Section 14.1
Dimensions
& Dimensional Compatibility

Remarks

In many physical and engineering applications, most quantities can be expressed in terms of the three fundamental units: mass(M), Length(L) and time(T).

Each quantity should have a unit and an associated dimension. Otherwise, it is called a dimensionless unit.

Notation

Let us denote by [x], the dimension operator of x.

Example:

$$[\text{distance}] = L \qquad [\text{Area}] = L^2$$

$$[\text{density}] = \left[\frac{M}{V}\right] = \frac{M}{L^3} \qquad [\text{speed}] = \frac{L}{T}$$

$$[\text{weight}] = [\text{mass} \times \text{gravity}] = \left[M\frac{m}{s^2}\right] = M\frac{L}{T^2}$$

Remarks: Units and dimensions help to:

(a) keep mathematical equations and models consistent. [LHS]=[RHS]

(b) pinpoint missing terms or units

(c) eliminate extra unrelated terms in equation

Remarks

$$[-\mathrm{x}] = [\mathrm{x}]$$

$$[\mathrm{x}y] = [x][y]$$

$$[x / y] = [x]/[y]$$

$$[\mathrm{x}^{\alpha}] = [\mathrm{x}]^{\alpha}$$

$$[\mathrm{C}] = 1, \text{ for any number C}$$

$$[C\mathrm{x}] = [\mathrm{x}], \text{ for any number C}$$

$$\text{if } [\mathrm{x}] = [y] = D, \text{ then } [\mathrm{x} \pm y] = [x] = [y] = D$$

$$[0] = \text{any dimensions}$$

Dimensions of Derivatives

$[\frac{dv}{dt}]$=[change in velocity]/[change time]=[V]/T=[distance/time]/T=L/T/T=L/T^2

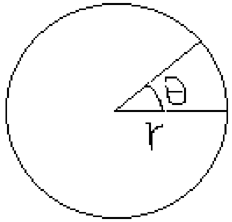
$$[\frac{d^2v}{dx^2}]=[\frac{d}{dx}(\frac{dv}{dx})]=[\frac{d(\frac{dv}{dx})}{dx}]=\frac{[d(\frac{dv}{dx})]}{[dx]}=\frac{[\frac{dv}{dx}]}{[dx]}=\frac{\frac{[dv]}{[dx]}}{[dx]}=\frac{[v]}{[dx]^2}=\frac{\frac{L}{T}}{L^2}=\frac{1}{LT}$$

In general: $\left[\frac{d^n f}{dz^n}\right]=\frac{[f]}{[z]^n}$

$$[\frac{\partial^2 s}{\partial x \partial t}]=[\frac{\partial}{\partial x}(\frac{\partial s}{\partial t})]=\frac{[\frac{\partial s}{\partial t}]}{[x]}=\frac{\frac{[s]}{[t]}}{[x]}=\frac{[s]}{[x][t]}=\frac{\frac{M}{L^3}}{L\cdot T}=\frac{M}{L^4 T}$$

Example

What is the dimension of an angle $[\theta]$?



$$[\theta] = \left[\frac{arclength}{radius}\right] = \frac{[arclength]}{[radius]} = \frac{L}{L} = 1$$

Example

Let P denote pressure and z denote length.

What is the dimension of $\dfrac{\partial P}{\partial z}$?

$$\left[\frac{\partial P}{\partial z}\right] = \frac{[\partial P]}{[\partial z]} = \frac{[P]}{[z]} = \frac{[Pascal]}{L} = \frac{\left[{}^{N}\!/_{m^2}\right]}{L} = \frac{\left[{}^{kg \cdot m}\!/_{s^2 m^2}\right]}{L} = \frac{M}{T^2 L^2}$$

<u>Note</u>: Pressure = $\dfrac{Force}{Area}$

Example

Find the dimension of the viscosity υ, where

$$\frac{\partial v}{\partial t} = -\upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

v is velocity, t is time and x, y are spatial variables.

Definition of Model…

- A model (equation) is dimensionally correct, consistent, homogeneous, or compatible if all terms in the equation (model) have the same physical dimension.

Example

Justify whether the model is dimensionally correct.

$$m\frac{dv}{dt} - F = 0$$

where m ~ mass, v ~ velocity, t ~ time, F ~ force

Example

Justify whether the model is dimensionally correct.

$$v^2 = \left(\frac{x}{t}\right)^2 + \left(\frac{1}{2}\right) g t^2$$

where v ~ velocity, x ~ distance, t~ time, g ~ gravity

Example

Justify whether the model is dimensionally correct.

$$\frac{de}{dt} = \frac{15}{18}\gamma\frac{Ne}{n}e\sqrt{1-e^2}\,sin^2(i)sin(2\omega)$$

where e ~ eccentricity of satellite (dimensionless)

i ~ inclination angle of a satellite

ω ~ angle of pengee

γ ~ mass ratio $\frac{M_{earth}}{M_{earth}+M_{moon}}$

G ~ universal gravitational constant $\left(\frac{L^3}{MT^2}\right)$

$$N_e = \sqrt{\frac{G(M_{earth} + \ M_{moon})}{a^3{}_{moon}}} \qquad n = \sqrt{\frac{GM_{moon}}{a^3{}_{satellite}}}$$

Example

Example

Find the dimension of k so that the model is dimensionally correct.

$$m\frac{d^2x}{dt^2} + kx = 0$$

Two types of fluid viscosity

a) Bulk Viscosity : $[\mu] = \dfrac{M}{LT}$

b) Kinematic Viscosity : $[\nu] = \dfrac{[\mu]}{[\rho]} = \dfrac{M/LT}{M/L^3} = \dfrac{L^2}{T}$

Specific Weights

$$\left[\frac{\text{Weight}}{\text{Volume}}\right] = \frac{ML/T^2}{L^3} = \frac{M}{L^2T^2}$$

Homework –Section 14.1

- # 1, 3, 5, 6