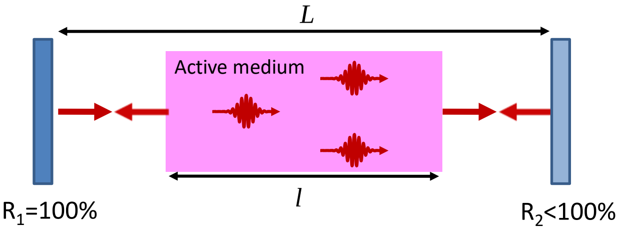
Photon lifetime



$$L_e = L + (n-1)l$$ Effective length of the cavity

L_i Internal cavity losses (for single pass)

$$t_1 = \frac{2L_e}{c}$$ (after a single pass back and forth) $$I(t_1) = I_0 R_1 R_2 (1-L_i)^2$$

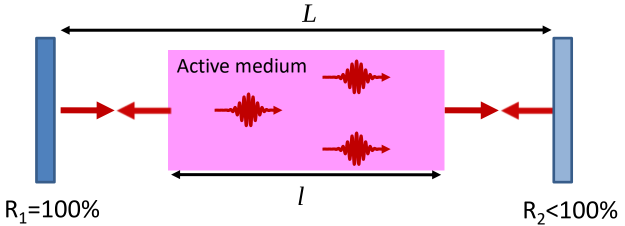
$$t_m = m\frac{2L_e}{c}$$ (after m passes) $$I(t_m) = I_0 [R_1 R_2 (1-L_i)^2]^m$$

$\phi(t) \propto I(t)$ since the mode (at frequency ν) keeps its shape at each pass

$$\phi(t_m) = \phi_0 [R_1 R_2 (1-L_i)^2]^m$$

ϕ_0: number of photons (at frequency ν) initially present in the cavity

Photon lifetime



$$\phi(t_m) = \phi_0 e^{-\frac{t_m}{\tau_c}} = \phi_0 e^{-\frac{m2L_e}{c\tau_c}}$$

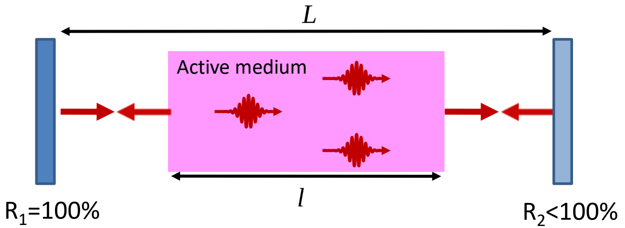
τ_c = lifetime of a photon in the cavity

$$[R_1 R_2 (1-L_i)^2]^m = e^{-\frac{m2L_e}{c\tau_c}} \quad \Rightarrow \quad \tau_c = -\frac{2L_e}{c\, ln[R_1 R_2 (1-L_i)^2]}$$

$$\phi(t_m) = \phi_0 [R_1 R_2 (1-L_i)^2]^m$$

ϕ_0: number of photons (at frequency ν) initially present in the cavity

Photon lifetime



(*) $$\phi(t_m) = \phi_0 e^{-\frac{t_m}{\tau_c}} = \phi_0 e^{-\frac{m2L_e}{c\tau_c}}$$

τ_c = lifetime of a photon in the cavity

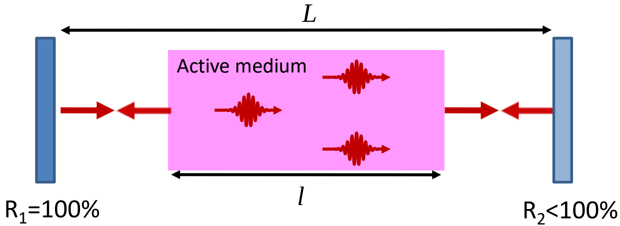
$$[R_1 R_2 (1 - L_i)^2]^m = e^{-\frac{m2L_e}{c\tau_c}} \Rightarrow \tau_c = -\frac{2L_e}{c\, ln[R_1 R_2 (1 - L_i)^2]}$$

Assuming equation (*) is valid $\forall\, t > 0$ $\qquad \phi(t) \cong \phi_0 e^{-\frac{t}{\tau_c}}$

Remembering that: $ln[R_1 R_2 (1 - L_i)^2] = -2\gamma \Rightarrow$ $\boxed{\tau_c = \frac{L_e}{c\gamma}}$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} \qquad \gamma_i = -ln(1 - L_i) \qquad \gamma_1 = -lnR_1 \qquad \gamma_2 = -lnR_2$$

Photon lifetime



For example:

τ_c = lifetime of a photon in the cavity

$$R_1 = R_2 = R = 0.98$$

$$L_i \cong 0 \qquad L_e = 90\ cm$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = \gamma_1 = -lnR = 0.02 \Rightarrow \tau_c = \frac{L_e}{c\gamma} = 150\ ns$$

$$\boxed{\tau_c = \frac{L_e}{c\gamma}}$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} \qquad \gamma_i = -ln(1 - L_i) \qquad \gamma_1 = -lnR_1 \qquad \gamma_2 = -lnR_2$$

Cavity quality factor

For any resonant system, and thus for the feedback optical cavity, it is possible to define a quality factor Q

$$Q = 2\pi\nu\left(\frac{E_\nu}{P}\right)$$

Cavity quality factor

$E_\nu = h\nu\,\phi$ Energy of the mode at frequency ν

$\phi =$ Number of photons with frequency ν (with energy $h\nu$)

$P = -\dfrac{dE_\nu}{dt} = -h\nu\dfrac{d\phi}{dt}$ Power dissipated by the resonator at frequency ν

$$Q = 2\pi\nu\left(\frac{h\nu\,\phi}{-h\nu\dfrac{d\phi}{dt}}\right) = 2\pi\nu\,\tau_c$$

$$\phi(t) = \phi_0 e^{-\frac{t}{\tau_c}}$$

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

Cavity quality factor

and given that

$$\Delta\nu_c = \frac{1}{2\pi\tau_c}$$ **Bandwidth** of the mode at frequency ν

(for the uncertainty principle: $\Delta t \Delta E = \hbar = \frac{h}{2\pi} \Longrightarrow \tau_c h \Delta\nu = \frac{h}{2\pi} \Longrightarrow \Delta\nu = \frac{1}{2\pi\tau_c}$)

➡ $$Q = \frac{\nu}{\Delta\nu_c}$$

$$\phi(t) = \phi_0 e^{-\frac{t}{\tau_c}}$$

$$Q = 2\pi\nu\left(\frac{h\nu\,\phi}{-h\nu\,\frac{d\phi}{dt}}\right) = 2\pi\nu\,\tau_c$$

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

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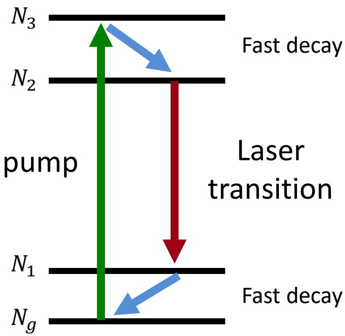
$$\Rightarrow \quad Q = \frac{\nu}{\Delta\nu_c}$$

Es:

$$\tau_c = 150\ ns \qquad @\ \lambda = 630\ nm \qquad \nu = \frac{c}{\lambda} = 4.8 \cdot 10^{14}\ Hz$$

$$Q = 2\pi\nu\,\tau_c = 4.5 \cdot 10^{8}$$

CW behavior



Working hypotheses:

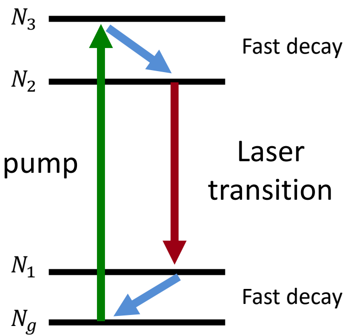
- **Four-level laser:**

$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

- **Single mode** (longitudinal and transverse)
- **Homogeneous broadening**
- **Uniform energy density** of the mode on the active medium (uniform transverse profile of the mode and standing-wave effects neglected)
- **Uniform pumping** and **constant R_p**

Space-independent rate equations

CW behavior



$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$

$$\begin{cases} \frac{dN_2}{dt} = R_p - WN_2 - \frac{N_2}{\tau} = R_p - B\phi N_2 - \frac{N_2}{\tau} \\ \frac{d\phi}{dt} = V_a W N_2 - \frac{\phi}{\tau_c} = V_a B\phi N_2 - \frac{\phi}{\tau_c} \end{cases}$$

$L_e = L + (n-1)l \qquad \tau_c = \frac{L_e}{\gamma c}$

$B\phi = W$

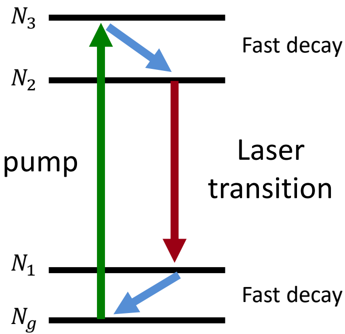
$V = \frac{L_e}{l} V_a$

V_a: Volume of the mode on the active medium

V: Volume of the mode in the cavity

$B \propto B_{21}$ Einstein's coefficient for stimulated emission

CW behavior



$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

$$\begin{cases} \dfrac{dN_2}{dt} = R_p - WN_2 - \dfrac{N_2}{\tau} = R_p - B\phi N_2 - \dfrac{N_2}{\tau} \\[2ex] \dfrac{d\phi}{dt} = V_a W N_2 - \dfrac{\phi}{\tau_c} = V_a B \phi N_2 - \dfrac{\phi}{\tau_c} \end{cases}$$

$$L_e = L + (n-1)l \qquad \tau_c = \frac{L_e}{\gamma c}$$

$$B\phi = W = \sigma F = \sigma \frac{I}{h\nu} = \sigma \frac{\phi}{t_t A_b}$$

$$V = \frac{L_e}{l} V_a$$

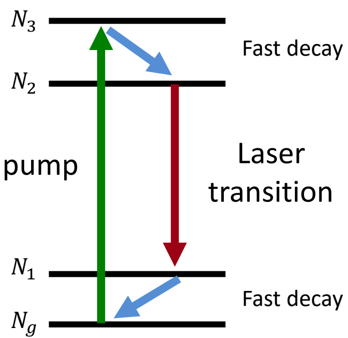
V_a: Volume of the mode on the active medium

V: Volume of the mode in the cavity

$$t_t = \frac{L_e}{c} \qquad A_b = \frac{V_a}{l}$$

$$\Rightarrow \quad B = \frac{\sigma l c}{V_a L_e} = \frac{\sigma c}{V}$$

CW behavior



$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

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$$L_e = L + (n-1)l \qquad \tau_c = \frac{L_e}{\gamma c}$$

$$B\phi = W = B_{21} h\nu\, n(\nu) = B_{21} h\nu \frac{\phi}{V_a}$$

$$V = \frac{L_e}{l} V_a$$

V_a: Volume of the mode on the active medium

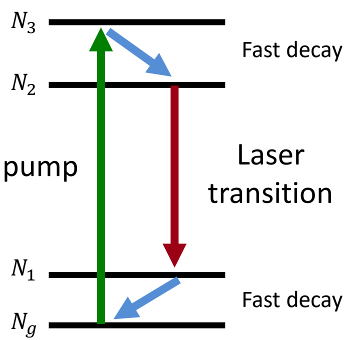
V: Volume of the mode in the cavity

$$n(\nu) = \frac{\phi}{V_a}$$

Number of photons (energy $h\nu$) per unit of volume

$\Rightarrow$ $B \propto B_{21}$

CW behavior



$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

$$\begin{cases} \dfrac{dN_2}{dt} = R_p - B\phi N_2 - \dfrac{N_2}{\tau} & (*) \\[2ex] \dfrac{d\phi}{dt} = V_a B\phi N_2 - \dfrac{\phi}{\tau_c} & (**) \end{cases}$$

Equation (**) does not contain any term that accounts for spontaneous emission.

It should contain only the fraction of spontaneously emitted light that contributes to the given mode.

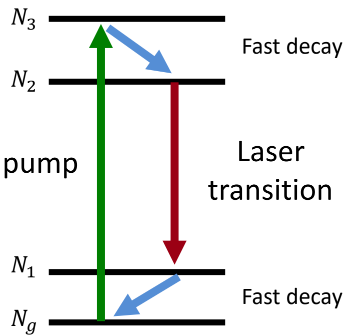
Extra-photon

$$\frac{d\phi}{dt} = V_a B(\phi + \cancel{1})N_2 - \frac{\phi}{\tau_c}$$

$\phi \approx 10^{10} - 10^{17}$

$\phi_i = 1$ for laser action to start!

CW behavior



$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

$$N \equiv N_2 - N_1 \cong N_2$$ **Population inversion**

$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

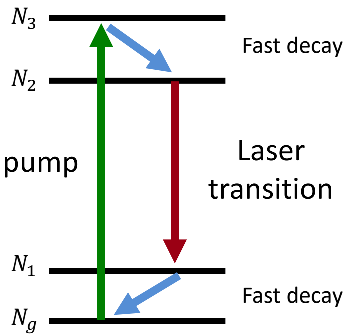
$$L_e = L + (n-1)l \qquad \tau_c = \frac{L_e}{\gamma c}$$

$$\frac{\phi}{\tau_c} = \frac{\phi c\gamma}{L_e} = \frac{\phi c\gamma_i}{L_e} + \frac{\phi c\gamma_1}{2L_e} + \frac{\phi c\gamma_2}{2L_e}$$

$$V = \frac{L_e}{l} V_a \qquad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$$

Rate of photons lost due to transmission through mirror 2 (**outcoupling**)

CW behavior



$N_1 \cong N_3 \cong 0$ $\quad \dfrac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} \\[2ex] \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

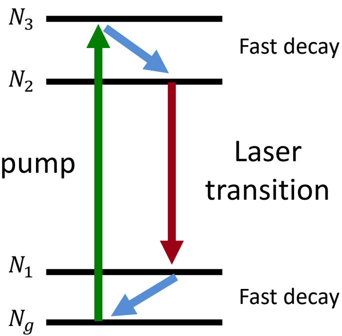
$$L_e = L + (n-1)l \qquad \tau_c = \frac{L_e}{\gamma c}$$

$$V = \frac{L_e}{l} V_a \qquad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$$

Output power

$$\boxed{P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi}$$

CW behavior



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Population inversion

$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} \\[2ex] \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

He-Ne laser $\quad P_{out} = 10\ mW$

$L_e = 50\ cm \qquad \lambda = 630\ nm$

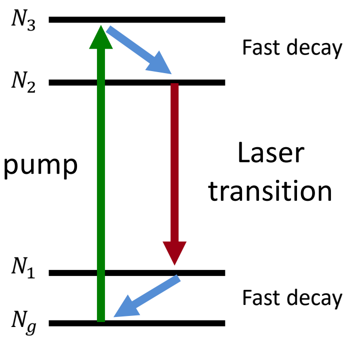
$R_2 = 99\% \qquad \gamma_2 = -lnR_2 \cong 0.01$

$$\phi = \left(\frac{2L_e}{\gamma_2 c}\right)\frac{P_{out}}{h\nu} \cong 1.06 \cdot 10^{10}$$ photons within the cavity!

Output power

$$\boxed{P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi}$$

CW behavior



$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$$\begin{cases} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{cases}$$

CO_2 laser $\quad P_{out} = 10\ kW$ $\quad$ High power example!

$L_e = 150\ cm \qquad \lambda = 10.6\ \mu m$

$R_2 = 55\% \qquad \gamma_2 = -lnR_2 = 0.598$

$\phi = \left(\frac{2L_e}{\gamma_2 c}\right)\frac{P_{out}}{h\nu} \cong 0.9 \cdot 10^{16}$ photons within the cavity!

Output power

$$\boxed{P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi}$$