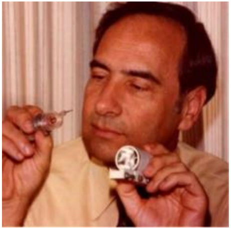
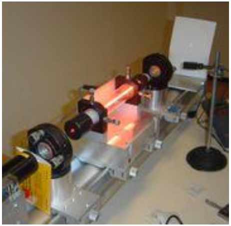
Light Amplification by Stimulated Emission of Radiation

A bit of history….

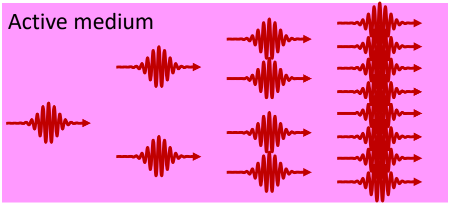
- 1917: A. Einstein studies the processes of spontaneous and stimulated emission;
- 1939 (and after the war in 1951): the Russian scientist V. A. Fabrikant suggests to use the stimulated emission process to amplify em radiation;
- 1952: the Russian scientists N.G. Basov e A.M. Prokhorov and the American physicist C.H. Townes suggest to use Fabrikant's idea in the microwaves range;
- 1954: the first MASER was built (by C. Townes and A. Schawlow);
- 1960: the American physicist Theodore Maiman put in operation the first LASER (ruby); A. Javan, W. Bennet and D. Harriott realize the first He-Ne laser at Bell's Labs;
- 1962-1963: the first semiconductors lasers were built.



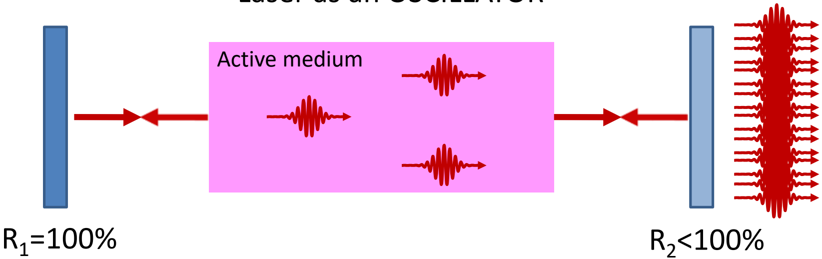


Schematic description

Laser as an AMPLIFIER



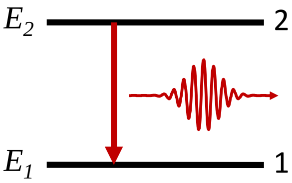
Laser as an OSCILLATOR



Laser properties

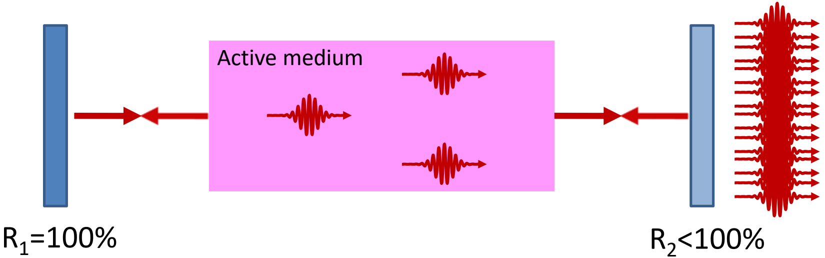
- Monochromaticity
- Directionality
- Brilliance
- Spatial and temporal coherence
- Ultra-short pulse generation

Monochromaticity



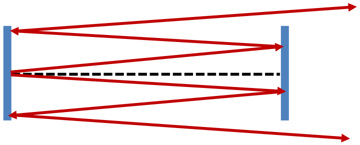
$$h\nu = E_2 - E_1$$

Only resonance frequencies of the laser feedback cavity are supported.



Directionality

Open feedback cavities: only modes that propagate along the cavity axis are sustained.



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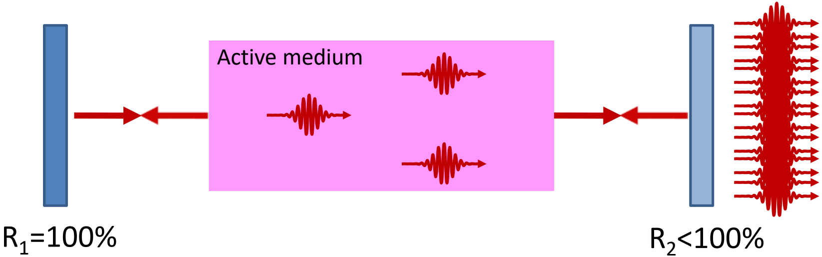


Brilliance

Emitted power per unit of surface (Σ) and unit of solid angle (Ω):

$$B = \frac{P}{\Sigma\,\Omega} = \frac{I}{\Omega}$$

Owing to the high directionality, a laser beam of limited power (mW) has a brilliance of several orders of magnitude higher than the most powerful conventional sources.

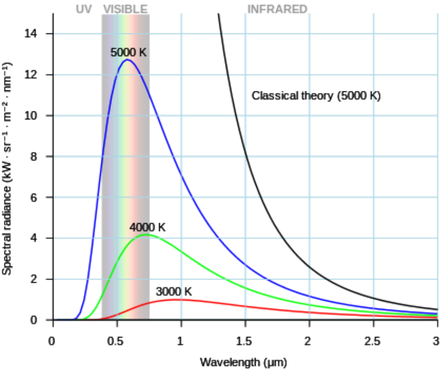
Thermal radiation

SPECTRAL RADIANCE (Specific emissive power):
Emitted energy per unit of time, unit of surface and unit of solid angle, at frequency ν

$$R_\nu(T) = \frac{2\pi}{c^2}\frac{h\nu^3}{e^{h\nu/kT}-1}$$

$h = 6.63\cdot 10^{-34}\,Js$ Planck's constant

$k = 1.38\cdot 10^{-23}\,J/K$ Boltzmann's constant



STEFAN-BOLTZMANN's LAW:
(Total emitted power per unit of surface: intensity)

$$I = \int_0^\infty R_\nu(T)d\nu \int d\Omega = e\sigma T^4$$

e = emissivity ($e \leq 1$)

$\sigma = 5.67\cdot 10^{-8}\,\mathrm{W/m^2K^4}$ Stefan's constant

WIEN's LAW: $\lambda_{max}T = \mathrm{cost} = 2898\ \mu\mathrm{m\ K}$ ➡ $\nu_{max} \propto T$

Conventional sources: Intensity ~ 10^4 K LASER: Intensity ~ 10^6-10^7 K !

Spatial coherence

Consider two points P_1, P_2 of a wavefront at $t = 0$.

By definition of wavefront, the phase difference between P_1, P_2 is null at $t = 0$:

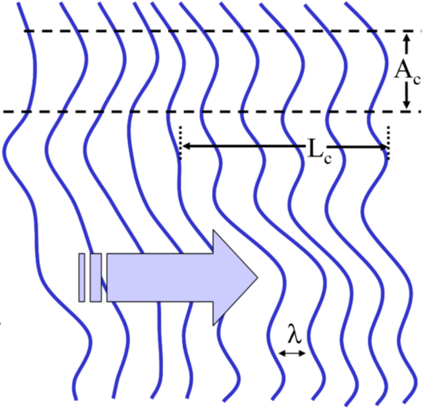
$$\Delta\phi_{P1P2}(t = 0) = 0$$

If $\Delta\phi_{P1P2}(t) = 0 \;\; \forall \;\; t > 0$

$\Longrightarrow$ **Perfect coherence between P_1 and P_2**

If this happens $\forall \; (P_1, P_2)$:

$\Longrightarrow$ **Perfect spatial coherence**



Typically for a given P_1 , P_2 has to be within a given area A_C to have a good phase correlation.

$\Longrightarrow$ **Partial spatial coherence** (within A_C)

A_C = coherence area

Spatial coherence

L_C = coherence lenght : propagation distance within which it is maintaned a given degree of spatial coherence.

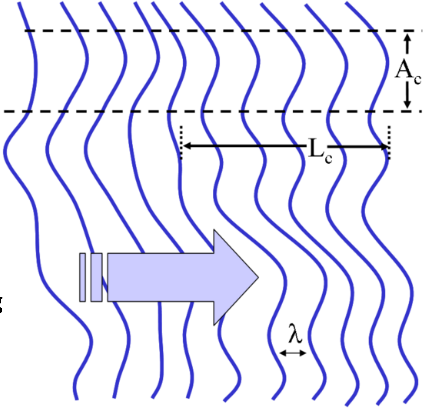
Van-Cittert-Zernike theorem

$$A_C = \frac{D^2\lambda^2}{\pi d^2}$$

D = distance from the source

d = diameter of the source

$\Longrightarrow$ Wavefronts tend to smooth-out going far away from the source



A_C = coherence area

Temporal coherence

Consider the electric field of an em wave in a given point P at time t and $t + \tau$

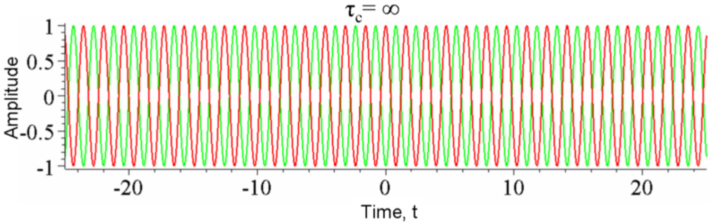
If for a given τ, the phase difference between the two electric fields is the same $\forall t$:

$\Longrightarrow$ **temporal coherence within time τ**

If this happens $\forall\, \tau$ $\Longrightarrow$ **Perfect temporal coherence**

If it happens for $0 < \tau < \tau_C$ $\Longrightarrow$ **Partial temporal coherence** (within τ_C)

τ_C = coherence time $\qquad L_C = c\, \tau_C$



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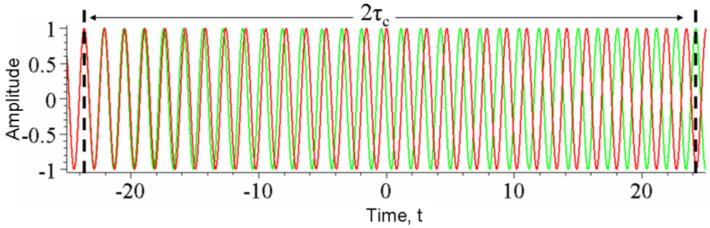


Spatial and temporal coherence

Spatial and temporal coherence are independent concepts: a wave can have perfect spatial coherence but partial temporal coherence, and viceversa.

Spatial coherence is associated with the wave's properties **transverse** to the direction of propagation.

Spatial coherence is associated with a **distribution of propagation vectors** $\vec{k}$ associated with the wave, i.e., with a departure of the wave from the ideal plane wave.

Temporal coherence is associated with the wave's properties **along** the direction of propagation.

It is associated with the **frequency distribution** of the source.

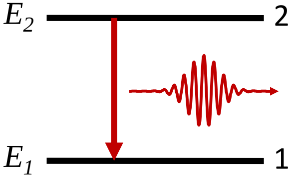
$$\tau_c \Delta\nu \approx 1 \quad \Longrightarrow \quad \tau_c \approx \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\Delta\lambda}$$

Ultra-short pulse generation

It is not an intrinsic property of any laser, but with special techniques (as the mode-locking) some lasers can produce ultra-short pulses down to fs duration.

The capability to produce ultra-short pulses, that implies the concentration of energy in time, can be considered as the counterpart of monochromaticity, that implies the concentration of energy in wavelength.

Spontaneous emission



$$h\nu = E_2 - E_1$$

Planck's constant

$$h = 6.63 \cdot 10^{-34}\,Js$$

$$\left(\frac{dN_2}{dt}\right)_{sp} = -A_{21}N_2$$

Einstein's coefficient for spontaneous emission

Spontaneous emission lifetime

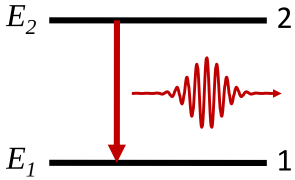
$$\tau_{sp} = \frac{1}{A_{21}}$$

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_{sp}} - \frac{N_2}{\tau'_{sp}} - \frac{N_2}{\tau_{nr}} = -\frac{N_2}{\tau}$$

$$N_2(t) = N_{20}e^{-t/\tau}$$

Lifetime of the level E_2

Spontaneous emission



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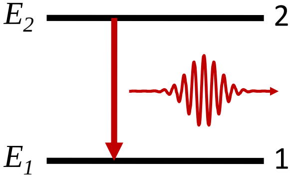
$$N_2(t) = N_{20}e^{-t/\tau}$$

$$P(t) = \frac{N_2(t)h\nu}{\tau_{sp}} \propto e^{-t/\tau}$$

Lifetime of the level E_2

Spontaneous emission



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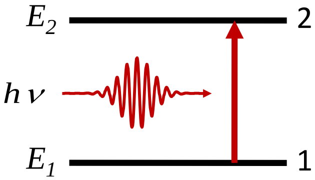
Fluorescence Quantum Yield

$$\Phi_{21} = \frac{\text{number of photons with energy } h\nu = E_2 - E_1}{\text{number of atoms at level } E_2} = \frac{\tau}{\tau_{sp}}$$

Absorption



Planck's constant

$$h = 6.63 \cdot 10^{-34}\ Js$$

$$\frac{dN_1}{dt} = -W_{12} N_1$$

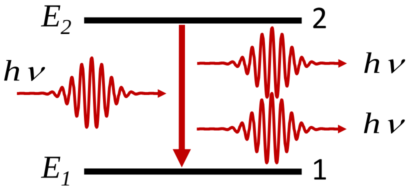
Photon flux $F = \dfrac{I}{h\nu}$

$$W_{12} = B_{12} n(\nu) h\nu = B_{12}\rho(\nu) = \sigma_{12} F$$

Einstein's coefficient for absorption

Absorption cross-section

Stimulated emission



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Photon flux $\quad F = \dfrac{I}{h\nu}$

$$W_{21} = B_{21} n(\nu) h\nu = B_{21}\rho(\nu) = \sigma_{21} F$$

Einstein's coefficient for stimulated emission

Stimulated emission cross-section