UNIVERSITÀ DEGLI STUDI DI PADOVA

Two-qubit CZ gate implementation with trapped neutral atoms: a numerical simulation

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Multi-qubit gates can be implemented with **trapped neutral atoms** by driving them to highly excited **Rydberg states**, in which nearby Rydberg atoms can interact very strongly and in a controlled way.

In this project, a physical implementation of a **two-qubit CZ gate** [1] (up to a global gauge choice) between individual neutral atoms trapped in optical tweezer is numerically simulated.

It maps the **computational basis states** as:

$$\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle\, e^{i\phi} \\
|10\rangle &\rightarrow |10\rangle\, e^{i\phi} \\
|11\rangle &\rightarrow |11\rangle\, e^{i(2\phi-\pi)}
\end{aligned} \tag{1}$$

- Individual neutral atoms are trapped in optical tweezers and organized in a one-dimensional array into groups of two.
- Qubits are encoded in hyperfine ground states of these atoms with $|0\rangle = |5S_{1/2}, F = 1, m_F = 0\rangle$ and $|1\rangle = |5S_{1/2}, F = 2, m_F = 0\rangle$.
- All qubits are initialized in $|0\rangle$ through a Raman-assisted optical pumping procedure.

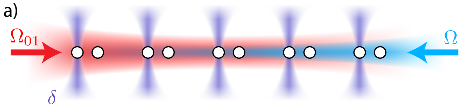

Figure: One-dimensional array of neutral atoms. Atoms are arranged in pairs and are globally driven with a 795 nm Raman laser (in red) which couples the qubit states $|0\rangle$ and $|1\rangle$. Local 420 nm (purple) beams are focused onto individual atoms. Atoms are globally excited by a bi-chromatic, 420 nm and 1013 nm, Rydberg laser (blue) from the $|1\rangle$ qubit state to $|r\rangle$.

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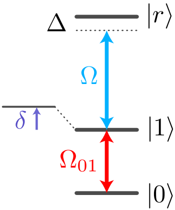

Figure: Relevant atomic levels. The qubit states $|0\rangle$ and $|1\rangle$ are coupled with Rabi frequency Ω_{01}. δ is the light shift used for individual addressing resulting from local 420 nm beams. The qubit state $|1\rangle$ is coupled to the Rydberg state $|r\rangle = |70S_{1/2}, m_J = -1/2\rangle$ with detuning Δ and effective Rydberg Rabi frequency Ω.

New protocol

- Two global laser pulses (with the laser phase of the second pulse shifted by ξ) of the same length τ and detuning Δ couple $|1\rangle$ to $|r\rangle$ and drive nearby atoms within the Rydberg blockade regime.
- Neighboring atoms cannot be simultaneously excited to the Rydberg state according to the **Rydberg blockade**.

If $V \gg |\Omega|, |\Delta|$, the dynamics of the system simplifies as:

- the **state** $|00\rangle$ does not evolve since it is uncoupled by the laser field;
- if one of the two atoms is in $|0\rangle$, only the system in $|1\rangle$ evolves. The dynamics can be described with a two-level system with states $|a_1\rangle \equiv |1\rangle$ and $|b_1\rangle \equiv |r\rangle$ and Hamiltonian:

$$H_1 = \frac{1}{2}(\Omega |a_1\rangle\langle b_1| + \Omega^* |b_1\rangle\langle a_1|) - \Delta |b_1\rangle\langle b_1| \tag{2}$$

- if **both** atoms are initially in $|1\rangle$, the dynamics can be described with a two-level system with states $|a_2\rangle \equiv |11\rangle$ and $|b_2\rangle \equiv \frac{1}{\sqrt{2}}(|r,1\rangle + |1,r\rangle)$ and Hamiltonian:

$$H_2 = \frac{\sqrt{2}}{2}(\Omega |a_2\rangle\langle b_2| + \Omega^* |b_2\rangle\langle a_2|) - \Delta |b_2\rangle\langle b_2| \tag{3}$$

Each **pulse** is mathematically represented by an unitary evolution of the state:

$$U = \exp(-iH\tau) \tag{4}$$

The **change of the laser phase** between the two pulses is represented as:

$$\Omega \to \Omega e^{i\xi} \tag{5}$$

Theoretical design of two-qubit CZ gate

Perfect blockade regime: optimal parameters

For a **fixed detuning $\boldsymbol{\Delta}$**:

- the **pulse length $\boldsymbol{\tau}$** is chosen such that the *first laser pulse* drives an incomplete oscillation on the $|01\rangle$ system, while the $|11\rangle$ system completes a full cycle of a detuned Rabi oscillation;
- the driving field of the *second laser pulse* is rotated around the *z*-axis by an **angle $\boldsymbol{\xi}$** such that a second pulse of length τ completes the oscillation for a $|01\rangle$ system returning to $|01\rangle$. It also drives a second complete oscillation on the $|11\rangle$ configuration.

$$\tau = \frac{2\pi}{\sqrt{\Delta^2 + 2\Omega^2}} \tag{6}$$

$$e^{-i\xi} = \frac{-\sqrt{(\Delta/\Omega)^2+1}\cos\left(\frac{1}{2}\Omega\tau\sqrt{(\Delta/\Omega)^2+1}\right) + i\Omega\tau\sin\left(\frac{1}{2}\Omega\tau\sqrt{(\Delta/\Omega)^2+1}\right)}{\sqrt{(\Delta/\Omega)^2+1}\cos\left(\frac{1}{2}\Omega\tau\sqrt{(\Delta/\Omega)^2+1}\right) + i\Omega\tau\sin\left(\frac{1}{2}\Omega\tau\sqrt{(\Delta/\Omega)^2+1}\right)} \tag{7}$$

The **optimal parameters** are:

$$\Omega\tau = 4.29268 \tag{8a}$$
$$\xi = 3.90242 \tag{8b}$$
$$\Delta/\Omega = 0.377371 \tag{8c}$$

The *finite value* of V only affects the dynamics if the system is initially in the state $|11\rangle$.

The dynamics of a **system initially in $|11\rangle$** can be described with a three-level system with states $|a_2\rangle \equiv |11\rangle$, $|b_2\rangle \equiv |1r\rangle + |r1\rangle$ and $|c_2\rangle = |rr\rangle$ and Hamiltonian:

$$
\begin{aligned}
H_2 = & \frac{\sqrt{2}}{2}\left(\Omega\,|a_2\rangle\langle b_2| + \Omega\,|b_2\rangle\langle c_2| + \Omega^*\,|c_2\rangle\langle b_2| + \Omega^*\,|b_2\rangle\langle a_2|\right) \\
& - \Delta\,|b_2\rangle\langle b_2| + (V - 2\Delta)\,|c_2\rangle\langle c_2|
\end{aligned} \tag{9}
$$

Small $V > 0$ and a given Δ

A value for Ω and τ can always be chosen such that a system initialized in $|11\rangle$ returns to itself after just the first pulse.

$V \gg |\Delta|, |\Omega|$

The effect for finite blockade simply reduces to the two-level system, $\{|a_2\rangle, |b_2\rangle\}$, where the detuning Δ is renormalized by $\Omega^2/(2V)$.

Code implementation of two-qubit CZ gate

Two-qubit system

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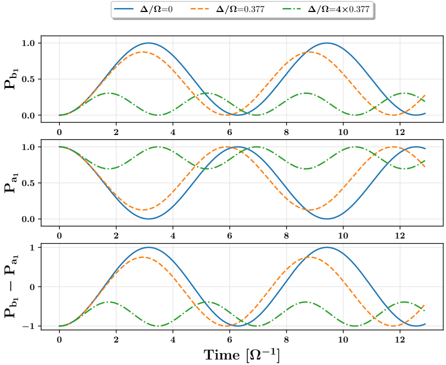
- The **system** is composed by **two atoms** each one can be in states $|\mathbf{0}\rangle$, $|\mathbf{1}\rangle$ or $|\boldsymbol{r}\rangle$. Thus system state vectors have dimension $d = 3^2$.
- To **simulate the dynamics** of the system under the application of the two-qubit CZ gate, the Python library **QuTip** is used.

```
# Hamiltonian definition
def hamiltonian(Omega,Delta):

    H0  = 0 * tensor(psi00.dag(),psi00)

    H01 = 1/2 * ( Omega*tensor(psi01.dag(),
     psi0r) + np.conj(Omega)*tensor(psi0r.
     dag(),psi01) )
        - Delta*tensor(psi0r.dag(),psi0r)

    H10 = ...
    H2  = ...

    H = H0 + H01 + H10 + H2

    return H
```

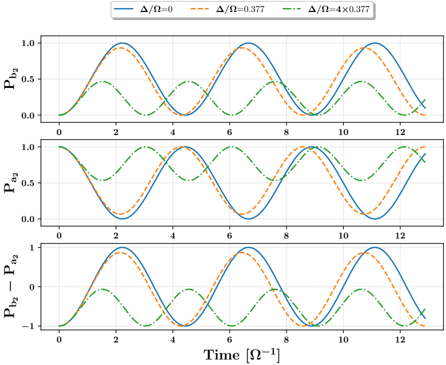
```
# CZ gate implementation
def CZ_gate(psi,Omega,Delta,tau):

    # Times discretization
    times = np.linspace(0.0, tau, 200)

    # Apply first pulse
    H = hamiltonian(Omega,Delta)
    result = mesolve(H, psi, times,[], [])
    psi = result.states[-1]

    # Apply second pulse rotated by Omega ->
      Omega exp(i xi)
    H = hamiltonian(Omega * exp_xi(Delta,
     Omega,tau), Delta)
    result = mesolve(H, psi, times,[], [])
    psi = result.states[-1]

    return result
```

Results

Perfect blockade regime: correct behavior check



(a) Dynamics of $|01\rangle$.

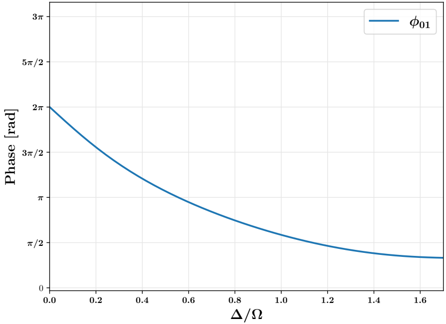


(b) Dynamics of $|11\rangle$.

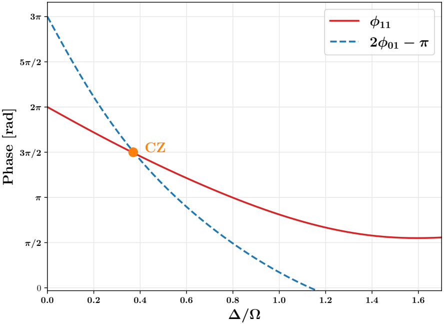
Figure: Time-dependence of the populations and the inversion for the two-level systems in Eqs. (2) and (3) under Rydberg perfect blockade assumption for different Δ/Ω ratios. The duration of the pulse is fixed by the product $\Omega\tau = 4.293$ (Eq. (8a)).

Results

Perfect blockade regime: correct behavior check

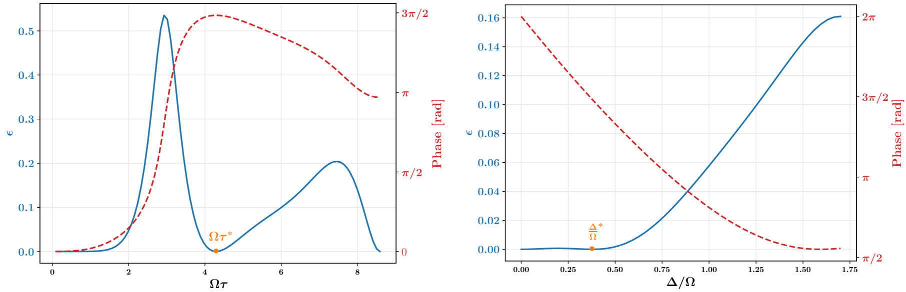


(a) Phase of $|01\rangle$ (or $|10\rangle$).



(b) Phase of $|11\rangle$.

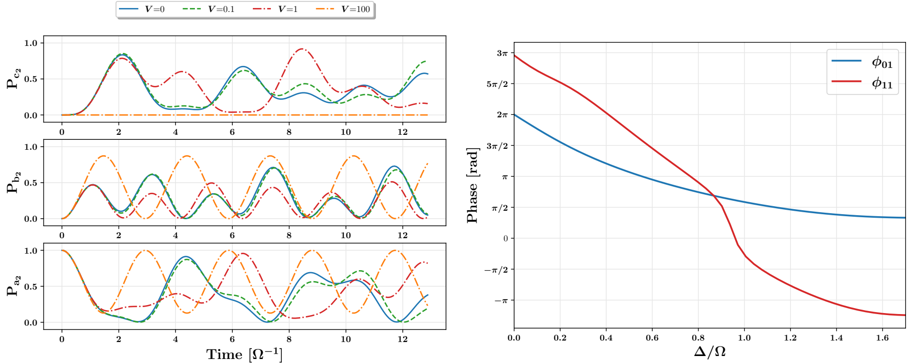
Figure: Dynamical phase ϕ_{01} (or ϕ_{10}) and ϕ_{11} acquired respectively by the states $|01\rangle$ (or $|10\rangle$) and $|11\rangle$ after the application of the two pulses. The intersection between ϕ_{11} and $2\phi_{01} - \pi$ shown in 4b realizes the CZ gate.



(a) $\Omega\tau$ variation.

(b) Δ/Ω variation.

Figure: In blue, the error computed as $\epsilon \equiv 1 - F(\rho_{init}, \rho_{fin})$ for a system initially in state $|11\rangle$ is plotted as a function of $\Omega\tau$ and Δ/Ω in 5a and 5b respectively. In red, also the dynamical phase ϕ_{11} acquired by the state $|11\rangle$ after the two pulses is reported.



(a) Population of $|11\rangle$.

(b) Dynamical phase of $|01\rangle$ and $|11\rangle$ for $V = 1$.

Figure: In 6a, the time-dependence of the population for the three-level system in Eq. (9) under finite blockade interactions assumption is reported. The parameters fixed for the simulation are the one in (8). In 6b, the dynamical phase of states in $|01\rangle$ (or $|10\rangle$) and $|11\rangle$ is illustrated for $V = 1$ as a function of the Δ/Ω ratio.

Perfect blockade regime

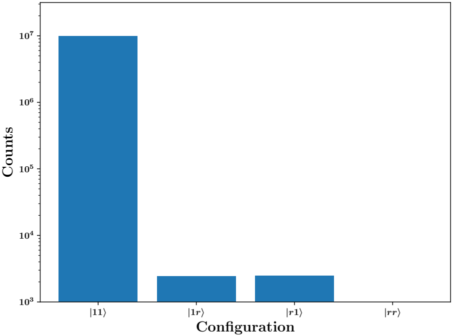


Figure: Bar plot of measurement process simulation for a system initially in $|11\rangle$ under the application of the CZ gate in perfect blockade assumption. A Gaussian noise with mean zero and standard deviation of 1% is introduced on the quantities $\Omega\tau$ and Δ/Ω. The experiment is ran 10^3 times and for each final wave-function 10^4 measurements are taken.

Results

Measurement process with Gaussian noise on τ and Δ

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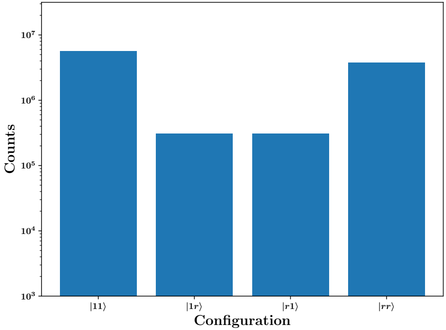
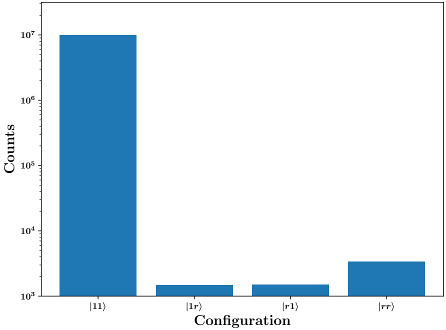
Imperfect blockade regime



(a) V=1



(b) V=100

Figure: Bar plot of measurement process simulation for a system initially in $|11\rangle$ under the application of the CZ gate in imperfect blockade assumption. A Gaussian noise with mean zero and standard deviation of 1% is introduced on the quantities $\Omega\tau$ and Δ/Ω. The experiment is ran 10^3 times and for each final wave-function 10^4 measurements are taken.

Conclusions

- We check the correctness of the numerical implementation by fixing the optimal parameters suggested and comparing the results with the one of Levine et al [1].
- The optimal parameters are varied in order to investigate the behavior of the CZ gate in response to this variation.
- A Gaussian noise is also introduced in the system and the measurement process is simulated, both in perfect and imperfect blockade regime.

- We check the correctness of the numerical implementation by fixing the optimal parameters suggested and comparing the results with the one of Levine et al [1].
- The optimal parameters are varied in order to investigate the behavior of the CZ gate in response to this variation.
- A Gaussian noise is also introduced in the system and the measurement process is simulated, both in perfect and imperfect blockade regime.

Thank you for the attention!

References

Harry c, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Tout T. Wang, Sepehr Ebadi, Hannes Bernien, Markus Greiner, Vladan Vuletić, Hannes Pichler, and Mikhail D. Lukin.
Parallel implementation of high-fidelity multiqubit gates with neutral atoms.
Phys. Rev. Lett., 123:170503, Oct 2019.