Section 14.5
Similitude (Model Similarity)
Continue

Modeling and Similitude in Astrodynamics

- A satellite moves in a trajectory around Earth. It is hypothesized that the period ***T*** depends upon its distance ***r,*** the mass of Earth ***m***, and the universal gravitational constant ***G***.

  a/ Use Buckingham-π theorem to model ***T*** as function of ***r***, ***m***, and ***G***.

Solution

Derived variables

T ~ Period ~ T

r ~ Distance ~ L

m ~ mass ~ M

G ~ Universal gravitational constant ~ L^3 / MT^2

4 derived variables
-
3 principle dimensions

1 dimensionless π-group

Choose *r, m,* and *G* as generators for the dimensions.

Solution

Check

$$\begin{array}{c c} & \begin{array}{ccc} \boldsymbol{r} & \boldsymbol{m} & \boldsymbol{G} \end{array} \\ \begin{array}{c} \boldsymbol{L} \\ \boldsymbol{M} \\ \boldsymbol{T} \end{array} & \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & -2 \end{bmatrix} \end{array}$$

det (Matrix) = -2,

Thus, the chosen variables are linearly independent.

Solution

The remaining derived variable can be modeled (generated) by by the dimensional basis elements.

$$T = \pi_1 r^{\alpha} m^{\beta} G^{\gamma}$$

$$\left[r^{\alpha}\right]\!.\left[m^{\beta}\right]\!.\left[G^{\gamma}\right] = L^{\alpha} M^{\beta} \left(\frac{L^3}{MT^2}\right)^{\gamma} = \frac{L^{\alpha+3\gamma} M^{\beta-\gamma}}{T^{2\gamma}}$$

Solution

$\pi_1 :$

$$\left.\begin{aligned} [T] = T \Rightarrow \quad \alpha + 3\gamma &= 1 \\ \beta - \gamma &= 0 \\ \gamma &= -\frac{1}{2} \end{aligned}\right\} \alpha = \frac{3}{2},\ \beta = -\frac{1}{2},\ \gamma = -\frac{1}{2}$$

$$T = \pi_1 r^{\frac{3}{2}} m^{-\frac{1}{2}} G^{-\frac{1}{2}}$$

$$T = \pi_1 r \sqrt{\frac{r}{mG}} \text{ and } \quad \pi_1 = r\sqrt{\frac{r}{mG}}$$

2. Modeling and Similitude in Astrodynamics

b/ If the same satellite is moving around Mars at the same altitude with the mass of Mars is roughly 1/10 that of Earth, find the relative period of the satellite around Mars.

Solution

- **Earth Model**

$$r$$

$$m_E$$

$$T_E$$

- **Mars Model**

$$r$$

$$m_M = \frac{1}{10} m_E$$

$$T_M$$

Solution

Assume the two models are similar. Then

$$\pi^E = \pi_1^M$$

$$\frac{T_E}{r}\sqrt{\frac{m_E G}{r}} = \frac{T_M}{r}\sqrt{\frac{m_M G}{r}}$$

$$T_E\sqrt{m_E} = T_M\sqrt{m_M}$$

$$T_M = T_E\sqrt{\frac{m_E}{m_M}} = T_E\sqrt{\frac{m_E}{\frac{1}{10}m_E}}$$

$$T_M = \sqrt{10}T_E = 3.16T_E$$