

Problem Set #2

MACSS 30100

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Problem 1

(a) The following graph is the histogram for the income of the MACSS cohort:

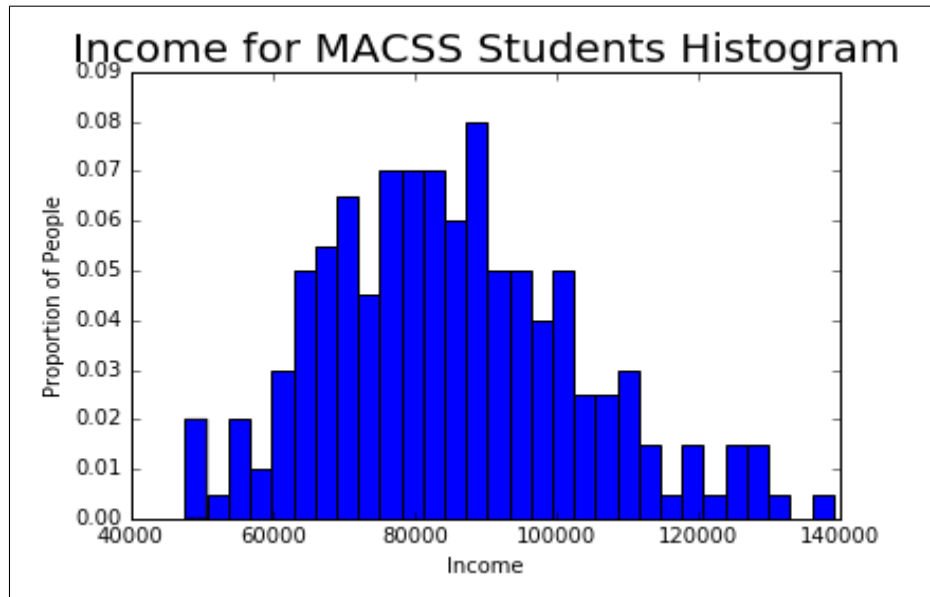


Figure 1: Histogram of Income of MACSS Students

(b) The value of the log likelihood value is -15829.239. The log normal pdf is as following.

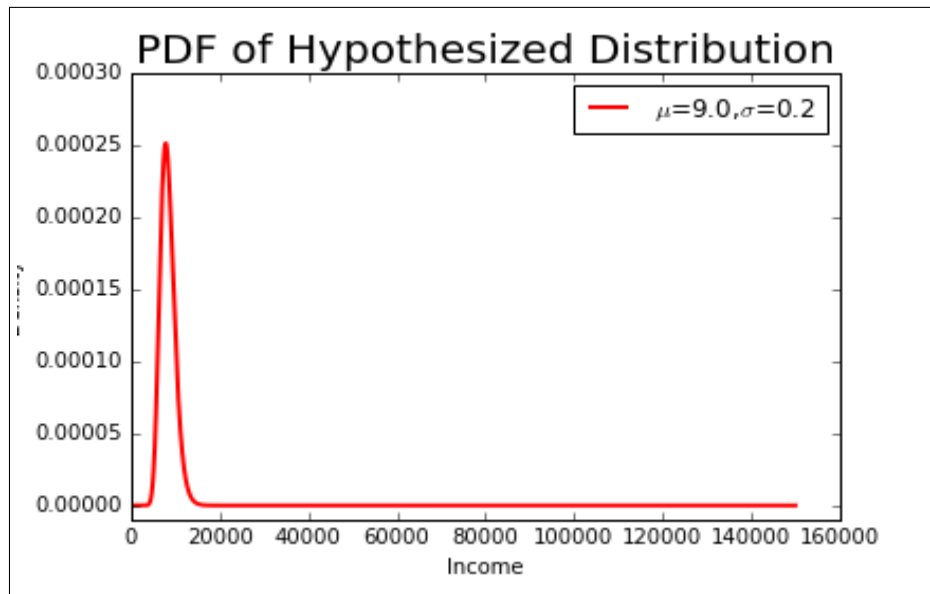


Figure 2: Hypothesized PDF Distribution

- (c) After the MLE estimation we have: $\mu_{MLE} = 11.33$; $\sigma_{MLE} = 0.21$. The value of the log likelihood function is -2239.53; the variance and covariance matrix is following:

$$\begin{bmatrix} 2.18801e-04 & 6.47943e-06 \\ 6.47943e-06 & 1.19570e-04 \end{bmatrix}$$

The following is the approximate value:

$$\begin{bmatrix} 0.00022 & 0.00001 \\ 0.00001 & 0.00012 \end{bmatrix}$$

The plotted graph is as following:

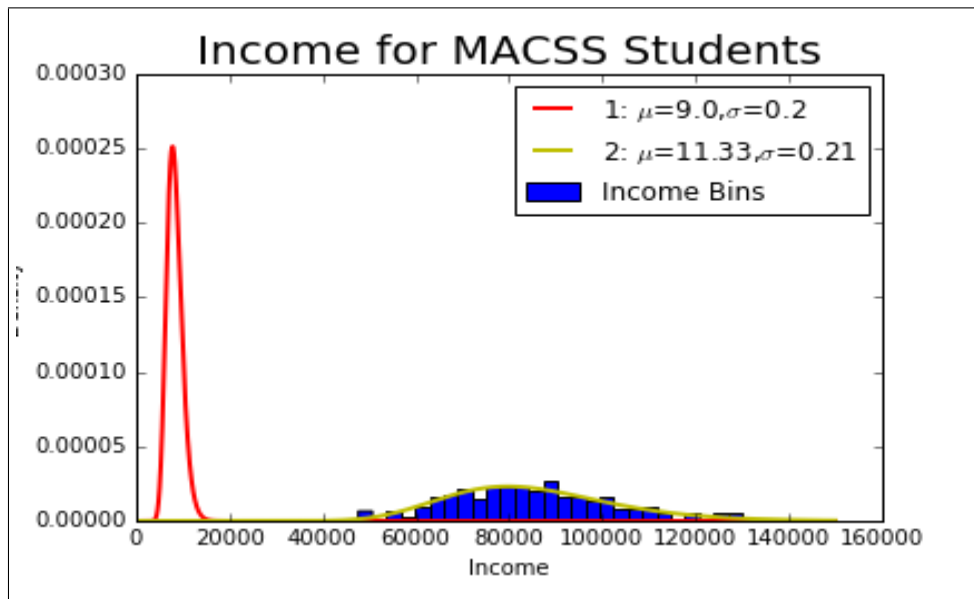


Figure 3: Distribution of Income

- (d) We have $H_0: \mu = 9.0$ and $\sigma = 0.3$;
Chi squared of H_0 with 2 degrees of freedom p-value = 0.0
We should reject the null hypothesis
- (e) We have the following result:

The possibility of having an income higher than \$100000 is 0.196
The possibility of having an income lower than \$75000 is 0.308

Problem 2

(a) The following is the estimate:

$$\begin{aligned}\sigma_{MLE} &= 0.00302 \\ \beta_0 &= 0.25164 \\ \beta_1 &= 0.01293 \\ \beta_2 &= 0.40050 \\ \beta_3 &= -0.00999\end{aligned}$$

The log likelihood function value is 876.86.

The variance and covariance matrix is:

$$\begin{bmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{bmatrix}$$

(b) The null hypothesis:

$$H_0 : \sigma^2 = 0.01, \beta_1, \beta_2, \beta_3, \beta_0 = 0$$

The p-value for the null hypothesis is approximately 0.00. We should reject the null hypothesis.