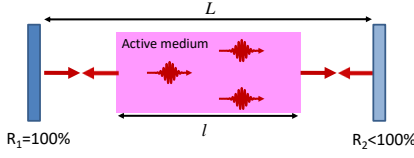


1 Lecture 12

Slide 1



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Photon lifetime

$$L_e = L + (n - 1)l \quad \text{Effective length of the cavity} \quad L_i \quad \text{Internal cavity losses (for single pass)}$$

$$t_1 = \frac{2L_e}{c} \quad (\text{after a single pass back and forth}) \quad I(t_1) = I_0 R_1 R_2 (1 - L_i)^2$$

$$t_m = m \frac{2L_e}{c} \quad (\text{after } m \text{ passes}) \quad I(t_m) = I_0 [R_1 R_2 (1 - L_i)^2]^m$$

$\phi(t) \propto I(t)$ since the mode (at frequency ν) keeps its shape at each pass

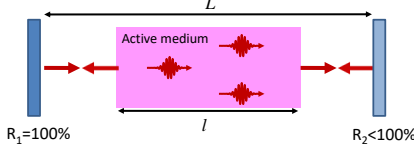
$$\phi(t_m) = \phi_0 [R_1 R_2 (1 - L_i)^2]^m$$

number of photons (at frequency ν) initially present in the cavity

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After m passes back and forth the time is $t_m = mt_1$. We can assume that the number of photon between the cavity is proportional to the intensity. So, the temporal dependence of the intensity is proportional to the number of photons in the cavity.

Slide 2



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Photon lifetime

$$\phi(t_m) = \phi_0 e^{-\frac{t_m}{\tau_c}} = \phi_0 e^{-\frac{m 2L_e}{c \tau_c}} \quad \tau_c = \text{lifetime of a photon in the cavity}$$

$$[R_1 R_2 (1 - L_i)^2]^m = e^{-\frac{m 2L_e}{c \tau_c}} \Rightarrow \tau_c = -\frac{2L_e}{c \ln[R_1 R_2 (1 - L_i)^2]}$$

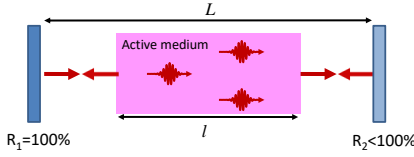
$$\phi(t_m) = \phi_0 [R_1 R_2 (1 - L_i)^2]^m$$

number of photons (at frequency ν) initially present in the cavity

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We rewrite the number of photons in the camera as an exponential decay with constant given by the photon lifetime.

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Photon lifetime

(*) $\phi(t_m) = \phi_0 e^{-\frac{t_m}{\tau_c}} = \phi_0 e^{-\frac{m 2L_e}{c \tau_c}} \quad \tau_c = \text{lifetime of a photon in the cavity}$

$$[R_1 R_2 (1 - L_i)^2]^m = e^{-\frac{m 2L_e}{c \tau_c}} \Rightarrow \tau_c = -\frac{2L_e}{c \ln[R_1 R_2 (1 - L_i)^2]}$$

Assuming equation (*) is valid $\forall t > 0$ $\phi(t) \cong \phi_0 e^{-\frac{t}{\tau_c}}$

Remembering that: $\ln[R_1 R_2 (1 - L_i)^2] = -2\gamma$ \Rightarrow $\tau_c = \frac{L_e}{c\gamma}$

$\gamma = \gamma_1 + \frac{\gamma_1 + \gamma_2}{2} \quad \gamma_1 = -\ln(1 - L_i) \quad \gamma_1 = -\ln R_1 \quad \gamma_2 = -\ln R_2$

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t_m are discrete values, but we can rewrite it in a continuous form.
 γ are the total logarithmic losses of the photon inside the cavity.

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Photon lifetime

For example:

$$R_1 = R_2 = R = 0.98$$

$$L_i \cong 0 \quad L_e = 90 \text{ cm}$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = \gamma_i = -\ln R = 0.02 \quad \Rightarrow \quad \tau_c = \frac{L_e}{c\gamma} = 150 \text{ ns}$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} \quad \gamma_i = -\ln(1 - L_i) \quad \gamma_1 = -\ln R_1 \quad \gamma_2 = -\ln R_2$$

$$\tau_c = \frac{L_e}{c\gamma}$$

τ_c = lifetime of a photon in the cavity

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These are some numbers to do some calculations. So, typically photons survive within the cavity 150 ns.

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Cavity quality factor

For any resonant system, and thus for the feedback optical cavity, it is possible to define a quality factor Q

$$Q = 2\pi\nu \left(\frac{E_\nu}{P} \right) \quad \text{Cavity quality factor}$$

$E_\nu = h\nu \phi$ Energy of the mode at frequency ν

ϕ = Number of photons with frequency ν (with energy $h\nu$)

$P = -\frac{dE_\nu}{dt} = -h\nu \frac{d\phi}{dt}$ Power dissipated by the resonator at frequency ν

$$Q = 2\pi\nu \left(\frac{h\nu \phi}{-h\nu \frac{d\phi}{dt}} \right) = 2\pi\nu \tau_c$$

$$\phi(t) = \phi_0 e^{-\frac{t}{\tau_c}}$$

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

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Another parameter is the **cavity quality factor** and it is defined by the ratio of the energy of the mode at the frequency ν divided by the power dissipated by the cavity at that frequency.

The quality factor is directly proportional to the lifetime of photons.

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Cavity quality factor

and given that

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} \quad \text{Bandwidth of the mode at frequency } \nu$$

(for the uncertainty principle: $\Delta t \Delta E = \hbar = \frac{h}{2\pi} \Rightarrow \tau_c h \Delta\nu = \frac{h}{2\pi} \Rightarrow \Delta\nu = \frac{1}{2\pi\tau_c}$)

$$\Rightarrow Q = \frac{\nu}{\Delta\nu_c}$$

$$Q = 2\pi\nu \left(\frac{h\nu \phi}{-h\nu \frac{d\phi}{dt}} \right) = 2\pi\nu \tau_c$$


$$\phi(t) = \phi_0 e^{-\frac{t}{\tau_c}}$$

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

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As said, we can define the **bandwidth** of the modes. If you have a finite lifetime you have a finite bandwidth to consider.

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Cavity quality factor

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and given that

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} \quad \text{Bandwidth of the mode at frequency } \nu$$

(for the uncertainty principle: $\Delta t \Delta E = \hbar = \frac{h}{2\pi} \Rightarrow \tau_c \hbar \Delta\nu = \frac{h}{2\pi} \Rightarrow \Delta\nu = \frac{1}{2\pi\tau_c}$)

→ $Q = \frac{\nu}{\Delta\nu_c}$

Es:


$$\tau_c = 150 \text{ ns} \quad @ \lambda = 630 \text{ nm} \quad \nu = \frac{c}{\lambda} = 4.8 \cdot 10^{14} \text{ Hz}$$

$$Q = 2\pi\nu\tau_c = 4.5 \cdot 10^8$$

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Just to have an idea about the numbers, we have $Q \sim 10^8$ which is extremely large. This means very narrow bandwidth.

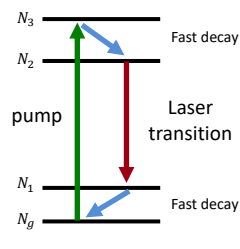
Slide 8



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CW behavior

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Fast decay

pump

Laser transition

Fast decay

Working hypotheses:

- **Four-level laser:**
 $N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$
- **Single mode** (longitudinal and transverse)
- **Homogeneous broadening**
- **Uniform energy density** of the mode on the active medium (uniform transverse profile of the mode and standing-wave effects neglected)
- **Uniform pumping and constant R_p**

↓

Space-independent rate equations

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Let us start describe the behavior of a laser in **continuous wave mode**.

We are adopting a phenomenological description with rate equations, not a description with quantum mechanics. It is a strong approach which in any cases provides most of the results.

We assume a **four-level laser** because they are the simplest system which can obtain laser action.

We will consider **single mode oscillation** (only one longitudinal mode with a given transverse profile).


We will assume that the active medium is **homogeneously broadened**: what is happening to an atom is happening to all the other atoms inside the active medium.

We have also **uniform energy density** of the mode on the active medium. We are assuming the transverse profile of the mode is uniform in the perpendicular plane wrt to the cavity.

The final assumption is the **uniform pumping** at a **constant pumping rate R_p** .

So, we study the CW behavior in a **space-independent rate equation** approach.

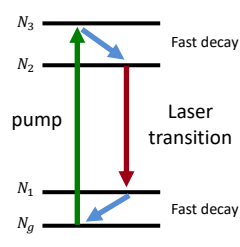
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CW behavior

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Fast decay

pump

Laser transition

Fast decay

$$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$$

$$\left\{ \begin{aligned} \frac{dN_2}{dt} &= R_p - W N_2 - \frac{N_2}{\tau} \\ &= R_p - B\phi N_2 - \frac{N_2}{\tau} \end{aligned} \right.$$

$$\frac{d\phi}{dt} = V_a W N_2 - \frac{\phi}{\tau_c} = V_a B\phi N_2 - \frac{\phi}{\tau_c}$$

$$L_e = L + (n-1)l \quad \tau_c = \frac{L_e}{\gamma c}$$

$$V = \frac{L_e}{l} V_a \rightarrow \text{Volume of the mode on the active medium}$$

↓
Volume of the mode in the cavity

$$B\phi = W$$

$$B \propto B_{21} \quad \text{Einstein's coefficient for stimulated emission}$$

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These are the rate equations.

The first one is about the number of photons in the 2 level.

The second is the one related to the number of photons inside the cavity.

V is the volume of the mode in the cavity.

Moreover, the **stimulated emission rate** can be written as a proportional function of the number of photons through the factor B . Hence, we can rewrite the rate equations.

B is not the same of the Einstein's coefficient for stimulated emission B_{21} !

A_b is the **transverse section of the mode**.

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$$\frac{dN_2}{dt} = R_p - W N_2 - \frac{N_2}{\tau}$$

$$= R_p - B \phi N_2 - \frac{N_2}{\tau}$$

$$\frac{d\phi}{dt} = V_a W N_2 - \frac{\phi}{\tau_c} = V_a B \phi N_2 - \frac{\phi}{\tau_c}$$

$L_e = L + (n-1)l \quad \tau_c = \frac{L_e}{\gamma c}$

$B\phi = W = \sigma F = \sigma \frac{I}{h\nu} = \sigma \frac{\phi}{t_t A_b}$

$t_t = \frac{L_e}{c} \Rightarrow B = \frac{\sigma l c}{V_a L_e} = \frac{\sigma c}{V}$

$V = \frac{L_e}{l} V_a$ Volume of the mode on the active medium

Volume of the mode in the cavity

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t_t is the **transit time**, which correspond to the time for a photon to go from a mirror to the other. We end up with a simplified expression for B .

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$$\frac{dN_2}{dt} = R_p - W N_2 - \frac{N_2}{\tau}$$

$$= R_p - B \phi N_2 - \frac{N_2}{\tau}$$

$$\frac{d\phi}{dt} = V_a W N_2 - \frac{\phi}{\tau_c} = V_a B \phi N_2 - \frac{\phi}{\tau_c}$$

$L_e = L + (n-1)l \quad \tau_c = \frac{L_e}{\gamma c}$

$B\phi = W = B_{21} h\nu n(\nu) = B_{21} h\nu \frac{\phi}{V_a}$

$n(\nu) = \frac{\phi}{V_a}$ Number of photons (energy $h\nu$) per unit of volume

$B \propto B_{21}$

$V = \frac{L_e}{l} V_a$ Volume of the mode on the active medium

Volume of the mode in the cavity

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But W can be written also as a function of the Einstein's coefficient.

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$$\frac{dN_2}{dt} = R_p - B \phi N_2 - \frac{N_2}{\tau} \quad (*)$$

$$\frac{d\phi}{dt} = V_a B \phi N_2 - \frac{\phi}{\tau_c} \quad (**)$$

Equation (**) does not contain any term that accounts for spontaneous emission.

It should contain only the fraction of spontaneously emitted light that contributes to the given mode.

$\frac{d\phi}{dt} = V_a B (\phi + \text{Extra-photon}) N_2 - \frac{\phi}{\tau_c}$

$\phi \approx 10^{10} - 10^{17}$

$\phi_i = 1$ for laser action to start!

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So, we have rewritten the rate equations in this way. An important point to stress is that in the second equation the possibility that the number of photons increases as a consequence of spontaneous emission is not considered. Just stimulated emission is considered. Instead, spontaneous emission was accounted in the first equation by the term $-N_2/\tau$. The reason why we are not considering any spontaneous emission term in the second equation is because we should consider only those photons that contribute to the given mode we are considering. It can be shown by quantum mechanics that the rate equation is this equation and to consider an extra photon to take into account spontaneous emission. However, we consider a very huge number of photons when the laser is active, so this extra photon can be neglected.

In order to have laser action starting, we will consider to have at least one photon in the cavity and it is produced by spontaneous emission. So, we will consider **spontaneous emission as a trigger for laser action**.

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$$\begin{cases} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{cases}$$

$L_e = L + (n-1)l \quad \tau_c = \frac{L_e}{\gamma c}$

$V = \frac{L_e}{l} V_a \quad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$

$\frac{\phi}{\tau_c} = \frac{\phi c \gamma}{L_e} = \frac{\phi c \gamma_i}{L_e} + \frac{\phi c \gamma_1}{2L_e} + \frac{\phi c \gamma_2}{2L_e}$

Rate of photons lost due to transmission through mirror 2 (**outcoupling**)

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$$\begin{cases} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{cases}$$

$L_e = L + (n-1)l \quad \tau_c = \frac{L_e}{\gamma c}$

$V = \frac{L_e}{l} V_a \quad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$

Output power

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e} \right) h\nu \phi$$

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CW behavior

$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$$\begin{cases} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{cases}$$

Output power

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e} \right) h\nu \phi$$

He-Ne laser $P_{out} = 10 \text{ mW}$

$L_e = 50 \text{ cm} \quad \lambda = 630 \text{ nm}$

$R_2 = 99\% \quad \gamma_2 = -\ln R_2 \cong 0.01$

$\phi = \left(\frac{2L_e}{\gamma_2 c} \right) \frac{P_{out}}{h\nu} \cong 1.06 \cdot 10^{10}$ photons within the cavity!

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Since we are dealing with a four-level system, the **population inversion** is equal to N_2 . So, we can rewrite the rate equations in terms of number of photon inside the cavity.

The term ϕ/τ_c is due to the lifetime, so due to the fact that we have losses inside the cavity. The last term is the number of photon per unit of time which are lost from the cavity due to the fact they are extracted from the cavity to produce the laser beam.

We can immediately calculate the **power of the beam**: the number of photon per unit of time that we are extracting from the cavity multiplied by the energy of those photons. The output beam is produced by those photons that are extracted from the cavity due to the outcoupling from the outcoupler mirror.

Just for this low power level you can gate a number of photons which is very large.

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CW behavior

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$N_1 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

$\left\{ \begin{aligned} \frac{dN}{dt} &= R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} &= V_a B\phi N - \frac{\phi}{\tau_c} \end{aligned} \right.$

Output power

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e} \right) h\nu \phi$$

$\phi = \left(\frac{2L_e}{\gamma_2 c} \right) \frac{P_{out}}{h\nu} \cong 0.9 \cdot 10^{16}$ photons within the cavity!

CO₂ laser $P_{out} = 10 \text{ kW}$ High power example!

$L_e = 150 \text{ cm}$ $\lambda = 10.6 \mu\text{m}$

$R_2 = 55\%$ $\gamma_2 = -\ln R_2 = 0.598$

This is another example.