1 Definitions

Definition 1. [2] A simple graph is *connected* if there is a path between every pair of vertices.

Definition 2. [3] A *topology* on a set X is a collection $\mathcal{T}$ of subsets of X having the following properties:

(a) $\emptyset$ and X are in $\mathcal{T}$.

(b) The union of any subcollection of $\mathcal{T}$ is in $\mathcal{T}$.

(c) The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$.

2 Theorems

Theorem 1 (Jordan Curve Theorem)**.** *[**?**] Removing the image of a simple closed curve from $\mathbb{R}^2$ leaves two connected components, the bounded **inside** and the un-bounded **outside**. The inside together with the image of the curve is homeo-morphic to a closed disk.*

Theorem 2 (Schönflies Theorem)**.** *[1] If J is a simple closed curve in $\mathbb{R}^2$, the closure of one of the compononents of $\mathbb{R}^2 - J$ is homeomorphic with the unit 2-ball.*

Theorem 3 (Kuratowski Theorem)**.** *[2] A simple graph is planar iff no subgraph is homeomorphic to K_5 or to $K_{3,3}$.*

Theorem 4 (Tuttes Theorem)**.** *[2] Let $G = (V, E)$ be the edge-skeleton of a triangulation of the disk and $f : V \rightarrow \mathbb{R}^2$ a strictly convex combination mapping that maps the boundary vertices to the corners of a strictly convex polygon. Then drawing straight edges between the image points gives a straight-line embedding.*

Theorem 5 (Geometric Realization Theorem)**.** *[2] Every abstract simplicial complex of dimension d has a geometric realization in $\mathbb{R}^{2d+1}$.*

Theorem 6 (Simplicial Approximation Theorem)**.** *[2] If g: $|K| \rightarrow |L|$ is continuous then there is a sufficiently large integer n such that g has simplicial approximation f: $Sd^n K \rightarrow L$.*

Theorem 7 (Helley's Theorem)**.** *[2] Let F be a finite collection of closed, convex sets in $\mathbb{R}^n$. Every $d + 1$ of the sets have a non-empty common intersection iff they all have a non-empty common intersection.*

Theorem 8 (Nerve Theorem)**.** *[2] Let F be a finite collection of closed, convex sets in Euclidean space. Then the nerve of F and the union of the sets in F have the same homotopy type.*

Theorem 9 (Brouwer's Fixed Point Theorem)**.** *[2] A continuous map f: $\mathbb{B}^{p+1} \rightarrow \mathbb{B}^{p+1}$ has at least one fixed point $x = f(x)$.*

Theorem 10 (Euler-Poincaré Theorem)**.** *[2] The Euler characteristic of a topological space is the alternating sum of its Betti numbers, $X = \sum_{p \geq 0} (-1)^p \beta_p$*

Theorem 11 (Excision Theorem). *[2] Let $K_0 \subseteq K$ and $L_0 \subseteq L$ be pairs of simplicial complexes that satisfy $L \subseteq K$ and $L - L_0 = K - K_0$. Then they have isomorphic relative homology groups, that is, $H_p(K, K_0) \simeq H_p(L, L_0)$ for all dimensions p.*

Theorem 12 (Exact Sequence of a Pair Theorem). *[2] Let K be a simplicial complex and $K_0 \subseteq K$ be a subcomplex. Then there is a long exact sequence*

$$... \to H_p(K_0) \to H_p(K) \to H_p(K, K_0) \to H_{p-1}(K_0) \to ...$$

References

[1] Schönflies theorem.

[2] Herbert Edelsbrunner and John Harer. *Computational topology: an introduction.* American Mathematical Soc., 2010.

[3] James R Munkres. *Topology.* Prentice Hall, 2000.