

# 1 Lecture 20

## Slide 1

Optics and  
Laser Physics  
T. Cesca

**Thick lens**

$f_e$  = effective focal length (EFL)  
 $f_f$  = forward focal length (FFL)  
 $f_b$  = back focal length (BFL)

$PP_1$  e  $PP_2$  = principal planes

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Let us apply the ABCD matrix approach for a **thick lens**. The thickness cannot be neglected. One important result we can no longer introduce just one focal length, but we have to associate to the lens three different focal lengths: EFL, FFL, BFL.

The **effective focal length** is the equivalent of the focal length for the thin lens provided that the distance of the image is calculated by the so called **principale planes**. At the distance of FFL, with a point source here we obtain a collimated beam (a beam which is parallel to the optical axis outside of the lens).

The same idea for BFL, a collimated beam will be converged to the focal point BFL.

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**Thick lens**

Spherical diopter of radius  $R_2$       Propagation in a medium with thickness  $d$       Spherical diopter of radius  $R_1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{n_2 - n_3}{n_3 R_2} & \frac{0}{n_3} \\ \frac{1}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{n_1 - n_2}{n_2 R_1} & \frac{0}{n_2} \\ \frac{1}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix}$$

$n_1 = n_3 = 1$        $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{n-1} & \frac{0}{R_2} \\ \frac{1}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1-n}{n R_1} & \frac{0}{n} \\ \frac{1}{n R_1} & \frac{1}{n} \end{bmatrix}$

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Let us consider the most general situation with two dioptrics with two different radii of curvature.

If the lens is not in air,  $n$  is the refractive index of the lens wrt the refractive index of the medium in which the lens is embedded.

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**Thick lens**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{n-1} & \frac{0}{R_2} \\ \frac{1}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1-n}{n R_1} & \frac{0}{n} \\ \frac{1}{n R_1} & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{d}{R_1} \frac{1-n}{n} & \frac{d}{n} \\ -\left( (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{(n-1)^2 d}{n R_1 R_2} \right) & 1 + \frac{d}{R_2} \frac{n-1}{n} \end{bmatrix}$$

$\frac{1}{f_e}$

$f_e = -\frac{1}{C}$

Effective focal length (EFL)

{

$A f_e = -\frac{A}{C} = f_b$       Back focal length (BFL)

$D f_e = -\frac{D}{C} = f_f$       Forward focal length (FFL)

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We can define the effective focal length. So, it is given by the entries  $C$ .

We are not demonstrating these results!

# Slide 4

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## Thick lens

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$

$p$  = distance **object**-PP<sub>1</sub>  
 $q$  = distance **image**-PP<sub>2</sub>

$$\frac{1}{f_e} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{(n-1)^2}{n} \frac{d}{R_1 R_2}$$

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## Optical systems

$f_f \neq f_1$        $f_b \neq f_2$

**Front Focal Length (FFL)**      **Back Focal Length (BFL)**

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## Effective focal length (EFL)

**Effective focal length**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$

The distances  $p$  and  $q$  are referred to the corresponding **principal planes**.

$$f_e = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{1}{C}$$

$$I = \frac{q}{p}$$

Transverse magnification

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We can now write an expression formally equivalent to the thin lens equation provided that the distances of the object and of the image are measured from the corresponding principal planes. So, you can apply for instance ray tracing if you compute the distances from the principal planes.

The thick lens is a peculiar optical system: two spherical diopters with propagation in free-space inside. Any optical system can be treated as we did for the thick lens. Any optical system can be considered as a black box, its needed only to the determine the most important parameters which are FFL, BFL and also principal planes.

Let us consider a generic optical system composed by two lenses at a distance  $d$  (or it can be even more complicated). The idea is that when you calculate the effective focal length you have an equation similar to the one of thin lenses.

Once you have determine  $p$  and  $q$ , you can compute the **magnification**.

## Slide 7

**Effective focal length (EFL)**

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**Effective focal length**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$

The distances  $p$  and  $q$  are referred to the corresponding **principal planes**.

**Principal planes**

$f_f = f_e - h_f$        $f_b = f_e + h_b$

$h_f = \frac{f_e d}{f_2} = \frac{D-1}{C}$

$h_b = -\frac{f_e d}{f_1} = -\frac{A-1}{C}$

The principal planes on the **left** of the corresponding lens have **negative distance**.

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You can calculate the position of the principal planes: the **forward principal plane**  $h_f$  and the **backward principal plane**  $h_b$ .

We should note the minus sign  $h_b$  which is due to the fact that the principal planes on the left of the corresponding lens have negative distance.

Positive distance for  $h_f$  and  $h_b$ , it means that the principal plane is on the right of the corresponding optical element.

If you get negative distance, it means that the principal plane is on the left.

## Slide 8

**Optical systems**

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**Q:** a ray parallel to the optical axis, at a height  $y_1$  from it, propagates in air ( $n = 1$ ) through an optical system made of a thin diverging lens with focal length  $f_1 = -10 \text{ cm}$  and a thin converging lens with focal length  $f_2 = 20 \text{ cm}$  at a distance  $d$ .

Determine: the distance  $d$  so that the ray will emerge from the converging lens still parallel to the optical axis, and the height  $y_2$  of the emerging ray.

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Let us make an exercise. We are considering thin lens.

## Slide 9

**Optical systems**

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**Q:** a ray parallel to the optical axis, at a height  $y_1$  from it, propagates in air ( $n = 1$ ) through an optical system made of a thin diverging lens with focal length  $f_1 = -10 \text{ cm}$  and a thin converging lens with focal length  $f_2 = 20 \text{ cm}$  at a distance  $d$ .

Determine: the distance  $d$  so that the ray will emerge from the converging lens still parallel to the optical axis, and the height  $y_2$  of the emerging ray.

**A:**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{d}{f_1} & d \\ -\left(\frac{1}{f_2} + \frac{1}{f_1}\right) + \frac{d}{f_1 f_2} & 1 - \frac{d}{f_2} \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ 0 \end{bmatrix}$$

incident ray parallel to the optical axis

$$\begin{cases} y_2 = \left(1 - \frac{d}{f_1}\right) y_1 \\ \theta_2 = \left(-\left(\frac{1}{f_2} + \frac{1}{f_1}\right) + \frac{d}{f_1 f_2}\right) y_1 \end{cases}$$

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## Slide 10

**Optical systems** Optics and Laser Physics  
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**Q:** a ray parallel to the optical axis, at a height  $y_1$  from it, propagates in air ( $n = 1$ ) through an optical system made of a thin diverging lens with focal length  $f_1 = -10 \text{ cm}$  and a thin converging lens with focal length  $f_2 = 20 \text{ cm}$  at a distance  $d$ .

Determine: the distance  $d$  so that the ray will emerge from the converging lens still parallel to the optical axis, and the height  $y_2$  of the emerging ray.

**A:**  
In order to have an emerging beam parallel to the optical axis:

$$\theta_2 = \left( -\left( \frac{1}{f_2} + \frac{1}{f_1} \right) + \frac{d}{f_1 f_2} \right) y_1 = 0 \quad \Rightarrow \quad \frac{d}{f_1 f_2} = \left( \frac{1}{f_2} + \frac{1}{f_1} \right) = \frac{f_1 + f_2}{f_1 f_2}$$

$$\Rightarrow d = f_1 + f_2 \quad \Rightarrow d = -10 \text{ cm} + 20 \text{ cm} = 10 \text{ cm}$$

$$y_2 = \left( 1 - \frac{d}{f_1} \right) y_1 = \left( 1 - \frac{10 \text{ cm}}{-10 \text{ cm}} \right) y_1 = 2y_1 \quad \text{Beam expander}$$

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Pay attention to the sign of the quantities!

We are realizing a **beam expander**: we have a double height.

## Slide 11

**Optical systems** Optics and Laser Physics  
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**Q:** a ray parallel to the optical axis, at a height  $y_1$  from it, propagates in air ( $n = 1$ ) through an optical system made of a thin diverging lens with focal length  $f_1 = -10 \text{ cm}$  and a thin converging lens with focal length  $f_2 = 20 \text{ cm}$  at a distance  $d$ .

Determine: the distance  $d$  so that the ray will emerge from the converging lens still parallel to the optical axis, and the height  $y_2$  of the emerging ray.

**Beam expander** **Galilean telescope**

$$d = |f_2| - |f_1|$$

$$d = f_2 + f_1$$

$$y_2 = \frac{|f_2|}{|f_1|} y_1 = 2y_1$$

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We can think it in this way.

If we look at it from left to right we have a beam expander, but if we look at it from right to left we have a **Galilean telescope**.

In the Keplerian telescope we have two convergence focal lens.

## Slide 12

**Optical systems** Optics and Laser Physics  
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**Q:** an optical system consists of a diverging thin lens with focal length  $f_1 = -20.0 \text{ cm}$  and a converging thin lens with focal length  $f_2 = 10.0 \text{ cm}$  at a distance  $d = 5.0 \text{ cm}$ . Determine: effective focal length ( $f_e$ ), forward focal length ( $f_f$ ) and back focal length ( $f_b$ ) of the optical system; position of the principal planes and transversal magnification of an object at a distance  $z = 20 \text{ cm}$  from lens 1.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \frac{d}{f_1} & d \\ -\left( \frac{1}{f_2} + \frac{1}{f_1} \right) + \frac{d}{f_1 f_2} & 1 - \frac{d}{f_2} \end{bmatrix}$$

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Let us make another exercise.

$p$  and  $q$  are the distances from the corresponding principal planes!

# Slide 13

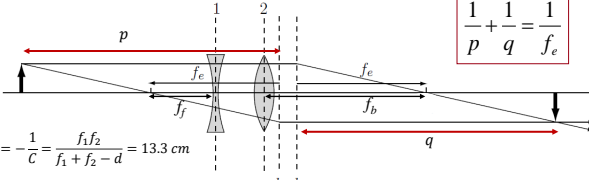


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$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$

$$f_e = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - d} = 13.3 \text{ cm}$$

$$f_b = A f_e = \left(1 - \frac{d}{f_1}\right) f_e = 16.7 \text{ cm} = (f_e + h_b)$$


$$f_f = D f_e = \left(1 - \frac{d}{f_2}\right) f_e = 6.7 \text{ cm} = (f_e - h_f)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \frac{d}{f_1} & d \\ -\left(\frac{1}{f_2} + \frac{1}{f_1}\right) + \frac{d}{f_1 f_2} & 1 - \frac{d}{f_2} \end{bmatrix}$$

430

Remember that the parameters have a given sign!

# Slide 14

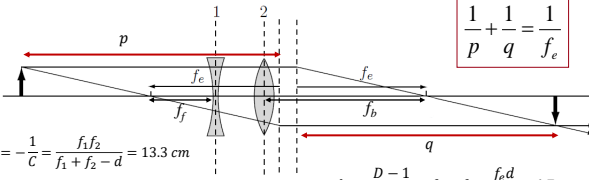


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$$f_f = D f_e = \left(1 - \frac{d}{f_2}\right) f_e = 6.7 \text{ cm} = (f_e - h_f)$$

$$h_f = \frac{D - 1}{C} = f_e - f_f = \frac{f_e d}{f_2} = 6.7 \text{ cm}$$

$$h_b = -\frac{A - 1}{C} = -(f_e - f_b) = -\frac{f_e d}{f_1} = 3.3 \text{ cm}$$


positive: PP on the right of the corresponding lens

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In this case the distances are both positive, it means that they are both on the right of the corresponding optical elements!

Positive  $h_f$  on the right of the first lens, positive  $h_b$  on the right of the second optical element! The distance is referred to the distance of the corresponding optical element! They can be in any position.

# Slide 15

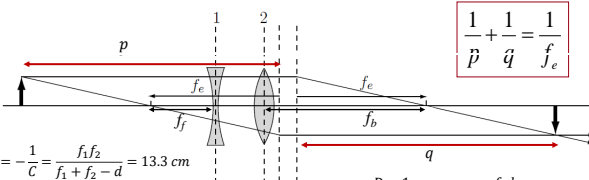


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$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}$$

$$f_e = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - d} = 13.3 \text{ cm}$$

$$p = (z + h_f) = (20 + 6.7) \text{ cm} = 26.7 \text{ cm}$$

$$p = q \quad \text{from the lens equation} \quad l = \frac{q}{p} = 1$$

$$h_f = \frac{D - 1}{C} = f_e - f_f = \frac{f_e d}{f_2} = 6.7 \text{ cm}$$

$$h_b = -\frac{A - 1}{C} = -(f_e - f_b) = -\frac{f_e d}{f_1} = 3.3 \text{ cm}$$

positive: PP on the right of the corresponding lens

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Let us determine the transverse magnification.

The image that you form has exactly the same size of the object. The object does not change the dimension.