CS394 R Homework 4

source available at https://github.com/alexskc/cs394r

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Most of this isn't done, and what is done is poor. Take off whatever marks are necessary.

1

a

$\int_0^1 c(1+x) = c(x + \frac{x^2}{2})|_0^1 = c(1+\frac{1}{2}) = 1$
$c = \frac{2}{3}$

b

$E[X] = c\int_0^1 x + x^2 = c(\frac{x^2}{2} + \frac{x^3}{3})|_0^1 = \frac{2}{3}(\frac{1}{2} + \frac{1}{3})$
$E[x^2] = c\int_0^1 x^2 + x^3 = c(\frac{x^3}{3} + \frac{x^4}{4})|_0^1 = \frac{2}{3}(\frac{1}{3} + \frac{1}{4})$
$Var(X) = E[X^2] - (E[X])^2 = \frac{13}{162}$
$\sigma = \sqrt{Var[X]} = \sqrt{\frac{13}{162}}$

c

$c\int_0^{\frac{1}{2}} = \frac{2}{3}(\frac{1}{2} + \frac{1}{8})$

d

$\frac{\int_0^{\frac{1}{2}}}{\int_0^{\frac{3}{4}}} = \frac{\frac{1}{2}+\frac{1}{8}}{\frac{3}{4}+\frac{9}{32}}$

2

3

a

$c\int_0^1(1 - x + 2x^2) = 1 = c(x - \frac{x^2}{2} + \frac{2x^3}{3}|_0^1) = 1 = c(1 - \frac{1}{2} + \frac{2}{3} = 1)$
$c = \frac{6}{7}$

b

$CDF = \frac{6}{7}(x - \frac{x^2}{2} + \frac{2x^3}{3})$

c

$E[X] = c\int_0^1 (x - x^2 + 2x^3) = \frac{6}{7}(\frac{x^2}{2} - \frac{x^3}{3} + x^4)|_0^1 = \frac{6}{7}(\frac{1}{2} - \frac{1}{3} + 1)$

d

$E[X^2] = c\int_0^1 (x^2 - x^3 + 2x^4) = \frac{6}{7}(\frac{x^3}{3} - \frac{x^4}{4} + \frac{2x^5}{5})|_0^1 = \frac{6}{7}(\frac{1}{3} - \frac{1}{4} + \frac{2}{5})$
$Var(x) = E[x^2] - (E[X])^2 =??$

e

$P[X < \frac{1}{2}] = c\int_0^{\frac{1}{2}} (1 - x + 2x^2) = \frac{6}{7}(x - \frac{x^2}{2} + \frac{2x^3}{3})|_0^{\frac{1}{2}} = \frac{6}{7}(\frac{1}{2} - \frac{1}{8} + \frac{1}{12})$

4

a

$c\int_0^4 \frac{y}{8} = 1$
$c(\frac{y^2}{16})|_0^4 = 1 = c(1) = 1.$
$CDF = \frac{y^2}{16}$

b

$\int_0^4 \frac{y^2}{8} = \frac{y^3}{24}|_0^4 = \frac{64}{24}$

c

$E[X^2] = \int_0^4 \frac{y^3}{8} = \frac{y^4}{32}|_0^4 = \frac{256}{32}$
$Var(X) = E[X^2] - (E[X])^2 = \frac{256}{32} - \frac{4096}{576} = \frac{120}{143}$

d

$P[2 < Y <= 3] = \int_2^3 \frac{y}{8} = \frac{y^2}{16}|_2^3 = \frac{5}{16}$

5

a

$C\int_{0.6}^{\infty} \frac{(0.6)^{2.5}}{x^{3.5}} = 1$
$C\frac{2}{5} = 1$
$C = \frac{5}{2}$

b

$E[X] = \frac{5}{2} \int_{0.6}^{\infty} \frac{(0.6)^{2.5}}{x^{2.5}} = 1$

c

$E[X^2] = \frac{5}{2} \int_{0.6}^{\infty} \frac{(0.6)^{2.5}}{x^{1.5}} = \frac{5}{2} * \frac{18}{25} = \frac{9}{5}$
$Var(X) = E[X^2] - (E[X])^2 = \frac{9}{5} - 1 = \frac{4}{5}$

d

$C \int_{0.6}^{10} = 1 - \frac{9\sqrt{6}}{25000}$