

1 Lecture 24

Slide 1

Cavity quality factor

$\phi_v(t) = \phi_v(0)e^{-(t/\tau_c)}$

τ_c = lifetime of a photon in the cavity

$$\tau_c = \frac{L_e}{c\gamma}$$

$\kappa = \frac{1}{\tau_c}$ cavity photon decay rate

$$Q = 2\pi\nu\tau_c = \frac{\nu}{\Delta\nu_c} = \frac{\omega}{\Delta\omega_c}$$

Cavity quality factor

$\Delta\nu_c = \frac{1}{2\pi\tau_c} = \frac{\nu}{Q}$ Bandwidth of the cavity resonance modes

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The cavity is a key aspect for realizing a laser. We have already introduced these quantities.

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Atoms in cavities

In resonance, the atom-cavity interaction is controlled by three parameters:

- κ : photon decay rate of the cavity
- Γ : non-resonant decay rate
- g_0 : atom-cavity coupling parameter

$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In the limit of **strong coupling**, the atom-photon interaction is faster than the irreversible processes due to loss of photons out of the cavity mode

⇒ the photon emission becomes a **reversible** process in which the photon is re-absorbed by the atom before being lost from cavity

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How can we move these concepts in the nanoscale world? Let us have a nanoscale cavity. We have an atom inside the cavity which can be schematized as a two-level system.

The interaction among the atom and the cavity is controlled by the three parameters k , Γ , g_0 .

We have the condition for **weak** and **strong** coupling. Also in the macroscopic world the interaction between these two regime is quite large.

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Atoms in cavities

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$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In the limit of **weak coupling**, photons are lost from the atom-cavity system faster than the characteristic interaction time between the atom and the cavity

⇒ the emission of light from the atom is an **irreversible** process, as for free-space spontaneous emission, but the emission rate is affected by the cavity

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Atoms in cavities

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$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$$\kappa = \frac{1}{\tau_c} = \frac{2\pi\nu}{Q} = \frac{\omega}{Q}$$

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Atoms in cavities

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In resonance, the atom-cavity interaction is controlled by three parameters:

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$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$$\Gamma = \tilde{\Gamma} + \Gamma'_{rad} + \Gamma_{nr}:$$

- emission of a resonant photon in a direction that does not coincide with a cavity mode ($\tilde{\Gamma}$);
- radiative emission at different frequencies (not resonant) (Γ'_{rad});
- non-radiative processes (Γ_{nr})

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Atoms in cavities

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In resonance, the atom-cavity interaction is controlled by three parameters:

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$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$$g_0 = \left(\frac{\mu_{12}^2 \omega}{2\epsilon_0 \hbar V_0} \right)^{1/2} \quad V_0 = \text{modal volume}$$

$$\vec{\mu}_{12} = -e(\langle 2|x|1\rangle \hat{i} + \langle 2|y|1\rangle \hat{j} + \langle 2|z|1\rangle \hat{k})$$

electric dipole moment of the transition

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This is the most difficult parameter to calculate. It depends on the kind of transition you are considering. It is inverse proportional to the volume of the mode inside the cavity.

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Atoms in cavities

In resonance, the atom-cavity interaction is controlled by three parameters:

κ : photon decay rate of the cavity	$g_0 \ll (\kappa, \Gamma)$: weak coupling
Γ : non-resonant decay rate	
g_0 : atom-cavity coupling parameter	$g_0 \gg (\kappa, \Gamma)$: strong coupling

If we assume that the cavity decay rate κ is the dominant loss mechanism ($\kappa > \Gamma$)

$$\Rightarrow \text{strong coupling occurs when: } g_0 \gg \kappa = \frac{\omega}{Q} \implies Q \gg \left(\frac{2\epsilon_0 V_0 \hbar \omega}{\mu_{12}^2} \right)^{1/2}$$

very strict!

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If we assume that $k > \Gamma$, in order to be in the **strong coupling** regime we can obtain a condition for the quality factor. So, it is not that easy to be in this regime, we have stringent condition to satisfy.

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Atoms in cavities

In resonance, the atom-cavity interaction is controlled by three parameters:

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Γ : non-resonant decay rate	
g_0 : atom-cavity coupling parameter	$g_0 \gg (\kappa, \Gamma)$: strong coupling

If N atoms are present in the cavity \implies The **strong coupling** condition becomes:

$$\sqrt{N}g_0 \gg (\kappa, \Gamma) \implies \text{the factor } \sqrt{N} \text{ makes it easier to observe strong coupling}$$

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If we have a certain number of atoms, the strong coupling is given by $\sqrt{N}g_0$. It makes the condition a little bit less stringent.

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Atoms in cavities

Q: An air-spaced symmetric planar cavity of length $L = 60 \mu\text{m}$ and modal volume $V_0 = 5 \times 10^{-14} \text{ m}^3$ is locked in resonance with a cesium transition at $\lambda = 852 \text{ nm}$ which has an electric dipole moment $|\vec{\mu}_{12}| = 3 \times 10^{-29} \text{ C m}$. The radiative lifetime of the transition is $\tau_R = 32 \text{ ns}$.

Estimate the smallest values of the cavity Q factor, the cavity finesse \mathcal{F} and the mirror reflectivity R required for strong coupling for a single atom, assuming that the cavity loss rate is the dominant loss mechanism.

$h = 6.63 \cdot 10^{-34} \text{ Js}$ $c = 3 \cdot 10^8 \text{ m/s}$ $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$

A:

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = 2.2 \times 10^{15} \text{ rad/s} \quad g_0 = \left(\frac{\mu_{12}^2 \omega}{2\epsilon_0 \hbar V_0} \right)^{1/2} = 1.46 \times 10^8 \text{ rad/s}$$

$$g_0 \gg \kappa = \frac{\omega}{Q} \implies Q \gg \frac{\omega}{g_0} = \left(\frac{2\epsilon_0 V_0 \hbar \omega}{\mu_{12}^2} \right)^{1/2} = 1.51 \times 10^7$$

$$Q = \frac{\omega}{\Delta\omega_c} \gg \frac{\omega}{g_0} \implies \Delta\omega_c = 2\pi\Delta\nu_c \ll g_0 \implies \Delta\nu_c \ll \frac{g_0}{2\pi}$$

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Let us try to solve this exercise.

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Atmos in cavities

Q: An air-spaced symmetric planar cavity of length $L = 60 \mu\text{m}$ and modal volume $V_0 = 5 \times 10^{-14} \text{ m}^3$ is locked in resonance with a cesium transition at $\lambda = 852 \text{ nm}$ which has an electric dipole moment $|\vec{\mu}_{12}| = 3 \times 10^{-29} \text{ C m}$. The radiative lifetime of the transition is $\tau_R = 32 \text{ ns}$.

Estimate the smallest values of the cavity Q factor, the cavity finesse \mathcal{F} and the mirror reflectivity R required for strong coupling for a single atom, assuming that the cavity loss rate is the dominant loss mechanism.

$$h = 6.63 \cdot 10^{-34} \text{ J s} \quad c = 3 \cdot 10^8 \text{ m/s} \quad \epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m} \quad n_2 = 1$$

A:

$$\Delta\nu_c \ll \frac{g_0}{2\pi} \quad \Delta\nu_c = \frac{c}{2n_2L} \frac{1}{\mathcal{F}} \quad \Rightarrow \quad \mathcal{F} \gg \frac{\pi c}{n_2 L g_0} = 5 \times 10^5$$

$$g_0 \gg \kappa = \frac{1}{\tau_c} \quad \tau_c = \frac{L_e}{c\gamma} \quad \gamma = \cancel{\gamma_i + \frac{\gamma_1 + \gamma_2}{2}} = \gamma_1 = \gamma_2 = -\ln R$$

$$\Rightarrow \quad \tau_c = \frac{L_e}{c\gamma} \gg \frac{1}{g_0} \quad \Rightarrow \quad \ln R \gg -\frac{L_e g_0}{c} = -3 \times 10^{-5} \quad \Rightarrow \quad R \gg 99.997\%$$

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We neglect internal losses.

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Weak coupling

In the limit of **weak coupling**, the effect of the cavity is small, so that the atom-cavity interaction can be treated by perturbation theory.

The main effect of the cavity is to enhance or suppress the photon density of states compared with the free-space value, depending on whether the cavity mode is resonant with the atomic transition or not.

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Weak coupling is more simple to satisfy also experimentally. The effect of the cavity in this regime is only to modify the density of states.

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Free-space spontaneous emission

Fermi's golden rule

$$W_{21} = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(hv)$$

Transition decay rate Transition matrix element Local Density of final States (LDOS)

$$M_{12} = \langle 2 | H' | 1 \rangle = \int \psi_2^*(\vec{r}) H'(\vec{r}) \psi_1(\vec{r}) d^3\vec{r}$$

$$M_{12} = -\vec{\mu}_{12} \cdot \vec{E}$$

↓ electric dipole moment

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800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Free-space spontaneous emission Optics and Laser Physics T. Cesca

V_0 g_0 κ Γ $g_0 \ll (\kappa, \Gamma)$: **weak coupling**

Fermi's golden rule

$$W_{21} = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(hv)$$

Transition decay rate Transition matrix element Local Density of final States (LDOS)

$$W_{21} = \frac{1}{\tau_R} = A_{21} = \frac{8\pi h\nu^3 n^3}{c^3} B_{21}$$

A_{21} = Einstein's coefficient for spontaneous emission
 B_{21} = Einstein's coefficient for stimulated emission (absorption)

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E. M. Purcell, 1946

V_0 g_0 κ Γ $g_0 \ll (\kappa, \Gamma)$: **weak coupling**

The Purcell factor:

$$F_P = \frac{W_{21}^{cav}}{W_{21}^{free}} = \frac{\tau_R^{free}}{\tau_R^{cav}}$$

For a single mode cavity (modal volume V_0) at exact resonance with the emission frequency of the atom (with the dipoles oriented along the field direction):

$$F_P = \frac{3Q(\lambda/n)^3}{4\pi^2 V_0}$$

λ is the free-space wavelength of the radiation
 n is the refractive index of the medium inside the cavity

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In the weak coupling regime the role of the cavity is to change the transition rate. The **Purcell factor** quantify how the cavity influence the transition rate. So, it is the ratio between the transition rate in the cavity and the transition rate when the atom is in free space.
Let us consider only electric-dipole transition, where the dipole is oriented along the field direction in the cavity. In order to have large Purcell factor you need to have large quality factor and small modal volume.

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E. M. Purcell, 1946

V_0 g_0 κ Γ $g_0 \ll (\kappa, \Gamma)$: **weak coupling**

The Purcell factor:

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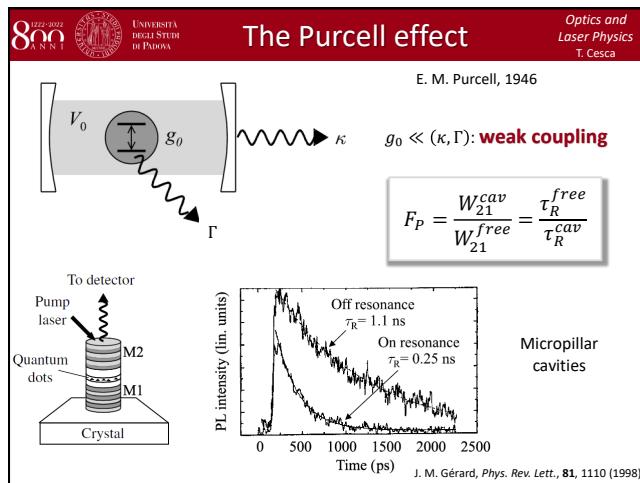
Spontaneous emission coupling factor

$$\beta = \frac{W_{21}^{cav}}{W_{21}^{free} + W_{21}^{cav}} = \frac{F_P}{1 + F_P}$$

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It is also possible to introduce the **spontaneous emission coupling factor**. In some cases is useful. But, from a physical point of view the most important is the Purcell factor.

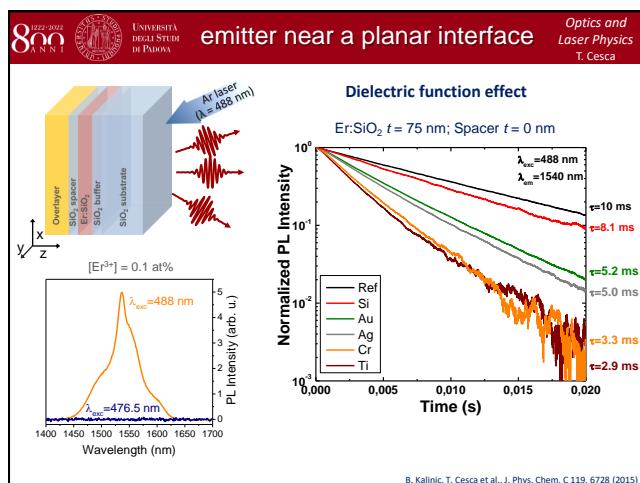
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This is one of the first experimental example in which we can observe a weak coupling between a cavity and an atom.

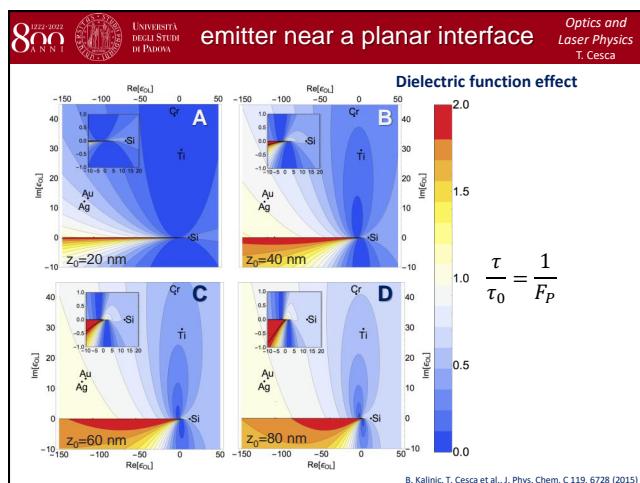
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The Purcell factor can be misurated also in other situation. The surfaces affect the local density of states. This example is already be shown.

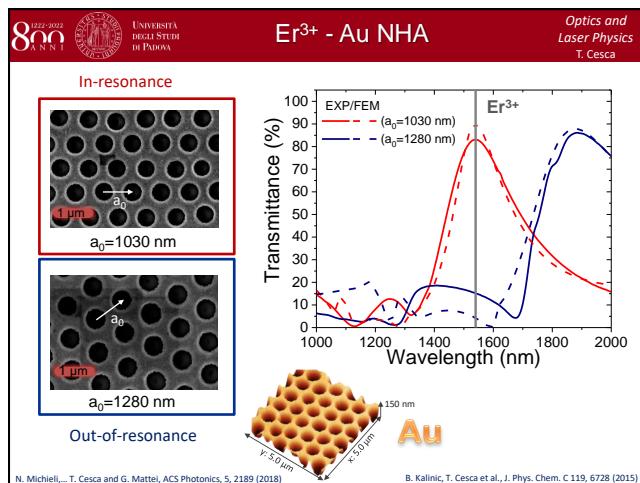
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It is possible to compute the Purcell factor for different values of the dielectric function of the material. So, we can see how the overlayer affect the density of states. These are simulation!
We vary the distance of the emitter.

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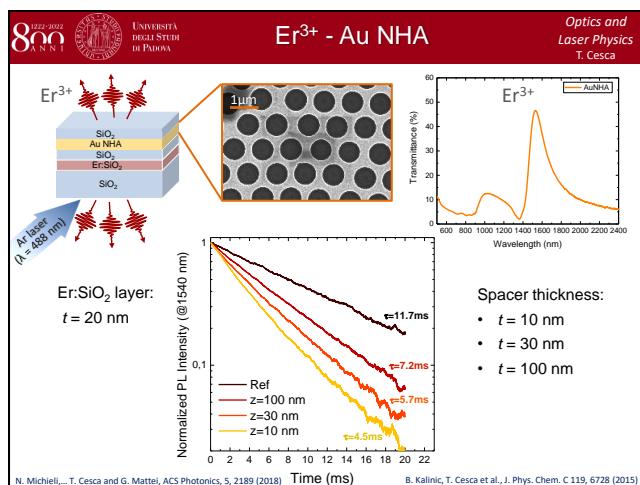


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Another way to affect the local density of states is to use nano-hole arrays.

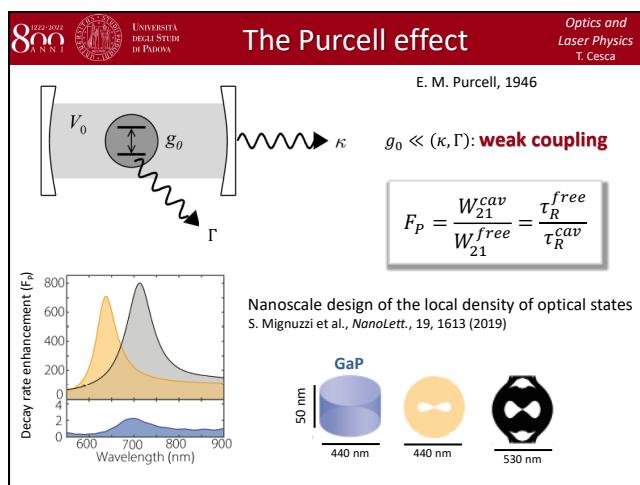
Just simply varing the lattice parameter of the nano-hole array we can have the system in resonance or out of resonance.

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Theoretically, is developed a code to obtain an increase of the Purcell factor.

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Strong coupling

V_0 g_0 κ $g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In the conditions of **strong coupling**, the interaction between the photons in the cavity mode and the atom is **reversible**.

The atom emits a photon into the resonant mode, which bounces between the mirrors and is re-absorbed by the atom faster than it is lost from the mode.

The reversible interaction between the atom and the cavity field is thus faster than the irreversible processes due to loss of photons.

This regime of reversible light-atom interactions is called **cavity quantum electrodynamics (CQED)**

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Strong coupling

V_0 g_0 κ $g_0 \gg (\kappa, \Gamma)$: **strong coupling**

Jaynes-Cummings model (1963)

$\Psi = |\psi; n\rangle$

ψ : state of the atom

n : number of photons

$\Delta E_n = 2\sqrt{n}\hbar g_0$ splitting

Energy in units of $\hbar\omega$

Atom Photons Dressed states

Jaynes-Cummings ladder

Uncoupled	Coupled
$ e; n-1\rangle g; n\rangle_{ n\rangle}$	$\pm \sqrt{n} \hbar g_0$
\vdots	\vdots
$ 3\rangle$	$\pm \sqrt{3} \hbar g_0$
$ 2\rangle$	$\pm \sqrt{2} \hbar g_0$
$ e; 0\rangle g; 1\rangle_{ 1\rangle}$	$\pm \hbar g_0$
$ e\rangle g; 0\rangle_{ 0\rangle}$	$\pm \hbar g_0$
$ g\rangle$	$\pm \hbar g_0$

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Strong coupling

V_0 g_0 κ $g_0 \gg (\kappa, \Gamma)$: **strong coupling**

vacuum Rabi splitting

$\Delta E^{vac} \equiv 2\hbar g_0$

$g_0 = \left(\frac{\mu_{12}^2 \omega}{2\epsilon_0 \hbar V_0} \right)^{1/2}$

$\Delta E^{vac} = \left(\frac{2\mu_{12}^2 \hbar \omega}{\epsilon_0 V_0} \right)^{1/2}$

Energy in units of $\hbar\omega$

Atom Photons Dressed states

Jaynes-Cummings ladder

Uncoupled	Coupled
$ n\rangle$	$\pm \sqrt{n} \hbar g_0$
\vdots	\vdots
$ 3\rangle$	$\pm \sqrt{3} \hbar g_0$
$ 2\rangle$	$\pm \sqrt{2} \hbar g_0$
$ 1\rangle$	$\pm \hbar g_0$
$ e\rangle$	$\pm \hbar g_0$
$ 0\rangle$	$\pm \hbar g_0$

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In a strong coupling regime the interaction between photons in the cavity and the atom is a reversible process: continuous exchange of energy between the cavity and the atom. This can be described by CQED.

We have a ladder which represent the different energy levels for the system. The atom can be in the ground state or in the excited state and you have a different number of photons.

The interaction of the cavity and the atom remove the degeneracy. These are called **dressed states**, it is like a Stark effect. The degeneracy is ΔE_n . The stronger is the coupling parameter the larger is the splitting.

The first split is called the **vacuum Rabi splitting**.

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For N atoms in the cavity:

$$\Delta E^{vac}(N) = \sqrt{N} \left(\frac{2\mu_{12}^2 \hbar \omega}{\epsilon_0 V_0} \right)^{1/2}$$

If the medium within the cavity has a relative permittivity ϵ_r , we should replace ϵ_0 with $\epsilon_r \epsilon_0 \equiv n^2 \epsilon_0$, where n is the refractive index:

$$\Delta E^{vac}(N) = \sqrt{N} \left(\frac{2\mu_{12}^2 \hbar \omega}{n^2 \epsilon_0 V_0} \right)^{1/2}$$

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In order to observe strong coupling it is necessary to have:

- cavities with small volumes to enhance the coupling constant g_0
- high Q -factors to reduce the photon loss rate
- other dissipative rates due to dephasing and non-resonant emission should be minimized
- the cavity should support only a single mode in resonance with the atom

challenging requirements!

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(a)

$\lambda = 589.0 \text{ nm}$ Atomic sodium beam
 Tunable Probe laser Detector
 $F = 26000$ $L = 3.2 \text{ mm}$

(b) Transmission vs Detuning (MHz) for $N = 0$. Shows a single peak at 0 MHz.

(c) Transmission vs Detuning (MHz) for $N = 200$. Shows two peaks at approximately +/- 10 MHz.

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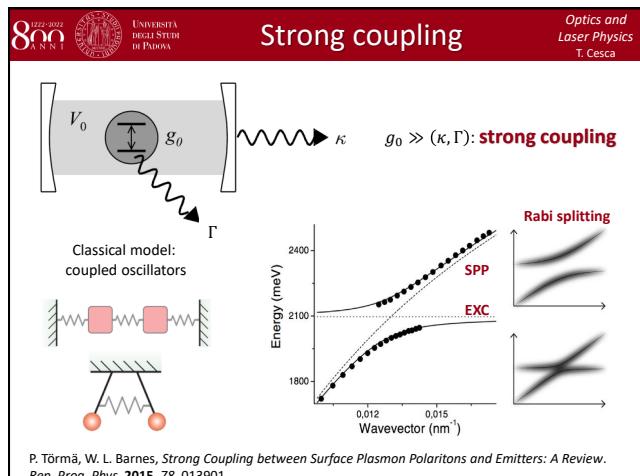
If you have N atoms in the cavity, we have to multiply the splitting by \sqrt{N} .

These are the condition to have strong coupling.
 Single mode: to maximize the interaction between the cavity and the atom.

This is an experiment in which a cavity with a large finesse is realized. A tunable laser is send and we have also an atomic sodium beam.

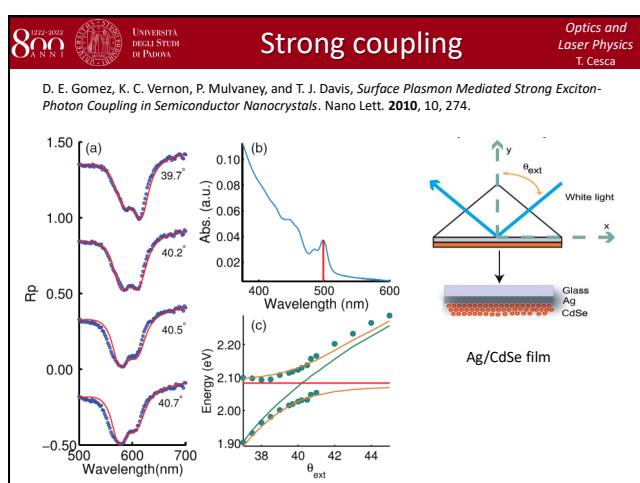
They detect the transmission of the tunable laser shine trough the cavity. When there is no atoms the cavity is designed such that to have just a narrow peak of transmission. When you have the beam of atoms, we have the splitting of the states in the cavity, so we can observe two transmission peaks.

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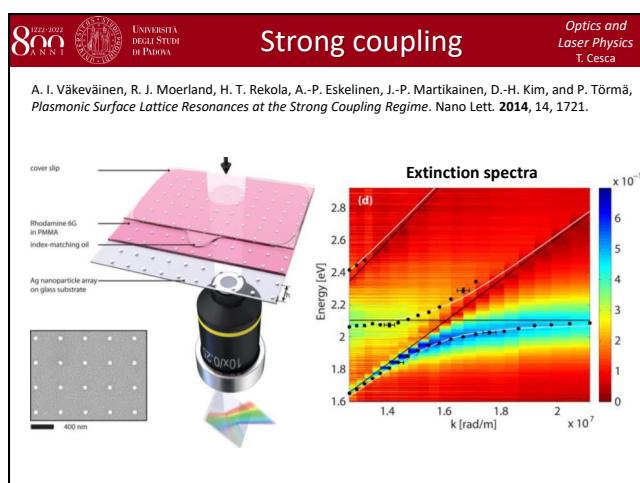
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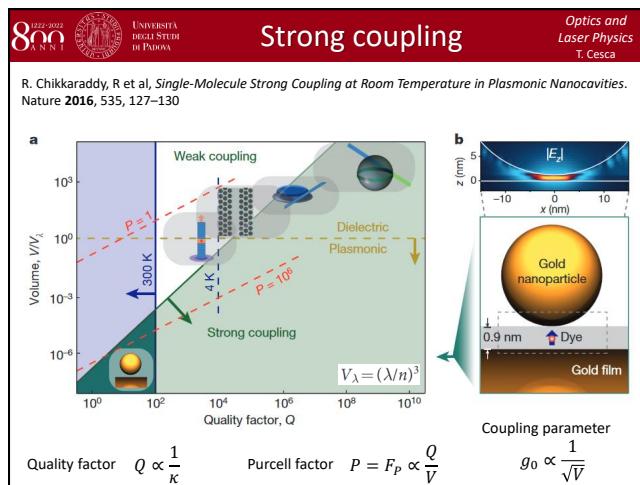


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From a classical point of view you cannot demonstrate all the effect. We can consider the strong coupling regime as if we have coupled oscillators (for instance a coupled pendulum).

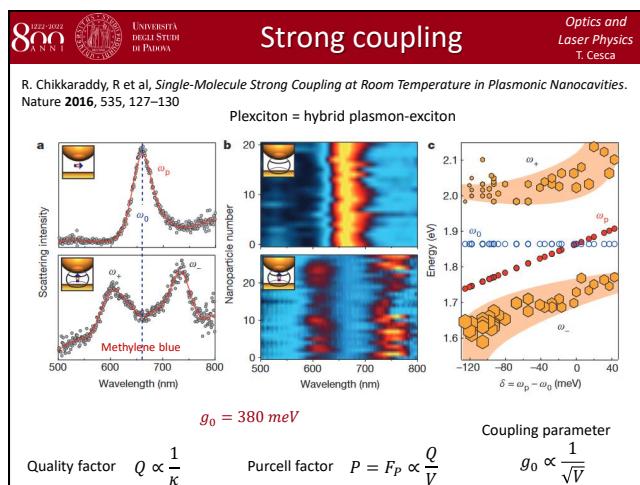
The exact solution requires a quantum mechanical approach.

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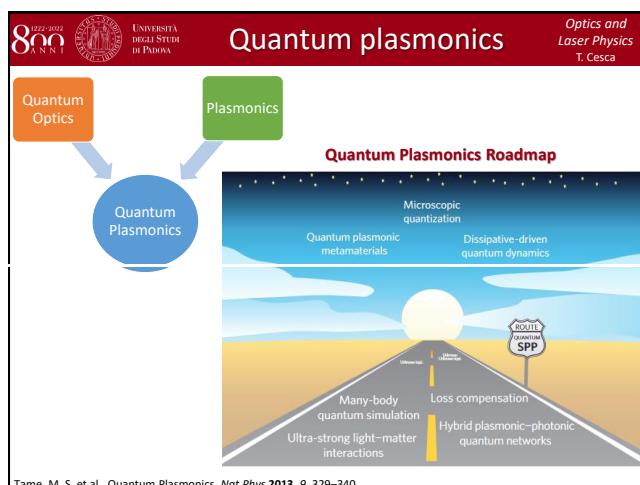
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Plasmonic system: you can have very small mode volumes.