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Exercise 1. Edelsbrunner Chapter 2, problem 1 *Classifying 2-manifolds*
Characterize the two surfaces depicted in Figure 11.15 in terms of genus, boundary, and orientability.

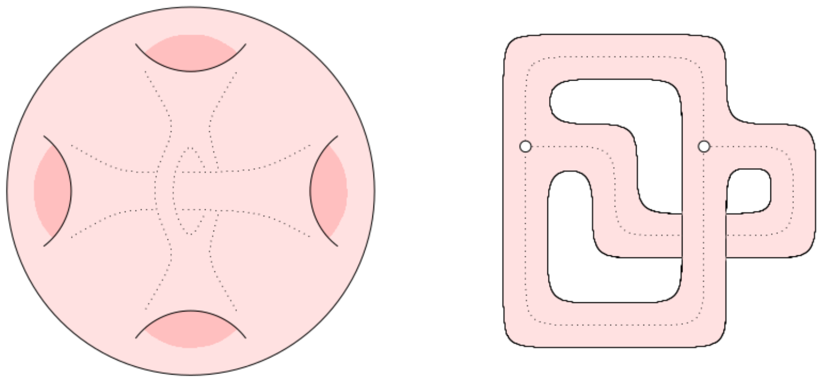

Figure II.15: Left: a 2-manifold without boundary obtained by adding tunnels inside the sphere. We see four tunnel openings and one tunnel passing though a fork of the other. Right: a 2-manifold with boundary obtained by thickening a graph.

The first manifold to the left has a genus of 3, since three closed curves (cuts) can be made to the manifold before creating two pieces. It has no boundary, but it is orientable. The second manifold to the right has a genus of 2, since two closed curves (cuts) can be made to the manifold before creating two pieces. Notice that the shape looks similar to a double tori except there are two vertices instead of one. The manifold has boundary, and it is orientable.

Exercise 2. Edelsbrunner Chapter 2, problem 2 *2-coloring*
Let K be a triangulation of an orientable 2-manifold without boundary. Construct L by decomposing each edge into two and each triangle into six. To do this, we add a new vertex in the interior of each edge. Similarly, we add a new vertex in the interior of each triangle, connecting it to the six vertices in the boundary of the triangle. The resulting structure is the same as the barycentric subdivision of K, which we will define in Chapter III.

(i) Show that the vertices of L can be 3-colored such that no two neighboring vertices recieve the same color.

(ii) Prove that the triangles of L can be 2-colored such that no two triangles sharing an edge receive the same color.

Exercise 3. Edelsbrunner Chapter 2, problem 3 *Klein bottle*
Cut and paste the standard polygonal schema for the Klein bottle (a, a, b, b) to obtain the polygonal schema in which opposite edges of a square are identified (a, b, a^-1, b); see Figure 11.3.

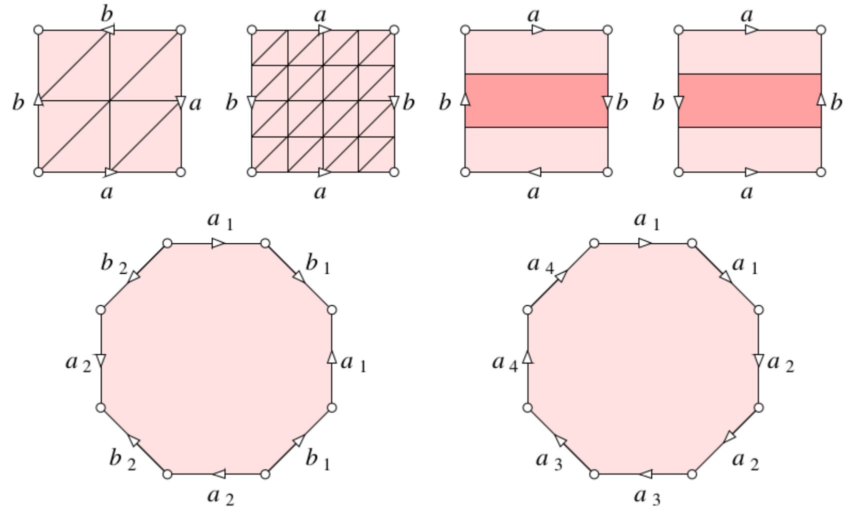

Figure II.3: Top from left to right: the sphere, the torus, the projective plane, and the Klein bottle. After removing the (darker) Möbius strip from the last two, we are left with a disk in the case of the projective plane and another Möbius strip in the case of the Klein bottle. Bottom: the polygonal schema in standard form for the double torus on the left and the double Klein bottle on the right.

Exercise 4. Edelsbrunner Chapter 2, problem 4 *Triangulation of 2-manifold*
Let $V = 1, 2, ..., n$ be a set of n vertices and $F \subseteq \vec{V, 3}$ a set of $l =$ card F triangles. Give an algorithm that takes time at most proportional to $n + l$ for the following tasks:

(i) decide whether or not every edge is shared by exactly two triangles;

(ii) decide wheter or not every vertex belongs to a set of triangles whose union is a disk.

Exercise 5. Edelsbrunner Chapter 2, problem 5 *Intersection tests in* $\mathbb{R}^3$
Let $a, b, c \in \mathbb{R}^3$ and $u, v, w \in \mathbb{R}^3$ be the vertices of two triangles in space. Write numerical tests for the following questions:

(i) does u see a, b, c form a left-turn or a right-turn?

(ii) does the line segment with endpoints u and v cross the plan that passes through a, b, c?

(iii) are the boundaries of the two triangles linked in $\mathbb{R}^3$?

Exercise 6. Edelsbrunner Chapter 2, problem 6 *Irreducible triangulations*
An *irreducible* triangulation is one in which every edge contraction changes its topological type. Prove that the only irreducible triangulation of $\mathbb{S}^2$ is the boundary of the tetrahedron, which consists of four triangles sharing six edges and four vertices.

Exercise 7. Edelsbrunner Chapter 2, problem 7 *Graphs on Möbius strip*
Is every graph that can be embedded on the Möbius strip planar?

Exercise 8. Edelsbrunner Chapter 2, problem 8, *Squared distance minimization*
Let S be a finite set of points in $\mathbb{R}^3$ and $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x) = \sum_{p \in S} ||x - p||^2$.

(i) Show that f is a quadratic function and has a unique minimum.

(ii) At which point does f attain its minimum?