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Robot Dynamics & Control – A.A. 2021/2022

Assignment 2: Recursive Inverse Dynamics

Inverse dynamics is the computation of the forces needed at a robot's joints to produce a given set of common accelerations. The main uses of inverse dynamics are in robot control and trajectory planning. In control operations it's generally incorporated as an element in a feedback loop to convert accelerations calculated according to some control law into the common forces which will achieve those accelerations. In trajectory planning, inverse dynamics can be used to check that the forces demanded to execute a proposed trajectory don't exceed the actuators' limits.

In the Newton- Euler approach, the equations of motion (Newton's Euler's equations) are applied to each link and the performing equations are combined with constraint equations from the joints in such a way as to give the common forces in terms of the common accelerations.

Thanks to forward recursion, we can obtain the geometric, kinetic and dynamic parameters of the robot. Then we can calculate the distances between the Q and P points of each link, their linear and angular velocities and accelerations. Using a for loop, starting with the first link to the last, we first calculate the position and orientation of each link, then the velocities, angular and linear, and finally the accelerations, angular and linear.

When joints are rotoidal, the following formulas are used:
$r_i = Q_{i-1} - P_{i-1}$
$w_i = w_{i-1} + k_i*q\_dot_i$
$v_i = v_{i-1} + w_{i-1} \times r_i$
$w\_dot_i = w\_dot_{i-1} + w_{i-1} \times k_i*q\_dot_i + k_i*q\_2dot_i$
$v\_dot_i = v\_dot_{i-1} + w\_dot_{i-1} \times r_i + w_{i-1} \times (w_{i-1} \times r_i)$

When, on the other hand, the joints are prismatic, the following formulae are used:
$r_i = Q_{i-1} - P_{i-1} + k_i*q_i$
$w_i = w_{i-1}$
$v_i = v_{i-1} + w_{i-1} \times r_i + k_i*q_i$
$w\_dot_i = w\_dot_{i-1}$
$v\_dot_i = v\_dot_{i-1} + w\_dot_{i-1} \times r_i + w_{i-1} \times (w_{i-1} \times r_i) + 2*w_{i-1} \times k_i*q\_dot_i + k_i*q\_2dot_i$

Through all these formulas, the linear velocity and acceleration of the centre of mass of each link are calculated, i.e:
$v\_C_i = v_i + w_i \times (C_i – P_i)$
$v\_Cdot = v\_dot_i + w\_dot_i \times (C_i – P_i) + w_i \times [w_i \times (C_i – Pi)]$

Using backward recursion instead, we can calculate the inverse dynamic joint torques of each link, having as input the geometric, inertia and topological structural parameters.
Using a for loop, starting from the last link to the first, we can first calculate the dynamic force and dynamic torque, then the total force and momentum acting on each link.

The formulas used are as follows:
$D_i = m_i*v\_Cdot_i$
$\Delta_i = Ic_i*w\_dot_i + w_i \times Ic_i*w_i$
$F = - m_i*g + F_{i+1} + D_i + Fext$
$M = - [(P_i – C_i) \times F_i] + [(Q_i^+ – C_i) \times F_{i+1}] + M_{i+1} + Mext + \Delta_i$

This finally results in τ, which for rotoidal and prismatic joints, is calculated, respectively, as:
$\tau_i = M_i*k_i$
$\tau_i = F_i*k_i$

Exercise 1 – Recursive Newton-Euler Implementation

The first exercise simply represents the implementation of the Newton-Euler recursion through a function.

As input to this function we have the parameters of the manipulator and then the variables q, q_dot and q_2dot which change according to the type of joint: if the joint is rotoidal q represents a rotation while q_dot and q_2dot represent the angular velocity and acceleration of the joint respectively; if the joint is prismatic q represents a translation, q_dot and q_2dot the linear velocity and acceleration.

Robot parameters are passed to the function via the struct Robot. In this structure the points C, P and Q of each link, the masses and inertia matrices, the type and number of joints and the direction of the k-axes are defined.

Initially, all data used in forward and backward computation are initialised, considering the link to which they refer. Each parameter is calculated with respect to the absolute reference system 0.

First the forward computation is performed by calculating the various velocities and acceleration according to the type of joint, then the backward computation is performed to calculate the forces and moments.

This function is called up in each subsequent exercise to calculate the inverse dynamic joint torques.

Exercise 2 – Inverse Dynamics of 2R robot

In the second exercise, we must find the inverse dynamic joint toques for a manipulator with two rotoidal joints first without considering gravity, then considering it acting along the Y_0 axis.

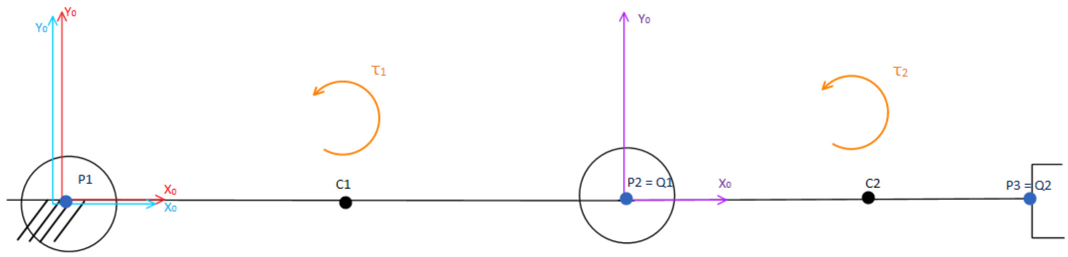
To perform the calculations, we assume that the links are prisms with a uniform distribution of mass, the COM of each being at the geometric centre.
At the beginning, the reference system 1 is considered coincident with the absolute system 0, while the reference system 2 is shifted by 1m with respect to the system 1.

P1 is located at the beginning of the first link and therefore at a distance of 0m from the centre of the absolute reference system, P2 coincides with Q1 and is located at the end of the first link, at a distance of 1m from the centre of the reference system 1, P3 coincides with Q2 and is located at a distance of 0.8m from the centre of the reference system of the second link.

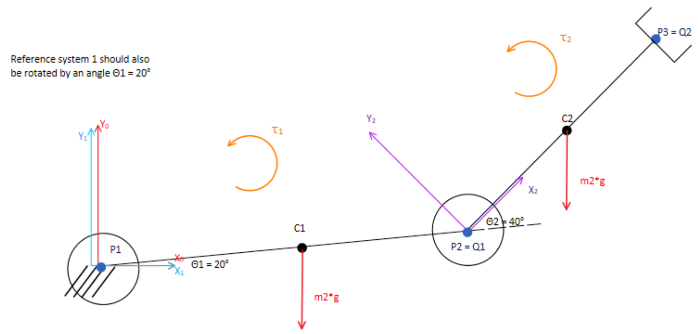
$l_1 = 1m$ $\quad$ $l_2 = 0.8m$
$m_1 = 22$ Kg $\quad$ $m_2 = 19$ Kg
$I_1 = diag\ (0\ Kgm^2, 0.4\ Kgm^2, 0.4\ Kgm^2)$
$I_2 = diag\ (0\ Kgm^2, 0.3\ Kgm^2, 0.3\ Kgm^2)$

Using the homogeneous transformation matrices, the coordinates of each point were calculated with respect to the absolute reference systems.
T0_1 represents the transformation matrix of the reference system 1 with respect to the absolute reference system 0, therefore only a rotation around the z axis of one angle ϑ_1 is taken into account.
T1_2 is the transformation matrix of the reference system 2 with respect to the reference system 1, the rotation takes place around the z axis by a ϑ_2 angle.

Being two rotoidal joints, the k-axes, calculated with respect to the absolute reference system, always remain positioned in the same way, [0 0 1]', rotate as a function of the angles ϑ but do not change direction.



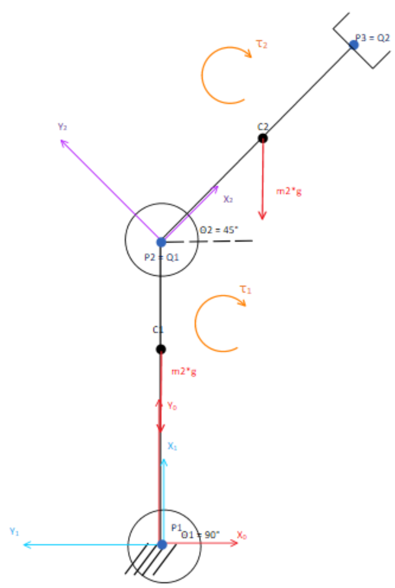
The points, in homogeneous coordinates, are given in [x,y,z] coordinates in order to calculate the vector product. Inertia matrices are given with respect to the centre of mass of each link, calculated in the local reference system. To be considered in the absolute reference system, they must be rotated with respect to the robot configuration.



2.1: In the first part ϑ_1 is 20° and ϑ_2 is 40°.
When gravity does not act on the robot, both components of τ are positive. The values do not vary much one from the other because the difference in configuration of the first link to the second is small.
When gravity acts on both centres of mass the component compared to the first link τ_1 and to the second link τ_2 are very different because, on the first link it must be taken into account that the reaction forces of the second link are also acting, so with a gravitational force acting at an angle of 60° and one of 40°, the component τ_1 is one order of magnitude greater than τ_2.
In both cases τ_{eq} is positive because must balance the clockwise movement of the manipulator.

$$\tau_{eq0} = \begin{bmatrix} 4.3641 \\ 1.3955 \end{bmatrix} \text{ [Nm]}$$

$$\tau_{eq} = \begin{bmatrix} 318.1937 \\ 38.6735 \end{bmatrix} \text{ [Nm]}$$



2.2: In the second part ϑ_1 is 90° and ϑ_2 is 45°.
When gravity does not act on the robot τ_1 is negative, while τ_2 is positive and very small. This is because one must take into account that the torque acting on the second link is not influenced by the forces and reaction moments due to the next link, as there are only two joints.
τ_{eq0} prevalent is negative because must balance the counter-clockwise movement of the manipulator.
When gravity acts on both centres of mass, the component with respect to the first link τ_1 and that with respect to the second link τ_2 are similar because, on the first link, the reaction forces of the second also act, but given the configuration of the robot the components of the first link along the Y_0 axis do not create moment, because perpendicular to the joint rotoidal.
τ_{eq} is negative because must balance the counter-clockwise movement of the manipulator.

$$\tau_{eq0} = \begin{bmatrix} -12.3727 \\ 0.2878 \end{bmatrix} \text{ [Nm]}$$

$$\tau_{eq} = \begin{bmatrix} -65.0917 \\ -52.4313 \end{bmatrix} \text{ [Nm]}$$

Exercise 3 – Inverse Dynamics of RP robot

In the third exercise, we must find the inverse dynamic joint toques for a manipulator with one rotoidal joint and one prismatic joint, first without considering gravity, then considering it acting along the Y_0 axis.

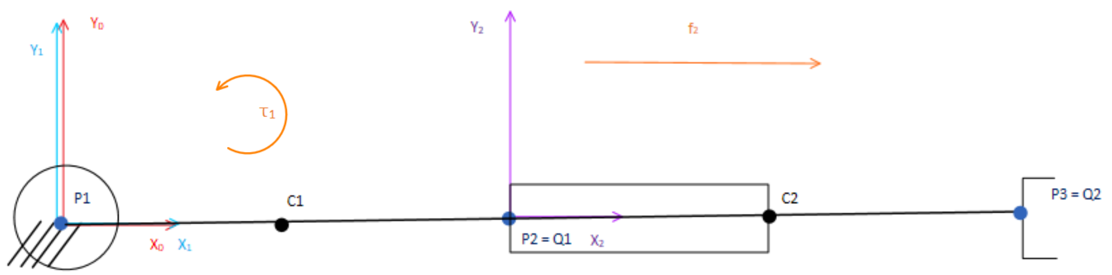
To perform the calculations, we assume that the links are prisms with a uniform distribution of mass, the COM of each being at the geometric centre.
At the beginning, the reference system 1 is considered coincident with the absolute system 0, while the reference system 2 is shifted with respect to the system 1.

P1 is positioned at the beginning of the first link, i.e., at a distance of 0m from the centre of the absolute reference system. P3 coincides with Q2 and is located at the end of the second link. Considering d_2, with a maximum value of 0.6m, the distance between Q1, at the end of the first link, and C2, link 2 is assumed to be 1.2m long. According to the considerations about prismatic joints, P2 varies as the elongation of the prismatic joint changes. In the first case P2 is found at 1.6m from the centre of reference system 1, in the second at 2m. This is because considering the centre of mass of the second link C2 at position 0.6m when the joint is not elongated, $q_2 = d_2 - 0.6$m

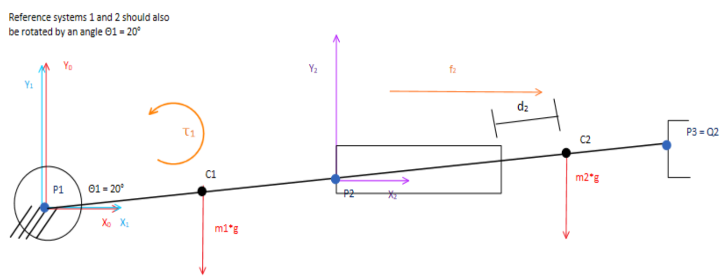
$l_1 = 1$m $\quad$ $l_2 = 1.2$m
$m_1 = 10$ Kg $\quad$ $m_2 = 6$ Kg
$I_1 = \text{diag}\,(0\ \text{Kgm}^2, 0.4\ \text{Kgm}^2, 0.4\ \text{Kgm}^2)$
$I_2 = \text{diag}\,(0\ \text{Kgm}^2, 0.3\ \text{Kgm}^2, 0.3\ \text{Kgm}^2)$

Using the homogeneous transformation matrices, the coordinates of each point were calculated with respect to the absolute reference systems.
T0_1 represents the transformation matrix of the reference system 1 with respect to the absolute reference system 0, therefore only a rotation around the z axis of one angle ϑ_1 is taken into account.
T1_2 is the transformation matrix of the reference system 2 with respect to the reference system 1, the rotation takes place around the z axis by a ϑ_2 angle, the translation varies according to the displacement of the prismatic joint.

Since there is a rotoidal joint and a prismatic joint, the k-axes vary. The system [i,j,k] of the rotoidal joint remains consistent with the reference system of the first link, so the k-axis can be written as a vector [0 0 1]'. As for the prismatic joint, the [i,j,k] system is rotated 90° around the y-axis with respect to the reference system of the second link, so the k-vector is defined as [1 0 0]'. Both axes are then rotated with respect to the robot configuration to be calculated with respect to the absolute reference system.



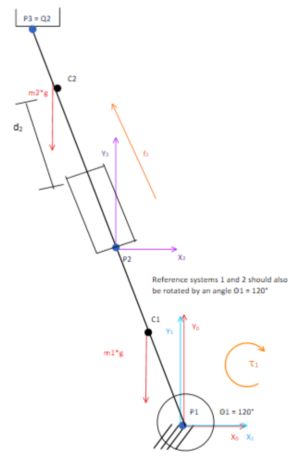
The points, in homogeneous coordinates, are given in [x,y,z] coordinates in order to calculate the vector product.
Inertia matrices are given with respect to the centre of mass of each link, calculated in the local reference system. To be considered in the absolute reference system, they must be rotated with respect to the robot configuration.



3.1: In the first part ϑ_1 is 20° and d_2 is 0.2m.
When gravity does not act on the robot τ_1 is positive, while f_2 is negative and very small. This is because one must take into account that the force acting on the second link is not influenced by the forces and reaction moments due to the next link, as there are only two joints.
τ_{eq0} prevalent is positive because must balance the clockwise movement of the manipulator.
When gravity acts on all the centres of mass, the component with respect to the first link τ_1 and the second link f_2 are very different because, on the first link the reaction forces of the second link also act, so with a gravitational force acting at an angle of 20° and a translation of 0.2m the component τ_1 is one order of magnitude greater than f_2.
τ_{eq} is positive because must balance the clockwise movement of the manipulator.

$$\tau_{eq0} = \begin{bmatrix} 4.0374 \\ -0.0245 \end{bmatrix} \begin{matrix} [Nm] \\ [N] \end{matrix}$$

$$\tau_{eq} = \begin{bmatrix} 217.9039 \\ 20.1068 \end{bmatrix} \begin{matrix} [Nm] \\ [N] \end{matrix}$$



3.1: In the second part ϑ_1 is 120° and d_2 is 0.6m.
When gravity does not act on the robot τ_1 and f_2 are negative.
τ_{eq0} is negative because must balance the counter-clockwise movement of the manipulator.
When gravity acts on all the centres of mass, the component with respect to the first link τ_1 and the second link f_2 are very different because, on the first link the reaction forces of the second link also act, so with a gravitational force acting at an angle of 120° and a translation of 0.6m the component τ_1 is one order of magnitude greater than f_2.
τ_{eq} prevalent is negative because must balance the counter-clockwise movement of the manipulator.

$$\tau_{eq0} = \begin{bmatrix} -4.1276 \\ -2.5560 \end{bmatrix} \begin{matrix} [Nm] \\ [N] \end{matrix}$$

$$\tau_{eq} = \begin{bmatrix} -129.6956 \\ 48.4183 \end{bmatrix} \begin{matrix} [Nm] \\ [N \end{matrix}$$

Exercise 4 – Inverse Dynamics of 3R robot

In the fourth exercise, we have to find the inverse dynamic joint toques for a manipulator with three rotoidal joints first without considering gravity, then considering it acting along the Z_0 axis.

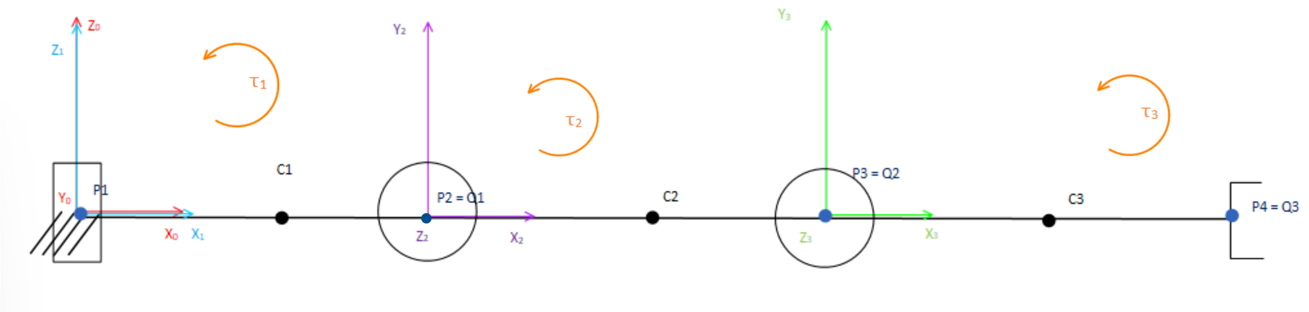
To perform the calculations, we assume that the links are prisms with a uniform distribution of mass, the COM of each being at the geometric centre.
At the beginning, the reference system 1 is considered coincident with the absolute system 0, the reference system 2 is shifted by 1m with respect to the system 1 and the reference system 3 is shifted by 0.8m with respect to the system 2.

P1 is located at the beginning of the first link and therefore at a distance of 0m from the centre of the absolute reference system, P2 coincides with Q1 and is located at the end of the first link, at a distance of 1m from the centre of the reference system 1, P3 coincides with Q2 and is located at a distance of 0.8m from the centre of the reference system of the second link and P4 coincides with Q3 and is located at the end of the third link.

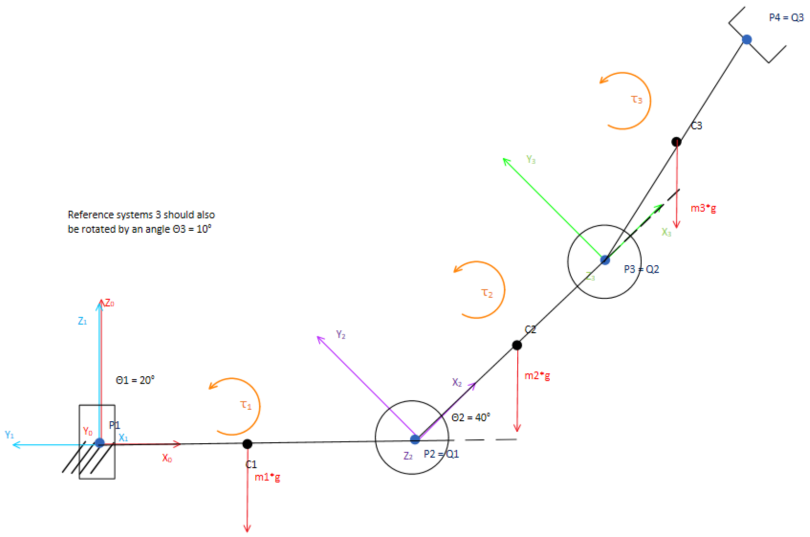
l_1 = 1m $\quad$ l_2 = 0.8m $\quad$ l_3 = 0.35m
m_1 = 22 Kg $\quad$ m_2 = 20 Kg $\quad$ m_3 = 6 Kg
I_1 = diag (0.2 Kgm^2, 0.2 Kgm^2, 0.8 Kgm^2)
I_2 = diag (0.2 Kgm^2, 0.2 Kgm^2, 0.8 Kgm^2)
I3 = diag (0.08 Kgm^2, 0.08 Kgm^2, 0.1 Kgm^2)

Using the homogeneous transformation matrices, the coordinates of each point were calculated with respect to the different reference systems.
T0_1 represents the transformation matrix of the reference system 1 with respect to the absolute reference system 0, therefore only a rotation around the z axis of one angle ϑ_1 is taken into account.
T1_2 is the transformation matrix of the reference system 2 with respect to the reference system 1, the rotation takes place around the z axis by a ϑ_2 angle. When transforming system 2 with respect to system 1, it must also be considered that system 2 must be rotated 90 degrees around the X_1 axis (T_x).
T2_3 is the transformation matrix of the reference system 2 with respect to the reference system 1, the rotation takes place around the z axis by a ϑ_3 angle.

The first rotoidal joint has the k-axis, calculated with respect to the absolute reference system, positioned in the same way, [0 0 1]', which rotates as a function of the angle ϑ_1 but does not change direction. The second and third rotoidal joints, on the other hand, are rotated with respect to the first one by 90 degrees around the X axis, so the k-axes [0 0 1]' must be turned according to the T_x transformation and then rotate as a function of the angles ϑ_2 and ϑ_3.



The points, in homogeneous coordinates, are given in [x,y,z] coordinates in order to calculate the vector product. Inertia matrices are given with respect to the centre of mass of each link, calculated in the local reference system. To be considered in the absolute reference system, they must be rotated with respect to the robot configuration.



4.1: In the first part ϑ_1 is 20°, ϑ_2 is 40° and ϑ_3 is 10°.

When gravity does not act on the robot, all components of τ are positive. The values do not vary much one from the others because the difference in link configuration is minimal.

When gravity acts on all the centres of mass, the component with respect to the second link τ_2 and the third link τ_3 are very different because, on the second link the reaction forces of the third link also act, so with a gravitational force acting at an angle of 50° and one of 40°, the component τ_2 is two orders of magnitude greater than τ_3. The gravitational force acts on link 1 and the reaction forces of link 2 are influenced by link 3. Since link 1 rotates about the Z axis, gravity, acting along the Z_0 axis, does not affect its configuration. The only forces actively acting on link 1 are those due to link 2 not perpendicular to the direction of the joint.

In both cases τ_{eq} is positive because must balance the clockwise movement of the manipulator.

$$\tau_{eq0} = \begin{bmatrix} 5.1128 \\ 1.3712 \\ 0.1532 \end{bmatrix} \text{ [Nm]}$$

$$\tau_{eq} = \begin{bmatrix} 5.1128 \\ 104.1829 \\ 6.7743 \end{bmatrix} \text{ [Nm]}$$

MATLAB script

To obtain the results, just run the MATLAB script. The program will first execute the exercises of point 2 (2R robot) asking 2 times which point of the exercise you want to solve. Then, following the same iteration, it will execute the exercises of point 3 (RP robot) and 4 (3R robot).