CS394 R Homework 3

source available at https://github.com/alexskc/cs394r

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September 21, 2018

1

a

First, we can find the probability that there is no such accident, one such accident, and two such accidents in a month.
$P(X = 0) = \frac{e^{-1.5}1.5^0}{0!}$
$P(X = 1) = \frac{e^{-1.5}1.5^1}{1!}$
$P(X = 2) = \frac{e^{-1.5}1.5^2}{2!}$
And then we simply take one minus the sum of these probabilities. $1 - e^{-1.5}\frac{29}{8} \approx 0.191$

b

This is simply the sum of the first two probabilities we found. $e^{-1.5}\frac{1+1.5}{1} \approx 0.558$
.

2

$P(X = 0) = P(Y = -3) = \frac{1}{16}$
$P(X = 1) = P(Y = -2) = \frac{4}{16}$
$P(X = 2) = P(Y = -1) = \frac{4}{16}$
$P(X = 3) = P(Y = 0) = \frac{4}{16}$
$P(X = 4) = P(Y = 1) = \frac{1}{16}$
Zero otherwise.

3

The probability of X is dependant on the number of students on each bus. So $P(X = 40) = \frac{40}{150}$, $P(X = 35) = \frac{35}{150}$, and so on. $E[X] = \frac{(40*40)+(35*35)+(25*25)+(50*50)}{150} = \frac{119}{3}$.
$var[X] = E[x^2] - (E[X])^2$
$var[X] = \frac{(40^2*40)+(35^2*35)+(25^2*25)+(50^2*50)}{150} - (\frac{119}{3})^2 = 1650 - \frac{14161}{9} = \frac{689}{9}$.

By contrast, the buses are all the same from the driver's perspective. The probability is simply $\frac{1}{4}$. $E[X] = \frac{(1*40)+(1*35)+(1*25)+(1*50)}{4} = \frac{75}{2}$.
$var[X] = \frac{(1*40^2)+(1*35^2)+(1*25^2)+(1*50^2)}{4} - (\frac{75}{2})^2 = \frac{2975}{2} - \frac{5625}{4} = \frac{325}{4}$.

4

a

$P[Y = 2] = \frac{1}{16}$
$P[Y = 3] = \frac{2}{16}$
$P[Y = 4] = \frac{3}{16}$
$P[Y = 5] = \frac{4}{16}$
$P[Y = 6] = \frac{3}{16}$
$P[Y = 7] = \frac{2}{16}$
$P[Y = 8] = \frac{1}{16}$
Zero otherwise.

b

$P[Y > 5] = \frac{3+2+1}{16} = \frac{3}{8}$

c

$P[Y > 5 | Y \geq 3] = \frac{\frac{3+2+1}{16}}{\frac{2+3+4+3+2+1}{16}} = \frac{2}{5}$

d

$E[Y] = \frac{(2*1)+(3*2)+(4*3)+(5*4)+(6*3)+(7*2)+(8*1)}{16} = 5$

e

$E[Y^2] = \frac{(2^2*1)+(3^2*2)+(4^2*3)+(5^2*4)+(6^2*3)+(7^2*2)+(8^2*1)}{16} = \frac{55}{2}$

f

$Var(Y) = E[Y^2] - (E[Y])^2 = \frac{55}{2} - 25 = \frac{5}{2}$

5

a

Let Y denote for a given number of days. We have to remember to consider that if we are paying for a day, we also have to pay for all the previous days. $E[Y] = \frac{(5*1000)+(4*2000)+(3*3000)+(2*3500)+(1*4000)}{15} = 2200$.

b

$E[Y^2] = \frac{(5*1000000)+(4*4000000)+(3*9000000)+(2*12250000)+(1*16000000)}{15} = 5900000$
$(E[Y])^2 = 4840000$
$Var(Y) = E[Y^2] - (E[Y])^2 = 1060000$

6

a

Probabilities awlays have to add up to 1. So to get $\alpha(5+8+11+14) = 1, \alpha = \frac{1}{38}$.

b

$E[X] = \frac{(0*5)+(1*8)+(2*11)+(3*14)}{38} = \frac{72}{38}$

c

$E[X^2] - \frac{(0*5)+(1*8)+(4*11)+(9*14)}{38} = \frac{178}{38}$
$(E[X]^2) = \frac{72^2}{38^2}$
$Var(X) = E[X^2] - (E[X]^2) = \frac{395}{361}$

d

$P[X \leq 2] = 1 - P[X = 3] = 1 - \frac{14}{38} = \frac{24}{38}$.