Section 14.5
Similitude

SIMILITUDE

Small Model:	**Real-Size Model:**
• Easy to model • Cheaper • Tractable	• Too costly • May not be implementable the first time

SIMILITUDE

<u>Idea</u>: If the 2 models (small vs. real) are similar, then all dimensionless π-groups must be equal.

<u>Ex</u> (cont): Detonating the same bomb on Earth vs. one on the Moon. How bigger/smaller is the size?

Solution:

$$\pi_2^{moon} = \pi_2^{earth}$$

$$\frac{P_{moon}}{V_{moon}^{7/6} \quad \rho_{moon} \quad g_{moon}^{3/2}} = \frac{P_{earth}}{V_{earth}^{7/6} \quad \rho_{earth} \quad g_{earth}^{3/2}}$$

Solution:

Suppose $g_e = 6g_m$, $\rho_e = 3\rho_m$.

$$V_m^{7/6} = V_e^{7/6} \frac{\rho_e}{\rho_m} \left(\frac{g_e}{g_m} \right)^{3/2}$$

$$\Rightarrow \quad V_m^{7/6} = V_e^{7/6} \frac{3\rho_m}{\rho_m} \left(\frac{6g_m}{g_m} \right)^{3/2}$$

Solution:

So,

$$V_m = \left(6^{3/2} \bullet 3\right)^{6/7} V_e = 25.67 \bullet V_e$$

Part (b) of explosive analysis example

If the same dynamite is exploded on the moon and its depth is measured to be 7 times of that on Earth. How is soil density of the Moon compared to that on Earth?

- Solution:

$$\Pi_2^{\mathrm{Moon}} = \Pi_2^{\mathrm{Earth}}$$

$$\frac{P_{\mathrm{M}}}{\sqrt{\mathrm{D}_{\mathrm{M}}^7 \rho_{\mathrm{M}}^2 g_{\mathrm{M}}^3}} = \frac{P_{\mathrm{E}}}{\sqrt{\mathrm{D}_{\mathrm{E}}^7 \rho_{\mathrm{E}}^2 g_{\mathrm{E}}^3}}$$

Now we solve for ρ_{Ml}

$$\rho_{\mathrm{M}} = \rho_{\mathrm{E}} \frac{P_{\mathrm{M}}}{P_{\mathrm{E}}} \sqrt{\frac{\mathrm{D}_{\mathrm{E}}^7 g_{\mathrm{E}}^3}{\mathrm{D}_{\mathrm{M}}^7 g_{\mathrm{M}}^3}}$$

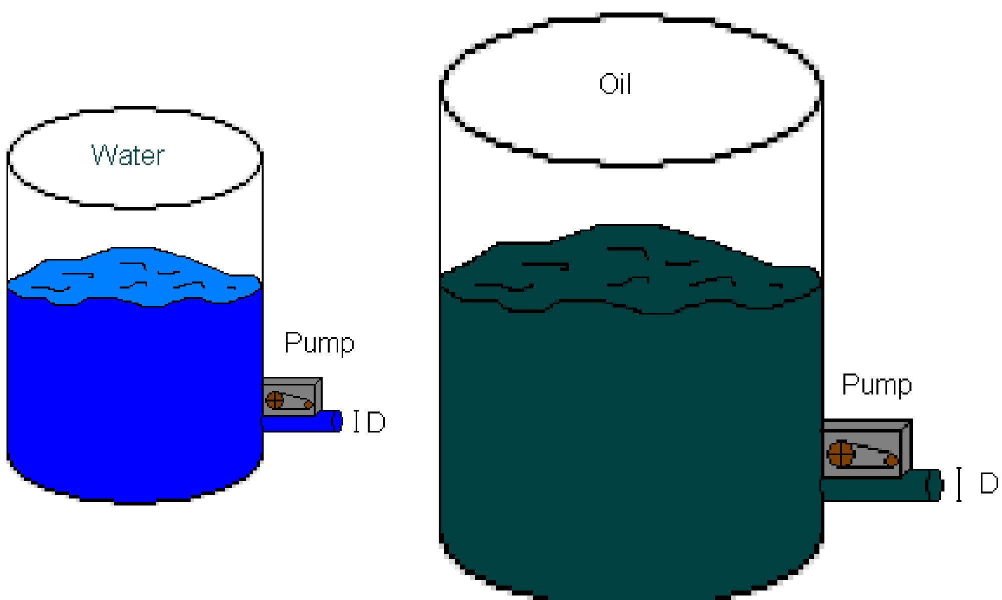
$$= \rho_{\mathrm{E}} \frac{P_{\mathrm{M}}}{P_{\mathrm{E}}} \left(\frac{\mathrm{D}_{\mathrm{E}}}{\mathrm{D}_{\mathrm{M}}}\right)^{7/2} \left(\frac{g_{\mathrm{E}}}{g_{\mathrm{M}}}\right)^{3/2}$$

$$= \quad \rho_{\mathrm{E}} \bullet 1 \bullet \left(\frac{1}{7}\right)^{7/2} \bullet \left(\frac{6}{1}\right)^{3/2} = \sqrt{\frac{6^3}{7^7}} \rho_{\mathrm{E}} = 0.0162 \bullet \rho_{\mathrm{E}}$$

So, the density of the area of impact on the moon is a hundredth of that of Earth.

Similitude

In many physical and engineering applications where the scaled model is too EXPENSIVE to build, the concept of similitude is extremely helpful in using the prototype to predict the true-scale values.



Example: Suppose the six derived variables are given as ρ, μ, L, D, V and P. There are three principle units L, M and T. L is the length the oil or water must be pumped, D is the diameter of the tube, V is the velocity it is pumped at, and P is the power of the pump. ρ and μ are the density and bulk viscosity of the oil and water.

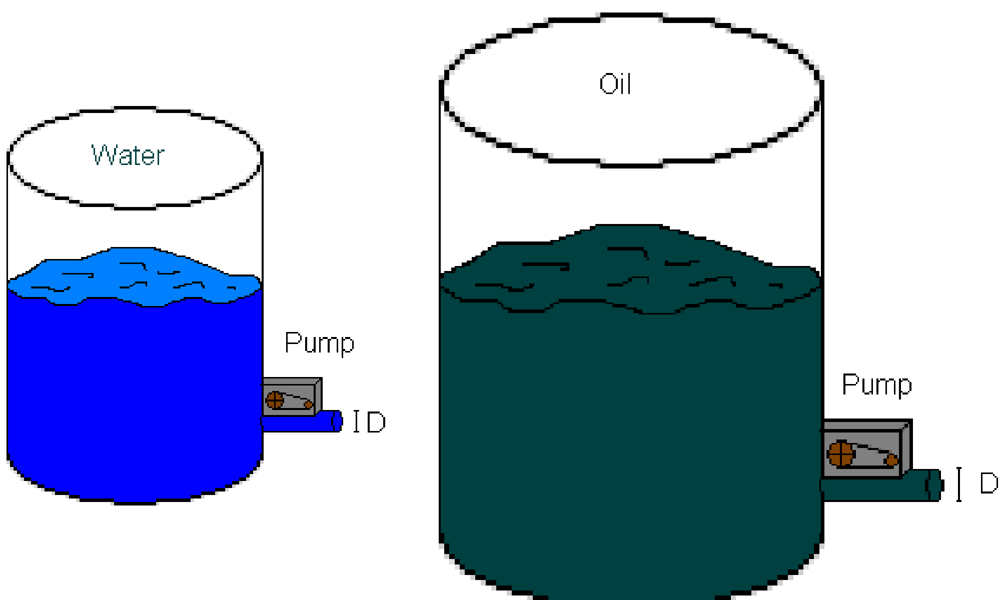
a) Given the $l_{\text{oil}} = 1\text{ km}$, $D_{\text{oil}} = 2.5\text{ m}$ and $D_{\text{water}} = 0.05\text{ m}$,

What is the l_{water}?

b) If $\mu_{\text{water}} = 0.98\mu_{\text{oil}}$, $\rho_{\text{water}} = \dfrac{10}{7}\rho_{\text{oil}}$, $V_{\text{water}} = 10\text{ m/s}$

what is V_{oil} ?

c) If Power to pump water at 10 m/s is 9.7 kwatts, what is the actual power required to pump oil?



In this case we choose D , μ , ρ
as our principal generators:

$$\begin{array}{c|ccc} & D & \rho & \mu \\ \hline L & 1 & 3 & -1 \\ M & 0 & 1 & 1 \\ T & 0 & 0 & -1 \end{array}$$

$$\begin{array}{c} \\ L \\ M \\ T \end{array} \begin{array}{c} \begin{array}{ccc} D & \rho & \mu \end{array} \\ \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

We can show that:

$$\Pi_1 = \frac{l}{D}, \Pi_2 = \frac{\rho V D}{\mu}, \Pi_3 = \frac{P\rho^2 D}{\mu^3}$$

a) Given the $l_{\text{oil}} = 1\ \text{km}$, $D_{\text{oil}} = 2.5\ \text{m}$ and $D_{\text{water}} = 0.05\ \text{m}$,

What is the l_{water}?

Solution:

$$\frac{l_{\text{oil}}}{D_{\text{oil}}} \overset{\Pi_1}{=} \frac{l_{\text{water}}}{D_{\text{water}}} \Rightarrow l_{\text{water}} = l_{\text{oil}} \left(\frac{D_{\text{water}}}{D_{\text{oil}}} \right)$$

$$= 1000\text{m} \left(\frac{0.05\text{m}}{2.5\text{m}} \right) = 20\text{m}$$

b) If $\mu_{\text{water}} = 0.98\mu_{\text{oil}}$, $\rho_{\text{water}} = \frac{10}{7}\rho_{\text{oil}}$, $V_{\text{water}} = 10$ m/s

what is V_{oil} ?

$$\frac{\rho_{\text{water}} V_{\text{water}} D_{\text{water}}}{\mu_{\text{water}}} \overset{\Pi_2}{=} \frac{\rho_{\text{oil}} V_{\text{oil}} D_{\text{oil}}}{\mu_{\text{oil}}}$$

Now we solve for V_{oil} :

$$V_{\text{oil}} = V_{\text{water}} \left(\frac{\rho_{\text{water}}}{\rho_{\text{oil}}} \right) \left(\frac{D_{\text{water}}}{D_{\text{oil}}} \right) \left(\frac{\mu_{\text{oil}}}{\mu_{\text{water}}} \right)$$

$$= 10 \left(\frac{10}{7} \right) \left(\frac{0.05}{2.5} \right) \left(\frac{1}{0.98} \right)$$

$$= 0.28 \text{ m/s}$$

c) If Power to pump water at 10 m/s is 9.7 kwatts, what is the actual power required to pump oil?

$$\frac{P_{water}\rho_{water}^{2}D_{water}}{\mu_{water}^{3}} \overset{\Pi_3}{=} \frac{P_{oil}\rho_{oil}^{2}D_{oil}}{\mu_{oil}^{3}}$$

Now we solve for P_{oil} :

$$P_{oil} = P_{water}\left(\frac{\rho_{water}}{\rho_{oil}}\right)^{2}\left(\frac{D_{water}}{D_{oil}}\right)\left(\frac{\mu_{oil}}{\mu_{water}}\right)^{3}$$

$$= 9.7\left(\frac{10}{7}\right)^{2}\left(\frac{0.05}{2.5}\right)\left(\frac{1}{0.98}\right)^{3}$$

$$= 0.420656 \quad \text{kwatts}$$

Homework

1) Section 14.5 problems 1 & 2
2) Derive the dimensionless pi-groups for the pump power.
3) In an example with a prototype and a full-scale hydroplane, consider F, ρ, V, L, and g. Given that they are both on water in the same gravity and the full-scale one is 25 times the length of the prototype along with:

$$V_{prot} = 6\frac{m}{s} \qquad F_{prot} = 1.8N$$

a) What is the velocity of the full-scale model?
b) What is the corresponding force at that speed?

Since we wish to find V and F we get the Π-groups to be:

$$\Pi_1 = \rho^{\alpha_1} L^{\beta_1} g^{\gamma_1} V, \quad \Pi_2 = \rho^{\alpha_2} L^{\beta_2} g^{\gamma_2} F$$