Recent advances in DiD Part I: Why is TWFE terrible & what could you use instead

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What is DID again?

- Canonical DiD is the difference of the differences between a treated group before and after (1st diff) and an untreated group before and after (2d diff)
- Causal interpretation of the ATT requires parallel trends & constant treatment effects
- Used in various contexts, including when multiple groups are receiving the treatment at different points of time
- Often estimated with OLS with two-way fixed effects - time + individual effects

What is the 2 x2 DiD estimator doing

- The 2 groups, 2 time periods estimator is a 2x2 comparison

$$\delta_{T,U} = (E[Y^1_{T,post}] - E[Y^0_{T,pre}]) - (E[Y^0_{U,post}] - E[Y^0_{U,pre}])$$

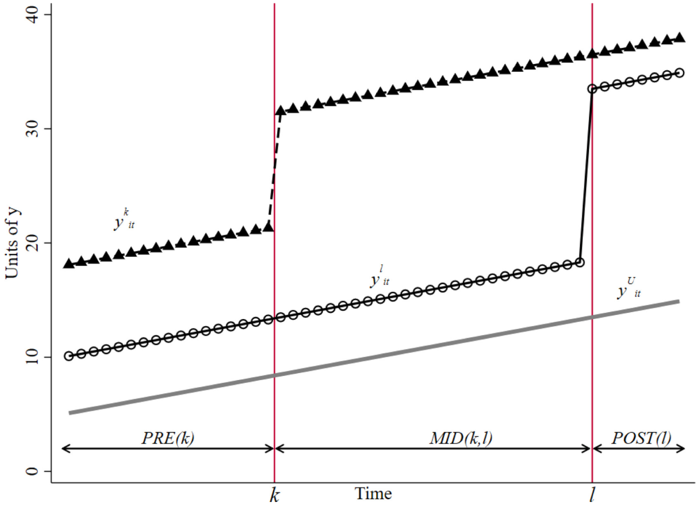
- Reorganizing

$$\delta_{T,U} = (E[Y^1_{T,post}] - E[Y^0_{T,post}]) + \underbrace{(E[Y^0_{T,post}] - E[Y^1_{T,pre}])}_{\text{Non-parallel trends bias}} - (E[Y_{U,post}] - E[Y_{U,pre}])$$

What is the multi group, multiple timing DiD estimator doing?

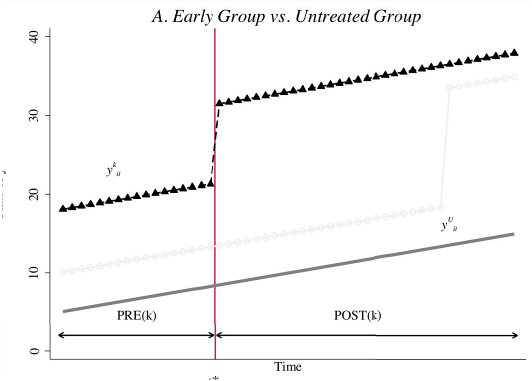
- The multiple groups, multiple time periods DiD estimator is less obvious...
- TWFE estimates a weighted average of all the 2x2 comparisons
- Goodman-Bacon (2021) TWFE weights are a function of sample sizes of each “group” and the variance of the treatment dummies for those groups

Example with 3 groups, 3 times periods



Example II

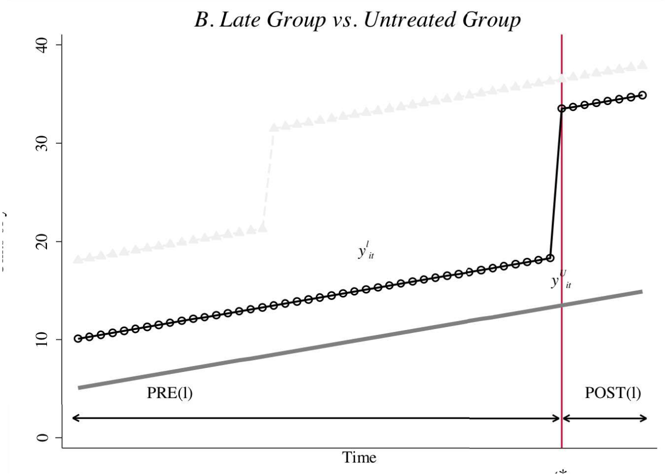
$$\widehat{\delta}_{kU}^{2x2} = \left(\overline{y}_k^{post(k)} - \overline{y}_k^{pre(k)}\right) - \left(\overline{y}_U^{post(k)} - \overline{y}_U^{pre(k)}\right)$$



Annotations courtesy of Scott Cunningham ☞ Check his substack

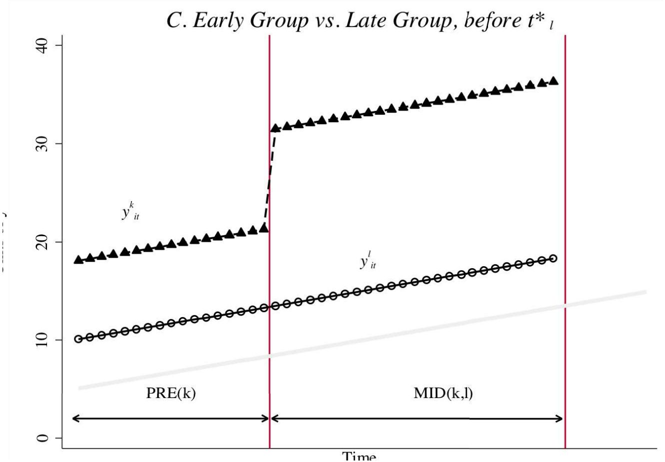
Example III

$$\widehat{\delta}_{lU}^{2x2} = \left(\overline{y}_l^{post(l)} - \overline{y}_l^{pre(l)}\right) - \left(\overline{y}_U^{post(l)} - \overline{y}_U^{pre(l)}\right)$$



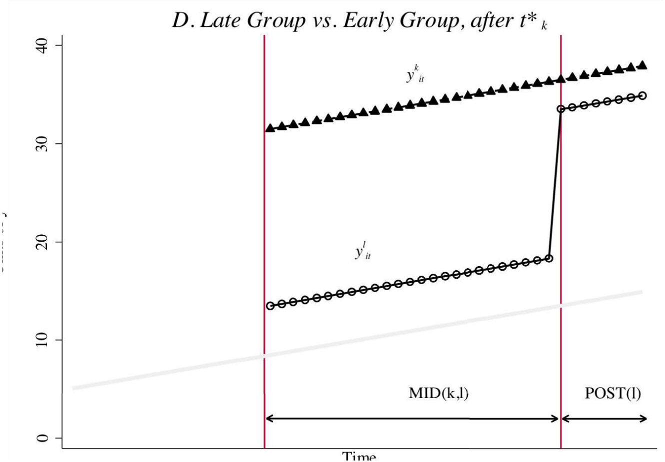
Example IV

$$\delta_{kl}^{2x2,k} = \left(\overline{y}_k^{MID(k,l)} - \overline{y}_k^{Pre(k,l)}\right) - \left(\overline{y}_l^{MID(k,l)} - \overline{y}_l^{PRE(k,l)}\right)$$



Example V

$$\delta_{lk}^{2x2,l} = \left(\overline{y}_l^{POST(k,l)} - \overline{y}_l^{MID(k,l)}\right) - \left(\overline{y}_k^{POST(k,l)} - \overline{y}_k^{MID(k,l)}\right)$$



Estimator decomposition

- Goodman-Bacon (2021) considers δ_{DiD} for $k = \{1, ..., K\}$ groups of units ordered by the time when they receive a binary treatment $k \in (1, T]$.
- The OLS estimates given by a TWFE regression is

$$\hat{\delta}_{D,D} = \sum_{k \neq U} s_{kU} \hat{\delta}_{k,U} + \sum_{k \neq U} \sum_{l > k} s_{kl} \left[\mu_{kl} \hat{\delta}^{l}_{k,l} + (1 - \mu_{kl}) \hat{\delta}^{k}_{k,l} \right]$$

- Weights s_{kU}, μ_{kl} , s_{kl} are the issue:
  - function of the groups' sample sizes & the time spent in treatment
  - Group variation matters more than unit variation, within-group treatment variance
  - Time spent in the panel matters

Weights

$$s_{kU} = \frac{n_k n_u \bar{D}_k (1 - \bar{D}_k)}{\widehat{Var}(\tilde{D}_{it})}$$

$$s_{kU} = \frac{n_k n_u (\bar{D}_k - \bar{D}_l)(1 - (\bar{D}_k - \bar{D}_l))}{\widehat{Var}(\tilde{D}_{it})}$$

$$\mu_{kl} = \frac{(1 - \bar{D}_k)}{(1 - (\bar{D}_k - \bar{D}_l))}$$

- n's are the sample sizes
- $\bar{D}$'s are the time spent in the panel
- $(1 - \bar{D}_k)$...variance of treatment - max weight when close to 0.5

What does it mean for the DiD assumptions?

- Let's look at the δ's highlighted above to show potential sources of bias

$$\hat{\delta}_{k,U} = \text{ATT}_k(Post) + \underbrace{\triangle Y_k^0(Post(k), Pre(k)) - \triangle Y_U^0(Post, Pre)}_{\text{Non parallel trends bias}}$$

$$\hat{\delta}_{k,l}^k = \text{ATT}_k(Mid) + \underbrace{\triangle Y_k^0(Mid(k), Pre(k)) - \triangle Y_l^0(Mid(l), Pre(l))}_{\text{Non parallel trends bias}}$$

$$\hat{\delta}_{k,l}^l = \text{ATT}_l(Post) + \underbrace{\triangle Y_l^0(Post(l), Mid) - \triangle Y_k^0(Post(l), Mid)}_{\text{Non parallel trends bias}} - \underbrace{ATT_k(Post) - ATT_k(mid)}_{\text{Heterogeneity bias}}$$

Weights- again

- Others have looked at the TWFE DID estimand Borusyak and Jaravel (2017), De Chaisemartin and d'Haultfoeuille (2020)
- Negative weights arise because the control group used is treated - this is an issues in non staggered design as well (De Chaisemartin and d'Haultfoeuille (2020))
- Negative weights if ATEs for early treated units are larger than the ATEs on later treated units
- Issue if ATEs are heterogeneous across periods- sign of the δ_{TWFE} can be the opposite of those of most ATEs.

Recap

- Need a lot of parallel trends assumptions
- Issue of heterogeneity of ATEs - both for the bias of the estimator and the estimand changing sign

What should we do?

- The literature on this is growing non stop - Exciting!
- Solutions based around changes in the grouping :
  - Callaway and Sant'Anna (2020) focuses treatment effect dynamics, parallel trends holds only after conditioning on observables
  - De Chaisemartin and d'Haultfoeuille (2020) is more general and focuses on cases where ATEs are heterogeneous across time or groups

Set-up

- T periods going from $t = 1; ...; T$
- Units are either treated $(D_t = 1)$ or untreated $(D_t = 0)$ but once treated cannot revert to untreated state
- G_g group dummy, =1 if treated in t
- C is a dummy =1 if the control group is never treated

Callaway and Sant'Ana (CS) estimator

- Define a ”Group-Time Average Treatment Effect Parameter”

$$ATT(g,t) = E\left[Y_t(g) - Y_t(0) | G_g = 1\right]$$

- Average treatment effect for units who are members of a particular group g at a particular time period t
- Inverse propensity weighted long-difference

CS - Assumptions

1. Irreversibility of Treatment
2. Sampling is iid (panel data)
3. Limited Treatment Anticipation
4. Conditional parallel trends - 2 versions on never treated and yet untreated

$$E\left[Y_t(0) - Y_{t-1}(0) | X, G_g = 1\right] = E\left[Y_t(0) - Y_{t-1}(0) | X, C = 1\right]$$

5. Common support (propensity score)

CS -III

$$ATT(g,t) = E\left[\left(\frac{G_g}{E[G_g]} - \frac{\frac{p_g(X)C}{1-p_g(X)}}{E\left[\frac{p_g(X)C}{1-p_g(X)}\right]}\right)(Y_t - Y_{g-1})\right]$$

- Can use outcome regression (OR), inverse probability weighting (IPW), or doubly robust (DR) estimands to recover the ATTs
- Can use the time period where g was untreated as reference time under Assumption 3 and either Assumption 4 or 5.
- Avoid using already treated as comparison group
- Available in R ☞ Github Repo

CS -IV

- Might need to aggregate the ATTs across cohort - how to pick weights?
  - Event study like: average effect of participating in the treatment t time periods for each t
  - Effect so far: cumulative ATE among the units that have been treated before a certain time
- Interpretation issues: composition of group changes, ...

Chaisemartin and d'Hautefeuille (DID_M) estimator

- Consider groups by treatment status
- Focus on the ATE of all switching ”cells”
  - For staggered designs, the average of TEs at the time when a group starts receiving the treatment, across all groups that become treated at some point
- Stata packages *multiplegt*, *twowayfeweights*
- Check it out ☞ here

DID_M assumptions

1. Strong exogeneity - ie, shock independents of treatment status
2. Common trends for $Y(1)$ - ie, expectation of the outcome with treatment follow the same evolution in each group
3. Existence of "stable groups": a treated group to compares with group leaving treatment, an untreated group for group switching to treated
4. Mean Independence between a group's outcome and other groups treatments

So, *what* should we do ?

- If staggered design, use on a non TWFE estimation:
  - De Chaisemartin and d'Haultfoeuille (2020)Sun and Abraham (2020), Callaway and Sant'Anna (2020), Borusyak and Jaravel (2017),...
- When using TWFE, check the weights!
  - De Chaisemartin and d'Haultfoeuille (2020) describe a test based on SDs of ATE and weights
  - When the statistic is close to 0, sensitivity to heterogeneity in TE (*twowayfeweights*)
- Try looking into ATEs and think about what goes into the aggregate

Borusyak, Kirill and Xavier Jaravel, "Revisiting event study designs," *Available at SSRN 2826228*, 2017.

Callaway, Brantly and Pedro HC Sant'Anna, "Difference-in-differences with multiple time periods," *Journal of Econometrics*, 2020.

Chaisemartin, Clément De and Xavier d'Haultfoeuille, "Two-way fixed effects estimators with heterogeneous treatment effects," *American Economic Review*, 2020, *110* (9), 2964–96.

Goodman-Bacon, Andrew, "Difference-in-differences with variation in treatment timing," *Journal of Econometrics*, 2021.

Sun, Liyang and Sarah Abraham, "Estimating dynamic treatment effects in event studies with heterogeneous treatment effects," *Journal of Econometrics*, 2020.