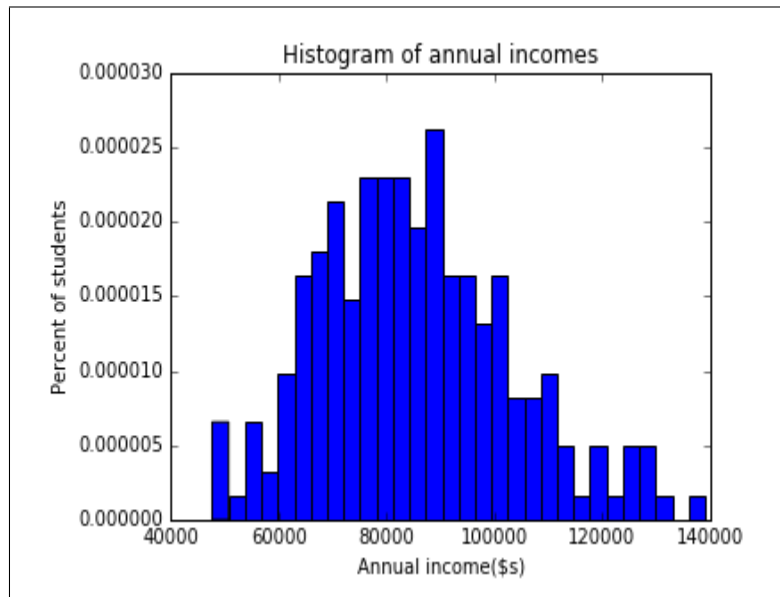


# Problem Set #3

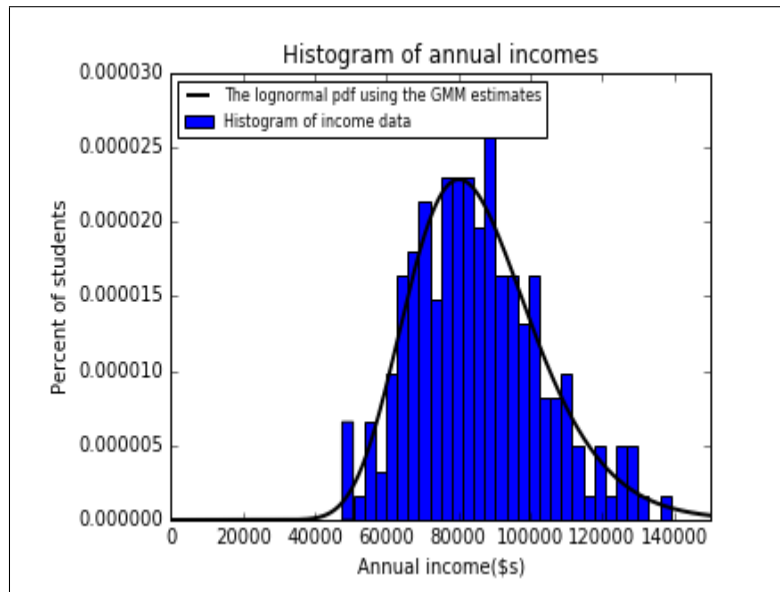
MACS 30100, Dr. Evans  
Huanye Liu

## Problem 1

### Part (a)

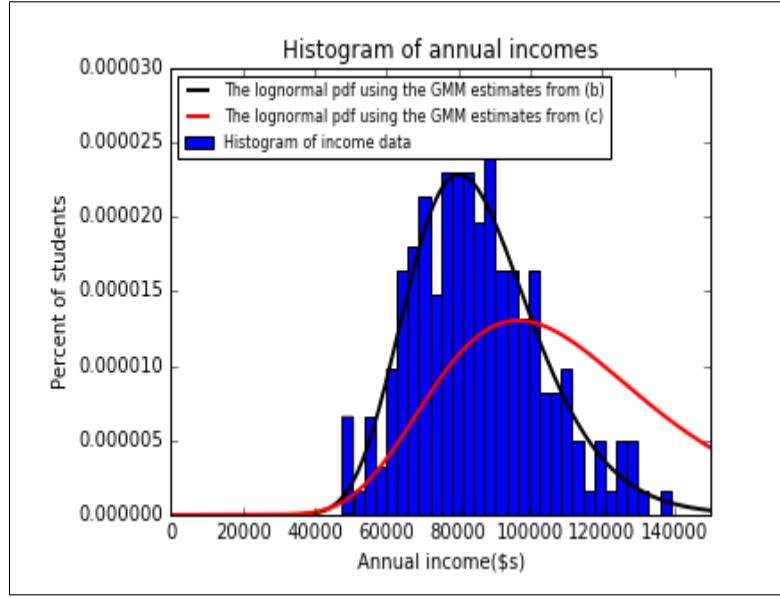


**Part (b)** The graph for comparison of histogram with lognormal pdf given the GMM estimates of parameters is shown below.



The estimates of the lognormal distribution parameters are  $\mu_{GMM1}= 11.3369084739$   $\sigma_{GMM1}= 0.213027407188$ , and the value of the GMM criterion function at the estimated parameter values is  $2.99828760e-12$ . The two data moments are Mean of incomes = 85276.8236063 , Variance of incomes = 323731572.23, and the two model moments are Mean of model = 85276.6760865 , Variance of model = 323731596.793, and we can see from the graph that the GMM estimates of parameters capture the data pretty closely.

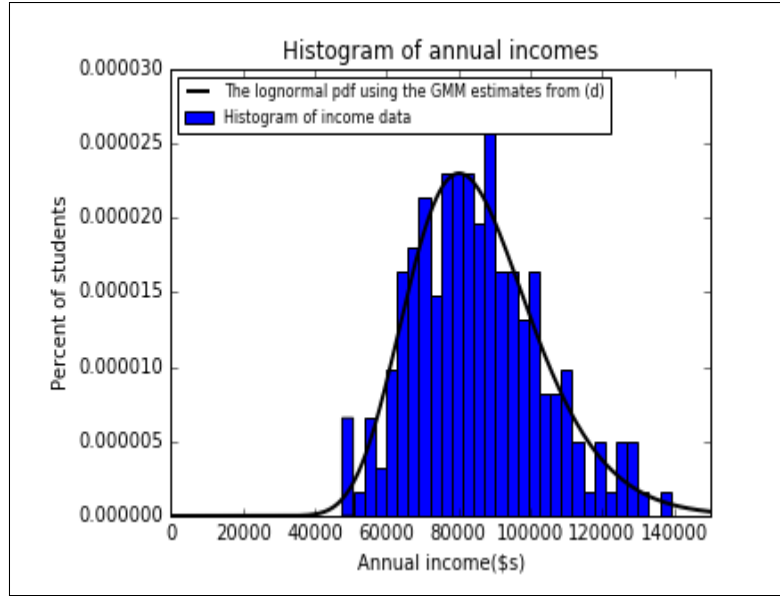
**Part (c)** The graph for comparison of histogram with lognormal pdf given the GMM estimates of parameters and lognormal pdf given the GMM estimates using optimal weighting matrix is show below.



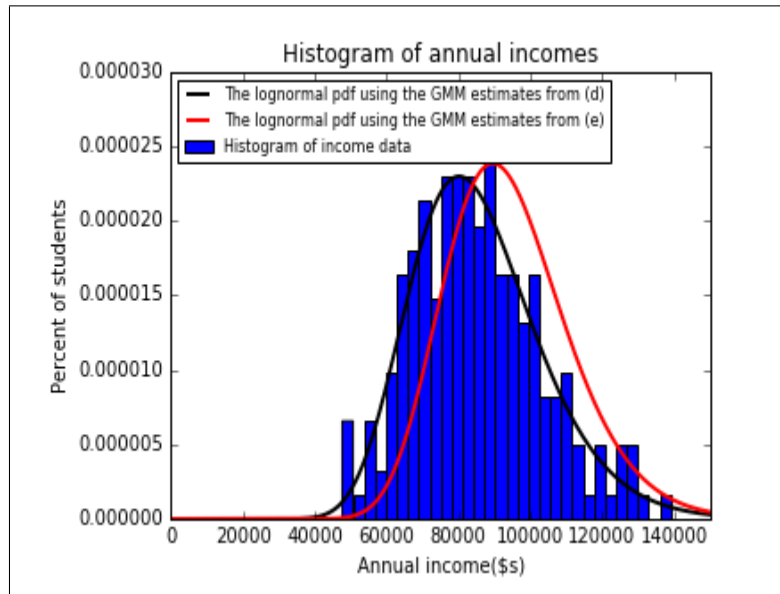
The estimates of the lognormal distribution parameters using optimal weighting matrix are  $\mu_{GMM2}= 11.5698612243$   $\sigma_{GMM2}= 0.302509175015$ , and the value of the GMM criterion function at the estimated parameter values is 0.93976374. The two data moments are Mean of incomes = 85276.8236063 , Variance of incomes = 323731572.23, and the two model moments are Mean of model = 88899.2607031, Variance of model = 637260094.247, and we can see from the graph that the GMM estimates of parameters using optimal weighting matrix does not capture the data pretty well.

**Part (d)** The graph for comparison of histogram with lognormal pdf given the GMM estimates of parameters is shown on the next page.

The estimates of the lognormal distribution parameters are  $\mu_{GMM3}= 11.3367266383$   $\sigma_{GMM3}= 0.211746444037$  and the value of the GMM criterion function at the estimated parameter values is 0.23818173. The three data moments are 0.3, 0.5 and 0.2 respectively, and the three model moments are 0.29927248776813703, 0.49805740594023 and 0.1996627959753557 correspondingly, and we can see from the graph that the GMM estimates of parameters capture the data pretty closely.



**Part (e)** The graph for comparison of histogram with lognormal pdf given the GMM estimates of parameters and lognormal pdf given the GMM estimates using optimal weighting matrix is shown below. The estimates of the lognormal distribution



parameters using optimal weighting matrix are  $\mu_{GMM4} = 11.4364107654$ ,  $\sigma_{GMM4} = 0.18367346379$ , and the value of the GMM criterion function at the estimated parameter values is  $2.20796285e-09$ . The three data moments are 0.3, 0.5 and 0.2 respectively, and the three model moments are 0.12513597988530095, 0.5363712158150175 and 0.3341490483589869 correspondingly, and we can see from the graph that the GMM estimates of parameters using optimal weighting matrix does not capture the data pretty well.

**Part (f)** Comparing four GMM estimates from (b), (c),(d),(e), we find that the criterion value is minimized when we use the GMM estimates from (b). But we can also see that the lognormal pdf using GMM estimates from (d) also fits the data pretty well. However, both the two-step estimator counterparts from (c) and (e) using the optimal weighting matrix do not perform as well as expected.

## **Problem 2**

### **Part (a).**

The four GMM estimates of the linear regression model parameters are  $\beta_{0_{GMM}}=0.25164486354$ ,  $\beta_{1_{GMM}}=0.0129334709489$ ,  $\beta_{2_{GMM}}=0.400500984897$ ,  $\beta_{3_{GMM}}=-0.00999170970069$ , and the value of GMM criterion function is 0.00182128980609.