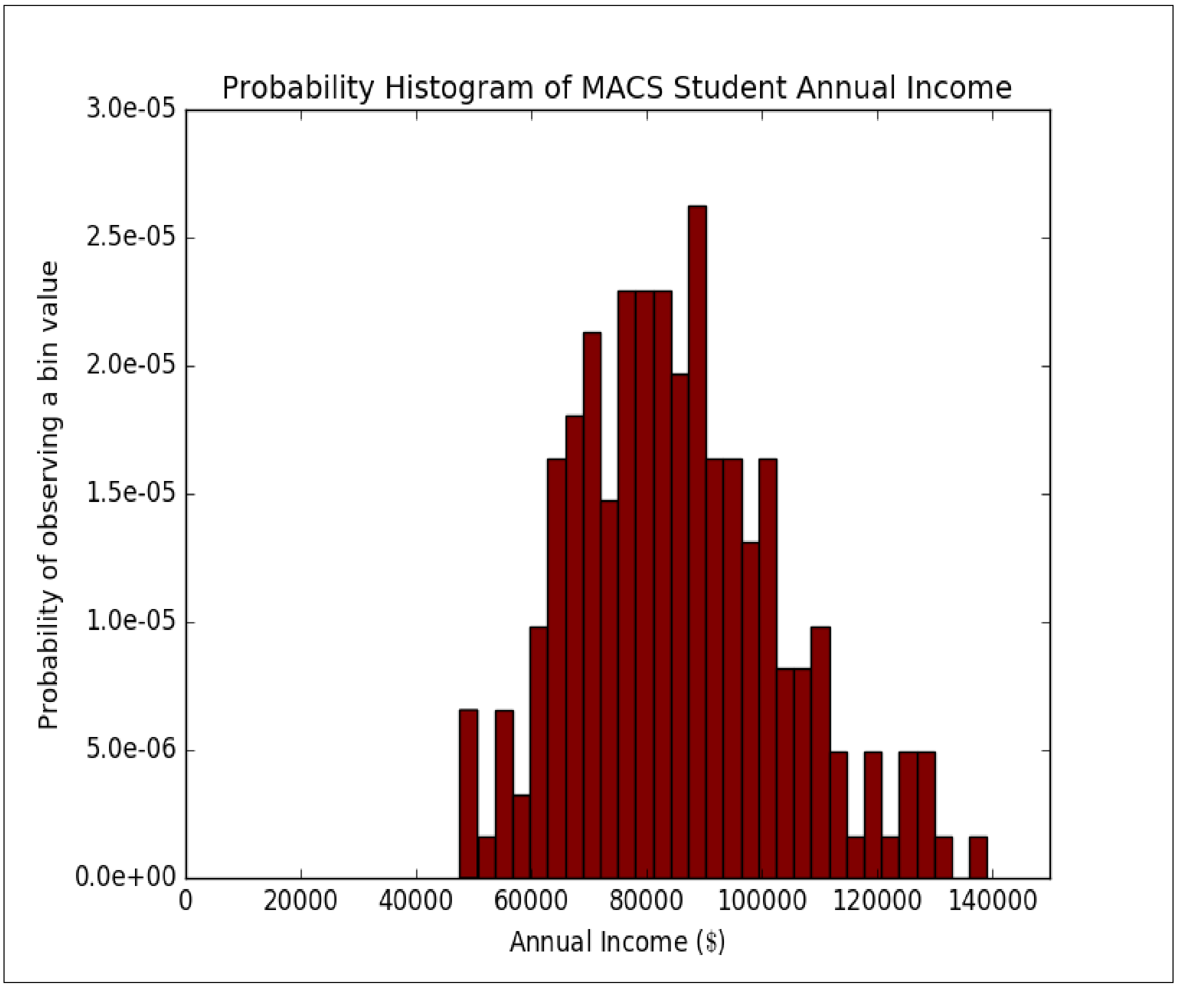
Problem Set #4

MACS 30100, Dr. Evans
Dongping Zhang

Problem 1: **Using Simulated Method of Moments (SMM) technique to estimate the parameters of a lognormal distribution that is supposedly resembling the actual distribution of UChicago MACSS students' future incomes.**

Part (a). Plot a histogram of percentages using *income.txt* data with 30 bins and *normed* = *True*.

Figure 1: Probability Histogram of MACSS Students' Annual Incomes



Part (b). Write a function for the lognormal PDF. This function should return an array pdf values that is the same size as input xvals and represents the lognormal PDF values of each element of xvals given the parameters μ and σ. Test your function by inputting the matrix **xvals = np.array([[200.0, 270.0], [180.0, 195.5]])** with parameter values $\mu = 5.0$ and $\sigma = 1.0$.

Input matrix:

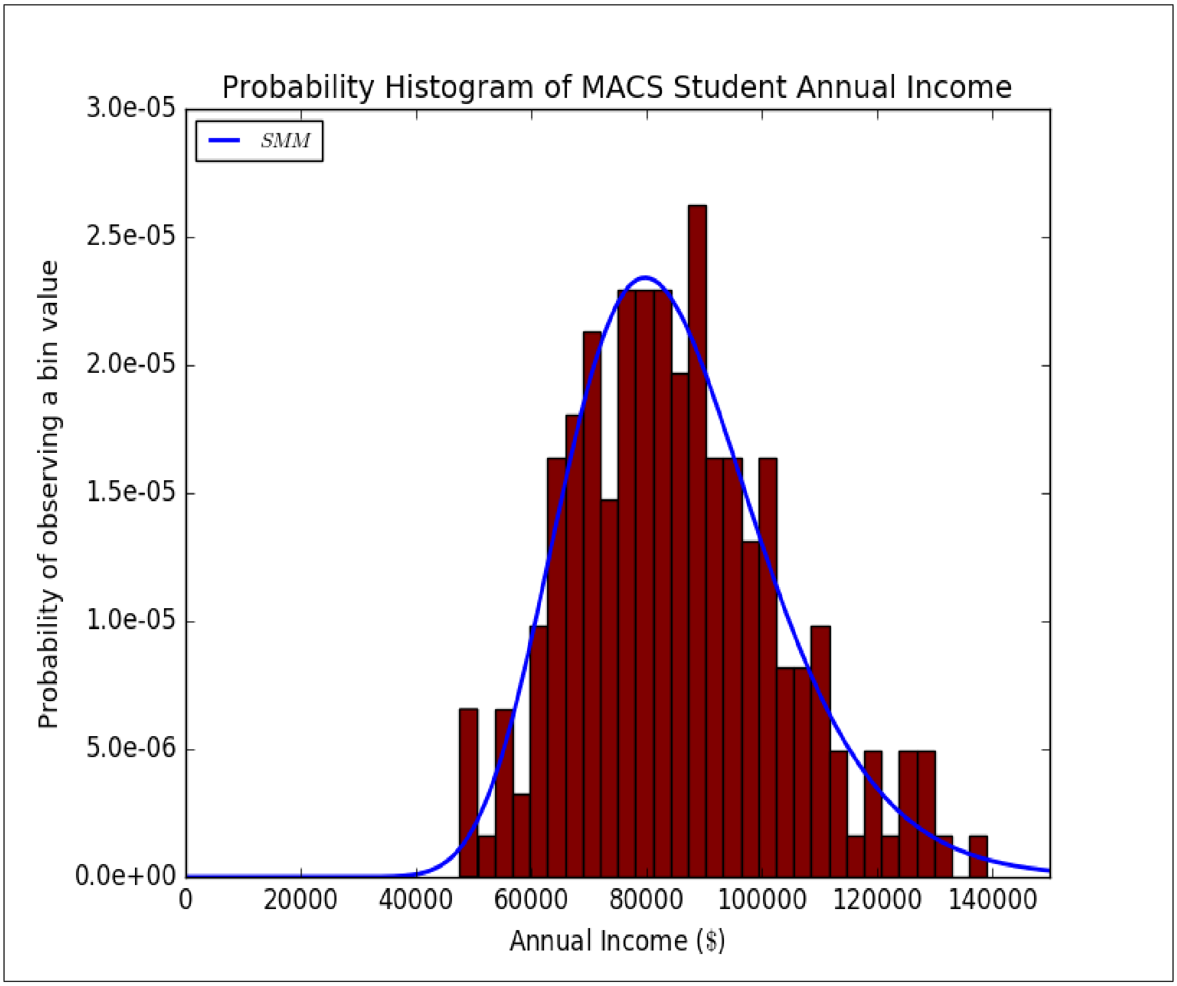
$$\begin{bmatrix} 200.0 & 270.0 \\ 180.0 & 195.5 \end{bmatrix}$$

Output matrix:

$$\begin{bmatrix} 0.0019079 & 0.00123533 \\ 0.00217547 & 0.0019646 \end{bmatrix}$$

Part (c). To estimate the parameters of the lognormal distribution by SMM. Create S = 300 simulations, each with N = 200 observations on income from the lognormal distribution above. Use the average income and standard deviation of income as your two moments. Use the identity matrix as your weighting matrix. Plot your estimated lognormal PDF against the histogram from part (a). Report the value of your SMM criterion function at the estimated parameter values. Report and compare your two data moments against your two model moments at the estimated parameter values.

Figure 2: SMM lognorm PDF on the income histogram



The estimated parameters of the lognormal distribution by SMM are:

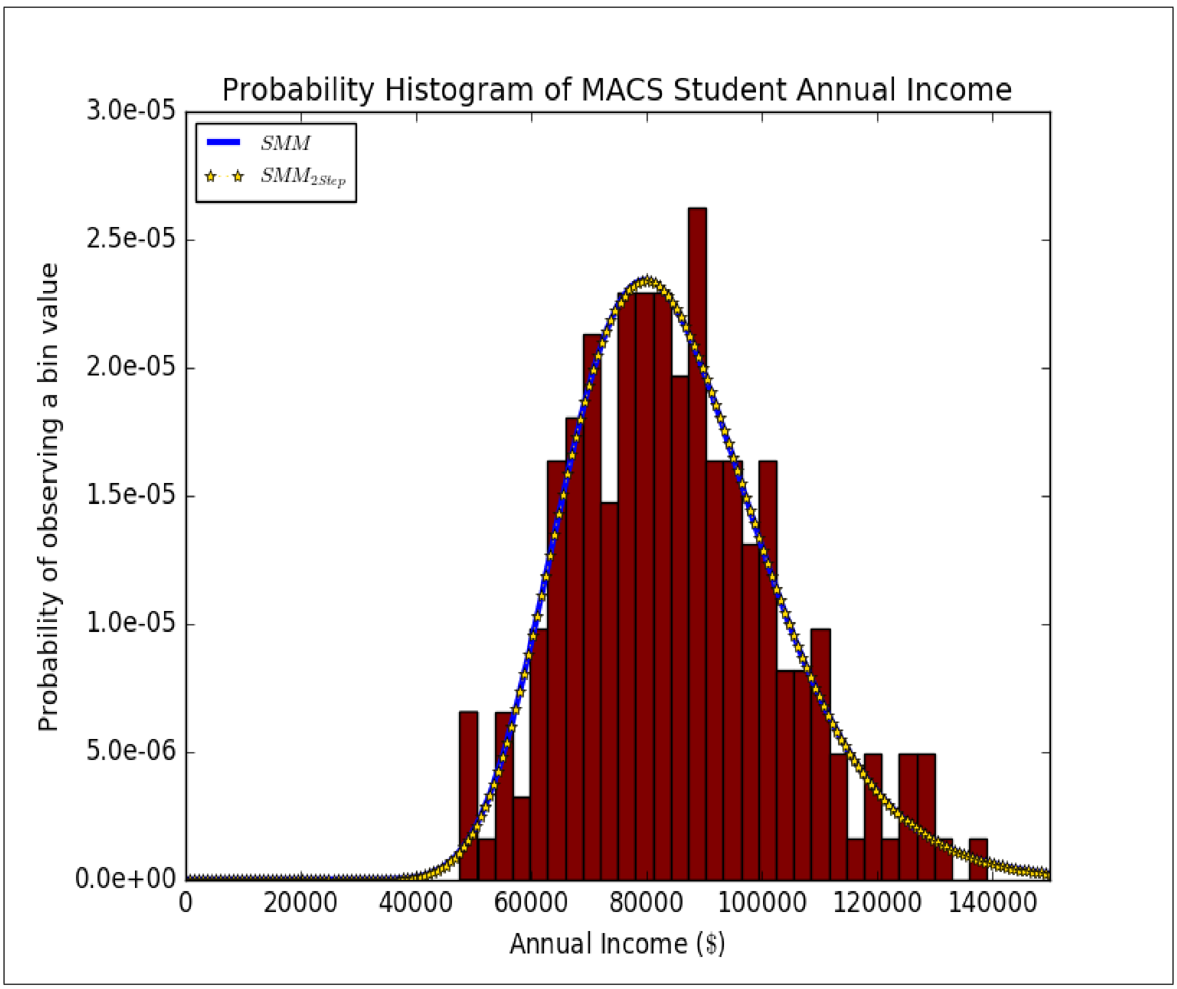
- $\hat{\mu}_{SMM}$ = **11.3306894798**
- $\hat{\sigma}_{SMM}$ = **0.208983844918**
- Value of the criterion function at using SMM parameters obtained is: $\mathbf{2.37960724 \times 10^{-13}}$
- Data Moments vs. Model Moments: from the table below, we are able to observe that the data moments and model moments are so similar that only

the decimal points vary. Thus we can claim that our optimization procedure is successful.

	μ	σ
data moments	85276.823606258113	17992.542128046523
model moments	85276.785425932292	17992.54561254603

Part (d). Perform the two-step SMM estimator by using your estimates from part (c) with two moments to generate an estimator for the variance covariance matrix, which you then use to get the two-step estimator for the optimal weighting matrix. Plot your estimated lognormal PDF against the histogram from part (a) and the estimated PDF from part (c). Report and compare your two data moments against your two model moments at the estimated parameter values.

Figure 3: SMM vs. SMM_{2Step}



The estimated parameters of the lognormal distribution by SMM are:

- $\hat{\mu}_{SMM}^{2step}$ = **11.3306904008**
- $\hat{\sigma}_{SMM}^{2step}$ = **0.208983840459**
- Value of the criterion function at using SMM parameters obtained is: $\mathbf{2.46223021 \times 10^{-14}}$
- Data Moments vs. Model Moments: from the table below, we are able to observe that the data moments and model moments are so similar that only the decimal points vary. Thus we can claim that our optimization procedure is successful.

	μ	σ
data moments	85276.823606258113	17992.542128046523
model moments	85276.863883245824	17992.561774277146