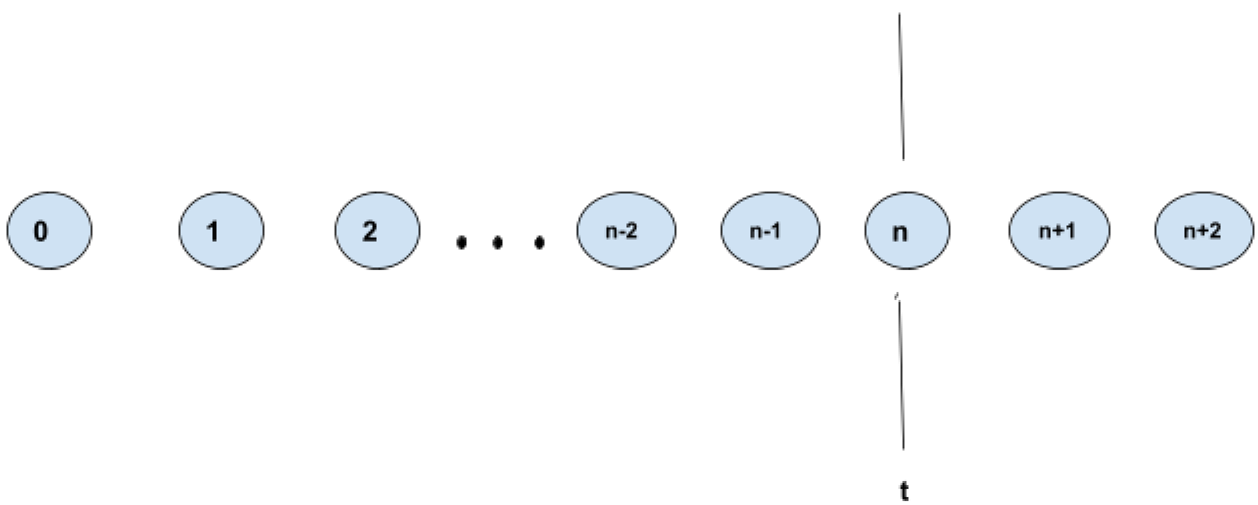
<u>Birth Death Processes (QUEUE)</u>

Birth means **arrival** of a new customer in the **queuing system**
Death means **departure/completion** of service to a customer

The states are 0,1,2…(n-2), (n-1), n, n+1, n+2
That means **N(t) = n** , denotes the **Number of customers** are **time t is equal to n**



Let the system be at **state n** , i.e n number of customers at **time t**
Now we can count the time until the **next arrival** of another customer or **completion of service** to any one of the customers may take place.

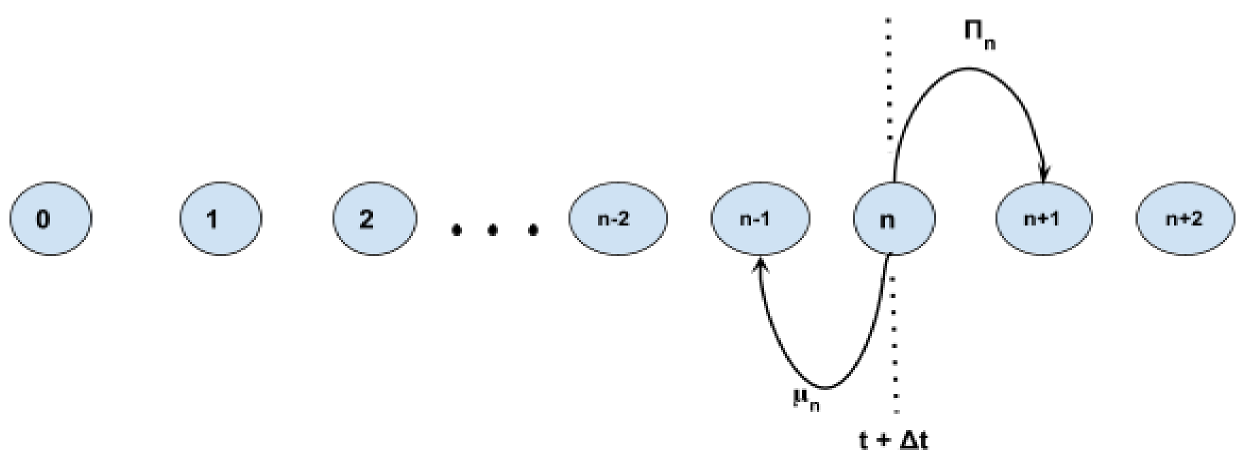
The **arrival time** of any **two consecutive customer** is a **random variable** and will some probability distribution.

Similarly, the service time needed by each customer will be different, thus that is also a random variable and will some probability distribution. .

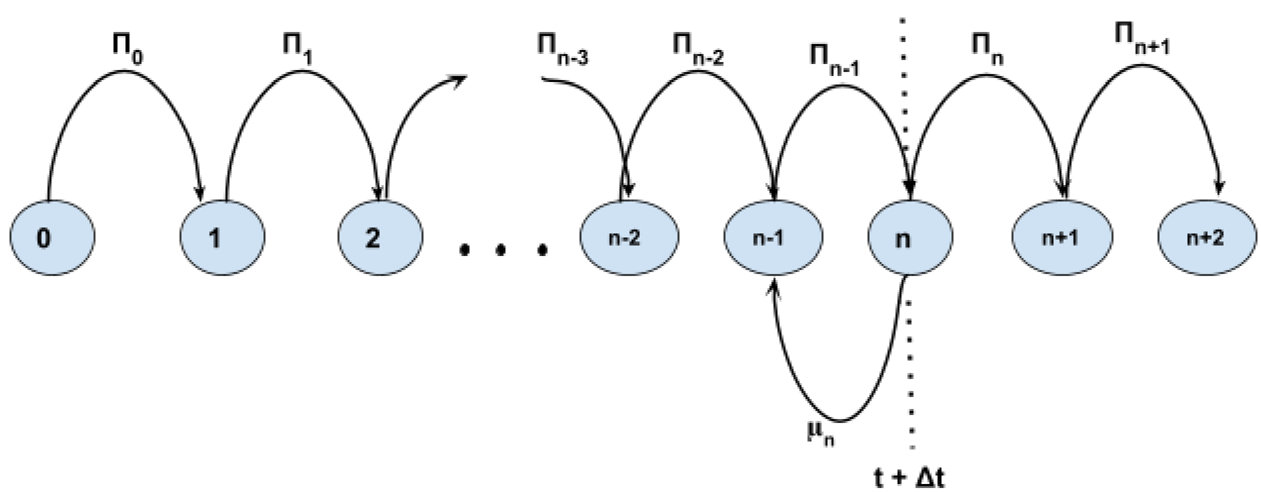
Given that N(t) = n

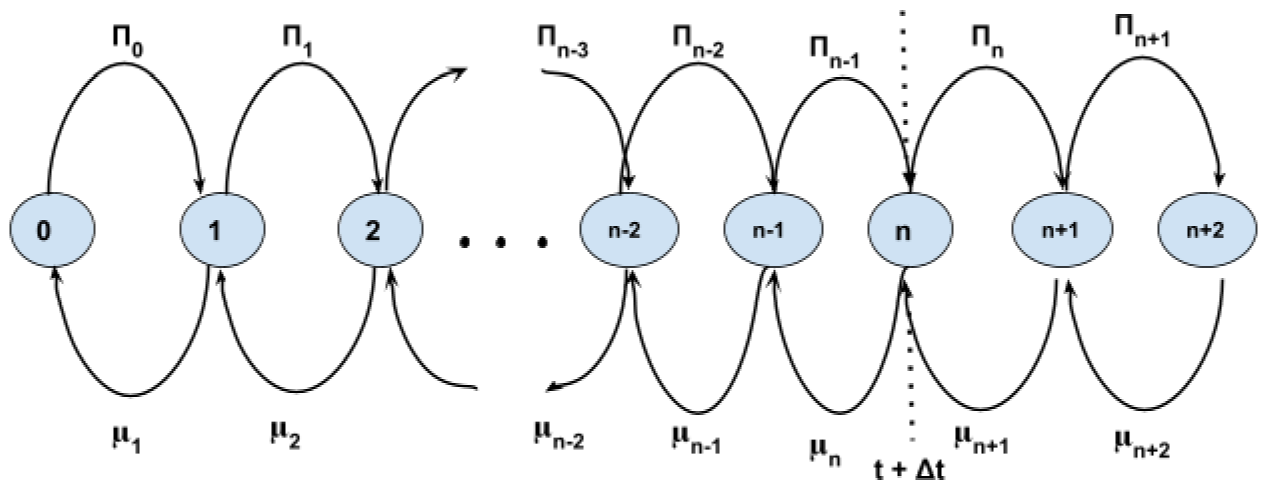
Then **Assumption 1** is, the current probability distribution of the remaining time until the next birth will be Π_n

Assumption 2 is that, it could be possible that a customer leaves the queue at time t, the current probability distribution of the remaining time until death takes placewill be μ_n



Π **and** μ are state dependant.





Now the series of random numbers for arrival and service completion are independent.

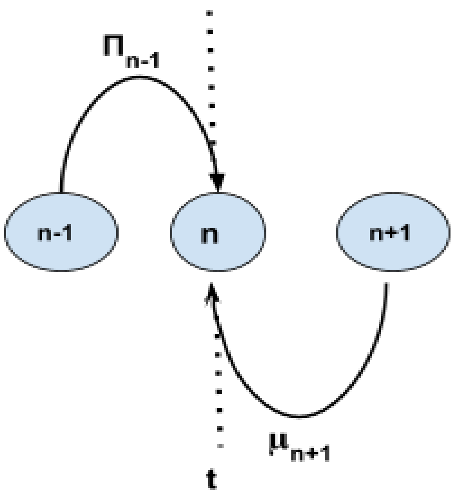
Assumption 3, the state may reach **n → n+1 with a birth**
or, n → n-1 with a death

The birth and death process is explained with these 3 assumptions.

<u>Deriving the balanced equation for any state n.</u>

Consider a state n at time t
Now, **E_n(t)** is **the number of times the system enters the state n**
L_n(t) is **the number of time the system leaves state n**

There are two possibilities to enter state n



New Arrival : from n-1 to n

Service Completion : from n+1 to n

Now, over a period of time

$| E_n(t) - L_n(t) | \le 1$

(It can be 0 or 1)

[Because if it leaves the state n as many times as it enters the state n, the equation will be 0 And, if it is in a position to get a new arrival from state n-1 or service completion from n+1, the difference will be -1 or +1.]

Applying limit from time **t to infinity**

$$\lim_{t\to\infty}\left|\frac{E_n(t)}{t}-\frac{L_n(t)}{t}\right|\le\frac{1}{t}$$

$$\lim_{t\to\infty}\left|\frac{E_n(t)}{t}-\frac{L_n(t)}{t}\right| = 0$$

$$\lim_{t\to\infty}\frac{E_n(t)}{t} = \lim_{t\to\infty}\left|\frac{L_n(t)}{t}\right|$$

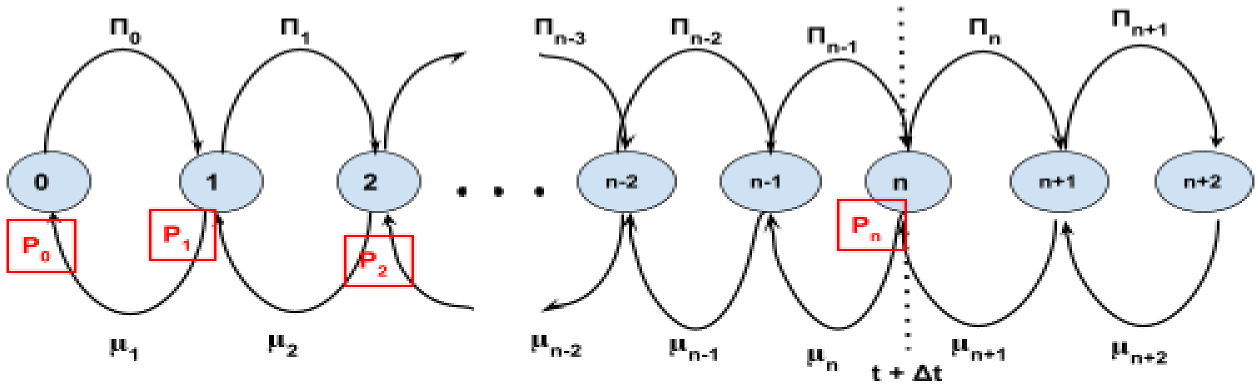
This shows that ,

Mean rate at which the system enters the state n = Mean rate at which the system leaves state n

Rate in = Rate out

This is the balanced equation for this continuous time markov chain

The probability that the system is in a particular state is P_n
Thus P_0 is the probability that the system is in state 0
P_1 is the probability that the system is in state 1, etc



Now, **as the total probability equals to 1,** using this we can find :

- The expected number of customers in the queuing system at any time t.
- The average time spent by a customer in a system
- The average waiting time of a customer in the queue