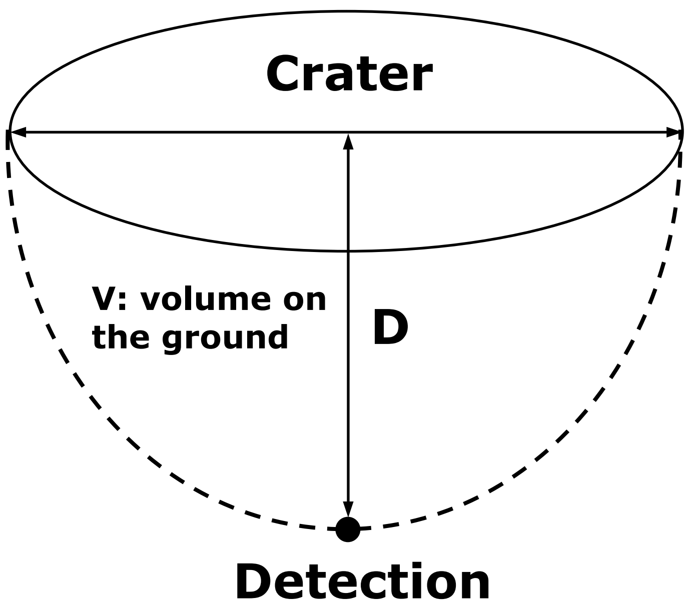
Section 14.4
Examples Illustrating Dimensional Analysis

Example 1
Explosion Analysis



Model the power P of the explosion based on the

Volume	V	(L^3)
Depth	D	(L)
Density of soil	ρ	(M/L^3)
Gravity	g	(L/T^2)
Power	P	(ML^2/T^3)

We have 5 derived variables and 3 principal dimensions.

2 Dimensionless Π-Groups

Generators are D,ρ,g

$$\underbrace{\det\begin{array}{c|c} \begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} & \begin{matrix} M \\ L \\ T \end{matrix} \end{array}}$$

2≠0

2 Dimensionless Π-Groups

My π-groups are

$$\left[V\right]=\left[\pi_1 D^{\alpha}\rho^{\beta}g^{\gamma}\right],$$

$$\left[P\right]=\left[\pi_2 D^{\alpha}\rho^{\beta}g^{\gamma}\right].$$

2 Dimensionless Π-Groups

For π_1;

$$L^3 = L^{\alpha} \frac{M^{\beta}}{L^{3\beta}} \frac{L^{\gamma}}{T^{2\gamma}} = \frac{L^{\alpha+\gamma-3\beta} \quad M^{\beta}}{T^{2\gamma}} \quad .$$

2 Dimensionless Π-Groups

$$\gamma = 0 \,,$$

$$\beta = 0 \,,$$

Therefore, $$\alpha = 3 \,.$$

2 Dimensionless Π-Groups

Then,

$$V = \pi_1 D^3 \rho^0 g^0 \quad .$$

So,

$$\pi_1 = \frac{V}{D^3} \quad .$$

2 Dimensionless Π-Groups

For π_2;

$$\frac{M \quad L^2}{T^3} = L^{\alpha} \frac{M^{\beta}}{L^{3\beta}} \frac{L^{\gamma}}{T^{2\gamma}} = \frac{L^{\alpha+\gamma-3\beta} \quad M^{\beta}}{T^{2\gamma}} .$$

2 Dimensionless Π-Groups

$$\gamma = {}^{3}\!/_{2} \quad ,$$

$$\beta = 1 \quad ,$$

Therefore, $\alpha + \frac{3}{2} - 3 = 2$

$$\Rightarrow \quad \alpha = 5 - \frac{3}{2} = {}^{7}\!/_{2} \quad .$$

2 Dimensionless Π-Groups

Then,

$$P = \pi_2 D^{7/2} \rho^1 g^{3/2} \quad .$$

So,

$$\pi_2 = \frac{P}{D^{7/2} \quad \rho \quad g^{3/2}} = \frac{P}{D^3 \quad \rho \quad \sqrt{D \quad g^3}} \quad .$$

2 Dimensionless Π-Groups

$\therefore$ By the B-π Theorem,

$$\pi_2 = \Phi(\pi_1)$$

or

$$P = D^3 \rho \sqrt{D \quad g^3}\, \Phi\left(\frac{V}{D^3}\right) .$$

Redefined Π-Groups

One can redefine π_2 as follows (get rid of D);

$$\pi_1 = \frac{V}{D^3} \qquad \pi_2 = \frac{P}{D^3 \;\; \rho \;\; \sqrt{D \;\; g^3}}$$

$$\tilde{\pi}_2 = \frac{\pi_2}{\pi_1^{7/6}} = \frac{P}{\rho V^{7/6} g^{3/2}}$$

Redefined Π-Groups

So, again by the B-π Theorem we have,

$$P = V^{7/6} \rho g^{3/2} \Phi\left(\frac{V}{D^3}\right) .$$

Redefined Π-Groups

Suppose one is interested in the volume of impact, then we can have

$$\pi_2 = \frac{P}{V^{7/6} \quad \rho \quad g^{3/2}} \quad ,$$

$$\Rightarrow \quad \pi_2' = \left(\frac{1}{\pi_2}\right)^{6/7} = \frac{V \quad \rho^{6/7} \quad g^{9/7}}{P^{6/7}} \quad .$$

Redefined Π-Groups

So, by the B-π Theorem again,

$$V = \sqrt[7]{\frac{P^6}{\rho^6 \quad g^9}}\,\Phi\left(\frac{V}{D^3}\right) .$$

Homework –Section 14.4

- # 4, 6a, 7a, 8