_0.$

$\downarrow R_a.\ X_a.\ \to Z_a\to Z_s$

力阻抗 $Z_m=Z_s\cdot S^2$

单位长度 $Z_m'=\frac{Z_m}{l}$

$Z_m'\bar{v}=(-\frac{\partial p}{\partial x})S$

5-24 $l_x\times l_y=0.4\times0.4$　　已知 $c_0=344m/s$

要求

$f_{n_xn_y}=\frac{c_0}{2}\sqrt{(\frac{n_x}{l_x})^2+(\frac{n_y}{l_y})^2}$

比 $f_{10}\cdot f_{01}$ 低

5-25 f_{10} 比较 f 与各次波的大小关系

f_{01} 判断所有波

$\therefore f_{02}$

$\therefore f_{20}$

5-26 振速 $u \to u_a$

$$\to \left.\begin{matrix} B_{n_x n_y} \\ B_{n_x 0} \\ B_{0 n_y} \\ B_{00} \end{matrix}\right\}$$ 振幅 $A_{n_x n_y} = \boxed{\omega \frac{\rho_0}{k_z}} B_{n_x n_y}$

5-27 $\times$ $p_{n_x n_y} = A_{n_x n_y} \cos\frac{n_x\pi}{l_x}x \cos\frac{n_y\pi}{l_y}y\, e^{-\alpha_{n_x n_y} z} e^{j\omega t}$

$p_{n_x n_y} = A_{n_x n_y} \cos k_x x \cos k_y y\, e^{j(\omega t - k_z z)}$

$z=0$ 处 $p_{n_x n_y} = A_{n_x n_y} \cos k_x x \cos k_y y$.

$z=0$ $u(t) \to u_a$

$\to \begin{cases} B_{00} \\ B_{10} \\ B_{01} \\ \cancel{B_{11}} \end{cases}$

$k_x = \frac{n_x}{l_x} \cdots\cdots$

$A_{n_x n_y} = \frac{\rho_0}{k_z} B_{n_x n_y}$

对角次简正波 (不再是c_0

$p_{00} = A_{00} = \frac{\rho_0 \omega}{k_z} B_{00} = \rho_0 \cdot c_0 \cdot \frac{2}{\lambda} u_0$

$k = \frac{2\pi}{\lambda}$ $\omega = 2\pi f = \frac{2\pi c}{\lambda}$ $= \frac{2\rho_0 c_0}{\pi} u_0$

$\frac{\omega}{k} = \frac{2\pi c}{\lambda} \frac{\lambda}{2\pi} = c_0$

$k_x l_x = n_x \pi$ 即 $n_x = \frac{k_x l_x}{\pi}$ $k_x = \frac{n_x l_x}{\pi}$

$k^2 = k_x^2 + k_y^2 + k_z^2$ $k = \frac{\omega}{c}$

则 $k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n_x l_x}{\pi}\right)^2 - \left(\frac{n_y l_y}{\pi}\right)^2}$

6-1 $|A| = \frac{\rho_0 c_0 k r_0^2 u_a}{\sqrt{1+(kr_0)^2}}$

① $kr_0 \ll 1$

$|A|_L \approx \rho_0 c_0 k r_0^2 u_a$

② $kr_0 \gg 1$

$|A|_H \approx \rho_0 c_0 r_0 u_a$

$$SPL = 20\log_{10} \frac{p_e}{p_{ref}}$$

6-2 (1) 声压幅值 $p_a = \left|\frac{A}{r}\right|$

$L = 20\lg \frac{p_e}{p_a}$

$L_1 - L_2 = 20\lg \frac{p_{e_1}}{p_{e_2}} = 20\lg \frac{p_{a_1}}{p_{a_2}} = 20\lg \frac{r_2}{r_1}$

① $20\lg \frac{2r}{r} = 20\lg 2$ dB

② $20\lg 4 = 40\lg 2$ dB

③ $20\lg 10 = 20$ dB.

(2) $\begin{cases} 1m \\ 2m \end{cases}$ $\Delta L = 20\lg 2$

$\begin{cases} 100m \\ 101m \end{cases}$ $\Delta L = 20\lg \frac{101}{100}$

6-3 $|A| = \frac{\rho_0 c_0 k r_0^2 u_a}{\sqrt{1+(kr_0)^2}}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ $r_0 = 0.01$ $u_a = 0.05 m/s.$

$\rho_0 c_0 = 415$

① $p_a = \frac{|A|}{r}$

$p = p_a e^{j(wt - kr + \theta)}$

$w = 2\pi f$ $\theta = \arctan(\frac{1}{kr_0})$

$SPL = 20\lg \frac{p_e}{p_{ref}}$ $p_e = \frac{p_a}{\sqrt{2}}$

② $v_{ra} = p_a \frac{\sqrt{1+(kr)^2}}{\rho_0 c_0 kr}$

$\xi = \frac{v_{ra}}{w}$

③ 辐射声功率

$\overline{W} = \frac{1}{2} R_r u_a^2$

$R_r = \rho_0 c_0 \frac{k^2 r_0^2}{1 + k^2 r_0^2}$

6-4 (1) 声压级 SPL = 74 dB $\rightarrow p_e \xrightarrow{\sqrt{2}} p_a$

$\rightarrow |A| = \frac{\rho_0 c_0 k r_0^2 u_a}{\sqrt{1+(kr_0)^2}} \Rightarrow u_a$

(2) $\overline{W} = \frac{1}{2} R_r u_a^2$ $\quad \overline{W} = \frac{p_a^2}{2\rho_0 c_0} 4\pi r^2$

$R_r = \rho_0 c_0 \frac{(kr_0)^2}{1+(kr_0)^2} S_0$

6-5 $p_e \rightarrow p_a$

$\overline{W_r} = \frac{p_a^2}{2\rho_0 c_0} 4\pi r^2$

$\parallel \frac{p_e^2}{\rho_0 c_0} 4\pi r^2$

6-6 (1) 同振质量 $M_r = \frac{X_r}{\omega}$.

$\begin{cases} X_r = \rho_0 c_0 \frac{kr_0}{1+(kr_0)^2} S_0 \\ \omega = 2\pi f \end{cases}$

(2) $M_m = 0.01 \times 2$

6-7 $K = \frac{2\pi f}{c} = \frac{2\pi \times 100}{344}$ $\quad r_0 = 0.005$

$kr_0 \ll 1$

$R_r \approx \rho_0 c_0 (kr_0)^2 S_0$

$\overline{W} = \frac{1}{2} R_r u_a^2$

6-8 $u_a = 0.08$ $f = 1000\text{Hz}$ $r_0 = 0.005\text{m}$

$\overline{W} = \overline{W_1} + \overline{W_2} = 2 \times \frac{1}{2} \rho_0 c_0 S_0 (kr_0)^2 \left(1 + \frac{\sin kl}{kl}\right) u_a^2$

$\therefore l$

6-9 $\overline{W} = \frac{2}{3} \pi \rho_0 c_0 k^4 r_0^4 l^2 u_a^2$

6-10 $\overline{W} = \frac{p_e^2}{\rho_0 c_0} 4\pi r^2 = \frac{2\pi}{\rho_0 c_0} |A|^2$

因为 $p_a = \frac{|A|}{r}$ $\quad p_e = \frac{|A|}{\sqrt{2} r}$ $\quad p_e^2 = \frac{|A|^2}{2r^2}$

(1) 由 $\overline{W} \rightarrow |A|$

$p_a = \frac{|A|}{r} e^{j(\omega t - kr + \theta)}$

$\theta = \arctan \frac{1}{kr_0}$ $\quad k = \frac{2\pi f}{c}$

(2) 水相对于空气是硬边界.
垂直入射水面：全反射

$p = 2p_a$.

6-11 证明：

$$p_1 = j\frac{k\rho_0 c_0}{4\pi r_1} Q_{01} e^{j(wt-kr_1)}$$

$$p_2 = j\frac{k\rho_0 c_0}{4\pi r_2} Q_{02} e^{j(wt-kr_2)}$$

$$r_1 = r - \frac{l}{2}\cos\theta = r-\delta$$

$$r_2 = r + \frac{l}{2}\cos\theta. = r+\delta \qquad \delta = \frac{l}{2}\cos\theta$$

r 对幅值不敏感，对相位敏感

$$p = p_1+p_2 \approx j\frac{k\rho_0 c_0 Q_{01}}{4\pi r} e^{j[wt-k(r-\delta)]} + j\frac{k\rho_0 c_0 Q_{02}}{4\pi r} e^{j[wt-k(r+\delta)]}$$

$$= j\frac{k\rho_0 c_0}{4\pi r} e^{j(wt-kr)}\left[Q_{01}e^{jk\delta} + Q_{02}e^{-jk\delta}\right]$$

$$= Q_{01}(\cos k\delta + j\sin k\delta) + Q_{02}(\cos k\delta - j\sin k\delta)$$

$$= \cos k\delta\,(Q_{01}+Q_{02}) + j\sin k\delta\,(Q_{01}-Q_{02}).$$

$$\cos k\delta = \cos\frac{kl}{2}\cos\theta \qquad k = \frac{2\pi}{\lambda}$$

$$= \cos\frac{\pi l}{\lambda}\cos\theta$$

得证！

6-13

$$p_1 = j\frac{k\rho_0 c_0 Q_0}{4\pi r_1} e^{j(wt-kr_1)}$$

$$p_2 = j\frac{k\rho_0 c_0 Q_0}{4\pi r_2} e^{j(wt-kr_2+\frac{\pi}{2})}$$

6-14 软边界 ⇒ 反相虚声源 相距 D 与 3D 两对

6-15 硬边界 ⇒ 同相虚声源.

解：由镜像原理知，绝对软边界对声源的影响等效于一个反相的的虚声源 . 由声压叠加原理得远场任意 p 点得声压表达式为

$$p = \left(\frac{A}{r_+}e^{j(wt-kr_+)} - \frac{A}{r_-}e^{j(wt-kr_-)}\right) + \left(\frac{A}{r'_+}e^{j(wt-kr'_+)} - \frac{A}{r'_-}e^{j(wt-kr'_-)}\right)$$

其中，$r_+ \approx r - \frac{3D}{2}\cos\theta$，$r_- \approx r - \frac{D}{2}\cos\theta$，$r'_+ \approx r + \frac{D}{2}\cos\theta$，$r'_- \approx r + \frac{3D}{2}\cos\theta$.

考虑远场的声压时，即假设 r>>D，则由四个小球源辐射的声波达到观察点 p 时，振幅差别甚小，可用 r 代替 r+，r-，r+'，r-'，但是它们对相位的差异不能忽略 .

证明：由镜像原理知，绝对硬边界对声源的影响等效于一个同相的的虚声源。根据同相小球声场叠加，分别得两个相距 3D 的正相小球声场

$$P_1 = \frac{A}{r} e^{j(wt-kr)} \cdot 2\cos(\frac{3kD}{2}\cos\theta)$$

两个相距 D 的负相小球声场

$$P_2 = -\frac{A}{r} e^{j(wt-kr)} \cdot 2\cos(\frac{kD}{2}\cos\theta)$$

则远场 $P = P_1 + P_2 = \frac{2A}{r} e^{j(wt-kr)} [\cos(\frac{3kD}{2}\cos\theta) - \cos(\frac{kD}{2}\cos\theta)]$

6-16 $D(\theta) = \left|\frac{\sin kn\Delta}{n\sin k\Delta}\right|$

$k = \frac{2\pi}{\lambda}$

$L = (n-1)l$

$\Delta = \frac{l}{2}\sin\theta$

$$\frac{\sin kn\Delta}{n\sin k\Delta} = \frac{\sin\frac{2\pi}{\lambda}(\frac{L}{l}+1)\frac{l}{2}\sin\theta}{(\frac{L}{l}+1)\sin\frac{2\pi}{\lambda}\frac{l}{2}\sin\theta}$$

$n\to\infty$

$L>>l$

$\frac{L}{l}+1\to\frac{L}{l}$

$$= \frac{\sin\frac{\pi L}{\lambda}\sin\theta}{\frac{\pi L}{\lambda}\sin\theta}.$$

6-17 $l = 20cm - 8cm$

$\frac{R_1}{R_{11}} > 1$ 影响为正 辐射阻增加

$\frac{R_1}{R_{11}} < 1$ 影响为负 辐射阻减小

6-18. ~~声柱的辐射声压为~~

$$p = \frac{A}{r} e^{j(wt-kr)} \frac{\sin kn\Delta}{\sin k\Delta}$$

~~$n\to\infty$ 近似为线声源~~

看作无数个点声源的组合

$$p(x,y) = \frac{A}{r_1} e^{j(wt-kr_1)}$$

$|A| = \frac{jk\rho_0 c_0 Q_0}{4\pi}$ $\quad r_1 = \sqrt{x^2-(y-y_i^2)}$

由声压的叠加原理

$$p(x,y) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{A}{\sqrt{x^2+(y-y_i)^2}} e^{j(wt-k\sqrt{x^2+(y-y_i)^2})} dy_i.$$

6-19 由于 $\bar{w} = \frac{p_e^2}{\rho_0 c_0} 4\pi r^2$ $p_e^2 = \frac{\bar{w}\rho_0 c_0}{4\pi r^2}$

→ p是对θ积分(因为它无限长)

→积分得 p_e 与 r_0 关系.

$r_0 = r_1\cos\theta$

$dx\cos\theta = r_1 d\theta$ $dx = \frac{r_1 d\theta}{\cos\theta}$

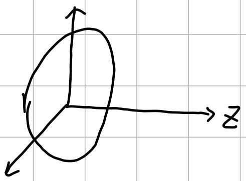
$p = \int_{x_1}^{x_1} \frac{\rho_0 c_0 w_1}{4\pi r_1^2} dx$

$= \int_{\theta_1}^{\theta_2} \frac{\rho_0 c_0 w_1}{4\pi}\left(\frac{\cos\theta}{r_0}\right)^2 \frac{r_0}{\cos\theta} d\theta$

6-20 $p_e^2 = \frac{\bar{w}\rho_0 c_0}{4\pi r^2}$ 每点对位置积分(直接).

$\Rightarrow p^2 = \int_{x_1}^{x_2} \frac{\bar{w}\rho_0 c_0}{4\pi(r_0^2+x^2)} dx.$

6-21



$\bar{p}^2 = \iint_D \frac{\rho_0 c_0}{4\pi r^2} dw$

$= \iint_D \frac{\rho_0 c_0}{4\pi r^2} \frac{w}{\pi a^2} ds = \iint_D \frac{\rho_0 c_0 w}{4\pi^2 a^2} \frac{1}{z^2+\rho^2} ds.$

$r = z^2 + \rho^2$

$= \int_0^{2\pi} d\theta \int_0^a \frac{\rho_0 c_0 w}{4\pi a^2} \frac{\rho}{z^2+\rho^2} d\rho.$

6-22 (1) $\begin{cases} D \to a = \frac{D}{2} \\ f \to k = \frac{2\pi f}{c} \end{cases}$ $ka \le 1$ $ka\ 1\sim 3.$

(2). 主声束角宽度 θ

$\theta = 2\arcsin \frac{0.61\lambda}{a}$ 无角斗.

(3) 扬声器临界距离 z_g

$a \ll \lambda$ $z_g = \frac{a^2}{\lambda}$

6-23 $p = j\omega \frac{\rho_0 u_a a^2}{2r}\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right] e^{j(\omega t - kr)}$

$u_a = u_0\left(1 - \frac{\rho^2}{a^2}\right)$

6-24 从 6-5-4 $\Rightarrow$ 6-5-5.

$$u_a = u_0\left(1-\frac{\rho^2}{a^2}\right)^n \cdot e^{j\omega t}$$

6-5-5 变成

$$p = j\frac{\omega\rho_0 u_0}{2\pi r} e^{j(\omega t - kr)} \int_0^a \rho\left(1-\frac{\rho^2}{a^2}\right)^n d\rho \int_0^{2\pi} e^{jk\rho\sin\theta\cos\varphi} d\varphi$$

用柱贝塞尔函数性质. 化简 积分

6-25 $p(b) - p(a)$

6-26

$$p = j\omega\frac{\rho_0 u_a a^2}{2r}\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]e^{j\omega - kr}$$

$\omega = 2\pi f$ $k = \frac{2\pi f}{c}$ $\theta = 0$

$u_a = 0.002\,m/s$

$a = 15cm = 0.15m$

$r = 1m$

$Z_g = \frac{a^2}{\lambda}$ $r = 1m > Z_g \Rightarrow$ 用远场近似公式得 p

(1) $L_p = 20\lg\frac{p_a/\sqrt{2}}{p_{ref}}$

(2) $\overline{W} = \frac{1}{2}R_r u_a^2$

u_a已知 $ka<1$ 时 $R_r = \frac{\rho_0 c_0 k^2}{2\pi}(\pi a^2)^2$

(3) 同振质量 $M_r = X_r/\omega$

$X_r = \rho_0 c_0 \pi a^2 \left(\frac{8}{3\pi}ka\right)$

6-27. ① 指向特性 $D(\theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta} = \frac{p_a(\theta)}{p_a(0)}$

$ka = 5$ 已知

声压级 $L_p = 20\log_{10}\frac{p_e}{p_{ref}}$

声强级 $L_I = 10\log_{10}\frac{I}{I_{ref}}$

声强 $I = \frac{\overline{W}}{S} = \frac{p_e^2}{\rho_0 c_0}$

$L_{I_1} - L_{I_2} = 3dB \Rightarrow 10\lg_{10} \frac{I_1}{I_2} = 3$

$I_1/I_1 = 10^{\frac{3}{10}}$

$\Rightarrow p_{e_1}/p_{e_2} = 10^{\frac{3}{20}}$

$D(\theta) = 10^{\frac{3}{20}}$

② $I = \frac{1}{8} \rho_0 c_0 u_a^2 (ka)^2 \frac{a^2}{r^2} \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$

$L_2 - L_1 = 3dB$.

$\Rightarrow 10\lg \frac{I_2}{I_1} = 3 = 10\lg \dfrac{\left[\frac{2J_1(ka\sin\theta_2)}{ka\sin\theta_2}\right]^2}{\left[\frac{2J_1(ka\sin\theta_1)}{ka\sin\theta_1}\right]^2}$

代入 $ka = 0 \quad \theta_1 = 0$

解得 θ_2

6-28 $a = 10cm \quad r = 20m \quad \theta = 0 \quad f = 100Hz \quad L_p = 80dB \to p_e \to p_a$

$k = \frac{2\pi f}{c} = \frac{2\pi \times 100}{344}$

$a = 0.1m$

$ka \approx 0.18 < 1 \Rightarrow R_r \ X_r$

$p_L \approx j\omega \frac{\rho_0 u_a a^2}{2r} e^{j(\omega t - kr)} \Rightarrow u_a$

声功率 $\bar{W}_L = 2\pi r^2 \frac{p_{Le}^2}{\rho_0 c_0}$

同振质量 $M_r = \frac{X_r}{\omega}$

$SWL = SPL + 10\lg \frac{2\pi}{\rho_0 c_0} + 20\lg r + 26$

$SWL = 10\lg \frac{\bar{W}}{\bar{W}_0}$

$\bar{W}_0 = 10^{-12} \Rightarrow \bar{W}$

$r > Z_g = \frac{a^2}{\lambda}$

用远场公式

6-29 半径 a. 振速幅值 u_a $ka < 1$ 低频情况

×

(1) $ka < 1$ 时 $D \approx 1$

$p_L = j\omega \frac{\rho_0 u_a a^2}{2r} e^{j(\omega t - kr)}$

辐射声功率 $\bar{W}_L = 2\pi r^2 \frac{p_{Le}^2}{\rho_0 c_0}$ $p_{Le} = \frac{p_L}{\sqrt{2}}$

(2) $r = 1m$ 代入 $|p_{La}| = \omega \frac{\rho_0 u_a a^2}{2r}$

(3) $|p_{La}| = \omega \frac{\rho_0 u_a a^2}{2r}$

$\omega = 2\pi f$ $\begin{cases} f_0 \\ >> f_0 \end{cases}$ $\lambda = \frac{c}{f_0}$ 大. $\lambda = \frac{c}{f_0}$ 小

$Z_g = \frac{a^2}{\lambda}$ 近声场 远声场

$$p_{Na} = \rho_0 c_0 k a u_a$$

$$p_{Fa} = 2\rho_0 c_0 u_a \frac{ka^2}{4z} = \frac{\rho_0 c_0}{2z} k a^2$$

(4) $L_{p_1} - L_{p_2} = 20\lg \frac{p_1}{p_2}$

(5) $p_e = 0.2\,Pa \rightarrow p_a = j\omega \frac{\rho_0 u_a a^2}{2r} e^{j(\omega t - kr)}$

$r = 1m \quad \omega = 2\pi f \quad a = 0.12m \quad \Rightarrow u_a$

同振质量 $M_r = \frac{X_r}{\omega} \quad \Rightarrow X_r, R_r.$

$u_a = \frac{F_{合}}{Z_r}$ 　声阻抗 $Z_r \rightarrow$ 力阻抗 $Z_r \cdot S^2$

$F = \frac{1}{2} Z \cdot u_a^2$

6-32 已知 $u \rightarrow p \rightarrow v_r$

$I = \overline{Re\,p \cdot Re\,v_r}$

$\overline{W} = \int I\,ds$

8-1 (1)平均自由程 $\bar{l}=\frac{4V}{S}$

(2)房间常数 $R=\frac{S\bar{\alpha}}{1-\bar{\alpha}}$

(3)混响时间 $T_{60}=0.161\frac{V}{-S\ln(1-\bar{\alpha})}$

8-2 (1) $\bar{\alpha}=(S-S_{木})\bar{\alpha}_1+S_{木}\bar{\alpha}_2$

修正 $T_{60}=0.161\frac{V}{S\bar{\alpha}+8\alpha V}$

$=0.161\frac{V}{S\bar{\alpha}^*}$

$\bar{\alpha}^*=\bar{\alpha}+\frac{4mV}{S}$

20℃ 50% : $4m=0.0244$ (4kHz)

<1kHz 忽略不计 $T_{60}=0.161\frac{V}{-S\ln(1-\bar{\alpha})}\approx 0.161\frac{V}{S\bar{\alpha}}$

8-3 $T_{60}=0.161\frac{V}{S\alpha}$

$T_{60}'=0.161\frac{V}{S'\alpha'+(S-S')\alpha}$

8-4

忽略

$SPL=10\lg\bar{W}+10\lg\rho_0c_0+94+10\lg\left(\boxed{\frac{1}{4\pi r^2}}+\frac{4}{R}\right)$

$R=\frac{S\bar{\alpha}}{1-\bar{\alpha}}$

$S=2(l_xl_y+l_yl_z+l_zl_x)\geq 6\sqrt[3]{l_x^2l_y^2l_z^2}$

8-5

稳态混响平均声能密度 $\bar{\varepsilon}_R=\frac{4\bar{W}}{Rc_0}$

声源辐射的平均能量密度 $\bar{\varepsilon}_D=\frac{\bar{W}}{4\pi r^2c_0}$

(直达声+混响声叠加) 总平均能量密度 $\bar{\varepsilon}=\frac{p_e^2}{\rho_0c_0^2}$

$\bar{\varepsilon}=\bar{\varepsilon}_D+\bar{\varepsilon}_R$

→忽略直达声 $\bar{\varepsilon}=\bar{\varepsilon}_R\Rightarrow p_e^2=\frac{4\rho_0c_0\bar{W}}{R}$

又 $\frac{4}{R}\approx\frac{4T_{60}}{0.161V}$ (8-1-3)

$p_e^2=\rho_0c_0\bar{W}\frac{4T_{60}}{0.161V}$

$\Rightarrow \bar{W}=p_e^2\frac{0.161V}{4\rho_0c_0T_{60}}$

$\approx 10^{-4}p_e^2\frac{V}{T_{60}}$

8-6 $T_{60} = 0.161 \frac{V}{S\bar{\alpha}}$

(1) $T_{60} = 2s$ $V.\ S \longrightarrow \bar{\alpha}$

(2) $SPL = 10\lg \bar{W} + 10\lg \rho_0 c_0 + 94 + 10\lg(\frac{4}{R})$

$R = \frac{S\bar{\alpha}}{1-\bar{\alpha}}$ $p_e^2 = \bar{W}\rho_0 c_0 \frac{4}{R}$ 声压级 $SPL = 10\lg\frac{p_e^2}{p_0^2}$

$SPL = 80dB$ $\rightarrow \bar{W}$

(3) $\bar{\alpha}' = (S_1\bar{\alpha} + S a_j \times 400)/S$

$T_{60}' = 0.161\frac{V}{-S\ln(1-\bar{\alpha})}$

(4) $R' = \frac{S\bar{\alpha}'}{1-\bar{\alpha}'}$ $L_p = 10\lg\frac{p_e^2}{p_0^2}$

(5) $p_e^2 = \bar{W}\rho_0 c_0(\frac{4}{R}+\frac{1}{4\pi r^2})$ $L_p = 10\lg\frac{p_e^2}{p_{ref}^2}$

$r = 3m\ /10m$

8-7 $T_{60} = 0.161\frac{V}{S\bar{\alpha}}$

$T_{60}'' = 0.161\frac{V}{S\bar{\alpha}''}$

$SPL = 10\lg \bar{W} + 10\lg\rho_0 c_0 + 94 + 10\lg\frac{4T_{60}}{0.161V}$

$\Delta L_p = 10\lg\frac{T_{60}''}{T_{60}}$

8-8 混响室 $L = L_w + 10\lg(\frac{1}{4\pi r^2} + \frac{4}{R})$

自由声场 $L = L_w + 10\lg\frac{1}{4\pi r^2}$

$$L - L_0 = 10\lg\frac{\frac{1}{4\pi r^2}+\frac{4}{R}}{\frac{1}{4\pi r^2}}$$

$$= 10\lg(1+\frac{16\pi r^2}{R})$$

8-9 $L_p = L_w + 10\lg(\frac{1}{S}+\frac{4}{R})$

$L_w = L_p - 10(\frac{1}{S}+\frac{4}{R}) \rightarrow \frac{R+4S}{SR}$

$= L_p + 10\frac{SR}{R+4S} = L_p + 10\frac{S}{1+\frac{4S}{R}}$ dB

8-10 (1) $SPL = 10\lg\frac{p_e^2}{p_0^2} = 60dB \Rightarrow p_e^2$

$p_e^2 = \overline{w}\rho_0 c_0\left(\frac{1}{4\pi r^2}+\frac{4}{R}\right) \Rightarrow \overline{w}$

$\overline{\varepsilon} = \overline{\varepsilon_D}+\overline{\varepsilon_R} = \frac{\overline{w}}{4\pi r^2 c_0} + \left(\frac{4\overline{w}}{Rc_0}\times 51\right)$

对话声 $p_e^2 = \overline{w}\rho_0 c_0\left(\frac{1}{4\pi r^2}+\frac{4}{R}\right)$

语噪声 $p_e^2 = \overline{w}\rho_0 c_0\left(\frac{4}{R}\times 50\right)$

$\Delta L = 10\lg\frac{\frac{4}{R}\times 50}{\frac{1}{4\pi r^2}+\frac{4}{R}}$

(2) 不能.

(3) $SPL' = 10\lg\frac{p_{e2}^2}{p_{e0}^2}$ $\quad p_e^2 = \overline{w}\rho_0 c_0\left(\frac{1}{4\pi r^2}+\frac{4}{R}\right)$

$\left.\Delta L = 10\lg\frac{\frac{4}{R}\times 50}{\frac{1}{4\pi r^2}+\frac{4}{R}}\right) r = 0.1m$

8-11 $L_p = L_w + 10\lg\left(\frac{1}{4\pi r^2}+\frac{4}{R}\right)$

法2. $\left.\begin{aligned}\overline{\varepsilon_D} &= \frac{\overline{w}}{4\pi r^2 c_0}\\ \overline{\varepsilon_R} &= \frac{4\overline{w}}{Rc_0}\\ \overline{\varepsilon} &= \overline{\varepsilon_D}+\overline{\varepsilon_R}\end{aligned}\right)$ $\quad p_e^2 \overset{\frac{4}{2}}{=} \overline{w}\rho_0 c_0\left(\frac{1}{4\pi r^2}+\frac{4}{R}\right)$

$L_p = 10\lg\frac{p_e^2}{p_0^2}$

8-12 S. V. $L = 4(L_x+L_y+L_z)$

df内的简正频率数:

$dN = \left(\frac{4\pi f^2 V}{c_0^3}+\frac{\pi f S}{2c_0^2}+\frac{L}{8c_0}\right)df$

频率f. 频带df

8-13 简正频率

$f_n = \frac{c_0}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2+\left(\frac{n_y}{L_y}\right)^2+\left(\frac{n_z}{L_z}\right)^2}$ $\quad$ 斜向波 $T_{60} = \frac{0.161V}{\Sigma\alpha S}$

8-14
8-15 混响时间 $T_{60} = \frac{0.161V}{\frac{1}{2}D_{nx}\alpha_x S_x + \frac{1}{2}D_{ny}\alpha_y S_y + \frac{1}{2}D_{nz}\alpha_z S_z}$

8-16 $$p = -\frac{\rho_0 c_0^2 \omega Q_0}{V} j \sum_{n_x=0} \sum_{n_y=0} \sum_{n_z=0} D_{n_x} D_{n_y} D_{n_z} \frac{\psi_{n_x n_y n_z}}{(\omega^2 - \omega^2_{n_x n_y n_z})} e^{j\omega t}$$

$\omega = \frac{2\pi f}{c}$ Q_0. V 已知.