Laboratory Session 07 : May 18, 2020

Exercises due : June 3, 2020

Exercise 1

- a researcher has collected $n = 15$ observations that are supposed to come from a `Norm`al distribution with known variance $\sigma^2 = 16$:

26.8	26.3	28.3	28.5	16.3
31.9	28.5	27.2	20.9	27.5
28.0	18.6	22.3	25.0	31.5

- assuming a normal prior for μ, $\texttt{Norm}(m = 20, s^2 = 25)$,

(a) determine the posterior distribution $P(\mu \mid y_1 \dots y_{15})$ and find the posterior mean and standard deviation

(b) find the 95% credibility interval for μ

(c) plot the posterior distribution, indicating on the same plot: the mean value, the standard deviation, and the 95% credibility interval

(d) repeat the analysis using a different prior $\texttt{Norm}(m = 30, s^2 = 16)$ and plot, on the same graph the likelihood, the prior and the posterior.

(e) compare the credibility intervals obtained with the two priors

Exercise 2

- a researcher has collected $n = 16$ observations that are supposed to come from a `Norm`al distribution with known variance $\sigma^2 = 4$:

4.09	4.68	1.87	2.62	5.58	8.68	4.07	4.78
4.79	4.49	5.85	5.09	2.40	6.27	6.30	4.47

- assuming the prior is a step funtion:

$$g(\mu) = \begin{cases} \mu & \text{for } 0 < \mu \leq 3 \ , \\ 3 & \text{for } 3 < \mu \leq 5 \ , \\ 8 - \mu & \text{for } 5 < \mu \leq 8 \ , \\ 0 & \text{for } \mu > 8 \ . \end{cases}$$

(a) find the posterior distribution, the posterior mean and standard deviation

(b) find the 95% credibility interval for μ

(c) plot the posterior distribution, indicating on the same plot: the mean value, the standard deviation, and the 95% credibility interval

(d) plot, on the same graph, the prior, the likelihood and the posterior distribution

Exercise 3

- A study on water quality of streams, a high level of bacter X was defined as a level greater than 100 per 100 ml of stream water. $n = 116$ samples were taken from streams having a high environmental impact on pandas. Out of these, $y = 11$ had a high bacter X level.
- indicating with p the probability that a sample of water taken from the stream has a high bacter X level,

(a) find the frequentist estimator for p

(b) using a $\texttt{Beta}(1, 10)$ prior for p, calculate and posterior distribution $P(p \mid y)$

(c) find the bayesian estimator for p, the posterior mean and variance, and a 95% credible interval

(d) test the hypotesis

$$H_\circ : p = 0.1 \text{ versus } H_1 : p \neq 0.1$$

at 5% level of significance with both the frequentist and bayesian approach

- a new measurement, performed one month later on $n = 165$ water samples, gives $y = 9$ high bacter X level

(e) find the frequentist estimator for p

(f) find a bayesian estimator for p, assuming both a $\texttt{Beta}(1, 10)$ prior for p, and assuming the posterior probability of the older measurement as the prior for the new one.

(g) find the bayesian estimator for p, the posterior mean and variance, and a 95% credible interval

(h) test the hypotesis

$$H_\circ : p = 0.1 \text{ versus } H_1 : p \neq 0.1$$

at 5% level of significance with both the frequentist and bayesian approach