Robot Dynamics & Control – A.A. 2021/2022

Assignment 1: Static equilibrium of serial manipulators

This assignment covers the basics of force/torque equilibrium for a robotic manipulator, a fundamental concept for dynamic control. You will be using with the following relation:

$$\boldsymbol{\tau}_{eq} = \boldsymbol{J}^T \boldsymbol{W}_{ext} \quad (1)$$

Where $\boldsymbol{W}_{ext}$ represents a cartesian wrench (torque, force) applied to an arbitrary position on the robot body, $\boldsymbol{J}^T$ is the transpose of the robot Jacobian and $\boldsymbol{\tau}$ is the vector of generalized actuator forces.

You are also asked to get familiar with basic parameter collection from CAD models, which is a fundamental step for the practical design of dynamic control algorithms.

Instructions for the CAD model implementation and parameter collection are described in the ***'CAD_parameters.pptx'*** file available on Teams.

Evaluation rules

You have to upload on Teams a '.zip' file named as **'SurnameName_Assignment_1.zip'** which contains:

- All the CAD files used to extract the inertial parameters
- A **'.pdf'** report (**'SurnameName_Assignment_1_report.pdf'**) with the motivated answers for each exercise and eventually drawings/diagrams to better motivate the solution. (**Note:** Any answer provided without a full motivation will not be considered for evaluation. All the steps towards the solution must be justified.).
- The MATLAB file (**'SurnameName_Assignment_1_solution.m'**) with the numerical implementations of the exercises.

The assignment is individual, so each student must deliver his/her .zip archive.

Exercise 1 – RR robot

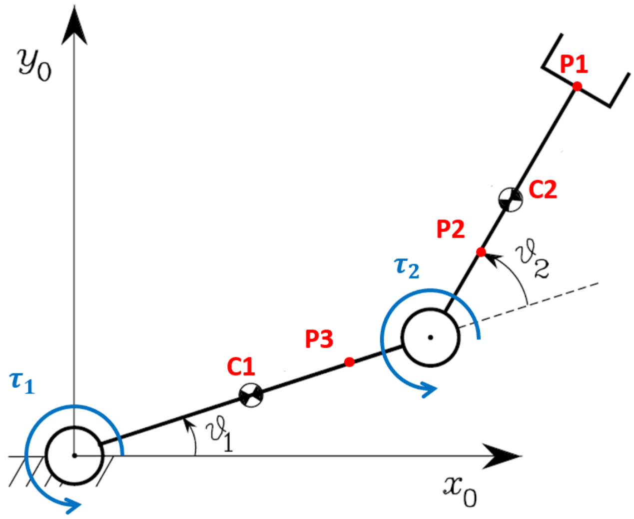

Figure 1: RR Planar Manipulator

Compute the robot generalized joint forces $\boldsymbol{\tau_1}, \boldsymbol{\tau_2}$ from Eq. (1), required to balance external forces acting on the robot under the following conditions, also remarked in Fig. 1. P1 is located at the end of the second link, P2 is located at -20 cm from C2 and P3 at 40 cm from C1. The link lengths and COM positions should be extracted from the CAD model.

Motivate each answer.

1) $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = -\frac{\pi}{2}$, *g = [0, -9.81]* acting on the COM of both links. Compute $\boldsymbol{\tau}_{\text{eq}}$.
2) $\theta_1 = 0$ and $\theta_2 = \frac{\pi}{2}$, *g = [0, -9.81]* acting on the COM of both links. Compute $\boldsymbol{\tau}_{\text{eq}}$.
3) $\theta_1 = \frac{\pi}{6}$ and $\theta_2 = \frac{\pi}{3}$, *no gravity, a pure force* F_{ext}*= [-0.7, -0.5]N acting on* P_1. Compute the equilibrium torques. What if F_{ext} acts on P_2? Compute $\boldsymbol{\tau}_{\text{eq}}$ also in this case.
4) $\theta_1 = \frac{\pi}{6}$ and $\theta_2 = \frac{\pi}{3}$, *no gravity, a pure force* F_{ext}*= [1.5, -0.3]N acting on* P_3 and a torque $\boldsymbol{\tau}_{\text{ext}}$= 1.2 Nm acting about an axis passing through P_1. Compute $\boldsymbol{\tau}_{\text{eq}}$.
5) $\theta_1 = \frac{\pi}{6}$ and $\theta_2 = -\frac{\pi}{3}$, *g = [0, -9.81]* acting on the COM of both links, *a pure force* $F_{ext,3}$*= [-0.4, 1.2]N acting on* P_3 *and a pure force* $F_{ext,2}$*= [1.2, -0.2]N acting on* P_2. Compute the equilibrium torques.

Exercise 2 – PR robot

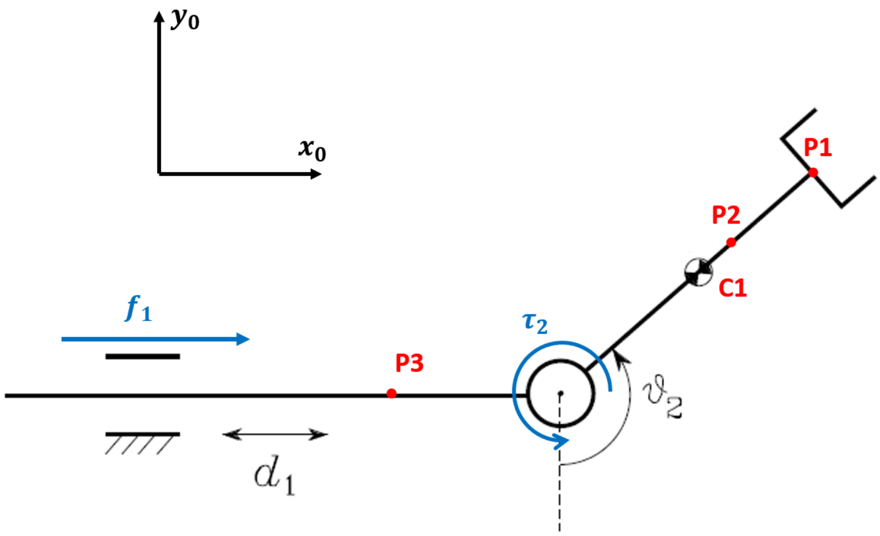

Figure 2: PR Planar Manipulator

Compute the robot generalized joint forces $\boldsymbol{f_1}$, $\boldsymbol{\tau_2}$ from Eq. (1), required to balance external forces acting on the robot under the following conditions, also remarked in Fig. 2. P1 is located at the end of the second link, P2 is located at 15 cm from C2 and P3 at -20 cm from the axis of the R joint. The link lengths and COM positions should be extracted from the CAD model.

Motivate each answer.

1) $\theta_2 = \frac{\pi}{4}$, *g = [0, -9.81]* acting on C1. Compute $\boldsymbol{\tau}_{eq}$.
2) $\theta_2 = \frac{\pi}{2}$, *g = [0, -9.81]* acting on C1. Compute $\boldsymbol{\tau}_{eq}$.
3) $\theta_2 = \frac{\pi}{4}$, *no gravity, a pure force* F_{ext}= *[-0.8, -0.8]N acting on* P_1. Compute $\boldsymbol{\tau}_{eq}$.
4) $\theta_2 = \frac{\pi}{4}$, no gravity, a pure force F_{ext}= [-0.8, -0.2]N acting on P_2, and a torque $\boldsymbol{\tau}_{ext}$= 0.5 Nm acting about an axis passing through P1. Compute $\boldsymbol{\tau}_{eq}$.
5) $\theta_2 = \frac{\pi}{4}$, *g = [0, -9.81]* acting on C1, *a pure force* $F_{ext,1}$= *[0.5, -0.6]N acting on* P_1 *and* $F_{ext,2}$= *[1.0, -0.4]N acting on* P_2. *Compute* $\boldsymbol{\tau}_{eq}$.