Problem Set #[3]

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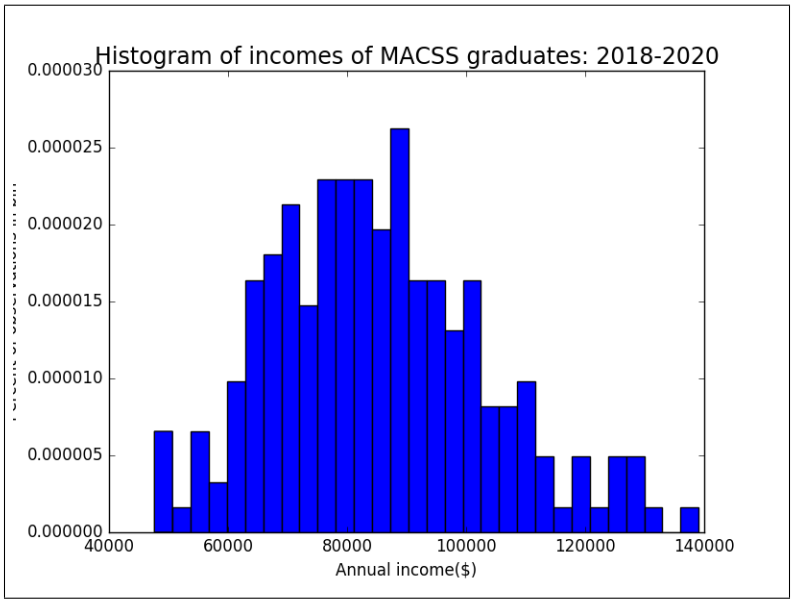
1. Some income data, lognormal distribution, and GMM.

Part (a). Plot a histogram of percentages of the income.txt data with 30 bins.

Answer: Figure 1 is the histogram of percentages of the income.txt data with 30 bins.

Figure 1: Histogram of incomes of MACSS graduates: 2018-2020



Part (b). Plot your estimated lognormal PDF against the histogram from part (a). Report the value of your GMM criterion function at the estimated parameter values. Report and compare your two data moments against your two model moments at the estimated parameter values.

Answer:

Figure 2 is the estimated lognormal PDF against the histogram from part (a). The value of GMM criterion function at the estimated parameter values is 6.63680004e-13. And the data moments are $\mu = 85276.8236$, $\sigma = 17992.5421$. And the model moments are $\mu = 85276.7551$, $\sigma = 17992.5433$.

Part (c). Perform the two-step GMM estimator by using your estimates from part (b) with two moments. Report your estimates as well as the criterion function value at these estimates. Plot your estimated lognormal PDF against the histogram from part (a) and the estimated PDF from part (b).

Answer: Figure 3 is the new estimated lognormal PDF against the histogram from part (a) and part (b).

Figure 2: Incomes of MACSS graduates: 2018-2020

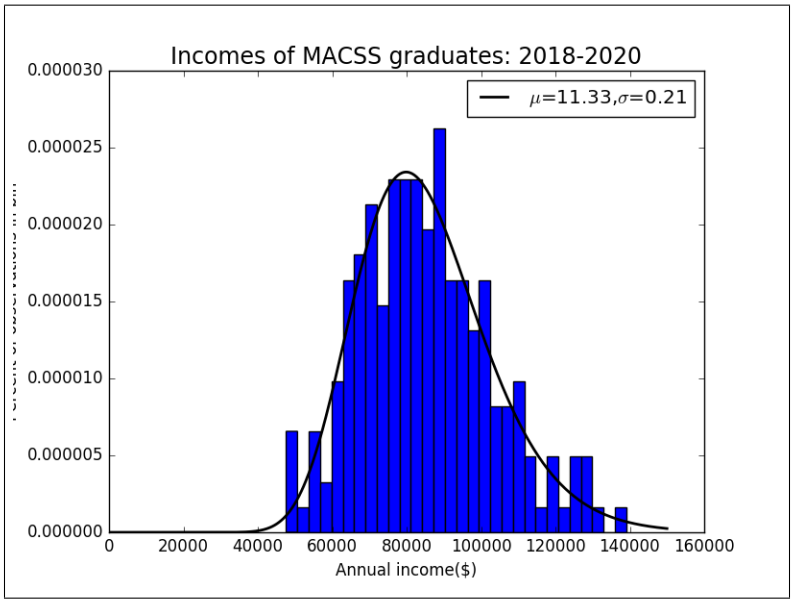
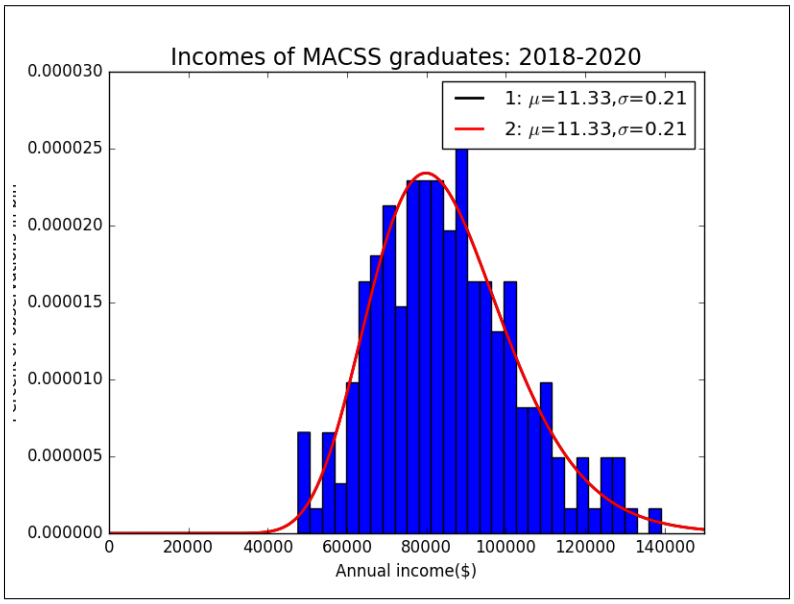


Figure 3: Incomes of MACSS graduates: 2018-2020

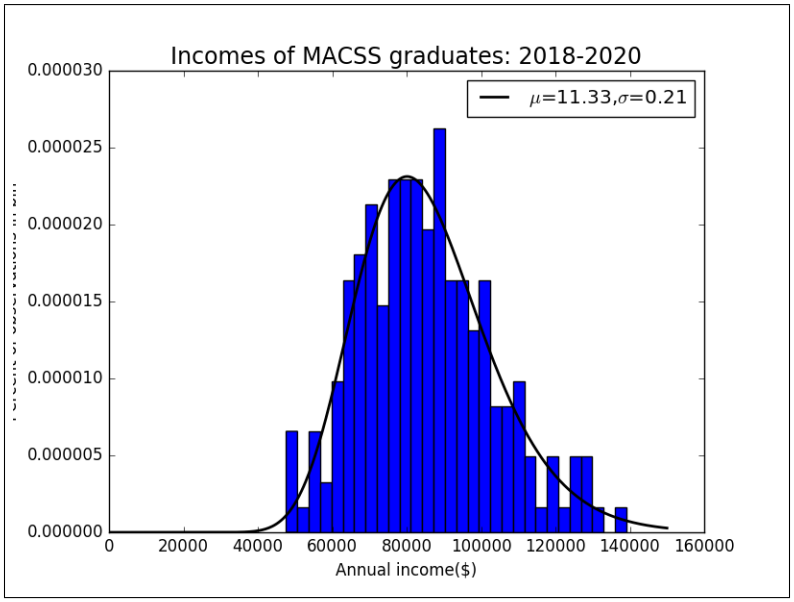


The value of GMM criterion function at the estimated parameter values is 4.62436995e-05. And the data moments are $\mu = 85276.8236$, $\sigma = 17992.5421$. And the model moments are $\mu = 85276.8333$, $\sigma = 17992.5480$

Part (d). Now estimate the lognormal PDF to fit the data by GMM using different moments. Plot your estimated lognormal PDF against the histogram from part (a).

Report the value of your GMM criterion function at the estimated parameter values. Report and compare your three data moments against your three model moments at the estimated parameter values.
Answer: Figure 4 is the estimated lognormal PDF with different moments against the histogram from part (a).

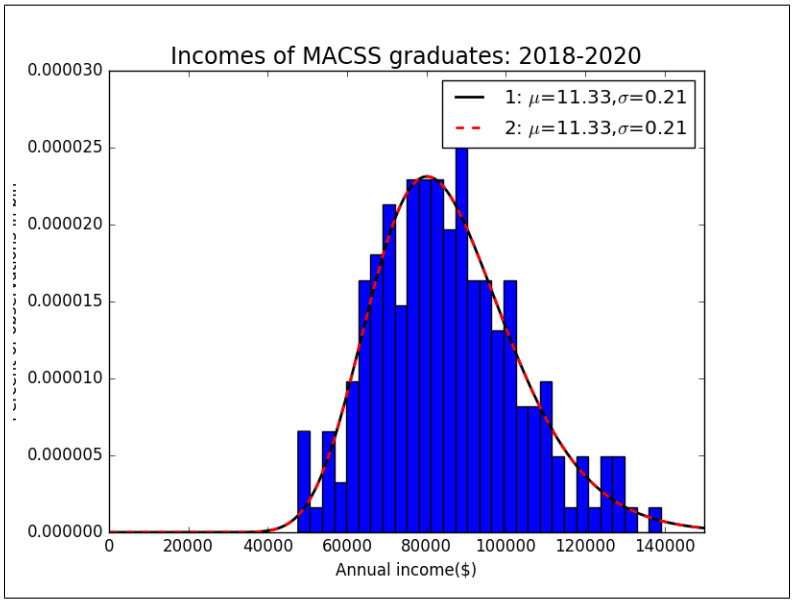
Figure 4: Incomes of MACSS graduates: 2018-2020



The value of GMM criterion function at the estimated parameter values is 2.44946079e-11. The data moments are 0.3, 0.5 and 0.2, and the model moments are (0.30000000, 0.50000000, 0.199999991), which are very close.
Part (e). Perform the two-step GMM estimator by using your estimates from part (d). Plot your estimated log- normal PDF against the histogram from part (a) and the estimated PDF from part (d).
Answer: Figure 5 is the estimated lognormal PDF with different moments against the histogram from part (a) and part (d).

The value of GMM criterion function at the estimated parameter values is 1.69712715e-11. The data moments are 0.3, 0.5 and 0.2, and the model moments are (0.30000000, 0.50000000, 0.199999992), which are very close.
Part (f). Which of the four estimations from parts (b), (c), (d), and (e) fits the data best? Justify your answer.
Answer: As we can see, the criterion function values from the four estimations are all below $5 * 10^{-}5$. However, from the comparison of criterion values, we can see that the estimation from part (b), (d), (e) would have a better fit for this model. Consider the comparison of model moments, we can see that part (e) would do better than part (d). Furthermore, from the figure, we can see that the plot from part (e) fits the

Figure 5: Incomes of MACSS graduates: 2018-2020



data better compared with part (b) estimates in the middle part. Then I would say that estimates from part (e) fits the data best.

2. Linear regression and GMM.

Part (a). Estimate the parameters of the model by GMM. Report your estimates and report the value of your GMM criterion function.

Answer: By GMM, we have $\beta_0 = 0.2516$, $\beta_1 = 0.0129$, $\beta_2 = 0.4005$, $\beta_3 = -0.00999$. The value of GMM criterion function is 0.0018.