Section 3.4
Choosing a Best Model

Nonlinear 2 Parameter Fit

Given a set of data points $\{(x_i, y_i)\}_{i=1}^{N}$ with nonlinear model : $y = Cx^A$

Steps :

Linearize $\mathrm{y = Cx^A \Rightarrow Y = AX + B \Rightarrow lny = lnC + Alnx}$

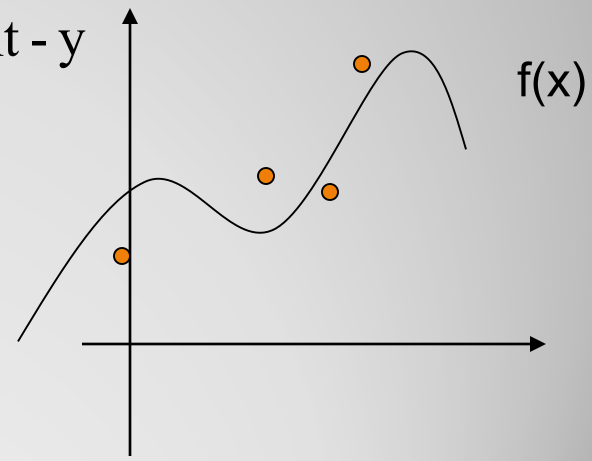
so $\mathrm{Y = lny; X = lnx;} (x_i, y_i)_{i=1}^{N}$ with the line fit

$\mathrm{[A\ B] = mylinefit(X, Y)}$ % get A & B, slope & int - y

$\mathrm{C = exp(B);\quad A = A;}$ % get value C & A

plug in C & A into best fitted model :

$\mathrm{yy = Cxx^A}$



Rules for linearizing:

1) X and Y transformed from x_i, y_i or both.

2) Vise versa A & B should contain NO x_i or y_i.

$$y = Cx^A$$

$$\downarrow$$

$$\ln y = \ln Cx^A$$

$$\ln y = A \ln x + \ln C$$

$$\underset{\substack{\downarrow \\ Y}}{\ln y} = \underset{A}{A}\, \underbrace{\ln x}_{X} + \underset{\substack{\downarrow \\ B}}{\ln C}$$

MATLAB

$$>> x = [.....],$$

$$>> y = [.....],$$

$$>> X = \ln x;\;\; Y = \ln y;\;\; [\mathrm{A},\mathrm{B}] = \mathrm{mylinefit}(X, Y);$$

$$>> C = \exp(B);$$

$$>> \mathrm{plot}(x, y, \text{'*'});\;\; \text{hold on}$$

$$>> \mathrm{plot}(x, C * x.\wedge A, \text{'s'});\;\; \text{hold off}$$

$$>> \mathrm{legend}(\text{'Data'}, [\text{'Fitted model } y = \text{'}\ \mathrm{num2str}(C) \ldots \text{'}\, x \wedge \{\text{'}\ \mathrm{num2str}(\mathrm{A})\text{'}\}]);$$

More Examples

$$y = Cxe^{kx}$$

$$y = \frac{f_o}{1 + Ce^{-kx}}$$

Non Linear Model	---→ Linear Model Y=AX+B
$y = Cx^{A}$	
$y = \frac{A}{x} + B$	
$y = Ce^{Ax}$	
$y = \frac{x}{Ax + B}$	
$y = \frac{A}{x - B}$	

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{bmatrix} \text{value from data}$$

$$yy = f(x) = \begin{bmatrix} f(x_1) \\ f(x_2) \\ . \\ . \\ f(x_N) \end{bmatrix} \text{predicted value from the model}$$

R^2 - MODEL ASSESSMENT

sum square (total Variance

$$SST \overset{def}{=} \sum_{i=1}^{N} \left(y_i - \overline{y} \right)^2$$

mean value of y_i's

$$\overline{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$$

sum of square errors

$$SSE \overset{def}{=} \sum_{i=1}^{n} \left(y(x_i) - y_i \right)^2$$

$$R^2 \overset{def}{=} 1 - \frac{SSE}{SST}$$

When R^2 is 1 (100%) – perfect fit.

Matlab

Function Rsq=myRsquare $(y_i \quad yy)$

```
SST=sum((y-mean(y)).^2)
SSE=sum((y-yy)).^2
Rsq=1-SSE/SST;
```

Single Parameter Model

Given a set of data points $\{(x_i, y_i)\}_{i=1}^{N}$

$$Ex: y = kx^3 \Rightarrow \qquad Y = AX \Rightarrow y(x) = ax^{1/2}$$

$$\text{Sum Square Error is } \mathrm{E(A)} = \sum_{i=1}^{N} (\mathrm{f(x_i)} - y_i)^2 = \sum_{i=1}^{N} (AX_i - Y_i)^2$$

Find best possible A(slope) to fit in for $\{(\mathrm{x_i}, y_i)\}$, so that the error E(A) is minimum

$$\mathrm{E'(A)} = 2\sum_{i=1}^{N} (\mathrm{Ax_i} - y_i)x_i = 0$$

$$A = \frac{\sum x_i y_i}{\sum x_i^2} < 0$$

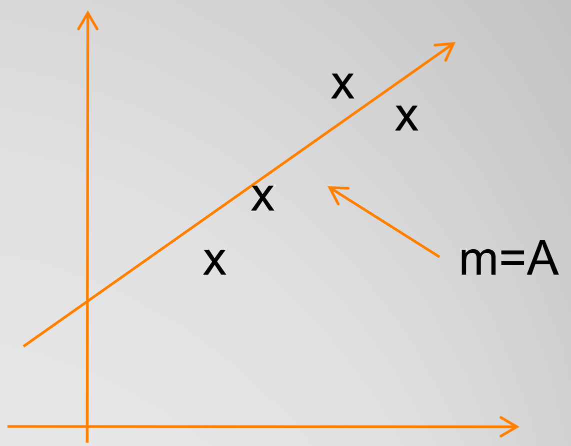
$$\mathrm{E''(A)} = 2\sum_{i=1}^{N} \mathrm{x_i^2} > 0$$

Matlab

```
function  A=myslopefit(X,Y)
A=sum(x.*y)/sum(x.^2);

>>x=[……..];
>>y=[……..];
>>X=sqrt(x); Y=y;

A=myslopefit(X,Y)
yy=A*sqrt(x)
Rsq=myRsquare(y,yy)
```



MODEL ASSESSMENT

Sum Square of the Total Variance

$$SST \overset{def}{=} \sum_{i=1}^{N} \left(y_i - \bar{y}\right)^2$$

the mean value of y_i's

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$$

sum of square errors

$$SSE \overset{def}{=} \sum_{i=1}^{n} \left(y(x_i) - y_i\right)^2$$

$$R^2 \overset{def}{=} 1 - \frac{SSE}{SST}$$

When R^2 is 1 (100%) ,it's a perfect fit. The closer R^2 is to 1, the better the model fits the data set

Homework

1. Write a MATLAB function that takes in the values of $(f(x_i),y_i)_{i=1}^{N}$ and returns R^2
2. $\oint$ Section 3.1 # 3, 4(a), 5(b), 6, 7

   a. Plot the data points

   b. Linearize the equation

   c. Fit the linear model

   d. Plot the data versus fitted nonlinear model

   e. Calculate R^2