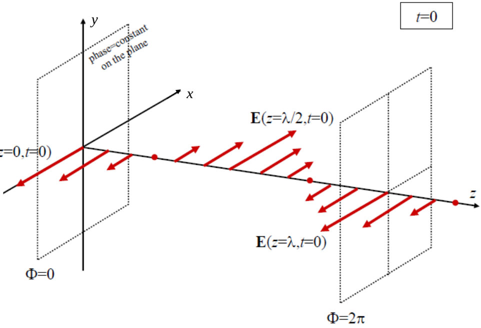
# Linear polarization

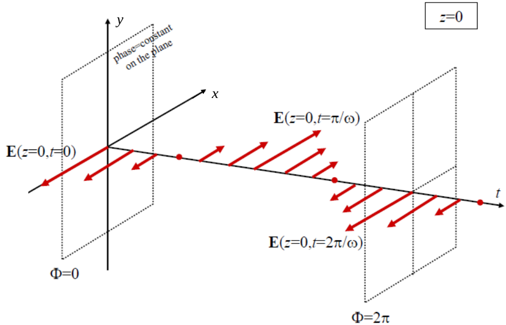
$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}$$



# Linear polarization



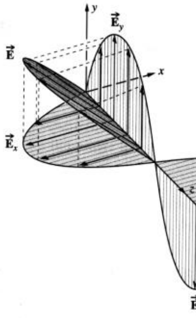
$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}$$

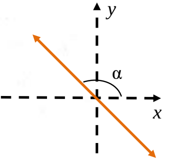


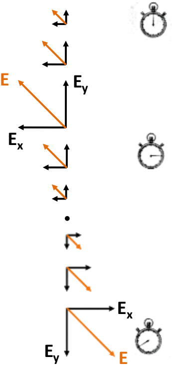
# Linear polarization: 135°



$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}$$



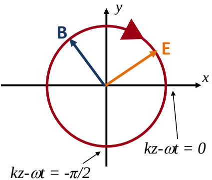



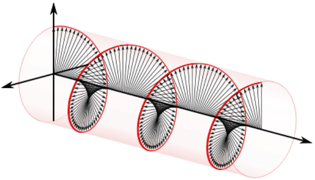
# Circular polarization



$$\vec{E} = \hat{\imath}E_{0x}e^{i(kz-\omega t)} + \hat{\jmath}E_{0y}e^{i(kz-\omega t+\delta)}$$

**Right-circularly** polarized (**R**): $E_{0x} = E_{0y}$ $\delta = -\dfrac{\pi}{2}$



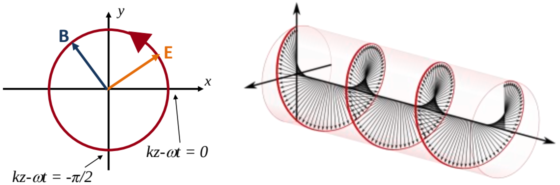



**Clockwise** rotation if looking from the receiver

# Circular polarization

$$\vec{E} = \hat{\imath}E_{0x}e^{i(kz-\omega t)} + \hat{\jmath}E_{0y}e^{i(kz-\omega t+\delta)}$$

**Left-circularly** polarized (**L**): $E_{0x} = E_{0y}$ $\delta = +\frac{\pi}{2}$




**Counter-clockwise** rotation if looking from the receiver

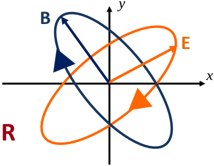
# Elliptical polarization

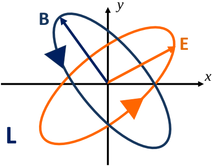


$$\vec{E} = \hat{\imath}E_{0x}e^{i(kz-\omega t)}+\hat{\jmath}E_{0y}e^{i(kz-\omega t+\delta)}$$

**Right-elliptically** polarized (**R**): $E_{0x} \neq E_{0y}$ $\delta = -\frac{\pi}{2}$

**Left-elliptically** polarized (**L**): $E_{0x} \neq E_{0y}$ $\delta = +\frac{\pi}{2}$

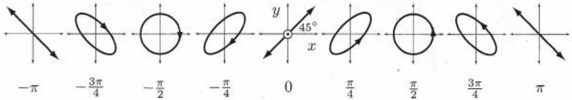


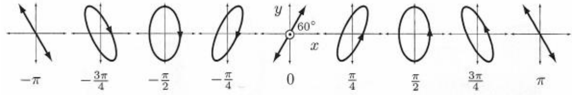

**Clockwise** (**counter-clockwise**) rotation if looking from the receiver

# Polarization

$E_{0x} = E_{0y}$

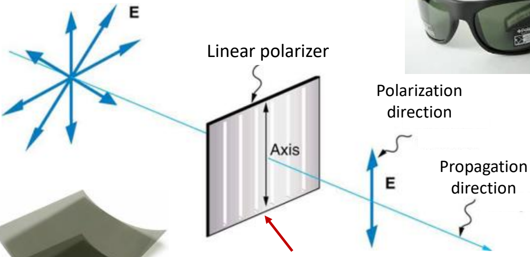


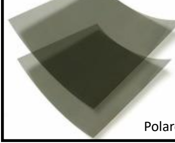

$E_{0x} \neq E_{0y}$



# Linear polarizer



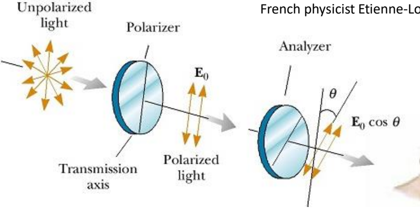


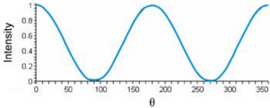
Polaroid film, Edwin H. Land, 1930

# Malus' law

French physicist Etienne-Louis Malus, 1809




$$\boxed{I(\theta) = I_0 \cos^2\theta}$$

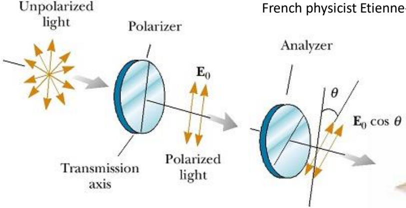



For unpolarized incident light: $I_1 = I_0 \langle \cos^2\theta \rangle = \frac{1}{2} I_0 \Rightarrow T = \frac{1}{2}$

# Partial polarization



French physicist Etienne-Louis Malus, 1809

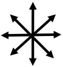



$$G_P = \frac{I_{POL}}{I_{POL} + I_{NON-POL}} = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}}$$

**Polarization degree**

(for partially linearly polarized light)

Determine the **transmitted intensity** and the **final polarization state** of a beam of unpolarized light with intensity $I_0$ impinging on 3 coaxial ideal linear polarizers; the transmission axis of the first polarizer is vertical, at 45° for the second and horizontal for the third.



Unpolarized light $I_0$

$$I_1 = T\, I_0 = \frac{1}{2} I_0 \qquad T = \frac{1}{2}$$

Transmission factor of an ideal linear polarizer for unpolarized light

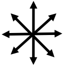
$$I_2 = I_1 cos^2 45$$

$$I_3 = I_2 cos^2 45 = I_1 cos^2 45 cos^2 45 = \frac{1}{2} I_0 \left(\frac{\sqrt{2}}{2}\right)^2 \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{I_0}{8}$$

If we remove the second polarizer, the transmitted intensity is 0!

A real linear polarizer with vertical transmission axis is made in such a way that it can transmit a fraction $f = 0.248$ of the intensity transmitted for vertical polarization when a beam with horizontal polarization is impinging on it.

Determine the **degree of polarization** of the output beam from this polarizer when the input beam is unpolarized.




$I_0$

$$I_y = T\, I_0 = \frac{1}{2} I_0$$

since the input beam is unpolarized

$$I_x = f\, I_y = 0.248\, I_y = \frac{0.248}{2} I_0$$ according to the text of the exercise

$$I_{MAX} = I_y = \frac{1}{2} I_0$$

$$I_{MIN} = I_x = \frac{0.248}{2} I_0$$

$$\Rightarrow \quad G_P = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}} = \frac{I_y - 0.248\, I_y}{I_y + 0.248\, I_y} = 0.603$$

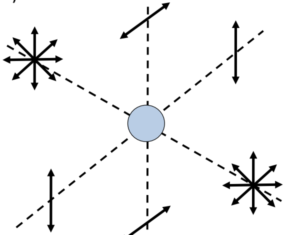
60.3%

# Polarization by scattering



Incident light (unpolarized)

Linearly polarized scattered light



Transmitted light

# Polarization by reflection


Without polarizer

With polarizer

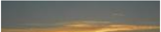
Linearly polarized scattered light

(unpolarized)



# Rayleigh scattering





$$I(\theta) \propto I_0 N \nu^4 (1 + \cos^2 \theta)$$