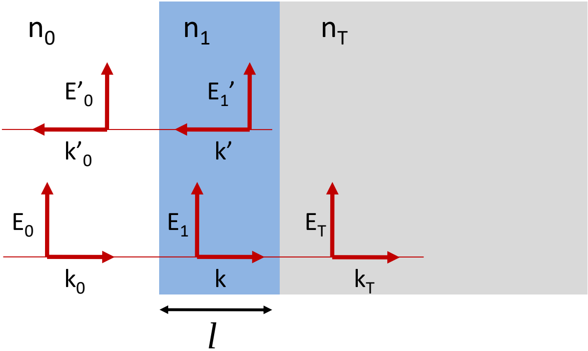
Multi-layers (normal incidence)



$$B = \frac{E}{v} = n\frac{E}{c}$$

First interface

$$E_0 + E_0' = E_1 + E_1'$$

$$B_0 - B_0' = B_1 - B_1'$$

$$n_0 E_0 - n_0 E_0' = n_1 E_1 - n_1 E_1'$$

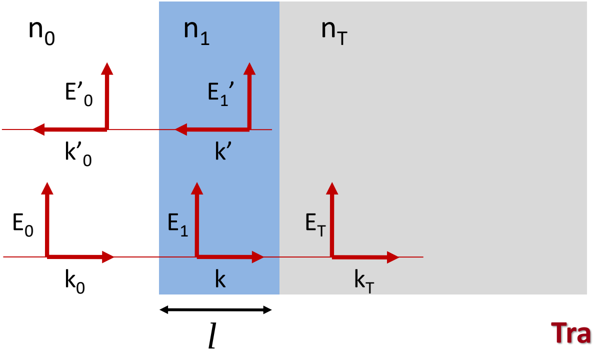
Second interface

$$E_1 e^{ikl} + E_1' e^{-ikl} = E_T$$

$$B_1 e^{ikl} - B_1' e^{-ikl} = B_T$$

$$n_1 E_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T$$

Multi-layers (normal incidence)



$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

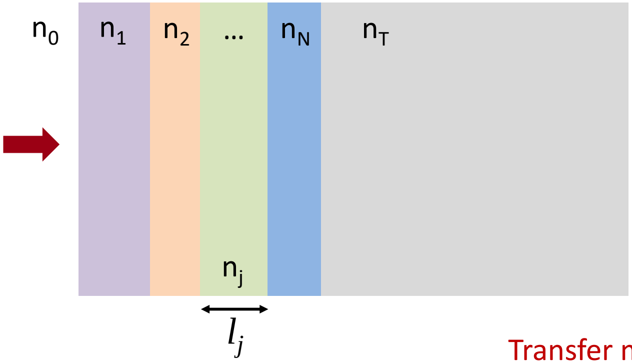
$$r = \frac{E_0'}{E_0} \qquad t = \frac{E_T}{E_0}$$

Transfer matrix

$$\boxed{M = \begin{bmatrix} \cos kl & -\frac{i}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix}}$$

$$R = |r|^2 \qquad T = |t|^2 \qquad \mathrm{R} + \frac{n_T}{n_0} T = 1$$

Multi-layers (normal incidence)



N layers

$$k_j = \frac{2\pi}{\lambda_j} = \frac{2\pi}{\lambda_0} n_j$$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$r = \frac{E_0'}{E_0} \qquad t = \frac{E_T}{E_0}$$

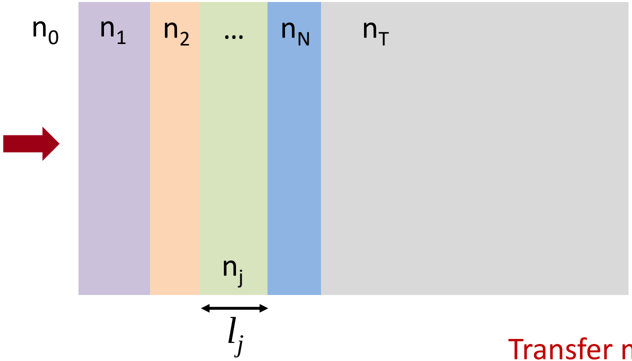
Transfer matrix (multi-layer)

$$M = M_1 M_2 \ldots M_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

First illuminated layer

$$R = |r|^2 \qquad T = |t|^2 \qquad \mathrm{R} + \frac{n_T}{n_0} T = 1$$

Multi-layers (normal incidence)



N layers

$$k_j = \frac{2\pi}{\lambda_j} = \frac{2\pi}{\lambda_0} n_j$$

Transfer matrix (multi-layer)

$$M = M_1 M_2 \dots M_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

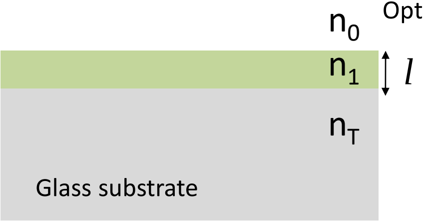
First illuminated layer

$$r = \frac{A n_0 + B n_T n_0 - C - D n_T}{A n_0 + B n_T n_0 + C + D n_T}$$

$$t = \frac{2 n_0}{A n_0 + B n_T n_0 + C + D n_T}$$

$$R = |r|^2 \qquad T = |t|^2 \qquad \mathrm{R} + \frac{n_T}{n_0} T = 1$$

Anti-reflection coating



Optical thickness of the layer: $l = \frac{\lambda}{4} \Rightarrow kl = \frac{\pi}{2}$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$M = \begin{bmatrix} \cos kl & -\frac{i}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix}$$

Reflectance of a $\lambda/4$ film

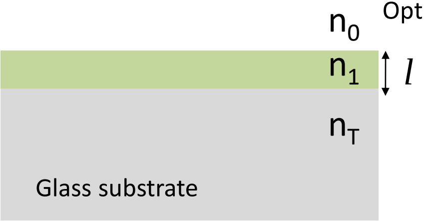
$$R = |r|^2 = \left(\frac{n_T n_0 - (n_1)^2}{n_T n_0 + (n_1)^2} \right)^2$$

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T}$$

$\sqrt{n_T n_0} = n_1 \Rightarrow R = 0$ ANTI-REFLECTION!

If $n_T = 1.5$ (glass) $\Rightarrow n_1 = \sqrt{n_T} = 1.22$ to get $R = 0$

Anti-reflection coating



Optical thickness of the layer: $l = \frac{\lambda}{4}$ ➡ $kl = \frac{\pi}{2}$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$M = \begin{bmatrix} \cos kl & -\frac{i}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix}$$

Reflectance of a $\lambda/4$ film

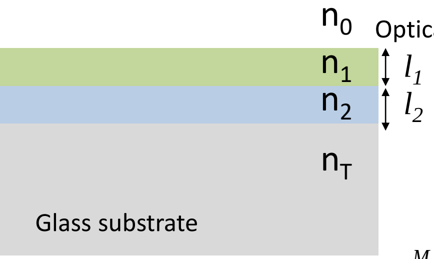
$$R = |r|^2 = \left(\frac{n_T n_0 - (n_1)^2}{n_T n_0 + (n_1)^2} \right)^2$$

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T}$$

$\sqrt{n_T n_0} = n_1$ ➡ $R = 0$ ANTI-REFLECTION!

Typically anti-reflection coatings of glass lenses ($n_T = 1.5$) are of MgF_2 ($n_1 = 1.38$) ➡ $R \simeq 0.014$ (1.4%)

Anti-reflection coating



Optical thickness of the layer: $l = \frac{\lambda}{4}$ ➡ $kl = \frac{\pi}{2}$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$M = M_1 M_2 = \begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{i}{n_2} \\ -in_2 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{n_2}{n_1} & 0 \\ 0_1 & -\frac{n_1}{n_2} \end{bmatrix}$$

$$R = |r|^2 = \left(\frac{n_0 (n_2)^2 - n_T (n_1)^2}{n_0 (n_2)^2 + n_T (n_1)^2} \right)^2$$

Reflectance of 2 $\lambda/4$ films

$R = 0$ ➡ $\sqrt{\frac{n_T}{n_0}} = \frac{n_2}{n_1}$

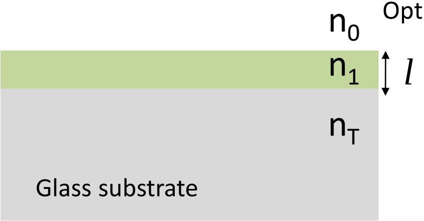
ANTI-REFLECTION!

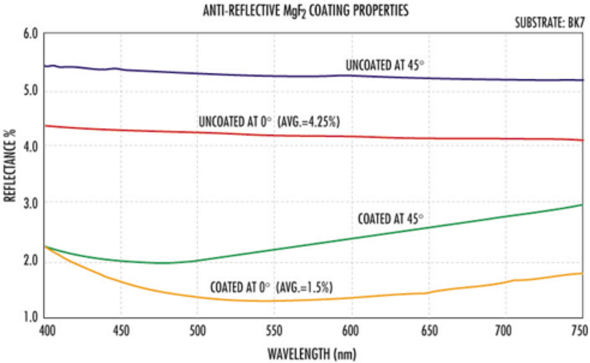
If $n_0 = 1$, $n_T = 1.5$, $n_1 = 1.38$ (MgF_2) ➡ $n_2 = 1.69$ ➡ $n_2 = 1.62$ (Al_2O_3)

$R = 0.0018$

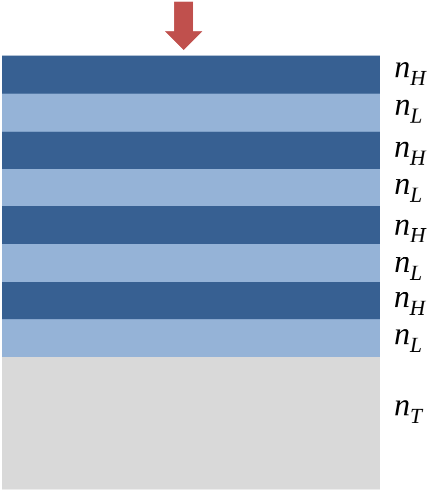
Anti-reflection coating

Optical thickness of the layer: $l = \frac{\lambda}{4}$ $\Rightarrow$ $kl = \frac{\pi}{2}$





High reflectance films



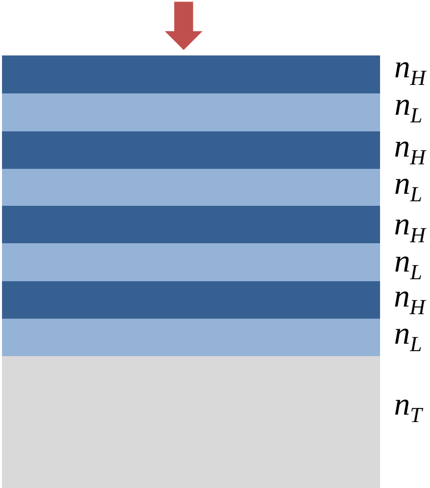
2N layers $\quad l = \dfrac{\lambda}{4}$

$$M = \left(M_H M_L\right)^N = \begin{bmatrix} -\dfrac{n_L}{n_H} & 0 \\ 0 & -\dfrac{n_H}{n_L} \end{bmatrix}^N$$

$$R = |r|^2 = \left(\frac{1 - \dfrac{n_T}{n_0}\left(\dfrac{n_H}{n_L}\right)^{2N}}{1 + \dfrac{n_T}{n_0}\left(\dfrac{n_H}{n_L}\right)^{2N}} \right)^2$$

$N \to \infty \quad \Rightarrow \quad R \to 1$

High reflectance films



2N layers $\quad l = \dfrac{\lambda}{4} \quad n_0 = 1 \quad n_T = 1.5$

2 layers of MgF_2 ($n_L = 1.38$)
2 layers of ZnS ($n_H = 2.39$) ➡ $R = 0.74$

4 layers of MgF_2 ($n_L = 1.38$)
4 layers of ZnS ($n_H = 2.39$) ➡ $R = 0.97$

30 layers of MgF_2 ($n_L = 1.38$)
30 layers of ZnS ($n_H = 2.39$) ➡ $R = 0.999$

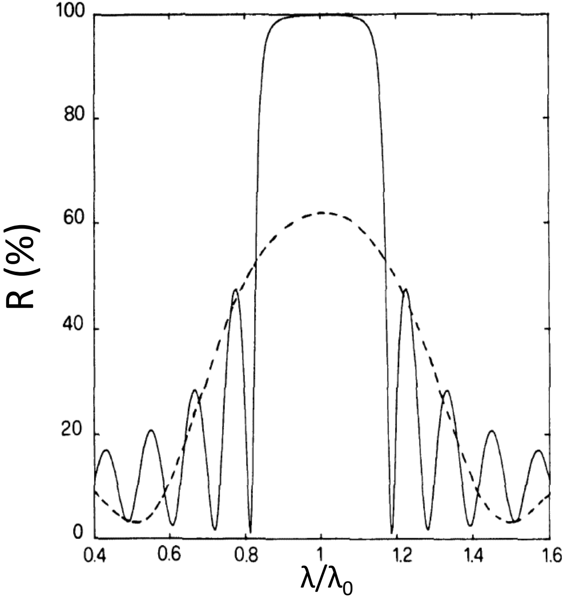
$$R = |r|^2 = \left(\frac{1 - \dfrac{n_T}{n_0}\left(\dfrac{n_H}{n_L}\right)^{2N}}{1 + \dfrac{n_T}{n_0}\left(\dfrac{n_H}{n_L}\right)^{2N}} \right)^2$$

$N \to \infty$ ➡ $R \to 1$

High reflectance films

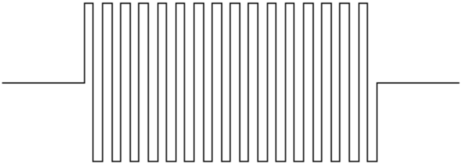
Stack of TiO_2 (rutile, $n_H = 2.72$) and SiO_2 ($n_L = 1.46$) layers of $\lambda/4$ thickness on BK7.

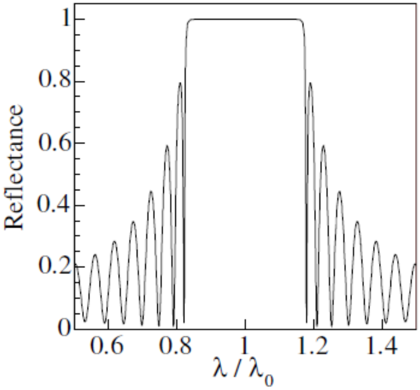
Number of layers (2N):
N = 3 dashed curve
N = 15 continuous curve



High reflectance films

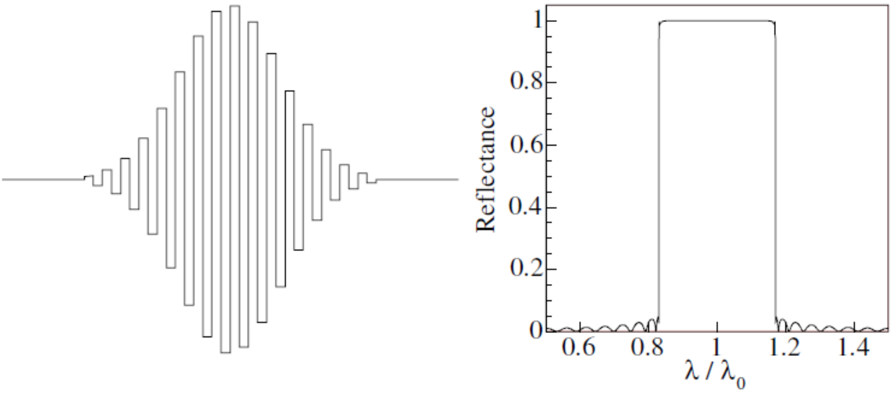
16 couples of MgF_2 ($n_L = 1.38$) and TiO ($n_H = 2.35$) layers of λ/4 thickness on glass substrate.





High reflectance films

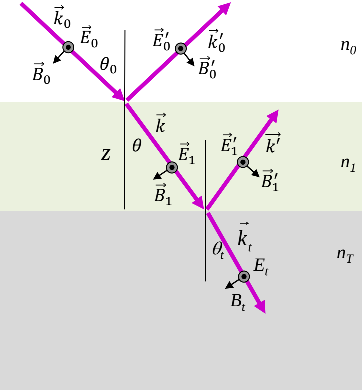
16 couples of MgF_2 ($n_L = 1.38$) and TiO ($n_H = 2.35$) layers of $\lambda/4$ thickness on glass substrate.



APODIZED Reflectivity

Multi-layers (oblique incidence)

Transverse Electric TE (s)



$$R = |r|^2 \qquad T = |t|^2$$

$$r = \frac{An_0 + Bn_0n_T - C - Dn_T}{An_0 + Bn_0n_T + C + Dn_T}$$

at normal incidence

$$t = \frac{2n_0}{An_0 + Bn_0n_T + C + Dn_T}$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\beta & -\frac{i}{p}\sin\beta \\ -i\,p\,\sin\beta & \cos\beta \end{pmatrix}$$

$$\beta = k\cos\theta\, z$$

$$n_0 \to n_0\cos\theta_0 \qquad r \to r$$

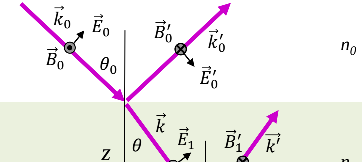
$$n_1 \to n_1\cos\theta = p \qquad t \to t$$

$$n_T \to n_T\cos\theta_t$$

$$\mathrm{R} + \frac{n_T\cos\theta_t}{n_0\cos\theta_0}T = 1$$

Multi-layers (oblique incidence)

Transverse Magnetic
TM (p)

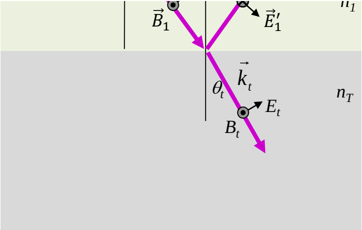


$$r = \frac{An_0 + Bn_0n_T - C - Dn_T}{An_0 + Bn_0n_T + C + Dn_T}$$

at normal incidence

$$t = \frac{2n_0}{An_0 + Bn_0n_T + C + Dn_T}$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\beta & -\frac{i}{p}\sin\beta \\ \end{pmatrix}$$



$$\begin{pmatrix} & \\ -i\,p\,sin\beta & cos\beta \end{pmatrix}$$

$$\beta = k\ cos\theta\ z$$

$$r \rightarrow r$$

$$n_0 \rightarrow \frac{n_0}{cos\theta_0}$$

$$t \rightarrow t\frac{cos\theta_t}{cos\theta_0}$$

$$n_1 \rightarrow \frac{n_1}{cos\theta} = p$$

$$R = |r|^2 \qquad T = |t|^2$$

$$n_T \rightarrow \frac{n_T}{cos\theta_t}$$

$$\mathrm{R} + \frac{n_T cos\theta_t}{n_0 cos\theta_0} T = 1$$