Chapter 7 Part 1

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Question 1:

$$\ddot{u} = \frac{v}{1+t^2} - sin(r)$$

$$\ddot{v} = \frac{-u}{1+t^2} + cos(r)$$

$$r = \sqrt{\dot{u}^2 + \dot{v}^2}$$

$$u(0) = 1$$

$$v(0) = \dot{u}(0) = \dot{v}(0) = 0$$

Express this ODE in Standard Form:

$$\dot{u} = f_1(t, u, v, \dot{u}, \dot{v}) = \dot{v},$$

$$\dot{v} = f_2(t, u, v, \dot{u}, \dot{v}) = \frac{v}{1+t^2} - \frac{\sin(\sqrt{\dot{u}^2 + \dot{v}^2})}{1+t^2},$$

$$\ddot{u} = f_3(t, u, v, \dot{u}, \dot{v}) = \frac{v}{1+t^2} - \frac{\sin(\sqrt{\dot{u}^2 + \dot{v}^2})}{1+t^2},$$

$$\ddot{v} = f_4(t, u, v, \dot{u}, \dot{v}) = -\frac{u}{1+t^2} + \frac{\cos(\sqrt{\dot{u}^2 + \dot{v}^2})}{1+t^2},$$

with initial conditions

$\& \ \mathrm{u}(0) = 1, \quad \mathrm{v}(0) = 0, \quad \dot{\mathrm{u}}(0) = 0, \quad \dot{\mathrm{v}}(0) = 0.$

Question 2:

$$\begin{aligned} \dot{y} &= ry \\ y(0) &= 100 \\ r &= 0.06 \end{aligned}$$

Continuous Compound Interest Problem:

```
% Initial Data
r = 0.06; y0 = 100;
T = 10; Tspan = [0 T];
hm = 1/12; hy = 1;
y_dot = @(t, y)r.*y;
y_cont = @(t) y0.*exp(r.*t);

% Integral Evaluations
[~, e_yy] = myEuler(y_dot, Tspan, y0, hy);
[~, e_ym] = myEuler(y_dot, Tspan, y0, hm);
[~, m_ym] = myMidpoint(y_dot, Tspan, y0, hm);
[~, t_ym] = myTrapezoidal(y_dot, Tspan, y0, hm);
```

```
options = odeset('MaxStep', hm);
[~, b_ym] = ode23(y_dot, Tspan, y0, options);

% Continuous compounding
c_yy = y_cont(T);

fprintf('Eulers Method     (Yearly): %.5f\n', e_yy(end));
```

```
Eulers Method     (Yearly): 179.08477
```

```
fprintf('Eulers Method    (Monthly): %.5f\n', e_ym(end));
```

```
Eulers Method    (Monthly): 181.93967
```

```
fprintf('Midpoint Method  (Monthly): %.5f\n', m_ym(end));
```

```
Midpoint Method  (Monthly): 182.21143
```

```
fprintf('Trapezoid Method (Monthly): %.5f\n', t_ym(end));
```

```
Trapezoid Method (Monthly): 182.21143
```

```
fprintf('BS23 Method      (Monthly): %.5f\n', b_ym(end));
```

```
BS23 Method      (Monthly): 182.21188
```

```
fprintf('Continuous Compounding    : %.5f\n', c_yy);
```

```
Continuous Compounding    : 182.21188
```

Question 3:

Part a:

```
% Initial Data
Tspan = [0 1];
f1 = @(t, y) 1;
f2 = @(t, y) t;
f3 = @(t, y) t^2;
f4 = @(t, y) t^3;

% Solving the ODEs
[t1, y1] = ode23(f1, Tspan, 0);
[t2, y2] = ode23(f2, Tspan, 0);
[t3, y3] = ode23(f3, Tspan, 0);
[t4, y4] = ode23(f4, Tspan, 0);

% Compute Exact Values
exact1 = t1;
exact2 = t2.^2 / 2;
exact3 = t3.^3 / 3;
exact4 = t4.^4 / 4;

% Determine Error in Reults
```

```
error1 = abs(y1 - exact1);
error2 = abs(y2 - exact2);
error3 = abs(y3 - exact3);
error4 = abs(y4 - exact4);

% Display Error Results:
fprintf('Max Error for f(t, y) = 1   : %.4e\n', max(error1));
```

```
Max Error for f(t, y) = 1   : 1.7347e-18
```

```
fprintf('Max Error for f(t, y) = f   : %.4e\n', max(error2));
```

```
Max Error for f(t, y) = f   : 1.1102e-16
```

```
fprintf('Max Error for f(t, y) = f^2 : %.4e\n', max(error3));
```

```
Max Error for f(t, y) = f^2 : 5.5511e-17
```

```
fprintf('Max Error for f(t, y) = f^3 : %.4e\n', max(error4));
```

```
Max Error for f(t, y) = f^3 : 4.3999e-06
```

Part b:

ode23 error estimator is exact as long as you are working with a function of relatively high stiffness. Having more varied results produces a more sizable error in the estimate of ode23.

Question 4:

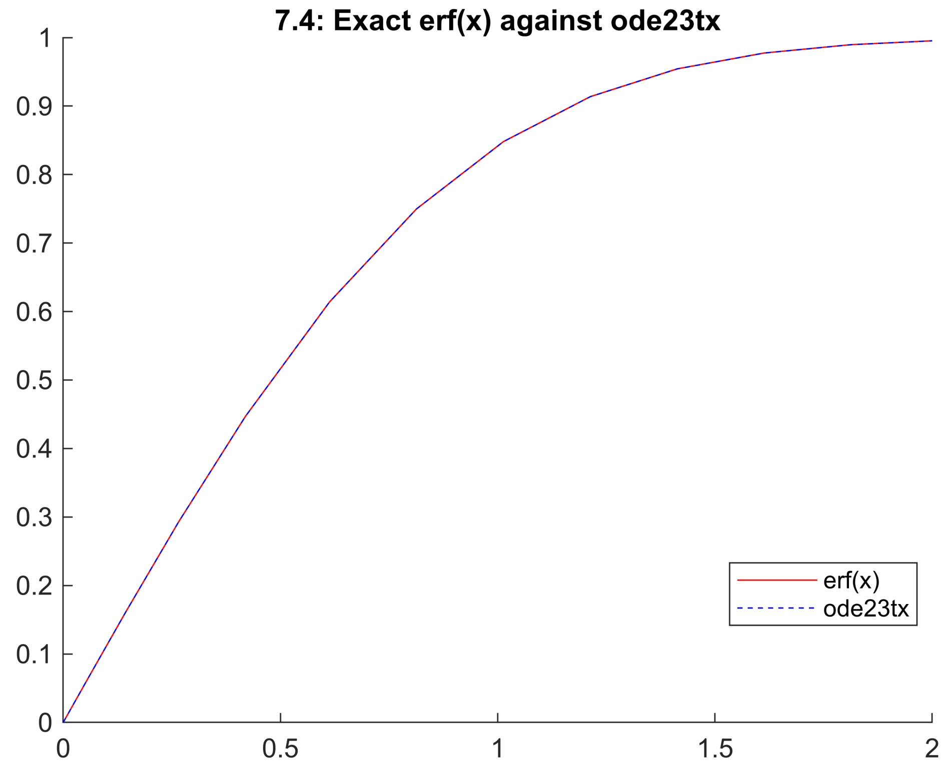
$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

$$y'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$y(0) = 0$$

```
% Initial Data
Tspan = [0 2]; y0 = 0;
f = @(x, y) 2/sqrt(pi) * exp(-x^2);
[t_tx, y_tx] = ode23tx(f, Tspan, y0);
y_exact = erf(t_tx);

% Plot Results
figure(2); clf; hold on;
plot(t_tx, y_exact, 'r', 'DisplayName', 'erf(x)');
plot(t_tx, y_tx, 'b--', 'DisplayName', 'ode23tx');
hold off;
legend('Location', 'best');
title('7.4: Exact erf(x) against ode23tx');
```



Question 5:

develop myrk4.m