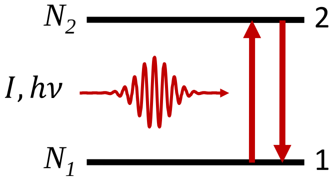
Saturation: homogeneous line

Absorption saturation ($N_1 > N_2$)



Homogeneously broadened line

cw regime

$$\frac{dN_2}{dt} = WN_1 - WN_2 - \frac{N_2}{\tau} = -W(N_2 - N_1) - \frac{N_2}{\tau}$$

absorption — stimulated emission — spontaneous emission

$$N_t = N_1 + N_2$$

Defining: $\Delta N = N_1 - N_2$

$$\Longrightarrow \quad N_2 = \frac{N_t - \Delta N}{2} \quad \text{and} \quad N_1 = \frac{N_t + \Delta N}{2}$$

Saturation: homogeneous line

Absorption saturation ($N_1 > N_2$)

$$\Longrightarrow \quad \frac{d\Delta N}{dt} = -\Delta N\left(\frac{1}{\tau} + 2W\right) + \frac{N_t}{\tau} \qquad \Longrightarrow \quad \frac{d\Delta N}{dt} = 0$$

Steady-state

$$\Longrightarrow \quad \Delta N = \frac{N_t}{1 + 2W\tau} \qquad W = \sigma F = \sigma \frac{I}{h\nu}$$

$$\Longrightarrow \quad \Delta N = \frac{N_t}{1 + \frac{I}{I_s}} \qquad \boxed{I_s = \frac{h\nu}{2\sigma\tau}}$$

Saturation intensity

$$I = I_s \quad \Longrightarrow \quad \Delta N = \frac{N_t}{2}$$

$$I \gg I_s \quad (I \to \infty) \quad \Longrightarrow \quad \Delta N \to 0$$

Absorption saturation ($N_1>N_2$)

$$\alpha = \sigma(N_1 - N_2) = \sigma\,\Delta N \quad \Longrightarrow \quad \alpha = \frac{\alpha_0}{1+\frac{I}{I_s}}$$

Absorption coefficient

$$\alpha_0 = \alpha(@N_1 = N_t\,; N_2 = 0) = \sigma N_t$$

unsaturated absorption coefficient

$$\Longrightarrow \quad \Delta N = \frac{N_t}{1+\frac{I}{I_s}} \qquad \boxed{I_s = \frac{h\nu}{2\sigma\tau}}$$

Saturation intensity

$$I = I_s \quad \Longrightarrow \quad \Delta N = \frac{N_t}{2}$$

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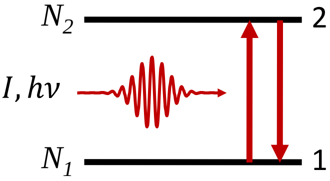
Saturation: homogeneous line

Absorption saturation ($N_1 > N_2$)

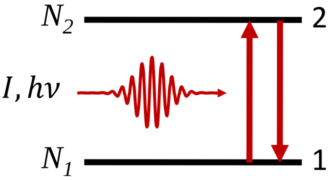


Homogeneously broadened line

Pulsed regime $I = I(t)$

1. The pulse duration is **much larger** than the upper level lifetime ($\Delta t \gg \tau$)
2. The pulse duration is **much smaller** than the upper level lifetime ($\Delta t \ll \tau$)

Absorption saturation ($N_1 > N_2$)



Homogeneously broadened line

Pulsed regime $I = I(t)$

1. The pulse duration is **much larger** than the upper level lifetime (***$\Delta t \gg \tau$***)

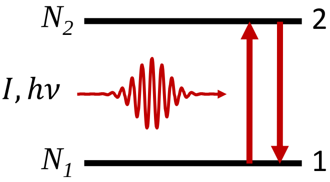
The temporal evolution of ΔN is very slow, and one can assume: $\left|\frac{d\Delta N}{dt}\right| \ll \frac{N_t}{\tau}$

At **steady-state** conditions, ΔN is still given by: $\Delta N = \dfrac{N_t}{1 + \frac{I}{I_s}}$

The saturation behavior is the same as for a cw beam:

$$\boxed{\alpha = \frac{\alpha_0}{1 + \frac{I}{I_s}}}$$

Absorption saturation ($N_1 > N_2$)



Homogeneously broadened line

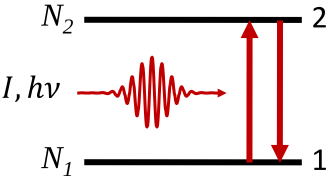
Pulsed regime $\quad I = I(t)$

2. The pulse duration is **much smaller** than the upper level lifetime ($\boldsymbol{\Delta t \ll \tau}$)

$$\frac{d\Delta N}{dt} = -\Delta N \left(\frac{1}{\tau} + 2W \right) + \frac{N_t}{\tau} = -2\mathrm{W}\,\Delta N + \cancel{\frac{N_t - \Delta N}{\tau}} = -2W\Delta N$$

The stimulated emission term ($2\mathrm{W}\Delta N$) dominates over the spontaneous emission one $\left(\frac{N_t - \Delta N}{\tau}\right)$: $\quad \Longrightarrow \quad \frac{N_t - \Delta N}{\tau} \ll 2W\Delta N$

Absorption saturation (N_1>N_2)



Homogeneously broadened line

Pulsed regime $I = I(t)$

2. The pulse duration is **much smaller** than the upper level lifetime ($\boldsymbol{\Delta t \ll \tau}$)

$$\frac{d\Delta N}{dt} = -2\mathrm{W}\,\Delta N = -\left(\frac{2\sigma}{h\nu}\right) I(t)\Delta N \qquad W = \sigma F = \sigma\frac{I}{h\nu}$$

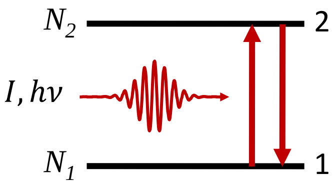
Integrating both terms with the initial condition:

$$\Delta N(0) = N_t \quad (t = 0, N_2 = 0, N_1 = N_t)$$

$$\Longrightarrow \quad \Delta N(t) = N_t\, e^{-\frac{2\sigma}{h\nu}\int_0^t I(t')dt'}$$

Saturation: homogeneous line

Absorption saturation ($N_1>N_2$)



Homogeneously broadened line

Pulsed regime $I = I(t)$

2. The pulse duration is **much smaller** than the upper level lifetime ($\Delta t \ll \tau$)

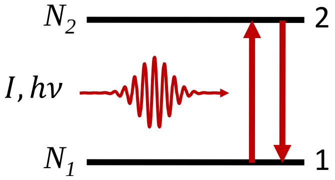
$$\Gamma(t) = \int_0^t I(t')dt'$$ **Fluence**

$$\boxed{\Gamma_s = \frac{h\nu}{2\sigma}}$$ **Saturation fluence**

$$\Downarrow \quad \Delta N(t) = N_t\, e^{-\frac{\Gamma(t)}{\Gamma_s}}$$

$$\Longrightarrow \quad \Delta N(t) = N_t\, e^{-\frac{2\sigma}{h\nu}\int_0^t I(t')dt'}$$

Absorption saturation (N_1>N_2)



Homogeneously broadened line

Pulsed regime $I = I(t)$

2. The pulse duration is **much smaller** than the upper level lifetime (***$\Delta t \ll \tau$***)

$$\Gamma(t) = \int_0^t I(t')dt'$$ **Fluence**

$$\boxed{\Gamma_s = \frac{h\nu}{2\sigma}}$$ **Saturation fluence**

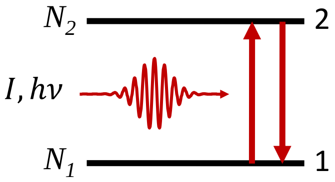
$$\Downarrow \quad \Delta N(t) = N_t\, e^{-\frac{\Gamma(t)}{\Gamma_s}}$$

$\Gamma_s =$ fluence needed to get $\Delta N_\infty = \frac{N_t}{e}$

$$\Delta N_\infty = \Delta N(t=\infty) = N_t\, e^{-\frac{\Gamma_t}{\Gamma_s}}$$

$\Gamma_t =$ Total fluence of the pulse

Absorption saturation ($N_1 > N_2$)



Homogeneously broadened line

Pulsed regime $I = I(t)$

2. The pulse duration is **much smaller** than the upper level lifetime ($\Delta t \ll \tau$)

$$\Gamma(t) = \int_0^t I(t')dt'$$ **Fluence**

$$\boxed{\Gamma_s = \frac{h\nu}{2\sigma}}$$ **Saturation fluence**

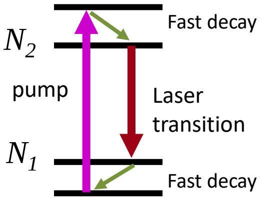
$\Downarrow$

$$\alpha = \alpha_0 \, e^{-\frac{\Gamma(t)}{\Gamma_s}}$$ **Absorption coefficient**

$$\alpha_0 = \sigma N_t$$

unsaturated absorption coefficient

Gain saturation ($N_1 < N_2$)



Homogeneously broadened line

cw regime

$N_1 \sim 0$ (4-level system)

$$\frac{dN_2}{dt} = R_P - WN_2 - \frac{N_2}{\tau}$$

pumping rate; stimulated emission; spontaneous emission

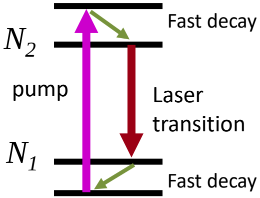
$$\frac{dN_2}{dt} = 0$$ **Steady-state**

$$N_2 = \frac{R_P \tau}{1 + W\tau} \implies N_2 = \frac{N_{20}}{1 + \frac{I}{I_s}} \implies N_{20} = R_P \tau$$

$$\boxed{I_s = \frac{h\nu}{\sigma\tau}}$$

Saturation intensity

Gain saturation ($N_1 < N_2$)



Homogeneously broadened line

cw regime

$N_1 \sim 0$ (4-level system)

$$g = \sigma(N_2 - N_1) = \sigma N_2$$

$$g = \frac{g_0}{1 + \frac{I}{I_s}}$$ **Gain coefficient**

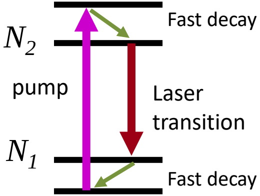
$$g_0 = \sigma N_{20}$$ unsaturated gain coefficient

$$N_2 = \frac{R_P \tau}{1 + W\tau} \Longrightarrow N_2 = \frac{N_{20}}{1 + \frac{I}{I_s}} \Longrightarrow N_{20} = R_P \tau$$

$$\boxed{I_s = \frac{h\nu}{\sigma\tau}}$$

Saturation intensity

Gain saturation (N_1<N_2)



Homogeneously broadened line

pulsed regime $I = I(t)$

$N_1 \sim 0$ (4-level system)

1. ($\Delta t \gg \tau$)

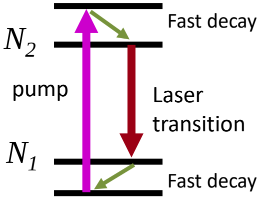
$$\frac{dN_2}{dt} = R_P - WN_2 - \frac{N_2}{\tau}$$

$\Longrightarrow$ The term $\frac{dN_2}{dt}$ is negligible with respect to the other terms and we get the steady-state condition:

$$N_2 = \frac{N_{20}}{1 + \frac{I}{I_s}} \qquad N_{20} = R_P\tau \qquad g = \frac{g_0}{1 + \frac{I}{I_s}}$$

$$I = I(t) \qquad I_s = \frac{h\nu}{\sigma\tau}$$

Gain saturation ($N_1<N_2$)



Homogeneously broadened line

pulsed regime $I = I(t)$

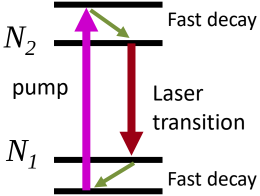
$N_1 \sim 0$ (4-level system)

2. ($\Delta t \ll \tau$) $$\frac{dN_2}{dt} = R_P - WN_2 - \frac{N_2}{\tau}$$

$\Longrightarrow$ During the interaction with the pulse the pumping rate (R_P) and the spontaneous decay term ($\frac{N_2}{\tau}$) can be neglected with respect to the stimulated emission term (WN_2):

$$\Longrightarrow \quad \frac{dN_2}{dt} = -WN_2 = -\left(\frac{\sigma I}{h\nu}\right)N_2 \quad \Longrightarrow \quad N_2(t) = N_{20}\, e^{-\frac{\Gamma(t)}{\Gamma_S}}$$

Gain saturation ($N_1<N_2$)



Homogeneously broadened line

pulsed regime $\quad I = I(t)$

$N_1 \sim 0 \quad$ (4-level system)

2. ($\Delta t \ll \tau$) $$\frac{dN_2}{dt} = R_P - WN_2 - \frac{N_2}{\tau}$$

$$g = g_0\, e^{-\frac{\Gamma(t)}{\Gamma_S}}$$

$$\boxed{\Gamma_S = \frac{h\nu}{\sigma}}$$ **Saturation fluence**

$$\Longrightarrow \quad \frac{dN_2}{dt} = -WN_2 = -\left(\frac{\sigma I}{h\nu}\right)N_2 \quad \Longrightarrow \quad N_2(t) = N_{20}\, e^{-\frac{\Gamma(t)}{\Gamma_S}}$$

Saturation: homogeneous line

Absorption saturation ($N_1>N_2$)

cw regime

$$\alpha = \frac{\alpha_0}{1+\frac{I}{I_s}}$$

$$\boxed{I_s = \frac{h\nu}{2\sigma\tau}}$$

Saturation intensity

pulsed regime

$$\alpha = \alpha_0 e^{-\Gamma(t)/\Gamma_s}$$

$$\boxed{\Gamma_s = \frac{h\nu}{2\sigma}}$$

Saturation fluence

Gain saturation ($N_1<N_2$)

cw regime

$$g = \frac{g_0}{1+\frac{I}{I_s}}$$

$$\boxed{I_s = \frac{h\nu}{\sigma\tau}}$$

Saturation intensity

pulsed regime

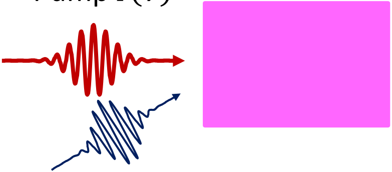
$$g = g_0 e^{-\Gamma(t)/\Gamma_s}$$

$$\boxed{\Gamma_s = \frac{h\nu}{\sigma}}$$

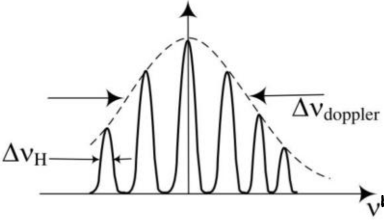
Saturation fluence

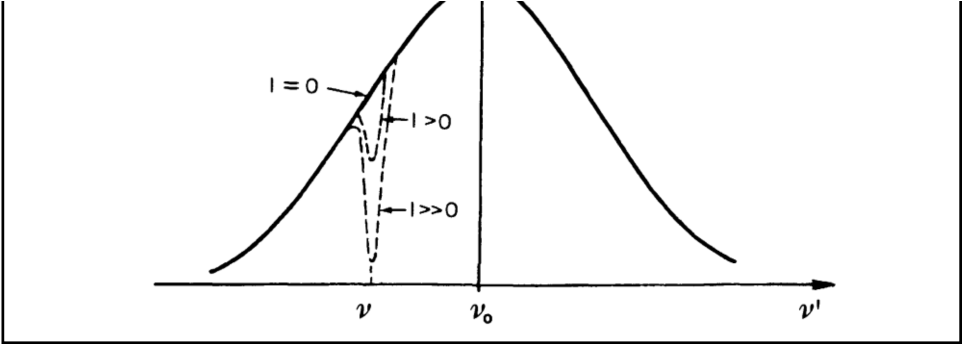
Spectral Hole Burning

Pump $I(\nu)$



Probe $I(\nu')$




I = 0
I >0
I>>0
ν
ν₀
ν'