CSE6706:
Advanced Digital Image Processing

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Image Enhancement using Sharpening Filter

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Objectives of Image Sharpening

- Highlight fine details
- Remove blurring

Smoothing Vs. Sharpening

Smoothing

⇩

Average

⇩

Integration

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Smoothing Vs. Sharpening

Smoothing	Sharpening
⇩	⇩
Average	Difference
⇩	⇩
Integration	Differentiation

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Image Sharpening

Sharpening

⇩

Difference

⇩

Differentiation

- Response of derivative is *proportional to* image discontinuity
- Sharp changes, noise point, edges, lines, grey ramp are easily detected

Image Sharpening

- 1^{st} and 2^{nd} order derivatives will be used
- Behavior to check
  - *Constant gray level*
  - *At onset and end of discontinuities (ramp and step)*
  - *Along gray-ramp*

Properties of Derivatives

- Value of 1st order derivative will be
  - 0 *at constant gray level*
  - *Nonzero at onset of step and ramp*
  - *Nonzero along ramp*

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Properties of Derivatives

- Value of 1st order derivative will be
  - 0 *at const gray level*
  - *Nonzero at onset of step and ramp*
  - *Nonzero along ramp*

- Value of 2nd order derivative will be
  - 0 *at const gray level*
  - *Nonzero at onset and end of step and ramp*
  - 0 *along ramp*

Digital 1st Order Derivative

$$\frac{\partial f}{\partial x} = \frac{\text{change of } f}{\text{change of } x}$$

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Digital 1st Order Derivative

$$\frac{\partial f}{\partial x} = \frac{f(x+1) - f(x)}{\text{change of } x}$$

Digital 1st Order Derivative

$$\frac{\partial f}{\partial x} = \frac{f(x+1) - f(x)}{x+1-x}$$

$$= f(x+1) - f(x)$$

Digital 2nd Order Derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Illustration of 1st and 2nd Order Derivatives

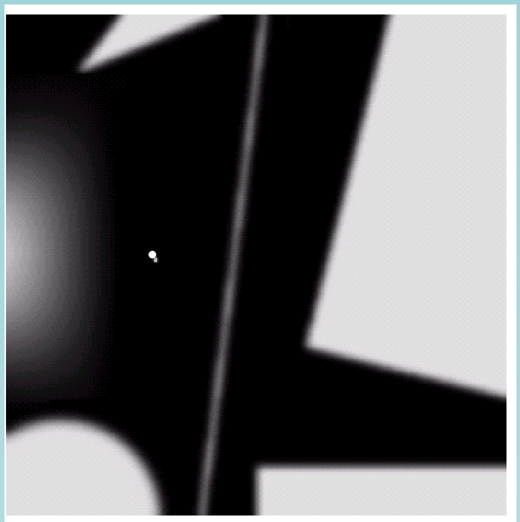

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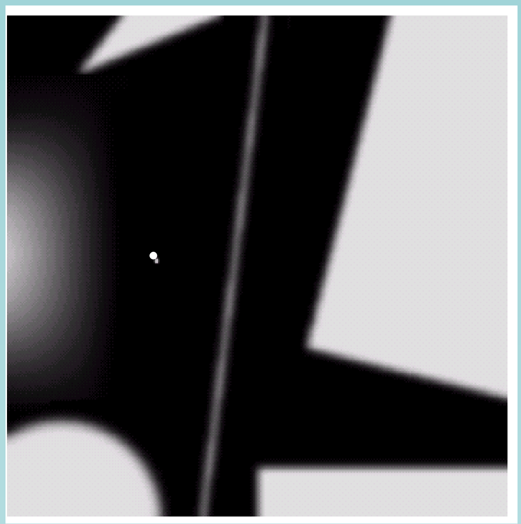
Illustration of 1st and 2nd Order Derivatives



- gray-ramp (smooth transition betn white and black)
- Isolated noise point
- Line
- Edge

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Illustration of 1st and 2nd Order Derivatives

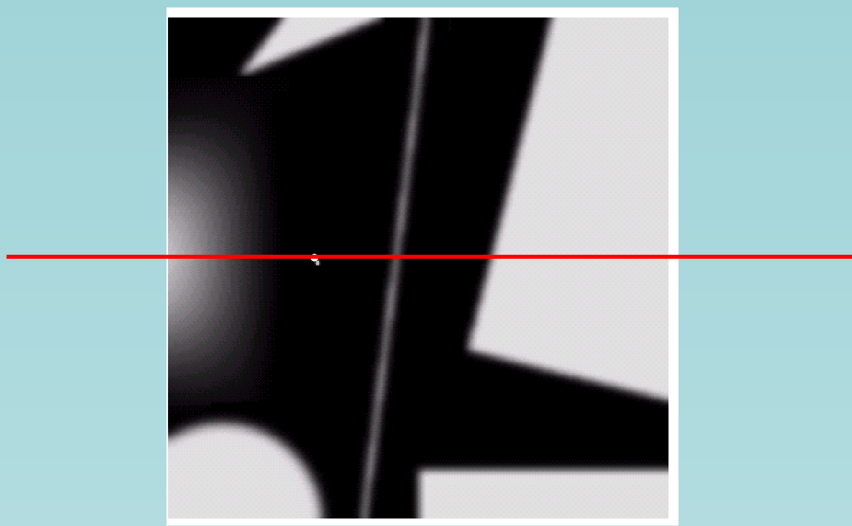

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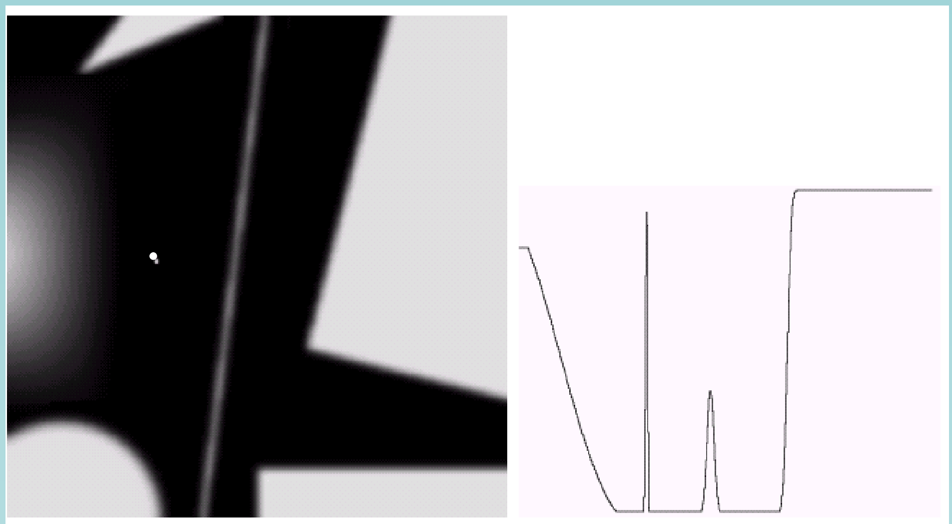
Illustration of 1st and 2nd Order Derivatives



Gray profile

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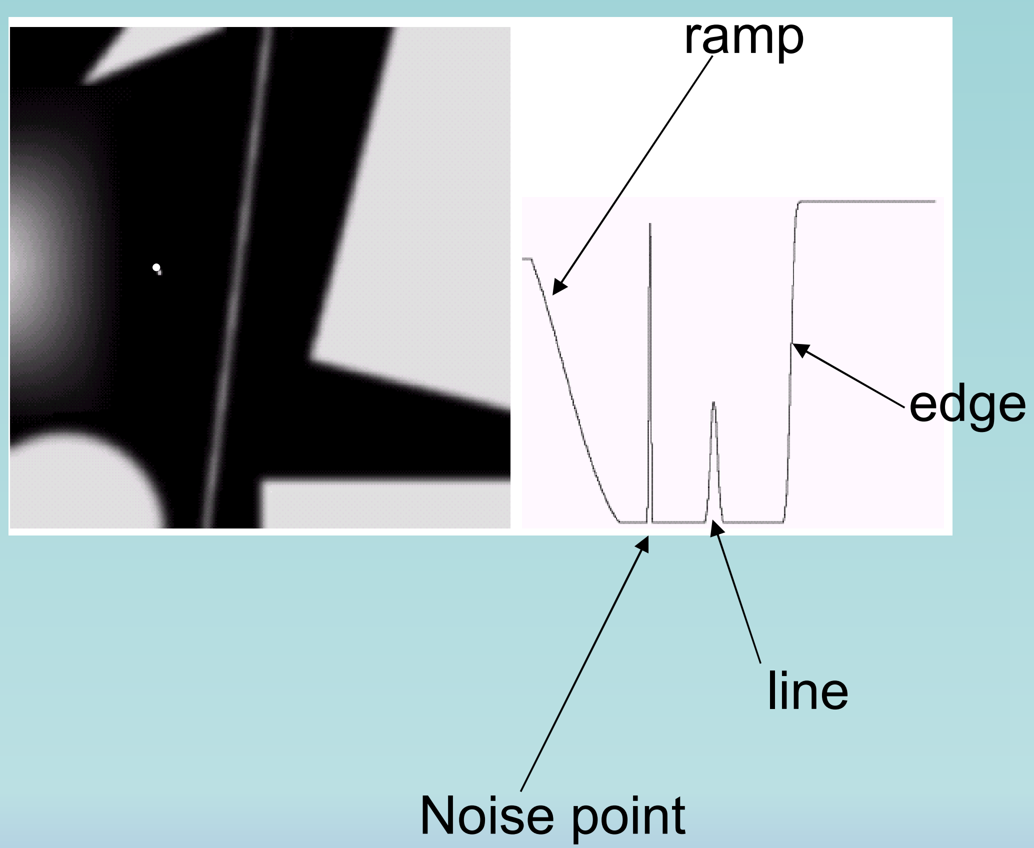
Illustration of 1^{st} and 2^{nd} Order Derivatives



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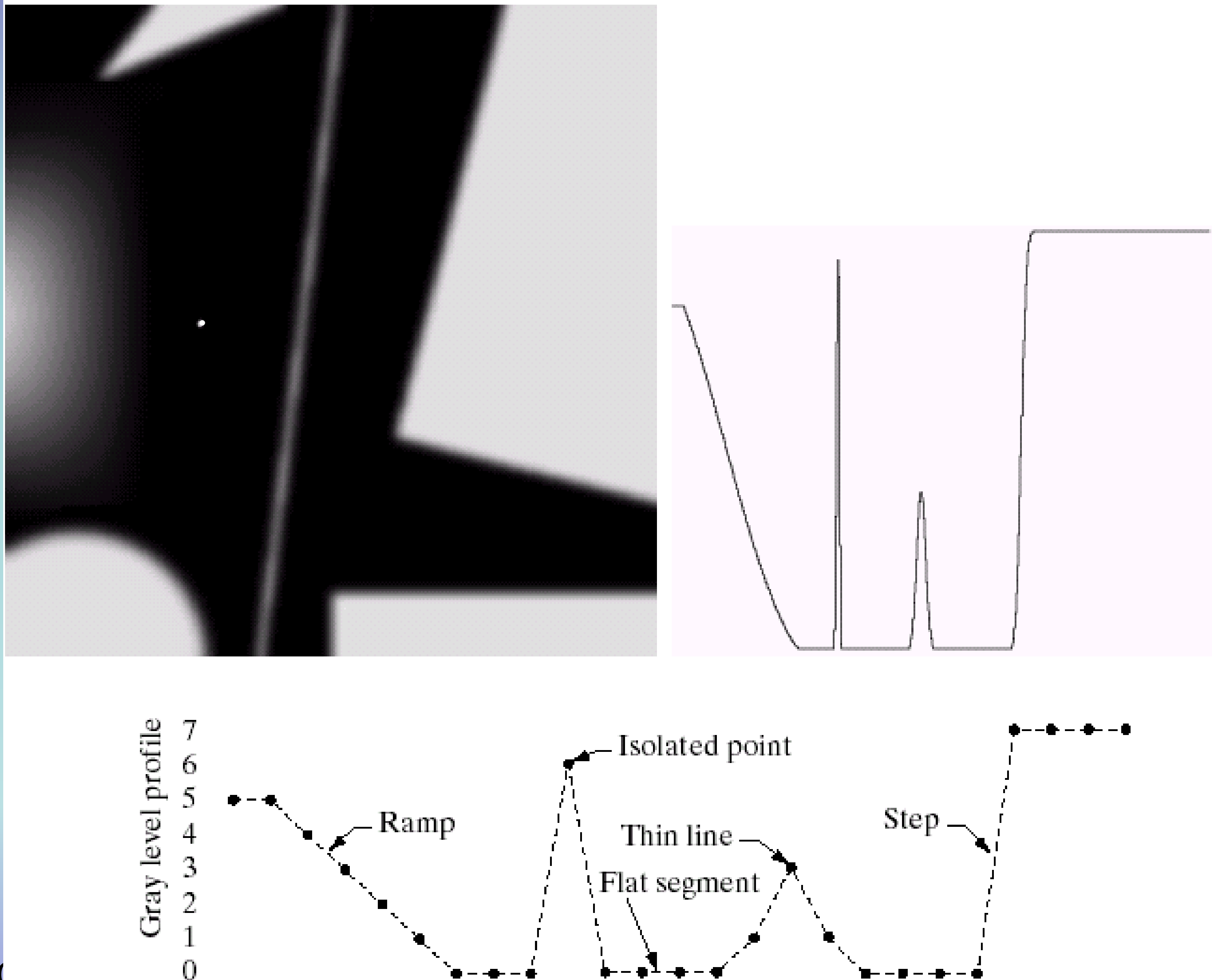
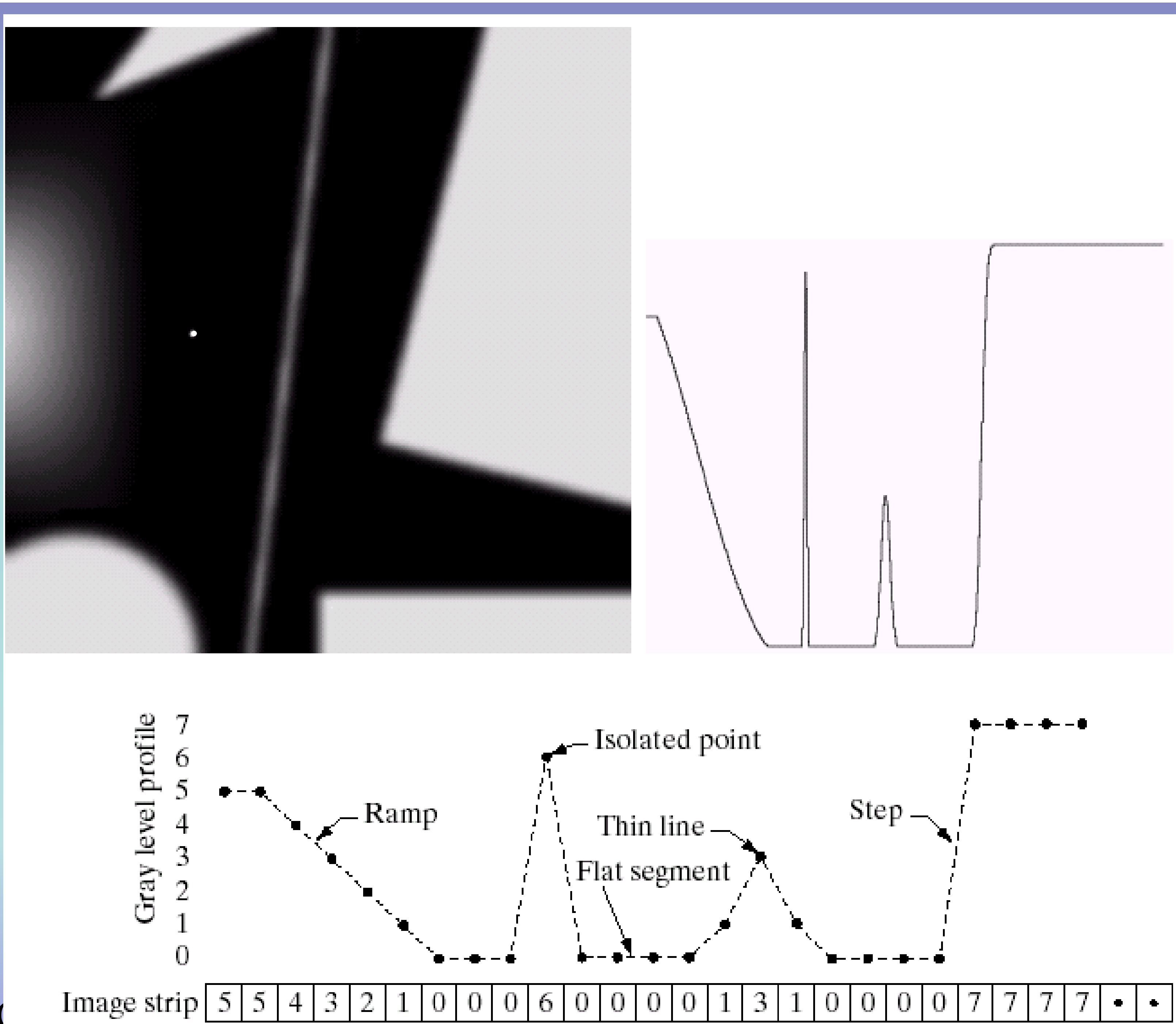
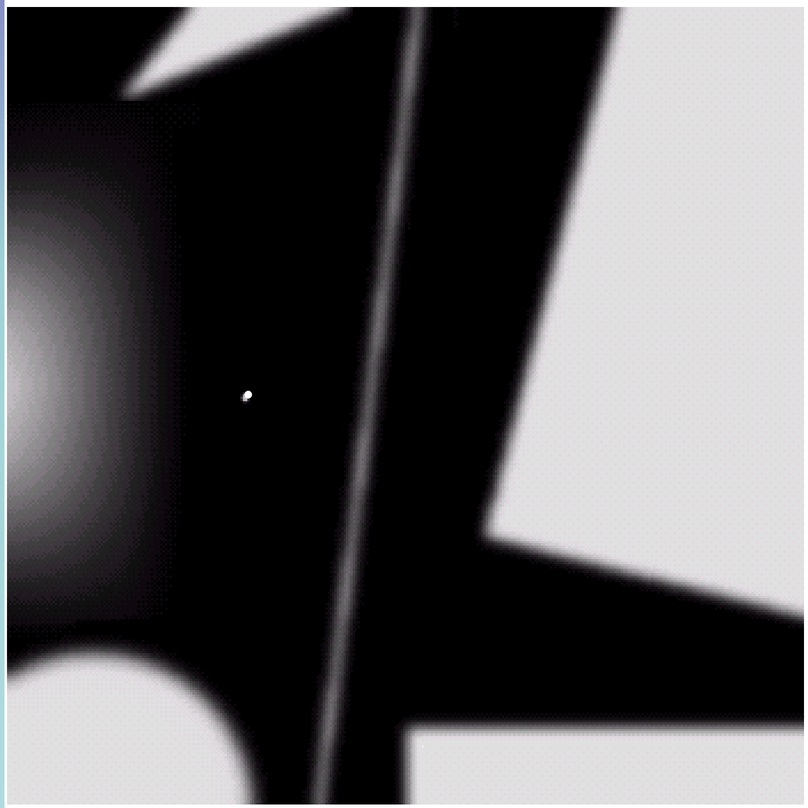
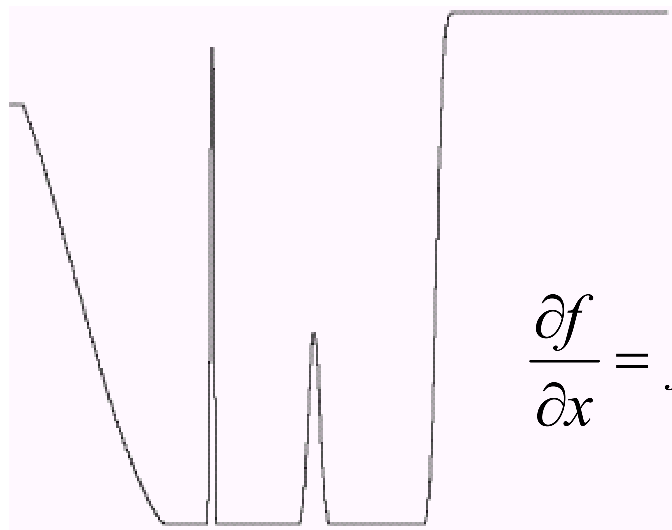

Gray level profile
7
6
5
4
3
2
1
0
Isolated point
Ramp
Thin line
Flat segment
Step


Gray level profile
7
6
5
4
3
2
1
0
Isolated point
Ramp
Thin line
Flat segment
Step
Image strip 5 5 4 3 2 1 0 0 0 6 0 0 0 0 1 3 1 0 0 0 0 7 7 7 7 • •





$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

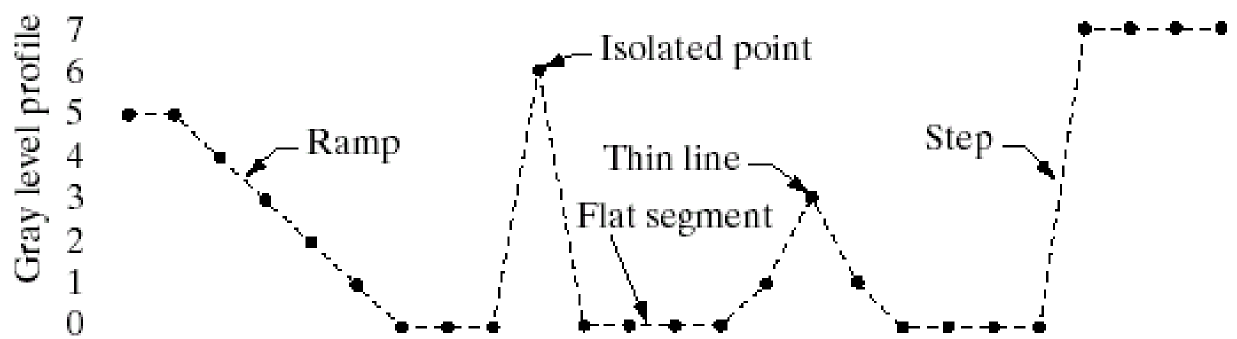
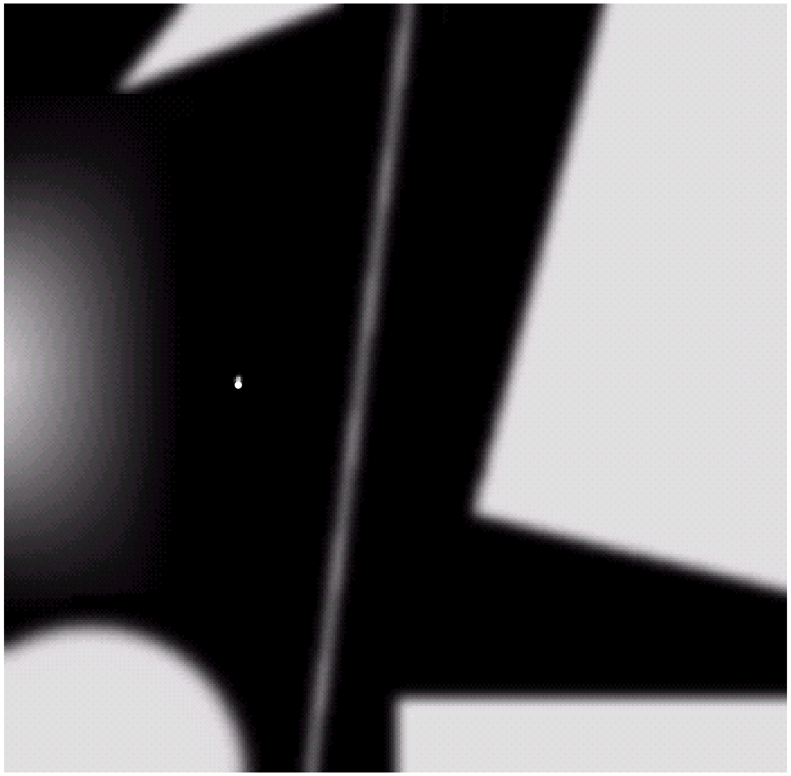
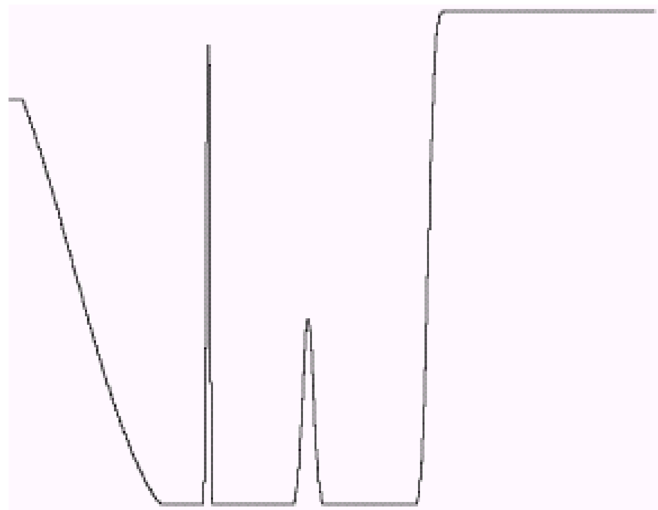
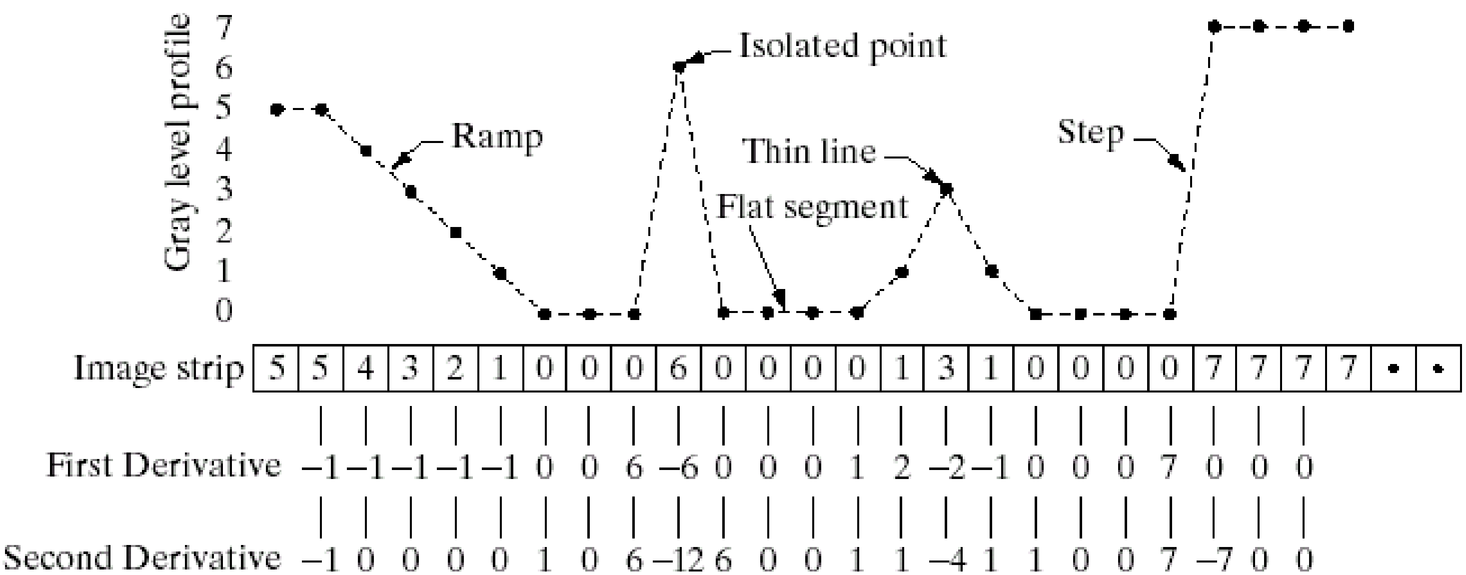


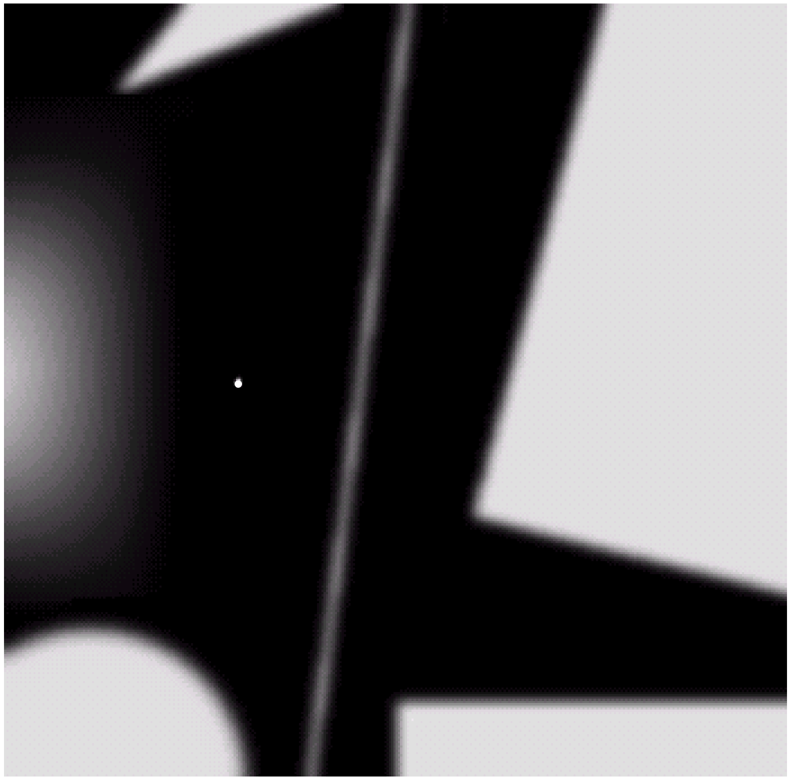
Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
First Derivative	−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0				

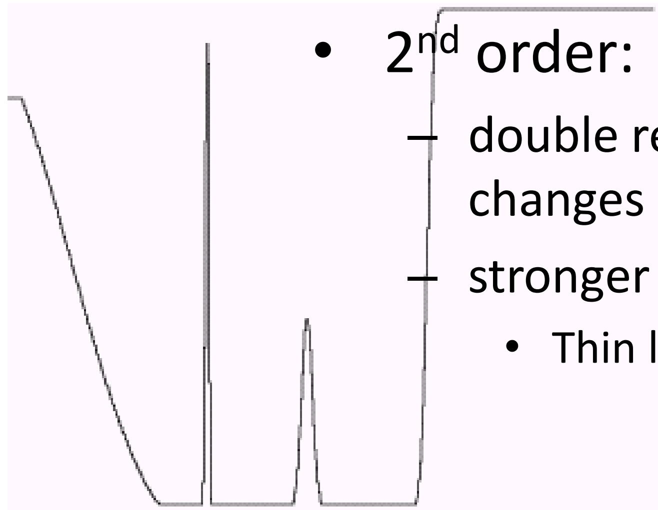
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



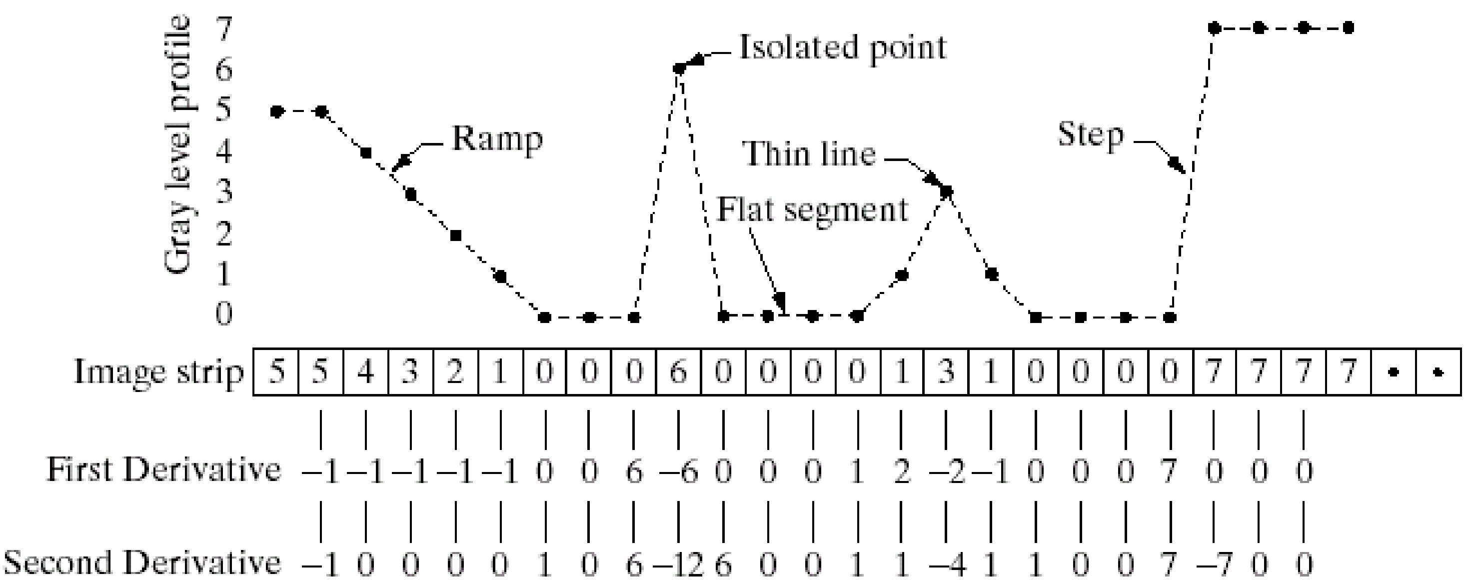








- 1^{st} order:
  - produce thicker edges
  - Strong response to step
- 2^{nd} order:
  - double response to step changes
  - stronger to fine details
    - Thin line, noise point



2^{nd} Order Derivative

- Laplacian 2^{nd} order derivative
  - Rotation invariant or isotropic

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

2^{nd} Order Derivative

- Laplacian 2^{nd} order derivative
  - Rotation invariant or isotropic

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

We know,

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and,

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

2^{nd} Order Derivative

Therefore,

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)$$

$$-4f(x, y)$$

Implementation of 2nd Order Derivative

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	–4	1
0	1	0

Implementation of 2nd Order Derivative

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	−4	1
0	1	0

- Isotropic or rotation invariant for 90º increments

Implementation of 2nd Order Derivative

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

- Rotation invariant for 45º increments

Other two Implementations of 2nd Order Derivatives

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Properties of Laplacian 2nd Order Derivatives

- Highlights discontinuities
- Deemphasizes slowly varying gray backgrounds
- Results in sharpened discontinuities superimposed on a dark featureless background

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Example of Image Sharpening using Laplacian Operator

Unsharpened
moon image

Details not clear

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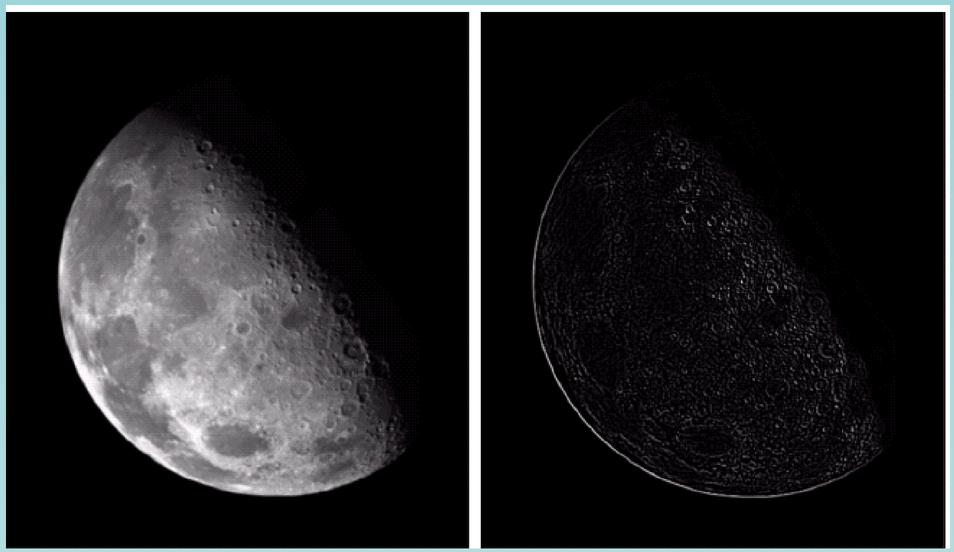
Example of Image Sharpening using Laplacian Operator



Original Image | After applying Laplacian Operator

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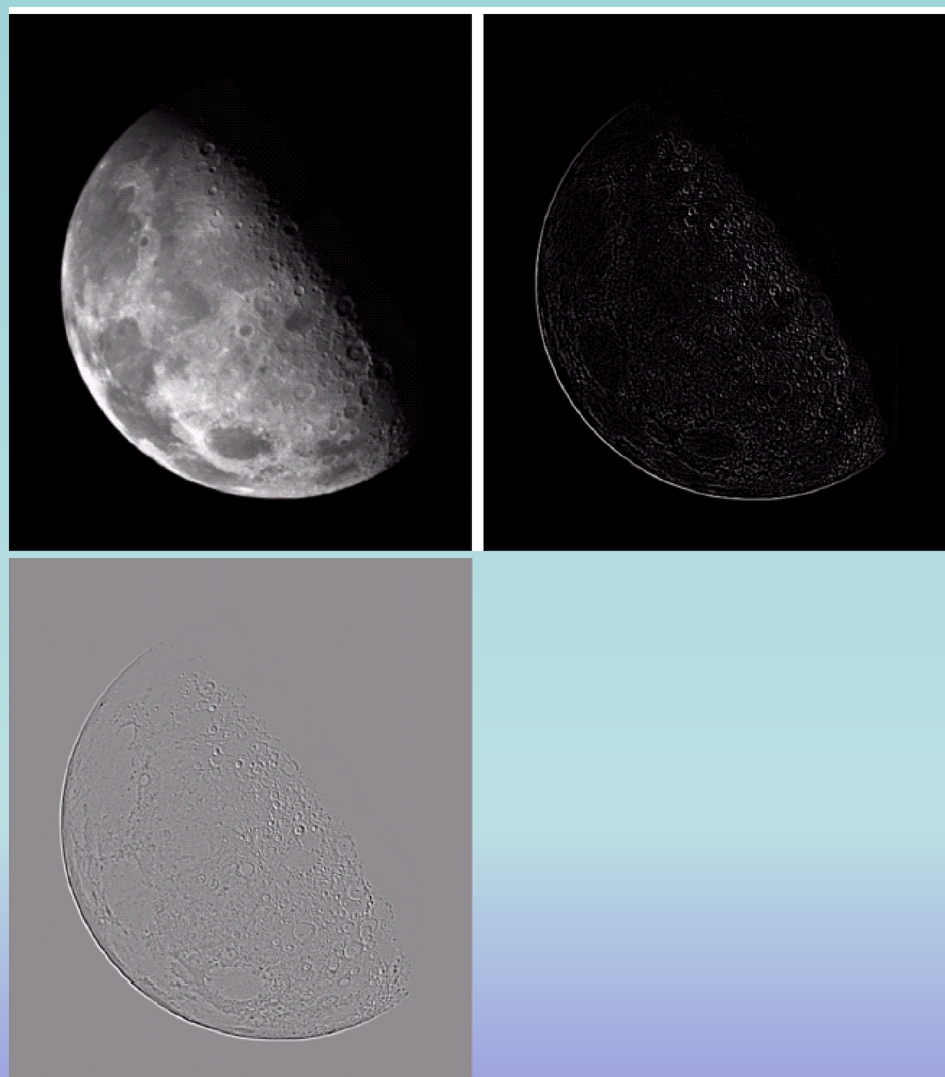
Example of Image Sharpening using Laplacian Operator

Original Image

After applying Laplacian Operator



Scaled for visualization

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Way out from the side effects of Laplacian Operator

- Add/ subtract the sharpened image from original image
- Recovers everything but still preserves the sharpening effect

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) & \text{if center coeff} < 0 \\ f(x,y)+\nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

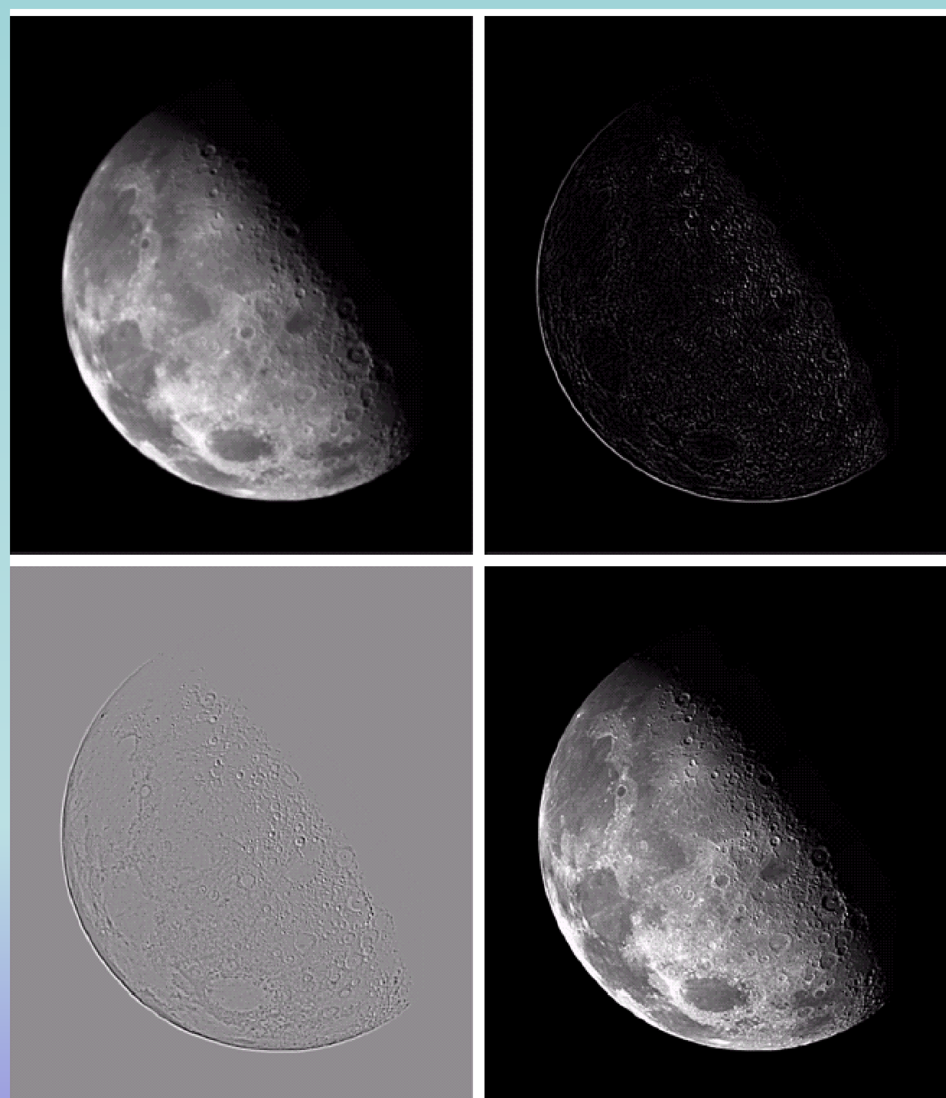
0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Example of Image Sharpening using Laplacian Operator

Original Image

After applying Laplacian Operator



Scaled for visualization

After added with the original

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Simplification of Laplacian Sharpening

- Original Laplacian sharpening requires two passes
- It can be reduced to a single pass

Simplification of Laplacian Sharpening

We know,

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) & \text{if center coeff} < 0 \\ f(x,y)+\nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}$$

and,

$$\nabla^2 f = f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)-4f(x,y)$$

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Simplification of Laplacian Operator

We know,

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if center coeff} < 0 \\ f(x,y) + \nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}$$

and,

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Therefore,

$$\begin{aligned} g(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)] \\ &= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] \end{aligned}$$

Simplification of Laplacian Sharpening

$$g(x,y) = f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$
$$= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

0	−1	0
−1	5	−1
0	−1	0

Simplification of Laplacian Sharpening

$$g(x,y) = f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$
$$= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

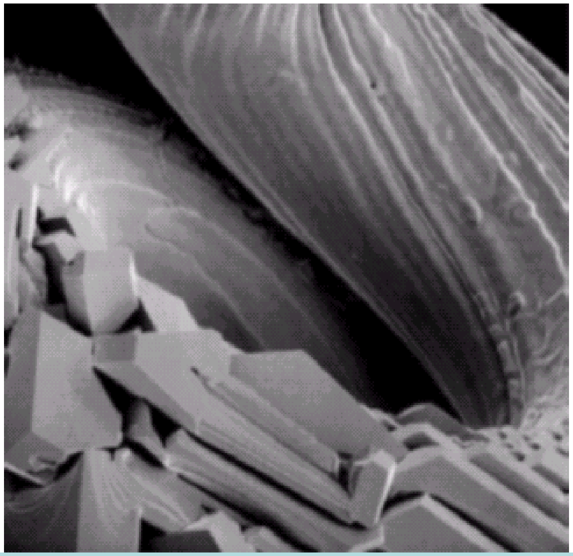
0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

Simplification of Laplacian Sharpening

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1



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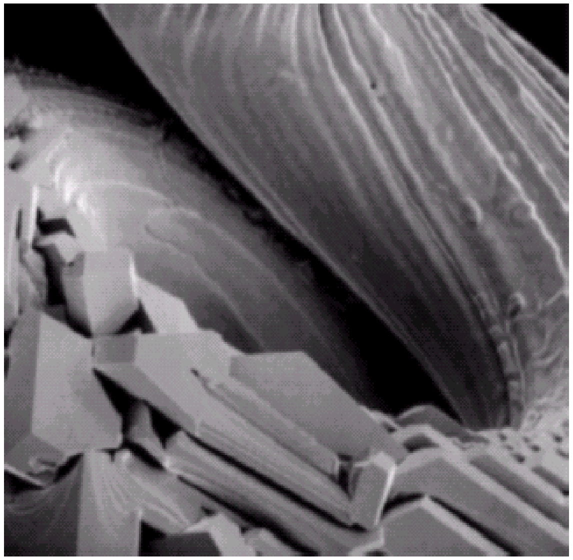
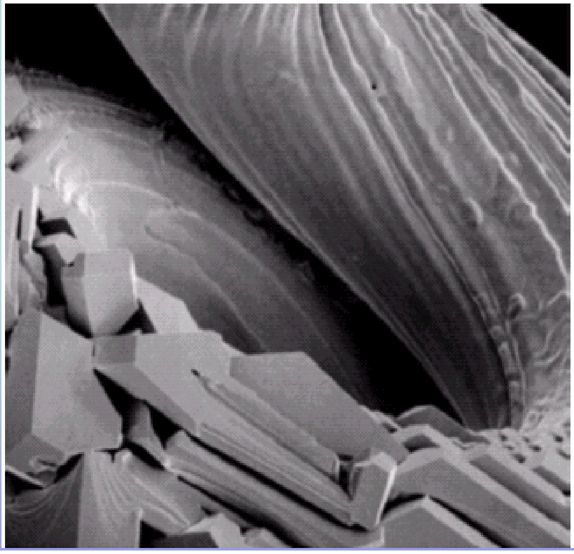
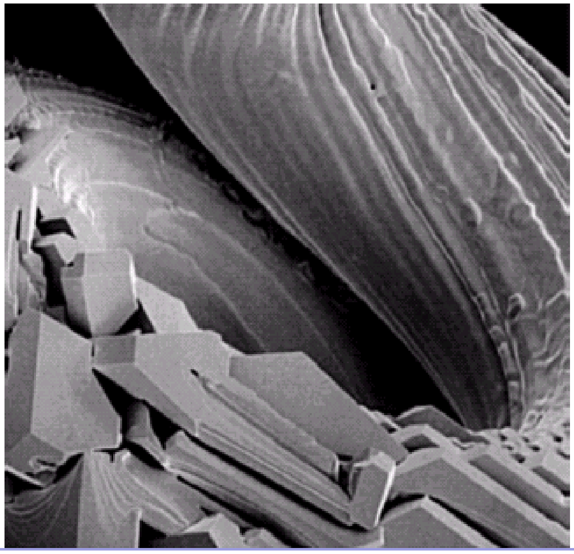
Simplification of Laplacian Sharpening

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1







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Sharpening through Unsharp masking

- Used in publishing industry for long
- A blur (unsharp) image is subtracted from the original image

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

Sharp image

Blurred or Unsharp or Average image

High Boost Filtering

- Generalization of unsharp masking

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$ Unsharp masking

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$$ High-boost filtering

High Boost Filtering

$$f_{hb}(x,y) = Af(x,y) - \bar{f}(x,y)$$
$$= (A-1)f(x,y) + f(x,y) - \bar{f}(x,y)$$

High Boost Filtering

$$
\begin{aligned}
f_{hb}(x,y) &= Af(x,y) - \bar{f}(x,y) \\
&= (A-1)f(x,y) + f(x,y) - \bar{f}(x,y) \\
&= (A-1)f(x,y) + f_s(x,y)
\end{aligned}
$$

Sharp image

High Boost Filtering

$$
\begin{aligned}
f_{hb}(x, y) &= Af(x, y) - \bar{f}(x, y) \\
&= (A-1)f(x, y) + f(x, y) - \bar{f}(x, y) \\
&= (A-1)f(x, y) + f_s(x, y)
\end{aligned}
$$

Sharp image
by any model

High Boost Filtering

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$$
$$= (A-1) f(x, y) + f(x, y) - \bar{f}(x, y)$$
$$= (A-1) f(x, y) + f_s(x, y)$$

Sharp image
by any model

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if center coeff} < 0 \\ f(x, y) + \nabla^2 f(x, y) & \text{if center coeff} > 0 \end{cases}$$

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High Boost Filtering

$$
\begin{aligned}
f_{hb}(x,y) &= Af(x,y) - \bar{f}(x,y) \\
&= (A-1)f(x,y) + f(x,y) - \bar{f}(x,y) \\
&= (A-1)f(x,y) + f_s(x,y)
\end{aligned}
$$

$$
g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if center coeff} < 0 \\ f(x,y) + \nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}
$$

$$
f_{hb}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{if center coeff} < 0 \\ Af(x,y) + \nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}
$$

High Boost Filtering

$$f_{hb}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{if center coeff} < 0 \\ Af(x,y) + \nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}$$

0	−1	0
−1	$A + 4$	−1
0	−1	0

High Boost Filtering

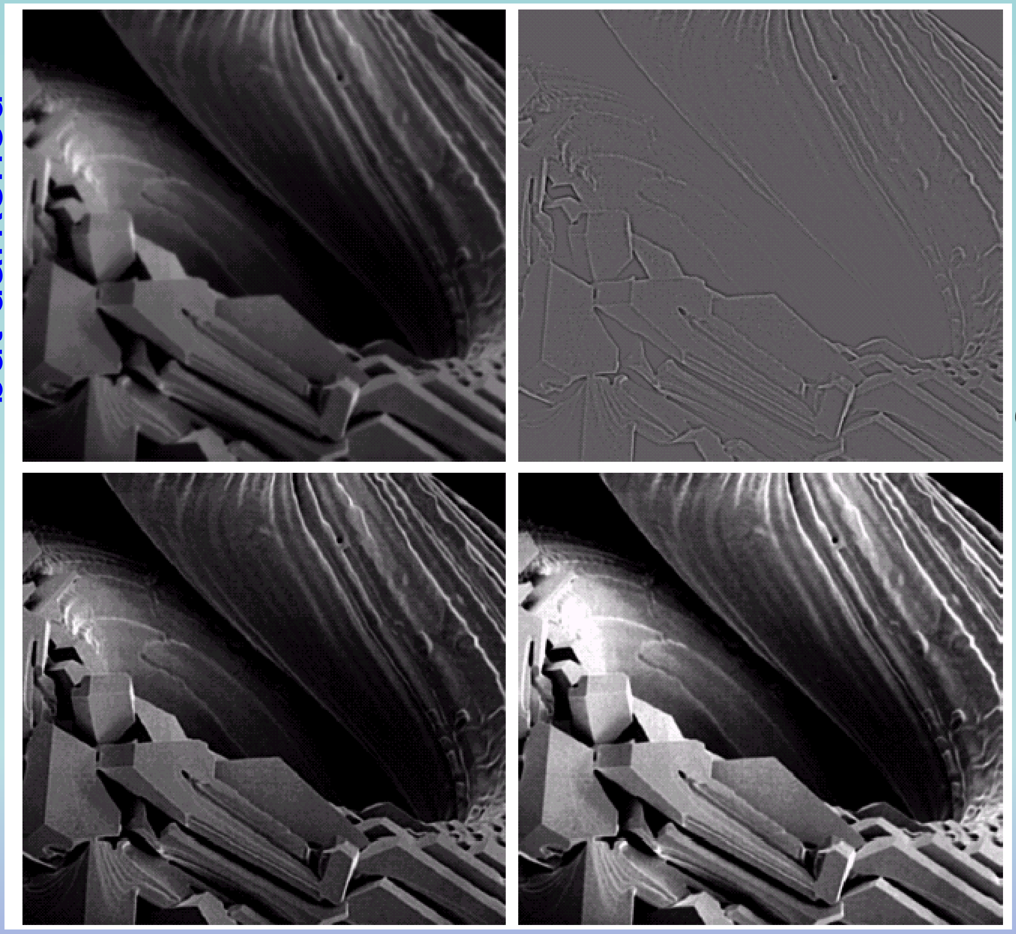
$$f_{hb}(x,y)=\begin{cases} Af(x,y)-\nabla^2 f(x,y) & \text{if center coeff} < 0 \\ Af(x,y)+\nabla^2 f(x,y) & \text{if center coeff} > 0 \end{cases}$$

0	−1	0
−1	$A+4$	−1
0	−1	0

−1	−1	−1
−1	$A+8$	−1
−1	−1	−1

Example of High Boost Filtering

-1	-1	-1
-1	$A + 8$	-1
-1	-1	-1



Previous Image but darkened

Sharpened with A=0

Sharpened with A=1

Sharpened with A=1.7

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1st Order Derivative –The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

1st Order Derivative –The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

- Individual elements are linear
- Not rotation invariant

1^{st} Order Derivative –The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \text{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2\right]^{1/2}$$

- rotation invariant, but NOT linear

1st Order Derivative –The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \mathrm{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2\right]^{1/2}$$

$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$

- approximation
- Linear, but rotation invariant for limited cases

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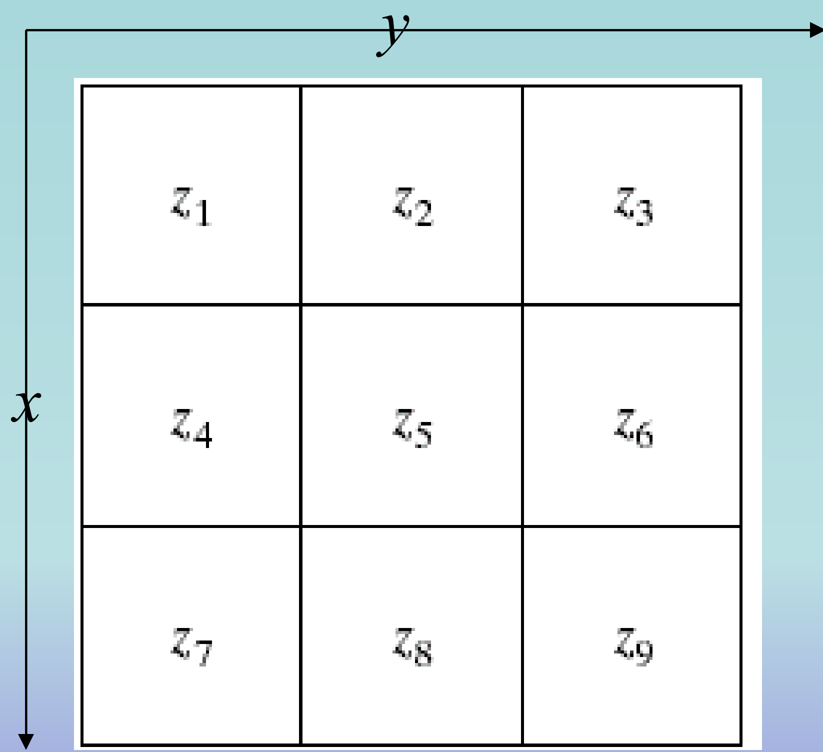
Digital Approximation of The Gradient

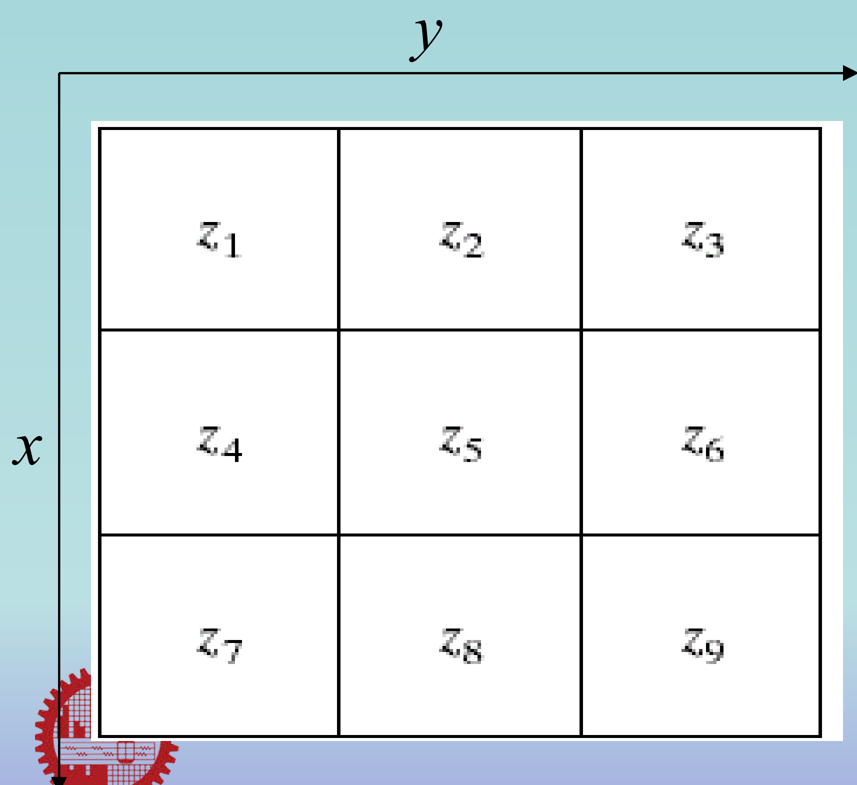
$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$



A 3X3 image region

Digital Approximation of The Gradient

$$\nabla f \approx |G_x| + |G_y|$$



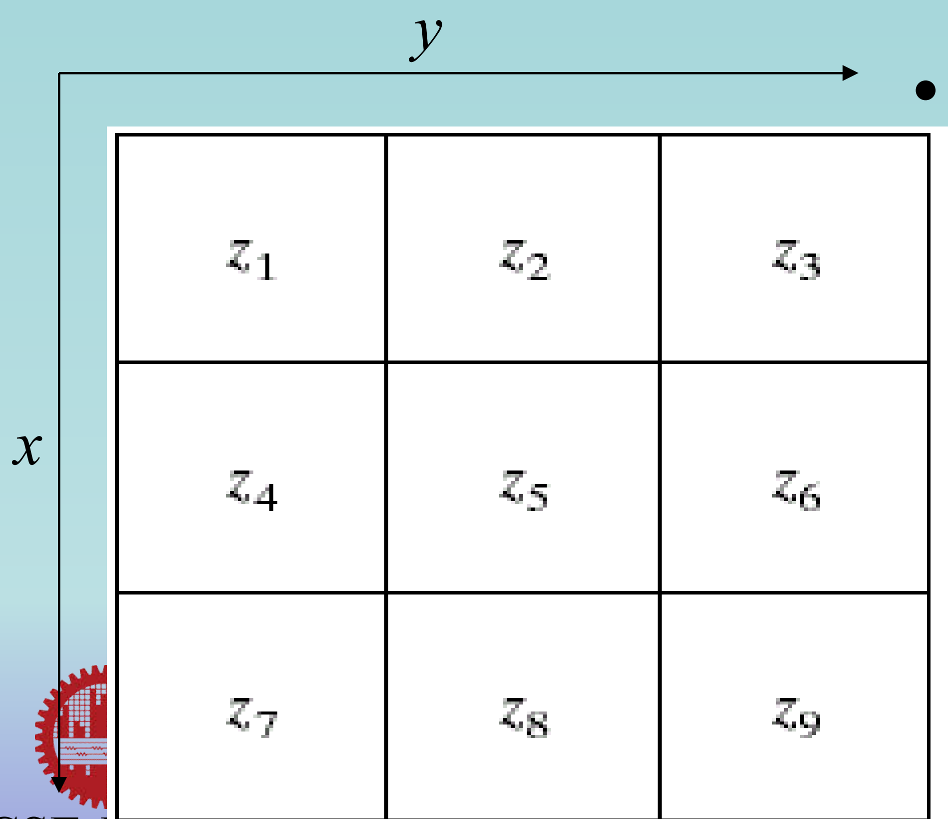
- Many implementations (1)

$$G_x = Z_8 - Z_5$$

$$G_y = Z_6 - Z_5$$

Digital Approximation of The Gradient

$$\nabla f \approx |G_x| + |G_y|$$



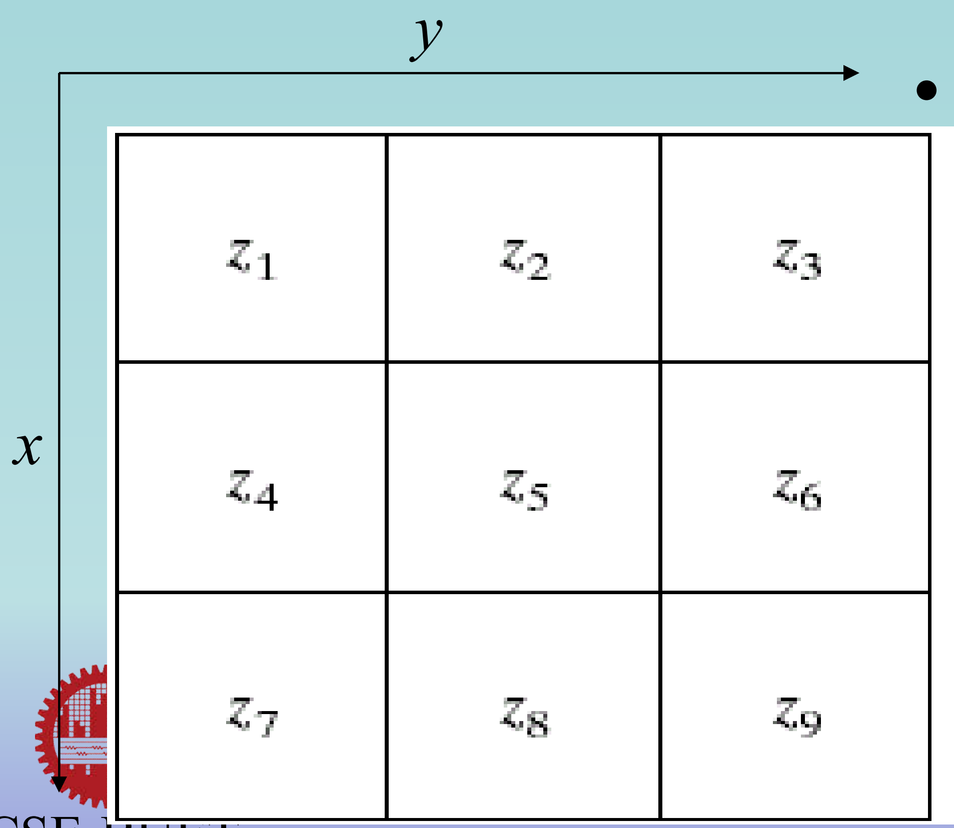
- Many implementations (2)

$$G_x = Z_9 - Z_5$$

$$G_y = Z_8 - Z_6$$

Digital Approximation of The Gradient

$$\nabla f = \left[G_x^2 + G_y^2 \right]^{1/2}$$



- Many implementations (2)

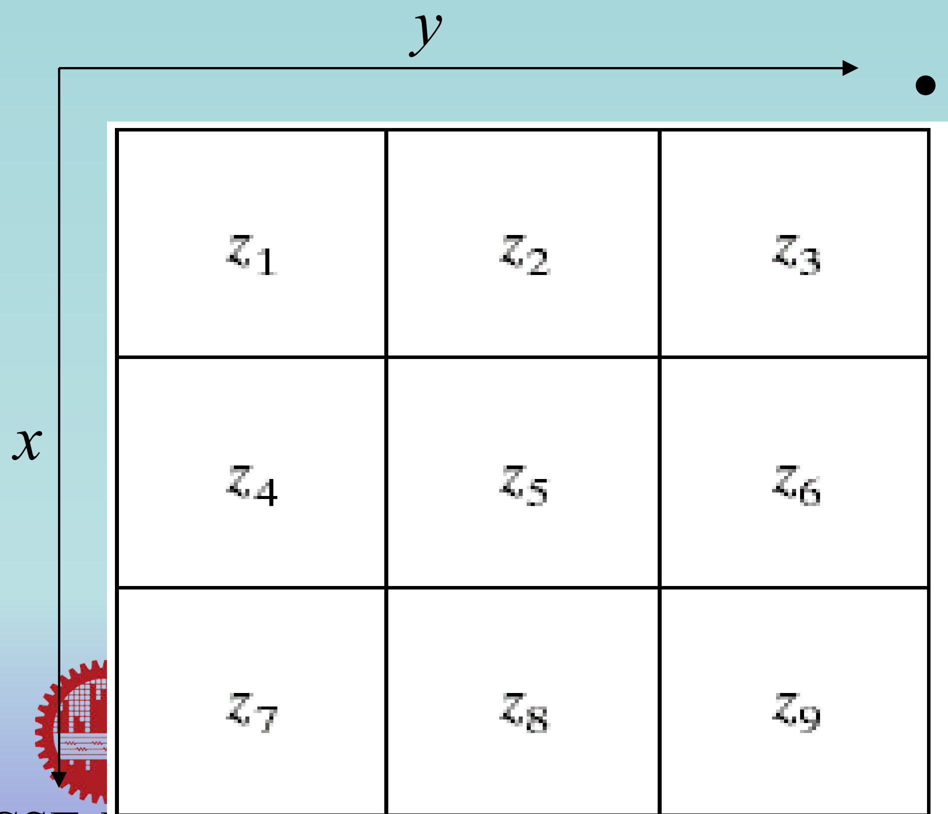
$$G_x = Z_9 - Z_5$$

$$G_y = Z_8 - Z_6$$

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}$$

Digital Approximation of The Gradient

$$\nabla f \approx |G_x| + |G_y|$$



- Many implementations (2)

$$G_x = Z_9 - Z_5$$

$$G_y = Z_8 - Z_6$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

Robert's Cross Gradient Operator

Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

$Z_9 - Z_5$ $\quad$ $Z_8 - Z_6$

- Many implementations (2)

$$G_x = Z_9 - Z_5$$

$$G_y = Z_8 - Z_6$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

Robert's Cross Gradient Operator

Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

- Many implementations (3)

$$G_x = (Z_7 + 2Z_8 + Z_9) - (Z_1 + 2Z_2 + Z_3)$$

$$G_y = (Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)$$

$$\nabla f = \left|(Z_7 + 2Z_8 + Z_9) - (Z_1 + 2Z_2 + Z_3)\right| + \left|(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)\right|$$

Sobel Operator

Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel Operators

$$\nabla f = \left|(Z_7 + 2Z_8 + Z_9) - (Z_1 + 2Z_2 + Z_3)\right| + \left|(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)\right|$$

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Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel Operators

- More importance to center pixel (z_5)
  - achieve some smoothing

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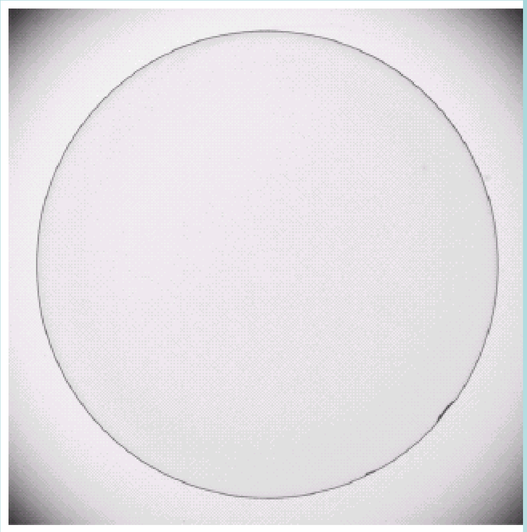
Example of Image Sharpening using Sobel Operator

- Automatic factory inspection



Contact lens

Any defects?

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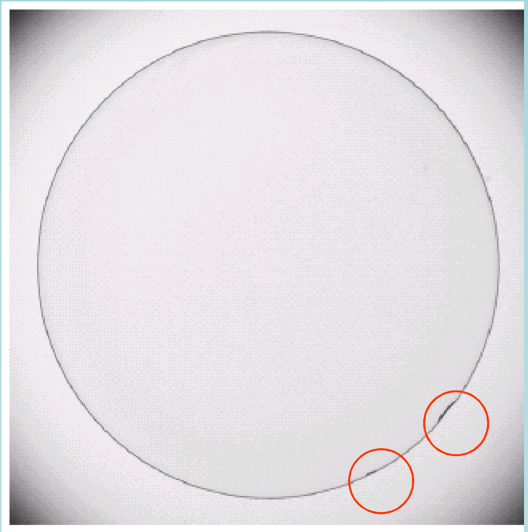
Example of Image Sharpening using Sobel Operator

- Automatic factory inspection



Contact lens

Note defects at 4 and 5 o'clock positions

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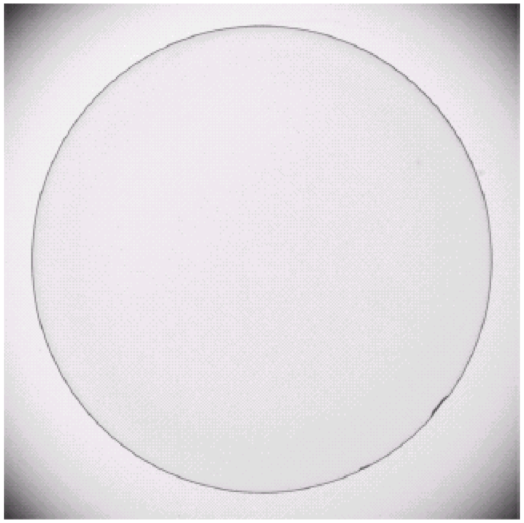
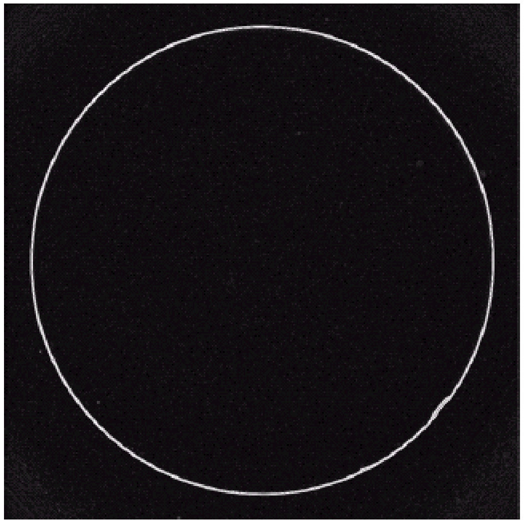
Example of Image Sharpening using Sobel Operator

- Automatic factory inspection



Original Image

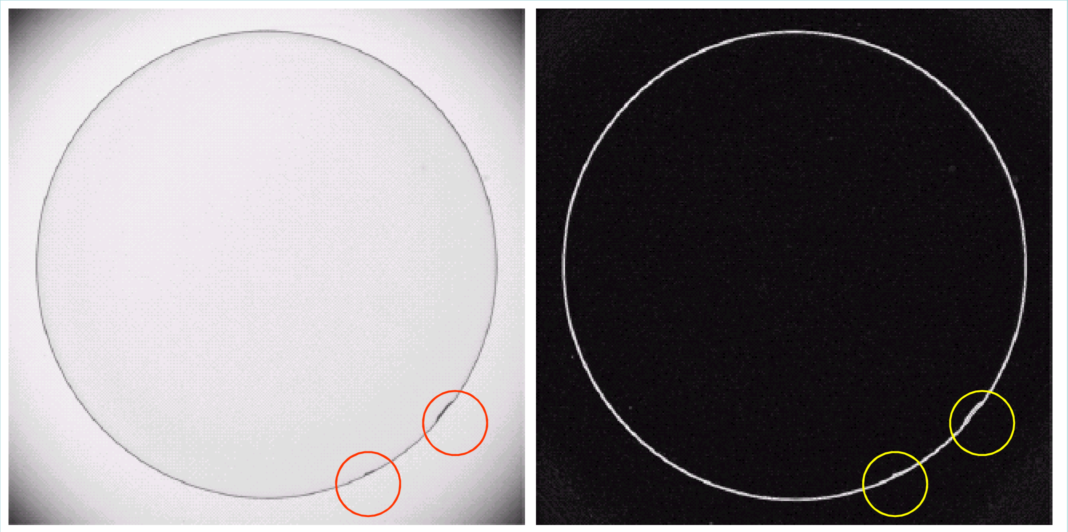


After Sobel Masking

Note: slowly varying background is removed and defects are clearer

Example of Image Sharpening using Sobel Operator

- Automatic factory inspection



Contact lens

Note defects at 4 and 5 o'clock positions

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Combining Spatial Filters

- A single approach often cannot achieve good enhancement

- A nuclear body scan image
- Objective: enhance by sharpening to get the fine details.
- Challenges:
  - Noise
  - dynamic range of gray scale

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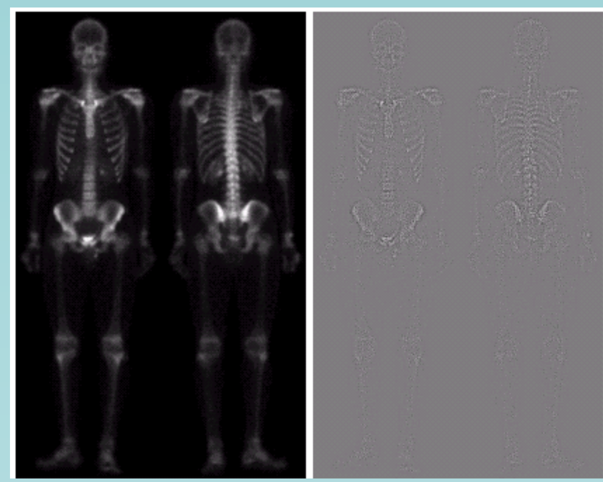
Combining Spatial Filters



Original image

After Laplacian applied

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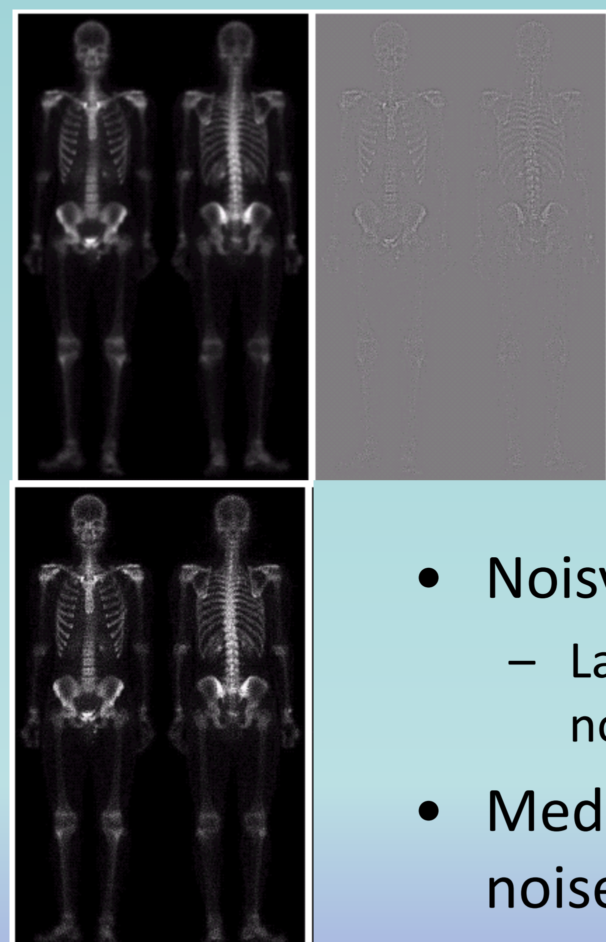
Combining Spatial Filters



- Noisy
  - Laplacian enhances the noise, too
- Median filter removes noises
  - But, it also removes other details

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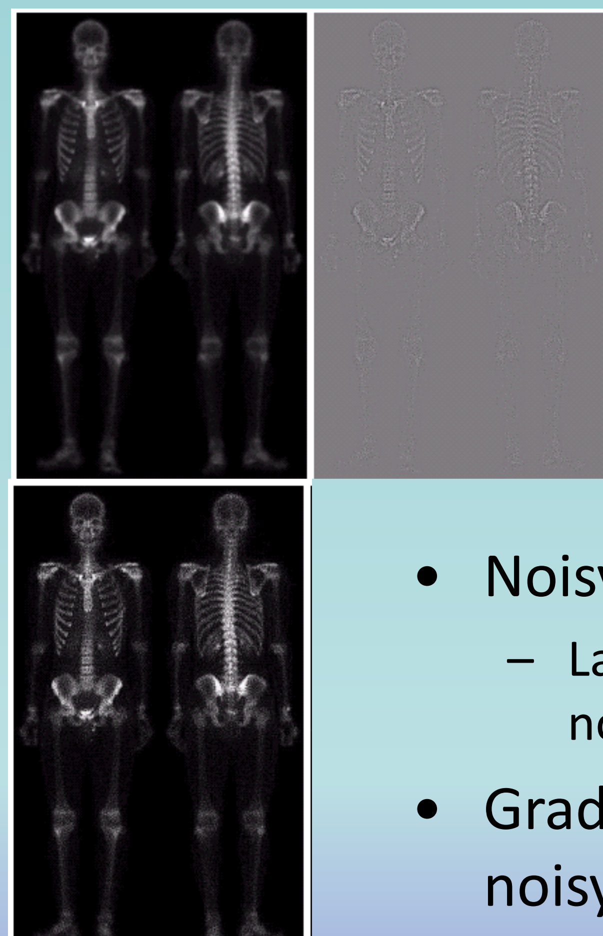
Combining Spatial Filters

Original image

After Laplacian applied



Added 2 images

- Noisy
  - Laplacian enhances the noise, too
- Gradient produces less noisy images
  - it also improves the edges

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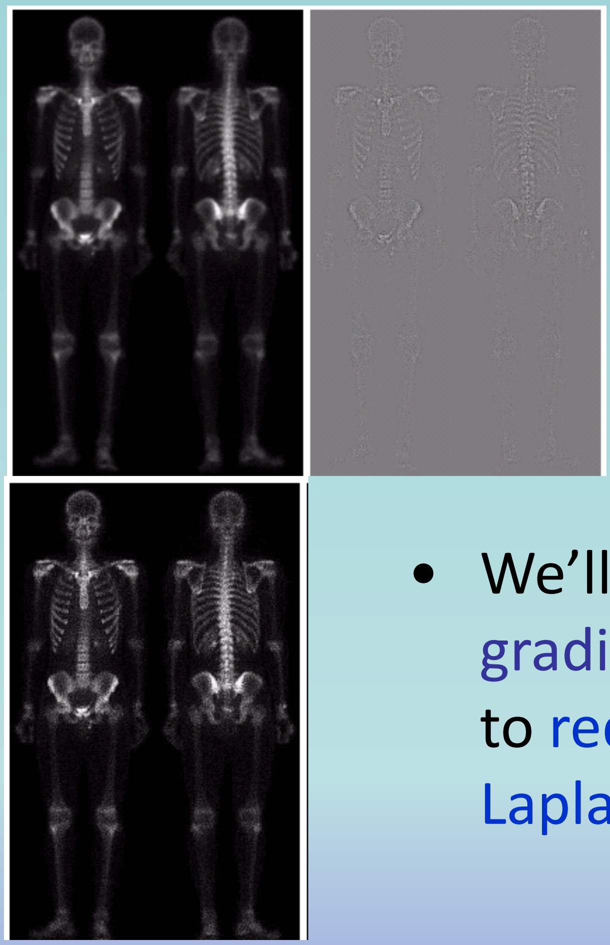
Combining Spatial Filters

Original image

After Laplacian applied

Added 2 images



- We'll use smoothed gradient image as a mask to reduce noise from Laplacian output

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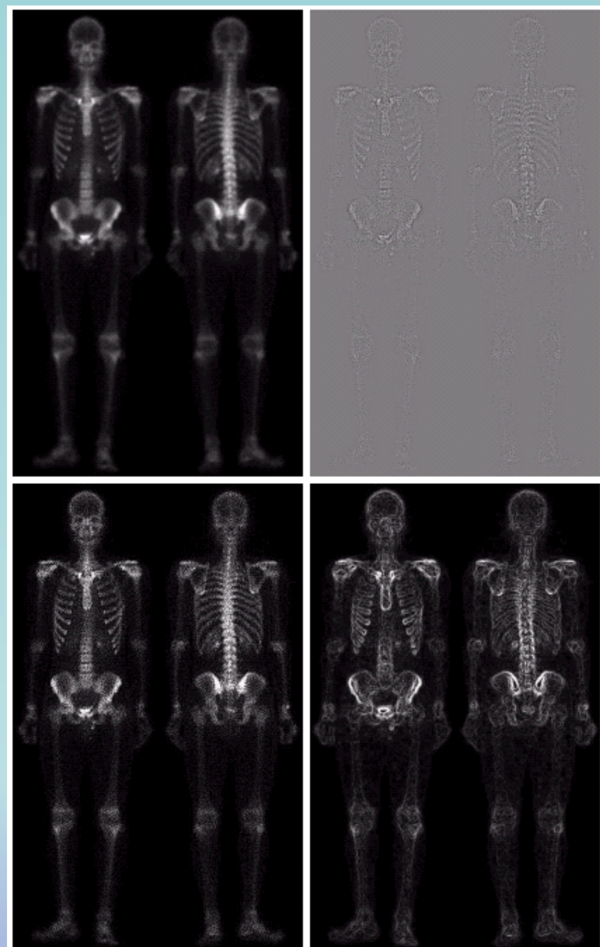
Combining Spatial Filters

Original image

After Laplacian applied



Added 2 images

After Sobel

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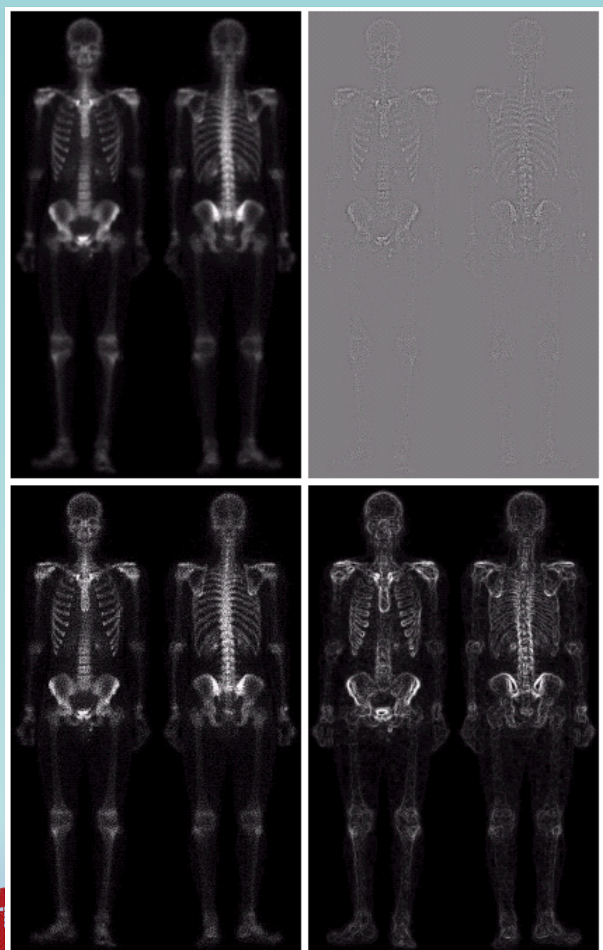
Combining Spatial Filters

Original image

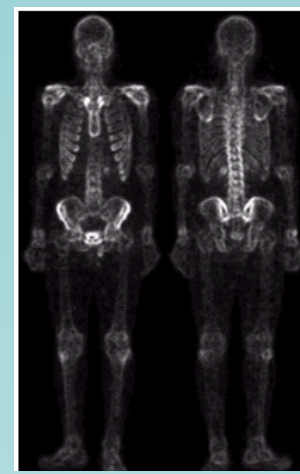
After Laplacian applied

Added 2 images

After Sobel



Sobel, smoothed by 5X5 avg. filter



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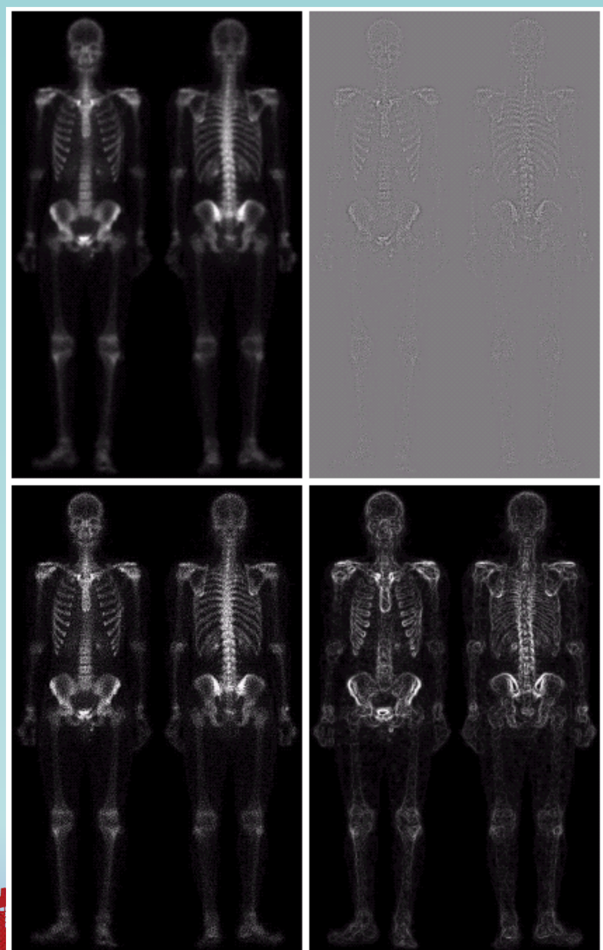
Combining Spatial Filters

Original image

After Laplacian applied

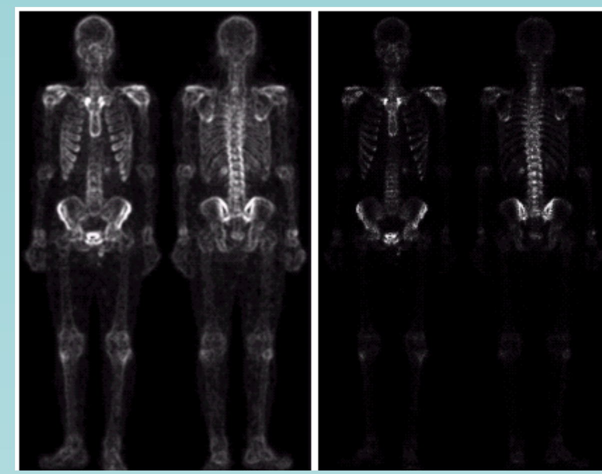
Added 2 images

After Sobel

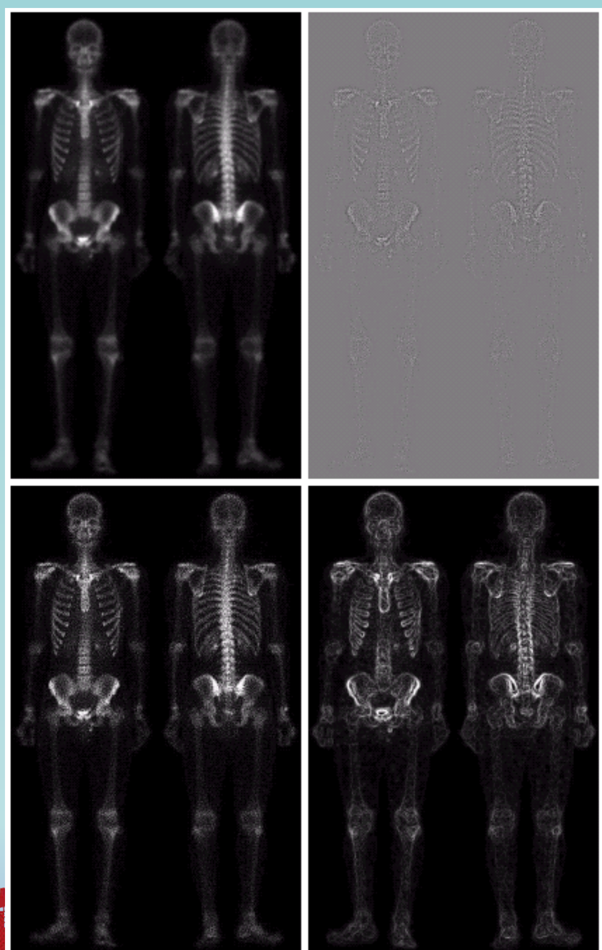


Sobel, smoothed by 5X5 avg. filter

After masking



Combining Spatial Filters

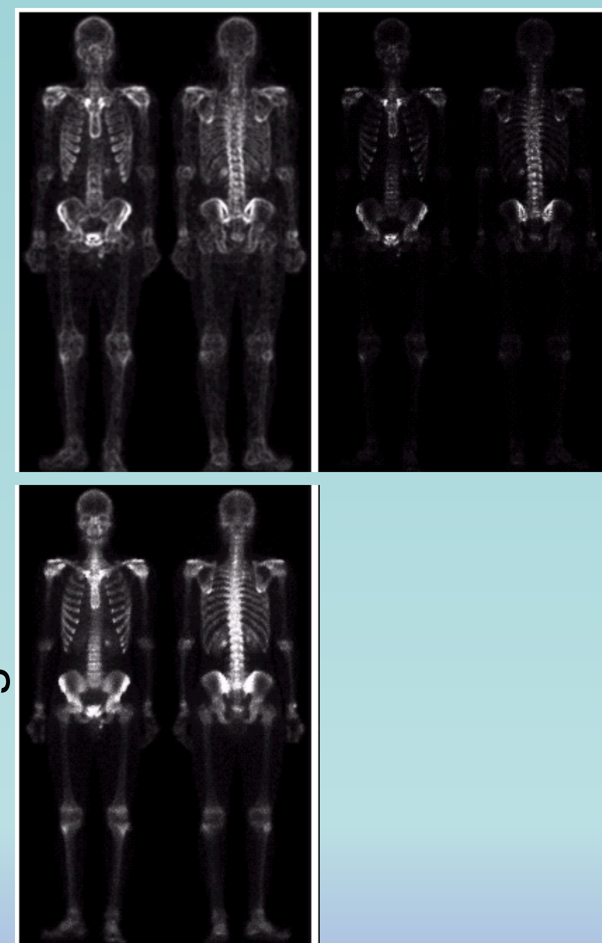


Original image

After Laplacian applied

Added 2 images

After Sobel

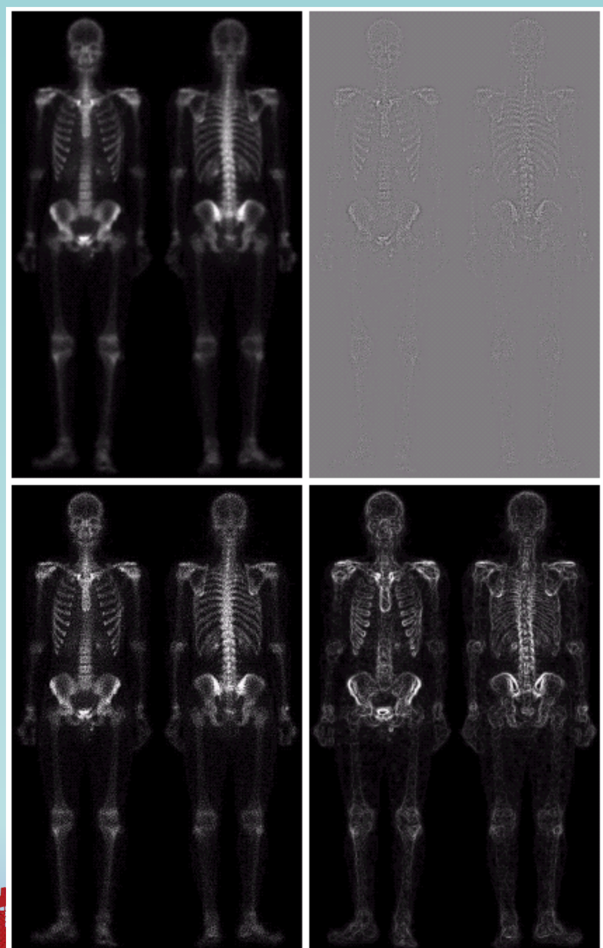


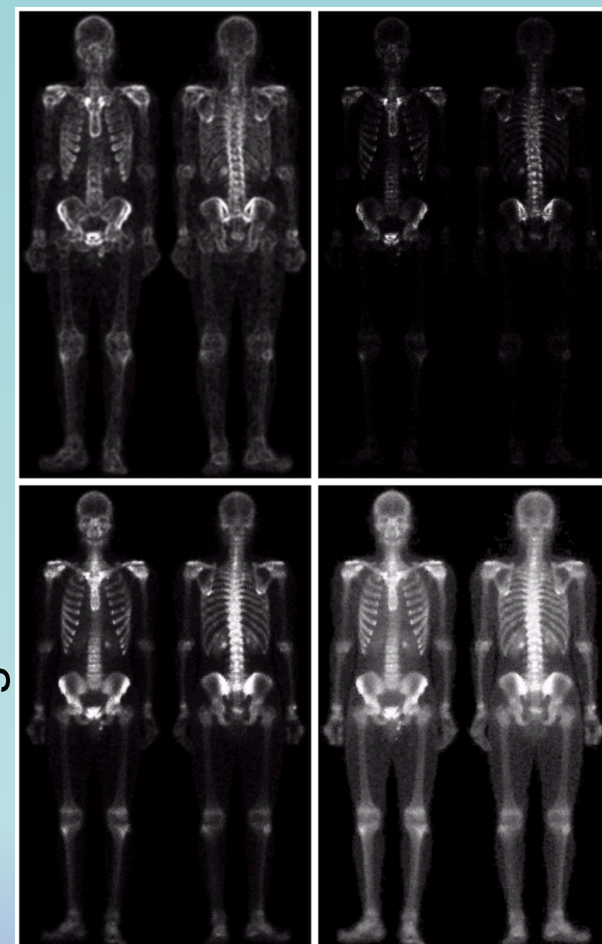
Sobel, smoothed by 5X5 avg. filter

After masking

Added masked result with the original

Combining Spatial Filters





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