Problem Set #4

MACS 30100, Dr. Evans
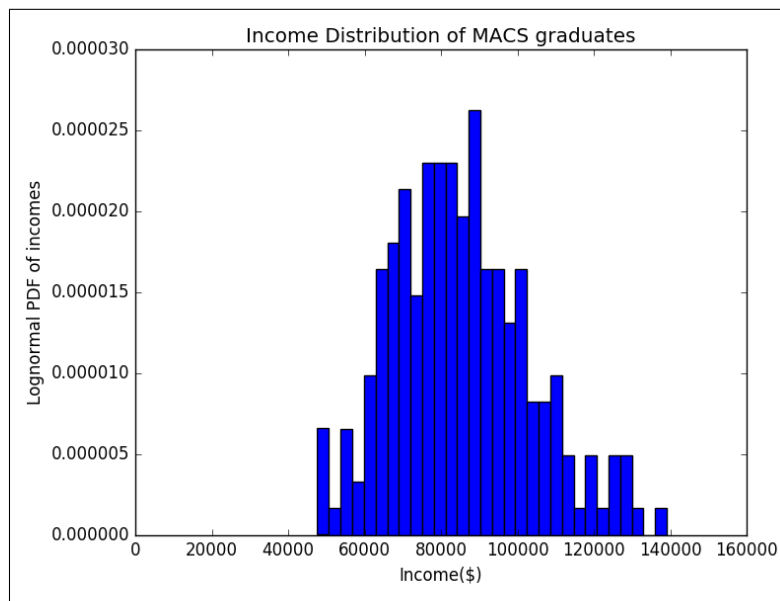
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Problem 1 Lognormal Distribution and SMM
Part (a).
The histogram of percentages of the annual income of MACS graduates is plotted in
Figure 1.

Figure 1:

Part (b).
The LN_pdf function I wrote takes as inputs a length N vector or an N by S matrix
`xvals`, the mean `mu` and the standard deviation `sigma` of the normal distribution on
which the lognormal is based, and then it returns `pdf_vals`, the lognormal PDF val-
ues of `xvals`. Inputting the matrix `xvals = np.array([[200.0, 270.0], [180.0,
195.5]])` with parameter values $\mu = 5.0$ and $\sigma = 1.0$, the function returns an array
of `pdf_vals` as below:
$[[0.0019079, 0.00123533], [0.00217547, 0.0019646]]$

Part (c).
Using the average income and standard deviation of income as two moments and
creating S = 300 simulations, each with N = 200 observation on income, I estimated
the parameters by SMM method with an identity matrix $\hat{W}$. The value of the SMM
criterion is 2.6075098e-14, and the estimated parameters are $\mu = 11.3306372358$ and
$\sigma = 0.209229329793$. (**Note**: the criterion value may change heavily, depending on the
inital guess of parameters inputting into the function and the minimization method

choosed. In PS4 I used `mu_init` $= 5.0$ and `sigma_init` $= 1.0$, and the minimization method = L-BFGS-B)

While the two data moments are average income \$85276.82361 and standard deviation \$17992.54213, the two model moments are \$85276.827303 and \$17992.539329, respectively. The SMM estimated lognormal PDF is plotted against the histogram from part (a) in Figure 2.

Figure 2:
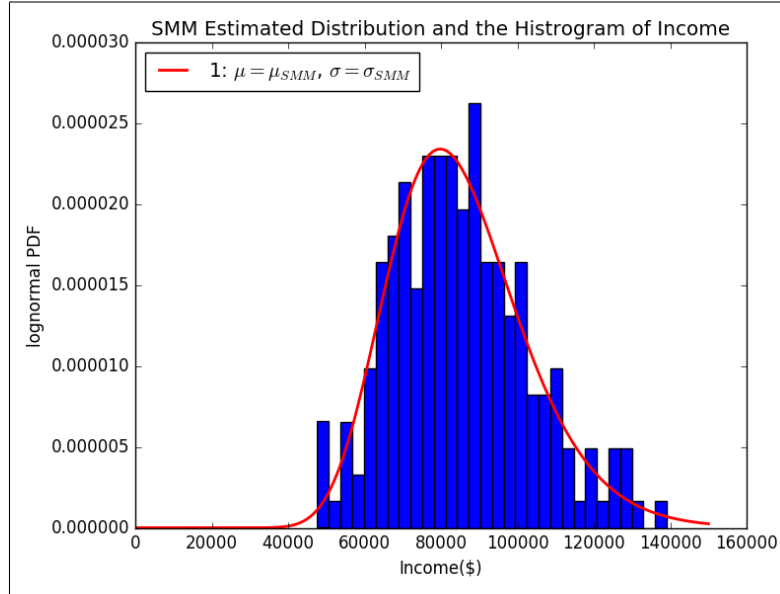
Part (d).
Performing the two-step SMM estimator by using the estimates from part (b) and replacing the indentity martix with an optimal weighting matrix $\hat{W}_2 step$, we can get a SMM criterion function value of 0.0009667949, and the estimated parameters are $\mu = 11.3306373072$ and $\sigma = 0.20922935278$. (minimization method = L-BFGS-B)

While the two data moments are average income \$85276.82361 and standard deviation \$17992.54213, the two model moments are \$85276.83381719 and \$17992.54272184 respectively. The two-step SMM estimated lognormal PDF is plotted against the PDF from part (b) and the histogram from part (a) in Figure 3.

Figure 3: