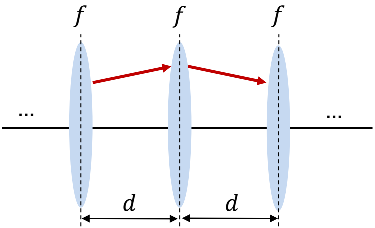
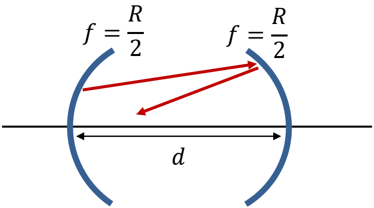
Optical resonators

Symmetrical period lens waveguide



Symmetrical two-mirror optical cavity



$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1 - \frac{d}{f} \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

Is there a ray vector that obeys the condition:

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \; ? \quad \Longrightarrow \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

Eigenvalues problem

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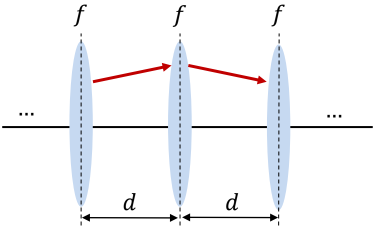
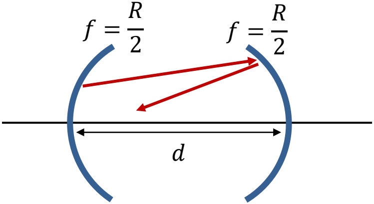


Symmetrical two-mirror optical cavity



$$\begin{vmatrix} A-\lambda & B \\ C & D-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & d \\ -\frac{1}{f} & 1-\frac{d}{f}-\lambda \end{vmatrix} = 0 \quad \Longrightarrow \quad \lambda^2 - \overbrace{\left(2-\frac{d}{f}\right)}^{2g}\lambda + 1 = 0$$

$$g = 1 - \frac{d}{2f}$$

$$\lambda_{1,2} = g \pm \sqrt{g^2-1} \qquad |g| \geq 1 \qquad \textbf{unstable}$$

$$\lambda_{1,2} = g \pm i\sqrt{1-g^2} = e^{\pm i\phi} \qquad |g| \leq 1 \qquad \textbf{stable}$$

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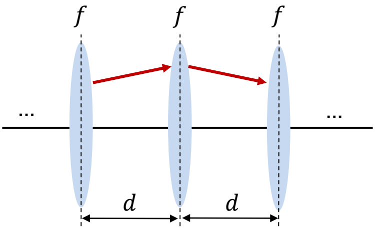
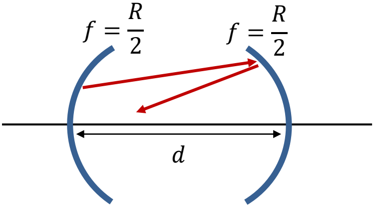
Symmetrical period lens waveguide



Symmetrical two-mirror optical cavity



$$|g| \leq 1 \quad \Longrightarrow \quad \begin{bmatrix} x_N \\ \theta_N \end{bmatrix} = \lambda^N \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \quad \Longrightarrow \quad \lambda^N = e^{\pm N i \phi} \qquad |\lambda^N| = 1$$

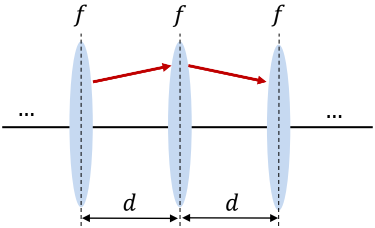
$|g| \geq 1$ the trajectory of the ray will diverge

$$g = 1 - \frac{d}{2f} \qquad \lambda_{1,2} = g \pm \sqrt{g^2 - 1} \qquad |g| \geq 1$$ **unstable**

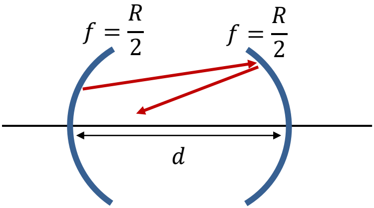
$$\lambda_{1,2} = g \pm i\sqrt{1 - g^2} = e^{\pm i\phi} \qquad |g| \leq 1$$ **stable**

Optical resonators

Symmetrical period lens waveguide



Symmetrical two-mirror optical cavity



$$|g| \le 1$$

Stability condition

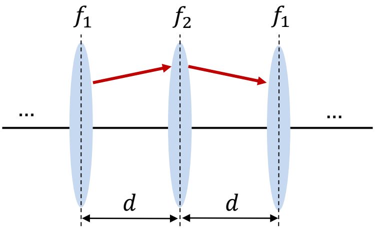
$$g = 1 - \frac{d}{2f}$$

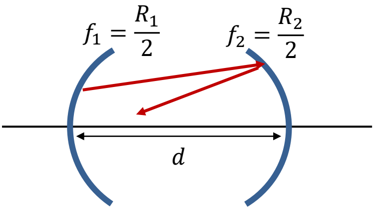
$$\left|1 - \frac{d}{2f}\right| \le 1 \qquad -1 \le 1 - \frac{d}{2f} \le 1 \qquad \Longrightarrow \qquad 0 \le d \le 4f$$

$$0 \le d \le 2R$$

with the convention adopted, a **concave mirror** has a **positive** radius of curvature $R > 0$

Asymmetrical resonators

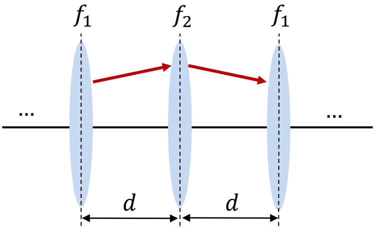


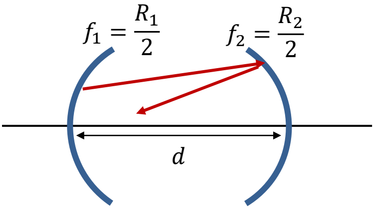


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{d}{f_1} - \frac{d}{f_2}\left(1 - \frac{d}{f_1}\right) - \frac{d}{f_1} & 2d - \frac{d^2}{f_2} \\ -\frac{1}{f_2}\left(1 - \frac{d}{f_1}\right) - \frac{1}{f_1} & 1 - \frac{d}{f_2} \end{bmatrix}$$

Asymmetrical resonators



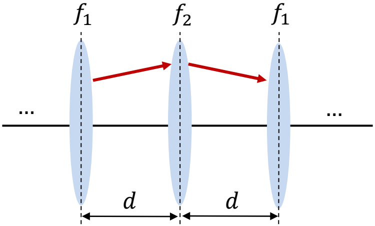


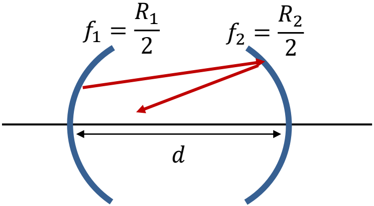
$$\lambda^2 - 2\underbrace{\left(1 - \frac{d}{f_2} + \frac{d^2}{2f_1 f_2} - \frac{d}{f_1}\right)}_{\alpha}\lambda + 1 = 0$$

$$\lambda_{1,2} = \alpha \pm \sqrt{\alpha^2 - 1}$$

Stability condition $\quad |\alpha| \leq 1 \quad \Longrightarrow \quad 0 \leq \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \leq 1$

Asymmetrical resonators





$$g_1 = 1 - \frac{d}{2f_1} = 1 - \frac{d}{R_1} \qquad g_2 = 1 - \frac{d}{2f_2} = 1 - \frac{d}{R_2}$$

$$\boxed{0 \le g_1 g_2 \le 1}$$

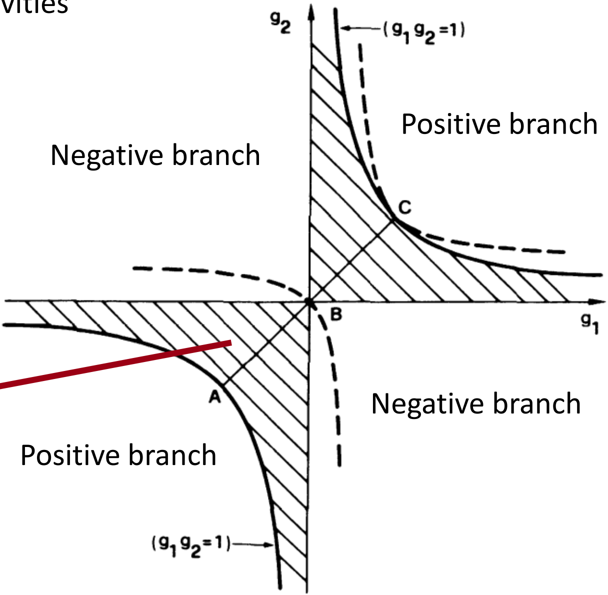
Stability condition $\quad |\alpha| \le 1 \quad \Longrightarrow \quad 0 \le \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \le 1$

Two-mirror optical cavities

$$g_1 = 1 - \frac{d}{R_1}$$

$$g_2 = 1 - \frac{d}{R_2}$$

$$\boxed{0 \le g_1 g_2 \le 1}$$



$$\left.\begin{aligned} g_1 g_2 &= 1 \\ g_1 g_2 &= 0 \end{aligned}\right\}$$ marginal stability

Stability

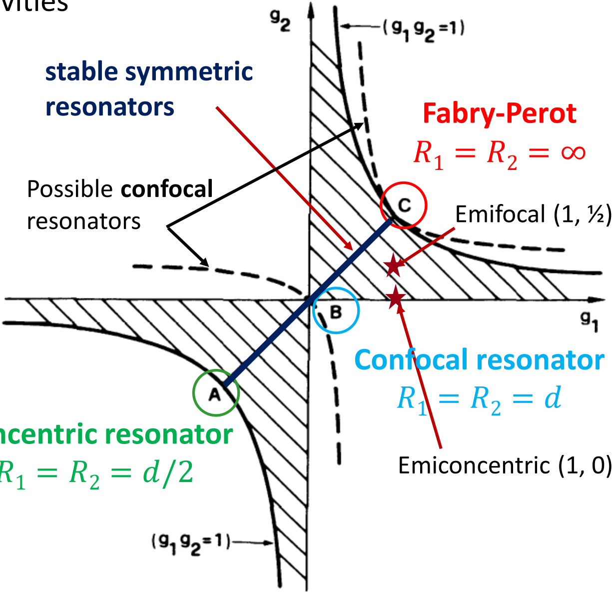
Two-mirror optical cavities

$$g_1 = 1 - \frac{d}{R_1}$$

$$g_2 = 1 - \frac{d}{R_2}$$

$$\boxed{0 \leq g_1 g_2 \leq 1}$$

$$\left.\begin{array}{l} g_1 g_2 = 1 \\ g_1 g_2 = 0 \end{array}\right\}$$ marginal stability

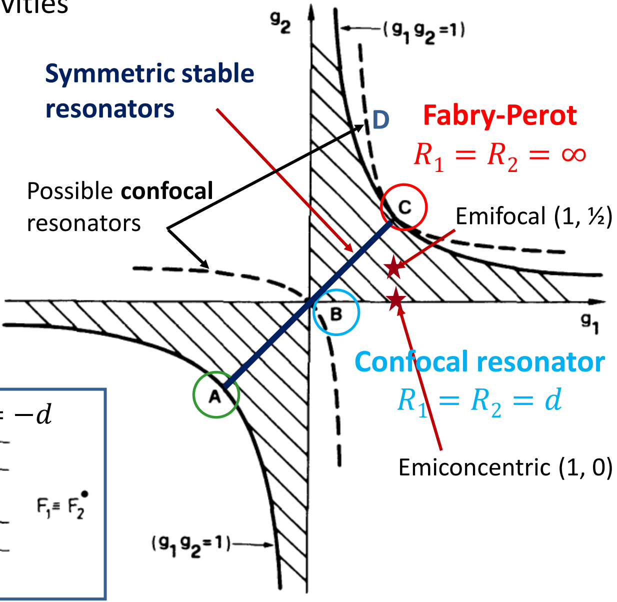


Stability

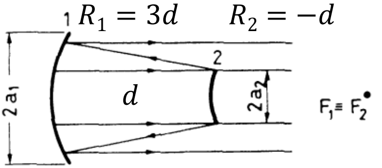
Two-mirror optical cavities

$$g_1 = 1 - \frac{d}{R_1}$$

$$g_2 = 1 - \frac{d}{R_2}$$

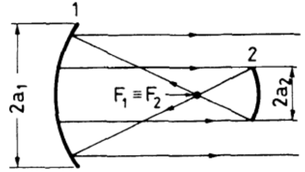


D Confocal telescopic resonator (2/3, 2)

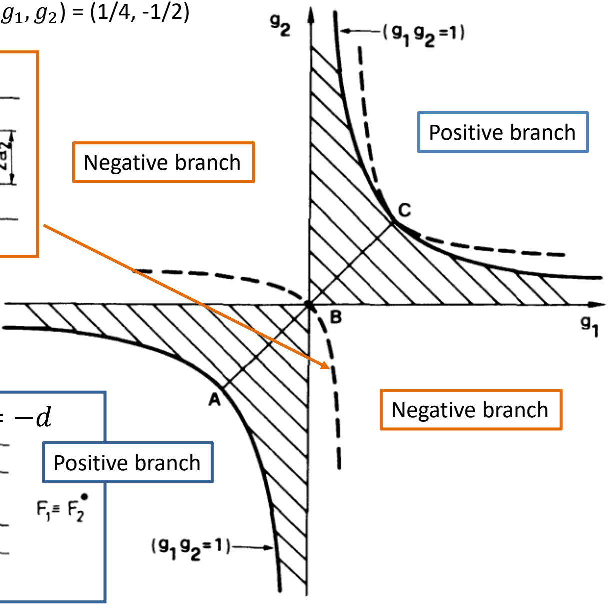


Stability

$$R_1 = \frac{4d}{3} \qquad R_2 = \frac{2d}{3} \qquad (g_1, g_2) = (1/4, -1/2)$$



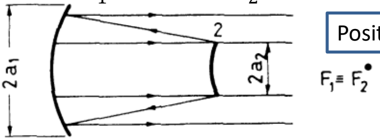
Negative branch



Positive branch

Negative branch

$$R_1 = 3d \qquad R_2 = -d$$

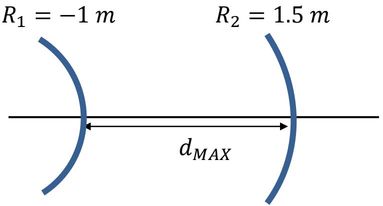


Positive branch

Optical resonators

Q: a two-mirror optical resonator is formed by a convex mirror of radius $R_1 = -1\ m$ and a concave mirror of radius $R_1 = 1.5\ m$. What is the maximum possible mirror separation if this is to remain a stable resonator?

A:



Stability condition $\quad 0 \leq g_1 g_2 \leq 1$

$$g_1 = 1 - \frac{d}{R_1} \qquad g_2 = 1 - \frac{d}{R_2}$$

$$g_1 g_2 \geq 0 \quad \left(1 - \frac{d}{R_1}\right)\left(1 - \frac{d}{R_2}\right) \geq 0 \quad R_1 < 0 \implies 1 - \frac{d}{R_2} \geq 0 \implies d \leq R_2$$

$$g_1 g_2 \leq 1 \quad \left(1 - \frac{d}{R_1}\right)\left(1 - \frac{d}{R_2}\right) \leq 1 \quad \frac{d}{R_1 R_2} \leq \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\implies \quad d \leq R_1 + R_2 \quad \implies \quad R_1 + R_2 \leq d \leq R_2 \quad \implies \quad 0.5\ m \leq d \leq 1.5\ m$$