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$M/G/1/GD/\infty/\infty$ (or) $M/G/1$

$M \rightarrow$ Interarrival time has exponential distribution

$G \rightarrow$ The service time can have ~~any~~ any distribution

$1 \rightarrow$ There is one server

- Service time is represented here by variable 's.'
- Avg. of service time, $E(s)$ (or) W_s $= \frac{1}{\mu}$, where μ is service rate
- variance of service time, $var(s) = \sigma^2$
- Traffic intensity, $\rho = \frac{\lambda}{\mu}$

$W_s = \frac{1}{\mu}$	$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$
$L_s = \lambda . W_s = \frac{\lambda}{\mu} = \rho$	$W_q = \frac{L_q}{\lambda}$
$P_0 = 1 - \rho$ $= 1 - \lambda/\mu$	Total Wait in the system (W) $= W_s + W_q$

- Total no. of customers in the system $(L) = \lambda * W$

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- if service time ~~can~~ can be determined, that means there is no variability.

$\therefore$ The variance, $var(s) = \sigma^2 = 0 =$ zero

Example 1

- average of 20 cars/hours arrive to a drive-in fast food.
- Service time of each car is 2 min $\rightarrow$ deterministic service time

Assume exponential inter arrival time. Find No. of cars waiting in line.

Ans:-

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$\rho = \frac{\lambda}{\mu} = \frac{20}{30}, \text{ it is } < 1 \text{ (less than)}$$

$\mu = 1 \text{ cars} / 2 \text{ min}$

$\therefore \frac{60}{2} = 30 \text{ cars/hour}$

$$\therefore L_q = \frac{20^2 * 0 + (2/3)^2}{2(1-2/3)}$$

$= 2/3$ customers

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$$W_q = \frac{L_q}{\lambda} = \frac{2/3}{20}$$

$$= 1/30 \text{ hour}$$ //Ans.

Example 2

Consider the following single server queue. The inter-arrival time is exponentially distributed with a mean of 10 min and the service time is also exponentially distributed with a mean of 8 min.

Find (i) Mean wait in the queue
(ii) Mean number in the queue
(iii) The mean wait in the system
(iv) mean number in the system
(v) Proportion of time the server is idle.

Sol: This is an M/M/1 system. $\rightarrow 1-\rho$

Arrival rate $(\lambda) = 1$ per 10 min

$= 1/10 = 6/\text{hour}$

Service rate $(\mu) = 1$ per 8 min

$= 1/8 = 7.5/\text{hour}$

$$\rho = \frac{\lambda}{\mu} = \frac{1/10}{1/8} = 8/10 = 0.8$$

(ii) Mean number is the queue

$$(L_q) = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$= \frac{6^2}{7.5(7.5-6)}$$

$$= 3.2$$

(i) Wait in the queue. (W_q)

$$= \frac{L_q}{\lambda} = \frac{3.2}{6}$$

$$\cancel{= \frac{\lambda}{\mu(\mu-\lambda)}} = 0.53 \text{ hours}$$

(iii) Wait in the system.

$$W_s = \frac{1}{\mu-\lambda}$$

$$= 0.667 \text{ hours}$$

(iv) $L_s = \frac{\lambda}{\mu-\lambda} = \frac{6}{7.5-6}$

$$= 4$$

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(v)

$$P_0 = 1 - \rho$$

$$= 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{6}{7.5}$$

$$= 0.2$$

Example 2

The inter-arrival time is exponentially distributed with a mean of 10 min. Service time ~~is exponentially~~ has uniform distribution with a max. of 9 min and min. of 7 min. Find (i) Mean wait in the queue

(ii) Mean number in the queue

(iii) Mean wait in the system

(iv) Mean number in the system

(v) proportion of time the server is idle.

Sol: $\boxed{\text{This is an } M/G/1 \text{ system}}$

$\lambda = 1/10$ min

$= 6/\text{hour}$

$\frac{60}{10} = 6$

~~$\mu = (8\ 1/(9+7)$~~

$\mu = 1/8$ min

$= 7.5/\text{hour}$

$\frac{9+7}{2} = 8$

$\frac{60}{8}$

variance, $\sigma^2 = \dfrac{(\text{max value} - \text{min value})^2}{12}$

$= \dfrac{(9-7)^2}{12}$

$= 1/3$

$\rho = \dfrac{\lambda}{\mu} = \dfrac{6}{7.5} = 0.8$

(ii) Number in the queue, $L_q = \dfrac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$

$= \dfrac{\left(\frac{1}{10}\right)^2 \left(\frac{1}{3}\right)^2 + (0.8)^2}{2(1-0.8)}$

$= \cancel{31.6} = 1.6083$ //

(i.) $\cancel{W_q = \dfrac{L_q}{\lambda} = \dfrac{31.6}{6}}$

$\cancel{= 5.267 \text{ hours}}$.

(i) $W_q = \dfrac{L_q}{\lambda} = 16.08$ min //

(iii) $W = W_q + 1/\mu = 16.08 + \dfrac{1}{(1/8)}$

$= 24.08$ min //

$\therefore$

(iv) $L = \lambda W = \left(\frac{1}{10}\right) * 24.08 = 2.$