Lecture 18. Friday 13th December, 2019. Compiled: Friday 13th December, 2019.

$$\left(-\boldsymbol{\nabla}^2 + \xi^{-2}(t)\right)G_c(\vec{\mathbf{r}} - \vec{\mathbf{r}}') = \frac{k_B T}{k}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \tag{1}$$

Let us try to do the Fourier transform. Let us define

$$\vec{\mathbf{x}} \equiv \vec{\mathbf{r}} - \vec{\mathbf{r}}' \tag{2}$$

e let us call $\widetilde{G}(q)$ the Fourier transform of the function G

$$\widetilde{G}(q) = \int_{-\infty}^{+\infty} \mathrm{d}|\vec{\mathbf{x}}|\, G_c(|\vec{\mathbf{x}}|)e^{-iq|\vec{\mathbf{x}}|} \tag{3}$$

we get

$$\widetilde{G}(q) = \frac{k_B T}{k}\frac{1}{q^2 + \xi^{-2}} \tag{4}$$

At $T = T_c$, we have $\xi \to \infty$ and $\widetilde{G}(q) \simeq \frac{1}{q^2}$. We have

$$G_c(|\vec{\mathbf{x}}|) = |\vec{\mathbf{x}}|^{2-D} \tag{5}$$

In this case we see immediately that $\eta = 0$. Go back and find why we have this.

$$G(\vec{\mathbf{x}}) = \int \mathrm{d}^D\vec{\mathbf{q}}\, \frac{1}{(2\pi)^D}\frac{1}{q^2 + \xi^{-2}}e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{x}}} \tag{6}$$

Let us do it for $D = 3$:

$$\Rightarrow G(|x|) = \frac{4\pi}{(2\pi)^3}\int_0^{\infty} \mathrm{d}q\, \frac{q^2}{q^2 + \xi^{-2}}\int_{-1}^{+1} \mathrm{d}(\cos\theta)\, e^{iq|\vec{\mathbf{x}}|\cos\theta} \tag{7}$$

we get

$$= \frac{4\pi}{(2\pi)^3}|\vec{\mathbf{x}}|\int_0^{\infty} \mathrm{d}q\, \frac{q\sin(q|\vec{\mathbf{x}}|)}{q^2 + \xi^{-2}} \tag{8}$$

At the end

$$\Rightarrow G(|\vec{\mathbf{x}}|) = \frac{1}{2\pi}\frac{e^{-\frac{|\vec{\mathbf{x}}|}{\xi}}}{|\vec{\mathbf{x}}|} \tag{9}$$

Can we rich the simple level of fluctuation? The simple level is the one that follow gaussian distribution. Let us introduce fluctuations at the Gaussian level.