Fresnel's formulas

Boundary conditions: continuity of tangential components at the interface

TE (s) polarization

$$\begin{cases} E_i + E_r = E_t \\ -B_i \cos\theta_i + B_r \cos\theta_r = -B_t \cos\theta_t \end{cases}$$

Fresnel's reflection and transmission coefficients

$$r_s = \left(\frac{E_r}{E_i}\right)_{TE} \qquad t_s = \left(\frac{E_t}{E_i}\right)_{TE}$$

TM (p) polarization

$$\begin{cases} B_i - B_r = B_t \\ E_i \cos\theta_i + E_r \cos\theta_r = E_t \cos\theta_t \end{cases}$$

$$r_p = \left(\frac{E_r}{E_i}\right)_{TM} \qquad t_p = \left(\frac{E_t}{E_i}\right)_{TM}$$

$$\theta_i = \theta_r \qquad B = \frac{E}{v} = \frac{E}{c} n$$

Fresnel's formulas

Reflection coefficients

$$n = \frac{n_2}{n_1}$$

$$r_s = \left(\frac{E_r}{E_i}\right)_{TE} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t}$$

$$r_p = \left(\frac{E_r}{E_i}\right)_{TM} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} = \frac{-n\cos\theta_i + \cos\theta_t}{n\cos\theta_i + \cos\theta_t}$$

Transmission coefficients

$$t_s = \left(\frac{E_t}{E_i}\right)_{TE} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + n\cos\theta_t}$$

$$t_p = \left(\frac{E_t}{E_i}\right)_{TM} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} = \frac{2\cos\theta_i}{n\cos\theta_i + \cos\theta_t}$$

Fresnel's formulas

Snell's law

$$\boxed{n = \frac{n_2}{n_1} = \frac{\sin\,\theta_i}{\sin\,\theta_t}}$$

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad t_s = \frac{2\cos\theta_i \sin\,\theta_t}{\sin(\theta_i + \theta_t)}$$

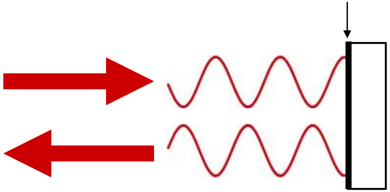
$$r_p = -\frac{tg(\theta_i - \theta_t)}{tg(\theta_i + \theta_t)} \qquad t_p = \frac{2\cos\theta_i \sin\,\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

At normal incidence ($\theta_i = \theta_r = \theta_t = 0$):

$$r_s = r_p = \frac{E_r}{E_i} = \frac{1-n}{1+n} \qquad t_s = t_p = \frac{E_t}{E_i} = \frac{2}{1+n}$$

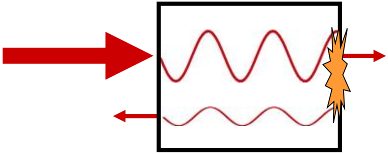
$r < 0$ ($n > 1$, es. from air to glass) indicates that the phase of the reflected wave is varied of 180° with respect to the incident wave.

High reflectance coating



Phase shift: 180°

Slightly increasing the intensity of a laser beam incident on a glass window, where will the window get damaged earlier (on the front or on the back)?



Phase shift: 0

$$r = \frac{1-n}{1+n} = \frac{1-(1/1.5)}{1+(1/1.5)} = 0.2 \qquad E_r = r\,E_i$$

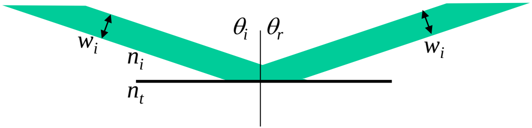
$$I = \left|E_i + E_r\right|^2 = \left|(1+r)E_i\right|^2 =$$

$$= (1.2)^2 I_i = 1.44 I_i$$

44% higher!

Reflectance (R)

$$I = \frac{1}{2} n \varepsilon_0 c |E|^2$$



$$\frac{A_r}{A_i} = \frac{w_r}{w_i} = 1$$

$$\boxed{R = \frac{I_r A_r}{I_i A_i} = \left| \frac{E_r}{E_i} \right|^2 = |r|^2}$$

$$R_s = \left| \frac{E_r}{E_i} \right|_{TE}^2 = |r_s|^2 \qquad R_p = \left| \frac{E_r}{E_i} \right|_{TM}^2 = |r_p|^2$$

At normal incidence ($\theta_i = \theta_r = \theta_t = 0°$)

$$R_s = R_p = R = \left(\frac{1-n}{1+n} \right)^2$$

From air ($n_1 = 1$) to glass ($n_2 = 1.5$) :
R=0.04 (4%)

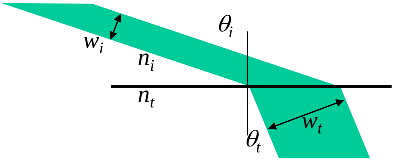
At grazing incidence ($\vartheta_i \sim 90°$)

$$R_s = R_p = R = 1$$

R is independent of n.

Transmittance (T)

$$I = \frac{1}{2} n \varepsilon_0 c |E|^2$$



$$\frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{\cos\theta_t}{\cos\theta_i}$$

$$\boxed{T = \frac{I_t A_t}{I_i A_i} = \frac{n_2}{n_1}\left|\frac{E_t}{E_i}\right|^2 \frac{w_t}{w_i} = n\,|t|^2 \frac{\cos\theta_t}{\cos\theta_i}}$$

At normal incidence ($\theta_i = \theta_r = \theta_t = 0°$)

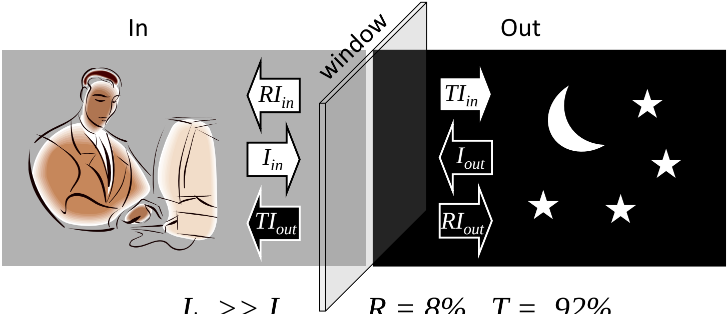
$$T_s = T_p = T = n\left(\frac{2}{1+n}\right)^2$$

From air ($n_1 = 1$) to glass ($n_2 = 1.5$): T=0.96 (96%)

For any angle of incidence:

$$\boxed{R + T = 1}$$

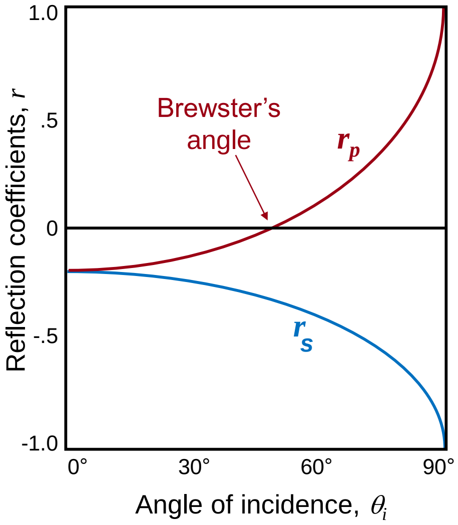
At night the windows seem mirrors when you are in an illuminated room!



$I_{in} >> I_{out}$ $R = 8\%$ $T = 92\%$

External reflection

(air-glass) n > 1



$$r_p = -\frac{tg(\theta_i - \theta_t)}{tg(\theta_i + \theta_t)} = 0$$

$$\theta_B + \theta_t = \frac{\pi}{2}$$

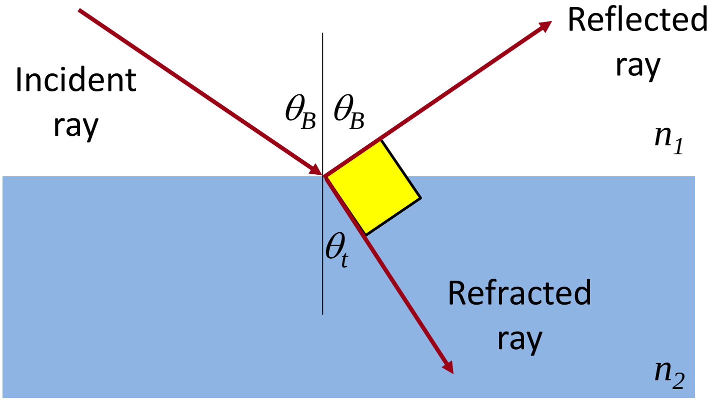
Brewster's angle

$$\boxed{\theta_B = arctg\left(\frac{n_2}{n_1}\right)}$$

air-glass $\quad \theta_B = arctg\left(\frac{1.5}{1}\right) \cong 57°$

glass-air $\quad \theta_B = arctg\left(\frac{1}{1.5}\right) \cong 33°$

Brewster's angle

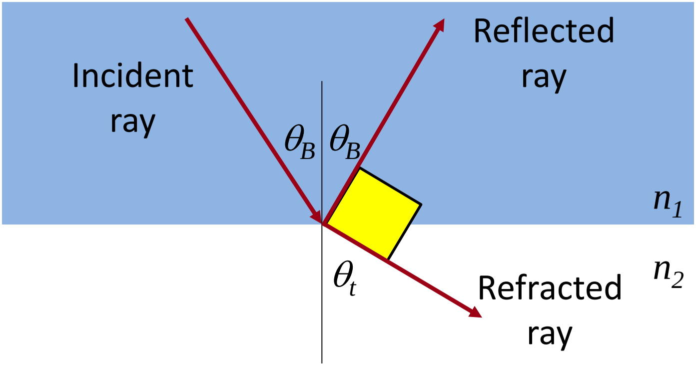


Brewster's angle

$$\theta_B = arctg\left(\frac{n_2}{n_1}\right) = arctg\left(\frac{1.5}{1}\right) = 57°$$

The reflected and refracted rays are perpendicular!

Brewster's angle

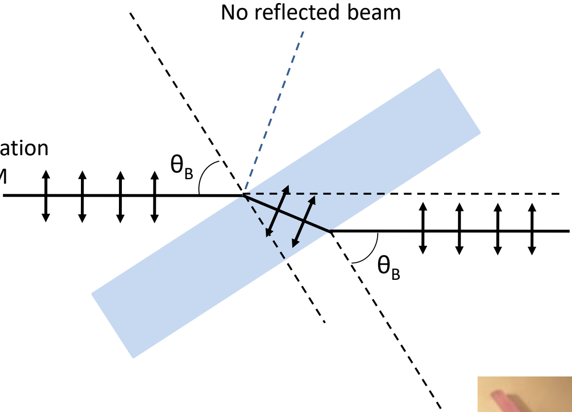


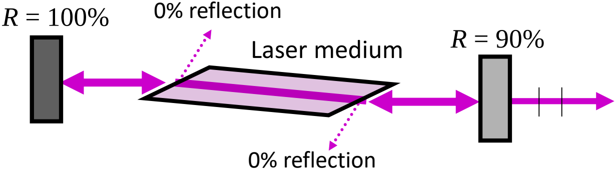
Brewster's angle

$$\theta_B = arctg\left(\frac{n_2}{n_1}\right) = arctg\left(\frac{1}{1.5}\right) = 33°$$

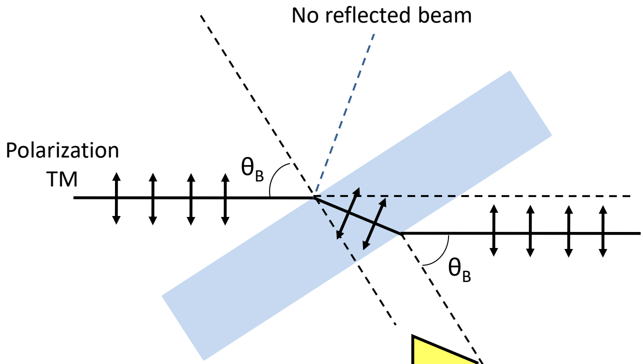
The reflected and refracted rays are perpendicular!

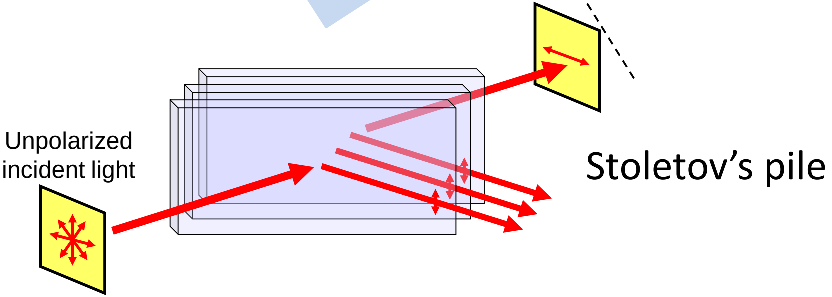
Brewster's window



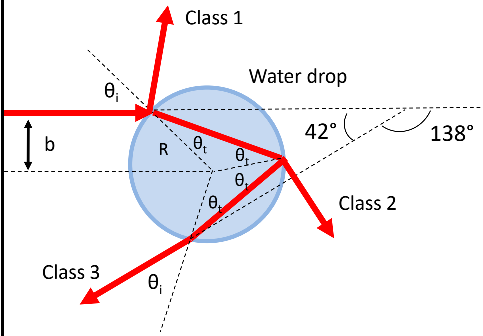


Brewster's window

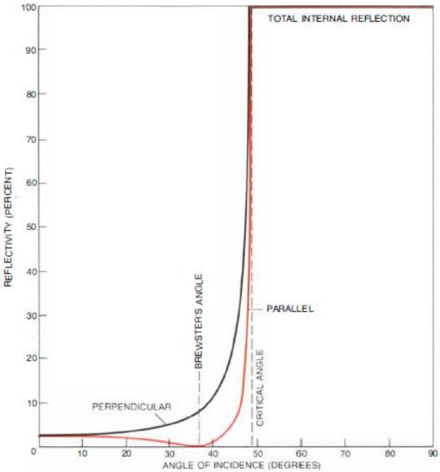




Rainbow



n = 1.33



$$\Delta_{min} = 138° \qquad \theta_i = 59° \qquad \theta_t = 40°$$

Brewster's angle

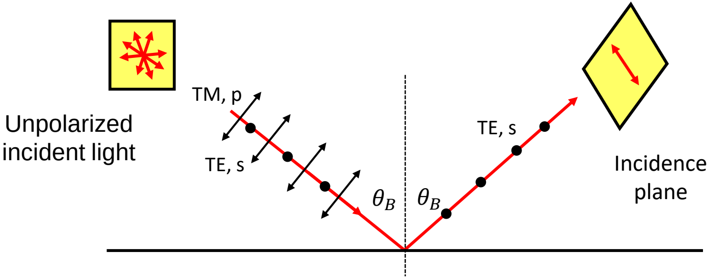
drop-air $$\theta_B = arctg\left(\frac{1}{1.33}\right) \cong 36.9°$$

Class 3 rays are TE (s) polarized.

Brewster's angle

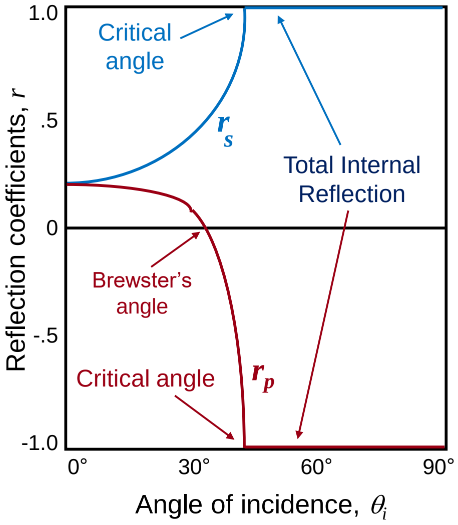
D: describe a method based on Brewster's angle to identify the transmission axis of an unknown linear polarizer.

R: at Brewster's angle $r_p = 0$, thus the reflected beam will be TE (s) polarized. If the linear polarizer is aligned to minimize the reflected beam, its transmittance axis will be orthogonal with respect to the polarization direction.



Internal reflection

(glass-air) n < 1



$$\sin\theta_c = \frac{n_2}{n_1} = n < 1$$

Critical angle

$$\boxed{\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)}$$

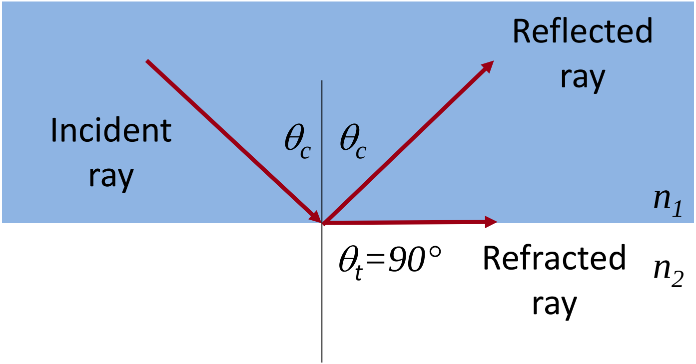
$\theta_i \geq \theta_c$ R = R_s=R_p=1

Total Internal Reflection (TIR)

$$n_{air} \sim 1 < n_{glass} \sim 1.5$$

$$\theta_c \cong 42°$$

Critical angle



Critical angle

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1}{1.5}\right) = 42°$$

Total Internal Reflection (TIR)

$$n = \frac{n_2}{n_1} < 1 \qquad \vartheta_i > \vartheta_c$$

Critical angle

$$\sin^2 \vartheta_t + \cos^2 \vartheta_t = 1 \qquad \sin \vartheta_i = n \sin \vartheta_t$$

$$r_s = \frac{\cos \vartheta_i - n\cos \vartheta_t}{\cos \vartheta_i + n\cos \vartheta_t} = \frac{\cos \vartheta_i - i\sqrt{\sin^2 \vartheta_i - n^2}}{\cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}} = e^{i\phi_s}$$

$$r_p = \frac{-n \cos \vartheta_i + \cos \vartheta_t}{n \cos \vartheta_i + \cos \vartheta_t} = \frac{-n^2 \cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}}{n^2 \cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}} = -e^{i\phi_p}$$

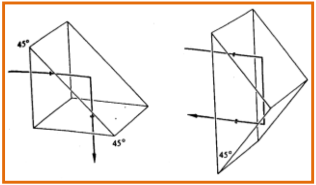
$$R = R_s = R_p = |r_s|^2 = \left|r_p\right|^2 = 1$$

Total Internal Reflection

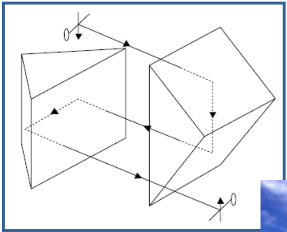
Total Internal Reflection (TIR)

Applications of TIR

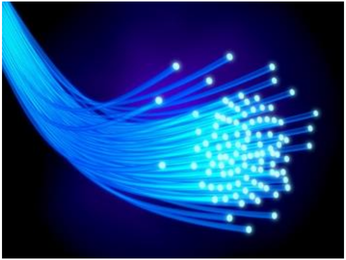
Beam steerers

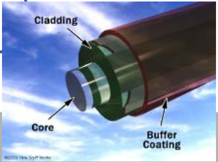


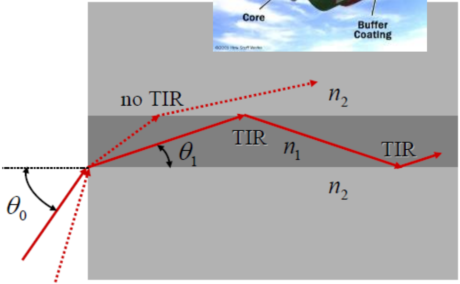
Porro's prisms



Optical fibers







Applications of TIR

Snell's law

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_1 \cos\theta_1^{\text{int}} = n_1\sqrt{1-\left(\sin\theta_1^{\text{int}}\right)^2} = n_1\sqrt{1-\left(\frac{n_2 \sin\theta_2}{n_1}\right)^2} =$$

$$= n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}$$

$\theta_2 = 90°$ (**TIR**)

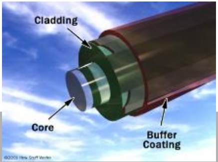
Optical fibers

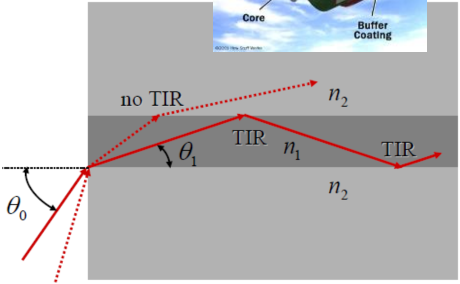
$n_1 = n_{core} > n_{cladding} = n_2$

θ_0 = acceptance angle
(Numerical Aperture, NA)

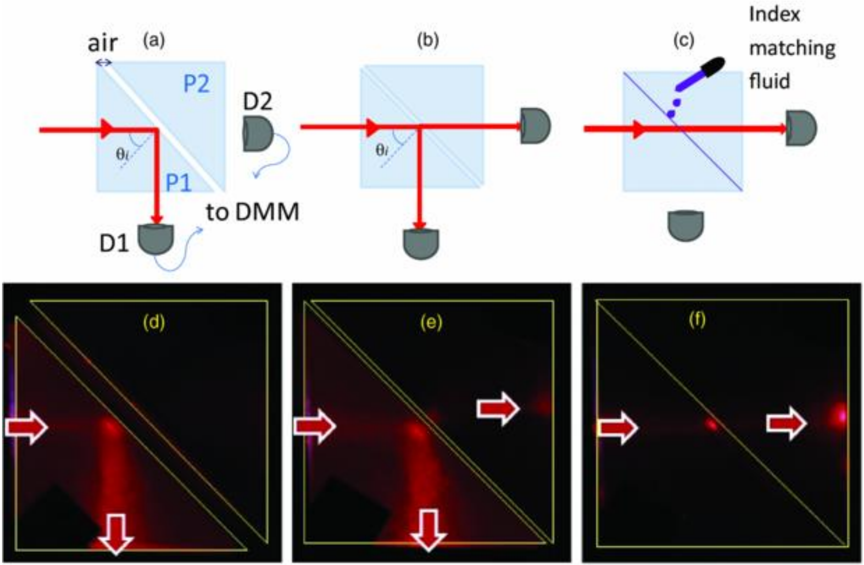
$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

High index contrast (n_1/n_2): high NA

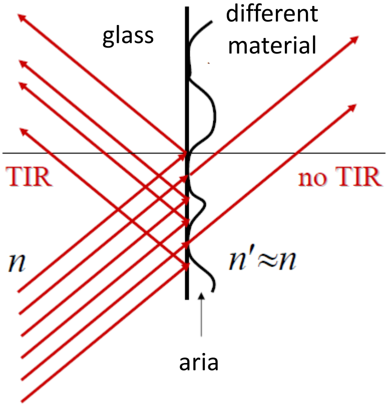




Frustrated Total Internal Reflection (FTIR)



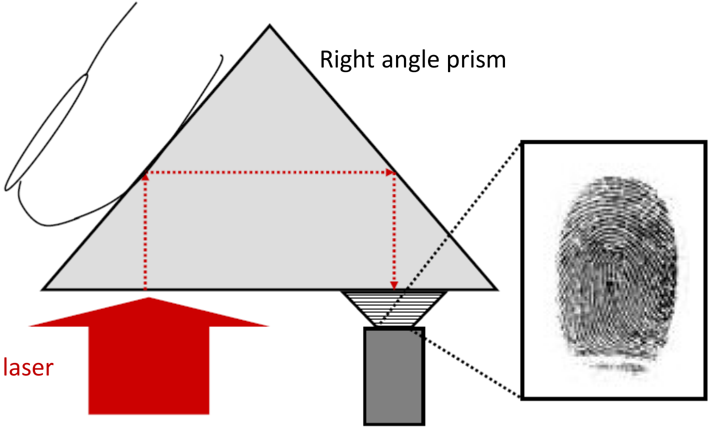
Frustrated Total Internal Reflection (FTIR)



Angle of incidence > critical angle for TIR

Where the two surfaces are in contact the total internal reflection is frustrated.

Sensors of fingerprints



See TIR from a fingerprint valley and FTIR from a ridge.

This works because the ridges are higher than the evanescent wave penetration.

Reflection and Transmission matrices

TM (p): horizontal

TE (s): vertical

Incidence plane: horizontal plane

$\begin{pmatrix} A \\ B \end{pmatrix}$ Jones vector

Reflection matrix $M_R = \begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}$

Transmission matrix $M_T = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$

$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

At **grazing** incidence: $M_R = \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(+): internal reflection, n<1

(-): external reflection, n>1

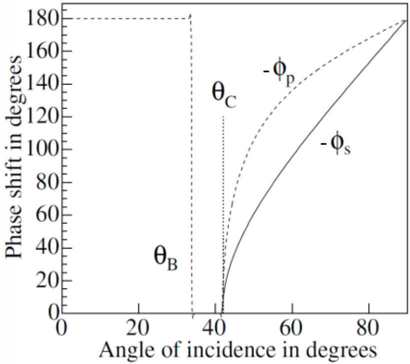
At **TIR**: $\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} e^{i\phi_p} & 0 \\ 0 & e^{i\phi_s} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = e^{i\phi_p} \begin{pmatrix} A \\ Be^{i\Delta} \end{pmatrix}$

$\boxed{\Delta = \phi_s - \phi_p}$

dephasing

At TIR the reflected wave is typically elliptically polarized!

Phase shift at TIR



$$r_s = \frac{\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}} = e^{i\phi_s}$$

$$r_p = \frac{-n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}} = -e^{i\phi_p}$$

$$\boxed{\Delta = \phi_s - \phi_p}$$

$$tg\frac{\Delta}{2} = \frac{\cos\theta_i\sqrt{\sin^2\theta_i - n^2}}{\sin^2\theta_i}$$

Fresnel's romb

$$tg\,\frac{\Delta}{2} = \frac{\cos\theta_i \sqrt{\sin^2\theta_i - n^2}}{\sin^2\theta_i}$$