The crystal lattice can be presented as a system of elastically bonded masses. In such system the resonant modes of the masses interact via the elastic bonding and passbands are generated.

The wave equation for inhomogenous crystals: [1]

$$\frac{\delta^2 u_j^i}{\delta t^2} = \frac{1}{\rho_j}\left[\frac{\delta}{\delta x_i}\left(\lambda \frac{\delta u_j^i}{\delta x_l}\right) + \frac{\delta}{\delta x_l}\left[\mu\left(\frac{\delta u_j^i}{\delta x_l} + \frac{\delta u_j^l}{\delta x_i}\right)\right]\right] \tag{1}$$

where u^i is the i^{th} component displacement vector. The subscript j is in reference to the medium; λ, μ are the Lame coefficients, ρ is the density.

The longitudinal and transverse speed of sound are given by

$$c_l = \sqrt{(\lambda + 2\mu)/\rho} \tag{2}$$

$$c_t = \sqrt{\mu/\rho} \tag{3}$$

The Lame coefficients can be expressed as Young's modulus E.

$$E_t = \rho c^2 = \mu \tag{4}$$

$$E_l = \rho c^2 = \lambda + 2\mu \tag{5}$$

The main constant in consideration of phononic distribution is Young's modulus.

The Floquet boundary conditions for the periodic structure:

$$u_{destination} = u_{source} \cdot exp[-ik \cdot (r_{destination} - r_{source})] \tag{6}$$

The band gap in periodic structures appears due to the Bragg's diffraction. An absolute phononic band gap, if one exists, can be a Bragg type gap, which appears at about an angular frequency ω of the order c/a where c is a typical velocity of sound in the structure and a is the lattice parameter. The frequency of a band gap can't me larger than the frequency for Bragg type lattice.

There are some effects that appear due to defects in phononic crystals: [2]

GUIDING

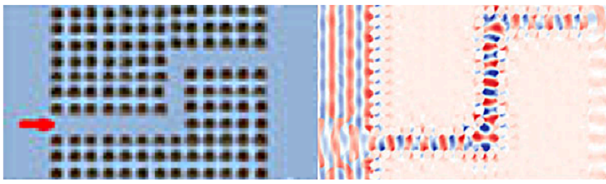

Figure 1

FILTERING

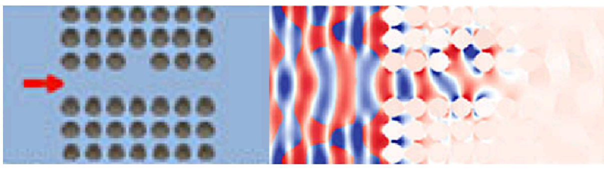

Figure 2

TUNABILITY

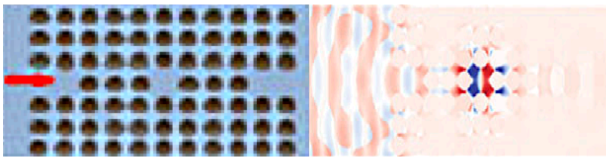

Figure 3

DEMULTIPLEXING

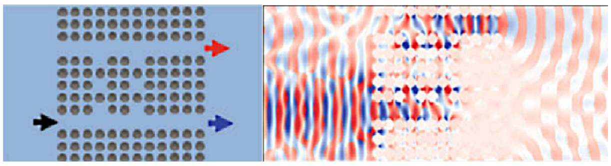

Figure 4

By matching the symmetry of the elements comprising the crystal (holes) with the lattice symmetry in the Brillouin zone, the band gap can be improved. The space vectors: $e1, e2, e3$. The Fourier transform over real space: $b_i = \frac{2\pi}{v} \cdot e_j \cdot e_k$. The first Brillouin zone of silicon is a quadrangle (diamond type lattice).

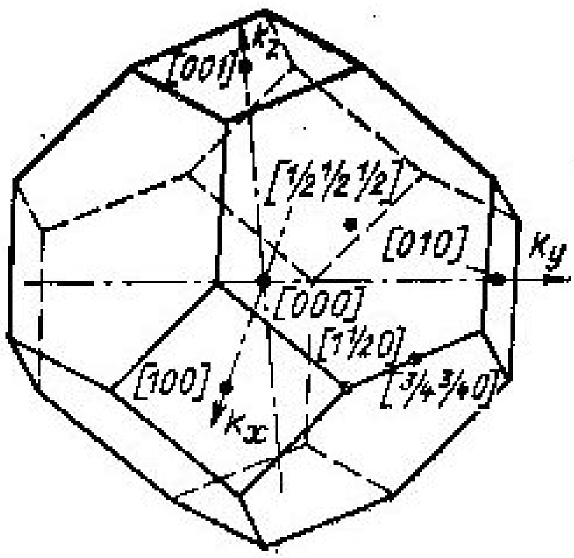

Figure 5

So the configuration of the holes that is more suitable to make a large band gap is circle. The center of the band frequency: $f = c/a$. The velocity in solids can be also calculated with equations:

$$c_l = \sqrt{\frac{E}{\rho}} \cdot \sqrt{\frac{1-\sigma}{1+\sigma} \cdot \frac{1}{1-2\sigma}} \tag{7}$$

$$c_t = \sqrt{\frac{E}{\rho}} \cdot \sqrt{\frac{1}{[2(1+\sigma)]}} \tag{8}$$

The transverse velocity in the silicon $c_t = 3770 m/s$. The diameter for the frequency $f = 1MHz$ is $a = 3.77mm$.

The first configuration of the holes for Comsol calculations is honey-comb pattern.

References

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