NAME: ____

Problem	Points
1	/20
2	/20
3	/20
4	/20
5	/20
BONUS	/10
Total	/100

INSTRUCTIONS:

1. Answer the following 5 problems. If you have time, attempt the bonus problem.
2. Write your answers in the space provided. If you do not have enough space, continue on the back side of the *previous* page.
3. Show all details of your work. Answers without justification will receive zero points.
4. Neither notes, books nor calculators are allowed in the exam. You may use a $3'' \times 5''$ notecard.
5. Relax. Think before (and after) doing.

1. (a) Show that $\mathbf{F}$ is conservative.

   (b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

$$\mathbf{F}(x, y) = (1 + xy)e^{xy}\,\mathbf{i} + (e^y + x^2 e^{xy})\,\mathbf{j}, \quad C : \mathbf{r}(t) = (t + \sin \pi t)\,\mathbf{i} + (2t + \cos \pi t)\,\mathbf{j},\ 0 \le t \le 1.$$

2. Use Green's Theorem to find the area bounded by one arc of the cycloid

$$x = a(t - \sin t),\ y = a(1 - \cos t),\ a > 0,\ 0 \le t \le 2\pi,$$

and the $x-$axis. (Hint: $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$.)

3. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

4. (a) A uniform fluid that flows vertically downward (heavy rain) is described by the vector field $\mathbf{F}(x, y, z) = \langle 0, 0, -1 \rangle$. Find the total flux through the cone $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 \leq 1$.

   (b) The rain is driven sideways by a strong wind so that it falls at a 45° angle, and it is described by $\mathbf{F}(x, y, z) = -\frac{1}{\sqrt{2}} \langle 1, 0, 1 \rangle$. Now what is the flux through the cone?

5. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$, oriented outward from the origin.

BONUS Suppose the components of a vector field $\mathbf{F}(x, y, z)$ have continuous second derivatives. Let S be the boundary surface of a simple solid region in $\mathbb{R}^3$. Show that

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = 0$$