Bayesian Modeling - BART task

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1 Problem

Model risk behavior of two persons in the BART task using Bayesian modeling.

Questions:

- Different people have different risk levels of behavior?
- A hierarchical model will be better to explain the data that account for different persons or a simpler model will be enough?
- If people are in the same contextual conditions, its behavior consistency is similar?

2 Data and Models

The Balloon Analogue Risk Task (BART) aims to measure risk taking behavior in a controlled setting (Ravenzwaaij, et. al, 2011). In this task, each trial begins with the presentation of a balloon symbolizing a small amount of money. The participant can decide to either transfer the money to a virtual bank account or pump the balloon, which inflates it slightly and increases its value. However, there's a risk that pumping the balloon too much will cause it to burst, resulting in the loss of all the money. The trial ends when the participant either transfers the money or the balloon bursts.

In the aim of understand the cognitive models of risk behavior (Ravenzwaaij, et. al, model), a simplified version of BART will be used in this project, in which the probability of the balloon bursting is constant, and the expected gain of every decision to pump is zero, allowing to look at the effects of risk-taking in isolation. Also, the data used in the project is part of a textbook exercises of Lee and Wagenmakers, 2013, which is based in Ravenzwaaij, et. al, research (2011), that take on count the before considerations in the task. The data accounts for two participants (i) that completed three different blocks (z) with 30 trials (j) each one, where each block had a different probability of the balloon bursting, although it is considered in this approach that just a general probability was informed to the participants at the beginning of the experiment for the three blocks (p = 0.15). For this, the three blocks will be gathered together in just one big block and used this way to analyze data.

The models that are going to be compered are: model A, from Lee et. al (2013); model C, also from Lee et. al, but with a few variation on it (instead of measure a group effect between conditions we used it between participants); and Model B, based on model C, but just with a hierarchical structure on γ_i^+ and not in β as in model C. This is because it is assumed that the contextual conditions that could affect the behavioral consistency of participants are equal for both of them, and it can be hypothesized that the behavioral consistency will be similar.

In the three models the parameter γ^+ controls the risk taking, β controls behavioral consistency (high values correspond to less variable responding), ω is the number of pumps the subject considers optimal on the z^{th} block, θ_{jk} is the probability that subjects choose to pump on the k^{th} opportunity within the j^{th} trial, and d_{jk} represents the observed decision made (transfer the money or pump the balloon) on the k^{th} choice within the j^{th} trial (parameter indexing can vary based on the model's structure).

[**Model A**]:

$$
\begin{aligned}
\gamma^+ &\sim \text{Uniform}(0,10)\\
\beta &\sim \text{Uniform}(0,10)\\
\omega &\longleftarrow \frac{-\gamma^+}{\log(1-p_z)}\\
\theta_{jk} &\longleftarrow \frac{1}{1+\exp\{\beta(k-\omega)\}}\\
d_{jk} &\sim \text{Bernoulli}(\theta_{jk})
\end{aligned}
$$

[**Model B**] :

$$
\begin{aligned}
\mu_{\gamma^+} &\sim \text{Uniform}(0,10)\\
\sigma_{\gamma^+} &\sim \text{Uniform}(0,10)\\
\gamma_i^+ &\sim \mathcal{N}\left(\mu_{\gamma^+}, \frac{1}{\sigma^2_{\gamma^+}}\right)\\
\beta &\sim \text{Uniform}(0,10)\\
\omega_i &\longleftarrow \frac{-\gamma_i^+}{\log(1-p)}\\
\theta_{ijk} &\longleftarrow \frac{1}{1+\exp\{\beta(k-\omega_{zi})\}}\\
d_{ijk} &\sim \text{Bernoulli}(\theta_{ijk})
\end{aligned}
$$

[**Model C**]:

$$
\begin{aligned}
\mu_{\gamma^+} &\sim \text{Uniform}(0,10)\\
\sigma_{\gamma^+} &\sim \text{Uniform}(0,10)\\
\gamma_i^+ &\sim \mathcal{N}\left(\mu_{\gamma^+}, \frac{1}{\sigma^2_{\gamma^+}}\right)\\
\mu_\beta &\sim \text{Uniform}(0,10)\\
\sigma_\beta &\sim \text{Uniform}(0,10)\\
\beta_i &\sim \mathcal{N}\left(\mu_\beta, \frac{1}{\sigma^2_\beta}\right)\\
\omega_i &\longleftarrow \frac{-\gamma_i^+}{\log(1-p)}\\
\theta_{ijk} &\longleftarrow \frac{1}{1+\exp\{\beta(k-\omega_i)\}}\\
d_{ijk} &\sim \text{Bernoulli}(\theta_{ijk})
\end{aligned}
$$

3 Analysis and Results

All models were adjusted in JAGS in R using the library R2jags and where checked that their traceplots, autocorrelation and Gelman-Rubinstein metric were adequate. Then, posterior graphics of the most important parameters were made, as well as a DIC analysis to compare the models. The code and data could be find in this link: https://github.com/Alicia-MJ/Bayesian_Modeling_BART.git

The following graphs shows, the histograms with the the number of pump decisions (left panels), posterior distribution for risk propensity γ^+ (middle-left panels), posterior distribution for the optimal number of pumps ω (middle-right panels) and posterior distribution for behavioral consistency β (right panels). Figure 1 has the parameters estimation for model A, while Figures 2 and 3 has the estimation of the parameters for subjects Bill (B) and George (G), for model B and C respectively.

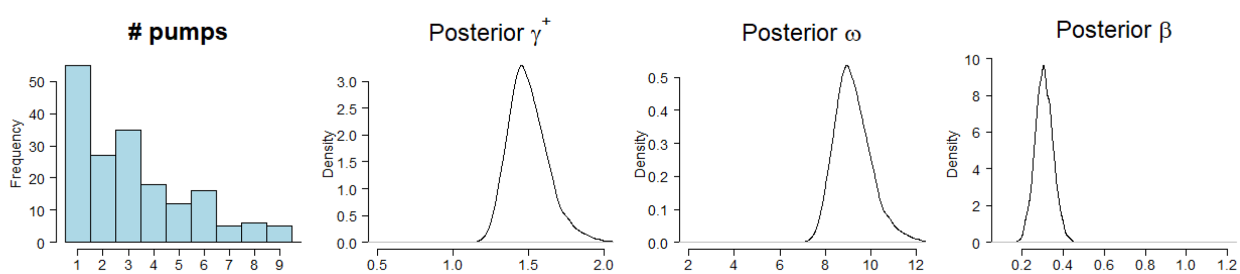

Figure 1: Model A - Histogram and posterior parameters estimation.

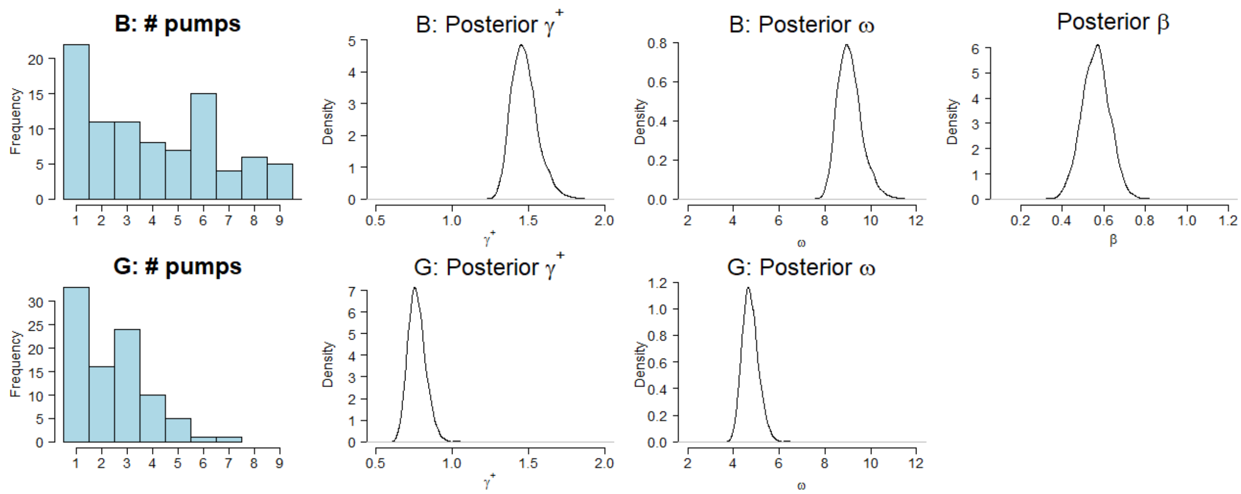

Figure 2: Model B - Histogram and posterior parameters estimation.

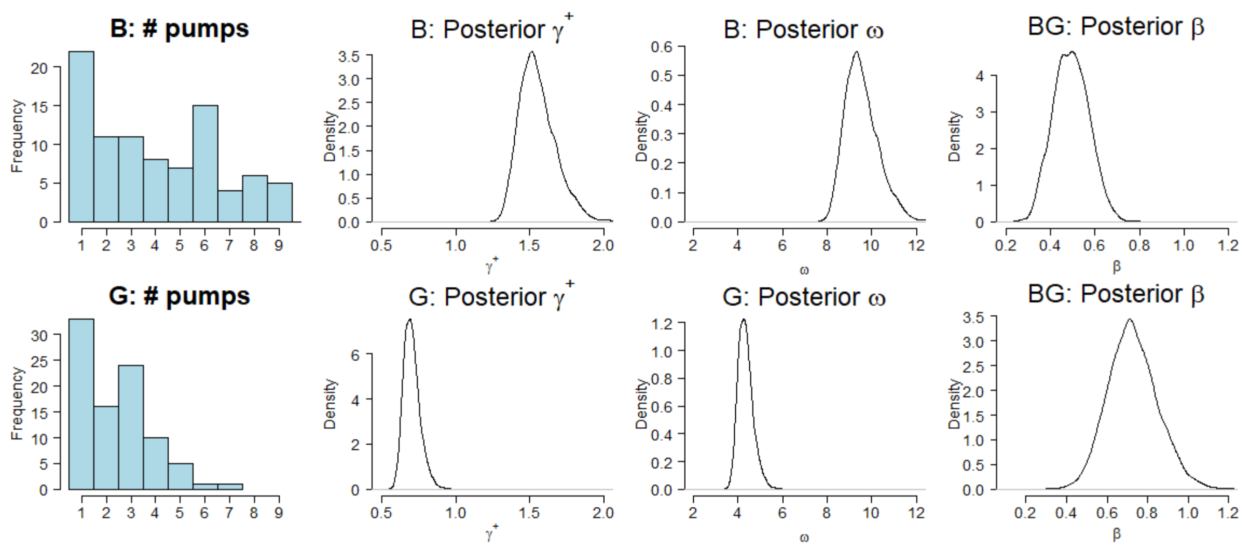

Figure 3: Model C - Histogram and posterior parameters estimation.

Modelo	DIC	Parameter penalty	Penalized deviance
A	550.6	1.7	548.9
B	475.9	4.1	471.8
C	473.5	4.4	469.1

Table 1: Model comparison criteria

From table 1 can be inferred that model C is the best model, although just for a few, as model B has a very similar DIC penalized deviance value. Also, is clear that model A can be discarded, as it has a much higher DIC value than the other two models. Besides, from the figure 3 is reasonable to think that is a clear difference between the two participants in the risk risk propensity γ^+, the optimal number of pumps ω and behavioral consistency β, even when the two participants do the experiments in the same conditions. So it can be concluded that Bill, has a higher risk propensity so in consequence also has a higher optimal number of pumps in the trials, but less behavioral consistency (related to low values of β), in contrast with George.

By other way, comparing the three figures is seen that in the model A, the estimation parameter is highly affected by Bill's data for γ^+ and ω, whereas β has a very thin distribution and with low values, indicating that the behavioral consistency of participants is very low. This makes sense with the betas graphs of figure 3, because the behavioral consistency of both participants is very different, so when the data is analyzed without a group effect it appears to be more sparse. Something similar happens with the β estimation from model B, although the distribution includes higher values and is a little bit more thick, so the hierarchical structure in γ_i^+ impacted β in a way. Finally, this is a great example of how important can be to analyze group effects in data.

4 Conclusions

Different people have different risk levels of behavior, even when they are in the same contextual conditions, so it can be assumed that the inner personal factors of each persons plays a crucial role on this. Something similar happens with behavioral consistency. Moreover, a hierarchical model with a hierarchical structure in γ_i^+ and β_i is better to explain the data that accounts for different participants in the BART task. Finally, is important to emphasize that the effects of groups on the data must be analyzed before discarding them.

5 Refrences

- van Ravenzwaaij, D., Dutilh, G., & Wagenmakers, E.-J. (2011). Cognitive model decomposition of the BART: Assessment and application. Journal of Mathematical Psychology, 55(1), 94–105. https://doi.org/10.1016/j.jmp.2010.08.010
- Lee, M.D., & Wagenmakers, E.-J. (2013). Bayesian cognitive modeling: A practical course. Cambridge University Press.