

1 Lecture 19

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Fabry-Perot: Two parallel plane mirrors separated by distance d . Light rays travel between them.

Longitudinal modes:

$$v_m = m \left(\frac{c}{2d} \right)$$

$$\Delta v_{sep} = \frac{c}{2d}$$

confocal: A spherical mirror of radius R and a plane mirror. Focal points F_1 and F_2 are at the same position. Distance $d=R$.

concentric: Two spherical mirrors of radius R with centers of curvature C_1 and C_2 at the same position. Distance $d=2R$.

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The Fabry-Perot is not the most practical one, because of the allineation of the two mirrors.
Another possibility is the **confocal cavity** composed by two spherical mirrors with the same *focal point*, where the distance between them is equal to the radius of curvature. This means that the focal point is at the same position. In a **concentric cavity** the center of curvature of these spherical mirror is in the same position.

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Fabry-Perot: Two parallel plane mirrors separated by distance d .

Longitudinal modes:

$$v_m = m \left(\frac{c}{2d} \right)$$

$$\Delta v_{sep} = \frac{c}{2d}$$

emifocal: A spherical mirror of radius R and a plane mirror. Focal point F_1 is at the position of the plane mirror. Distance $d=R/2$.

emiconcentric: A spherical mirror of radius R and a plane mirror. Plane mirror is in the center of the spherical mirror. Distance $d=R$.

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One example of asymmetric configuration is the **emifocal cavity**, the focal point is at the position of the plane mirror.

From now R is indicating the radius of curvature of the spherical mirror!

We have also an **emiconcentric cavity** in which the spherical mirror and the plane mirror are at distance R such that the plane mirror is in the center of the spherical mirror.

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Ring resonators

3-mirrors: Three mirrors forming a triangle.

4-mirrors: Four mirrors forming a square.

Longitudinal modes:

$$v_m = m \left(\frac{c}{L_p} \right)$$

$$\Delta v_{sep} = \frac{c}{L_p}$$

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We can have more complex configurations as **ring resonators**. In this case we should consider the effective perimeter of the cavity.

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Unstable resonators

Confocal telescopic resonator

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For **unstable resonators** means that the photons after a certain number of passes leave the cavity. In any optical cavity the photon go back and forth thousand of times, so with an unstable resonator you can have the photons leaving the cavity. However for unstable resonator is possible to realize very large beams with a hole in the middle visible when you see the spot near the outcoupling mirror.

Another example of unstable resonator is the **confocal telescopic resonator**.

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Geometrical optics

Sign convention

R>0
p>0
q<0
y'>0

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In order to establish if a cavity is stable or unstable, its important to remember some concepts of *geometrical optics*.

Firstly, we should define the **sign convention**. We consider always light coming from left to right. We will work always with spherical surfaces with radius of curvature R (we can approximate any surface as a sphere locally). The two media between the spherical surfaces have different refractive index.

We will work with the **central optical system**: all the optical elements are aligned in the same optical axis.

In the vertex the surface intercepts the optical axis. If the surface is convex from the light coming from left, the radius is positive.

Object: p and y .

Image: q and y' .

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Spherical dioptric

Paraxial approximation $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

Equation of the spherical dioptric

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All the expression that we are writing are in the **paraxial approximation**: all the angles are small that you can approximate the sin of the angle and the tangent with the angle itself (so the distance from the optical axis are so small).

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Thin lens

Thin lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$s \rightarrow 0$

$n_1 = n_3$

Lens-maker's equation

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

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A thin lens is a block of material with two spherical surfaces. It separates two media with different refractive indexes.

A thin lens is when s is so small wrt other distances that can be neglected.

In this case from the equation of the **spherical diopter** we can write the **thin lens equation**. We are assuming $n_1 = n_3$.

f is the **focal length of the lens**. We can determine it by the **Lens-maker's equation**.

All the quantities have a sign! Also f can have one! Moreover, the focal length of most of the situation is given even when $n_1 = n_3$!

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Types of lenses

Converging

- Biconvex
- Plano-convex
- Meniscus-converging

Diverging

- Meniscus-diverging
- Plano-concave
- Biconcave

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If the focal length is positive, the lens is a **converging lens**.

If the focal length is negative, the lens is a **diverging lens**.

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Thin lens

Thin lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Lens-maker's equation

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

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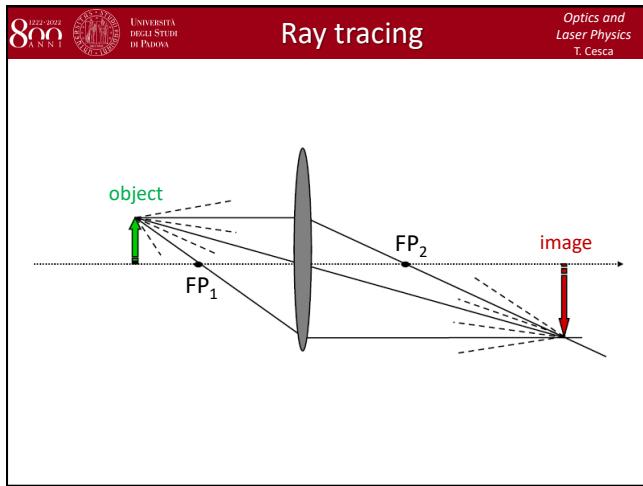
Let us imagine to have a **thin lens**. When the tips of the arrow are in this way, we have a **converging thin lens**.

We have the **forward focal point** and the **backward focal point**.

There is a ray parallel to the optical axis (it means $p = \infty$). From the lens equation we have that $q = f$. So the image will be formed at the focal point of the lens!

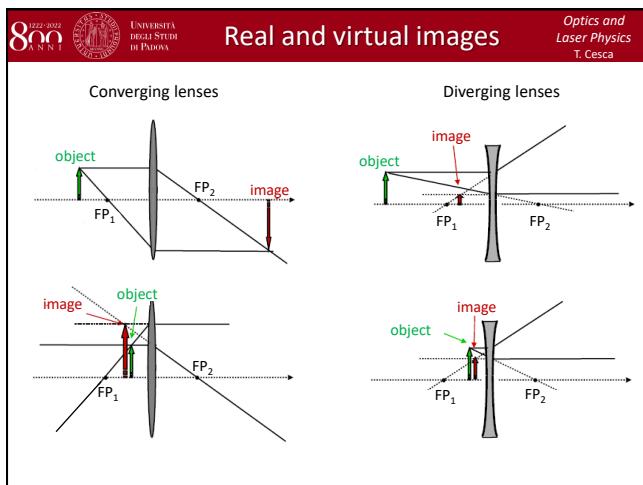
Let us consider an object placed at P_1 , we have that $p = f$. We obtain $q = \infty$. The ray is coming parallel to the optical axis!

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Magnification

Thin lenses equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$$q = \frac{fp}{p-f} \quad p = \frac{fq}{q-f}$$

Transverse magnification

$$I = \frac{y'}{y} = \frac{q}{p} = \frac{q-f}{f} = \frac{f}{p-f}$$

Longitudinal magnification

$$I_L = \frac{\Delta q}{\Delta p} = -\left(\frac{f}{p-f}\right)^2 = -I^2$$

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Graphical tool to obtain the position of an image formed when the rays of light coming from an object impinge on a thin lens.

We can get both quality and quantitative informations if we do the drawing in scale.

Three specific rays are enough to determine the position where the image is formed both for converging and diverging lenses.

We have a virtual image in the second case. There is no energy! It is just virtual.. it is formed by the extension of the rays on the back.

So the converging lens behaves as before the focal point. Otherwise it behaves as diverging.

Instead, the diverging lens is always behaving as diverging. You always have the formation of virtual images.

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Spherical mirror

Paraxial approximation

$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{R}$$

Equation of the spherical mirror

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Spherical mirror

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We can do ray tracing also in this case.

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Spherical mirror

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Try to do these exercises!

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$$\begin{bmatrix} x \\ \theta \end{bmatrix} \text{ Ray vector}$$

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

$$\begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} \xrightarrow{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix}$$

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Let us introduce the tool of **ABCD matrices**.

x is the hight wrt the optical axis and θ is the inclination.

These two elements form a **ray vector**.

Moreover, for any optical system we can introduce a matrix. The order of the matrix is important: the first matrix on the right is the first optical element the beam encounters.

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Free-space propagation (for a distance d)

$$x_2 = x_1 + d \tan \theta \approx x_1 + d \theta$$

$$\begin{cases} x_2 = x_1 + d\theta \\ \theta_2 = \theta_1 = \theta \end{cases}$$

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

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Let us consider the first simpler situation. **Free-space propagation** means the propagation inside the same medium (not in vacuum!). So, there is no change in refractive index, independent on its value. So light beam is not deflected by anything.

We will work with the paraxial approximation.

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Plane dioptr

Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \theta_1 = n_2 \theta_2$$

paraxial approximation

$$\begin{cases} x_2 = x_1 = x \\ \theta_2 = \theta_1 \frac{n_1}{n_2} \end{cases}$$

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

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Let us consider a **plane dioptr**: plane surface separating two media with different refractive index.

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Spherical dipter (with curvature radius R)

$$\begin{aligned} n_1 \theta_i &= n_2 \theta_t \\ \theta_i &= \theta_1 + \theta_s \\ \theta_t &= \theta_2 + \theta_s \\ x/R &= \theta_s \end{aligned}$$

$$\left\{ \begin{array}{l} x_2 = x_1 = x \\ \theta_2 = \theta_t - \theta_s = \theta_i \frac{n_1}{n_2} - \theta_s = (\theta_1 + \theta_s) \frac{n_1}{n_2} - \theta_s = \left(\theta_1 + \frac{x_1}{R} \right) \frac{n_1}{n_2} - \frac{x_1}{R} \end{array} \right.$$

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

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Propagation for a length d in a homogeneous and isotropic medium	$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
Crossing of a spherical dipter of radius R (that divides two media of indeces n_1 and n_2)	$\begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$
Plane dipter	$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$
Thin lens (of focal f)	$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$
Spherical mirror (of radius R)	$\begin{bmatrix} 1 & 0 \\ +\frac{2}{R} & 1 \end{bmatrix}$

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Let us consider a **spherical dipter** with radius of curvature R . It separates the two media with different refractive index.

These are the most relevant matrices for most of the optical elements.

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Thin lens

It is formed by two spherical dipters (with curvature radii R_1 and R_2) that delimit a block of material with refractive index n_2 (embedded in a material with refractive index n_1) and negligible thickness ($s \rightarrow 0$)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n_2 \frac{1}{n_1} & 0 \\ \frac{n_2 - n_1}{n_1 R_2} & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{Lens' maker equation}$$

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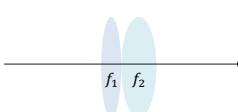
Let us compute the ABCD matrix for a **thin lens**. We can compute the matrix as the combination of the matrices of two spherical dipters with two different radii.

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ABCD matrices

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Thin lens



Combining two thin lenses with focal lengths f_1 and f_2 we get:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\left(\frac{1}{f_1} + \frac{1}{f_2}\right) & 1 \end{bmatrix}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{Effective length}$$

In the same case we can solve the situation of two thin lenses. We can define an **effective focal length**.