Problem Set #3

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Python Version: 3.5.2

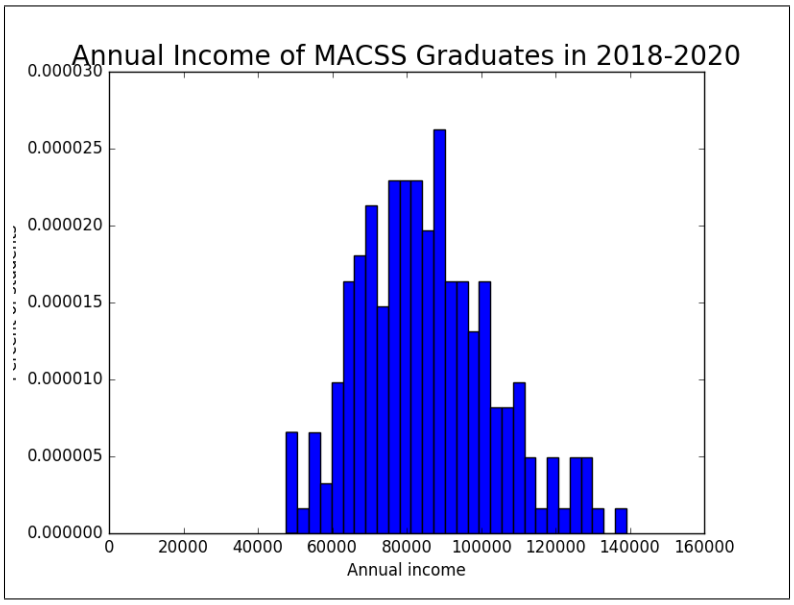
Problem 1

The minimization processes in the 4 estimations in (b) to (e) all succeeded. For acquiring converged results from the algorithms, the estimation settings are as follow:
(b). error simple mode = False; minimization method = L-BFGS-B.
(c). error simple mode = False; minimization method = TNC.
(d). error simple mode = True; minimization method = L-BFGS-B.
(e). error simple mode = True; minimization method = TNC.

Part (a). histogram

A histogram of annual incomes of students who graduated in 2018, 2019, and 2020 from the University of Chicago M.A. Program in Computational Social Science.

Figure 1:



Part (b). One step GMM with mean and std

One step GMM is estimated with mean and standard deviation as moments and identity weighting matrix. The initial guess of μ is 11 and σ is 0.2.
The resulted criterion value is 8.97474397e-15. The estimated parameters are as follow:

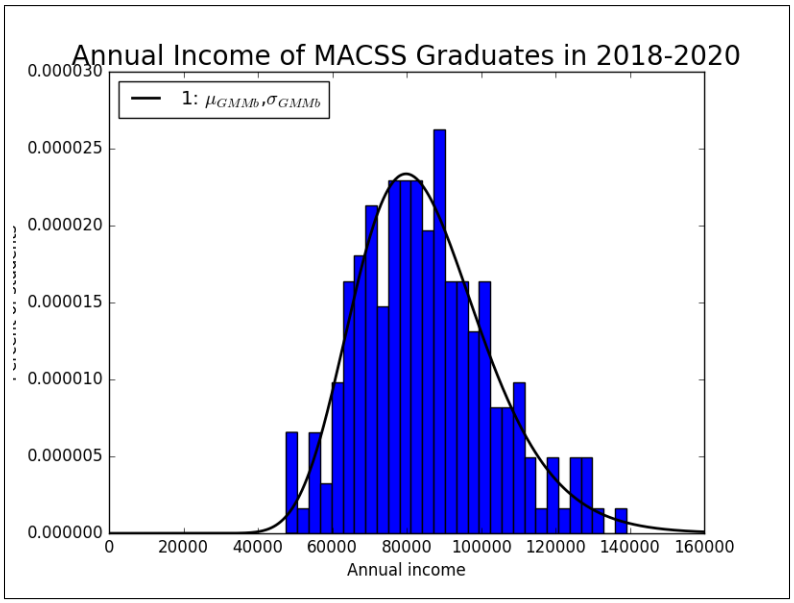
$$\mu_{GMM1} = 11.3317738625$$

$$\sigma_{GMM1} = 0.209209064254$$

moments	mean	var
data	85276.82360625808	325358364.0497777
model	85276.8309999	325358351.628
data - model	-0.00739366149355	12.4213916063

A lognormal pdf with estimated parameters is plotted.

Figure 2:



Part (c). Two step GMM with mean and std

Two step GMM is estimated with mean and standard deviation as moments and weighting matrix derived from (b). The initial guess of μ and σ is the estimated results from (b).
The resulted criterion value is 6.98194849. The estimated parameters are as follow:

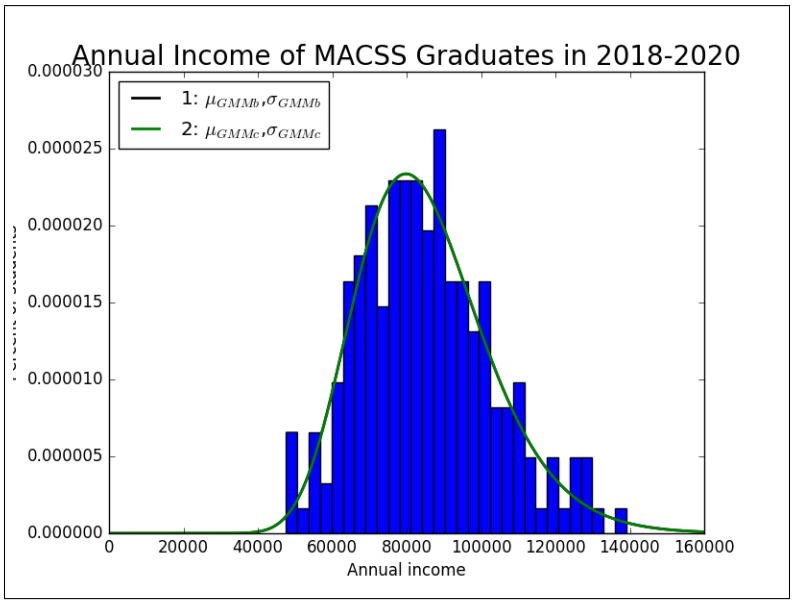
$$\mu_{GMM2} = 11.3317737968$$

$$\sigma_{GMM2} = 0.209209082101$$

moments	mean	var
data	85276.82360625808	325358364.0497777
model	85276.8257185	325358368.061
data - model	-0.00211221084464	-4.01102161407

Lognormal pdfs with estimated parameters in previous and this questions are plotted.

Figure 3:



Part (d). One step GMM with three proportion moments

One step GMM is estimated with percent of individuals who earn less than $75,000, between $75,000 and $100,000, and more than $100,000 as moments and identity weighting matrix. The initial guess of μ is 11 and σ is 0.2.
The resulted criterion value is 6.36574754e-15. The estimated parameters are as follow:

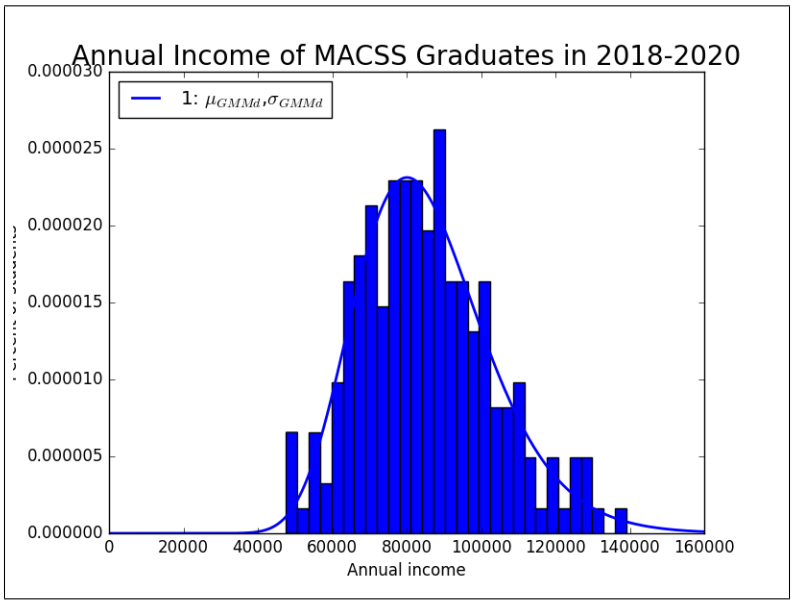
$$\mu_{GMM1_3} = 11.3356813687$$

$$\sigma_{GMM1_3} = 0.210598464642$$

moments	less than 75,000	75,000-100,000	more than 100,000
data	0.3	0.5	0.2
model	0.2999999448933023	0.4999999974649346	0.2000000576417631
data - model	5.510669770503185e-08	2.535065379838386e-09	-5.7641763084870234e-08

A lognormal pdf with estimated parameters is plotted.

Figure 4:



Part (e). Two step GMM with three proportion moments

Two step GMM is estimated with percent of individuals who earn less than $75,000, between $75,000 and $100,000, and more than $100,000 as moments and weighting matrix derived from (d). The initial guess of μ and σ is the estimated results from (d).
The resulted criterion value is 2.6828065. The estimated parameters are as follow:

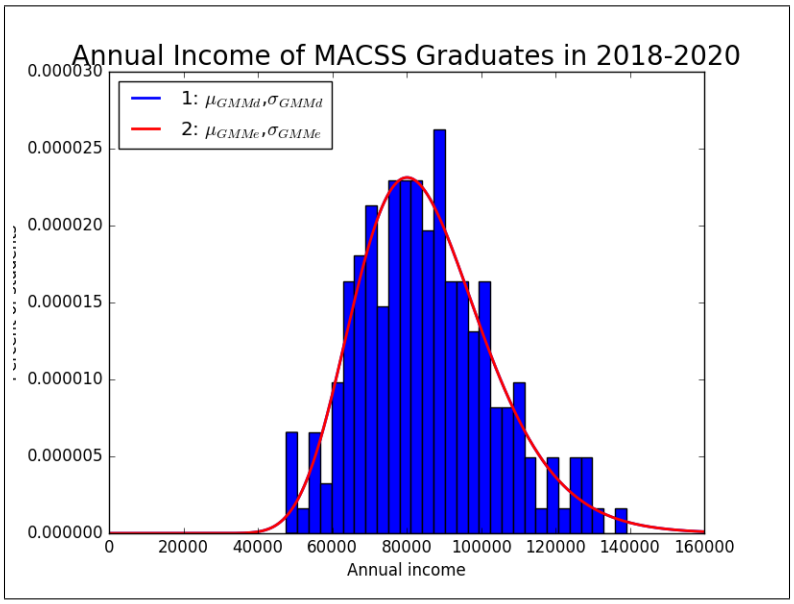
$$\mu_{GMM2_3} = 11.335681335$$

$$\sigma_{GMM2_3} = 0.210598464603$$

moments	less than 75,000	75,000-100,000	more than 100,000
data	0.3	0.5	0.2
model	0.3000000005650838	0.4999999866894511	0.20000001274546508
data - model	-5.650838130755176e-10	1.3310548885314688e-08	-1.274546507223917e-08

Lognormal pdfs with estimated parameters in previous and this questions are plotted.

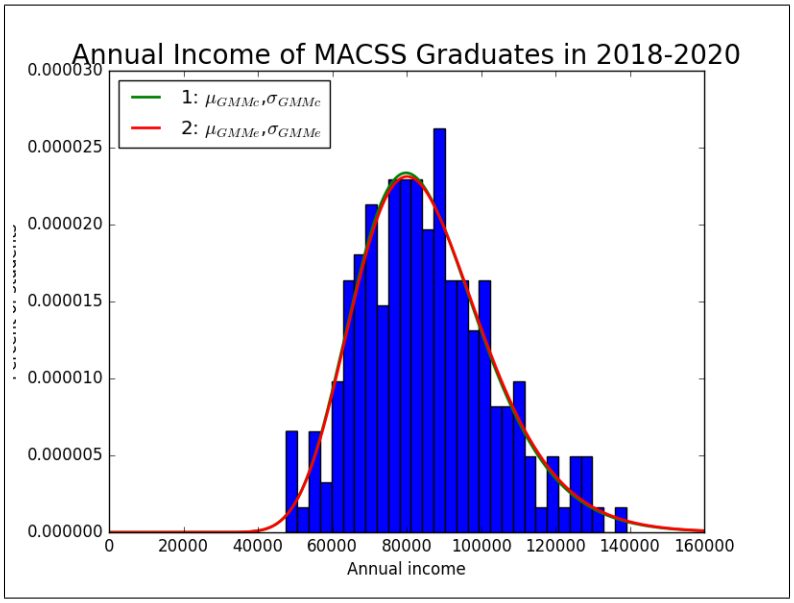
Figure 5:



Part (f). Estimation comparison

From the following reasoning, I deem the estimation from (e) fits the data best.
The pdf from (b) and (c) estimations are visually indistinguishable in the plot. Though, comparing the difference of data moment and model moment of these two estimations, we can still find that the two-step estimation in (c) provides a slightly better result than (b), while the difference of data moment and model moment in (c) is generally smaller than (b).
Similarly, the pdf from (d) and (e) estimations are also visually indistinguishable in the plot. Though, comparing the difference of data moment and model moment of these two estimations, we can still find that the two-step estimation in (e) provides a slightly better result than (d), while the difference of data moment and model moment in (e) is generally smaller than (d).
Visually comparing estimations from (c) and (e), one can tell that (c) covers more data in the middle range while (e) more in the tails. Based on the real data distribution, I would say estimation from (e) is slightly better than (c) since it covers more and has better description around the range 100,000 to 140,000, where the two pdfs generally can not describe the real data very well and I believe should be better addressed.

Figure 6:



Note:
The above estimations in (c) and (e) are based on the parameters estimated in (b) and (d) respectively as initial guess.
I also tried applying $\mu = 11$ and $\sigma = 0.2$ as the initial guess in (c) and (e); this yielded worse fits as the images shown below and thus are not considered in the answers of (c), (e), and (f). The code applying constant initial guess values to (c) and (e) can be accessed in "PS3_with_constant_initial_guess.py". The corresponding estimation results and criterion values are also computed when running the mentioned Python file.

Figure 7:

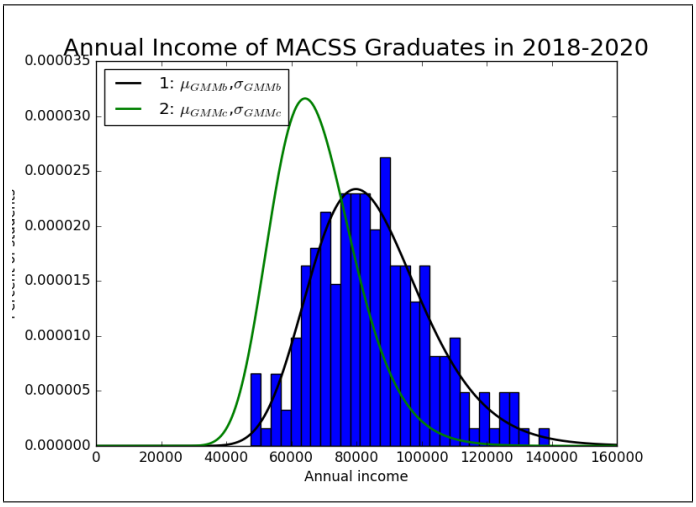
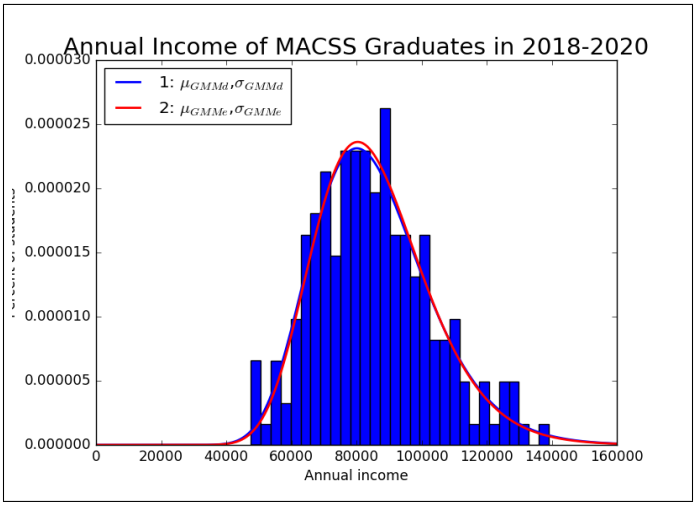


Figure 8:



Problem 2
Part (a). Linear regression and GMM

GMM is estimated with real and estimated sick data as moments and identity weighting matrix. The initial guesses for β_0, β_1, β_2, β_3 are 0, 0, 0, 1.
The resulted criterion value is 0.0018212898429. The estimated parameters are as follow:

$$\beta_{0,GMM} = 0.251644806982$$
$$\beta_{1,GMM} = 0.0129334340231$$
$$\beta_{2,GMM} = 0.400501320535$$
$$\beta_{3,GMM} = -0.00999168732927$$