

Technical Report

November 27, 2017

Outline

1 Diffusion on networks

1.1 Motivation

Diffusion on networks is one of the dynamic processes that occur on networks. Diffusion processes on networks are used to develop models for the study of real-world processes such as the spread of infections with in a group of people, spread of information in a social network, studying failures in power supply on the grid with in an area, etc.

1.2 Ideas

- a) Diffusion on networks through direct interactions
- b) Accounting for long-range interaction in diffusion process on network
- c) k-path Laplacians, that is, the generalised Laplacian matrices. Considerations of Mellin and Laplace transforms of the k-path Laplacians to account for longrange interactions
- d) Illustration of Diffusion of heat on a lattice. First, considering direct interactions only and then accounting both direct and long-range interactions.

2 Image Segmentation

2.1 Motivation

Image Segmentation is the problem of localising regions of an image relative to content. It aids in the extraction of objects of interest from an image. Image segmentation has applications in medical field, engineering field, computer science among others. A number of segmentation techniques have been developed based on graph theory concepts which include intelligent scissors, Normalised cut algorithm, random walker algorithm, etc. Our interest is in the segmentation of images using the random walk process on graph as put forward by Grady.

2.2 Ideas

- a) To extend the idea of image segmentation using random walks on networks by accounting for long-range links in the network. Our aim is to ascertain whether a better image segmentation can be obtained on including long-range links.

3 Systemic Risk and Contagion in Financial Systems

3.1 Motivation

The financial crisis of 2008 stimulated intense research in the area of financial systems. The use of complex networks in the study of financial systems is one of the avenues that have been used by a number of research in uncovering the root causes of the crisis, which financial institutions sparked off the cascaded failure leading to system break down, the impact of the network structure, strategies to either prevent the re-occurrence or to guarantee minimal impact of the crisis etc.

3.2 Ideas

- a) The concept of "too interconnected to fail", i.e. which measure (centrality) can we use to identify nodes that are considered too interconnected to fail.
- b) Can the centrality based on generalised degree centrality capture the node which is too interconnected to fail?
- c) How can we apply the concept of long-range interactions to financial networks? What does the long-range interactions mean?

4 Laplacian centrality of an edge

4.1 Motivation

Centrality measures in networks have proved to be relevant tools in network analysis. They are indicators of the 'importance' of nodes and edges in the networks. Though most work has been geared towards the study of importance of nodes (i.e. degree, closeness, betweenness, subgraph, eigenvector, Laplacian centralities, etc.), interest in the study of edge centralities is now gaining ground with prominent works of Girvan and Newman. Some of the known edge centralities include edge-betweenness, k-path edge centrality, among others. The motivation for the introduction of edge centrality measures lies in real-world applications in a wide range of context such as in identifying communities in networks, identifying strong relationships among people in social networks, etc.

4.2 Ideas

- a) Extend the concept of Laplacian centrality of nodes to edges, obtain the graph theoretical description of the edge centrality.
- b) What is the relation between Laplacian centrality of the node and that of an edge?
- c) Can we apply the concept of edge centrality to the minimum cut problem in graphs?
- d) Yang et.al, in his work "Air traffic network optimization via Laplacian energy maximization" highlights the change in Laplacian energy due to edge removal as a promising fair measure of network robustness. We consider comparing this measure with other measures of network robustness.

- e) How can we apply the edge laplacian centrality to the concept of robustness in targeted edge attacks
- f) Application of laplacian centrality of an edge to edge consensus, that is, addition or removal of edges so the relative difference in centrality of edges is small.
- g) Application to electric flow in circuits

5 Minimum-maximum cut

5.1 Motivation

The minimum cut of a network is the minimum weight or number of edges whose removal results into a disconnected network. The max-flow min-cut theorem states that in a flow network, the maximum amount of flow passing from the source to the sink is equal to the total weight of the edges in the minimum cut, i.e. the smallest total weight of the edges which if removed would disconnect the source from the sink. This theorem has applications in image segmentation, and in optimisation problems such as project selection by guiding the decision regarding the purchase of machines that can be used in a number of projects so as to maximise profits of the company.

5.2 Ideas

- a) Investigate whether there exists a unique feature or behaviour for the nodes around the minimum cut.
- b) Whether the minimum cut edges can be obtained by their Laplacian centrality.
- c) relation between Laplacian energy and the minimum cut of a network.

6 Noise on networks

6.1 Motivation

In systems made of interconnections of sensors, it's observed that the reported results from the systems are normally subject to errors due to noise. In order to obtain accurate results, we need to ascertain the effect of noise on the reported results. Since these systems can be represented as networks, we can then formulate this problem as one of studying the impact of noise on sensor network.

6.2 Ideas

- a) How do we define noise on networks? Could the noise be applied to nodes or edges?
- b) What could be the effect of noise on the structure of a network
- c) What insights do we draw from simulation of noise on networks by considering additive White Gaussian noise for instance?

7 Communicability in Networks

7.1 Motivation

Communicability in networks is a concept introduced by Estrada & Hatano based on the motivation that most real networks, communication between nodes does not necessarily follow shortest paths only

as normally assumed. Evidence shows that communication can occur along any paths which are not shortest path. Communicability between a pair of nodes accounts for all possible walks through which a given pair of nodes can communicate. Some of the applications of this concept include community detection in networks. We are looking forward to research possibilities in this area which include the following:

7.2 Ideas

- a) Consider the impact of long range interactions to communicability in networks
- b) Other possible real-world application of communicability in networks