Hashtables

Implementations for dictionaries and sets

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See https://github.com/parrt/msds692/blob/master/notes/hashtable.ipynb and
https://github.com/parrt/msds692/blob/master/notes/dict.ipynb

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How to search big collections quickly

- *Hashtables* are data structures that efficiently implement search/lookup operations for sets and dictionaries
- Sets and dictionaries are abstract data structures, hashtables are concrete implementations of those structures
- Simple lists of elements and lists of tuples work but are slow
- Hashtable's **key idea**: partition the search space into well-defined regions so we don't have to search linearly through the entire collection to find an element
- We use a (hash) function of the values to partition into buckets

Review: Sets

- A set is just an unordered, unique collection of elements; here is an example using integers:
  `ids = {100, 103, 121, 102, 113, 113, 113, 113}`
- We can do lots of fun set arithmetic:

```
{100,102}.union({109})
```

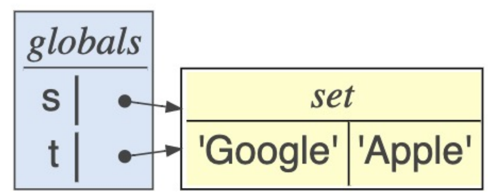
```
{100, 102, 109}
```

```
{100,102}.intersection({100,119})
```

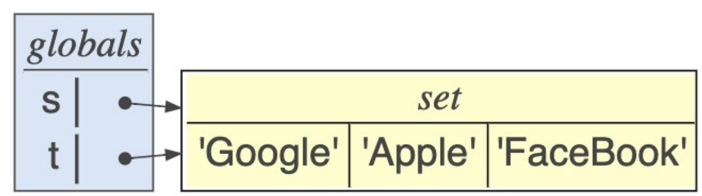
```
{100}
```

Watch out for aliasing!

```
s = {"Apple", "Google"}
t = s
```




```
s.add("FaceBook")
```



Review: Dictionaries map keys to values

- If we arrange two lists side-by-side and kind of glue them together, we get a *dictionary* (type is **dict**)
- Dictionaries map one value to another, just like a dictionary in the real world maps a word to a definition
- Here is a sample dictionary:

```
movies = {'Amadeus':1984, 'Witness':1985}
```

'Amadeus' → 1984
'Witness' → 1985

- Index by key to get the value; e.g., `movies['Amadeus']`

List implementation for sets

```
import numpy as np
n = 5_000_000
A = list(np.random.randint(low=0,high=1_000_000,size=n))
A[0:10]
```

```
[509385, 571020, 998421, 173251, 567339, 229005, 614066, 89806, 878866, 496601]
```

```
def lsearch(A,x):
    for a in A:
        if a==x:
            return True
    return False
```

```
%time lsearch(A, 999)
```

```
CPU times: user 362 ms, sys: 8.02 ms, total: 370 ms
Wall time: 378 ms

True
```

```
%time for a in range(50): lsearch(A, a)
```

```
CPU times: user 8.65 s, sys: 102 ms, total: 8.75 s
Wall time: 8.9 s
```

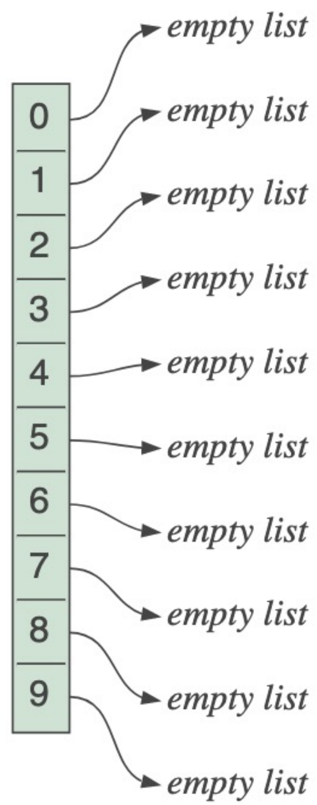
Searching linearly is pretty slow

Can we do better than linear search?

- Rather than search through every element, let's partition the set of numbers into 10 buckets so that, on average, we only need to search 1/10 of the elements
- We partition with a *hash* function that operates on set values
- We are effectively using something about the value to hint at the location
- E.g., where does Eric Erickson live in US?
- Imagine a hash function that gave the postal code given a name (set value)

Partitioning requires a new data structure

- Rather than a list of set values, we need a list of regions called buckets where each bucket is a list of values
- The hash of a value leads to the bucket index
- For sets of integers, let's use the value modulo 10 to uniquely place values into one of 10 buckets, indexed 0..9

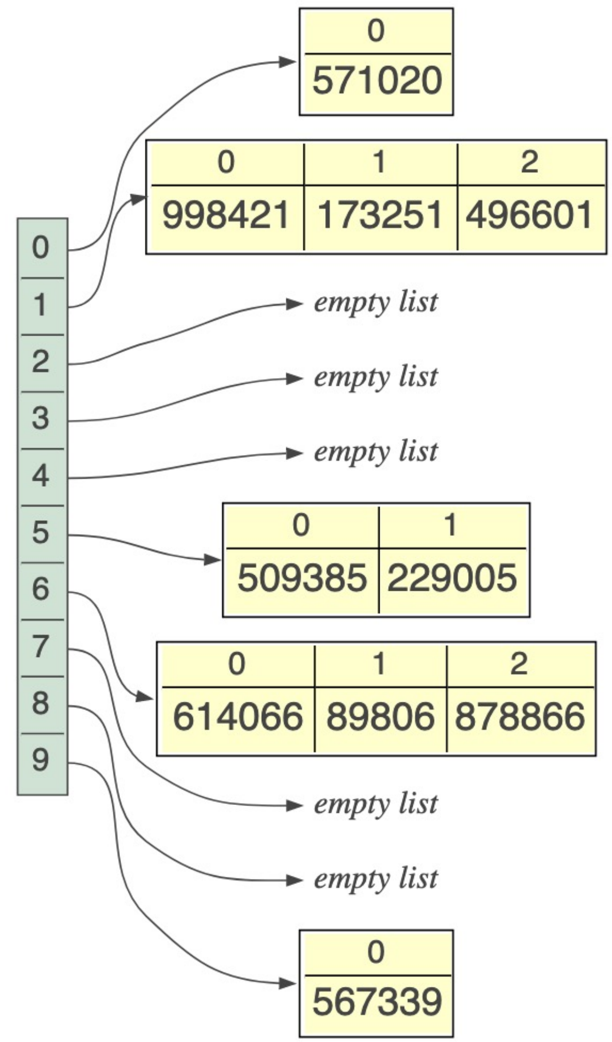


Partitioning with modulo hash

```python
def hash(x):
    return x % 10

[(a,hash(a)) for a in A[0:10]]
```

```
[(509385, 5),
 (571020, 0),
 (998421, 1),
 (173251, 1),
 (567339, 9),
 (229005, 5),
 (614066, 6),
 (89806, 6),
 (878866, 6),
 (496601, 1)]
```



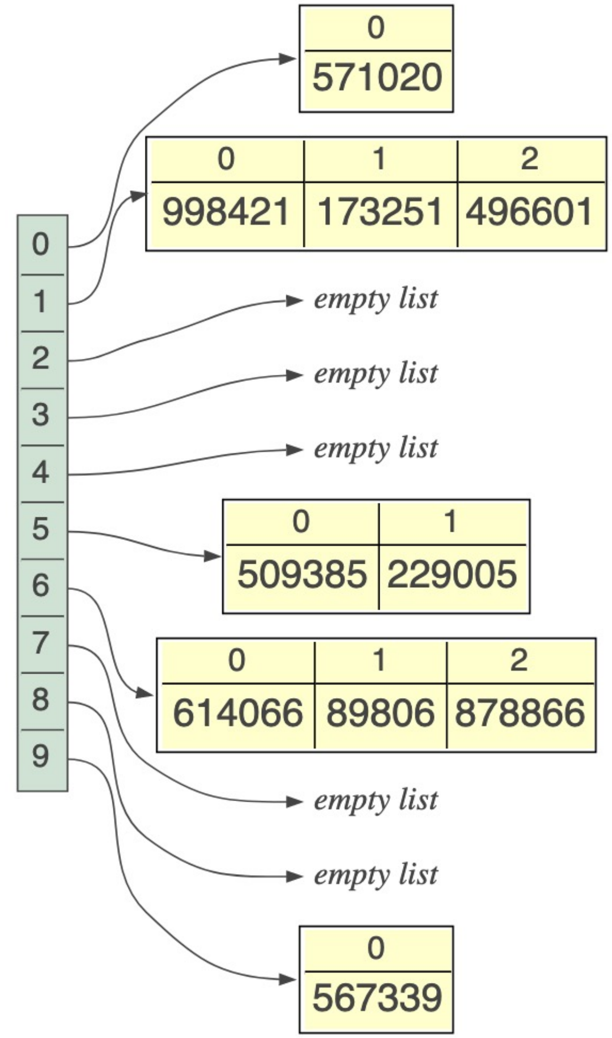
Hashtable construction

- Make 10 empty buckets (lists)
- Add each set element to correct bucket
- Amounts to appending element to one of 10 lists

```
def hash(x):
    return x % 10
```

```
def htable(A):
    "Build hashtable for integer values"
    buckets = [[] for i in range(10)]
    for a in A:
        buckets[hash(a)].append(a)
    return buckets
```

```
buckets = htable(A)
```



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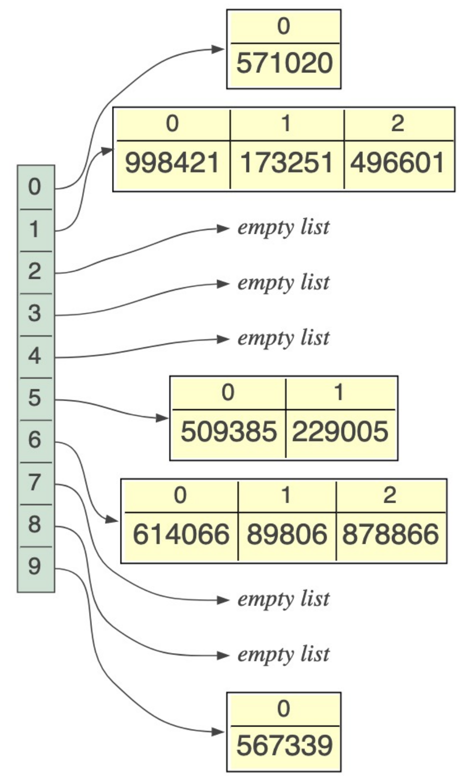
Searching hashtable set implementation

- To find x, look in the bucket indicated by hash(x)



Linear

```
def lsearch(A,x):
    for a in A:
        if a==x:
            return True
    return False
```

```
%time lsearch(A, 999)
```

```
CPU times: user 190 ms, sys:
Wall time: 209 ms
```

Hashtable

```
def hsearch(buckets,x):
    i = hash(x)
    for a in buckets[i]:
        if a==x:
            return True
    return False
```

```
buckets = htable(A)
%time hsearch(buckets, 999)
```

```
CPU times: user 18 ms, sys:
Wall time: 18.7 ms
```

Speed difference is dramatic

Linear

```
%time for a in range(50): lsearch(A, a)
```

```
CPU times: user 6.97 s, sys: 93.9 ms, total: 7.06 s
Wall time: 7.38 s
```

Hashtable

```
%time for a in range(50): hsearch(buckets, a)
```

```
CPU times: user 823 ms, sys: 14.3 ms, total: 837 ms
Wall time: 861 ms
```

Exercise

- What would happen if we used a hash function like these?

```
def hash(x):
    return x % 2
```

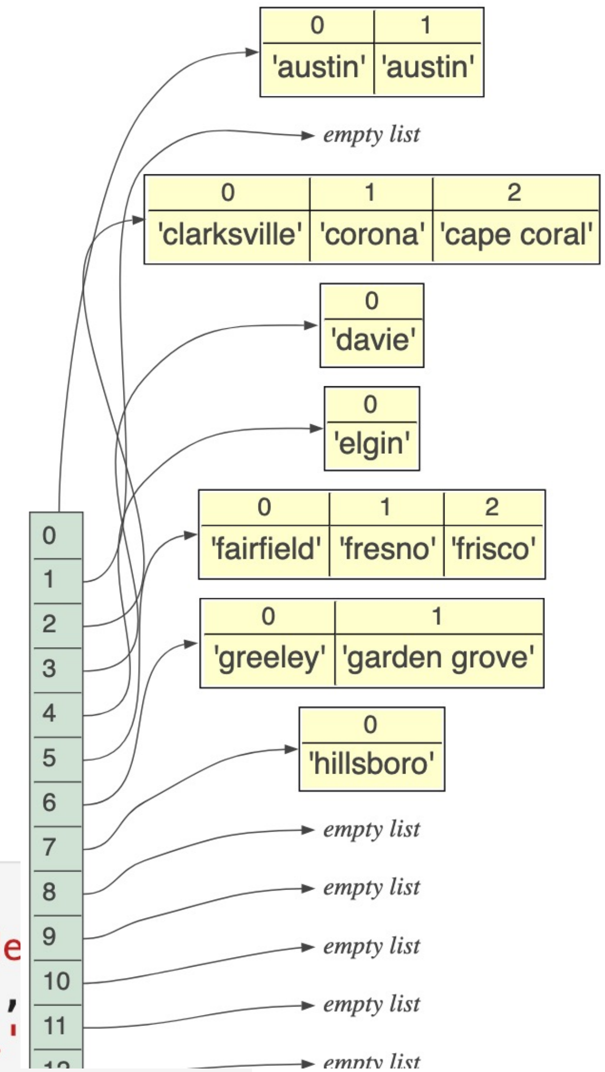
```
def hash(x):
    return 0
```

Sets of strings

- Hashtables work for any value for which we can define a good hash function, one that partitions the space evenly
- What hash function am I using here?

Hash is distance of char code from ‘a’s code

```
cities = ['elgin', 'tyler', 'austin', 'hillsboro', 'greeley',
          'davie', 'rockford', 'orange', 'sandy springs', 'garde
          'paterson', 'clarksville', 'fairfield', 'victorville',
          'palmdale', 'frisco', 'corona', 'austin', 'cape coral'
```

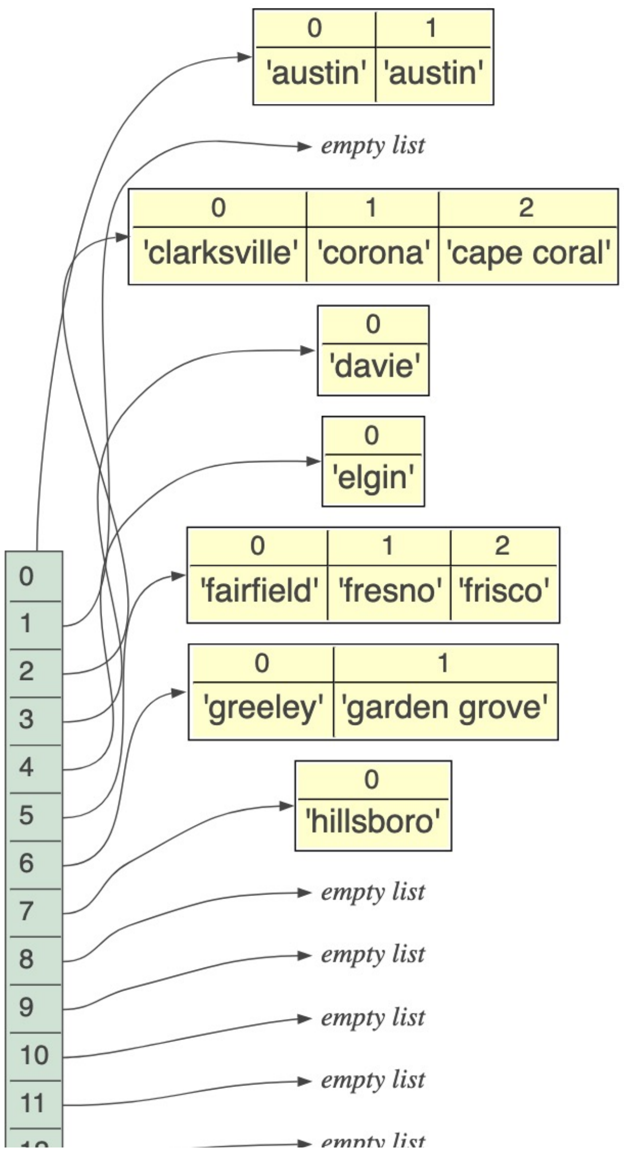


Hashing strings

```python
def hash(s):
    # convert first char to int in [0,25]
    return ord(s[0]) - ord('a')

[(c,hash(c)) for c in cities[0:10]]
```

```
[('elgin', 4),
 ('tyler', 19),
 ('austin', 0),
 ('hillsboro', 7),
 ('greeley', 6),
 ('davie', 3),
 ('rockford', 17),
 ('orange', 14),
 ('sandy springs', 18),
 ('garden grove', 6)]
```



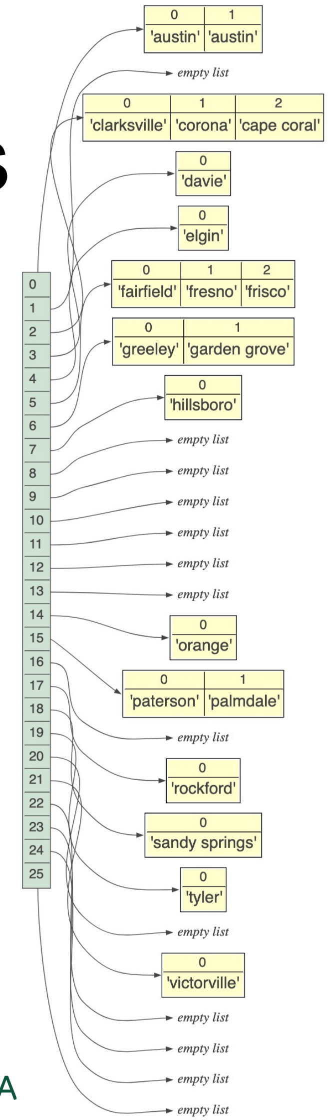
Hashtable impl. differs only in buckets

```python
def htable(A):
    buckets = [[] for i in range(26)]
    for a in A:
        buckets[hash(a)].append(a)
    return buckets
```

```python
def hsearch(buckets,x):
    i = hash(x)
    for a in buckets[i]:
        if a==x:
            return True
    return False
```

```python
buckets = htable(cities)
%time hsearch(buckets, "austin")
```

```
CPU times: user 5 µs, sys: 0 ns, total: 5 µs
Wall time: 7.15 µs
```



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An important implementation detail

- The hash function does not directly give the bucket index: it converts values to integers, then we make sure that the hash value fits into the table by doing modulo the number of buckets
- For our int sets, the hash(x) is just x; the modulo 10 just puts it in one of 10 buckets
- Same for strings; the hash(x) is in 0..25 but we could stick it into 10 buckets by taking modulo 10

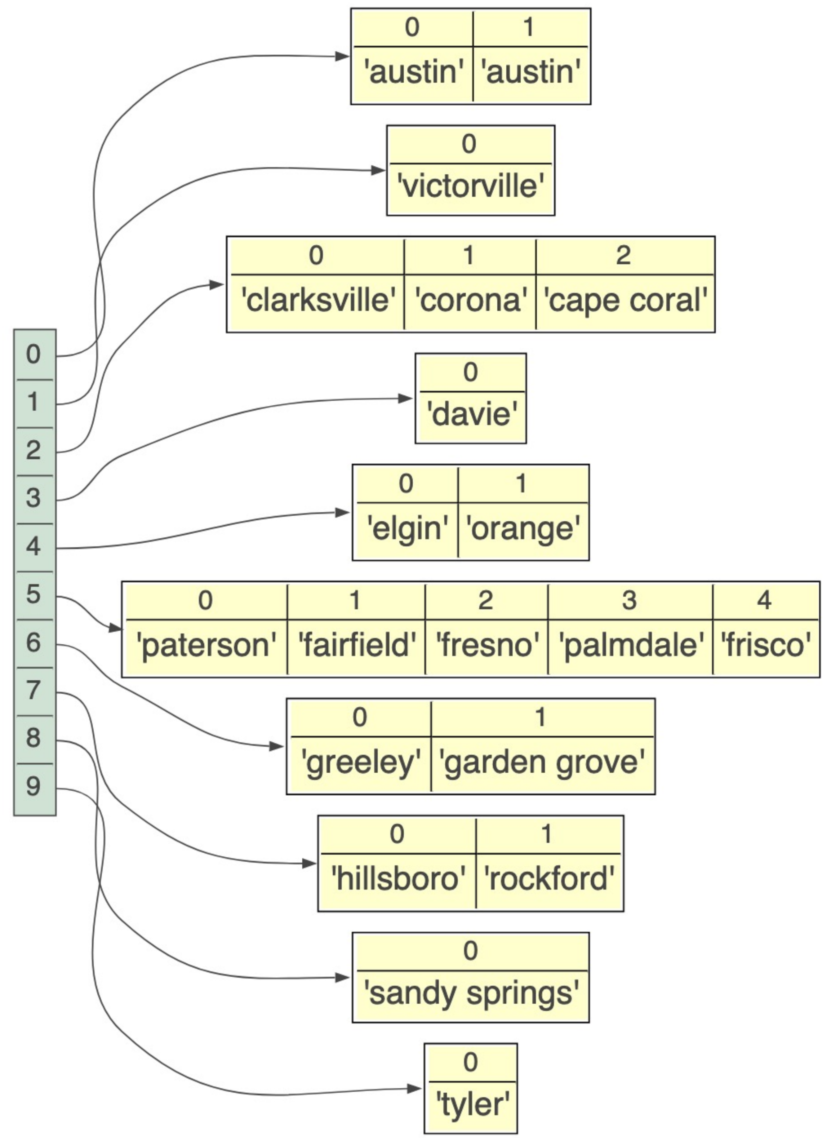
Hash vs bucket index

- Compute the hash and mod with the number of buckets we have

```python
def hash(s):
    # convert first char to int in [0,25]
    return ord(s[0]) - ord('a')

def htable(A):
    buckets = [[] for i in range(10)]
    for a in A:
        # fit in 10 buckets
        b = hash(a) % 10
        buckets[b].append(a)
    return buckets
```

Previously: `buckets[hash(a)].append(a)`



How much faster are hash tables?

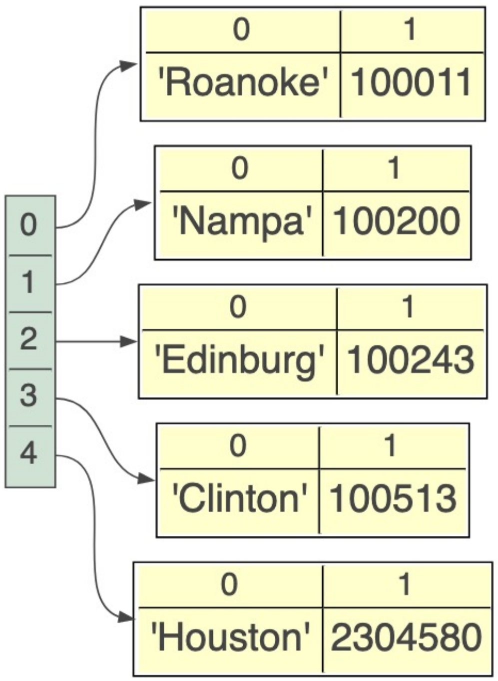
- With a uniform distribution, we would expect roughly *N*/*B* associations in each bucket for *B* buckets and *N* total elements in the dictionary
- A complexity of *N*/*B* is much better than *N* and, with sufficiently large *B*, we would say that *N*/*B* approaches 1, giving complexity *O(1)* versus *O(n)*

Dictionary implementations

A simple dictionary implementation

- Let's represent the set of key→value pairs as a list of tuples:

```
pop = [
    ('Roanoke', 100011),
    ('Nampa', 100200),
    ('Edinburg', 100243),
    ('Clinton', 100513),
    ('Houston', 2304580)
]
```



- The key operation is to look up a value by key
- How would you implement this?

Linearly search through the list of tuples and compare the first value and the tuple to the key of interest; return the associated value if key is found

Linear lookup

- Looking for a key is a simple matter of examining the first element of every tuple stored in the list
- Return None if the key is not found

```
def llookup(A,x):
    for k,v in A:
        if k==x:
            return v
    return None
```

```
llookup(pop, 'Clinton')
```

```
100513
```

```
llookup(pop, 'SF')
```

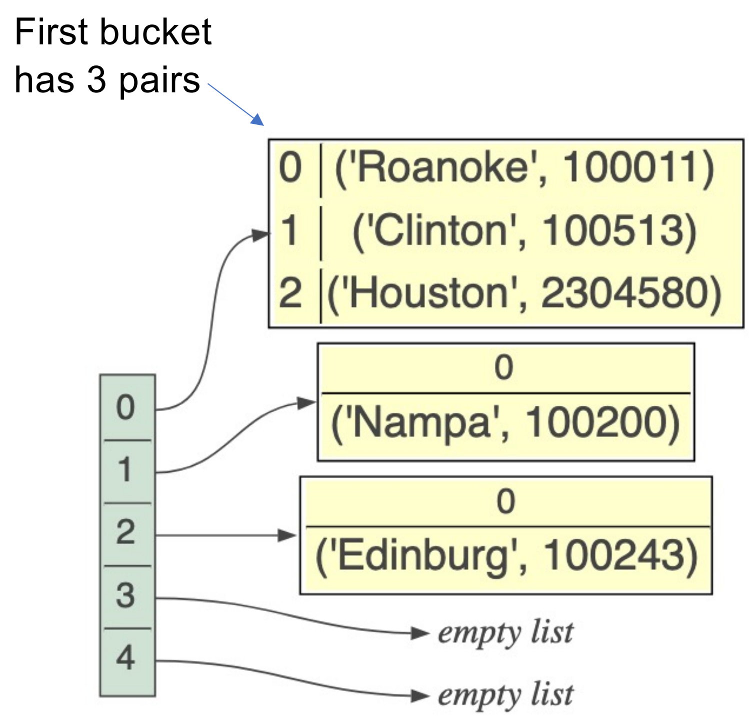
Hashtable dictionary implementation

- Split pairs into, say, 5 buckets (use our string hash function)

```
def hash(s):
    # convert first char to int in [0,25]
    return ord(s[0]) - ord('a')

def htable_dict(A,nbuckets):
    buckets = [[] for i in range(nbuckets)]
    for k,v in A:
        b = hash(k) % nbuckets
        buckets[b].append((k,v))
    return buckets
```

```
buckets = htable_dict(pop, 5)
```



Hashtable key look up

- Compute the hash, modulo a number of buckets, to get the bucket index
- Linear search within the bucket
- If key found, return value
- Else return None

```python
def hlookup(buckets,x,nbuckets):
    i = hash(x) % nbuckets
    for k,v in buckets[i]:
        if k==x:
            return v
    return None
```

```python
buckets = htable_dict(pop, 3)
```

```python
hlookup(buckets, 'Clinton', nbuckets=3)
```

```
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```
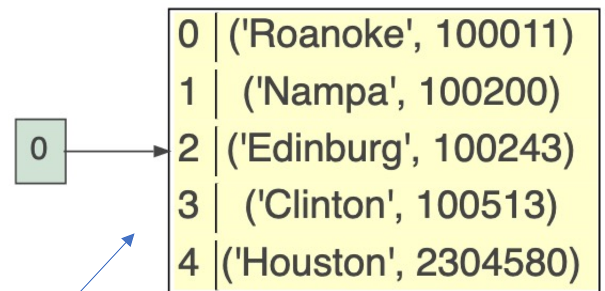

```python
hlookup(buckets, 'SF', nbuckets=3)
```

Degenerate case of one bucket

- With only one bucket, all pairs hash to the same bucket, which means doing a linear search of all elements to look up a key

```python
buckets = htable_dict(pop, 1)
```



One and only bucket

Some details relevant to the search project

Review: tuples

- A tuple is an *immutable* list and uses parentheses rather than square brackets for notation
- Tuples are often used to group related elements:

```
me = ('parrt',607)
userid,office = me
print(userid)
print(office)
print(me[0], me[1])
```

```
parrt
607
parrt 607
```

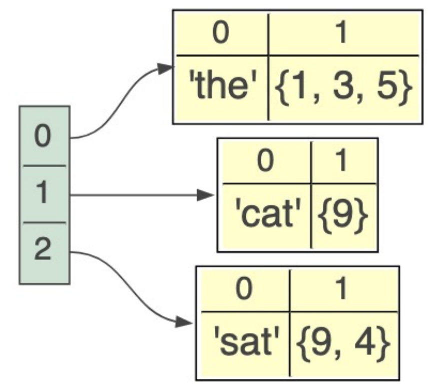
```
me = ('parrt', 607)
me[1] = 525 # change office
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-36-e02b2267f45c> in <module>
      1 me = ('parrt', 607)
----> 2 me[1] = 525 # change office

TypeError: 'tuple' object does not support item assignment
```

Values can be anything including sets

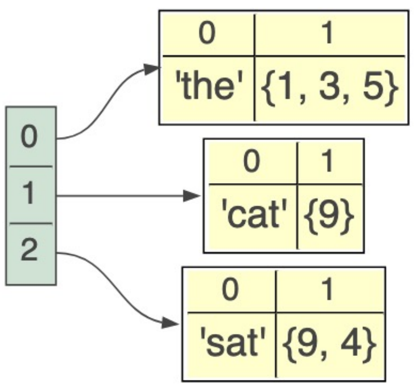
- The tuples used to represent key and value pairs are immutable, but the pair's value can point at a mutable data structures such as a set
- Consider a simple list of tuples implementation that maps words to sets of integers

```
words = [('the', {3,1,5}), ('cat',{9}), ('sat',{4,9})]
```



Modifying dictionary set values

- If you extract a mutable value from a data structure, you can modify it without having to delete and add an updated version



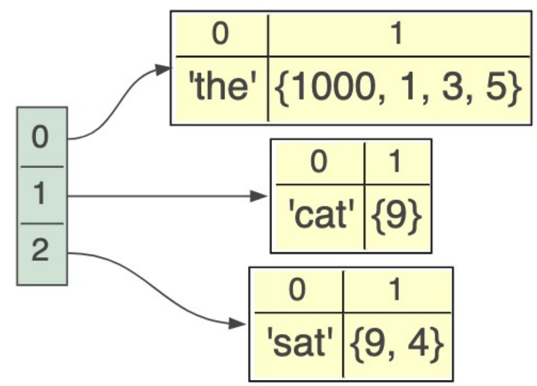

```python
the = llookup(words, 'the')
the
```

```
{1, 3, 5}
```

```python
the.add(1000)
```

```python
llookup(words, 'the')
```

```
{1, 3, 5, 1000}
```



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Summary

- Dictionaries and sets are typically implemented with a form of hashtable because the key lookup operation is so much faster
- The speed comes from a partitioning of the search space into a large number of small regions, which are searched linearly
- If we make enough buckets so that at most there are three keys in each bucket, lookup takes three operations no matter how many keys have been added to the dictionary