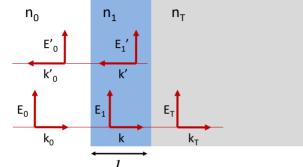
Multi-lavers (normal incidence)

Optics and



 $B = \frac{E}{v} = n\frac{E}{c}$

$$E_0 + E_0' = E_1 + E_1'$$

$$R = R'$$

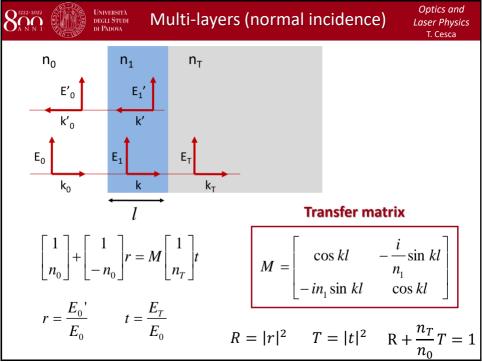
$$B_0 - B_0' = B_1 - B_1'$$

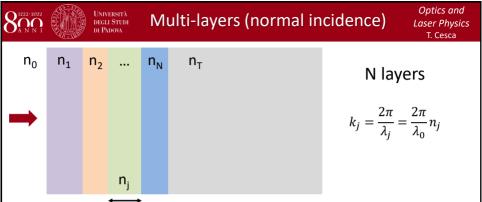
$$n_0 E_0 - n_0 E_0' = n_1 E_1 - n_1 E_1'$$

Second interface

$$E_1 e^{ikl} + E_1' e^{-ikl} = E_T$$

$$\begin{bmatrix}
B_1 e^{ikl} - B_1' e^{-ikl} = B_T \\
n_1 E_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T
\end{bmatrix}$$





$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

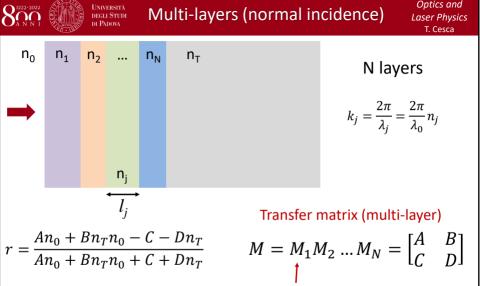
$$Transfer matrix (multi-layer)$$

$$M = M_1 M_2 \dots M_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$r = \frac{E_0'}{E_0}$$

$$t = \frac{E_T}{E_0}$$

$$R = |r|^2 \quad T = |t|^2 \quad R + \frac{n_T}{n_0} T = 1$$



$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T}$$

$$t = \frac{2n_0}{An_0 + Bn_T n_0 + C + Dn_T}$$

$$R = |r|^2 \qquad T = |t|^2 \qquad R + \frac{n_T}{n_0} T = 1$$



Glass substrate

 $\sqrt{n_T n_0} = n_1$

Anti-reflection coating

Optics and Laser Physics T. Cesca

n₀ Optical thickness of the layer:
$$l = \frac{\lambda}{4}$$
 $\implies kl = \frac{\pi}{2}$

Reflectance of a
$$\lambda/4$$
 film

Reflectance of a
$$\lambda/4$$
 film
$$R = |r|^2 = \left(\frac{n_T n_0 - (n_1)^2}{n_T n_0 + (n_1)^2}\right)^2$$

R = 0

$$\begin{bmatrix} 1 \end{bmatrix}_{+} \begin{bmatrix} 1 \end{bmatrix}_{r} =$$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_0 \end{bmatrix} t$$

$$\begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ -n_0 \end{bmatrix}$$

$$r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$-\frac{i}{n_1}$$

$$M = \begin{bmatrix} \cos kl & -\frac{i}{n_1} \sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix}$$

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T}$$
NTI-REFLECTION!

If
$$n_T = 1.5$$
 (glass) \longrightarrow $n_1 = \sqrt{n_T} = 1.22$ to get $R = 0$



Anti-reflection coating

 $\begin{vmatrix} 1 \\ n_- \end{vmatrix} + \begin{vmatrix} 1 \\ -n_- \end{vmatrix} r = M \begin{vmatrix} 1 \\ n_- \end{vmatrix} t$

Optical thickness of the layer: $l = \frac{\lambda}{4}$ $\implies kl = \frac{\pi}{2}$

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$$\begin{array}{c|c}
 & n_1 \\
 & n_{\tau}
\end{array}$$

$$M = \begin{bmatrix} \cos kl & -\frac{i}{n_1} \sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\sin kl \\ n_1 \\ \cos kl \end{bmatrix} =$$

$$n_T n_0 - C -$$

 \rightarrow $R \simeq 0.014 (1.4\%)$

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T}$$

ANTI-REFLECTION!

$$r = \frac{An_0 + R}{An_0 + R}$$

$$R = |r|^2 = \left(\frac{n_T n_0 - (n_1)^2}{n_T n_0 + (n_1)^2}\right)^2$$

 $\sqrt{n_T n_0} = n_1$

Glass substrate

$$\frac{1}{5}$$

R = 0

Typically anti-reflection coatings of glass lenses

 $(n_T = 1.5)$ are of MgF₂ $(n_1 = 1.38)$

Reflectance of a
$$\lambda/4$$
 film





$$n_0$$



Anti-reflection coating

Laser Physics T. Cesca

Optics and

 $\mathbf{n_0}$ Optical thickness of the layer: $l = \frac{\lambda}{4}$ $\implies kl = \frac{\pi}{2}$

$$\begin{array}{c|c}
\mathbf{n_2} & \downarrow & l_2 & \begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t \\
M = M M = \begin{bmatrix} 0 & -\frac{i}{n} \end{bmatrix} \begin{bmatrix} 0 & -\frac{i}{n} \end{bmatrix} = \begin{bmatrix} -\frac{i}{n} \end{bmatrix} = \begin{bmatrix}$$

Glass substrate
$$M = M_1 M_2 = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{i}{n_2} \\ -in_2 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{n_2}{n_1} & 0 \\ 0_1 & -\frac{n_1}{n_2} \end{bmatrix}$$

$$(n_0(n_2)^2 - n_T(n_1)^2)^2$$

$$M = M_1 M_2 = \begin{bmatrix} 0 & -\frac{1}{n_1} & 0 & -\frac{1}{n_2} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{n_2} \\ -in_2 & 0 \end{bmatrix} = \begin{bmatrix} n_1 \\ 0_1 & -\frac{n_1}{n_2} \end{bmatrix}$$

$$R = |r|^2 = \left(\frac{n_0(n_2)^2 - n_T(n_1)^2}{n_2(n_2)^2 + n_T(n_1)^2}\right)^2 \quad \text{Reflectance of 2 } \lambda/4 \text{ films}$$

$$R = |r|^2 = \left(\frac{n_0(n_2)^2 - n_T(n_1)^2}{n_0(n_2)^2 + n_T(n_1)^2}\right)^2$$
 Reflectance of 2 $\lambda/4$ films

$$R = |r|^2 = \left(\frac{n_0(n_2)^2 - n_T(n_1)^2}{n_0(n_2)^2 + n_T(n_1)^2}\right)^2$$
 Reflectance of 2 $\lambda/4$ films

$$R = |r|^2 = \left(\frac{n_0(n_2)^2 - n_T(n_1)^2}{n_0(n_2)^2 + n_T(n_1)^2}\right)^2$$
 Reflectance of 2 $\lambda/4$ films

$$=|r|^2 = \left(\frac{6(2)^2 + n_1(n_1)^2}{n_0(n_2)^2 + n_1(n_1)^2}\right)$$
 Reflectance of 2 $\lambda/4$ films

$$R = 0$$
 \longrightarrow $\sqrt{\frac{n_T}{n_0}} = \frac{n_2}{n_1}$ ANTI-REFLECTION!

$$R = 0$$
 \longrightarrow $\sqrt{\frac{n_T}{n_0}} = \frac{n_2}{n_1}$ ANTI-REFLECTION!

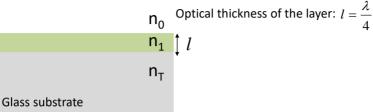
$$R = 0 \qquad \sqrt{n_0} = \frac{1}{n_1}$$
If $n_0 = 1$, $n_T = 1.5$, $n_1 = 1.38$ (MgF₂) $\qquad \qquad n_2 = 1.69 \qquad \qquad n_2 = 1.62$ (Al₂O₃)

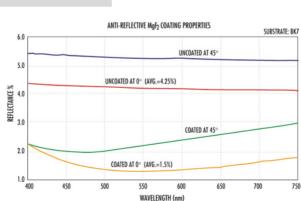
R = 0.0018



Anti-reflection coating

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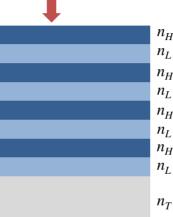




 n_H

 n_L

 n_L



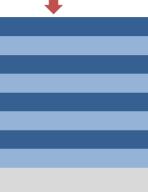
2N layers
$$l = \frac{\lambda}{4}$$

$$\begin{array}{ccc}
n_L \\
n_L \\
n_H \\
n_L
\end{array}
M = \left(M_H M_L\right)^N = \begin{bmatrix}
-\frac{n_L}{n_H} & 0 \\
0 & -\frac{n_H}{n_H}
\end{bmatrix}^N$$

$$R = |r|^{2} = \left(\frac{1 - \frac{n_{T}}{n_{0}} \left(\frac{n_{H}}{n_{L}}\right)^{2N}}{1 + \frac{n_{T}}{n_{0}} \left(\frac{n_{H}}{n_{L}}\right)^{2N}}\right)^{2}$$



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2N layers
$$l = \frac{\lambda}{4}$$
 $n_0 = 1$ $n_T = 1.5$

2 layers of MgF₂ (
$$n_L = 1.38$$
) $\Rightarrow R = 0.74$

$$n_H$$
 n_L

 n_H

 n_L

2 layers of ZnS (
$$n_H = 2.39$$
)

4 layers of MgF₂ ($n_L = 1.38$)

4 layers of ZnS ($n_H = 2.39$)

 $R = 0.97$

$$n_H$$
 n_L

$$n_{L} = n_{L} = n_{H}$$

30 layers of MgF₂ (
$$n_L = 1.38$$
) $R = 0.999$ R = 0.999

$$n_H$$
 n_L

 n_T

$$n_L = 1.$$
 $n_H = 2.3$

$$R = \left| r \right|^2 = \left(\frac{1 - \frac{n_T}{n_0} \left(\frac{n_H}{n_L} \right)^{2N}}{1 + \frac{n_T}{n_0} \left(\frac{n_H}{n_L} \right)^{2N}} \right)^2$$

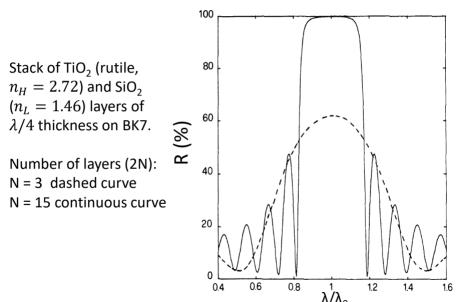
$$(x - 0)^2$$

$$R \rightarrow$$

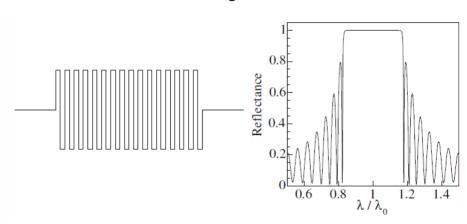
$$R \rightarrow$$



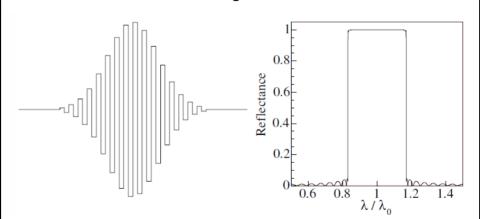
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16 couples of MgF₂ ($n_L=1.38$) and TiO ($n_H=2.35$) layers of $\lambda/4$ thickness on glass substrate.



16 couples of MgF₂ ($n_L=1.38$) and TiO ($n_H=2.35$) layers of $\lambda/4$ thickness on glass substrate.



APODIZED Reflectivity



Multi-layers (oblique incidence)

T. Cesca

Optics and

Laser Physics

TE(s)

$$n_0$$
 n_1

$$t = \frac{1}{2}$$
 $M = \frac{1}{2}$

$$r = \frac{An_0 + Bn_0n_T - C - Dn_T}{An_0 + Bn_0n_T + C + Dn_T}$$

$$t = \frac{2n_0}{An_0 + Bn_0n_T + C + Dn_T}$$

$$=\begin{pmatrix} cc \\ cc \end{pmatrix}$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\beta & -\frac{i}{p}\sin\beta \\ -i p \sin\beta & \cos\beta \end{pmatrix}$$

$$\beta = k \cos\theta z$$

$$r \rightarrow r$$

$$\rightarrow r$$

$$\vec{k}_{t}$$
 E_{t}

$$\rightarrow n_0 cos$$

 $n_T \rightarrow n_T cos\theta_t$

$$n_0 \to n_0 cos \theta_0$$

$$n_0 \rightarrow n$$

$$n_0 \to n_0 cos \theta_0$$

 $n_1 \to n_1 cos \theta = p$

$$r \to r$$
$$t \to t$$

$$\rightarrow n_0 cos$$

$$B_t$$

 $R = |r|^2 \qquad T = |t|^2$

$$n_T$$

$$n_I$$

at normal incidence

Multi-layers (oblique incidence)

Università

DEGLI STUDI

DI PADOVA

TM (p)

Transverse Magnetic

 n_0

 $r = \frac{An_0 + Bn_0n_T - C - Dn_T}{An_0 + Bn_0n_T + C + Dn_T}$

 $t = \frac{1}{An_0 + Bn_0n_T + C + Dn_T}$

at normal incidence

Optics and

Laser Physics

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$$\beta = k \cos\theta z$$

$$n_0 \to \frac{n_0}{\cos\theta_0}$$

$$n_1 \to \frac{n_1}{\cos\theta} = p$$

$$r \to r$$

$$t \to t \frac{\cos\theta_t}{\cos\theta_0}$$

$$r \to r$$

$$t \to t \frac{\cos\theta_t}{\cos\theta_0}$$

$$R = |r|^2 \quad T = |t|^2$$

$$n_T \to \frac{n_T}{\cos\theta_t}$$

$$R + \frac{n_T \cos\theta_t}{n_0 \cos\theta_0} T = 1$$

 $-\iota p sin \beta$

cosß /