QED matrix element

$$M_\gamma = -\frac{e^2}{3s}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right]$$

$$|M_\gamma|^2 = K\left[\bar{v}_a(p_2)\gamma^\mu_{ab}u_b(p_1)\right]\left[\bar{u}_c(p_3)\gamma_\mu^{cd}v_d(p_4)\right]\left[\bar{v}_e(p_4)\gamma_\nu^{ef}u_f(p_3)\right]\left[\bar{u}_g(p_1)\gamma^\nu_{gh}v_h(p_2)\right], \qquad K = \frac{e^4}{9s^2}$$

$$\frac{4}{K}\langle|M_\gamma|^2\rangle = \sum\left[u_b(p_1)\bar{u}_g(p_1)\right]\left[v_h(p_2)\bar{v}_a(p_2)\right]\left[u_f(p_3)\bar{u}_c(p_3)\right]\left[v_d(p_4)\bar{v}_e(p_4)\right]\gamma^\mu_{ab}\gamma_\mu^{cd}\gamma_\nu^{ef}\gamma^\nu_{gh}$$

$$= \mathrm{Tr}\left[(\not{p}_1 + m_\mu)\gamma^\nu(\not{p}_2 - m_\mu)\gamma^\mu\right]\mathrm{Tr}\left[(\not{p}_3 + m_b)\gamma_\mu(\not{p}_4 - m_b)\gamma_\nu\right]$$

Using $\not{p}_1 = \gamma^\sigma p_{1\sigma}$ and $\not{p}_2 = \gamma^\rho p_{2\rho}$

$$\begin{aligned}
\mathrm{Tr}[(p_1+m)\gamma^\nu(p_2-m)\gamma^\mu] &= \mathrm{Tr}[(\not{p}_1\gamma^\nu + m\gamma^\nu)(p_2\gamma^\mu - m\gamma^\mu)] \\
&= \mathrm{Tr}[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu - \not{p}_1\gamma^\nu m\gamma^\mu + m\gamma^\nu\not{p}_2\gamma^\mu - m^2\gamma^\nu\gamma^\mu] \\
&= \mathrm{Tr}[\gamma^\sigma p_{1\sigma}\gamma^\nu\gamma^\rho p_{2\rho}\gamma^\mu] - m^2\,\mathrm{Tr}[\gamma^\nu\gamma^\mu] \\
&= p_{1\sigma}p_{2\rho}\,\mathrm{Tr}[\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu] - m^2\,4g^{\nu\mu} \\
&= 4p_{1\sigma}p_{2\rho}(g^{\sigma\nu}g^{\rho\mu} - g^{\sigma\rho}g^{\nu\mu} + g^{\sigma\mu}g^{\nu\rho}) - 4m^2g^{\nu\mu} \\
&= 4(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu - m^2g^{\nu\mu})
\end{aligned}$$

$$\mathrm{Tr}[(p_3+m)\gamma_\mu(p_4-m)\gamma_\nu] = 4(p_{3\mu}p_{4\nu} - g_{\mu\nu}(p_3\cdot p_4) + p_{3\nu}p_{4\mu} - m^2g_{\mu\nu})$$

$$\begin{aligned}
\frac{4}{K}\langle|M_\gamma|^2\rangle &= 16\left[(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu - m_\mu^2g^{\nu\mu})(p_{3\mu}p_{4\nu} - g_{\mu\nu}(p_3\cdot p_4) + p_{3\nu}p_{4\mu} - m_b^2g_{\mu\nu})\right] \\
&= 16\big[\cancel{(p_1\cdot p_4)(p_2\cdot p_3)} - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + \cancel{(p_1\cdot p_3)(p_2\cdot p_4)} - \cancel{m_b^2(p_1\cdot p_2)} \\
&\quad - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + 4\cancel{(p_1\cdot p_2)(p_3\cdot p_4)} - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + \cancel{4m_b^2(p_1\cdot p_2)} \\
&\quad + \cancel{(p_1\cdot p_3)(p_2\cdot p_4)} - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + \cancel{(p_1\cdot p_4)(p_2\cdot p_3)} - \cancel{m_b^2(p_1\cdot p_2)} \\
&\quad - \cancel{m_\mu^2(p_3\cdot p_4)} + 4\cancel{m_\mu^2(p_3\cdot p_4)} - \cancel{m_\mu^2(p_3\cdot p_4)} + \cancel{4m_\mu^2m_b^2}\big] \\
&= 16[2(p_1\cdot p_4)(p_2\cdot p_3) + 2(p_1\cdot p_3)(p_2\cdot p_4) + 2m_b^2(p_1\cdot p_2) + 2m_\mu^2(p_3\cdot p_4) + 4m_\mu^2m_b^2] \\
&= 32[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2m_b^2]
\end{aligned}$$

$$\langle|M_\gamma|^2\rangle = \frac{8e^4}{9s^2}\left[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2m_b^2\right]$$

Higgs term

$$M_H = \underbrace{\left(\frac{g_w}{2m_w}\right)^2 \frac{m_\mu m_b}{s - m_H^2}}_{K} [\bar{v}(p_2)u(p_1)][\bar{u}(p_3)v(p_4)]$$

$$\frac{4}{K^2}\langle |M_H|^2\rangle = \sum [\bar{v}_a(p_2)u_b(p_1)][\bar{u}_c(p_3)v_d(p_4)][\bar{v}_e(p_4)u_f(p_3)][\bar{u}_g(p_1)v_h(p_2)] \ \mathbb{1}_{ab}\ \mathbb{1}_{cd}\ \mathbb{1}_{ef}\ \mathbb{1}_{gh}$$

$$= \sum [u_b(p_1)\bar{u}_g(p_1)][v_h(p_2)\bar{v}_a(p_2)][u_f(p_3)\bar{u}_c(p_3)][v_d(p_4)\bar{v}_e(p_4)] \quad \text{— " —}$$

$$= [(\not{p}_1 + m_\mu)_{bg}\mathbb{1}_{gh}(\not{p}_2 - m_\mu)_{ha}\mathbb{1}_{ab}][(\not{p}_3 + m_b)_{fc}\mathbb{1}_{cd}(\not{p}_4 - m_b)_{de}\mathbb{1}_{ef}]$$

$$= \mathrm{Tr}[(\not{p}_1 + m_\mu)(\not{p}_2 - m_\mu)]\ \mathrm{Tr}[(\not{p}_3 + m_b)(\not{p}_4 - m_b)] = \mathcal{S}_1 \times \mathcal{S}_2$$

$$\mathcal{S}_1 = \mathrm{Tr}[\not{p}_1\not{p}_2 - \not{p}_1 m_\mu + m_\mu\not{p}_2 - m_\mu^2] \qquad \mathcal{S}_2 = 4[(p_3\cdot p_4) - m_b^2]$$

$$= p_{1\sigma}p_{2\rho}\,\mathrm{Tr}(\gamma^\sigma\gamma^\rho) - m_\mu^2\,\mathrm{Tr}(I)$$

$$= 4(p_1\cdot p_2) - 4m_\mu^2$$

$$\mathcal{S}_1 \times \mathcal{S}_2 = 16[(p_1\cdot p_2) - m_\mu^2][(p_3\cdot p_4) - m_b^2]$$

$$= 16[(p_1\cdot p_2)(p_3\cdot p_4) - m_b^2(p_1\cdot p_2) - m_\mu^2(p_3\cdot p_4) + m_\mu^2 m_b^2]$$

$$\langle |M_H|^2\rangle = \frac{g_w^2 m_\mu^2 m_b^2}{4m_w^2(s-m_H^2)^2}\, 4[(p_1\cdot p_2)(p_3\cdot p_4) - m_b^2(p_1\cdot p_2) - m_\mu^2(p_3\cdot p_4) + m_\mu^2 m_b^2]$$

$$= \frac{g_w^2 m_\mu^2 m_b^2}{m_w^2(s-m_H^2)^2}[(p_1\cdot p_2)(p_3\cdot p_4) - m_b^2(p_1\cdot p_2) - m_\mu^2(p_3\cdot p_4) + m_\mu^2 m_b^2]$$

$$K^2 = \left(\frac{g_w}{2m_w}\right)^4 \frac{m_\mu^2 m_b^2}{(s - m_H^2)^2} \qquad \frac{16}{4} = 4 \qquad 2^4 = 2\cdot 2\cdot 2\cdot 2 = 4\cdot 4$$

$$\langle |M_H|^2\rangle = \frac{K^2}{4}\mathcal{S}_1\mathcal{S}_2 = \cancel{g_w} \left(\frac{g_w}{2m_w}\right)^4 \frac{m_\mu^2 m_b^2}{(s - m_H^2)^2}\, 4[\qquad]$$

$$= \frac{g_w^4 m_\mu^2 m_b^2}{4m_w^4(s - m_H^2)^2} \rightarrow \frac{g_w^4 m_\mu^2 m_b^2}{4m_w^4[(s-m_H^2)^2 + m_H^2\Gamma_H^2]}$$

$$\langle |M_{fi}|^2\rangle$$

Interference $\chi = \frac{1}{2}\sum_{spin} Re(M_\gamma M_Z^*)$ $\quad \langle |M_\gamma|^2\rangle \neq \ldots$

$$M_\gamma = \underbrace{-\frac{e^2}{3s}}_{\alpha}[\bar v(p_2)\gamma^\mu u(p_1)][\bar u(p_3)\gamma_\mu v(p_4)]$$

$$M_Z = \underbrace{-\frac{g_Z^2}{s-m_Z^2}}_{K}[\bar v(p_2)\gamma^\mu \underbrace{\tfrac{1}{2}(c_V - c_A\gamma^5)}_{C_1} u(p_1)][\bar u(p_3)\gamma_\mu \underbrace{\tfrac{1}{2}(c_V^b - c_A^b\gamma^5)}_{C_2} v(p_4)]$$

$$M_Z^* = K[\bar v(p_4)\gamma_\nu C_2 u(p_3)][\bar u(p_1)\gamma^\nu C_1 v(p_2)]$$

$$\frac{2}{K\alpha}\chi = \sum[\bar v_a(p_2)\gamma^\mu_{ab} u_b(p_1)][\bar u_c(p_3)\gamma_{\mu\,cd} v_d(p_4)][\bar v_e(p_4)(\gamma_\nu C_2)_{ef} u_f(p_3)][\bar u_g(p_1)(\gamma^\nu C_1)_{gh} v_h(p_2)]$$

$$= \sum u_b(p_1)\bar u_g(p_1)\, v_h(p_2)\bar v_a(p_2)\, u_f(p_3)\bar u_c(p_3)\, v_d(p_4)\bar v_e(p_4)\; \gamma^\mu_{ab}\;\gamma_{\mu\,cd}\;(\gamma_\nu C_2)_{ef}\;(\gamma^\nu C_1)_{gh}$$

$$[(\not{p}_1 + m_\mu)_{bg}(\gamma^\nu C_1)_{gh}(\not{p}_2 - m_\mu)_{ha}\gamma^\mu_{ab}][(\not{p}_3 + m_b)_{fc}\gamma_{\mu\,cd}(\not{p}_4 - m_b)_{de}(\gamma_\nu C_2)_{ef}]$$

$$= Tr[(\not{p}_1 + m_\mu)\gamma^\nu C_1(\not{p}_2 - m_\mu)\gamma^\nu]\; Tr[(\not{p}_3 + m_b)\gamma_\mu(\not{p}_4 - m_b)\gamma_\nu C_2] = \mathcal{L}_1 \times \mathcal{L}_2$$

$$\mathcal{L}_1 = Tr[(\not{p}_1 + m_\mu)\gamma^\nu(c_L P_L + c_R P_R)(\not{p}_2 - m_\mu)\gamma^\mu]$$
$$= Tr[\not{p}_1\gamma^\nu C_1\not{p}_2\gamma^\mu] - m_\mu Tr[\not{p}_1\gamma^\nu C_1\gamma^\mu] + m_\mu Tr[\gamma^\nu C_1\not{p}_2\gamma^\mu] - m_\mu^2 Tr[\gamma^\nu C_1\gamma^\mu]$$
$$= \tfrac{1}{2}(Tr[\not{p}_1\gamma^\nu c_V\not{p}_2\gamma^\mu] - Tr[\not{p}_1\gamma^\nu c_A\gamma^5\not{p}_2\gamma^\mu]) - \tfrac{1}{2}m_\mu^2(Tr[\gamma^\nu c_V\gamma^\mu] - Tr[\gamma^\nu c_A\gamma^5\gamma^\mu])$$
$$= \tfrac{1}{2}(c_V Tr[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu] - c_A Tr[\not{p}_1\gamma^\nu\gamma^5\not{p}_2\gamma^\mu] - m_\mu^2 c_V 4 g^{\nu\mu} + 0)$$
$$= \tfrac{1}{2}[4c_V(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu) - c_A p_{1\sigma}p_{2\rho}4i\varepsilon^{\sigma\nu\rho\mu} - 4m_\mu^2 c_V g^{\nu\mu}]$$
$$= 2[c_V(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu) - i c_A p_{1\sigma}p_{2\rho}\varepsilon^{\sigma\nu\rho\mu} - m_\mu^2 c_V g^{\nu\mu}]$$

$$\mathcal{L}_1 = 2[c_V(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu - m_\mu^2 g^{\nu\mu}) - i c_A p_{1\sigma}p_{2\rho}\varepsilon^{\sigma\nu\rho\mu}]$$

$$\mathcal{L}_2 = 2[c_V^b(p_{3\mu}p_{4\nu} - g_{\mu\nu}(p_3\cdot p_4) + p_{3\nu}p_{4\mu} - m_b^2 g_{\mu\nu}) - i c_A^b p_3^\alpha p_4^\beta\varepsilon_{\alpha\mu\beta\nu}]$$

$$\frac{\mathcal{L}_1\mathcal{L}_2}{4} = c_V c_V^b[2(p_1\cdot p_4)(p_2\cdot p_3) + 2(p_1\cdot p_3)(p_2\cdot p_4) + 2m_b^2(p_1\cdot p_2) + 2m_\mu^2(p_3\cdot p_4) + 4m_\mu^2 m_b^2]$$
$$- i c_V c_A^b(p_1^\nu p_2^\mu + p_1^\mu p_2^\nu)p_3^\alpha p_4^\beta\varepsilon_{\alpha\mu\beta\nu} \quad \Rightarrow 0$$
$$- i c_A c_V^b(p_{3\mu}p_{4\nu} + p_{3\nu}p_{4\mu})p_{1\sigma}p_{2\rho}\varepsilon^{\sigma\nu\rho\mu} \quad \Rightarrow 0$$
$$+ i^2 c_A c_A^b p_{1\sigma}p_{2\rho}p_3^\alpha p_4^\beta\varepsilon^{\sigma\nu\rho\mu}\varepsilon_{\alpha\mu\beta\nu}$$

$$= 2c_V c_V^b[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2 m_b^2]$$
$$+ c_A c_A^b p_{1\sigma}p_{2\rho}p_3^\alpha p_4^\beta(\varepsilon^{\nu\mu\sigma\rho}\varepsilon_{\nu\mu\alpha\beta}$$

$$= 2c_V c_V^b[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2 m_b^2]$$
$$- 2c_A c_A^b[(p_1\cdot p_3)(p_2\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3)]$$

$$\frac{\mathcal{L}_1\mathcal{L}_2}{4} = 2(p_1\cdot p_4)(p_2\cdot p_3)[c_V c_V^b + c_A c_A^b] + (p_1\cdot p_3)(p_2\cdot p_4)[c_V c_V^b - c_A c_A^b] + c_V c_V^b[m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2 m_b^2]$$

$$\chi = \frac{K\alpha}{2}\mathcal{L}_1\mathcal{L}_2 \qquad \frac{\varepsilon}{2}$$
$$= \frac{e^2 g_Z^2 4}{3s(s-m_Z^2)}[(c_V c_V^b + c_A c_A^b)(p_1\cdot p_4)(p_2\cdot p_3) + (c_V c_V^b - c_A c_A^b)(p_1\cdot p_3)(p_2\cdot p_4) + c_V c_V^b(m_b^2(p_1\cdot p_2) + m_\mu^2(p_3\cdot p_4) + 2m_\mu^2 m_b^2$$

NC WI term

$$M_Z = -\overbrace{\frac{g_Z^2}{s-m_Z^2}}^{K}\left[\bar v(p_2)\gamma^\mu \tfrac12(c_V - c_A\gamma^5)u(p_1)\right]\left[\bar u(p_3)\gamma_\mu \tfrac12(c_V^b - c_A^b\gamma^5)v(p_4)\right]$$

$$= -K\left[\bar v(p_2)\gamma^\mu \underbrace{(c_L P_L + c_R P_R)}_{C_1} u(p_1)\right]\left[\bar u(p_3)\gamma_\mu \underbrace{(c_L^b P_L + c_R^b P_R)}_{C_2} v(p_4)\right]$$

$$= -K\left[\bar v(p_2)\gamma^\mu C_1 u(p_1)\right]\left[\bar u(p_3)\gamma_\mu C_2 v(p_4)\right]$$

$$M_Z M_Z^* = K^2\left[\bar v_a(p_2)\gamma^\mu C_{1\,ab} u_b(p_1)\right]\left[\bar u_c(p_3)\gamma_\mu C_{2\,cd} v_d(p_4)\right]\left[\bar v_e(p_4)\gamma_\nu C_{2\,ef} u_f(p_3)\right]\left[\bar u_g(p_1)\gamma^\nu C_{1\,gh} v_h(p_2)\right]$$

$$\frac{4}{K^2}\langle|M_Z|^2\rangle = \sum\left[u(p_1)\bar u(p_1)\right]_{bg}\left[v(p_2)\bar v(p_2)\right]_{ha}\left[u(p_3)\bar u(p_3)\right]_{fc}\left[v(p_4)\bar v(p_4)\right]_{de}\,(\gamma^\mu C_1)_{ab}\,(\gamma_\mu C_2)_{cd}\,(\gamma_\nu C_2)_{ef}\,(\gamma^\nu C_1)_{gh}$$

$$\left[(\not p_1 + m_\mu)\gamma^\nu C_1(\not p_2 - m_\mu)\gamma^\mu C_1\right]\left[(\not p_3 + m_b)\gamma_\mu C_2(\not p_4 - m_b)\gamma_\nu C_2\right]$$

$$= \mathrm{Tr}\left[(\not p_1 + m_\mu)\gamma^\nu C_1(\not p_2 - m_\mu)\gamma^\mu C_1\right] \times \mathrm{Tr}\left[(\not p_3 + m_b)\gamma_\mu C_2(\not p_4 - m_b)\gamma_\nu C_2\right] = \mathcal{L}_1 \times \mathcal{L}_2$$

$$\mathcal{L}_1 = \mathrm{Tr}\left[(\not p_1\gamma^\nu C_1 + m_\mu\gamma^\nu C_1)(\not p_2\gamma^\mu C_1 - m_\mu\gamma^\mu C_1)\right] = \mathrm{Tr}\left[\not p_1\gamma^\nu C_1\not p_2\gamma^\mu C_1\right] - m_\mu\mathrm{Tr}$$

$$= \mathrm{Tr}\left[\not p_1\gamma^\nu C_1\not p_2\gamma^\mu C_1\right] - m_\mu\underbrace{\mathrm{Tr}\left[\not p_1\gamma^\nu C_1\gamma^\mu C_1\right]}_{0} + m_\mu\underbrace{\mathrm{Tr}\left[\gamma^\nu C_1\not p_2\gamma^\mu C_1\right]}_{0} - m_\mu^2\mathrm{Tr}\left[\gamma^\nu C_1\gamma^\mu C_1\right]$$

$$= \mathrm{Tr}\left[\not p_1\gamma^\nu C_1\not p_2\gamma^\mu C_1\right] - m_\mu^2\mathrm{Tr}\left[\gamma^\nu C_1\gamma^\mu C_1\right]$$

$$= \mathrm{Tr}\left[\not p_1\gamma^\nu(c_L P_L + c_R P_R)\not p_2\gamma^\mu(c_L P_L + c_R P_R)\right] - m_\mu^2\mathrm{Tr}\left[\gamma^\nu(c_L P_L + c_R P_R)\gamma^\mu(c_L P_L + c_R P_R)\right]$$

$$= p_{1\sigma}p_{2\rho}\mathrm{Tr}\left[(\gamma^\sigma\gamma^\nu c_L P_L + \gamma^\sigma\gamma^\nu c_R P_R)(\gamma^\rho\gamma^\mu c_L P_L + \gamma^\rho\gamma^\mu c_R P_R)\right] - m_\mu^2\mathrm{Tr}\left[(\gamma^\nu c_L P_L + \gamma^\nu c_R P_R)(\gamma^\mu c_L P_L + \gamma^\mu c_R P_R)\right]$$

$$= p_{1\sigma}p_{2\rho}\mathrm{Tr}\left[\gamma^\sigma\gamma^\nu c_L P_L\gamma^\rho\gamma^\mu c_L P_L + \underbrace{\gamma^\sigma\gamma^\nu c_L P_L\gamma^\rho\gamma^\mu c_R P_R}_{0} + \gamma^\sigma\gamma^\nu c_R P_R\gamma^\rho\gamma^\mu c_L P_L + \gamma^\sigma\gamma^\nu c_R P_R\gamma^\rho\gamma^\mu c_R P_R\right]$$
$$- m_\mu^2\mathrm{Tr}\left[\underbrace{\gamma^\nu c_L P_L\gamma^\mu c_L P_L}_{0} + \gamma^\nu c_L P_L\gamma^\mu c_R P_R + \gamma^\nu c_R P_R\gamma^\mu c_L P_L + \underbrace{\gamma^\nu c_R P_R\gamma^\mu c_R P_R}_{0}\right]$$

$$= p_{1\sigma}p_{2\rho}\left(\mathrm{Tr}\left[c_L^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu P_L\right] + \mathrm{Tr}\left[c_R^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu P_R\right]\right) - m_\mu^2\left(\mathrm{Tr}\left[c_L c_R\gamma^\nu\gamma^\mu P_R\right] + \mathrm{Tr}\left[c_R c_L\gamma^\nu\gamma^\mu P_L\right]\right)$$

$$= p_{1\sigma}p_{2\rho}\left(\mathrm{Tr}\left[c_L^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\tfrac12\right] - \mathrm{Tr}\left[c_L^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\tfrac12\gamma^5\right] + \mathrm{Tr}\left[c_R^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\tfrac12\right] + \mathrm{Tr}\left[c_R^2\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\tfrac12\gamma^5\right]\right)$$
$$- m_\mu^2\left(\mathrm{Tr}\left[c_L c_R\gamma^\nu\gamma^\mu\tfrac12\right] + \mathrm{Tr}\left[c_L c_R\underbrace{\gamma^\nu\gamma^\mu\tfrac12\gamma^5}_{0}\right] + \mathrm{Tr}\left[c_R c_L\gamma^\nu\gamma^\mu\tfrac12\right] - \mathrm{Tr}\left[c_R c_L\underbrace{\gamma^\nu\gamma^\mu\tfrac12\gamma^5}_{0}\right]\right)$$

$$= p_{1\sigma}p_{2\rho}\left[\tfrac12(c_L^2 + c_R^2)\mathrm{Tr}\left[\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\right] - \tfrac12(c_L^2 - c_R^2)\mathrm{Tr}\left[\gamma^\sigma\gamma^\nu\gamma^\rho\gamma^\mu\gamma^5\right]\right] - m_\mu^2\left[\tfrac12(c_L c_R + c_R c_L)\mathrm{Tr}\left[\gamma^\nu\gamma^\mu\right]\right]$$

Comparing with calculations for QED transition amplitude

$$\mathcal{L}_1 = 2\left[\overbrace{(c_L^2 + c_R^2)}^{C_1^+}(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu) - 2m_\mu^2\overbrace{c_L c_R}^{C_1} g^{\nu\mu} - p_{1\sigma}p_{2\rho}\overbrace{(c_L^2 - c_R^2)}^{C_1^-} i\,\varepsilon^{\sigma\nu\rho\mu}\right]$$

with $C_2^+ = (c_L^{b2} + c_R^{b2})$, $C_2 = c_L^b c_R^b$ and $C_2^- = c_L^{b2} - c_R^{b2}$ we have

$$\mathcal{L}_1 = 2\left[C_1^+(p_1^\nu p_2^\mu - g^{\nu\mu}(p_1\cdot p_2) + p_1^\mu p_2^\nu) - 2m_\mu^2 C_1 g^{\nu\mu} - i C_1^- p_{1\sigma}p_{2\rho}\varepsilon^{\sigma\nu\rho\mu}\right]$$

$$\mathcal{L}_2 = 2\left[C_2^+(p_{3\mu}p_{4\nu} - g_{\mu\nu}(p_3\cdot p_4) + p_{3\nu}p_{4\mu}) - 2m_b^2 C_2 g_{\mu\nu} - i C_2^- p_3^\alpha p_4^\beta\varepsilon_{\alpha\mu\beta\nu}\right]$$

Neutral current weak interaction term continued

Still comparing with QED calculations

$$\frac{\mathcal{L}_1 \times \mathcal{L}_2}{4} = C_1^+ C_2^+ [2(p_1\cdot p_4)(p_2\cdot p_3) + 2(p_1\cdot p_3)(p_2\cdot p_4)]$$
$$- 2m_b^2 C_1^+ C_2 [(p_1\cdot p_2) - 4(p_1\cdot p_2) + (p_1\cdot p_2)] \Rightarrow + 4m_b^2 C_1^+ C_2 (p_1\cdot p_2)$$
$$- i C_1^+ C_2^- [p_1^\nu p_2^\mu p_3^\alpha p_4^\beta \varepsilon_{\alpha\mu\beta\nu} + p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \varepsilon_{\alpha\mu\beta\nu}] \Rightarrow 0$$
$$- 2m_\mu^2 C_1 C_2^+ [(p_3\cdot p_4) - 4(p_3\cdot p_4) + (p_3\cdot p_4)] \Rightarrow + 4 m_\mu^2 C_2^+ C_1 (p_3\cdot p_4)$$
$$+ 4 m_\mu^2 C_1 m_b^2 C_2 4 \Rightarrow 16 m_\mu^2 m_b^2 C_1 C_2$$
$$+ 0$$
$$- i C_1^- C_2^+ [p_{3\mu} p_{4\nu} p_{1\sigma} p_{2\rho} \varepsilon^{\sigma\nu\rho\mu} + p_{3\nu} p_{4\mu} p_{1\sigma} p_{2\rho} \varepsilon^{\sigma\nu\rho\mu}] \Rightarrow 0$$
$$+ 0$$
$$+ i^2 C_1 C_2^- p_{1\sigma} p_{2\rho} p_3^\alpha p_4^\beta \varepsilon^{\sigma\nu\rho\mu} \varepsilon_{\alpha\mu\beta\nu}$$

$$\begin{array}{l} -\varepsilon_{\mu\alpha\beta\nu} \\ +\varepsilon_{\mu\alpha\nu\beta} \\ -\varepsilon_{\mu\nu\alpha\beta} \\ +\varepsilon_{\nu\mu\alpha\beta} \\ \\ -\varepsilon^{\nu\sigma\rho\mu} \\ +\varepsilon^{\nu\sigma\mu\rho} \\ -\varepsilon^{\nu\mu\sigma\rho} \end{array}$$

The last term: $(\varepsilon^{\alpha\beta\mu\nu}\varepsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu_\rho \delta^\nu_\sigma - \delta^\mu_\sigma \delta^\nu_\rho)$

$$= -C_1 C_2^- p_{1\sigma} p_{2\rho} p_3^\alpha p_4^\beta (-\varepsilon^{\nu\mu\sigma\rho}\varepsilon_{\nu\mu\alpha\beta}) = + C_1^- C_2^- p_{1\sigma} p_{2\rho} p_3^\alpha p_4^\beta [-2(\delta^\sigma_\alpha \delta^\rho_\beta - \delta^\sigma_\beta \delta^\rho_\alpha)$$
$$= -2 C_1^- C_2^- [(p_1\cdot p_3)(p_2\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3)]$$

$$C_V = C_L + C_R$$
$$C_A = C_L - C_R$$
$$C_V^2 + C_A^2 = 2(C_L^2 + C_R^2)$$
$$C_V C_A = C_L^2 - C_R^2$$
$$C_V^2 - C_A^2 = 4 C_L C_R$$

$$\frac{\mathcal{L}_1 \times \mathcal{L}_2}{4} = 2C_1^+ C_2^+ [(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)]$$
$$+ 2 C_1^- C_2^- [(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4)]$$
$$+ 4 m_b^2 C_1^+ C_2 (p_1\cdot p_2)$$
$$+ 4 m_\mu^2 C_2^+ C_1 (p_1\cdot p_2)$$
$$+ 16 m_\mu^2 m_b^2 C_1 C_2$$

$$\mathcal{L}_1 \times \mathcal{L}_2 = 2(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2)[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)]$$
$$+ 2(C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2) m_b^2 (p_1\cdot p_2)$$
$$+ 2(C_V^2 - C_A^2)(C_{Vb}^2 + C_{Ab}^2) m_\mu^2 (p_3\cdot p_4)$$
$$+ 4(C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2) m_\mu^2 m_b^2$$
$$+ 8 C_V C_A C_{Vb} C_{Ab} [(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4)]$$

The spin-averaged matrix element squared is

$$\langle |M_Z|^2 \rangle = \frac{K^2}{4} \mathcal{L}_1 \times \mathcal{L}_2$$
$$= \frac{g_Z^4}{2(s-M_Z^2)^2}\Big[(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2)[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)]$$
$$+ (C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2) m_b^2 (p_1\cdot p_2)$$
$$+ (C_V^2 - C_A^2)(C_{Vb}^2 + C_{Ab}^2) m_\mu^2 (p_3\cdot p_4)$$
$$+ (C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2) 2 m_\mu^2 m_b^2$$
$$+ 4 C_V C_A C_{Vb} C_{Ab} [(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4)] \Big]$$

$$= \frac{g_Z^4}{2(s-M_Z^2)^2}\Big[(p_1\cdot p_4)(p_2\cdot p_3)[(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2) + C_V C_A C_{Vb} C_{Ab}]$$
$$+ (p_1\cdot p_3)(p_2\cdot p_4)[(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2) - C_V C_A C_{Vb} C_{Ab}]$$
$$+ m_b^2 (p_1\cdot p_2)(C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2)$$
$$+ m_\mu^2 (p_3\cdot p_4)(C_V^2 - C_A^2)(C_{Vb}^2 + C_{Ab}^2)$$
$$+ 2 m_\mu^2 m_b^2 (C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2) \Big]$$

$$
\begin{aligned}
\frac{\delta_1 \times \delta_2}{8} = {} & C_1^+ C_2^+ \left[p_{14} p_{23} + p_{13} p_{24} \right] \\
& + C_1^- C_2^- \left[p_{14} p_{23} - p_{13} p_{24} \right. \\
& + 2 m_b^2 \, C_1^+ C_2 \, (p_1 p_2) \\
& + 2 m_\mu^2 \, C_2^+ C_1 \, (p_3 p_4) \\
& + 8 \, m_\mu^2 m_b^2 \, C_1 C_2
\end{aligned}
$$

$$
\begin{aligned}
= {} & (p_{14} p_{23} + p_{13} p_{24}) \, \tfrac{1}{4} (C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2) \\
& + (p_{14} p_{23} - p_{13} p_{24}) \, C_V C_A C_{Vb} C_{Ab} \\
& + 2 \, p_{12} \, m_b^2 \, \tfrac{1}{8} (C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2) \\
& + 2 \, p_{34} \, m_\mu^2 \, \tfrac{1}{8} (C_{Vb}^2 + C_{Ab}^2)(C_V^2 - C_A^2) \\
& + 8 \, m_\mu^2 m_b^2 \, \tfrac{1}{16} (C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2)
\end{aligned}
$$

$$
\begin{aligned}
\delta_1 \times \delta_2 = {} & 2 (C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2)(p_{14} p_{23} + p_{13} p_{24}) \\
& + 8 \, C_V C_A C_{Vb} C_{Ab} \, (p_{14} p_{23} - p_{13} p_{24}) \\
& + 2 (C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2) \, p_{12} \, m_b^2 \\
& + 2 (C_V^2 - C_A^2)(C_{Vb}^2 + C_{Ab}^2) \, p_{34} \, m_\mu^2 \\
& + 4 (C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2) \, m_\mu^2 m_b^2
\end{aligned}
$$

$$
\begin{aligned}
= 2 \Big(& p_{14} p_{23} \left[(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2) + 4 C_V C_A C_{Vb} C_{Ab} \right] \\
& + p_{13} p_{24} \left[(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2) - 4 C_V C_A C_{Vb} C_{Ab} \right] \\
& + 1 - " - \\
& + 1 - " - \\
& + 2 (C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2) \, m_\mu^2 m_b^2
\end{aligned}
$$

$$
= 2 \left[f_1 (p_1 \cdot p_4)(p_2 \cdot p_3) + f_2 (p_1 \cdot p_3)(p_2 \cdot p_4) + f_3 m_b^2 (p_1 \cdot p_2) + f_4 m_\mu^2 (p_3 \cdot p_4) + 2 f_5 m_\mu^2 m_b^2 \right]
$$

$$C_1^+ = C_L^2 + C_R^2, \quad C_1^- = C_L^2 - C_R^2, \quad C_1 = C_L C_R$$

$$C_V^2 + C_A^2 = 2(C_L^2 + C_R^2), \quad C_V C_A = C_L^2 - C_R^2, \quad C_V^2 - C_A^2 = 4 C_L C_R$$

$$C_1^+ C_2^+ = (C_L^2 + C_R^2)(C_{Lb}^2 + C_{Rb}^2) = \frac{1}{2}(C_V^2 + C_A^2)\,\frac{1}{2}(C_{Vb}^2 + C_{Ab}^2) = \frac{1}{4}(C_V^2 + C_A^2)(C_{Vb}^2 + C_{Ab}^2)$$

$$C_1^- C_2^- = (C_L^2 - C_R^2)(C_{Lb}^2 - C_{Rb}^2) = C_V C_A C_{Vb} C_{Ab}$$

$$C_1^+ C_2 = (C_L^2 + C_R^2)(C_{Lb} C_{Rb}) = \frac{1}{2}(C_V^2 + C_A^2)\,\frac{1}{4}(C_{Vb}^2 - C_{Ab}^2) = \frac{1}{8}(C_V^2 + C_A^2)(C_{Vb}^2 - C_{Ab}^2)$$

$$C_2^+ C_1 = (C_{Lb}^2 + C_{Rb}^2)\, C_L C_R = \frac{1}{8}(C_{Vb}^2 + C_{Ab}^2)(C_V^2 - C_A^2)$$

$$C_1 C_2 = C_L C_R C_{Lb} C_{Rb} = \frac{1}{4}(C_V^2 - C_A^2)\,\frac{1}{4}(C_{Vb}^2 - C_{Ab}^2) = \frac{1}{16}(C_V^2 - C_A^2)(C_{Vb}^2 - C_{Ab}^2)$$