COMPUTER ENGINEERING MASTER DEGREE

COMPUTER ARCHITECTURE

PROJECT DISCUSSION

INTEGER FACTORIZATION

PROFESSORS

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SYLLABUS

ALGORITHM > GOALS > CPU > GPU

ALGORITHM

INTEGER FACTORIZATION

«Every positive integer can be written uniquely as a product of primes»

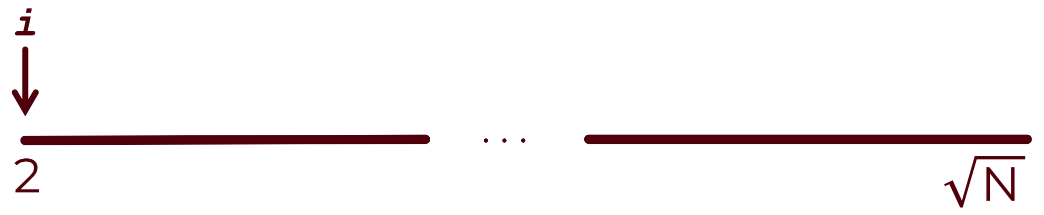
MATHEMATICAL FORMULATION

TRIAL DIVISION

[SEQUENTIAL]

INPUT

N : Number to be Factorized



```
    while N mod $i$ == 0
        N = N / $i$
        Add $i$ to Primes
if N ≠ 1
    Add N to Primes
```

OUTPUT

Primes : Product of primes

INTEGER FACTORIZATION

«Every positive integer can be written uniquely as a product of primes»

MATHEMATICAL FORMULATION

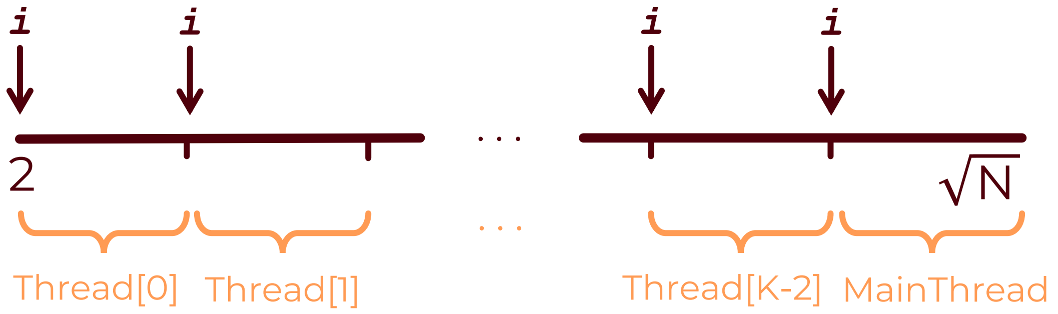
TRIAL DIVISION

[PARALLEL]

INPUT

N : Number to be Factorized

K : Number of Threads



OUTPUT

Primes : Product of primes 🔒 Mutex

GOALS

GOALS

END USER

Execution Time

≤ **1** second

For numbers up to

18

DIGITS

e.g. **975734686214396237**

DEVELOPER

>

Develop a GPU version that **outperforms** the CPU version

CPU

IMPLEMENTATION

11th Gen intel® Core(TM) **i5**-11400

x Cores per socket: **6**

= Threads per core: **2**

TOTAL # LOGICAL CORES: **12**

L1**d** Cache: **288** KiB

L1**i** Cache: **192** KiB

L2 Cache: **3** MiB

L3 Cache: **12** MiB

Base Frequency: **2.60** GHz

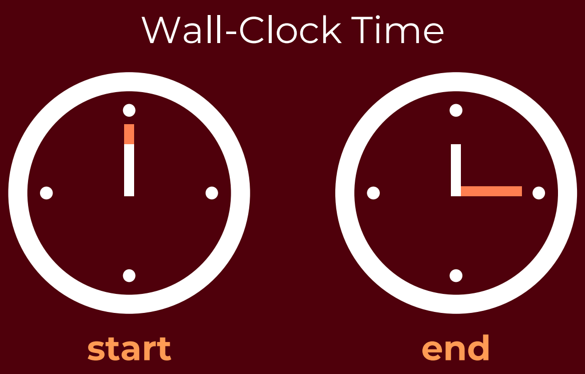
Max Turbo Frequency: **4.40** GHz

HARDWARE DETAILS

TOOLS

Execution Time

```
chrono::steady_clock::time_point start = chrono::steady_clock::now();
parallelTrialDivision(NUMBER, NUM_THREADS);
chrono::steady_clock::time_point end = chrono::steady_clock::now();

chrono::milliseconds duration = chrono::duration_cast<chrono::milliseconds>(end - start);
```



Intel **VTune** Profiler

Visual Studio Code

FIRST EXECUTION

#THREADS : **12**

$$\underbrace{975734686214396237}_{\textbf{18}\text{ DIGITS}} = 748609 * 1303396948493$$

EXECUTION TIME **568340** MILLISECONDS ≈ **9** MINUTES >> **1** SECOND

⚠ REQUIREMENT **NOT** SATISFIED

LET'S GO INTO **DETAIL!**

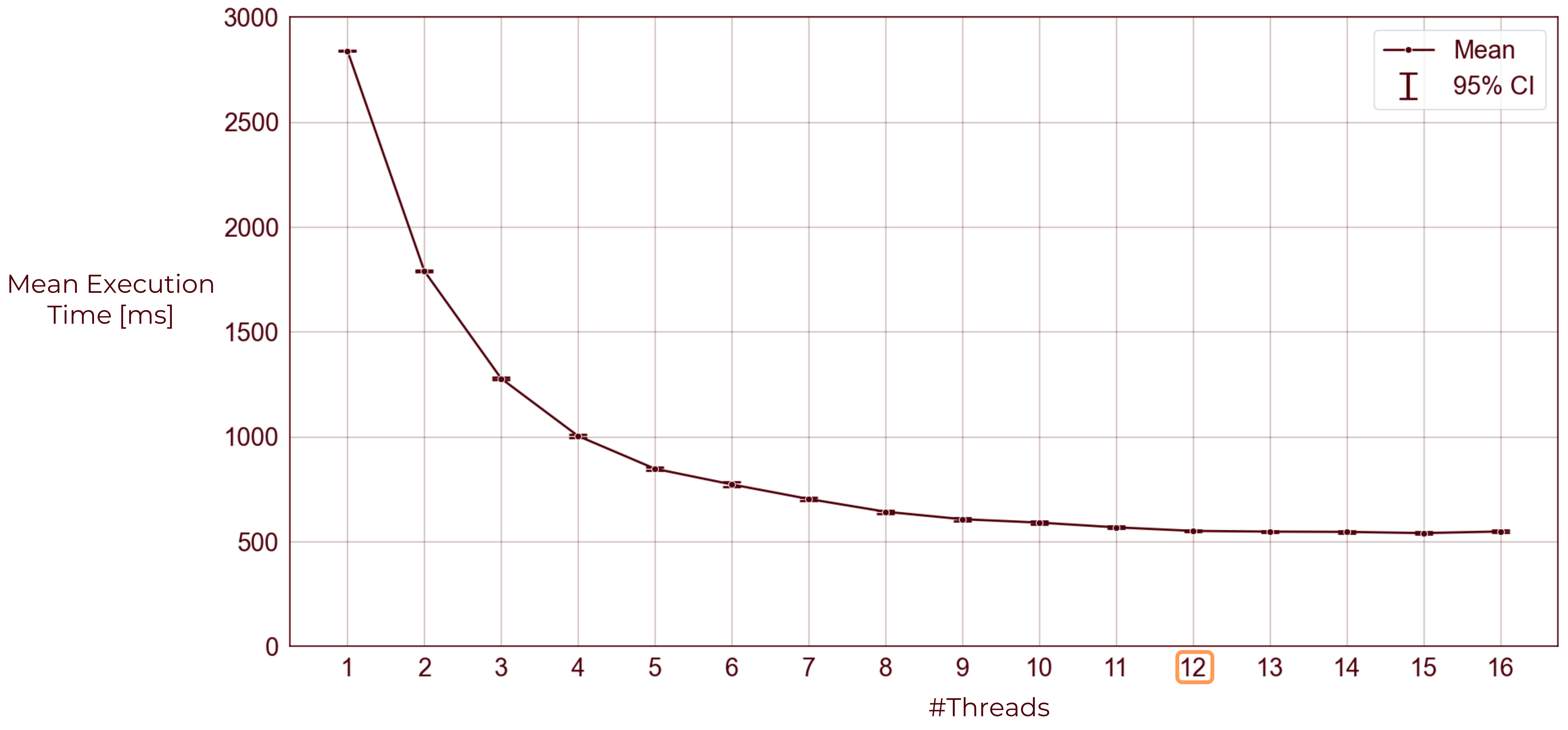
EXECUTION TIME

FIRST VERSION

#ITERATIONS : **30**

59999999999991 = 3 * 19999999999997

(**14** DIGITS)



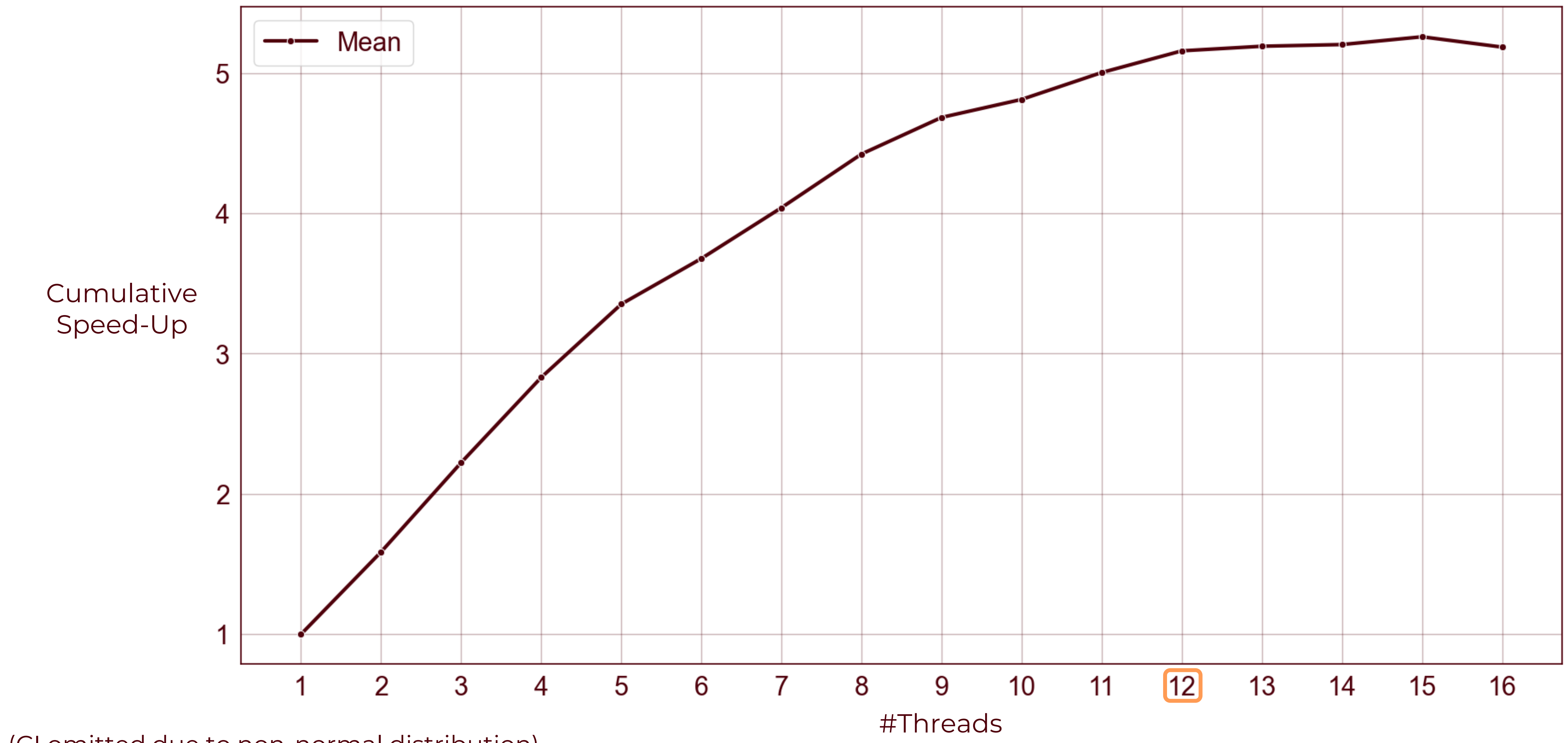
SPEED-UP

FIRST VERSION

#ITERATIONS : **30**

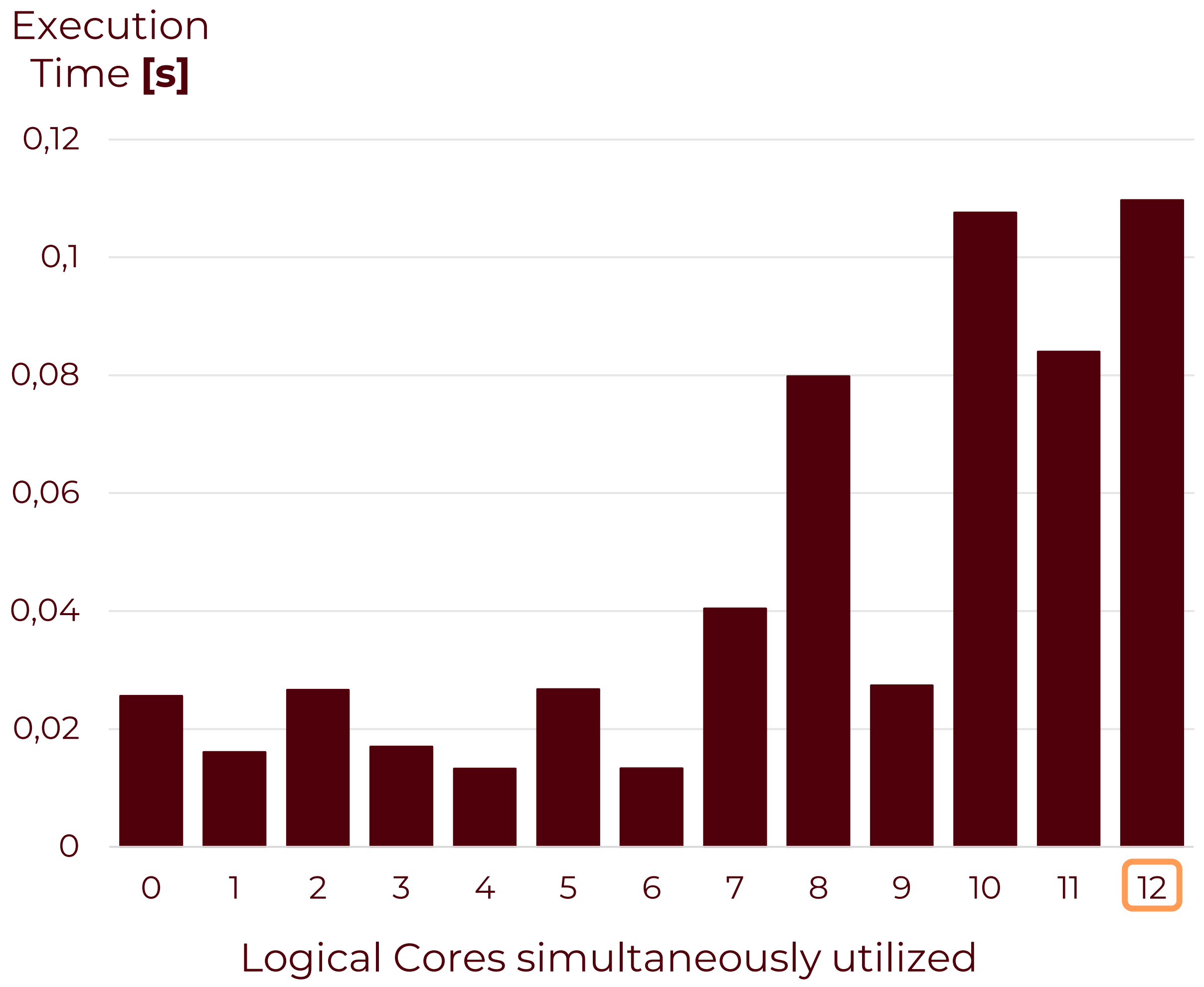
59999999999991 = 3 * 19999999999997

(**14** DIGITS)



(CI omitted due to non-normal distribution)

EFFECTIVE CPU UTILIZATION



TOP **HOTSPOTS**

#THREADS : **12**

0.588s
EXECUTION TIME

4.829s
CPU TIME

0s
PAUSED TIME

0s (0.0% of CPU Time)
THREAD OVERSUBSCRIPTION

Function	**CPU Time**	**% CPU Time**
isPrime	4.808s	99.6%
findPrimesInRange	0.013s	0.3%
others	0.008s	0.2%

FUNCTION	CPU Time	% CPU Time
→ isPrime	4.808s	99.6%

```
bool isPrime(unsigned long long n) {

    …

    for(unsigned long long i = 2; i * i <= n; ++i) {
            if(n % i == 0) {
                    is_prime = false;
                    break;
            }
    }

    …
}
```

CPU TIME

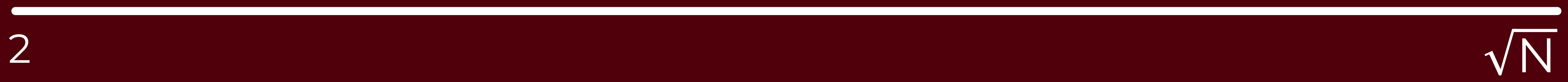
Total (%)	Self (s)
92,8%	4,483s

HOTSPOT

LET’S

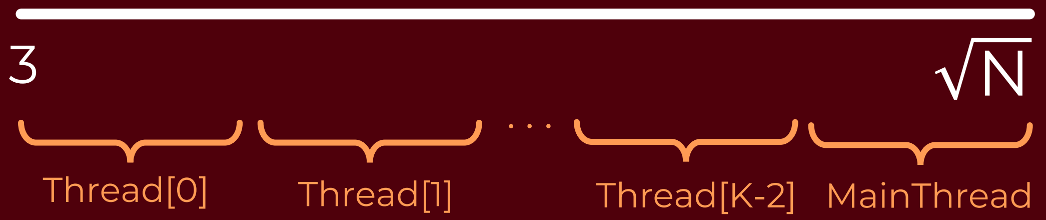
OPTIMIZE!

OPTIMIZATIONS **#1**



MainThread: N mod 2 == 0

OPTIMIZATIONS **#1**



OPTIMIZATIONS **#2**

findPrimesInRange

```
for (unsigned long long i = start; i <= end;  ++i ) {
     if (isPrime(i) && num % i == 0) {

          int exponent = 0;
          while (num % i == 0) {
               exponent++;
               num /= i;
          }

          {
               lock_guard<mutex> lock(mtx);
               primes.push_back({i, exponent});
          }
     }
}
```

OPTIMIZATIONS **#2**

findPrimesInRange

```
for (unsigned long long i = start; i <= end; i += 2) {
    if (isPrime(i) && num % i == 0) {

        int exponent = 0;
        while (num % i == 0) {
            exponent++;
            num /= i;
        }

        {
            lock_guard<mutex> lock(mtx);
            primes.push_back({i, exponent});
        }
    }
}
```

OPTIMIZATIONS **#2**

findPrimesInRange

```
for (unsigned long long i = start; i <= end; i += 2) {
    if (isPrime(i) && num % i == 0) {

        int exponent = 0;
        while (num % i == 0) {
            exponent++;
            num /= i;
        }

        {
            lock_guard<mutex> lock(mtx);
            primes.push_back({i, exponent});
        }
    }
}
```

OPTIMIZATIONS **#2**

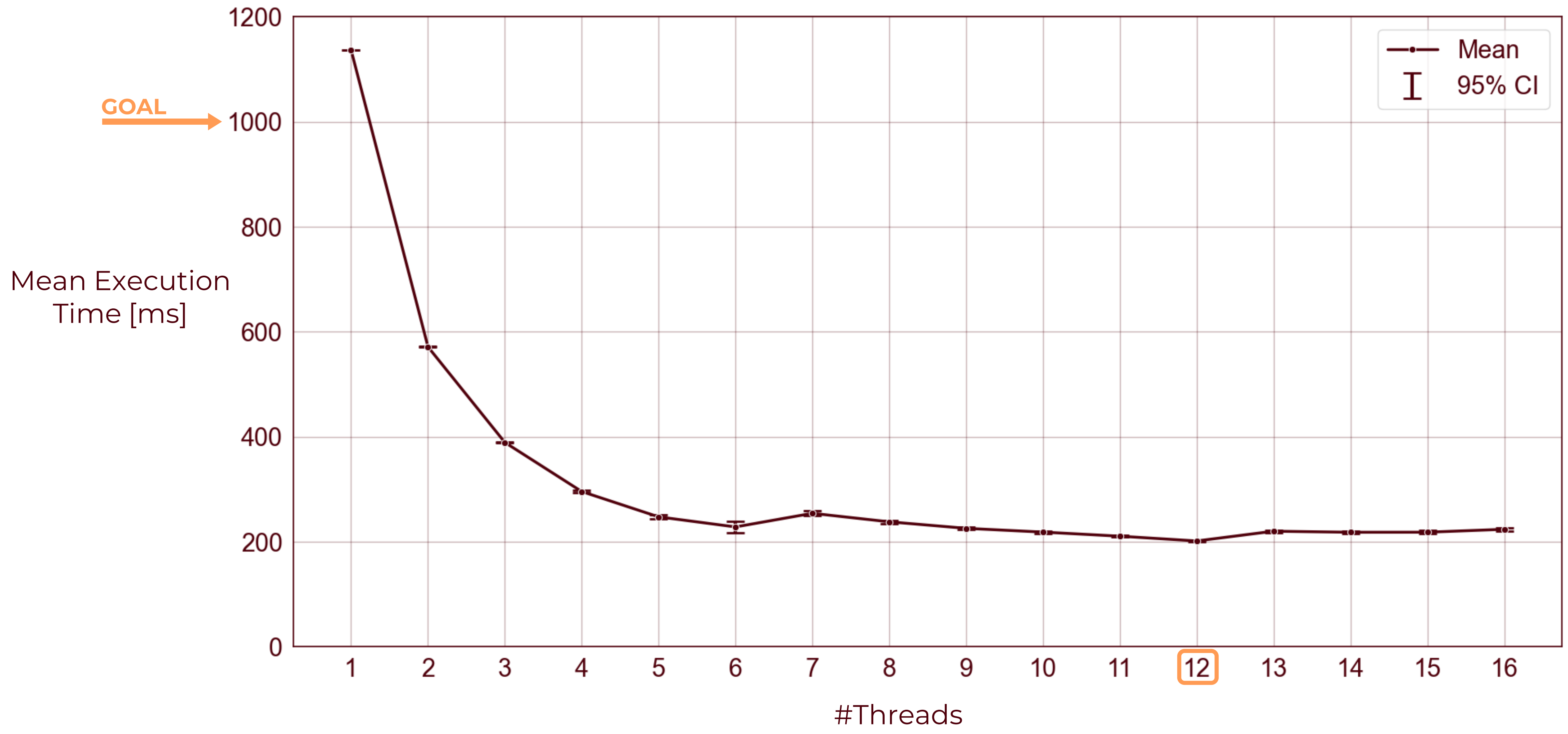
findPrimesInRange

```
for (unsigned long long i = start; i <= end; i += 2) {
        if (num % i == 0) {

                int exponent = 0;
                while (num % i == 0) {
                        exponent++;
                        num /= i;
                }

                if(isPrime(i)){
                        lock_guard<mutex> lock(mtx);
                        primes.push_back({i, exponent});
                }
        }
}
```

EXECUTION TIME
OPTIMIZED VERSION

#ITERATIONS : **30**

975734686214396237 = 748609 * 1303396948493
(**18** DIGITS)



FIRST VERSION

EXECUTION TIME
568340
MILLISECONDS

≈

9
MINUTES

OPTIMIZED VERSION

MAX (OVER 30 ITERATIONS)
EXECUTION TIME
211
MILLISECONDS

<

1
SECOND

x2500
SPEED-UP

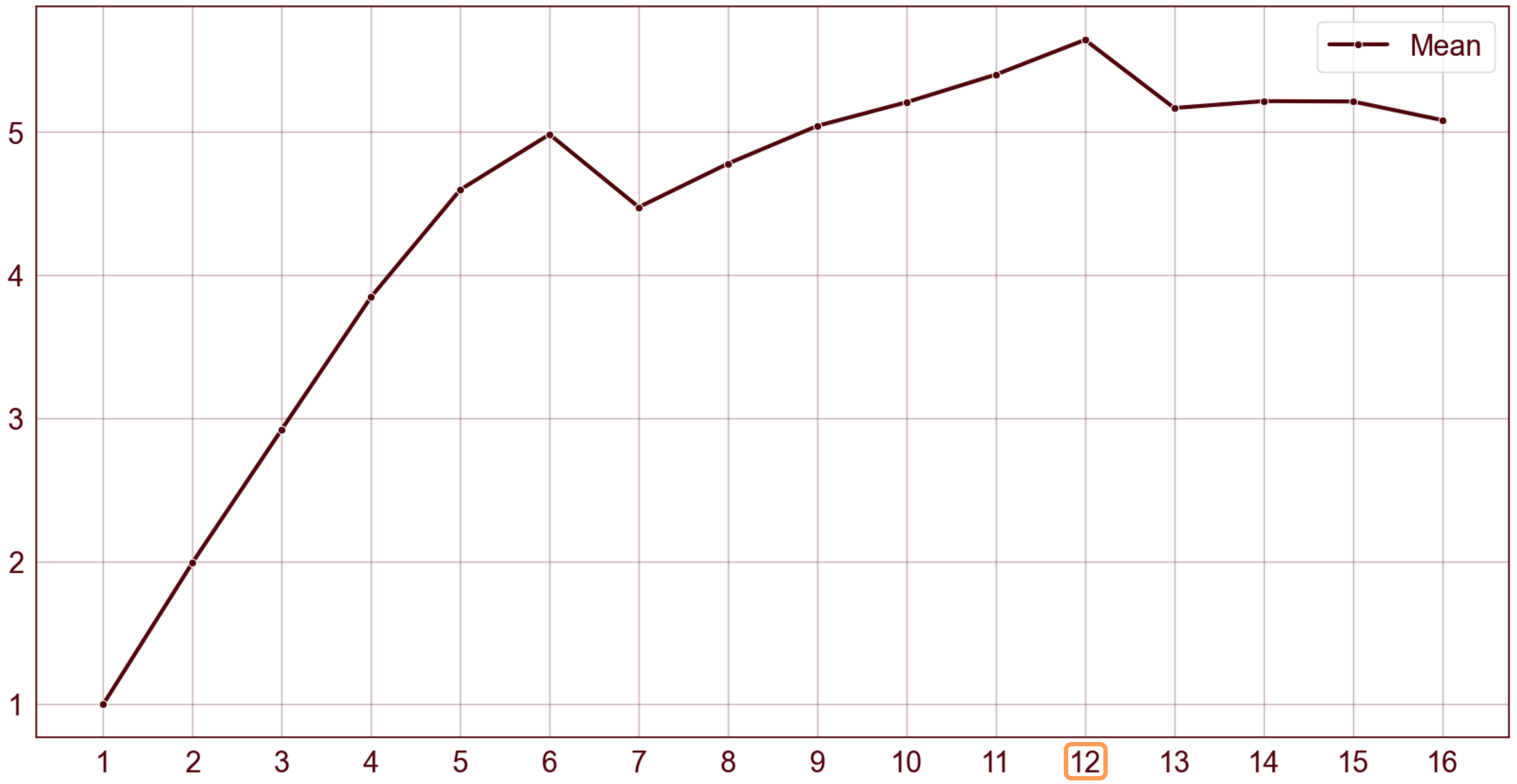
#THREADS : **12**

SPEED-UP
OPTIMIZED VERSION

#ITERATIONS : **30**

975734686214396237 = 748609 * 1303396948493
(**18** DIGITS)

Cumulative Speed-Up



#Threads

(CI omitted due to non-normal distribution)

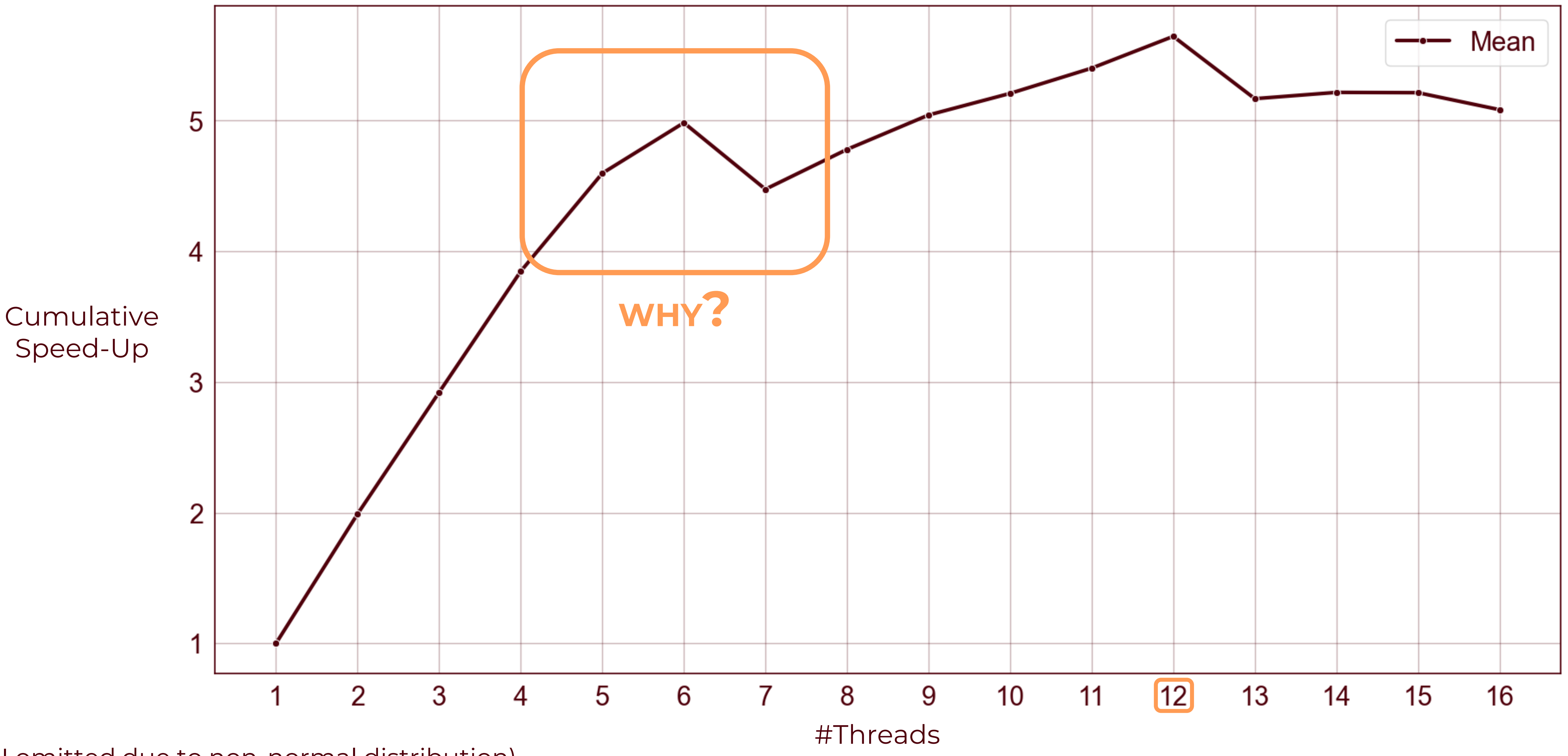
SPEED-UP

OPTIMIZED VERSION

#ITERATIONS : **30**

975734686214396237 = 748609 * 1303396948493

(**18** DIGITS)



(CI omitted due to non-normal distribution)

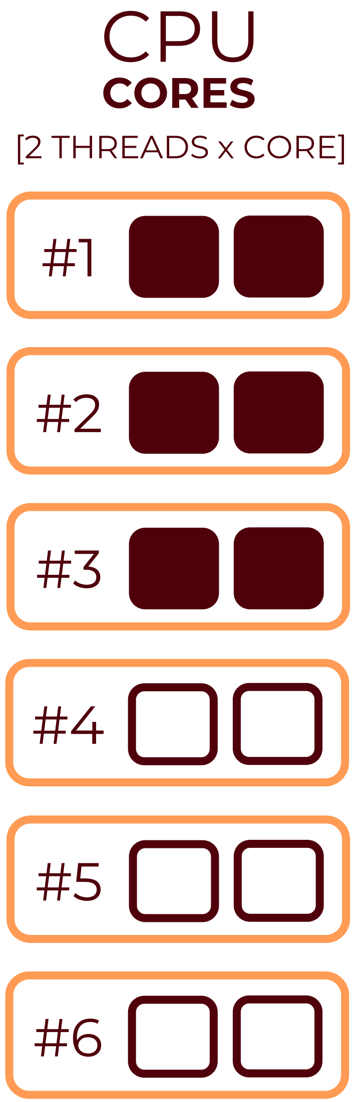
SPEED-UP + AFFINITY

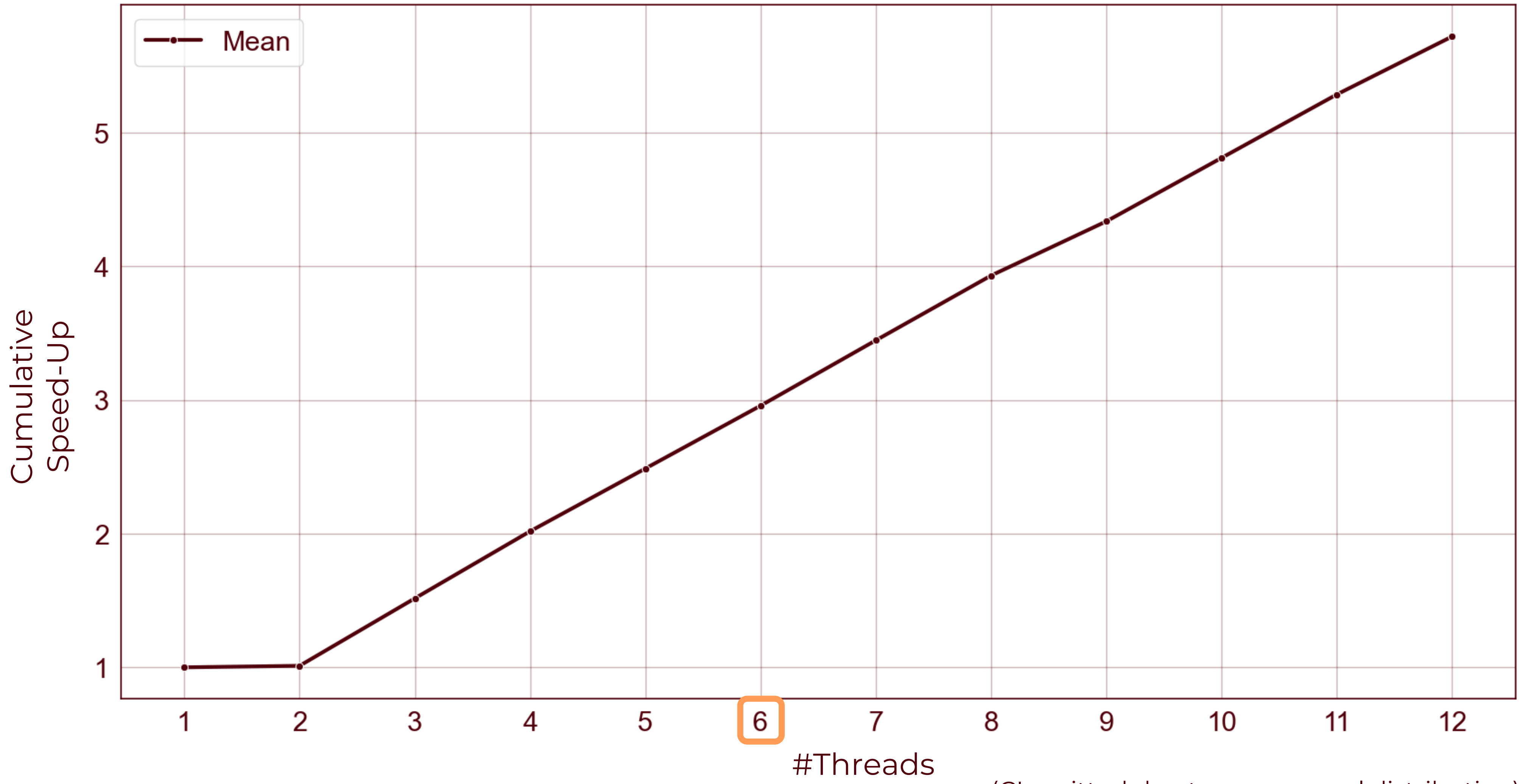
OPTIMIZED VERSION

#ITERATIONS : **30**

975734686214396237 = 748609 * 1303396948493

(**18** DIGITS)





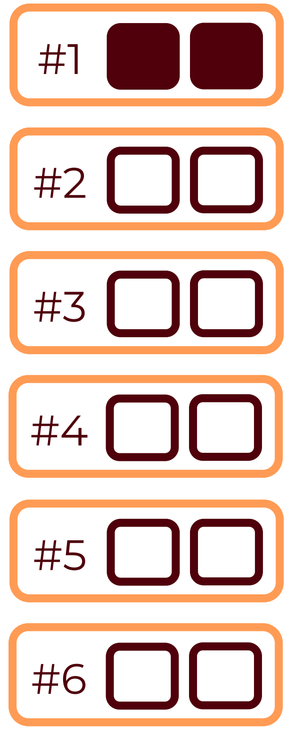
(CI omitted due to non-normal distribution)

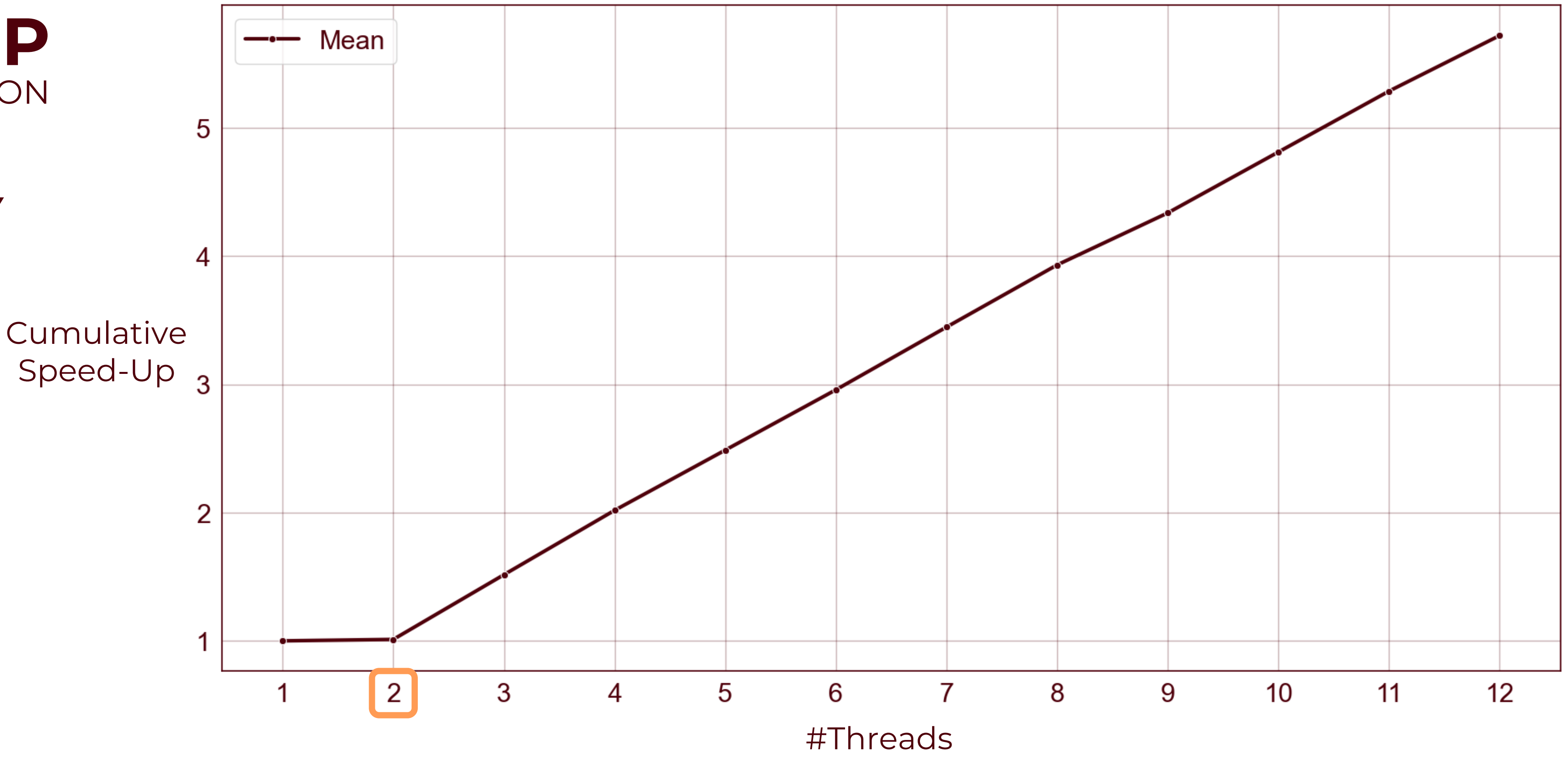
SPEED-UP
OPTIMIZED VERSION

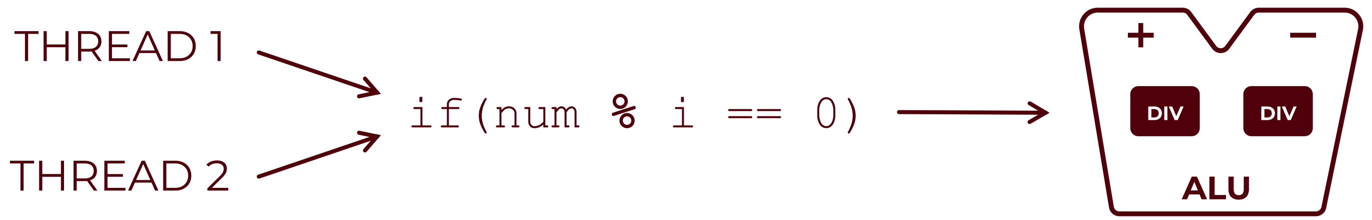
+

AFFINITY

CPU
CORES
[2 THREADS x CORE]







DIVIDER
UTILIZATION
~ **56**%

SPEED-UP
OPTIMIZED VERSION

+ **AFFINITY**

#ITERATIONS : **30**

975734686214396237 = 748609 * 1303396948493

(**18** DIGITS)

CPU
CORES
[2 THREADS x CORE]

#1 ■ □

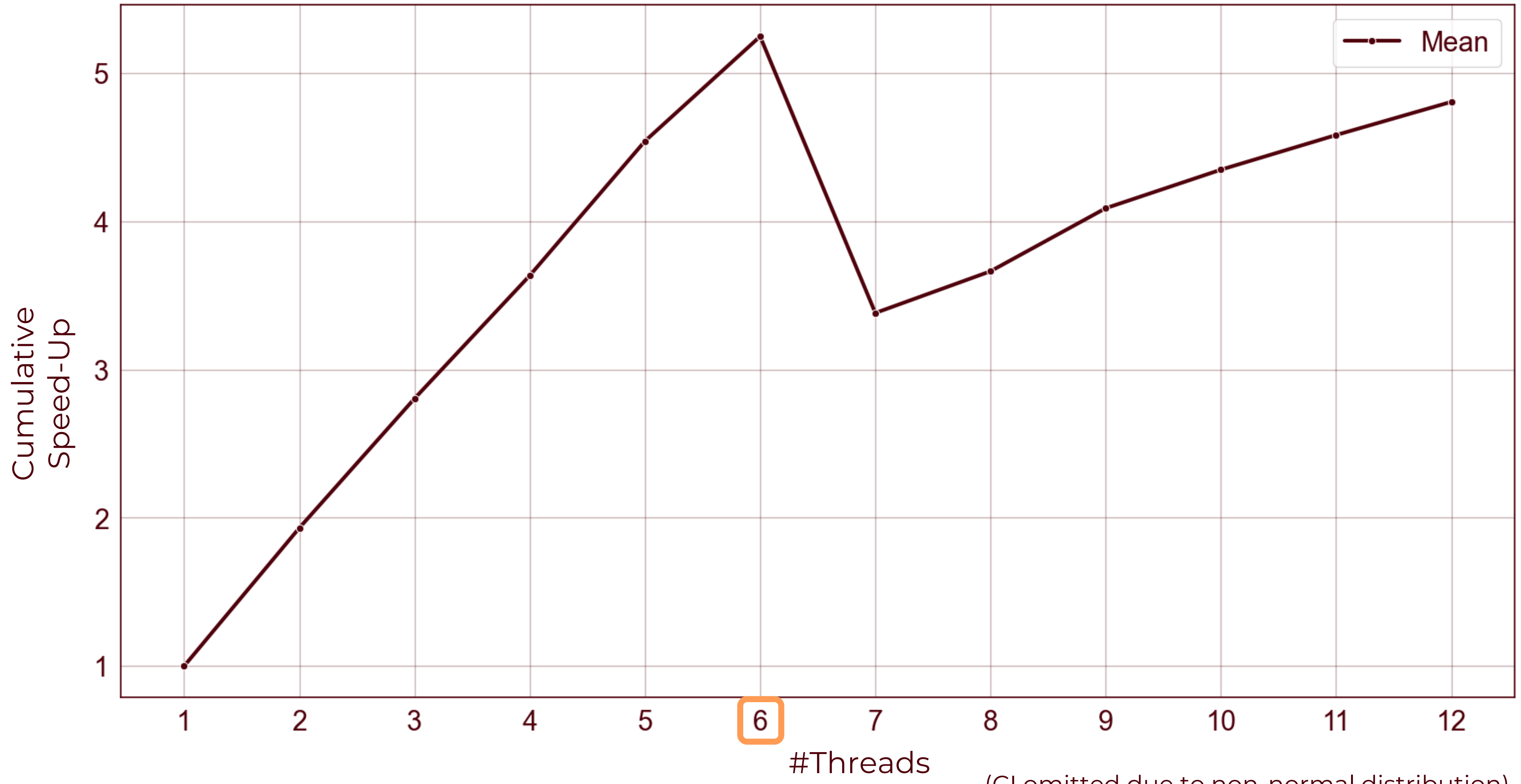
#2 ■ □

#3 ■ □

#4 ■ □

#5 ■ □

#6 ■ □



(CI omitted due to non-normal distribution)

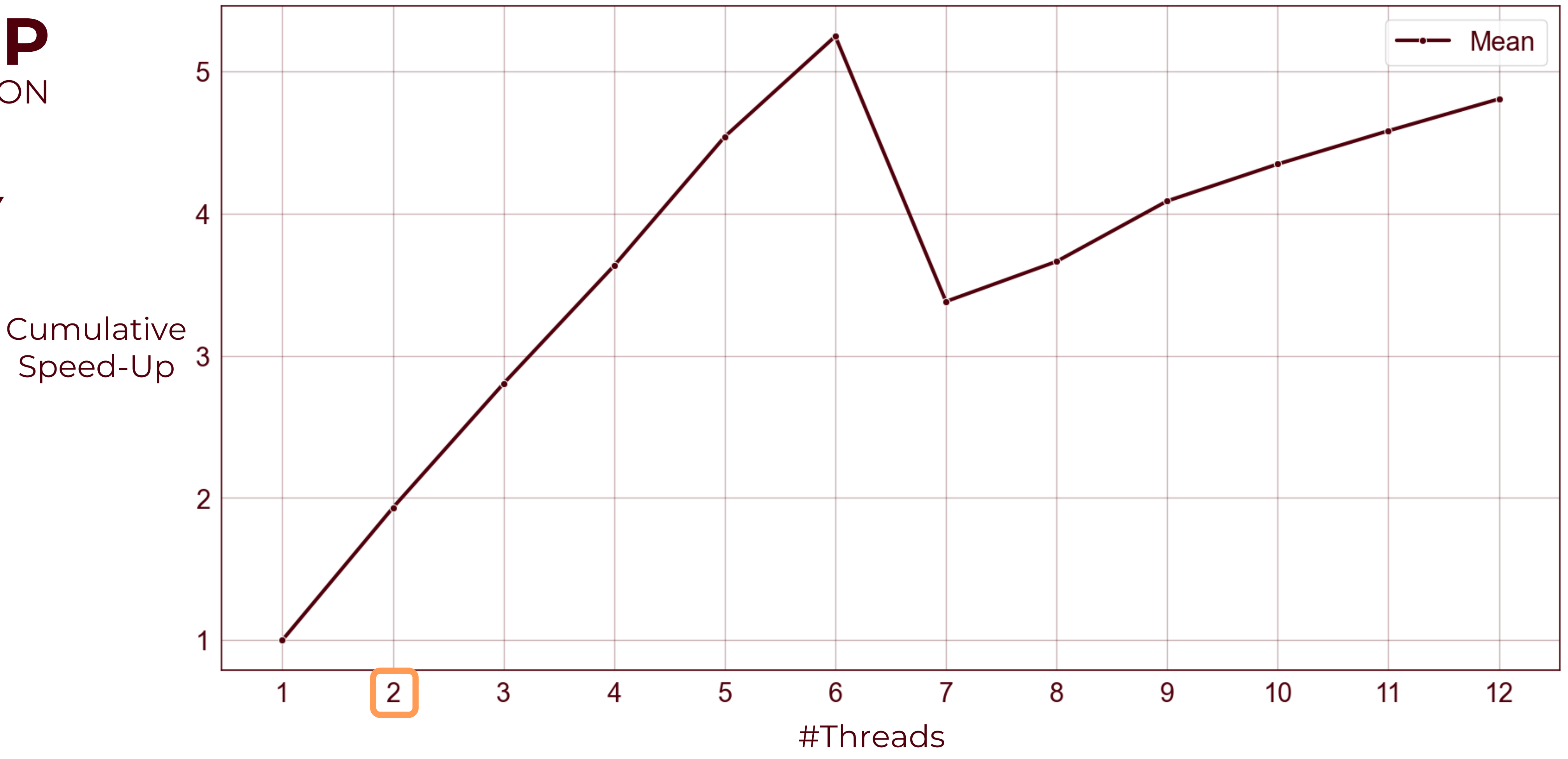
SPEED-UP
OPTIMIZED VERSION

+

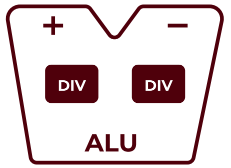
AFFINITY

CPU
CORES
[2 THREADS x CORE]

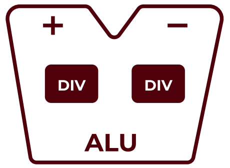
#1	■	□
#2	■	□
#3	□	□
#4	□	□
#5	□	□
#6	□	□



THREAD 1 → `if(num % i == 0)` →



THREAD 2 → `if(num % i == 0)` →



DIVIDER
UTILIZATION
~ **92**%

EXECUTION TIME - SCALABILITY

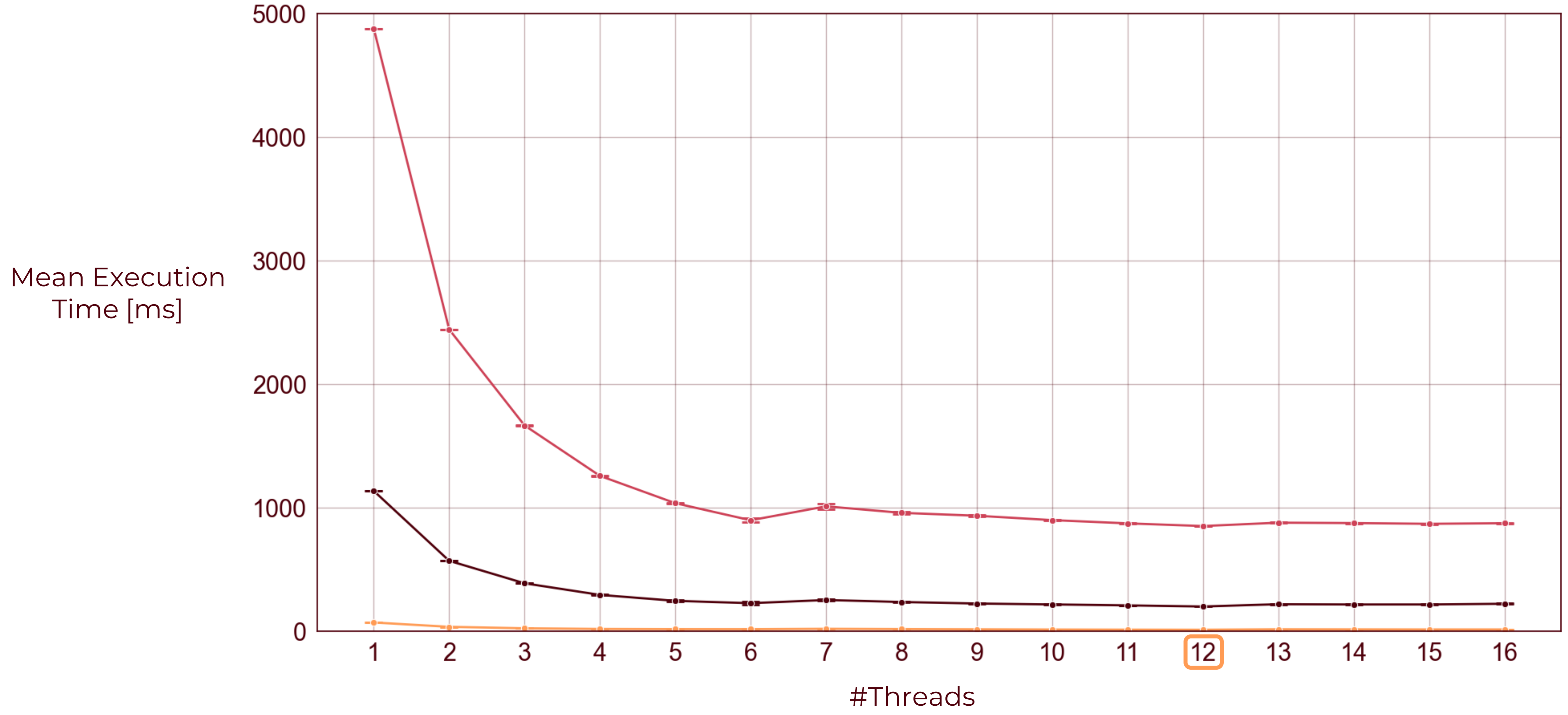
OPTIMIZED VERSION

#ITERATIONS : **30**

20 DIGITS **17975734686214396237**

18 DIGITS **975734686214396237**

16 DIGITS **3934686214396237**



SPEED-UP - SCALABILITY

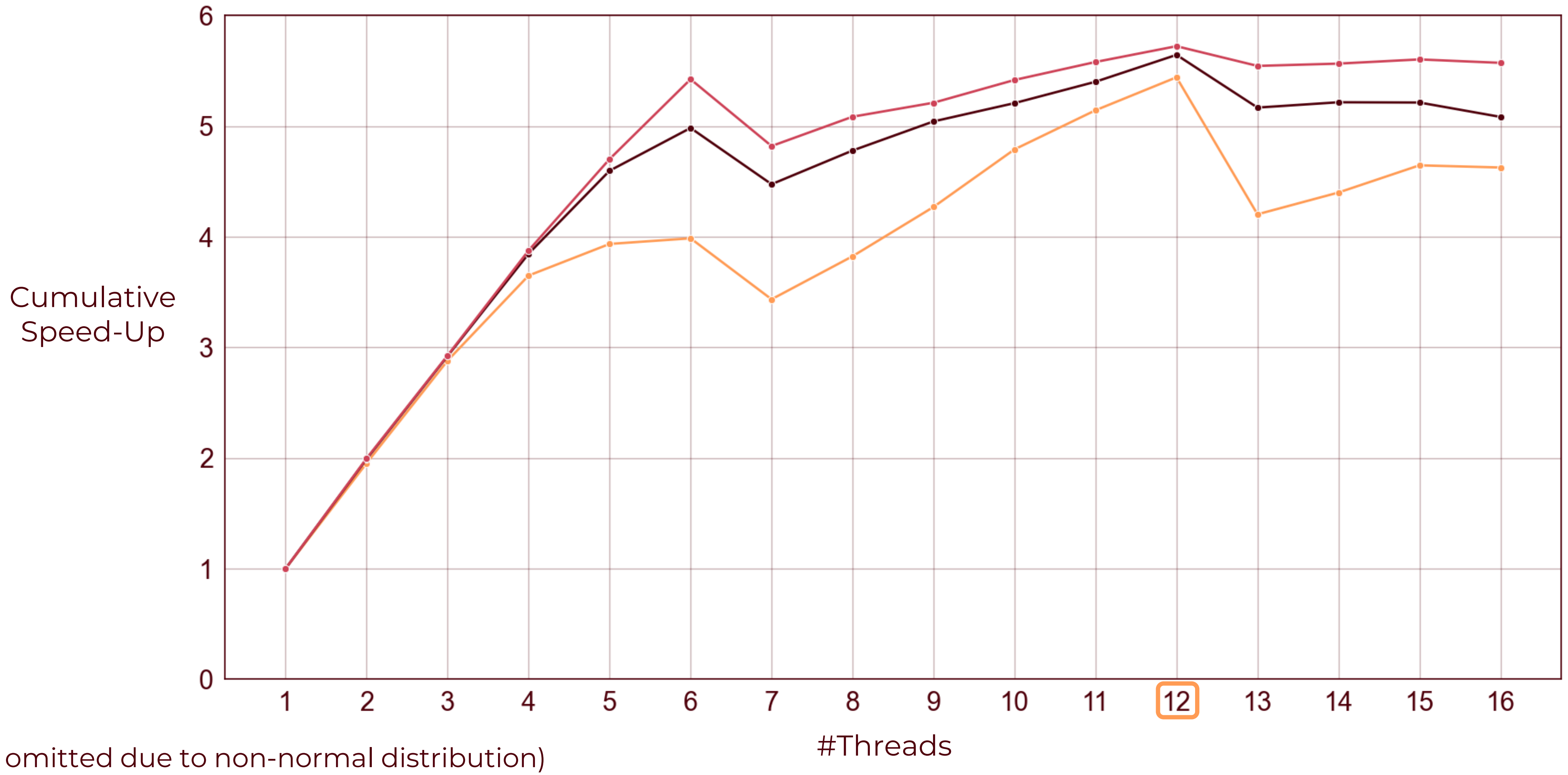
OPTIMIZED VERSION

#ITERATIONS : **30**

20 DIGITS **17975734686214396237**

18 DIGITS **975734686214396237**

16 DIGITS **3934686214396237**



(CI omitted due to non-normal distribution)

GPU
IMPLEMENTATION

NVIDIA Tesla T4

Streaming Multiprocessors (SM): **40** x
CUDA Cores per SM: **64** =

TOTAL # CUDA Cores: **2560**

Warp Size: **32**

Memory: **16 GB GDDR6 300 GB/s**

Interconnection bandwidth: **32 GB/s**

Max Threads per Block: **1024**

GPU Max Clock Rate: **1590 MHz**

Memory Max Clock Rate: **5001 MHz**

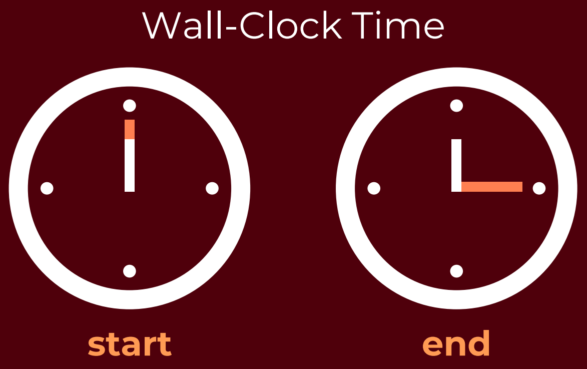
HARDWARE DETAILS

TOOLS

Execution Time

TOTAL
CPU + GPU → chrono : : steady_clock

GPU
TIME → cudaEvent_t



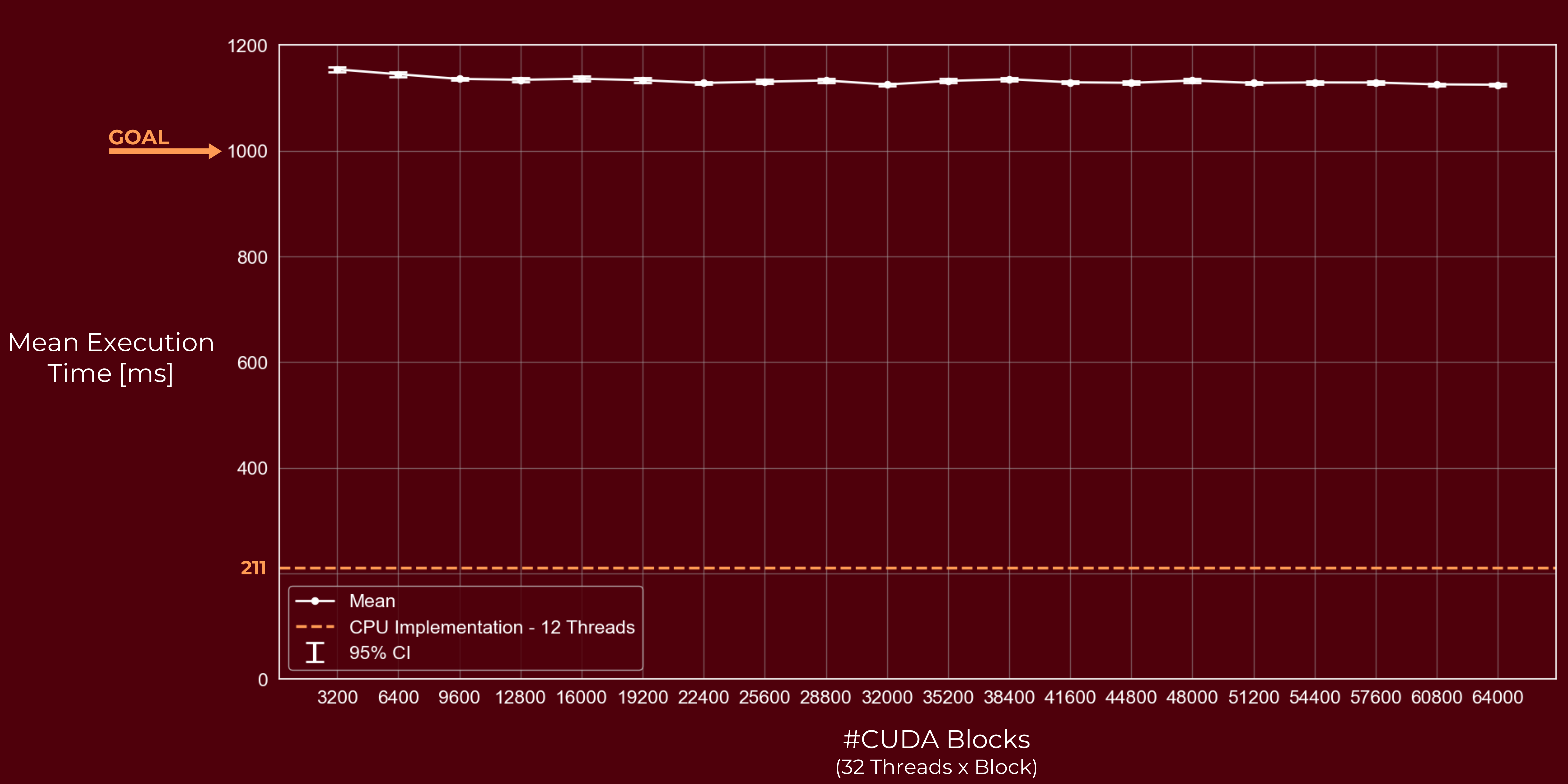
NVIDIA **Nsight Compute** | Visual Studio Code

EXECUTION TIME

FIRST VERSION ON GPU

#ITERATIONS : **30**

18 DIGITS **975734686214396237**



OPTIMIZATIONS

KERNEL

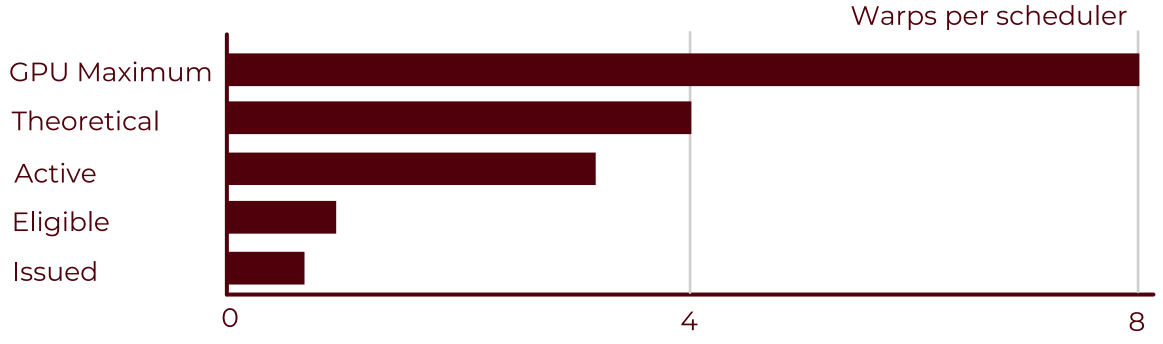
```
__global__ void findPrimesInRange(...)
```

JUST **1 THREAD** PER WARP WAS WORKING

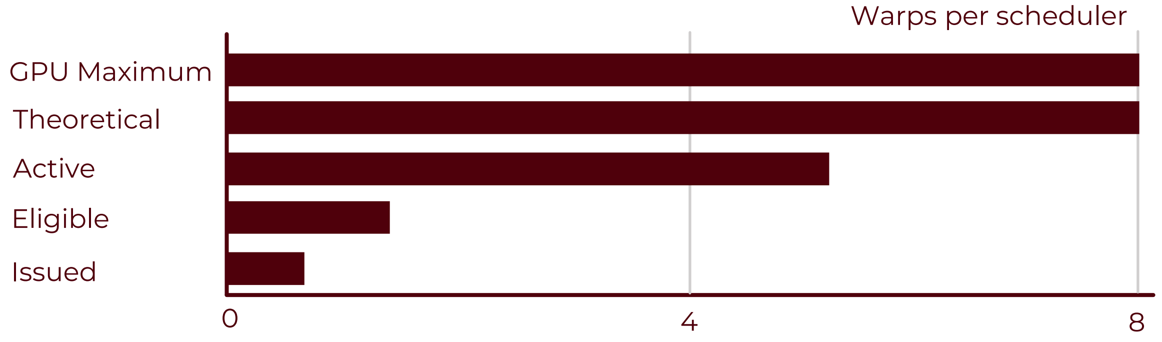
32 THREADS PER WARP*

*DIVERGENCE CAN HAPPEN BUT ITS EFFECT IS NEGLIGIBLE AS THE AVERAGE NUMBER OF NOT PREDICATED OFF THREADS PER WARP IS **31.64**

32 THREADS x BLOCK



64 THREADS x BLOCK



max_blocks_per_multiprocessor **16**

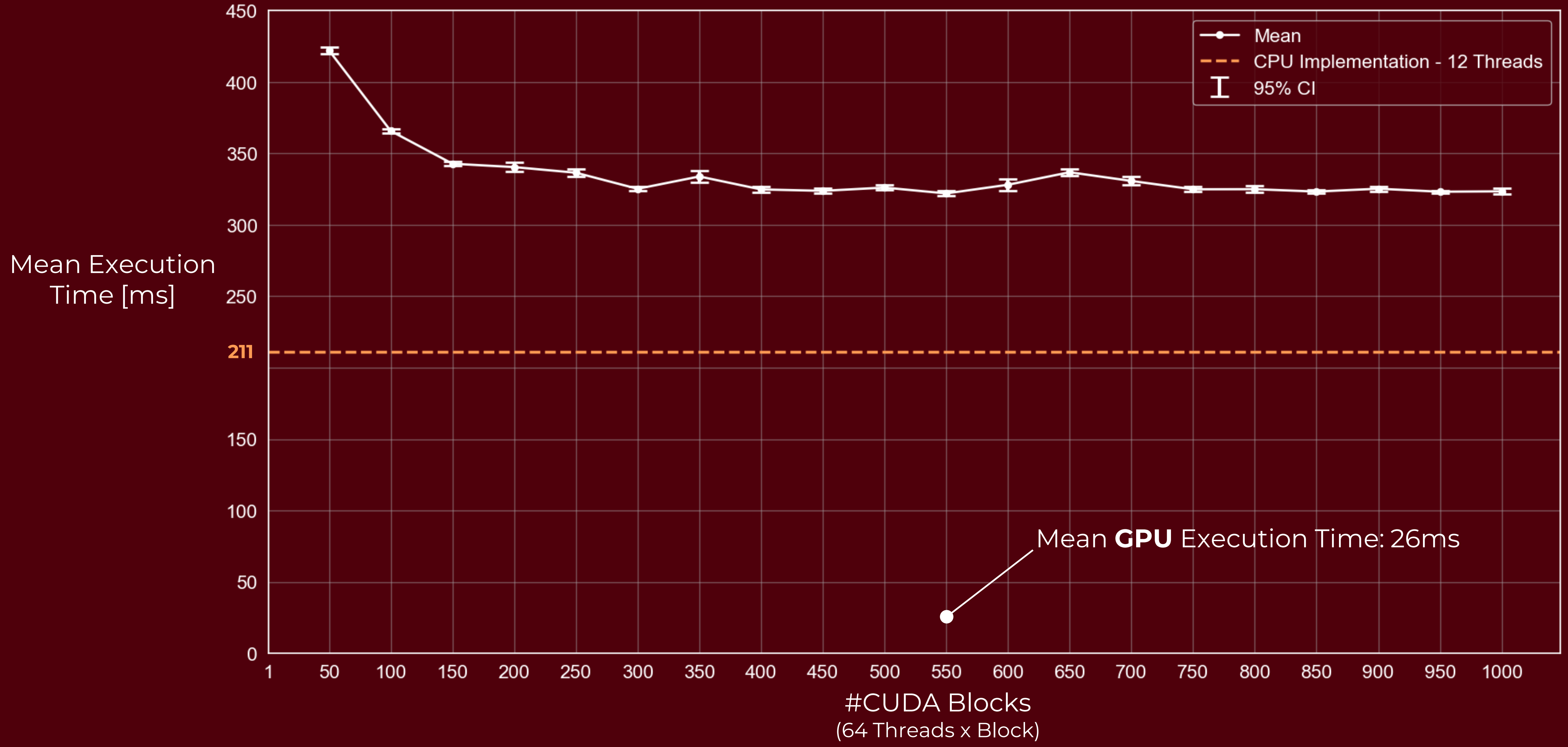
max_warps_per_multiprocessor **32**

max_warps_per_scheduler **8**

NVIDIA Nsight™ Compute

EXECUTION TIME
OPTIMIZED VERSION ON GPU

#ITERATIONS : **30**
18 DIGITS **975734686214396237**

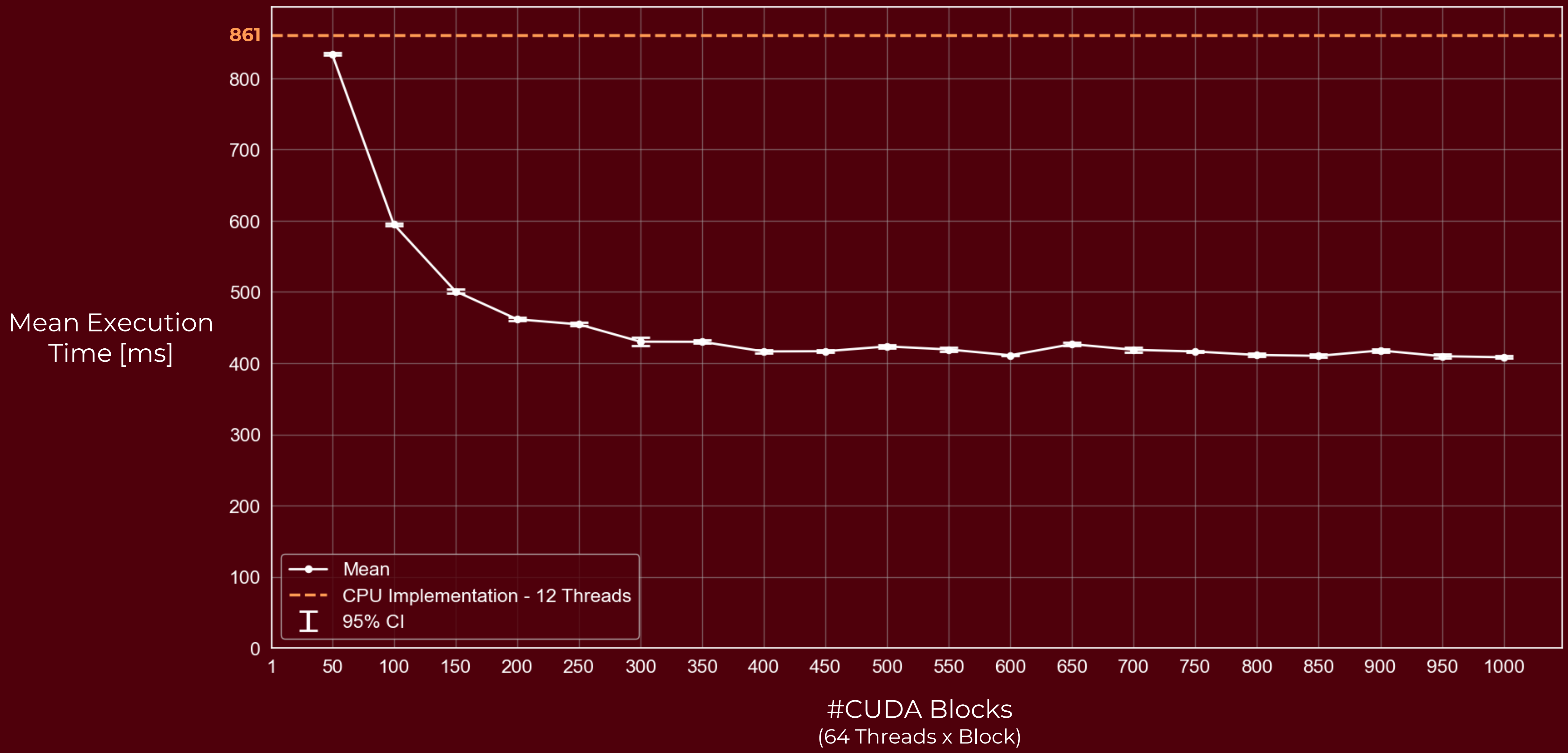


EXECUTION TIME

OPTIMIZED VERSION ON GPU

#ITERATIONS : **30**

20 DIGITS **17975734686214396237**

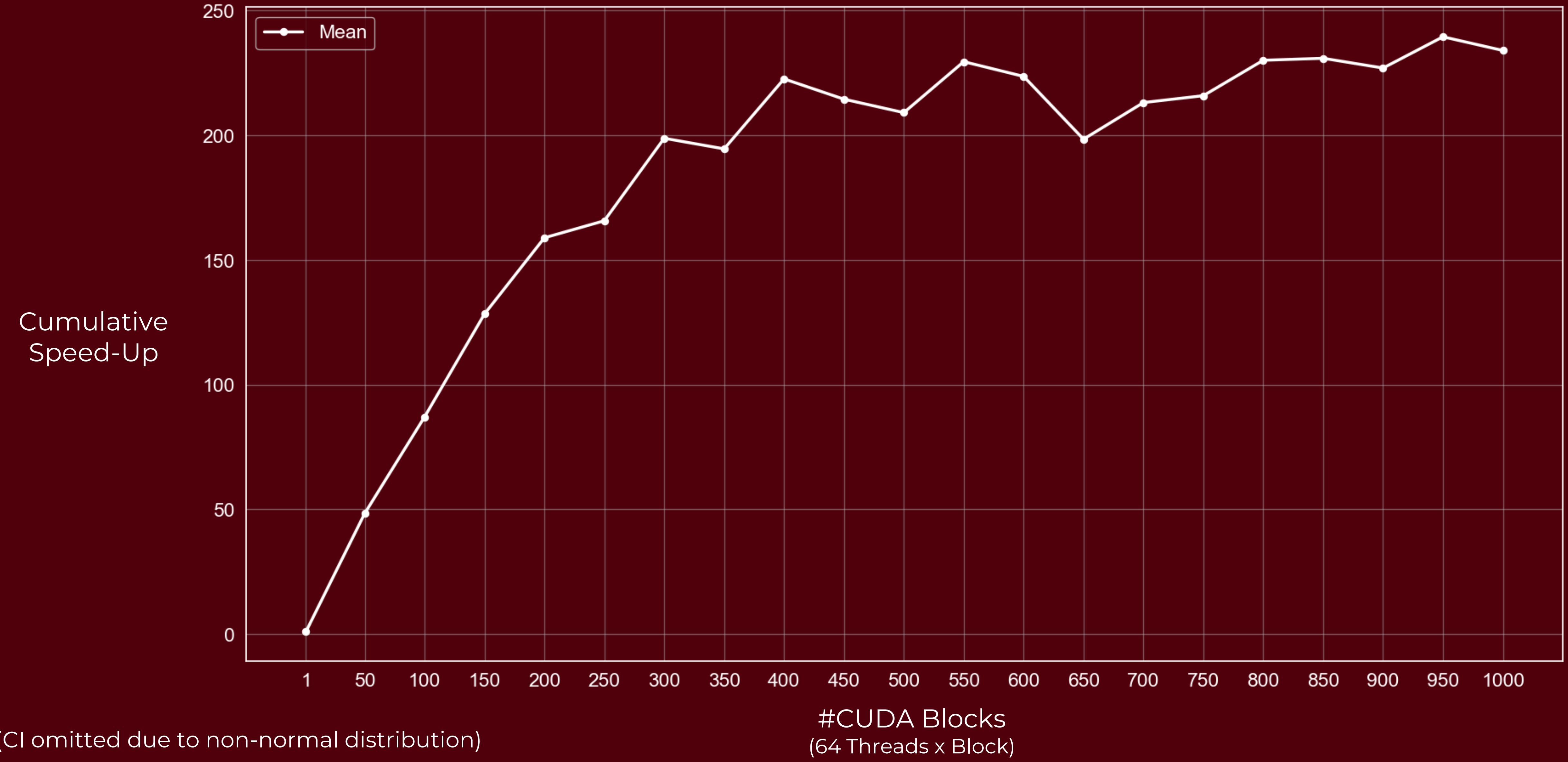


SPEED-UP

OPTIMIZED VERSION ON GPU

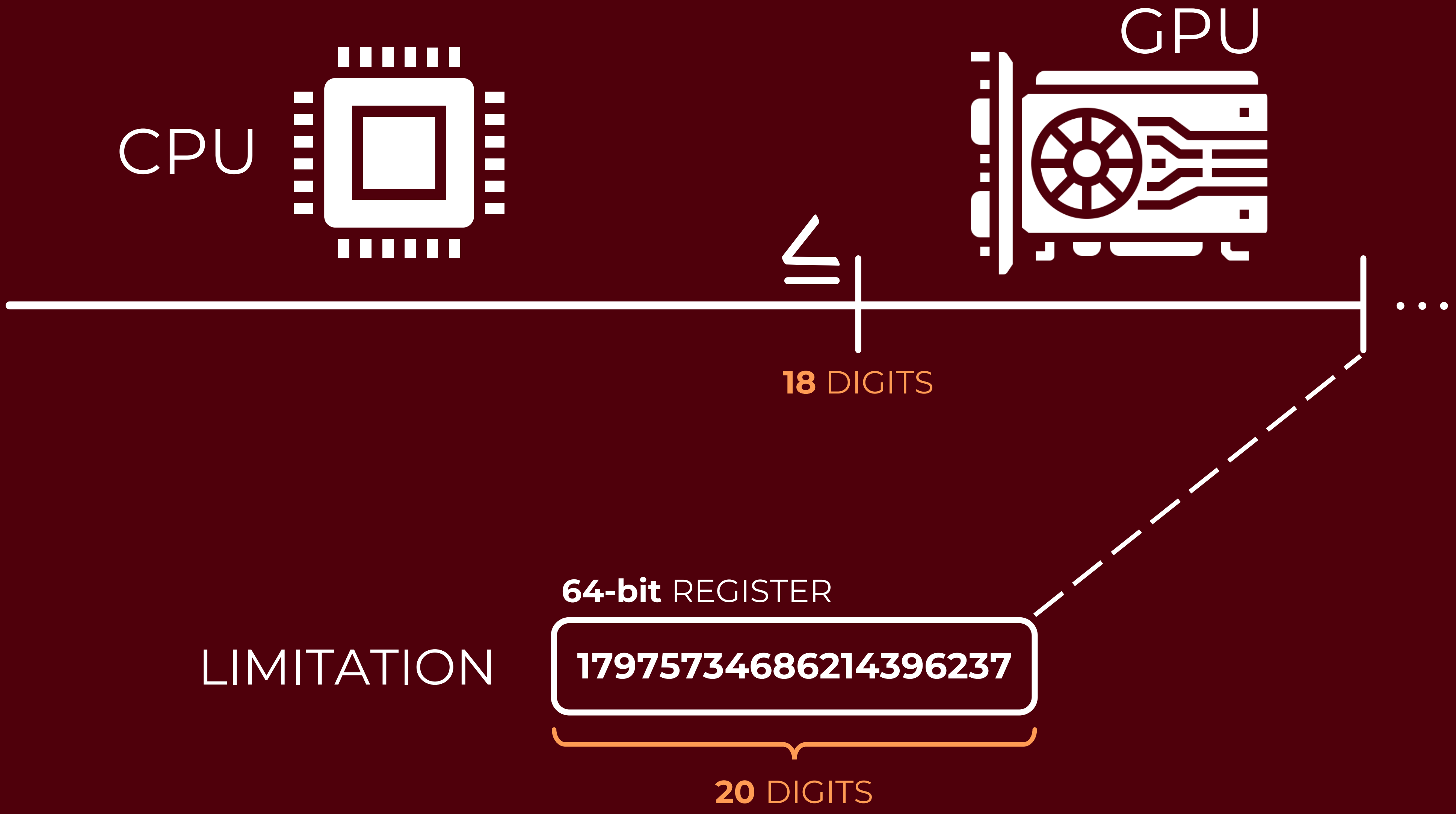
#ITERATIONS : **30**

20 DIGITS **17975734686214396237**



(CI omitted due to non-normal distribution)

CONCLUSIONS



THANKS
FOR THE
ATTENTION