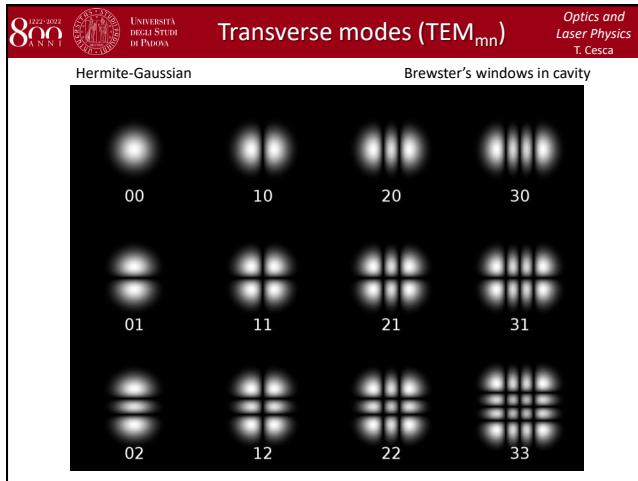


# 1 Lecture 22

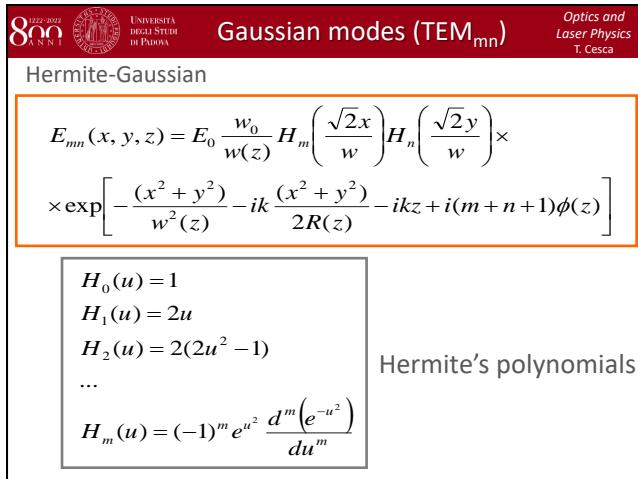
## Slide 1



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Let us see how the property of the cavity control the properties of the beam. These are simulation for transverse modes.

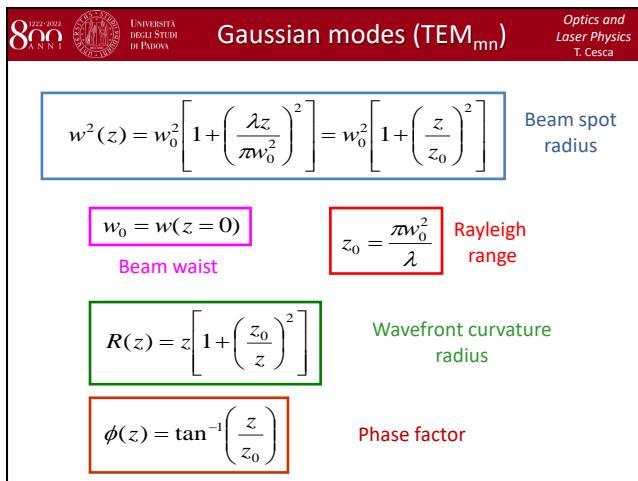
## Slide 2



This is the expression of the electromagnetic field.

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## Slide 3



It is controlled by these main parameters.  
At  $w_0$  the radius of curvature is infinite.

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## Slide 4

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Let's consider a symmetrical cavity formed by two concave mirrors with radius of curvature  $R$ , at a distance  $d$ :

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right] \Rightarrow R \left( z = \frac{d}{2} \right) = \frac{d}{2} + \frac{2z_0^2}{d} = R$$

$$R = \frac{d^2 + 4z_0^2}{2d} \Rightarrow z_0^2 = \frac{d(2R - d)}{4} \Rightarrow z_0 = \frac{\sqrt{d(2R - d)}}{2} = \frac{\pi w_0^2}{\lambda}$$

$$\Rightarrow w_0^2 = \frac{\lambda}{2\pi} \sqrt{d(2R - d)}$$

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## Slide 5

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For a **confocal resonator**:  $d = R$

$$z_0 = \frac{\sqrt{d(2R - d)}}{2} = \frac{d}{2} \Rightarrow w_0^2 = \frac{\lambda}{2\pi} \sqrt{d(2R - d)} = \frac{\lambda d}{2\pi} \quad w_0 = \sqrt{\frac{\lambda d}{2\pi}}$$

$$w^2(z) = w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right] \Rightarrow w^2 \left( z = \frac{d}{2} \right) = w_0^2 \left[ 1 + \left( \frac{d/2}{d/2} \right)^2 \right]$$

$$\Rightarrow w \left( \frac{d}{2} \right) = \sqrt{2} w_0 = \sqrt{\frac{\lambda d}{\pi}}$$

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## Slide 6

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For a **confocal resonator**:  $d = R$

$$d = R = 1 \text{ m}$$

$$\lambda = 532 \text{ nm}$$

$$z_0 = \frac{d}{2} = 0.5 \text{ m} \quad w_0 = \sqrt{\frac{\lambda z_0}{\pi}} = \sqrt{\frac{\lambda d}{2\pi}} = 0.29 \text{ mm}$$

$$w \left( \frac{d}{2} \right) = w(z_0) = \sqrt{2} w_0 = 0.41 \text{ mm}$$

$$\vartheta = \frac{2\lambda}{\pi w_0} = 1.2 \cdot 10^{-3} \text{ rad} = 0.07^\circ \quad \text{Far-field beam divergence}$$

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Let us see how we can determine the properties starting from the characteristic of the cavity. Let us consider a symmetric two-mirror optical cavity.

$R(z)$  is the curvature radius of the electromagnetic field. The wavefront at the position of the mirror has to be equal to the radius of the mirror!

If we make the calculation we have an expression for the radius of curvature. We can reverse the expression for the Rayleigh range of the beam.

We have imposed  $R(z = d/2) = R$ !

They control the characteristic of the beam also outside the cavity.

Let us consider a **confocal resonator**. The focal point of the two mirror is the same and its at the center of the cavity.

We obtain that the Rayleigh range is half the distance between the two mirrors.

Once you have the beam waist you want compute the beam spot radius at any position  $z$ .

Just to have some realistic numbers.

We can also calculate the **far-field beam divergence**. We have a very small divergence, that is why we image a laser beam as a collimated beam!

## Slide 7

**Optical resonators**

Let's consider a two-mirror **asymmetrical** cavity:

$$\left\{ \begin{array}{l} d = z_2 + z_1 \\ R(z_2) = z_2 \left[ 1 + \left( \frac{z_0}{z_2} \right)^2 \right] = R_2 \\ R(-z_1) = -z_1 \left[ 1 + \left( \frac{z_0}{-z_1} \right)^2 \right] = -R_1 \end{array} \right.$$

The “-” sign has to be considered here since the system of reference is unique (from left to right) and thus mirror 1 is seen having an opposite radius with respect to how it is seen from inside the cavity (e.g., convex instead of concave in the example in the figure) for a light beam coming from left.

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## Slide 8

**Optical resonators**

Let's consider a two-mirror **asymmetrical** cavity:

$$\left\{ \begin{array}{l} z_2^2 + z_0^2 = R_2 z_2 \\ z_1^2 + z_0^2 = R_1 z_1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} z_0^2 = R_2 z_2 - z_2^2 \\ z_1^2 + R_2 z_2 - z_2^2 - R_1 z_1 = 0 \end{array} \right. \quad (*)$$

$$(*) \quad z_0^2 = R_2 z_2 - z_2^2 = z_2(R_2 - z_2) \Rightarrow z_0 = \sqrt{z_2(R_2 - z_2)}$$

$z_1 = d - z_2 \Rightarrow$  substituting in  $(**)$  we get

$$(d - z_2)^2 + R_2 z_2 - z_2^2 - R_1(d - z_2) = 0 \Rightarrow z_2 = \frac{d(R_1 - d)}{R_1 + R_2 - 2d}$$

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Let us consider an **asymmetrical cavity**. We want to determine the parameters of the beam from this cavity with two mirror with different radii of curvature.

We have to determine where is the zero for this system of reference. It is easy if we apply the sign convention which we said at the beginning. We have the  $z$  axis which is the propagation axis. We think that we are no more inside the cavity, but the light comes from the left and go to the right.

Let us suppose that the zero is placed here.

We have to impose the boundary conditions. The radius of curvature at the two points  $z_1$  and  $z_2$  has to be equal to  $R$ .

We have three unknown parameters:  $z_0$ ,  $z_1$  and  $z_2$ .

We obtain an expression for the Rayleigh range  $z_0$ . We determine also  $z_2$  given the two radii of curvature. Hence, we know where is the zero of the coordinate of reference.

## Slide 9

**Optical resonators**

Let's consider a two-mirror **asymmetrical** cavity:

$$z_2 = \frac{d(R_1 - d)}{R_1 + R_2 - 2d} \quad z_0 = \sqrt{z_2(R_2 - z_2)} \quad z_1 = d - z_2$$

$0 < z_2 < d \Rightarrow$  the beam waist is inside the cavity

If  $z_0$  is complex number ( $z_0 \in \mathbb{C}$ ) the cavity becomes **unstable**.

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Once we have determined  $z_2$ , we have determined all the important parameters.

If  $z_0$  is a real number, the cavity is a **stable cavity**. If  $z_0$  is a complex number, the cavity is **unstable**.

## Slide 10

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Let's consider a two-mirror **asymmetrical** cavity:

$$R_1 = \infty$$

$$z_2 = \frac{d(R_1 - d)}{R_1 + R_2 - 2d}$$

$$z_0 = \sqrt{z_2(R_2 - z_2)}$$

$$z_1 = d - z_2$$

If one of the two mirrors is a plane mirror ( $R = \infty$ ), then the beam waist will be at the plane mirror and its position will be at  $z = 0$

$$w(z = 0) = w_0 \quad R(z = 0) = \infty \quad \Rightarrow \quad \begin{cases} z_2 = d \\ z_0 = \sqrt{d(R_2 - d)} \end{cases}$$

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## Slide 11

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**Q:** A CO<sub>2</sub> laser ( $\lambda = 10.6 \mu\text{m}$ ) has an asymmetric two-mirror optical cavity formed by two concave mirrors with curvature radii of  $R_1 = 20 \text{ m}$  and  $R_2 = 10 \text{ m}$ , at a distance  $d = 1.5 \text{ m}$ .

Determine:

- size and position of the beam waist
- beam radius at each mirror
- Rayleigh range of the beam extracted from the cavity

Is the resonator stable or unstable?

**A:** Stability condition  $0 \leq g_1 g_2 \leq 1$

$$g_1 = 1 - \frac{d}{R_1} = 1 - \frac{1.5}{20} = 0.925$$

$$g_2 = 1 - \frac{d}{R_2} = 1 - \frac{1.5}{10} = 0.85$$

$$0 \leq g_1 g_2 = 0.925 \cdot 0.85 = 0.786 \leq 1 \quad \Rightarrow \quad \text{stable}$$

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If one of the two mirrors is a plane mirror, the beam waist will be at the plane mirror.

Let us make an example.

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**Q:** A CO<sub>2</sub> laser ( $\lambda = 10.6 \mu\text{m}$ ) has an asymmetric two-mirror optical cavity formed by two concave mirrors with curvature radii of  $R_1 = 20 \text{ m}$  and  $R_2 = 10 \text{ m}$ , at a distance  $d = 1.5 \text{ m}$ .

Determine:

- size and position of the beam waist
- beam radius at each mirror
- Rayleigh range of the beam extracted from the cavity

Is the resonator stable or unstable?

**A:**

$$z_2 = \frac{d(R_1 - d)}{R_1 + R_2 - 2d} = \frac{1.5(20 - 1.5)}{20 + 10 - 3} = 1.03 \text{ m}$$

$$0 < z_2 < d = 1.5 \text{ m}$$

$$\Rightarrow \text{the beam waist is inside the cavity}$$

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## Slide 13

**Optical resonators**

**Q:** A CO<sub>2</sub> laser ( $\lambda = 10.6 \mu\text{m}$ ) has an asymmetric two-mirror optical cavity formed by two concave mirrors with curvature radii of  $R_1 = 20 \text{ m}$  and  $R_2 = 10 \text{ m}$ , at a distance  $d = 1.5 \text{ m}$ .

Determine:

- size and position of the beam waist
- beam radius at each mirror
- Rayleigh range of the beam extracted from the cavity

Is the resonator stable or unstable?

**A:**

$$\begin{aligned} z_0 &= \sqrt{z_2(R_2 - z_2)} = \\ &= \sqrt{1.03(10 - 1.03)} = 3.04 \text{ m} \\ &= \frac{\pi w_0^2}{\lambda} \\ \Rightarrow w_0 &= \sqrt{\frac{\lambda z_0}{\pi}} = 3.2 \text{ mm} \end{aligned}$$

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## Slide 14

**Optical resonators**

**Q:** A CO<sub>2</sub> laser ( $\lambda = 10.6 \mu\text{m}$ ) has an asymmetric two-mirror optical cavity formed by two concave mirrors with curvature radii of  $R_1 = 20 \text{ m}$  and  $R_2 = 10 \text{ m}$ , at a distance  $d = 1.5 \text{ m}$ .

Determine:

- size and position of the beam waist
- beam radius at each mirror
- Rayleigh range of the beam extracted from the cavity

Is the resonator stable or unstable?

**A:**

$$\begin{aligned} w_2 &= w(z_2) = w_0 \sqrt{1 + \left(\frac{z_2}{z_0}\right)^2} = 3.38 \text{ mm} \\ w_1 &= w(z_1) = w_0 \sqrt{1 + \left(\frac{z_2 - d}{z_0}\right)^2} = 3.24 \text{ mm} \end{aligned}$$

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Once we have the beam waist and the curvature radius in one specific point, we can determine for every point. We have just to set  $R(z) = R$ .

## Slide 15

**Gaussian beam propagation**

The properties of a **Gaussian beam** are totally determined **in every point** of the space if it is known the **beam waist** and the **curvature radius** **in one specific point**.

The ABCD matrix method allows us to calculate the propagation of a Gaussian beam through any optical element.

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)}$$

**Complex parameter**  
of the Gaussian beam

$q(z)$  contains information on  $\lambda$ , the radius of curvature  $R(z)$  and the waist  $w(z)$  of the beam in the point  $z$

$$w^2(z) = w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right] \quad z_0 = \frac{n\pi w_0^2}{\lambda} \quad R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$

$n$  = refractive index of the medium

Let us see what happens when a Gaussian beam is interacting with optical elements. The propagation of a Gaussian beam through any optical element is different from what we have seen in geometrical optics. We have techniques which allow us to calculate the propagation of a Gaussian beam through any optical element if we know the ABCD matrix of the optical elements. We have to introduce the **complex parameter** of the Gaussian beam.

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## Slide 16

**Gaussian beam propagation**

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$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)} = \frac{1}{q_0 + z}$$

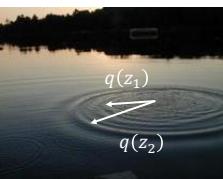
$$q_0 = q(z=0) = i \frac{n\pi w_0^2}{\lambda} = iz_0$$

$$\Rightarrow q(z) \text{ is the } \mathbf{\text{complex curvature radius}} \text{ of the Gaussian beam}$$

it transforms itself as the curvature radius of a spherical wave:

$$q(z_2) = q(z_1) + (z_2 - z_1)$$

*n = refractive index of the medium*

$$w^2(z) = w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right] \quad z_0 = \frac{n\pi w_0^2}{\lambda} \quad R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$


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## Slide 17

**Gaussian beam propagation**

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Given the parameter  $q_1 = q(z_1)$  in a point  $z_1$ , the parameter  $q_2 = q(z_2)$  in the point  $z_2$  after the interaction of the beam with an optical element with  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$  matrix can be obtained applying the **propagation relation**:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad \Rightarrow \quad \frac{1}{q_2} = \frac{C + D \left( \frac{1}{q_1} \right)}{A + B \left( \frac{1}{q_1} \right)} = \frac{1}{R_2(z)} - i \frac{\lambda}{n\pi w_2^2(z)}$$

**Free-space propagation** (for a distance  $z$ )

Let's assume that at the coordinate  $z_1 = 0$  the beam has plane wavefront  $R(z_1) = \infty$  and beam waist  $w(z_1) = w_0$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \quad \frac{1}{q_1} = -i \frac{\lambda}{n\pi w_0^2} = -\frac{i}{z_0} = \frac{1}{iz_0} \quad z_0 = \frac{n\pi w_0^2}{\lambda}$$

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## Slide 18

**Gaussian beam propagation**

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$$\begin{aligned} \frac{1}{q_2} &= \frac{C + D \left( \frac{1}{q_1} \right)}{A + B \left( \frac{1}{q_1} \right)} = \frac{0 + \frac{1}{q_1}}{1 + z \frac{1}{q_1}} = \frac{\frac{1}{iz_0}}{1 + \frac{z}{iz_0}} = \frac{1}{z + iz_0} = \frac{z - iz_0}{(z + iz_0)(z - iz_0)} \\ &= \frac{z - iz_0}{z^2 + z_0^2} = \frac{z}{z^2 + z_0^2} - i \frac{z_0}{z^2 + z_0^2} = \frac{1}{z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]} - i \frac{1}{z_0 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]} \\ &= \frac{1}{R(z)} - i \frac{\lambda}{n\pi w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)} \\ \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \quad \frac{1}{q_1} = -i \frac{\lambda}{n\pi w_0^2} = -\frac{i}{z_0} = \frac{1}{iz_0} \quad z_0 = \frac{n\pi w_0^2}{\lambda} \end{aligned}$$

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Consider that at  $z = 0$  we have  $R(z) = \infty$ , the complex parameter at  $z = 0$  as a simple expression.  
 $q(z)$  is the **complex curvature radius** of the Gaussian beam. It transforms itself as the curvature radius of a spherical wave.

How this complex parameter can tell us? We do not demonstrate these results.

Let us consider the free-space propagation (this is our optical element).

We obtain the same results that we already knew by applying propagation relation!

## Slide 19

**Gaussian beam propagation**

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**Gaussian beam with plane wavefront impinging on a thin lens (with focal length  $f$ )**

$R(z_1) = \infty$

$w(z_1) = w_{01}$

$z_{01} = \frac{n\pi w_{01}^2}{\lambda}$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{z}{f} & z \\ -\frac{1}{f} & 1 \end{bmatrix}$

$\frac{1}{q_1} = -i \frac{\lambda}{n\pi w_{01}^2} = -i \frac{1}{z_{01}} \Rightarrow \frac{1}{q_2} = \frac{C + D \left( \frac{1}{q_1} \right)}{A + B \left( \frac{1}{q_1} \right)} = \frac{-\frac{1}{f} + \frac{1}{q_1}}{1 - \frac{z}{f} + \frac{z}{q_1}}$

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## Slide 20

**Gaussian beam propagation**

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$\frac{1}{q_2} = \frac{-\frac{1}{f} + \frac{1}{q_1}}{1 - \frac{z}{f} + \frac{z}{q_1}} = \frac{-\frac{1}{f} - \frac{i}{z_{01}}}{1 - \frac{z}{f} - \frac{iz}{z_{01}}} = \frac{\left( -\frac{1}{f} - \frac{i}{z_{01}} \right) \left( 1 - \frac{z}{f} + \frac{iz}{z_{01}} \right)}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2}$

$= \frac{-\frac{1}{f} + \frac{z}{f^2} - \cancel{i\frac{z_{01}}{f}} - \frac{i}{z_{01}} + \cancel{i\frac{z_{01}}{f}} + \frac{z}{z_{01}^2}}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2}$

$= \frac{-\frac{1}{f} + z \left( \frac{1}{f^2} + \frac{1}{z_{01}^2} \right)}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2} - i \frac{\frac{1}{z_{01}}}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2}$

$= \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)} \Rightarrow z = z_{min} \quad R(z_{min}) = \infty$

plane wavefront

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Let us assume a Gaussian beam with a plane wavefront which is impinging on a thin lens. We will have the propagation after the interaction.

Our optical system is the lens and the propagation after the lens.

$z_{min}$  is when the radius of curvature is infinite after the interaction with the lens.

## Slide 21

**Gaussian beam propagation**

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$\frac{1}{R(z_{min})} = 0$

$R(z_{min}) = \infty$

$\frac{1}{q_2} = \frac{-\frac{1}{f} + z \left( \frac{1}{f^2} + \frac{1}{z_{01}^2} \right)}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2} - i \frac{\frac{1}{z_{01}}}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2} = \boxed{\frac{1}{R(z)}} - i \frac{\lambda}{n\pi w^2(z)}$

$\Rightarrow -\frac{1}{f} + z_{min} \left( \frac{1}{f^2} + \frac{1}{z_{01}^2} \right) = 0 \Rightarrow z_{min} = \frac{\frac{1}{f}}{\frac{1}{f^2} + \frac{1}{z_{01}^2}} = \frac{f}{1 + \frac{f^2}{z_{01}^2}} < f \quad !!$

$z_{01} \gg f \Rightarrow z_{min} \approx f$

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In principle  $z_{min}$  is smaller than the focal length! It will be focalized in principle at distances smaller than the focal length!

In general  $z_{01} \gg f$ , in this case we obtain  $z_{min} \approx f$ .

## Slide 22

**Gaussian beam propagation**

To determine the beam spot at  $z_{min}$ :

$$w_{02} = w(z_{min})$$

$$z_{01} = \frac{n\pi w_{01}^2}{\lambda}$$

$$\frac{1}{q_2} = \frac{-\frac{1}{f} + z \left( \frac{1}{f^2} + \frac{1}{z_{01}^2} \right)}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2} - i \frac{\frac{1}{z_{01}}}{\left( 1 - \frac{z}{f} \right)^2 + \left( \frac{z}{z_{01}} \right)^2} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)}$$

$$\Rightarrow \frac{1}{z_{01} \left[ \left( 1 - \frac{z_{min}}{f} \right)^2 + \left( \frac{z_{min}}{z_{01}} \right)^2 \right]} = \frac{\lambda}{n\pi w_{01}^2 \left[ \left( 1 - \frac{z_{min}}{f} \right)^2 + \left( \frac{z_{min}}{z_{01}} \right)^2 \right]} = \frac{\lambda}{n\pi w^2(z_{min})}$$

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## Slide 23

**Gaussian beam propagation**

To determine the beam spot at  $z_{min}$ :

$$w_{02} = w(z_{min})$$

$$z_{01} = \frac{n\pi w_{01}^2}{\lambda}$$

$$w_{02}^2 = w^2(z_{min}) = w_{01}^2 \left[ \left( 1 - \frac{z_{min}}{f} \right)^2 + \left( \frac{z_{min}}{z_{01}} \right)^2 \right] \quad z_{min} = \frac{f z_{01}^2}{f^2 + z_{01}^2}$$

$$= \frac{w_{01}^2 f^2}{f^2 + z_{01}^2} \quad \Rightarrow \quad w_{02} = \frac{w_{01} f}{z_{01} \sqrt{1 + \left( \frac{f}{z_{01}} \right)^2}} \cong \frac{w_{01} f}{z_{01}} \quad z_{01} \gg f$$

$$= \frac{w_{01}^2 f^2}{z_{01}^2 \left[ 1 + \left( \frac{f}{z_{01}} \right)^2 \right]} \quad w_{02} \cong \frac{\lambda f}{n\pi w_{01}}$$

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This is the new beam waist!

The larger is the spot of the input beam, the smaller is the beam waist that is focalized after thin lens!

## Slide 24

**Gaussian beam propagation**

Determine the output waist of a Gaussian beam impinging on a thin lens with focal length  $f$ , whose input waist is at a distance  $d$  before the lens.

$$z_{0in} = \frac{n\pi w_{0in}^2}{\lambda} \quad R(z=0) = \infty \quad \frac{1}{q_{in}} = -i \frac{\lambda}{n\pi w_{0in}^2} = -i \frac{1}{z_{0in}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{z}{f} & d + z \left( 1 - \frac{d}{f} \right) \\ -\frac{1}{f} & 1 - \frac{d}{f} \end{bmatrix}$$

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Let us make a slightly more complicated example. We have no more a plane wavefront which is impinging on the lens.

First, we have the propagation in free space, then we have the interaction with the thin lens and then the propagation in free space after the interaction.

## Slide 25

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Determine the output waist of a Gaussian beam impinging on a thin lens with focal length  $f$ , whose input waist is at a distance  $d$  before the lens.

$$z_{0in} = \frac{n\pi w_{0in}^2}{\lambda} \quad R(z=0) = \infty \quad \frac{1}{q_{in}} = -i \frac{\lambda}{n\pi w_{0in}^2} = -i \frac{1}{z_{0in}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \frac{z}{f} & d + z \left(1 - \frac{d}{f}\right) \\ -\frac{1}{f} & 1 - \frac{d}{f} \end{bmatrix} \quad \frac{1}{q_{out}} = \frac{C + D \left(\frac{1}{q_{in}}\right)}{A + B \left(\frac{1}{q_{in}}\right)} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi w^2(z)}$$

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## Slide 26

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Determine the output waist of a Gaussian beam impinging on a thin lens with focal length  $f$ , whose input waist is at a distance  $d$  before the lens.

$$\frac{1}{R(z_{min})} = 0 \implies z[(f-d)^2 + z_{0in}^2] + fd(f-d) - z_{0in}^2 f = 0$$

$$z_{min} = \frac{z_{0in}^2 f - fd(f-d)}{(f-d)^2 + z_{0in}^2}$$

Let's assume the lens is converging ( $f > 0$ ) and  $d = f$

$$\Rightarrow z_{min} = \frac{z_{0in}^2 f - f^2(f-f)}{(f-f)^2 + z_{0in}^2} = \frac{z_{0in}^2 f}{z_{0in}^2} = f$$

!! A Gaussian beam with input waist at a distance  $d = f$  from a converging lens will be focalized at  $z_{min} = f$  independently of the Rayleigh range!

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To determine the new beam waist we consider again the condition  $R(z_{min})$ .

Let us consider the case in which  $f > 0$  and  $d = f$ . We obtain that  $z_{min} = f$ . A gaussian beam with input waist at a distance  $d = f$  from a converging lens will be focalized at  $z_{min} = f$  independently on the input Rayleigh range!

## Slide 27

**800 ANNI** UNIVERSITÀ DEGLI STUDI DI PADOVA Gaussian beam propagation Optics and Laser Physics T. Cesca

Determine the output waist of a Gaussian beam impinging on a thin lens with focal length  $f$ , whose input waist is at a distance  $d$  before the lens.

$$\frac{\lambda}{n\pi w^2(z_{min})} = \frac{1}{z_{0out}} = z_{0in} \frac{fd + f(f-d)}{[fd + z_{min}(f-d)]^2 + z_{0in}^2(f-z_{min})^2}$$

$$d = f \implies z_{min} = f = z_{0in} \frac{f^2}{f^4} = \frac{z_{0in}}{f^2} \implies z_{0out} = \frac{f^2}{z_{0in}}$$

$$\Rightarrow \frac{n\pi w_{0out}^2}{\lambda} = \frac{n\pi w_{0in}^2}{\lambda} = f^2 \implies w_{0out} = \frac{\lambda f}{n\pi w_{0in}}$$

The larger the input waist at  $d = f$ , the smaller the output waist!

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Let us determine also the new beam spot.

We obtain exactly the same expression that we determine before, but now it is exact! The larger is the input beam waist and the smaller is the beam waist at the output.