CS303 Data Structures Assignment #1

August 26, 2018

1.

Choose an arbitrary n_0, eg. $n_0 = 1$.

$Cn_0^3 > n_0^3 - 5n_0^2 + 20n_0 - 10$

$C * 1^3 > 1^3 - 5 * 1^2 + 20 * 1 - 10$

$C > 1 - 5 + 20 - 10$

$C = 6, n_0 = 1$

For all n, where $n > 1$, $6n^3 > n^3 - 5n^2 + 20n - 10$.

2.

See attached `comparegrowth.cpp`. Output:

```
y1:  10
y2:  2
y1:  1010
y2:  502
y1:  2010
y2:  2002
y1:  3010
y2:  4502
y1:  4010
y2:  8002
y1:  5010
y2:  12502
y1:  6010
```

```
y2:  18002
y1:  7010
y2:  24502
y1:  8010
y2:  32002
y1:  9010
y2:  40502
y1:  10010
y2:  50002
```

The results here are to be expected. y1 is initially larger because of the constant 100 rather than 5, but y2 quickly overtakes it because of how much faster n^2 grows than n. This would be true regardless of what constant y1 used. If $y1 = 10000n + 20$, y2 would still outgrow it.

3.

3.1

The inner loop is run i^2 times, with i being every number from 0 to n-1. Therefore, we get the sum: $T(n) = 1^2 + 2^2 + 3^2 \cdots + n^2$ or
$T(n) = \sum_{i=1}^{n} i^2$, which is simply a geometric series, so $T(n) = \frac{1}{6}n(n+1)(2n+1)$, or $O(n) = n^3$.

3.2

This is the same as a simple "i to n" loop, but backwards, which makes no difference, and skipping every other number, which halves the iterations. For loops with odd numbers, we get to 1, and then decrement again, so we take the ceiling of that fraction.
$T(n) = \lceil \frac{1}{2}n \rceil$, and $O(n) = n$.