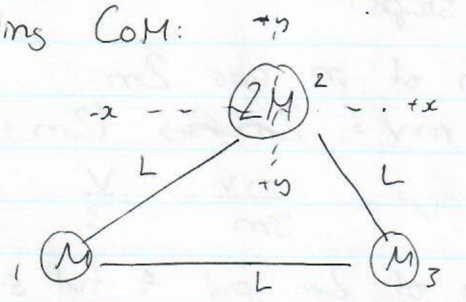
# ECE 105 TIPS: POST MIDTERM

## MOMENTUM:

1. Always conserve momentum before considering energy.
2. Do collisions in steps.
3. Be wary of relative velocities.
4. Centre of Mass:
    1. Finding CoM:



Find COM.

1. Put origin at mass ~~2M~~
    Let origin be at 2M
2. Put axes + directions.
3. Find coordinates of other masses:

$$(x_1, y_1) = (-L\sin 30°, L\cos 30°) = \left(-\frac{L}{2}, \frac{\sqrt{3}}{2}L\right)$$

$$(x_2, y_2) = (0, 0)$$

$$(x_3, y_3) = (L\sin 30°, L\cos 30°) = \left(\frac{L}{2}, \frac{\sqrt{3}}{2}L\right)$$

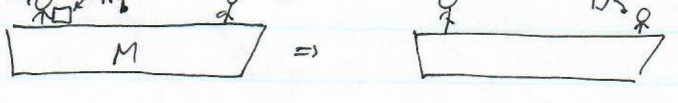
4. CoM equation: split into components

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 0 \checkmark$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{\sqrt{3}}{4}L \checkmark$$

2. Change via CoM:
    CoM stays constant so if mass on one side moves, other side mass must also move.

5. Ramp collisions: consider them together! Don't separate.
6. Use momentum if objects jumping/separating from each other.
7. CoM movements:

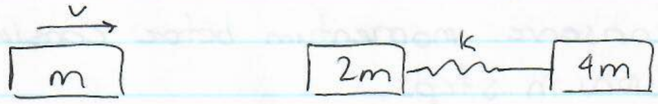


$$\therefore (m_1 + m_2 + M)\Delta x_{boat} = m_b \Delta x_b$$

↳ Consider boat + ppl (stationary) as 1 object.

Problems:

1.



Find max compression of spring.

Consider in steps:

①: collision of m and 2m:

$$mv = (2m + m)v'$$

$$v' = \frac{mv}{3m} = \frac{v}{3}$$

②: Collision of 2m and 4 full system:

$$3m \cdot \frac{v}{3} = (m + 2m + 4m)\, v''$$

$$v'' = \frac{mv}{7m} = \frac{v}{7}$$

Note the systematic way we took in collision! Step-by-step.

③: All collisions finished. Consider energy:

$$\cancel{\frac{1}{2}\, 3m}$$

$$\frac{1}{2}(m_1 + m_2)v'^2 = \frac{1}{2}(m_1 + m_2 + m_3)\, v''^2 + \frac{1}{2}kx^2$$

$$3m \cdot \left(\frac{v}{3}\right)^2 = \cancel{\frac{1}{2}} \cdot 7m \cdot \left(\frac{v}{7}\right)^2 + kx^2$$

$$\frac{mv^2}{3} = \frac{mv^2}{7} + kx^2$$

Solve for x:

$$kx^2 = \frac{4mv^2}{21}$$

$$x = \sqrt{\frac{4mv^2}{21k}}$$

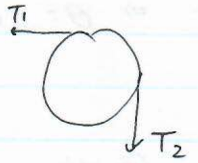
# ROTATION

1. Remember to use parallel axis theorm when changing pivot.

$$I_p = I_{cm} + mr^2.$$

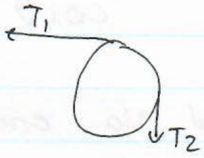
2. Pulleys:

Massless/frictionless:



$T_1 = T_2$

With inertia:



$T_1 \neq T_2$

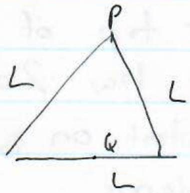
3. Same axle: same angular velocity + acceleration.
Connected by chain/teeth: same linear velocity. } Connecting equations.

4. If rotating: include rotational energy

$$E_{rot} = \frac{1}{2} I \omega^2$$

5. Conserve both angular + linear momentum ($L =$ [illegible] $m\overset{\text{liner motion}}{v}L$ )

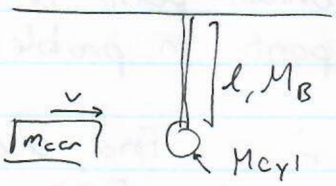
6. Be able to find $I_{CM}$ of composite/complex objects.



$I_{CM}$ of equilateral triangle: $\frac{2}{3} PQ = \frac{2}{3} L \sin 60°$

7. When taking energy, use CoM for gravity.

Ex:// Find $\theta$ when car collides with rod-cylinder system below:



⇐ Inelastic:

1. Find $I_{total}$

$$I_{total} = I_{rod} + I_{cyl} + I_{car}$$
$$= \frac{1}{3} M_B l^2 + m_{cyl} l^2 + m_{car} l^2$$

↙ Coud have used ~~I~~ PAT but assume point mass.

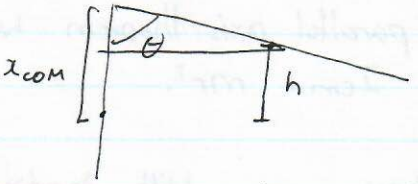
2. Conservation of angular momentum:

$$mvl = I_{total} \cdot \omega$$

3. Find new CoM:

$$x_{cm} = \frac{M_B\left(\frac{l}{2}\right) + m_{cyl}\, l + m_{car}\, l}{m_B + m_{cyl} + m_{car}}$$

4. Angle note:



$$\therefore \quad \cos\theta = \frac{x_{COM} - h}{x_{COM}} \Rightarrow \theta = \cos^{-1}\left(\frac{x_{COM} - h}{x_{COM}}\right)$$

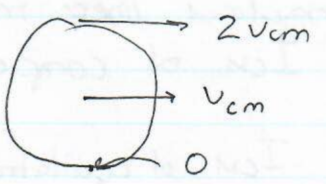
h found via energy:

$$mgh = \frac{1}{2} I\omega^2 \Leftarrow \text{Use } x_{COM}$$

## ROLLING MOTION

~~w=r~~ $v = r\omega$

1. At rolling stage:

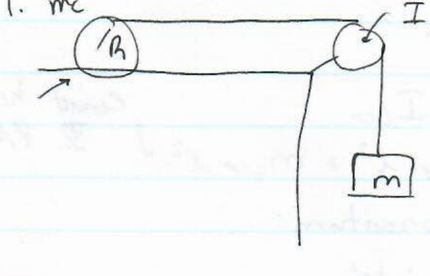


} If smth connected to top of obj., then it has $2a_{CM}$, $2v_{cm}$

2. Since object not moving at instant on ground, use $f_s$
3. How to solve rolling motion problems
    1. Determine motion types
    2. Construct equations via torque + forces.
    3. Connecting equation: $V = r\omega$, $a_p = 2a$ } equate linear + angula.
4. Put pivot on contact point to eliminate force if not moving
5. Change pivot points in problem to find unknown forces.
   ↳ ~~#~~ I must change!

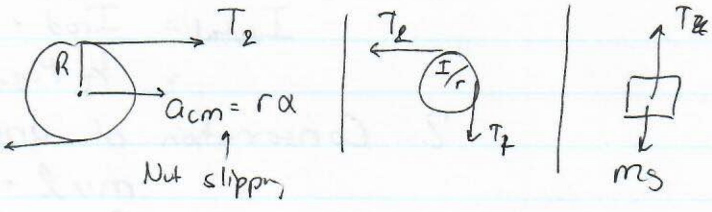
- Ex://

1.



Find acceleration of m.

1. FBD:



2. Create equations for linear + rotational motion.

① : $mg - T_1 = ma$

② : $r(T_1 - T_2) = I\alpha_p$

③: Pivot on bottom:

$$2RT_2 = I_c \alpha \Rightarrow \text{Eliminated } f_s \text{ (don't reintroduce)}$$

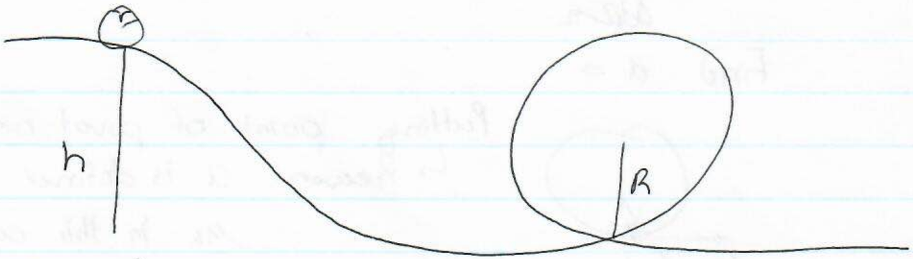
Find $I_c$:

$$I_c = I_{cm} + m_c R^2 = \frac{1}{2} m_c R^2 + m_c R^2$$
$$= \frac{3}{2} m_c R^2$$

④: $a = r\alpha_p \Rightarrow$ Pulley connected to mass.

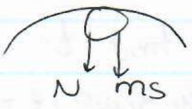
⑤: $a = 2R\alpha_c \Rightarrow a_{top} = 2 \times a_{cm}$ (rolling)

2.



Find h such that ball makes to top of loop.

1. Investigate constraints via uniform circular motion.



$$N + mg = \frac{mv^2}{R} \Leftarrow \text{Min @ } N = 0$$
$$v = \sqrt{gR}$$

* Change to ball cm b/c rolling *

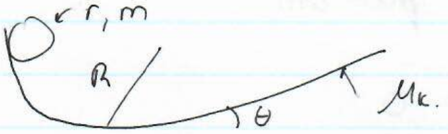
$$v^2 = g(R - r)$$

2. Energy:

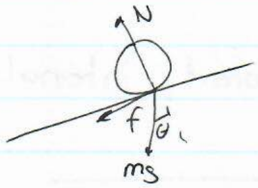
$$mg(h + r) = mg(2R - r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

We know rolling w/out slip $\Rightarrow v = r\omega$

3.



a) Find acceleration on ramp:



$$\Rightarrow f + mg\sin\theta = ma$$
$$\mu_k mg\cos\theta + mg\sin\theta = ma$$
$$a = g\sin\theta + g\cos\theta \cdot \mu_k$$

b) Find the angular speed when ball starts to roll.

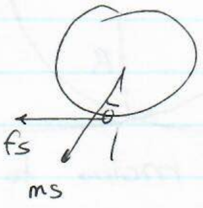
1. Find the condition for roll: $v = r\omega$
2. Divide into linear + angular speeds on ramp.

$$\therefore \left.\begin{aligned} v_f &= v_i + at \\ \omega_f &= \omega_i + \alpha t \end{aligned}\right\} \rightarrow \text{Find } t \text{ when } v_f = \omega_f$$

Find $v_i \Rightarrow mg(R-r) = \frac{1}{2}mv^2$

$v^2 = 2g(R-r) \Leftarrow$ initial velocity

Find $\alpha \Rightarrow$



Putting point of pivot on middle:

↳ Reason: a is defined w/ ~~[illegible]~~ $\mu_k$. Include $\mu_k$ in this equation too.

$$\therefore \alpha = \frac{\tau}{I} = \frac{\mu_k mg\cos\theta\, r}{\frac{2}{5}mr^2} = \frac{5\mu_k mg\cos\theta}{2}$$

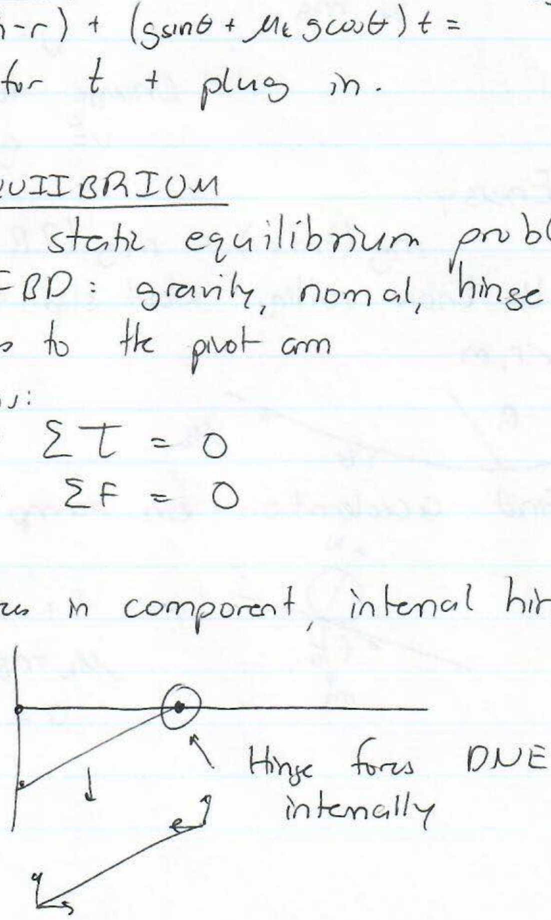
3. Equate $v_f = \omega_f$ to find $t$:

$$\sqrt{2g(R-r)} + (g\sin\theta + \mu_k g\cos\theta)t = \frac{5\mu_k mg\cos\theta}{2}t$$

4. Solve for $t$ + plug in.

## Osc STATIC EQUILIBRIUM

1. How to solve a static equilibrium problem.
   - a) Draw a FBD: gravity, normal, hinge force.
   - b) Find angles to the pivot arm
   - c) 2 equations:
     - ① $\sum\tau = 0$
     - ② $\sum F = 0$
   - d) Solve.
2. FBD: hinge forces in component, internal hinge forces DNE

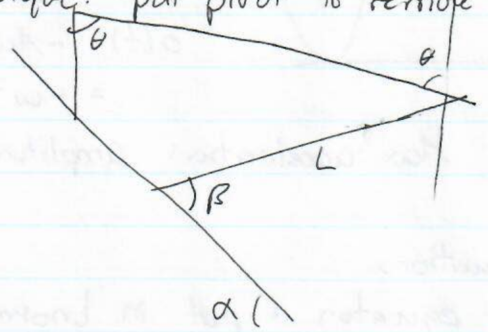


3. Angles:
    1. Find angles to vertical + horizontal
    2. Heavy use of parallel law
    3. Combine angles.

4. Force equation: take it with regular axes. (x and y equations)

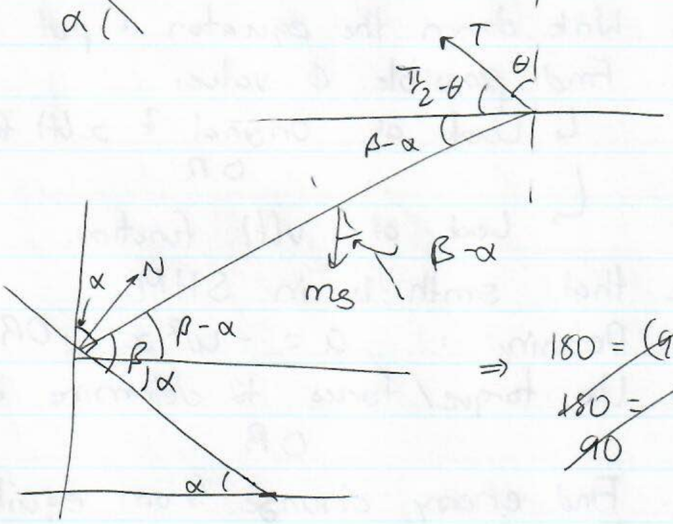
5. Torque: put pivot to remove as many hinge force as possible.

Ex:



a) Find friction between bar + incline
b) Tension
c) Normal for.

1. FBD:



⇒ ~~$180 = (90-\alpha) + \alpha + \beta - \alpha + ?$~~

~~$180 = 90 + \beta + ?$~~

~~90~~

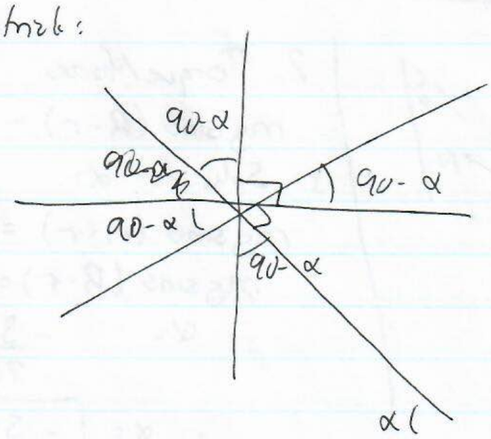
2. Equations:

τ: $TL\sin\left(\frac{\pi}{2}-\theta+\beta-\alpha\right) = mg\cos(\beta-\alpha)\cdot\frac{L}{2}$

Fy: $T\cos\theta + N\cos\alpha = mg + f_s\sin\alpha$

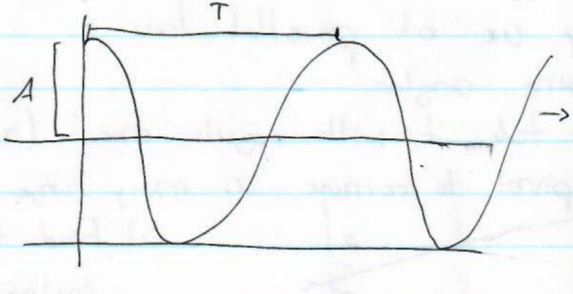
Fx: $T\sin\theta = N\sin\alpha + f_s\cos\alpha$

Ultimate angle trick:



# SIMPLE HARMONIC MOTION

1. Undestand simple harmonic motion diagram.



→ $x(t) = A\cos(\omega t + \phi)$

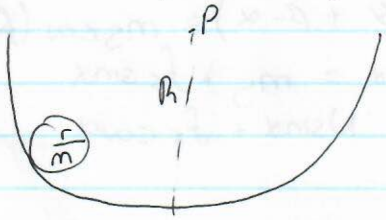
$v(t) = -A\omega\sin(\omega t + \phi)$

$a(t) = -A\omega^2\cos(\omega t + \phi)$

$= -\omega^2 x(t)$

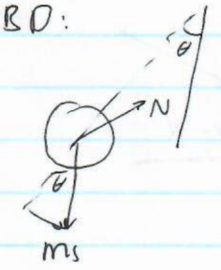
- Max velocity: origin. Max acceleration: amplitudes.

2. Types of problem.
    1. Determinings SHM equations.
        1. Write down the equation + put in knowns.
        2. Find possible $\phi$ values
            - ↳ Look at original & $x(t)$ function
            - OR
            - ↳ Look at $v(t)$ function.
            - } Plus in points that you already know.
    2. Prove that smth is in SHM.
        1. Defining : $a = -\omega^2 x$ OR $\alpha = -\omega^2\theta$
        2. Use torque/forces to determine equations (moment of inertia)
        - OR
        3. Find energy change from equilibrium + diffentiate.
        4. Keep at equilibrium → change. Approximate!



Find the frequency of the ball moving.

1. FBD:



2. Torque/forces

$mg\sin\theta(R-r) = I\alpha$ ⇐ Pivot at bottom.

3. Solve for $\alpha$:

$mg\sin\theta(R-r) = \left(\frac{2}{5}mr^2 + m\overset{r^2}{(\cancel{R-r})}\right)\alpha$

$\cancel{m}g\sin\theta(R-r) = \frac{7}{5}\cancel{m}r^2\alpha$

$\alpha = \frac{-5g\sin\theta}{7R-r}$ ← $\theta$ in small angl.

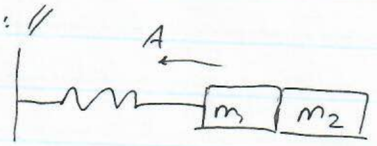
$\therefore \alpha = \boxed{\frac{-5g}{7(R-r)}}\theta$

$\alpha$ $\omega^2$

3. Energy:

1. Use equation to convert energy to speed (max).
2. Esp. useful w/ 2 blocks. If not attached, block will remove itself at equilibrium.

- Ex: //



a) $m_2$ detaches at equilibrium. Find its speed.

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}m_1 v^2 + \tfrac{1}{2}m_2 v^2$$

$$kA^2 = (m_1 \cancel{v} + m_2) v_2$$

$$v = \sqrt{\frac{kA2}{m_1+m_2}}$$

b) How far apart are objects when fully stretched.

Initial energy: energy of $m_1$

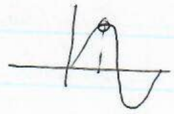
$$\tfrac{1}{2}m_1 v^2 = \tfrac{1}{2}kA'^2$$

$$\cancel{\tfrac{1}{2}} m_1 \cdot \frac{\cancel{k}A2}{m_1+m_2} = \cancel{\tfrac{1}{2}}\cancel{k}A'^2$$

$$A' = A\sqrt{\frac{m_1}{m_1+m_2}} \Rightarrow \text{Distance.}$$

Time:

At amplitude: 1/4 of period



$$\therefore d_{block} = v \cdot \frac{T}{4}$$

spring eqn.

$$= \sqrt{\frac{kA2}{m_1+m_2}} \cdot \frac{1}{4} \cdot 2\pi\sqrt{\frac{m}{k}}$$

Separation distan.

$$\cancel{A'}\ d_{block} - A' = \cancel{A\sqrt{\frac{m_1}{m_1+m_2}}}\ \left(\frac{\pi}{2} - 1\right) A \left(\sqrt{\frac{m_1}{m_1+m_2}}\right)$$