NAME: (Signature) ______ Amelia Rotondo - 887925113

Problem	Points
1	/25
2	/25
3	/25
Total	/75

INSTRUCTIONS:

1. Any form of collaboration on the exam is strictly forbidden. All work must be yours and yours alone. Include this sheet with your signature attesting that you did not receive any help on the exam.
2. You may use any of the built-in MATLAB functions, and the help files, as well as the course textbook and notes. Use of the internet is not allowed.
3. Clearly mark the number of the problem.
4. Show all details of your work. Include any code used in your solutions, as well as all figures and explanations.
5. Compile your exam into a single pdf file before submitting to Canvas.
6. The exam is due at 1:00 PM, Tuesday, Oct 3, 2023.

1. The point of this exercise is to illustrate an interesting phenomenon about eigenvalues known as "avoidance of crossing." Recall that if A is a symmetric matrix, the spectral theorem says that the eigenvalues of A are real numbers. Another result says that "generically" the eigenvalues are distinct. That is, with probability 1, if the entries are chosen at random, the eigenvalues will be distinct. To illustrate this, we will choose two matrices at random, A and B, and calculate the eigenvalues of

$$A + tB$$

The eigenvalues $\lambda_i(t)$ are functions of t. We will plot them as a function of t from $t = 0$ to $t = 1$.

Write a code to do the following:

(a) Choose two different matrices, A and B, at random, and then make symmetric matrices, by

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A = randn(10);
A = A + A';
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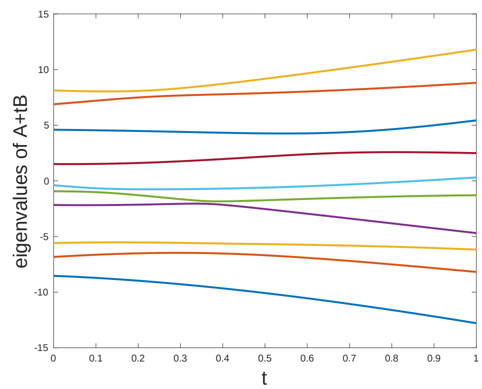
and similarly for B.

(b) Make a vector t of values between 0 and 1. For each entry in t, calculate the eigenvalues of $A+tB$.

(c) Plot the eigenvalues as a function of t. You should get a figure that looks something like this:



Notice that sometimes the eigenvalue trajectories look they are going to cross, but veer away at the last moment. This is "avoidance of crossing."

2. The following are census figures for the population of the U.S. in millions in the years 1910, 1920, . . . , 2010:

1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
91.97	105.71	122.78	131.67	151.33	179.32	203.30	226.54	248.72	281,42	308.75

(a) Plot the data, marked with circles. On the same graph, plot the polynomial of degree 10, the shape-preserving pchip, and the spline interpolating the points.

(b) Plot the polynomial to the year 2020. Determine the year of "doomsday."

(c) Which of these interpolations would you trust? Which would you not trust, and why?

(d) Based on the data, estimate the population in 2005.

3. *Halley's method* is another iterative method for solving $f(x) = 0$. The Halley iteration formula for finding a simple zero is $x_{n+1} = g(x_n)$, where

$$g(x) = x - \frac{2f(x)f'(x)}{2[f'(x)]^2 - f(x)f''(x)}$$

(a) Use Halley's method to find a solution of $2 - e^x = 0$.

(b) Compare the performance of Halley's method with that of Newton's method by computing the iterates in both, starting from the same initial guess.

(c) From the previous part, deduce the order of convergence of Halley's method.

(d) Show that Halley's method for solving $a - e^x = 0$ is

$$x_{n+1} = x_n + 2\left(\frac{ae^{-x_n} - 1}{ae^{-x_n} + 1}\right)$$

Comment on this as a practical means for computing $\log a$.