

 $R_p$ 

 $\gamma(t)$ 

N(t)

φ(t)

# Q-switch

Laser Physics T. Cesca Population inversion

Optics and

$$t$$

$$d\tau_{p}$$

$$0|_{\tau}$$

$$N \equiv N_2 - N_1 \cong N_2$$

$$\frac{dN}{dt} = -B\phi N$$

$$\frac{dN}{dt} = -B\phi N$$

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$

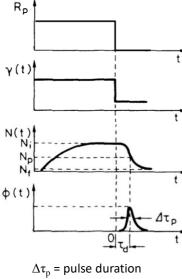
$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$N_P=$$
 population inversion at the peak of the pulse  $rac{d\phi}{dt}=0 \implies N_P=rac{1}{V_aB au_c}=rac{\gamma}{\sigma l}=N_C$ 

$$\Delta au_{
m p}$$
 = pulse duration  $au_{
m d}$  = build-up time  $au_{
m d}$  = build-up time  $au_{
m p}$  at Q-switch element open!



#### Optics and Laser Physics T. Cesca



 $\tau_d$  = build-up time

$$N\equiv N_2-N_1\cong N_2$$
 Population inversion  $\dfrac{dN}{dt}=-B\phi N$   $N(0)=N_i$   $\phi(0)=\phi_ipprox 1$ 

$$rac{d\phi}{dt} = V_a B \phi N - rac{\phi}{ au_c}$$
  $P_P = \;\;$  peak power of the output pulse

$$P_P = \left(\frac{\gamma_2 c}{2L_e}\right) h \nu \phi_P$$
  $\phi_P = \text{number of photons at the peak}$  of the laser pulse

to calculate  $\phi_P$  let's make the ratio of the two rate equations



#### Q-switch $N \equiv N_2 - N_1 \cong N_2$

T. Cesca Population inversion

Optics and

Laser Physics

$$\frac{d\phi}{dt} = V_a B \phi N - \frac{\phi}{\tau_c}$$

 $\phi = V_a \left| N_i - N - N_P \ln \left( \frac{N_i}{N} \right) \right| = \phi(N)$ 

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \qquad N_P = \frac{1}{V_a B \tau_c}$$

$$\frac{d\phi}{dN} = -V_a \left[ 1 - \frac{1}{V_a B \tau_c N} \right] = -V_a \left[ 1 - \frac{N_P}{N} \right]$$

$$d\phi = -V_a dN + V_a N_P \frac{dN}{N}$$

 $N(0) = N_i$ 

$$\frac{d\phi}{dN} = -V_a \left[ 1 - \frac{1}{V_a B \tau_c N} \right] = -V_a \left[ 1 - \frac{N}{N} \right]$$

$$d\phi = -V_a dN + V_a N_P \frac{dN}{N}$$

$$\phi - \phi_i = V_a (N_i - N) + V_a N_P \ln \left( \frac{N}{N_i} \right)$$

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 $\gamma(t)$ 

φ(t)  $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_d$  = build-up time



Laser Physics T. Cesca Population inversion

Optics and

$$\begin{array}{c} \gamma(t) \\ N(t) \\ N_{p} \\ N_{p} \\ \lambda(t) \\$$

 $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_d$  = build-up time

$$\int \frac{dN}{dt} = -B\phi N$$

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$\phi(0) = \phi_i \approx 1$$

$$N_P = \frac{1}{V_a B \tau_c}$$

 $N(0) = N_i$ 

$$V_a \left[ N_i - N_P - N_P \right]$$

 $N \equiv N_2 - N_1 \cong N_2$ 

$$-\frac{N_P}{c}$$

$$\phi_P = V_a \left[ N_i - N_P - N_P \ln \left( \frac{N_i}{N_P} \right) \right] = \phi(N_P)$$

$$\phi_P = V_a N_P \left[ \frac{N_i}{N_P} - 1 - \ln \left( \frac{N_i}{N_P} \right) \right]$$

 $\phi = V_a \left| N_i - N - N_P \ln \left( \frac{N_i}{N} \right) \right| = \phi(N)$ 

$$\phi_{P} = V_{a} \left[ N_{i} - N_{P} - N_{P} \ln \left( \frac{N_{i}}{N_{P}} \right) \right] = \phi(N_{P})$$

$$\phi(t)$$

$$\phi(t)$$

$$\phi_{P} = V_{a} N_{P} \left[ \frac{N_{i}}{N_{P}} - 1 - \ln \left( \frac{N_{i}}{N_{P}} \right) \right]$$

$$\phi_{P} = V_{a} N_{P} \left[ x - 1 - \ln(x) \right]$$

$$\chi = \frac{N_{i}}{N_{P}}$$

 $R_p$ 



#### Q-switch $N \equiv N_2 - N_1 \cong N_2$

Population inversion

 $N(0) = N_i$ 

 $\phi(0) = \phi_i \approx 1$ 

Optics and

Laser Physics T. Cesca

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$\phi_P = V_a N_P [x - 1 - \ln(x)]$$

 $P_P = \left(\frac{\gamma_2 c}{2L_a}\right) h v V_a N_P [x - 1 - \ln(x)]$ 

 $P_{P} = \left(\frac{\gamma_{2}}{2}\right) \left(\frac{A_{b}}{\sigma}\right) \left(\frac{h\nu}{\tau_{c}}\right) \left[x - 1 - \ln(x)\right]$ 

$$\frac{\phi}{\tau_c} \qquad N_P = \frac{1}{V_a B \tau_c}$$

$$m(x) \qquad x = \frac{N_i}{T_c}$$

$$h(t)$$
 $h(t)$ 
 $h(t)$ 

 $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_d$  = build-up time

$$\phi_P = V_a N_P [x - 1 - \ln(x)] \qquad x = \frac{N_i}{N_P}$$

$$P_P = \left(\frac{\gamma_2 c}{2L_o}\right) h \nu \phi_P \qquad A_b = \frac{V_a}{l} \quad \tau_c = \frac{L_e}{c\nu}$$

 $\gamma(t)$ 



 $R_p$ 

## Q-switch

Laser Physics T. Cesca Population inversion

 $N_P = \frac{1}{V_c B \tau_c}$ 

Optics and

 $\Delta \tau_{\rm p}$  = pulse duration  $\tau_d$  = build-up time

$$N \equiv N_2 - N_1 \cong N_2$$

$$dN = R dN$$

$$N(0) = N_i$$
$$\phi(0) = \phi_i \approx$$

The output energy per pulse can be calculated as 
$$E = \int_0^\infty P(t) dt = \left(\frac{\gamma_2 c}{2L_e}\right) h v \int_0^\infty \phi(t) dt$$
 Integrating both terms of (\*\*) we get: 
$$\phi(\infty) - \phi(0) = \int_0^\infty \frac{d\phi}{dt} dt = \int_0^\infty V_a B \phi N dt - \int_0^\infty \frac{\phi}{\tau_c} dt$$



 $N \equiv N_2 - N_1 \cong N_2$ 

Laser Physics T. Cesca Population inversion

Optics and

$$N = N_2 - N_1 \cong N_2 \quad \text{Population inversion}$$

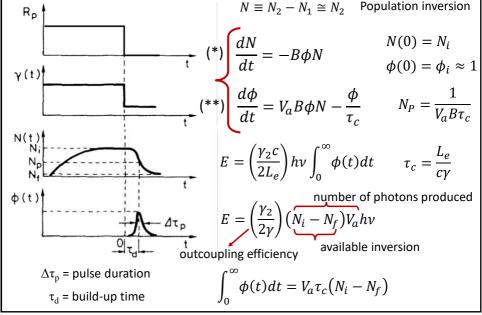
$$\gamma(t) \quad t \quad (*) \quad \frac{dN}{dt} = -B\phi N \quad N(0) = N_i \\ \phi(0) = \phi_i \approx 1$$

$$V(t) \quad \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \quad N_p = \frac{1}{V_a B\tau_c} \quad N$$



Laser Physics
T. Cesca

Optics and





 $\gamma(t)$ 

#### Q-switch $N \equiv N_2 - N_1 \cong N_2$

T. Cesca Population inversion

Optics and

Laser Physics

$$\int \frac{dN}{dt} = -B\phi h$$

 $N(0) = N_i$ 

$$(**) \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$N_P = \frac{1}{V_a B \tau_c}$$

$$-N_f)V_ah$$

 $\phi(N_f) = V_a \left| N_i - N_f - N_P \ln \left( \frac{N_i}{N_f} \right) \right| = 0$ 

$$V_a B$$

 $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_d$  = build-up time

$$E = \left(\frac{\gamma_2}{2\gamma}\right) (N_i - N_f) V_a h v$$

$$E$$
 is necessary

To calculate 
$$E$$
 is necessary to determin  $\phi(N) = V_a \left[ N_i - N - N_P \ln \left( \frac{N_i}{N} \right) \right]$ 

$$(2\gamma)^{C}$$
To calculate  $E$  is

$$N_f)V_ahv$$

$$E=\left(rac{\gamma_2}{2\gamma}
ight)(N_i-N_f)V_ahv$$
 To calculate  $E$  is necessary to determine  $N_f$ :



Laser Physics T. Cesca Population inversion

Optics and

 $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_d$  = build-up time

$$N \equiv N_2 - N_1 \cong N_2$$

$$(*) \int \frac{dN}{dt} = -B\phi N$$

$$(**) \int \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

$$N(0) = N_i$$
$$\phi(0) = \phi_i$$

$$N_P = \frac{1}{V_2 B \tau_2}$$

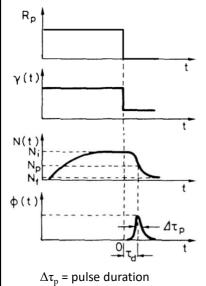
$$\eta_E = \frac{N_i - N_f}{N_i} = \frac{N_P}{N_i} \ln \left(\frac{N_i}{N_f}\right)$$
Inversion (energy)-utilization factors

$$n\left(\frac{N_i}{N_f}\right)$$

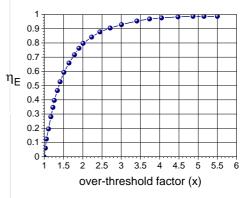
 $\eta_E \frac{N_i}{N_P} = -\ln(1 - \eta_E) \qquad x = \frac{N_i}{N_P}$ 

$$\eta_E = \frac{N_i - N_f}{N_i} = \frac{N_P}{N_i} \ln\left(\frac{N_i}{N_f}\right)$$
Inversion (energy)-utilization factor
$$\eta_E \frac{N_i}{N_P} = -\ln\left(\frac{N_f}{N_i}\right) = -\ln\left(\frac{N_i - N_i + N_f}{N_i}\right)$$

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 $\tau_{\rm d}$  = build-up time

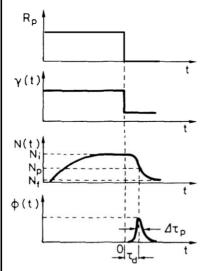


Inversion (energy)-utilization factor

$$\eta_E \frac{N_i}{N_P} = -\ln(1 - \eta_E)$$

$$x = \frac{N_i}{N_P}$$





 $\Delta \tau_{\rm p}$  = pulse duration

 $\tau_{\rm d}$  = build-up time

$$F = \left(\frac{\gamma_2}{\gamma_2}\right) (N_1)$$

$$E = \left(\frac{\gamma_2}{2\gamma}\right) (N_i - N_f) V_a h v$$

$$E = \left(\frac{\gamma_2}{2\gamma}\right) \left(\frac{N_i - N_f}{N_i}\right) \left(\frac{N_i}{N_P}\right) N_P V_a h v$$

$$N_P = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l} = N_C \qquad A_b = \frac{V_a}{l}$$

$$E = \eta_E \left(\frac{\gamma_2}{2}\right) \left(\frac{A_b}{\sigma}\right) \left(\frac{N_i}{N_P}\right) h \nu \quad \begin{array}{c} \text{Pulse} \\ \text{energy} \end{array}$$

Inversion (energy)-utilization factor 
$$\eta_E \frac{N_i}{N_D} = -\ln(1-\eta_E) \qquad \qquad x = \frac{N_i}{N_D}$$



## T. Cesca $\Delta \tau_p = \frac{E}{P_p}$

Optics and

Laser Physics

Pulso 
$$E = \eta$$

$$P_P = 0$$

$$E = \eta_E \left(\frac{\gamma_2}{2}\right) \left(\frac{A_b}{\sigma}\right) \left(\frac{N_i}{N_P}\right) h\nu$$

$$P_{P} = \left(\frac{\gamma_{2}}{2}\right) \left(\frac{A_{b}}{\sigma}\right) \left(\frac{h\nu}{\tau_{c}}\right) \left[x - \frac{1}{2}\right]$$

For x = 2 - 10  $\Longrightarrow$ 

$$N(t)$$
 $N_p$ 
 $N_t$ 
 $\phi(t)$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 
 $\Delta \tau_p$ 

 $\tau_d$  = build-up time

$$\downarrow \qquad \qquad \downarrow$$

 $\Delta \tau_p = \eta_E \tau_c \frac{\lambda}{x - 1 - \ln(x)}$ 

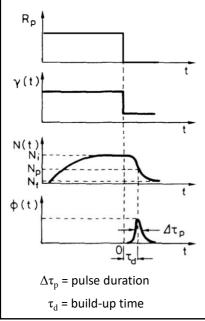
 $P_{P} = \left(\frac{\gamma_{2}}{2}\right) \left(\frac{A_{b}}{\sigma}\right) \left(\frac{h\nu}{\tau_{c}}\right) \left[x - 1 - \ln(x)\right]$  $x = \frac{N_i}{N_D}$ 

 $\Delta \tau_p \approx (5-1.5)\tau_c$ 

$$-\eta_E$$

 $R_p$ 

 $\gamma(t)$ 



#### **Build-up time**

 $au_d = egin{array}{l} ext{time necessary for the number} \ ext{of photons to reach a given} \ ext{fraction of the peak value } \phi_P \end{array}$ 

Let's assume such a fraction to be 1/10:

$$\Rightarrow N(t) \approx N_i \qquad N_P = \frac{1}{V_a B \tau_c}$$

$$\frac{d\phi}{dt} = (V_a B N \tau_c - 1) \frac{\phi}{\tau_c} = \left(\frac{N(t)}{N_P} - 1\right) \frac{\phi}{\tau_c}$$

$$\frac{d\phi}{dt} = (x-1)\frac{\phi}{\tau_c} \qquad x = \frac{N_i}{N_P}$$



 $\gamma(t)$ 

N(t)

#### **Build-up time**

$$au_d = ext{time necessary for the number}$$
of photons to reach a given

fraction of the peak value  $\phi_P$ 

$$\phi(t) = \phi_i e^{-\frac{(x-1)t}{\tau_c}}$$

$$\phi(t) = \phi_i e^{\frac{\tau_c}{\tau_c}}$$

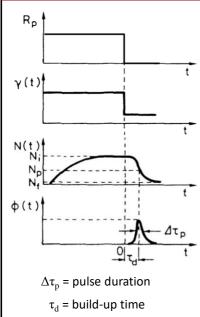
$$\phi_i \approx 1 \qquad \phi(\tau_d) = \phi_i e^{\frac{(x-1)\tau_d}{\tau_c}}$$

$$\tau_c = \tau_c \left( \phi_P \right)$$

 $\phi_{P} = V_{\alpha} N_{P} [x - 1 - \ln(x)]$ 

$$\tau_d = \frac{\tau_c}{x - 1} \ln \left( \frac{\phi_P}{10} \right)$$

φ(t)  $\Delta \tau_{\rm p}$  = pulse duration  $\tau_d$  = build-up time



#### Build-up time

 $\tau_d = \begin{array}{l} \text{time necessary for the number} \\ \text{of photons to reach a given} \\ \text{fraction of the peak value } \phi_P \end{array}$ 

$$\tau_d = \frac{\tau_c}{x - 1} \ln \left( \frac{\phi_P}{10} \right)$$

Es: x = 5,  $\phi_P = 10^{17}$ 

Since  $\phi_P$  can be very large ( $\phi_P \approx 10^{17}$ ) and it appears in  $\tau_d$  in the logarithm, it doesn't make a real difference to consider  $\frac{\phi_P}{10}$  or  $\frac{\phi_P}{20}$ :

$$\tau_d = \frac{\tau_c}{x - 1} \ln \left( \frac{\phi_P}{10} \right) = \frac{\tau_c}{4} 36.84 = 9.2 \tau_c$$

$$\tau_d = \frac{\tau_c}{x - 1} \ln \left( \frac{\phi_P}{20} \right) = \frac{\tau_c}{4} 36.15 = 9.0 \tau_c$$