M/M/1 QUEUING MODEL

The M/M/1 queuing model is a queuing model where the **arrivals follow a Poisson process**, **service times are exponentially distributed** and **there is one server.**

A Poisson process means the arrival of an event is independent of the event before (waiting time between events is memoryless).

The assumptions of M/M/1 queuing model are as follows:

- The **number of customers** arriving in a **time interval t** follows a Poisson Process with **parameter λ**
- The **interval between any two successive arrivals** is **exponentially distributed** with **parameter λ**
- The **time taken to complete a single service** is **exponentially distributed** with **parameter μ.**
- The number of **server is one**.
- Although not explicitly stated both the **population and** the **queue size** can be **infinity**.
- The **order of service** is assumed to be **FIFO.**

Example 1

Students arrive at the head office of Universal Teacher Publications according to a Poisson input process with a mean rate of 40 per hour. The time required to serve a student has an exponential distribution with a mean of 50 per hour. Assume that the students are served by a single individual, find the average waiting time of a student.

Given,

Student's arrival rate $(\lambda) = 40/\text{hour}$

Time required to serve a student $(\mu) = 50/\text{hour}$

Average waiting time of a student before receiving service (W_q)

$$= \frac{\lambda}{\mu(\mu-\lambda)} * 60 \text{ min}$$

$$= \frac{40}{50(50-40)} * 60 \text{ min}$$

$$= 0.08 * 60$$

$$= 4.8 \text{ minutes}$$

or, it can be 0.08 hours also.

Example 2

Universal Bank is considering opening a drive in window for customer service. Management estimates that customers will arrive at the rate of 15 per hour. The teller whom it is considering to staff the window can service customers at the rate of one every three minutes.

Assuming Poisson arrivals and exponential service find

1. Average number in the waiting line.
2. Average number in the system.
3. Average waiting time in line.
4. Average waiting time in the system.

Solution:

Given

λ = 15/hour,

μ = 3/60 hour

or 20/hour

Average number in the waiting line = $\dfrac{(15)^2}{20(20 - 15)}$ = 2.25 customers

Average number in the system = $\dfrac{15}{20 - 15}$ = 3 customers

Average waiting time in line = $\dfrac{15}{20(20 - 15)}$ = 0.15 hours

Average waiting time in the system = $\dfrac{1}{20 - 15}$ = 0.20 hours

Example 3

At a petrol pump, customers arrive according to a Poisson process with an average time of 5 minutes between arrivals. The service time is exponentially distributed with mean time = 2 minutes. On the basis of this information, find out:

1. What would be the average queue length?
2. What would be the average number of customers in the system?
3. What is the average time spent by a car in the petrol pump?
4. What is the average waiting time of a car before receiving petrol?

customer's arrival rate (λ) = 5 minutes b/w arrivals

$$= \cancel{\frac{60}{60}} = \frac{60}{5} \text{ per hour}$$

$$= 12/\text{hour}.$$

service time (μ) = 2 minutes

$$= \frac{60}{2} \text{ per hour}$$

$$= 30/\text{hour}.$$

1. Average queue length $(L_q) = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$$= \frac{(12)^2}{30(30-12)}$$

$$= \frac{4}{15}$$

2. Average no. of customers in the system $(L_s) = \dfrac{\lambda}{\mu - \lambda}$

$$= \frac{12}{30-12}$$

$$= 2/3$$

3. Average time spent at the petrol pump $(W_s) = \dfrac{1}{\mu - \lambda}$

$$= \frac{1}{30-12}$$

$$= \cancel{3.33}\ 0.056 \text{ hour}$$

4. Average waiting time of a car before receiving petrol $(W_q) = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$$= \frac{12}{30(30-12)}$$

$$= 0.022 \text{ hour}$$

P_0 = probability that there are no customers in the system, $1 - \rho = 1 - \frac{\lambda}{\mu}$

Example 4

New Delhi Railway Station has a single ticket counter. During the rush hours, customers arrive at the rate of 10 per hour. The average number of customers that can be served is 12 per hour. Find out the Probability that the ticket counter is free.

Solution.

Given

λ = 10/hour, μ = 12/hour

Probability that the counter is free = $1 - \dfrac{10}{12} = 1/6$