Lecture 22.
Wednesday 8th January, 2020.
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0.1 Renormalization group

The idea is to consider a partition function $Z_N(\vec{\mathbf{k}})$ and then integrate, in order to obtain another partition function $Z_{N(/l^d)}(\vec{\mathbf{k}}')$. We make the assumptions:

1. $R_l(\vec{\mathbf{k}}) = \vec{\mathbf{k}}'$, R_l is analytic
2. $R_l \dot{R}_{l'} = R_{ll'}$ semigroup

Procedure:

- You find the fixed point
- You linearized around the fixed point.
- You find the behaviour.

0.2 Ising model

we use l = 2.
We have $\sigma_i = \pm 1$ and a weight

$$w(\sigma_i, \sigma_j) = -\hat{g} - \frac{h}{z}(\sigma_i + \sigma_j) - J\sigma_i\sigma_j \tag{1}$$

In that sense, the partition function can be written as

$$Z = \sum_{\{\sigma\}} e^{\sum_{\langle ij\rangle} w(\sigma_i, \sigma_j)} \tag{2}$$

$$Z(g, h, k) = \sum_{\{\sigma\}} = \sum_{\{\sigma'\}} \sum_{\{S\}} e^{\sum_i w(\sigma_i, \sigma_{i+1})} \tag{3}$$

(the σ_1' are the new red bubble, while the S_1 the rejected one in the second graph. In the first graph we had σ_1)

$$w(\sigma_i, \sigma_{i+1}) = g + \frac{h}{z}(\sigma_i + \sigma_{i+1}) + k\sigma_i\sigma_{i+1} \tag{4}$$

Hence,

$$Z(g, h, k) = \sum_{\{\sigma'\}_{N/2}} \sum_{\{S\}_{N/2}} e^{\sum_i^{N/2} \left[w(\sigma_i', S_i) + w(S_i, \sigma_{i+1}')\right]} = \sum_{\{\sigma'\}_{N/2}} \prod_{i=1}^{N/2} \left(\sum_{S=\pm 1} e^{w(\sigma_i', S_i) + w(S_i, \sigma_{i+1}')} \right) \tag{5}$$

and we define

$$f(\sigma_i', \sigma_{i+1}') = e^{w(\sigma_i', S_i) + w(S_i, \sigma_{i+1}')} \tag{6}$$

Hence,

$$f(\sigma_i', \sigma_{i+1}') = \exp\left[w'(\sigma_i', \sigma_{i+1}')\right] \tag{7}$$

So

$$Z_N(g, h, k) = \sum_{\{\sigma'\}_{N/2}} e^{\sum_i^{N/2} w'(\sigma_i', \sigma_{i+1}')} = Z_{N/2}(g', h', k') \tag{8}$$

It means that this equations must holds for each σ'_i and σ'_{i+1}.

$$e^{g'+\frac{h'}{2}(\sigma'_i+\sigma'_{i+1})+k'\sigma'_i\sigma'_{i+1}} = \sum_{S_i=\pm 1} e^{g+\frac{h}{2}(\sigma'_i+S_i)+k\sigma'_iS_i+g+\frac{h}{2}(S_i+\sigma_{i+1})+kS_i\sigma'_{i+1}} \tag{9}$$

$\forall(\sigma'_i, \sigma'_{i+1})$

$$\begin{cases} x \equiv e^K \\ y \equiv e^h \\ z \equiv e^g \end{cases} \tag{10}$$

$$z'y'^{\frac{\sigma'_i+\sigma'_{i+1}}{2}}x'^{\sigma'_i\sigma'_{i+1}} = \sum_{S_i} zy^{\frac{\sigma'_i+S_i}{2}}x^{\sigma'_iS_i}zy^{S_i+\sigma'_{i+1}}x^{S_i\sigma'_{i+1}} \tag{11}$$

Copiare tabella da qualcuno: (da 4 equazioni) σ'_i σ'_{i+1}

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$$z'y'x' = z^2y(x^2y + x^{-2}y^{-1}) \quad \text{for } S_i = +1, S_i = -1 \tag{12}$$

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$$z'y'^{-1}x' = z^2y^{-1}(x^{-2}y + x^2y^{-1}) \tag{13}$$

+-

$$z'x'^{-1} = z^2y(yy^{-1}) \tag{14}$$

-+

$$z'x'^{-1} = z^2y(yy^{-1}) \tag{15}$$

Now we multiply (1)(2)(3)(4)

$$(1)(2)(3)(4) \Rightarrow z'^4 = \ldots (see notes) \tag{16}$$

$$(1)/(2) \Rightarrow y'^2 = y^2\frac{x^2y + x^{-2}y^{-1}}{x^{-2}y + x^2y^{-1}} \tag{17}$$

$$\frac{(1)(2)}{(3)(4)} \Rightarrow x'^4 = \frac{(x^2y + x^{-2}y^{-1})(x^{-2}y + x^2y^{-1})}{(y + y^{-1})^2} \tag{18}$$

We obtain:

$$2h' = 2h + \log() - \log() \tag{19}$$

$$4k' = \log() + \log() - 2\log() \tag{20}$$

Those are the normalization equation.

$$(k', h') = \mathcal{R}_2(k, h) \tag{21}$$