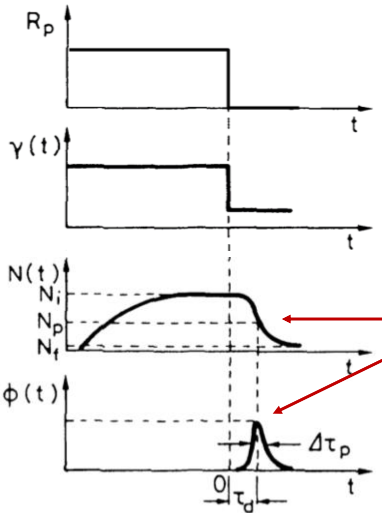
Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$N(0) = N_i$

$\phi(0) = \phi_i \approx 1$

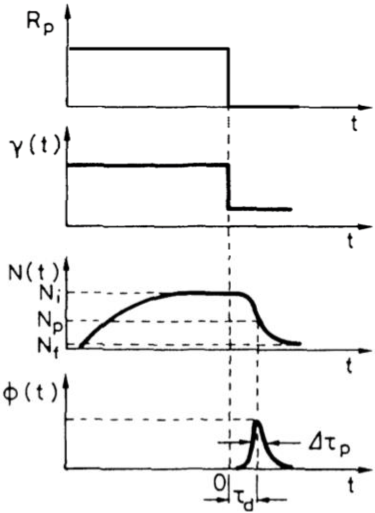
$N_P =$ population inversion at the peak of the pulse

$$\frac{d\phi}{dt} = 0 \implies N_P = \frac{1}{V_a B\tau_c} = \frac{\gamma}{\sigma l} = N_C$$

at Q-switch element open!

$$\implies x = \frac{N_i}{N_P}$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N \\[2ex] \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$N(0) = N_i$

$\phi(0) = \phi_i \approx 1$

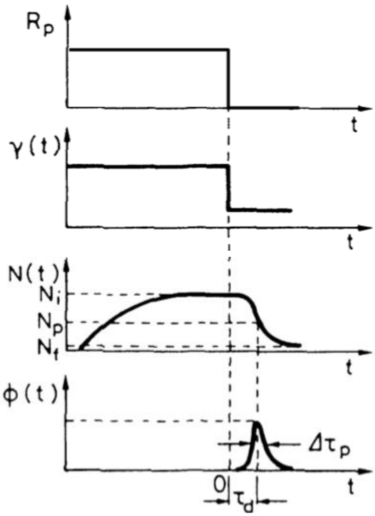
$P_P =$ peak power of the output pulse

$$P_P = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi_P$$

$\phi_P =$ number of photons at the peak of the laser pulse

to calculate ϕ_P let's make the ratio of the two rate equations

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$$N \equiv N_2 - N_1 \cong N_2$$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

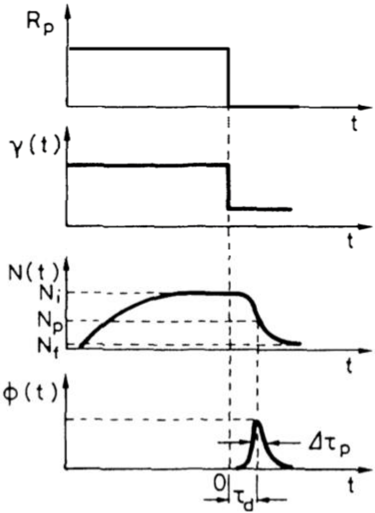
$$\frac{d\phi}{dN} = -V_a\left[1 - \frac{1}{V_a B \tau_c N}\right] = -V_a\left[1 - \frac{N_P}{N}\right]$$

$$d\phi = -V_a dN + V_a N_P \frac{dN}{N}$$

$$\phi - \cancel{\phi_i} = V_a(N_i - N) + V_a N_P \ln\left(\frac{N}{N_i}\right)$$

$$\phi = V_a\left[N_i - N - N_P \ln\left(\frac{N_i}{N}\right)\right] = \phi(N)$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

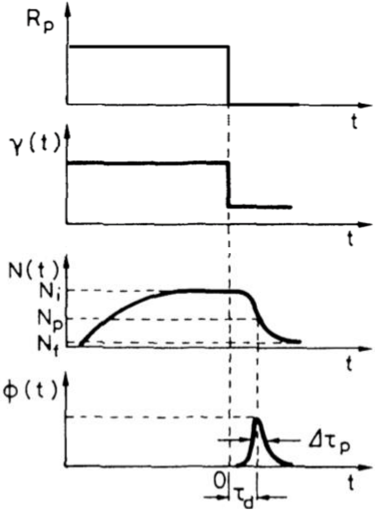
$$\phi_P = V_a\left[N_i - N_P - N_P \ln\left(\frac{N_i}{N_P}\right)\right] = \phi(N_P)$$

$$\phi_P = V_a N_P\left[\frac{N_i}{N_P} - 1 - \ln\left(\frac{N_i}{N_P}\right)\right]$$

$$\phi_P = V_a N_P[x - 1 - \ln(x)] \qquad x = \frac{N_i}{N_P}$$

$$\phi = V_a\left[N_i - N - N_P \ln\left(\frac{N_i}{N}\right)\right] = \phi(N)$$

Q-switch



$\Delta\tau_p$ = pulse duration

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$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

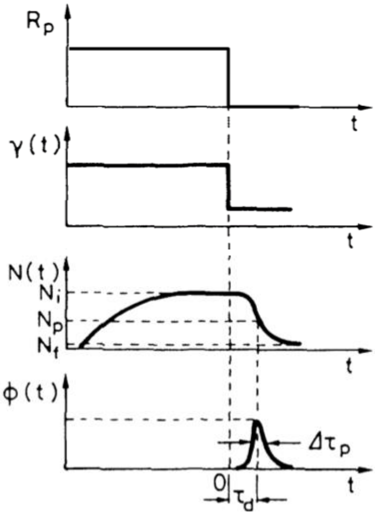
$$\phi_P = V_a N_P[x - 1 - \ln(x)] \qquad x = \frac{N_i}{N_P}$$

$$P_P = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi_P \qquad A_b = \frac{V_a}{l} \quad \tau_c = \frac{L_e}{c\gamma}$$

$$P_P = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\, V_a N_P[x - 1 - \ln(x)]$$

$$P_P = \left(\frac{\gamma_2}{2}\right)\left(\frac{A_b}{\sigma}\right)\left(\frac{h\nu}{\tau_c}\right)[x - 1 - \ln(x)]$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$(*)\quad \frac{dN}{dt} = -B\phi N \qquad\qquad N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$(**)\quad \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \qquad\qquad N_P = \frac{1}{V_a B\tau_c}$$

The output energy per pulse can be calculated as

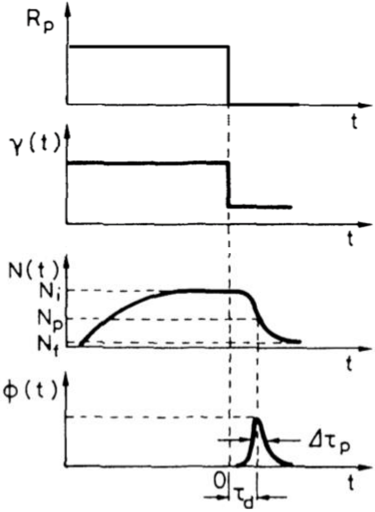
$$E = \int_0^\infty P(t)dt = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu \int_0^\infty \phi(t)dt$$

Integrating both terms of (**) we get:

$$\phi(\infty) - \phi(0) = \int_0^\infty \frac{d\phi}{dt}dt = \int_0^\infty V_a B\phi N dt - \int_0^\infty \frac{\phi}{\tau_c}dt$$

$0 =$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$
\begin{cases}
(*) \quad \dfrac{dN}{dt} = -B\phi N \\[2ex]
(**) \quad \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c}
\end{cases}
$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

$$\int_0^\infty \phi(t)dt = V_a\tau_c \int_0^\infty B\phi N dt$$

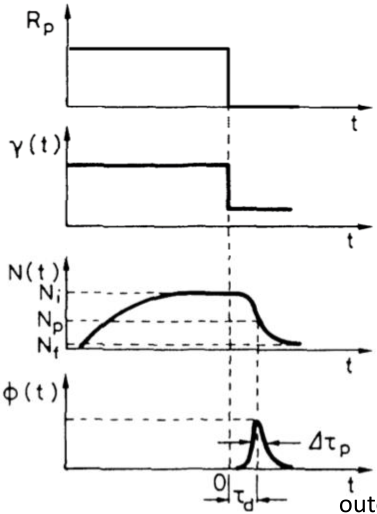
Integrating both terms of (*) we get:

$$\int_0^\infty \frac{dN}{dt}dt = \int_0^\infty -B\phi N dt = N_f - N_i$$

final inversion

$$\int_0^\infty \phi(t)dt = V_a\tau_c(N_i - N_f)$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = -B\phi N & (*) \\[2ex] \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} & (**) \end{cases}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

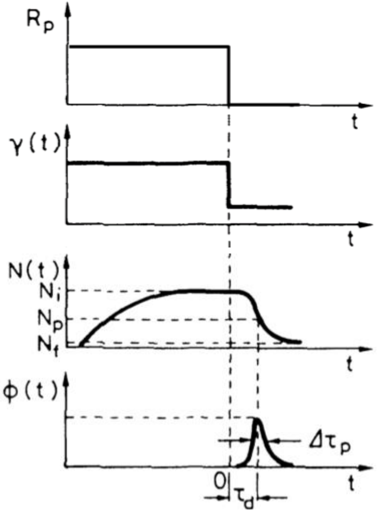
$$N_P = \frac{1}{V_a B \tau_c}$$

$$E = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu \int_0^\infty \phi(t)dt \qquad \tau_c = \frac{L_e}{c\gamma}$$

$$E = \underbrace{\left(\frac{\gamma_2}{2\gamma}\right)}_{\text{outcoupling efficiency}} \overbrace{\underbrace{(N_i - N_f)}_{\text{available inversion}} V_a h\nu}^{\text{number of photons produced}}$$

$$\int_0^\infty \phi(t)dt = V_a \tau_c (N_i - N_f)$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$$N \equiv N_2 - N_1 \cong N_2$$ Population inversion

$$\begin{cases} (*) & \dfrac{dN}{dt} = -B\phi N \\ (**) & \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

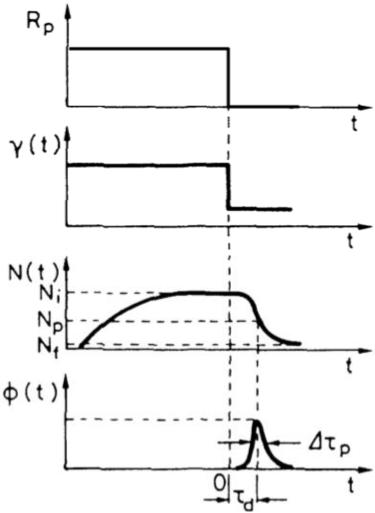
$$E = \left(\frac{\gamma_2}{2\gamma}\right)(N_i - N_f)V_a h\nu$$

To calculate E is necessary to determine N_f:

$$\phi(N) = V_a\left[N_i - N - N_P \ln\left(\frac{N_i}{N}\right)\right]$$

$$\phi(N_f) = \cancel{V_a}\left[N_i - N_f - N_P \ln\left(\frac{N_i}{N_f}\right)\right] = 0$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$(*)\quad \frac{dN}{dt} = -B\phi N$$

$$(**)\quad \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

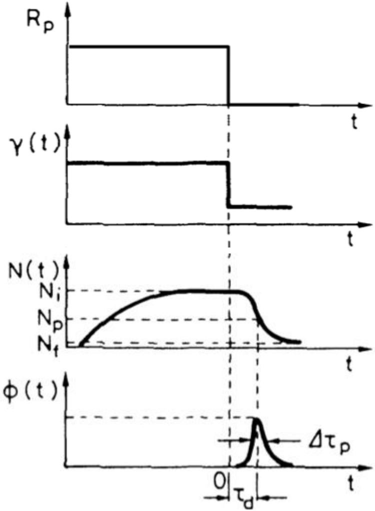
$$N_P = \frac{1}{V_a B \tau_c}$$

$$\eta_E = \frac{N_i - N_f}{N_i} = \frac{N_P}{N_i}\ln\left(\frac{N_i}{N_f}\right)$$

Inversion (energy)-utilization factor

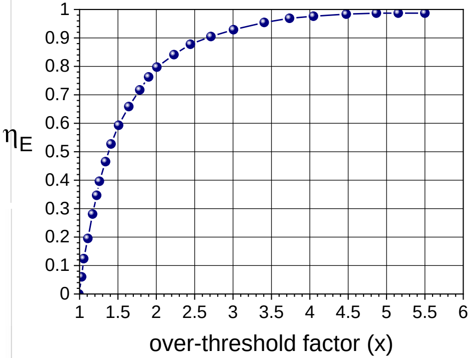
$$\eta_E \frac{N_i}{N_P} = -\ln\left(\frac{N_f}{N_i}\right) = -\ln\left(\frac{N_i - N_i + N_f}{N_i}\right)$$

$$\eta_E \frac{N_i}{N_P} = -\ln(1-\eta_E) \qquad x = \frac{N_i}{N_P}$$



$\Delta\tau_p$ = pulse duration

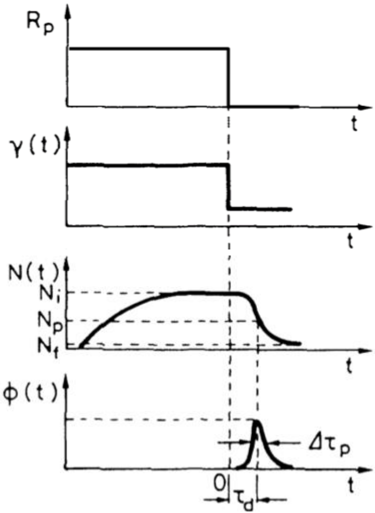
τ_d = build-up time



Inversion (energy)-utilization factor

$$\eta_E \frac{N_i}{N_P} = -\ln(1-\eta_E) \qquad x = \frac{N_i}{N_P}$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$$E = \left(\frac{\gamma_2}{2\gamma}\right)(N_i - N_f)V_a h\nu$$

$$E = \left(\frac{\gamma_2}{2\gamma}\right)\left(\frac{N_i - N_f}{N_i}\right)\left(\frac{N_i}{N_P}\right)N_P V_a h\nu$$

$$N_P = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l} = N_C \qquad A_b = \frac{V_a}{l}$$

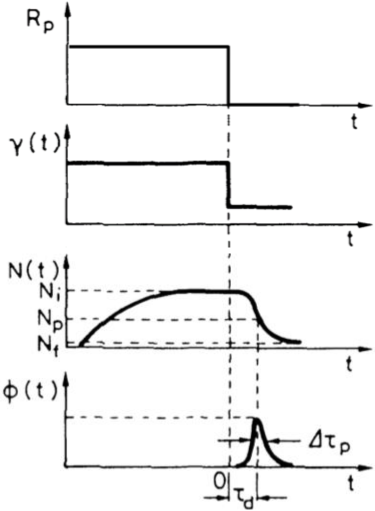
$$E = \eta_E \left(\frac{\gamma_2}{2}\right)\left(\frac{A_b}{\sigma}\right)\left(\frac{N_i}{N_P}\right)h\nu$$

Pulse energy

Inversion (energy)-utilization factor

$$\eta_E \frac{N_i}{N_P} = -\ln(1 - \eta_E) \qquad x = \frac{N_i}{N_P}$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

Pulse duration $\Delta\tau_p = \dfrac{E}{P_P}$

$$E = \eta_E \left(\frac{\gamma_2}{2}\right)\left(\frac{A_b}{\sigma}\right)\left(\frac{N_i}{N_P}\right) h\nu$$

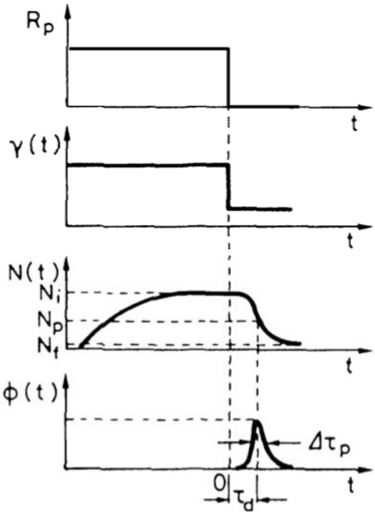
$$P_P = \left(\frac{\gamma_2}{2}\right)\left(\frac{A_b}{\sigma}\right)\left(\frac{h\nu}{\tau_c}\right) [x - 1 - \ln(x)]$$

$$\Downarrow \qquad\qquad x = \frac{N_i}{N_P}$$

$$\Delta\tau_p = \eta_E \tau_c \frac{x}{x - 1 - \ln(x)}$$

For $x = 2 - 10 \quad \Longrightarrow \quad \Delta\tau_p \approx (5 - 1.5)\tau_c$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

Build-up time

$\tau_d =$ time necessary for the number of photons to reach a given fraction of the peak value ϕ_P

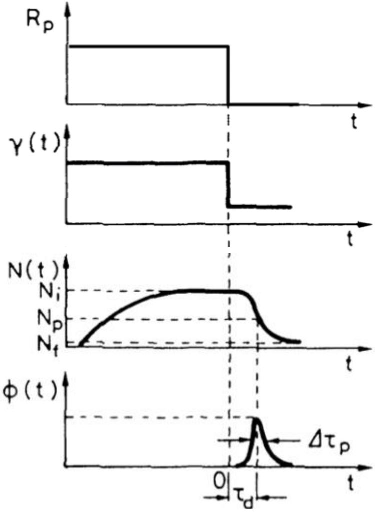
Let's assume such a fraction to be 1/10:

$$\Longrightarrow \quad N(t) \approx N_i \qquad N_P = \frac{1}{V_a B \tau_c}$$

$$\frac{d\phi}{dt} = (V_a B N \tau_c - 1)\frac{\phi}{\tau_c} = \left(\frac{N(t)}{N_P} - 1\right)\frac{\phi}{\tau_c}$$

$$\frac{d\phi}{dt} = (x-1)\frac{\phi}{\tau_c} \qquad x = \frac{N_i}{N_P}$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

Build-up time

τ_d = time necessary for the number of photons to reach a given fraction of the peak value ϕ_P

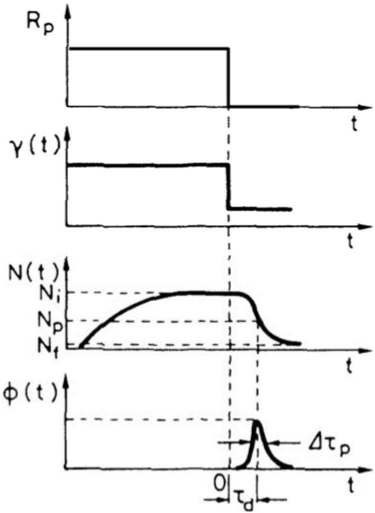
$$\phi(t) = \phi_i e^{-\frac{(x-1)t}{\tau_c}}$$

$$\phi_i \approx 1 \qquad \phi(\tau_d) = \phi_i e^{-\frac{(x-1)\tau_d}{\tau_c}} = \frac{\phi_P}{10}$$

$$\tau_d = \frac{\tau_c}{x-1} \ln\left(\frac{\phi_P}{10}\right)$$

$$\phi_P = V_a N_P [x - 1 - \ln(x)]$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

Build-up time

$\tau_d =$ time necessary for the number of photons to reach a given fraction of the peak value ϕ_P

$$\tau_d = \frac{\tau_c}{x-1}\ln\left(\frac{\phi_P}{10}\right)$$

Since ϕ_P can be very large ($\phi_P \approx 10^{17}$) and it appears in τ_d in the logarithm, it doesn't make a real difference to consider $\frac{\phi_P}{10}$ or $\frac{\phi_P}{20}$:

Es: $x = 5, \quad \phi_P = 10^{17}$

$$\tau_d = \frac{\tau_c}{x-1}\ln\left(\frac{\phi_P}{10}\right) = \frac{\tau_c}{4}36.84 = 9.2\tau_c$$

$$\tau_d = \frac{\tau_c}{x-1}\ln\left(\frac{\phi_P}{20}\right) = \frac{\tau_c}{4}36.15 = 9.0\tau_c$$