

Random walks with long-range interactions

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July 11, 2018

1 Random walks on graphs

Random walks are a fundamental concept as they are used to model dynamics of systems in numerous fields ranging from physics, biology, engineering, economics and mathematics. Recently, random walks on networks have been extensively studied [[Newman\(2003\)](#), [Estrada\(2011\)](#)] and various applications of this concept have been explored. For example modeling diffusion on networks, image segmentation [[Grady\(2006\)](#)], measuring centrality of nodes in networks [[Chung\(2007\)](#)], community detection [[Pons and Latapy\(2005\)](#)], and many others.

1.1 Simple random walk

Let $G = (V, E)$ be a simple undirected graph with V as a set of nodes and E as the set of edges. Suppose v_0 is the initial node at which the random walker sets off from, at each time step the random walker moves along the edges to the nearest-neighbour nodes with a uniform probability $p_{u,v}$ where u is the node at which the random walker is positioned and v is any of the nodes to which u is connected by an edge. The resultant sequence of nodes and edges traversed is referred to as random walk. The probability of a random walker moving from one node to another is given by the transition matrix whose entries are given by

$$\mathbf{P}_{uv} = \begin{cases} \frac{1}{k_u} & \text{if } u \text{ is adjacent to } v \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where k_u is the degree of node u .

1.2 Random walk with long-range interactions

Here, we extend the notion of random walk on a network by considering the possibility that at every step, the random walker can not only move to the nearest nodes via an edge but can as well hop to the any other node which is connected to the current node through a path in the network. The maximum length of any hop is the diameter of the network. The transition matrix associated with this random walk is defined as

$$\mathbf{P}_G(uv) = \begin{cases} \frac{1}{\sigma_u} & \text{if } d_{u,v} = k \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where σ_u is the k -path degree for node u (definition ??).

References

- [Chung(2007)] F. Chung. The heat kernel as the pagerank of a graph. *Proceedings of the National Academy of Sciences*, 104(50):19735–19740, 2007.
- [Estrada(2011)] E. Estrada. *The structure of complex networks: theory and applications*. OUP Oxford, 2011.
- [Grady(2006)] L. Grady. Random walks for image segmentation. *IEEE transactions on pattern analysis and machine intelligence*, 28(11):1768–1783, 2006.
- [Newman(2003)] M. E. Newman. The structure and function of complex networks. *SIAM review*, 45(2):167–256, 2003.
- [Page et al.(1999)Page, Brin, Motwani, and Winograd] L. Page, S. Brin, R. Motwani, and T. Winograd. The pagerank citation ranking: Bringing order to the web. Technical report, Stanford InfoLab, 1999.
- [Pons and Latapy(2005)] P. Pons and M. Latapy. Computing communities in large networks using random walks. In *International symposium on computer and information sciences*, pages 284–293. Springer, 2005.