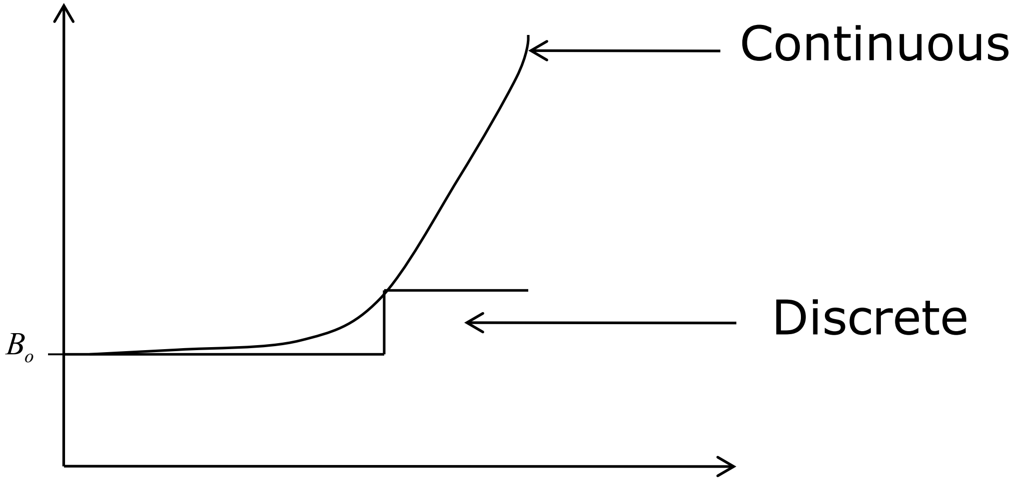
Section 1.2
Approximating Change with Difference Equations

Discrete Model Vs. Continuous Model

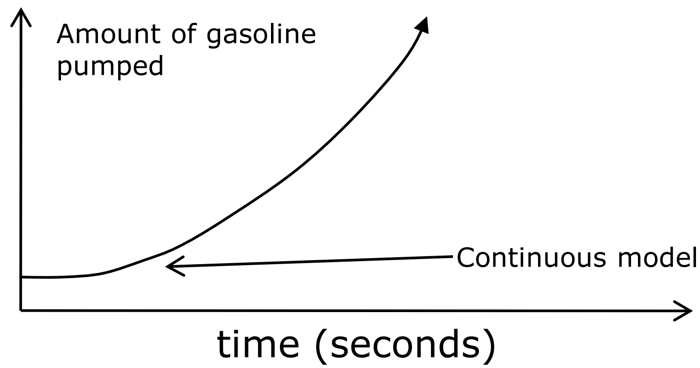
$$P(n+1) - P(n) = \Delta P \quad (Discrete)$$

$$P(t+\Delta t) - P(t) = \Delta P \quad (Continuous)$$



Discrete Model Vs. Continuous Model

Pumping gasoline into your vehicle



Remarks:

- One could be using one of the other, but the change must reflect the value difference from the current and previous time instance.
- Let us at the moment work with discrete models.

Working with discrete models

Various ways to model the change in the difference equation.

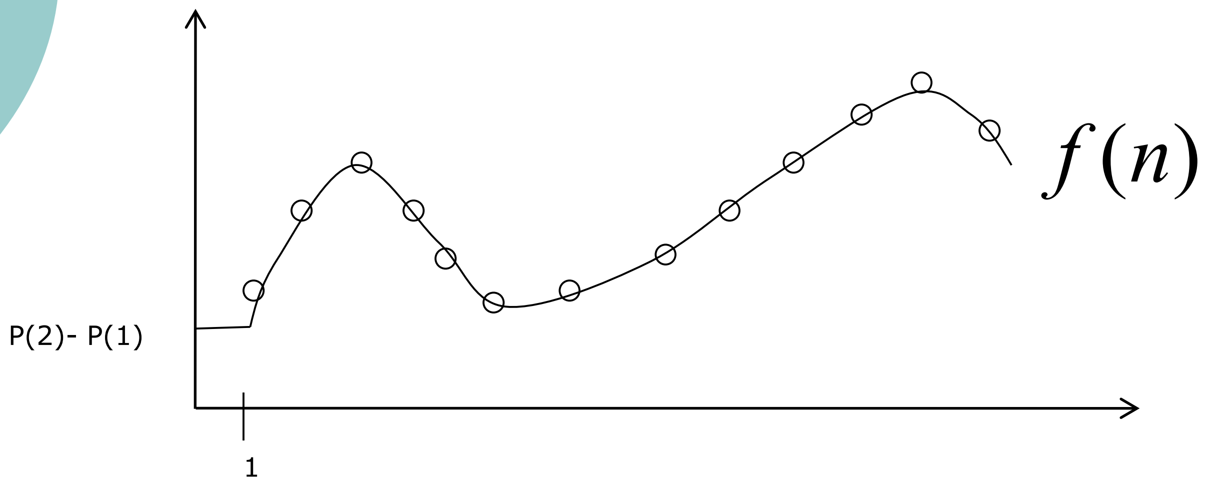
Given a set of data points,

$$\left\{P(n)\right\}_{n=1}^{N}$$

e.g. this is the number of people infected with the Swine Flu; where n is the number of days since the outbreak

Working with discrete models cont.

One can plot the difference of P(ΔP) as a function of n or P(n)



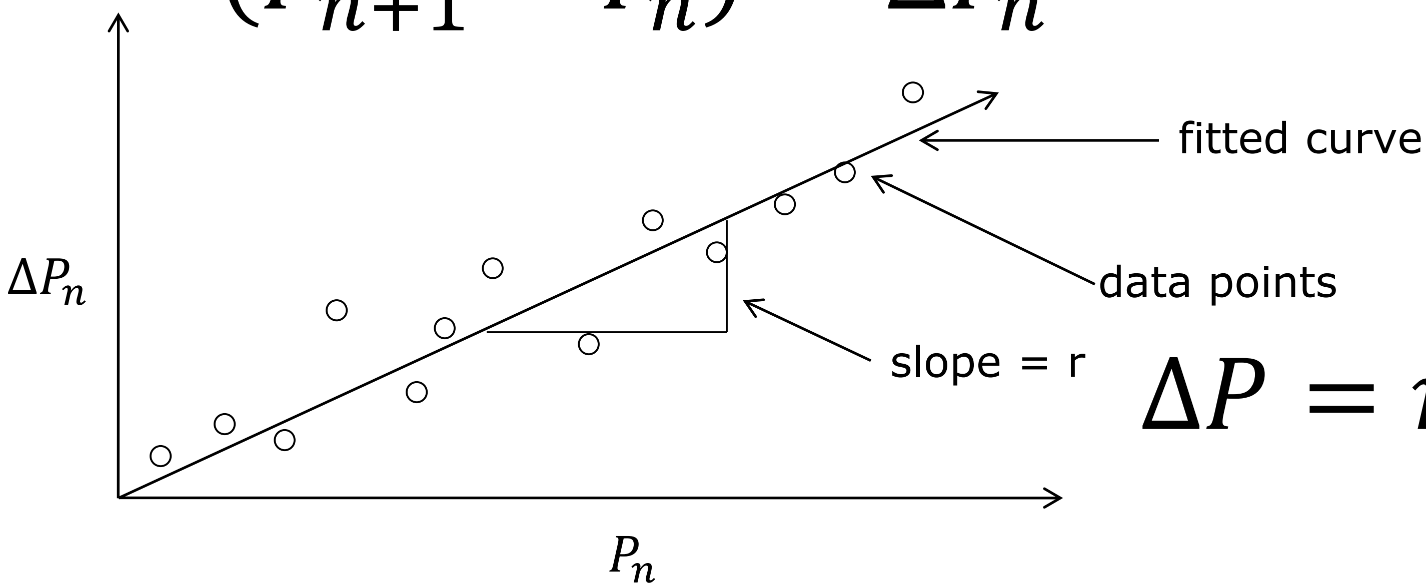
We assume $\Delta P = f(n)$.

Then one can solve this difference equation to get P(n) for n=1, 2, . . . , N

Model ΔP as a function of P

One can also model ΔP as a function of P.

Ex. $(P_{n+1} - P_n) = \Delta P_n$



$$\Delta P = r * P_n$$

Model ΔP as a function of P

So that

$$\Delta P = r * P_n$$

$$P_{n+1} - P_n = r * P_n$$

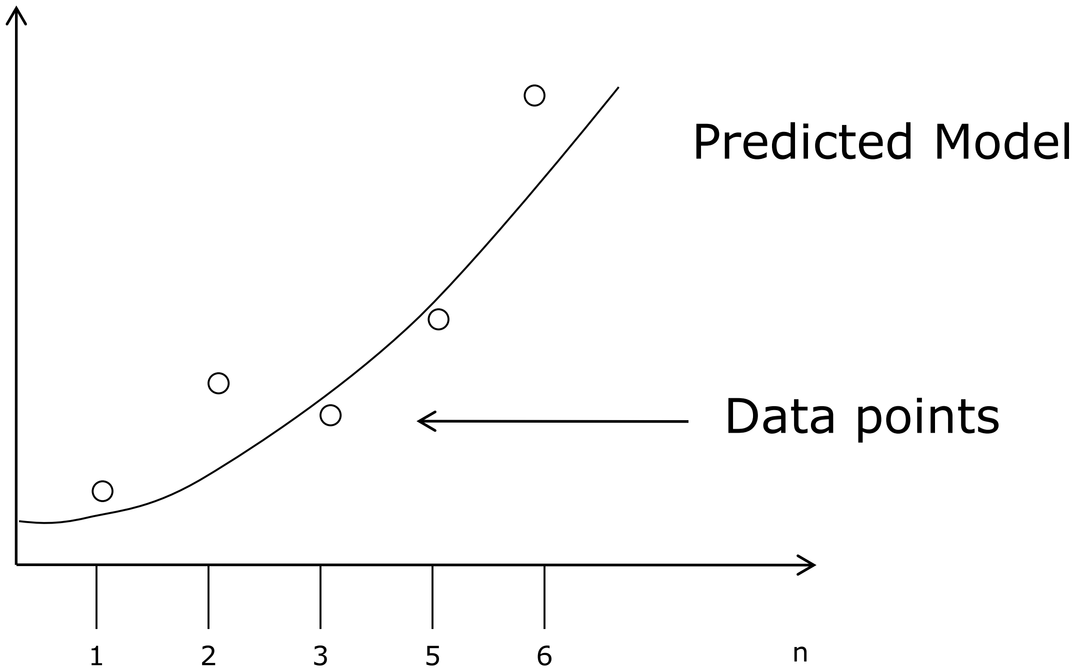
$$P_{n+1} = (1 + r)P_n$$

$$= (1 + r)P_{n-1}$$

$$\vdots$$

$$= (1 + r)^{n+1} P_0$$

Model ΔP as a function of P



Model ΔP as a function of P (+MATLAB)

Time in hours n	Observed yeast biomass p_n	Change in biomass $p_{n+1}-p_n$
0	9.6	8.7
1	18.3	10.7
2	29.0	18.2
3	47.2	23.9
4	71.1	48.0
5	119.1	55.5
6	174.6	82.7
7	257.3	

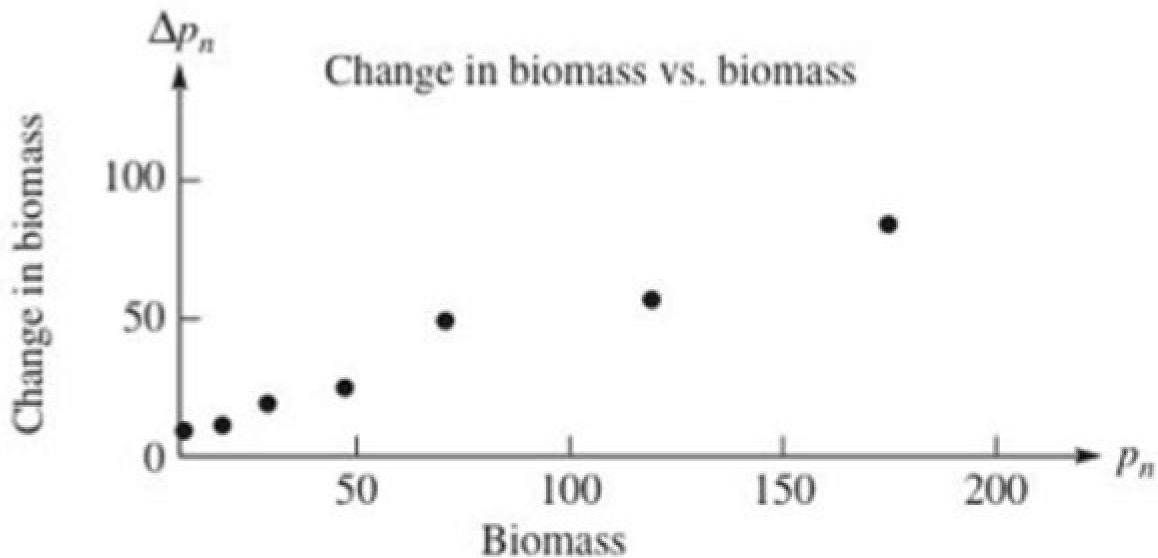


Figure 1.7

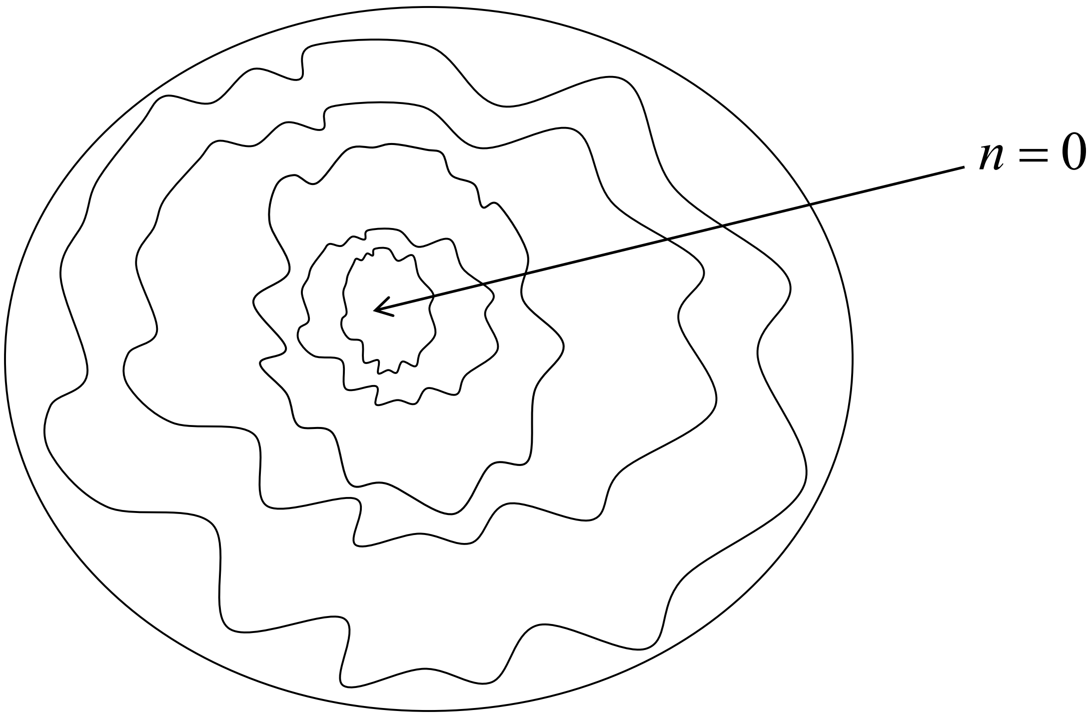
Growth of a yeast culture versus time in hours; data from R. Pearl, "The Growth of Population," *Quart. Rev. Biol.* 2(1927): 532–548

$$\Delta p_n = p_{n+1} - p_n = 0.5p_n \qquad p_{n+1} = 1.5p_n.$$

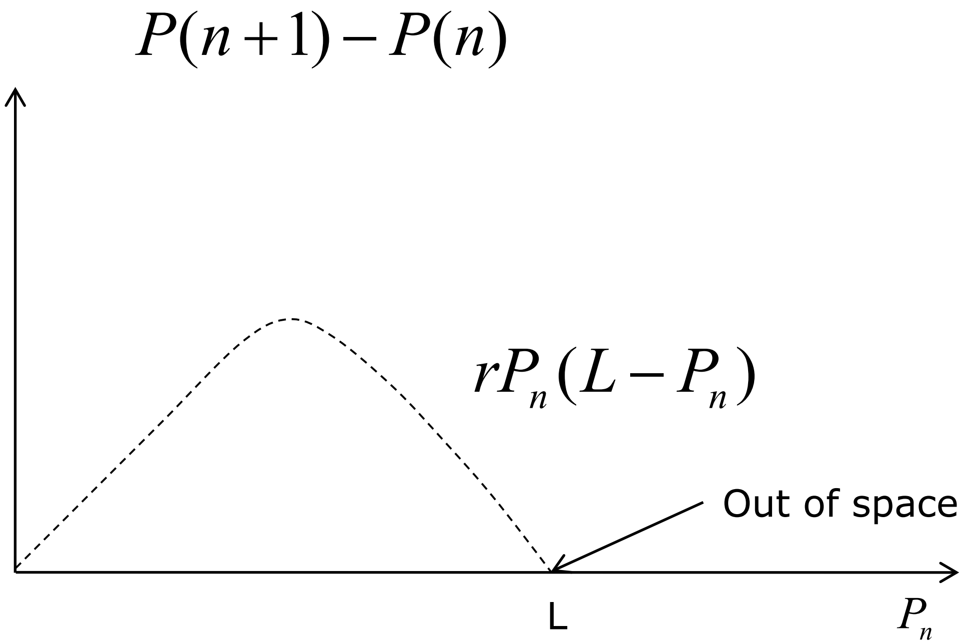
Discrete Limited Growth Model

The previous example has P_n grow indefinitely. That is not the case for bacteria or animal species.

biomass:



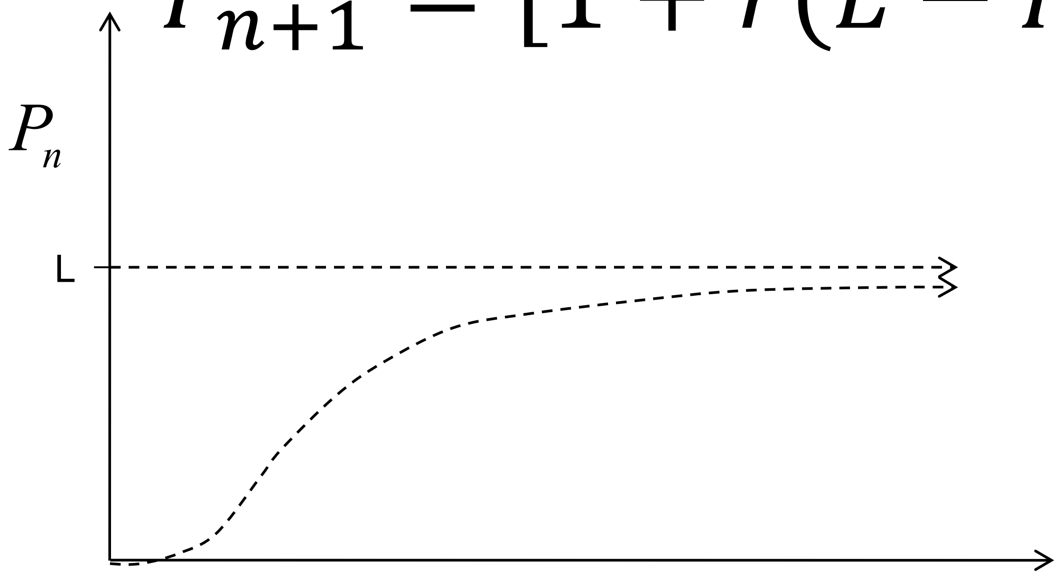
Discrete Limited Growth Model (cont.)



Discrete Limited Growth Model (cont.)

$$P_{n+1} - P_n = \Delta P_n = rP_n(L - P_n)$$

$$P_{n+1} = [1 + r(L - P_n)] * P_n$$



Discrete Limited Growth Model (cont.)

Time in hours n	Yeast biomass p_n	Change/hour $p_{n+1} - p_n$
0	9.6	8.7
1	18.3	10.7
2	29.0	18.2
3	47.2	23.9
4	71.1	48.0
5	119.1	55.5
6	174.6	82.7
7	257.3	93.4
8	350.7	90.3
9	441.0	72.3
10	513.3	46.4
11	559.7	35.1
12	594.8	34.6
13	629.4	11.4
14	640.8	10.3
15	651.1	4.8
16	655.9	3.7
17	659.6	2.2
18	661.8	

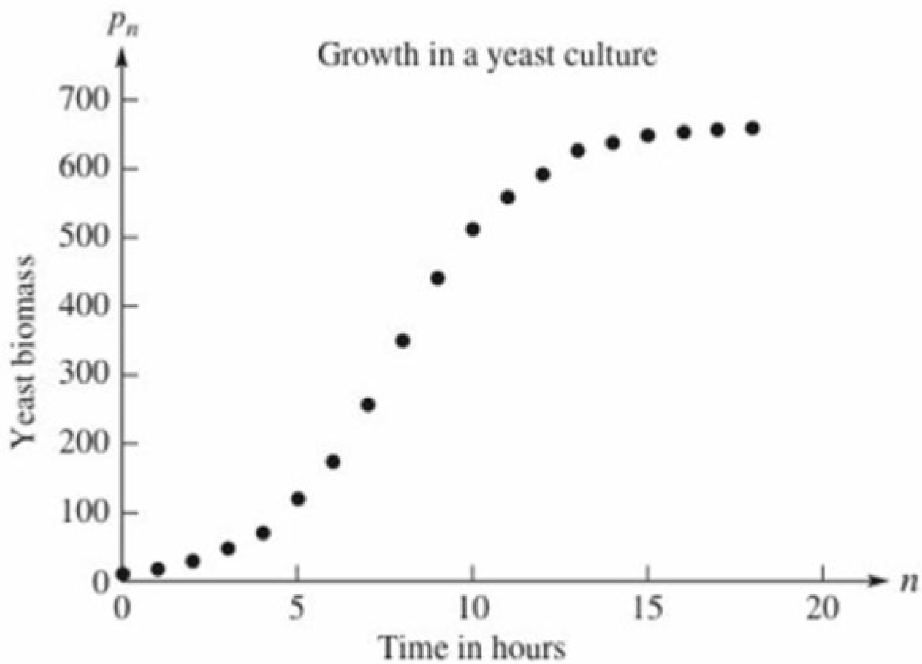


Figure 1.8

Yeast biomass approaches a limiting population level

Discrete Limited Growth Model (cont.)

Yeast biomass approaches a limiting population level

$p_{n+1} - p_n$	$p_n(665 - p_n)$
8.7	6291.84
10.7	11,834.61
18.2	18,444.00
23.9	29,160.16
48.0	42,226.29
55.5	65,016.69
82.7	85,623.84
93.4	104,901.21
90.3	110,225.01
72.3	98,784.00
46.4	77,867.61
35.1	58,936.41
34.6	41,754.96
11.4	22,406.64
10.3	15,507.36
4.8	9050.29
3.7	5968.69
2.2	3561.84

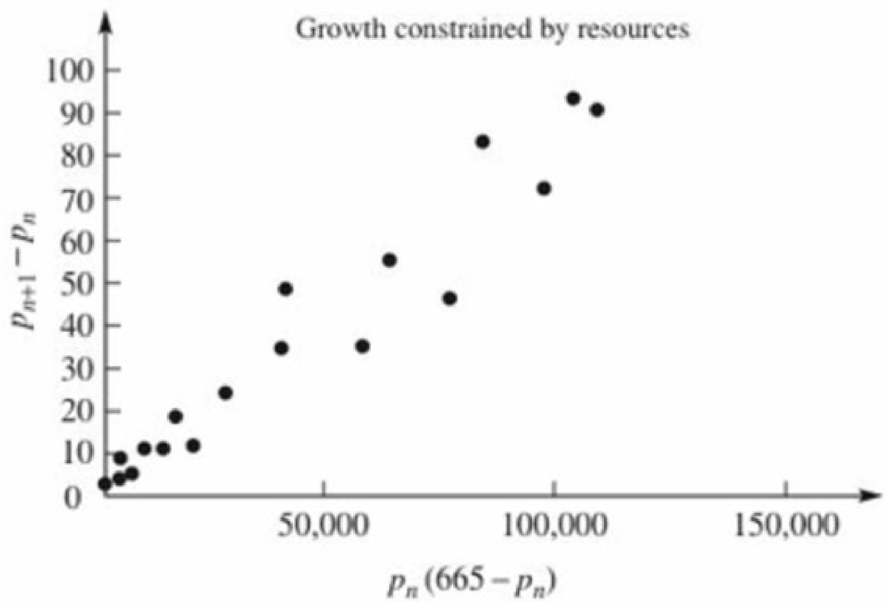


Figure 1.9

Testing the constrained growth model

$$k \approx 0.00082,$$

$$p_{n+1} - p_n = 0.00082(665 - p_n)p_n$$

$$p_{n+1} = p_n + 0.00082(665 - p_n)p_n$$

Homework –Section 1.2

- # 1, 2, 6, 7, 9