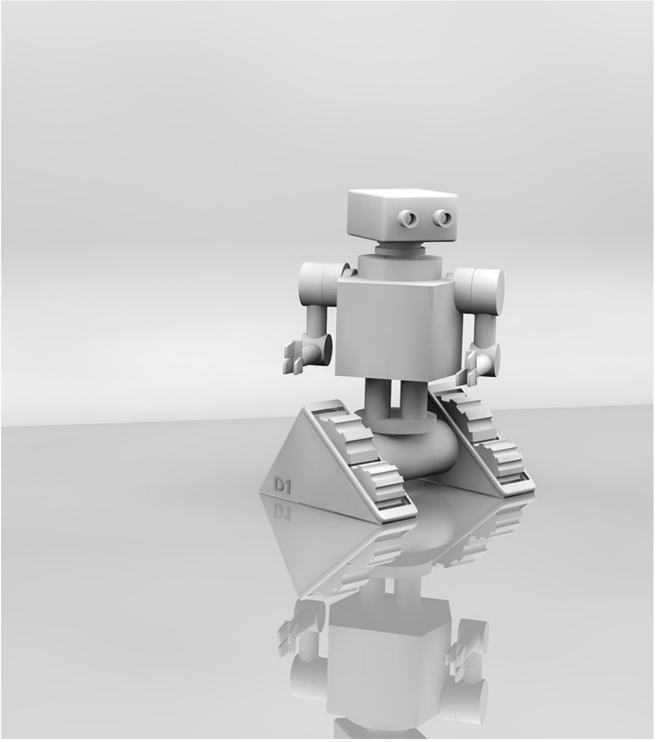

CPSC 481 Artificial Intelligence

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What we will cover this week

- Predicate logic

Predicate Logic or First Order Logic

- In addition to propositions, Predicate Logic provides:
  - Variables
    - Represent objects in the world
    - x, y
  - Predicates
    - Represent relationships between objects
    - father-of (Mary, John)
    - father-of (x, y) child(y,x)
    - Predicate is either True or False
  - Functions
    - father-of(Mary) = John
    - Function input and output are objects

Building a FOL sentence

- *Constant symbols* correspond to objects ("individuals") in the Universe
  - E.g., Today, Mary
  - Convention: begin with a Capital letter
- *Variable symbols* represent one of the objects
  - E.g., x, y, z
  - Convention: use small letters
- *Function symbols*
  - E.g., mother-of (Bill); maximum-of (x,y)
- A *term* is either a constant or variable

Building a FOL sentence

- *Predicates*
  - Represents a specific relationship between objects
  - father-of (Mary, John)
  - "John is the father of Mary"
- An *atomic sentence* is a predicate (with *n* terms)
- The truth values, True and False, are also atomic sentences.

Building a FOL sentence

- Every atomic sentence is a *sentence*.
- 1. If s is a sentence, then so is its negation, $\neg s$
- If s_1 and s_2 are sentences, then so is their
- 2. Conjunction, $s_1 \wedge s_2$
- 3. Disjunction, $s_1 \vee s_2$
- 4. Implication, $s_1 \rightarrow s_2$
- 5. Equivalence, s_1 s_2
- If x is a variable and s is a sentence, then so are
- 6. Universal quantification, $\forall x\ s$
- 7. Existential quantification, $\exists x\ s$

Quantifiers

- Universal **quantification**
  - $\forall x\ P(x)$ means that P holds for **all** values of x in the domain associated with that variable
  - E.g., $\forall x\ \text{dolphin}(x) \rightarrow \text{mammal}(x)$
- Existential **quantification**
  - $\exists x\ P(x)$ means that P holds for **some** value of x in the domain associated with that variable
  - E.g., $\exists\ x\ \text{mammal}(x) \wedge \text{lays-eggs}(x)$
  - Permits one to make a statement about some object without naming it

Universal quantifiers

- "All students are smart"
- Define predicates: student(x), smart(x)

$\forall$x student(x)^smart(x)

or

$\forall$x student(x) $\rightarrow$ smart(x)

- Universal quantifiers are often used with $\rightarrow$ (implies) to form "rules":
  - $\forall$x student(x) $\rightarrow$ smart(x) means "All students are smart"
- Universal quantification is *rarely* used to make statements about *every* individual in the world:
  - $\forall$x student(x)^smart(x) means "Everyone in the world is a student and is smart"

Existential quantifiers

- “There is a student who is smart”
- Predicates: student(x), smart(x)

$\exists x$ student(x) ^ smart(x)

or

$\exists x$ student(x) $\rightarrow$ smart(x)

- Existential quantifiers are usually used with ^(and) to specify a list of properties about an individual:

  $\exists x$ student(x) ^ smart(x) means “There is a student who is smart”

- A common mistake:

  $\exists x$ student(x) $\rightarrow$ smart(x) to represent “There is a student who is smart”

  – Why? (what happens when there is a person who is *not* a student?)

Quantifier Scope

- Switching the order of universal quantifiers *does not* change the meaning:
  - $\forall x \, \forall y \; P(x,y) \;\leftrightarrow\; \forall y \, \forall x \; P(x,y)$
- Can switch the order of existential quantifiers:
  - $\exists x \exists y \; P(x,y) \;\leftrightarrow\; \exists y \exists x \; P(x,y)$
- Switching the order of universals and existentials *does* change meaning:
  - $\forall x \; \exists f \; food(f) \wedge \; likes\,(x,f)$ “Everybody likes some food”
  - $\exists f \; \forall x \; food(f) \wedge \; likes\,(x,f)$ “There is a (one specific) food that everyone likes”

Connections between All and Exists

Can rewrite sentences involving $\forall$ and $\exists$ using De Morgan's laws:

$$\forall x\, P(x) \leftrightarrow \exists x\, P(x)$$

$$\exists x\, P(x) \leftrightarrow \forall x\, P(x)$$

Translating English to FOL

Predicates

tall(x)
gardener(x)
likes(x, y) – x likes y
mushroom(x)
purple(x)
poisonous(x)
teacher(x)
wears-shorts(x)
wears-tshirt(x)
loves(x, y) – x loves y

Translating English to FOL

Clinton is not tall.

$\neg$tall(Clinton)

Every gardener likes the sun.

$\forall$x gardener(x) $\rightarrow$ likes(x,Sun)

All purple mushrooms are poisonous.

$\forall$x (mushroom(x) $\wedge$ purple(x)) $\rightarrow$ poisonous(x)

No purple mushroom is poisonous.

$\neg\exists$x purple(x) $\wedge$ mushroom(x) $\wedge$ poisonous(x)

$\forall$x (mushroom(x) $\wedge$ purple(x)) $\rightarrow$ $\neg$poisonous(x)

There is somebody whom everybody loves

$\exists$y $\forall$x loves(x, y)

In-Class Exercise

Write FOL sentences representing:

1. There are poisonous mushrooms.
2. No teacher wears shorts and t-shirt.
3. There is someone who Jake does not love

What about Time?

- Is Time represented? **No!**
- Can include time in propositions:

Explicit time: L(i,j,t) means agent at (i,j) at time t

Also: P(i,j,t) B(i,j,t)

Many more sentences: $O(TN^2)$

What about Actions?

- move(i ,j ,k ,l ,t)
  - Represents move from (i,j) to (k,l) at time t
  - E.g. Move(1, 1, 2, 1, 0): move from (1,1) to (2,1) at time 0

- What knowledge axioms capture the effect of an Agent's move?
  - $move(i, j, k, l ,t) \wedge L(i, j, t) \Rightarrow \sim L(i, j, t+1) \wedge L(k, l, t+1)$
  - For all tuples (i, j, k, l) that represent legitimate possible moves
    - E.g. (1, 1, 2, 1) or (1, 1, 1, 2)

- Some subtleties when representing time and actions
  - What happens to propositions at time *t+1* compared to at time *t*, that are *not* involved in any action?

The frame problem

- How to specify that all conditions ***not*** affected by an action are ***not changed*** while executing that action?
- In predicate logic, all such static properties need to be explicitly specified for every possible action
- **Frame axioms**
- Example:
  - If robot moves to (x,y) and is *not holding* a ball, then the location of the ball does *not* change
- What is the problem?
  - A very large number of frame axioms is often necessary

Qualification problem

- How to define every possible precondition of an action
- Including exceptions that might occur
- Example
  - When robot picks up the ball, it will hold the ball unless
    - The robot battery runs out, or
    - The gripper slips, or
    - The ball rolls away, or
    - Another robot picks the ball first, or …

Ramification problem

- How to define every possible effect of an action
- Example
  - When robot picks up the ball, all of these are possible
    - The battery drains a little bit, and
    - The gripper is closed, and
    - The ball does not roll, and
    - No other robot holds the ball, and …

Knowledge engineering

- Modeling the “right” conditions and the “right” effects at the “right” level of abstraction is difficult
- Automated knowledge acquisition and machine learning
  - Intelligent systems should be able to **learn** about the conditions and effects
  - Learn from?
  - Data: sensor measurements (of both causes and effects)