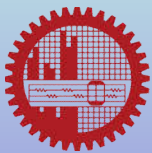


CSE6706: *Advanced Digital Image Processing*

Dr. Md. Monirul Islam



CSE-BUET



Image Segmentation



Image Segmentation

- Divides an image into semantically meaningful regions
- Doesn't need to go for unnecessary detail



Segmentation



Image Segmentation

- Nontrivial task
- Some control over the environment/background can help



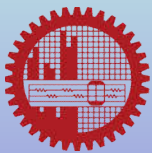
Approaches in Image Segmentation

- Look for discontinuity and similarities



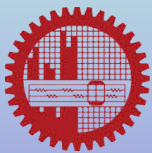
Approaches in Image Segmentation

- Look for discontinuity and similarities
- Discontinuity
 - Abrupt changes in gray level
 - Example: edges



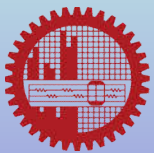
Approaches in Image Segmentation

- Look for discontinuity and similarities
- Discontinuity
 - Abrupt changes in gray level
 - Example: edges
- Similarity
 - divides into regions which are similar based on some criteria
 - Example: thresholding,
region growing
region splitting and merging



Approaches in Image Segmentation

- Look for discontinuity and similarities
- Discontinuity
 - Abrupt changes in gray level
 - Focus on detecting point, line and edges
 - Try to join edges to form regions/segments



Detection of Discontinuity

- Remember the mask

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9



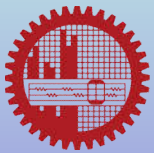
Detection of Discontinuity

- Remember the mask

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

The response is given by

$$R = \sum_{i=1}^9 w_i z_i$$



Detection of Isolated Point

- Isolated point: whose gray level is significantly different from its homogeneous/nearly homogeneous background



Detection of Isolated Point

- Isolated point: whose gray level is significantly different from its homogeneous/nearly homogeneous background

mask to isolated
detect point

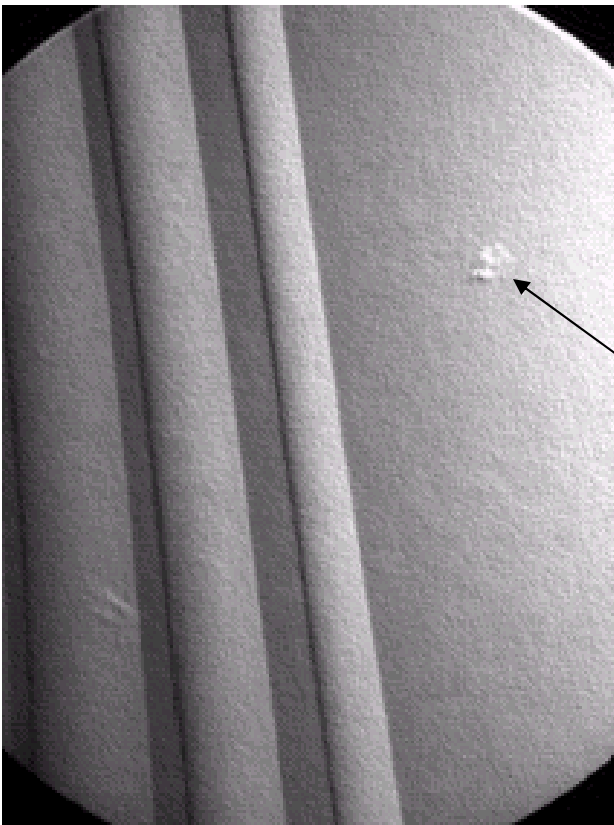
-1	-1	-1
-1	8	-1
-1	-1	-1

A point is detected if

$$|R| \geq T$$



Detection of Isolated Point

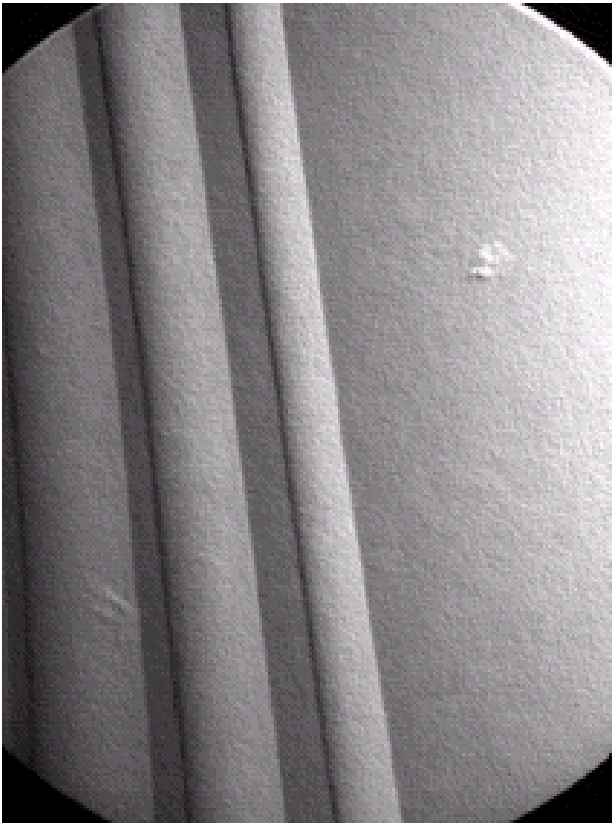


Isolated point



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Detection of Isolated Point

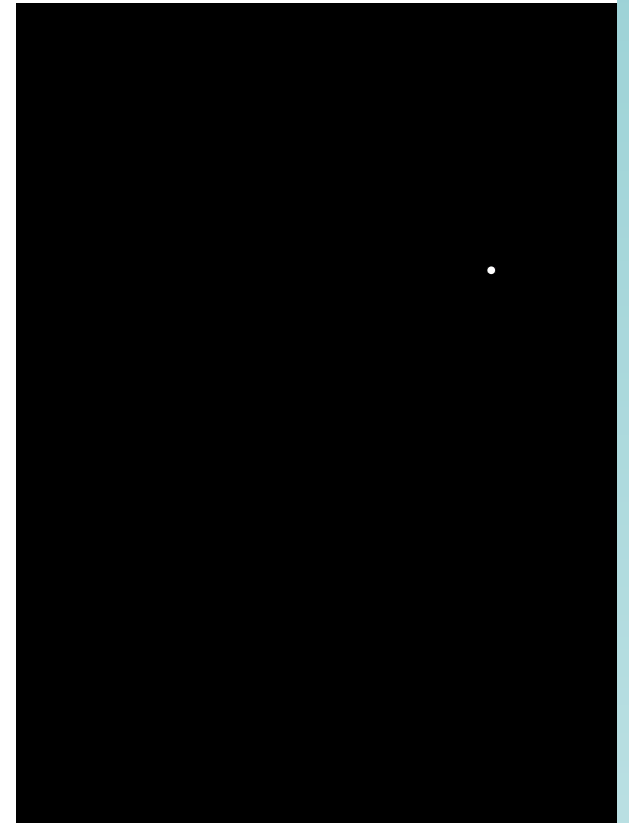
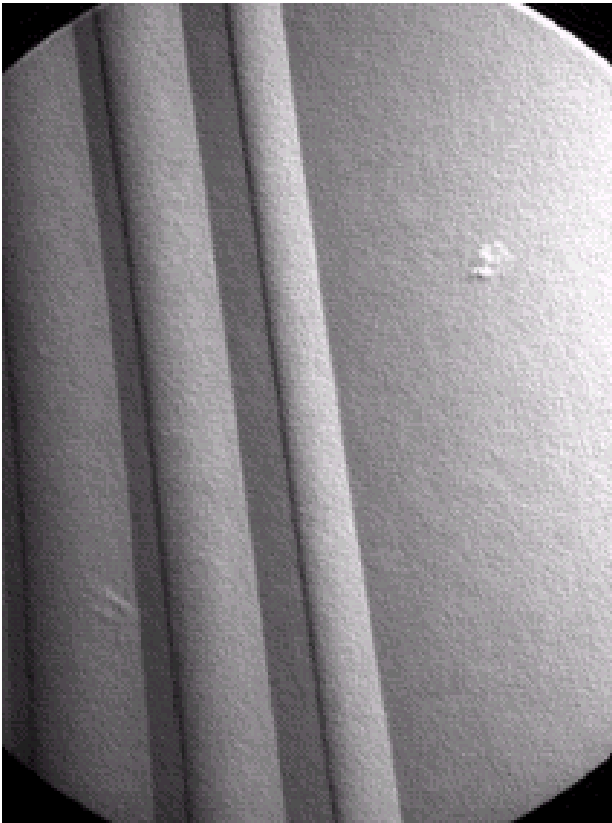


After applying the
mask



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Detection of Isolated Point



After thresholding
with $T = 90\%$ of max
gray level



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Line Detection

-1	-1	-1
2	2	2
-1	-1	-1

Which direction will it detect?



Line Detection

-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		



Line Detection

-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		

- Detects line of single pixel thick
- 2 ways to detect lines



Line Detection

-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		

- Line detection (1)
 - To detect lines in every directions
 - Find masked (**filtered or convolved**) image with each mask
 - Let the responses be R_1, R_2, R_3, R_4



Line Detection (1)

-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		

- A point is oriented to the **direction of mask i** if

$$|R_i| > |R_j| \text{ for all } j$$



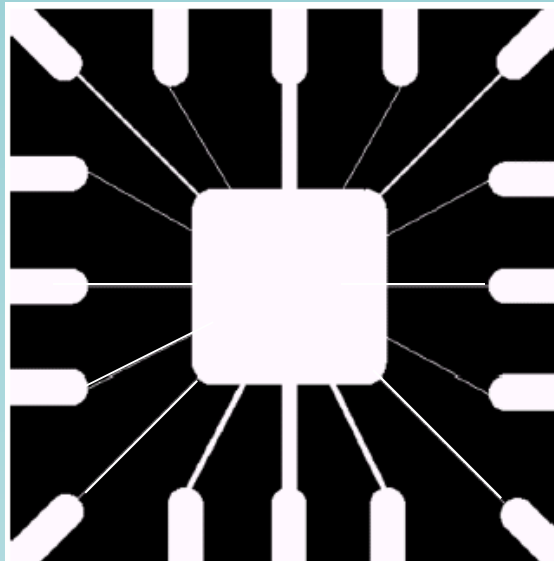
Line Detection (2)

-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
Horizontal			+45°			Vertical			-45°		

- Use a single mask and use thresholding



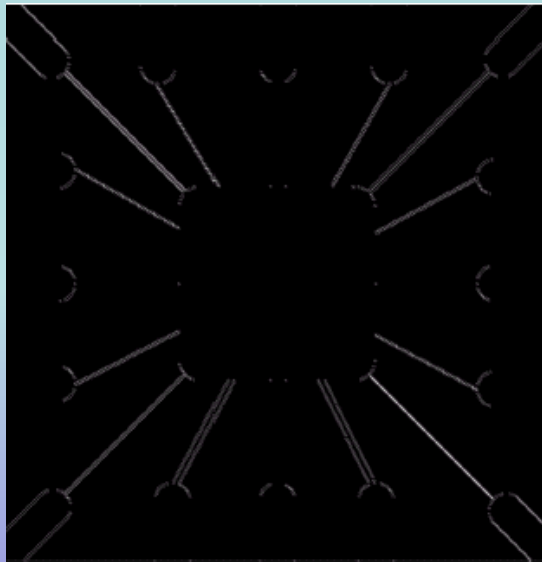
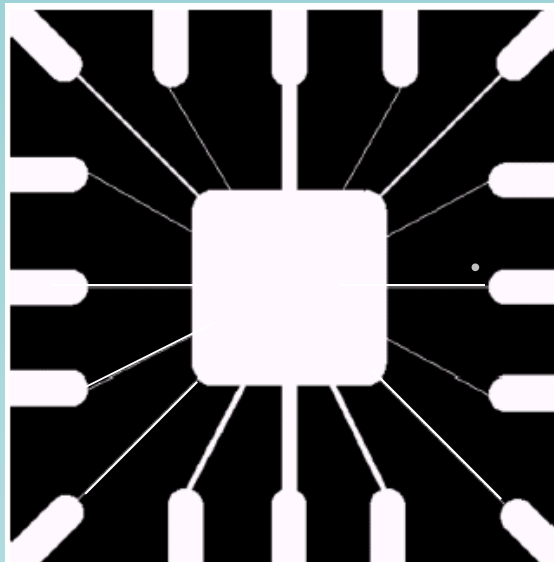
Line Detection (2)



Circuit board: binarized



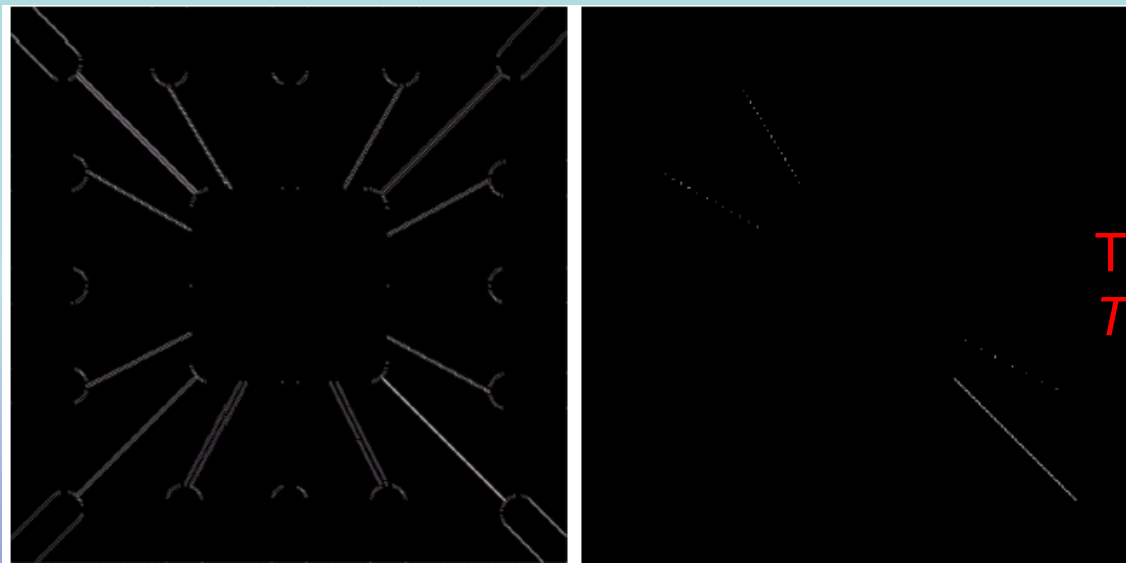
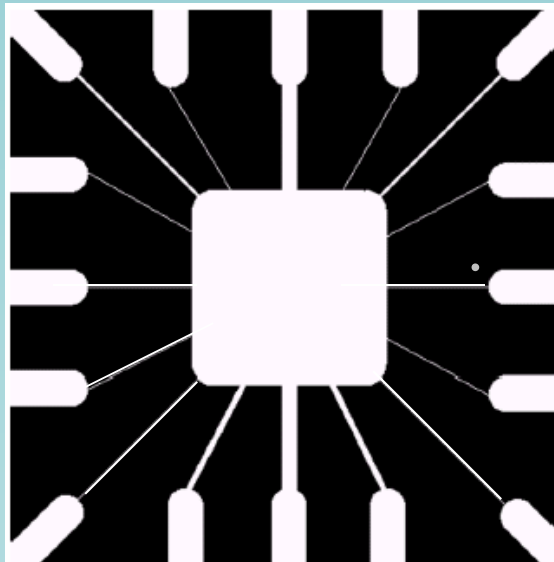
Line Detection (2)



Masked with
-45° line detector



Line Detection (2)



Thresholded with
 $T = \text{max gray level}$



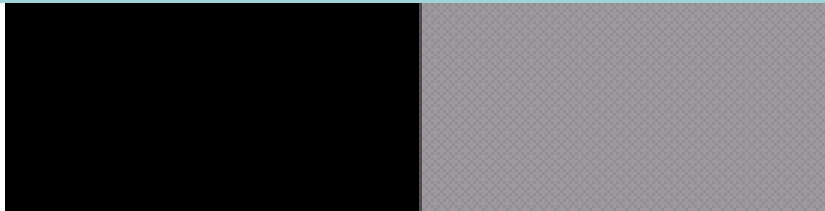
Edge Detection

- Review from Chapter 3:
 - Use 1st and 2nd order derivative
- Indicates boundary between regions



Edge Modeling

Model of an **ideal edge**

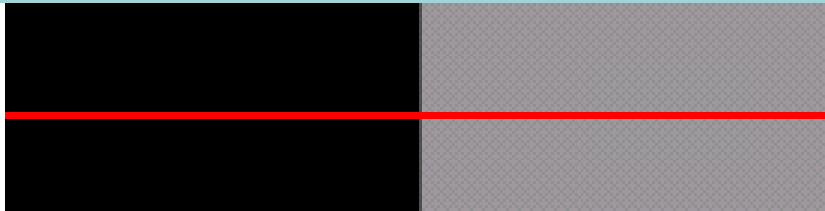


- **Ideal edge** is a set of connected pixels located at an orthogonal step transition



Edge Modeling

Model of an **ideal edge**

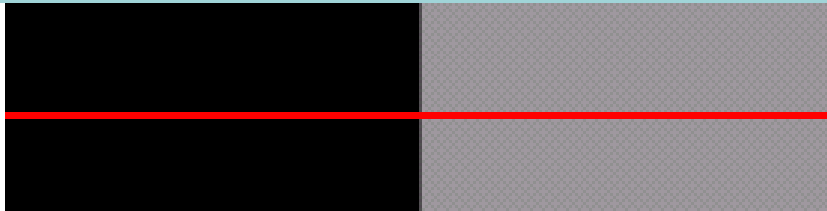


- **Ideal edge** is a set of connected pixels located at an orthogonal step transition

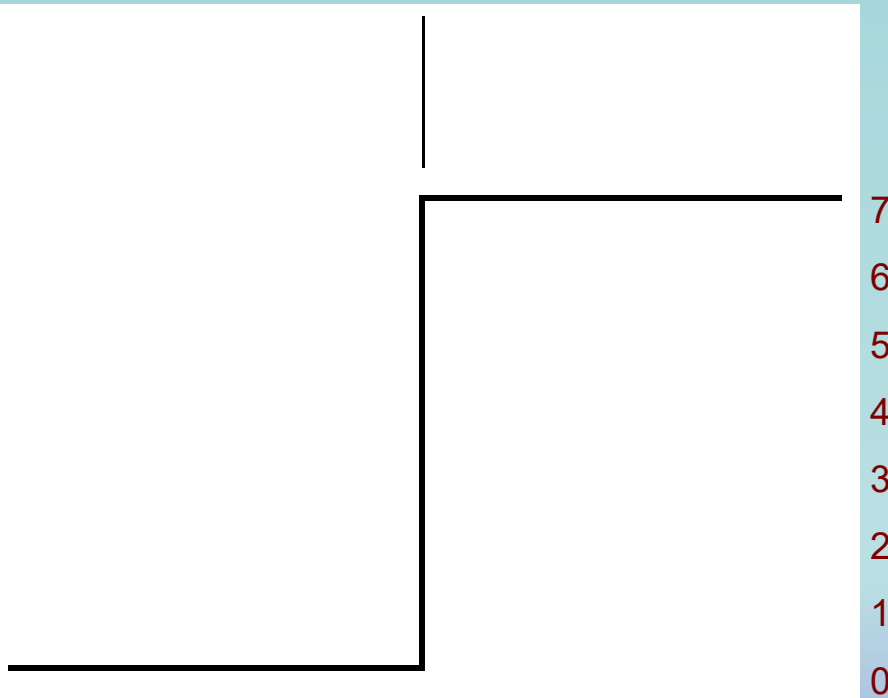


Edge Modeling

Model of an **ideal edge**



- **Ideal edge** is a set of connected pixels located at an orthogonal step transition

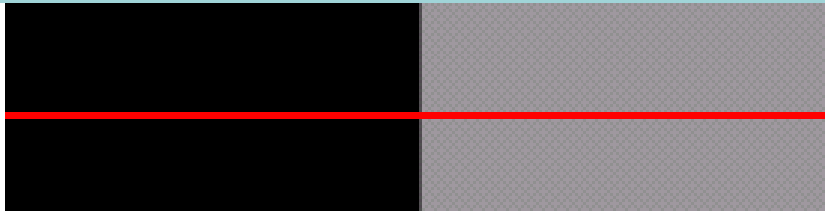


Gray-level profile
of a horizontal line
through the image

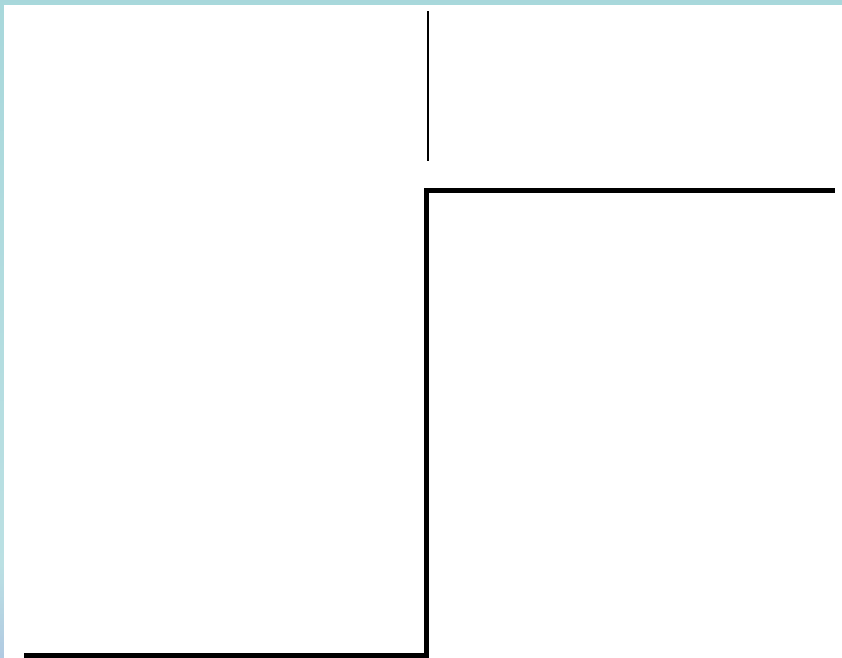
gray level
values

Edge Modeling

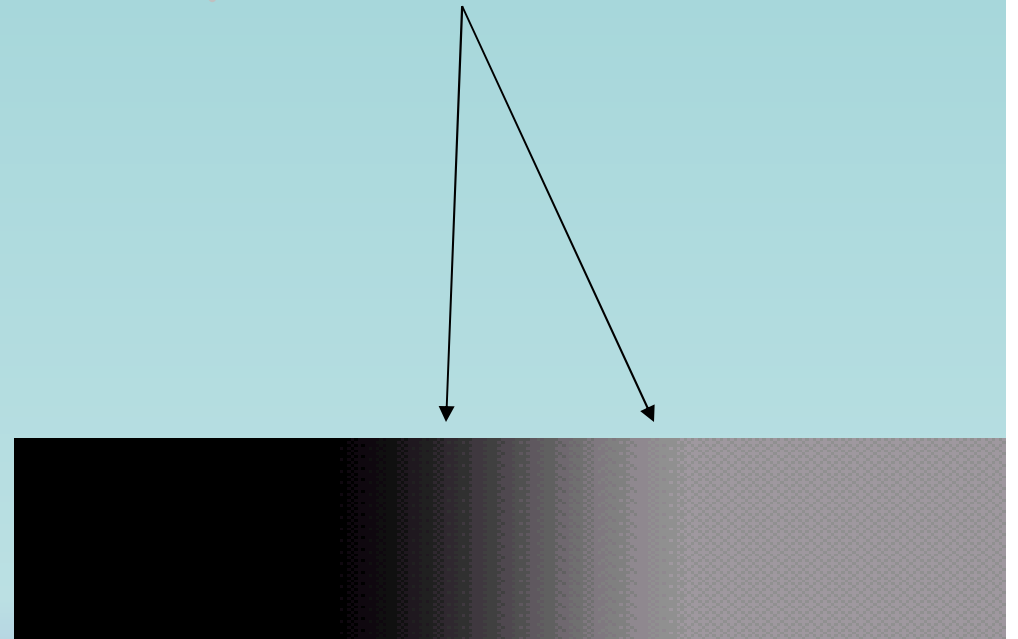
Model of an **ideal edge**



- **Imperfection** (noise, acquisition) leads to blurred and smooth transition



Gray-level profile
of a horizontal line
through the image

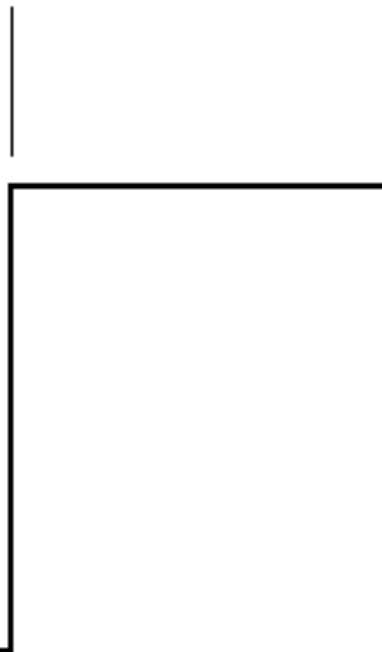


Edge Modeling

Model of an ideal digital edge

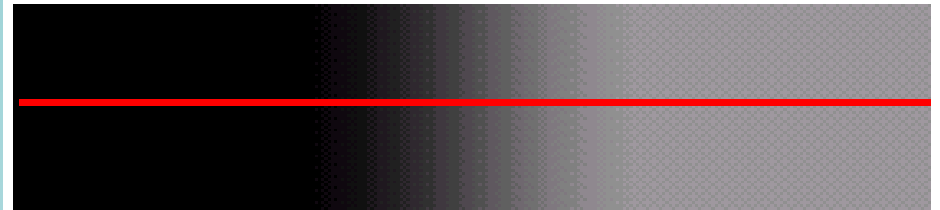


Gray-level profile
of a horizontal line
through the image

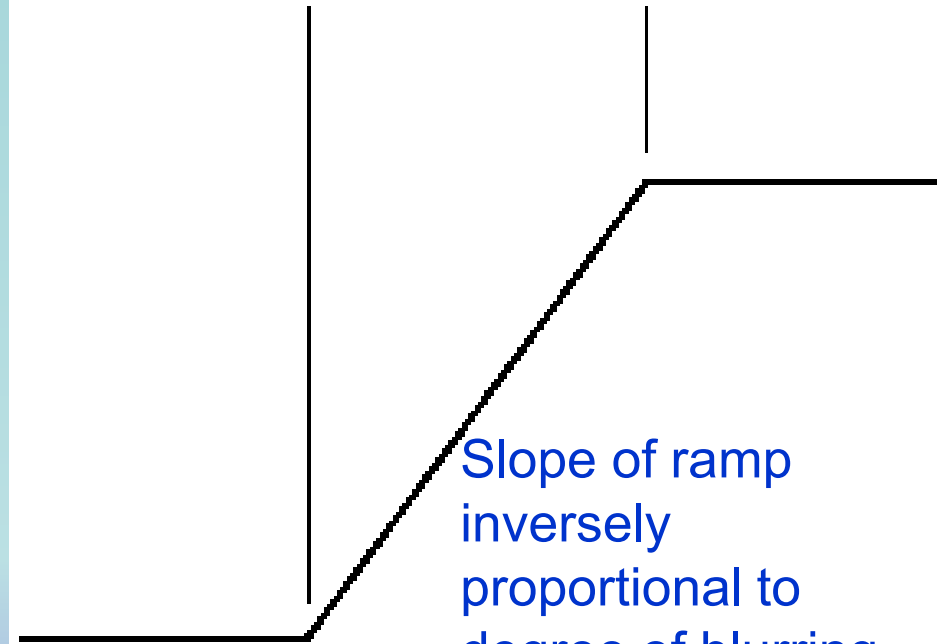


7
6
5
4
3
2
1
0

Model of a ramp digital edge

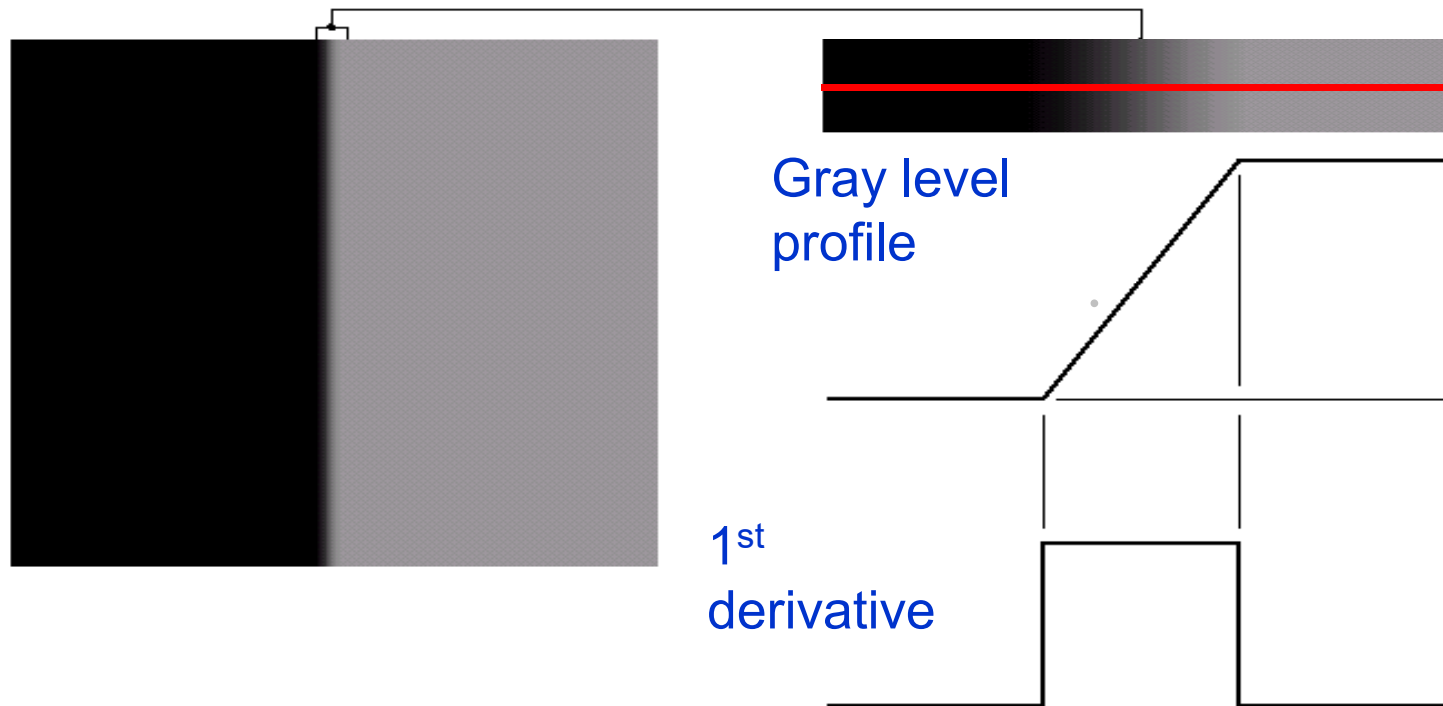


Gray-level profile
of a horizontal line
through the image

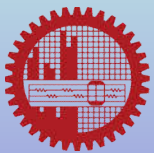
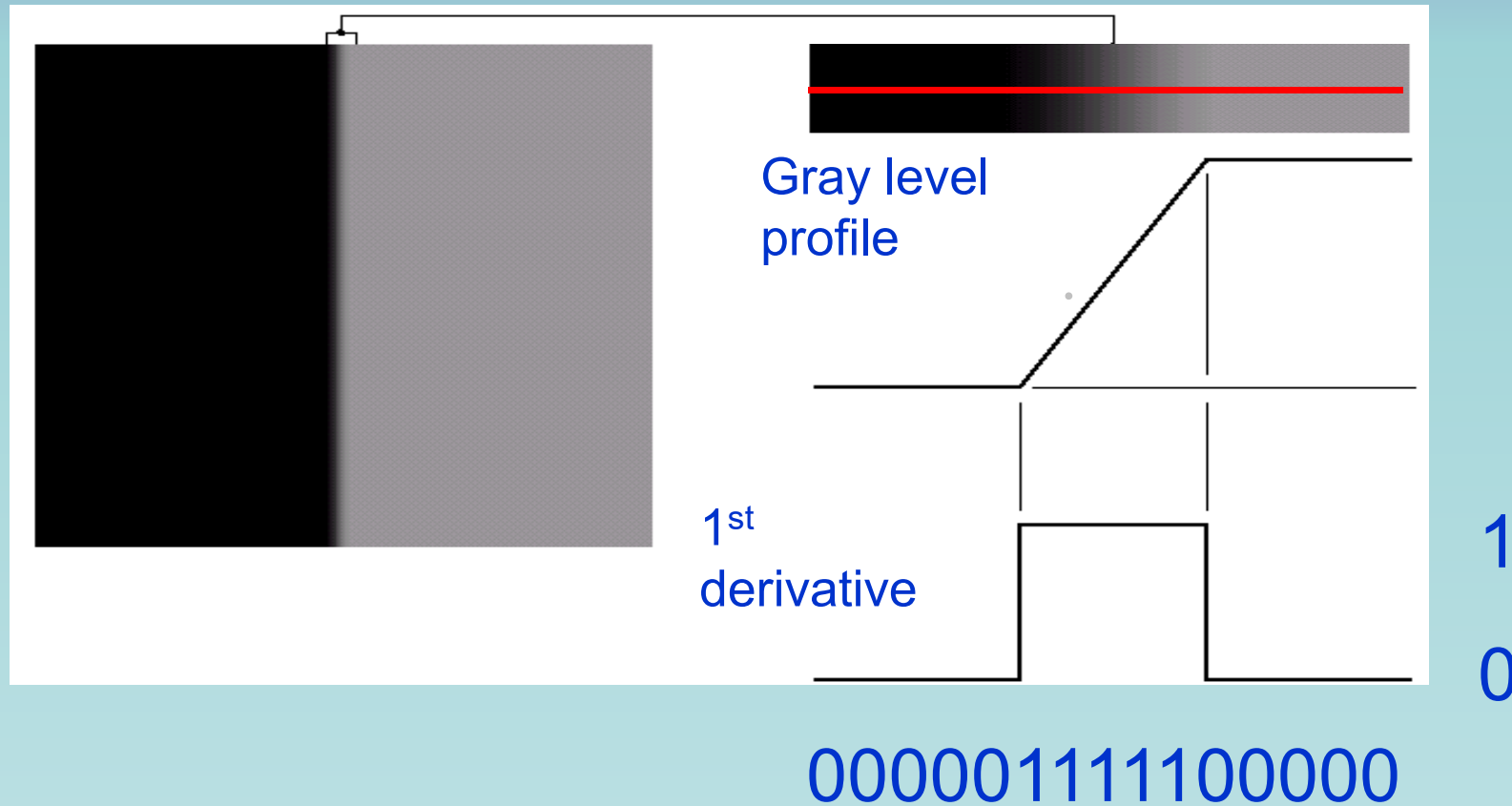


Slope of ramp
inversely
proportional to
degree of blurring

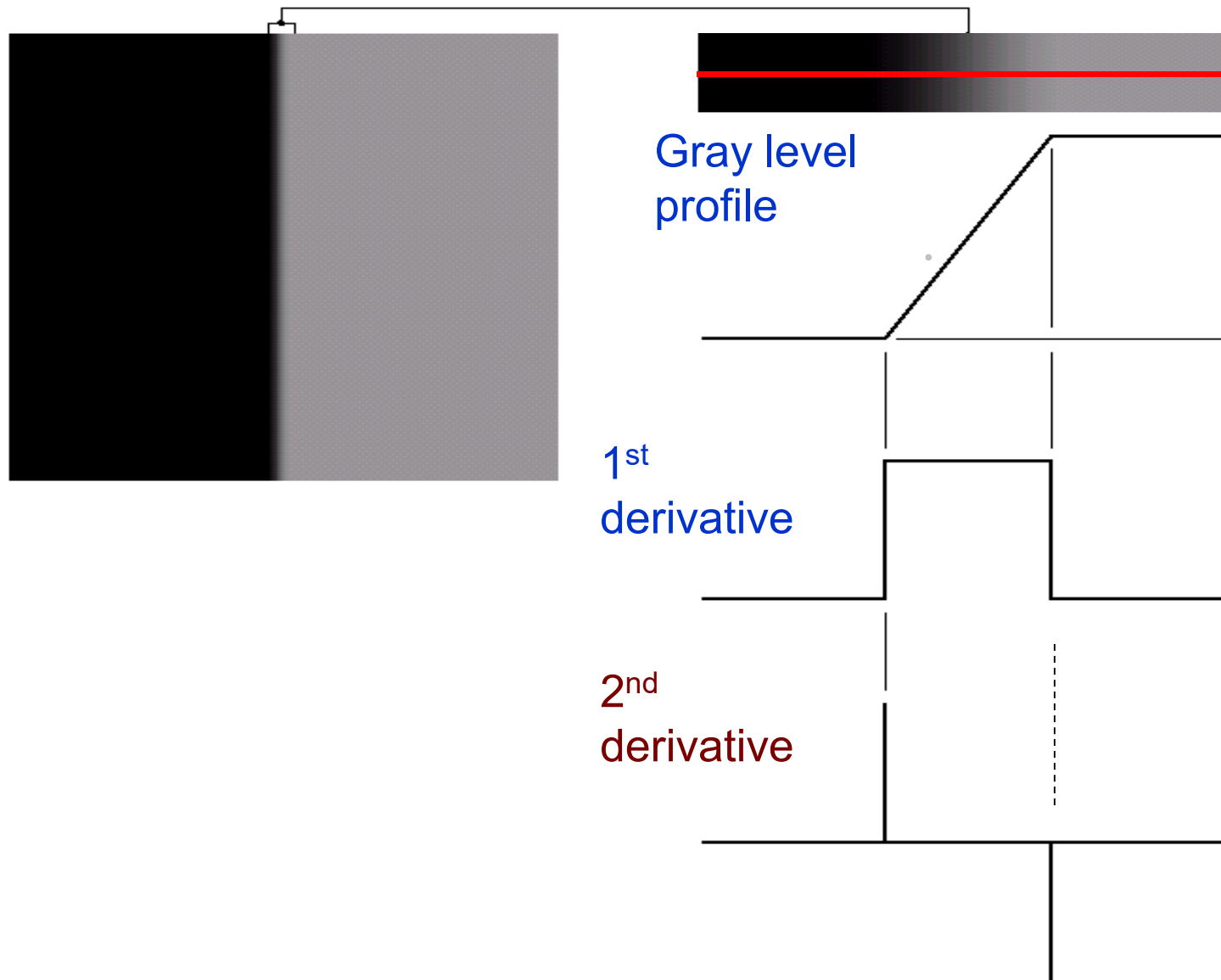
Edge Modeling



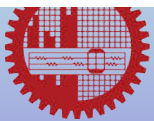
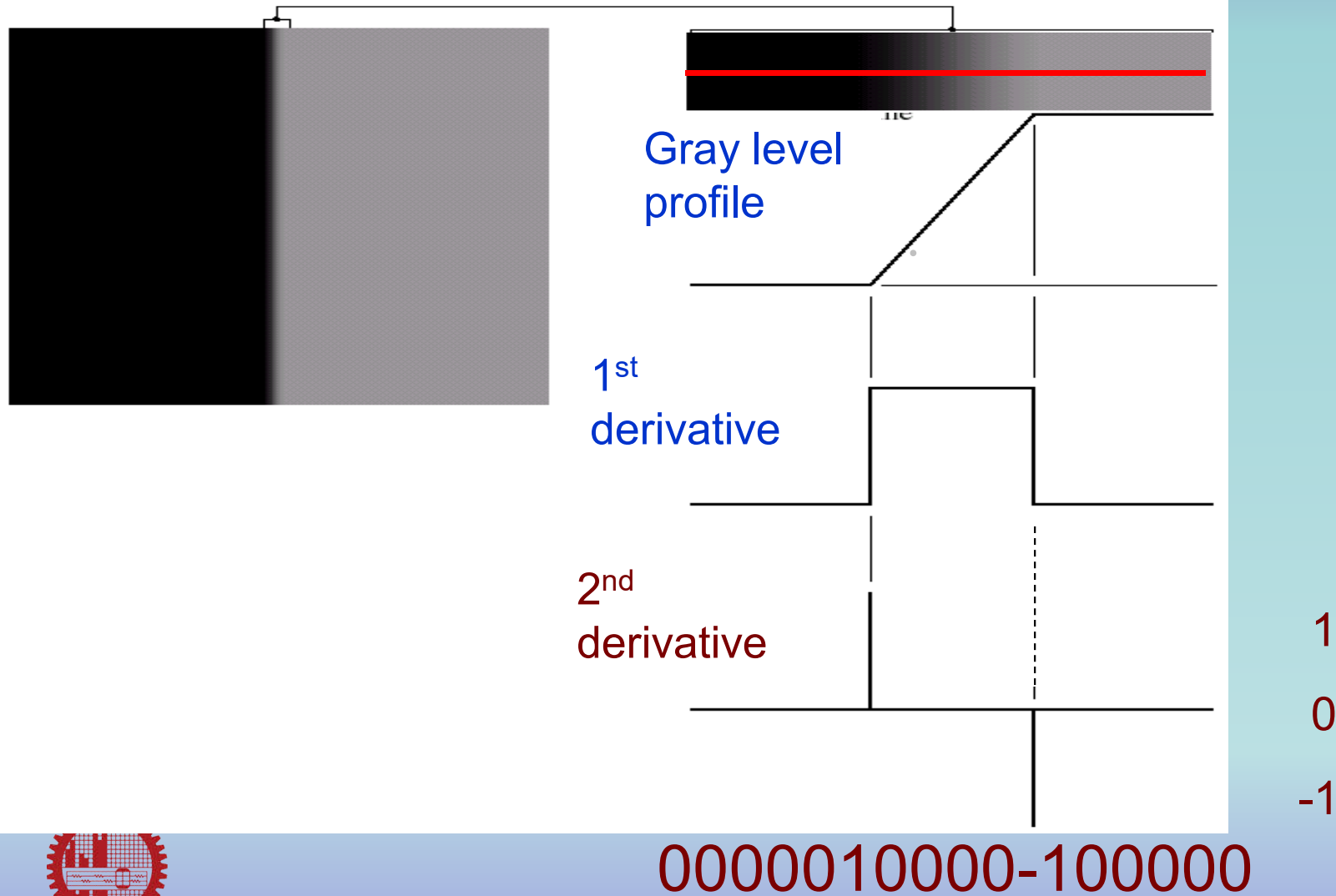
Edge Modeling



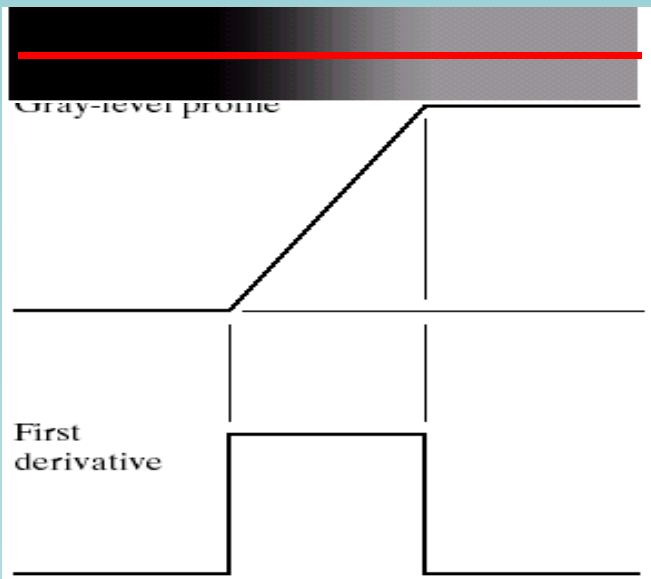
Edge Modeling



Edge Modeling



Edge Modeling



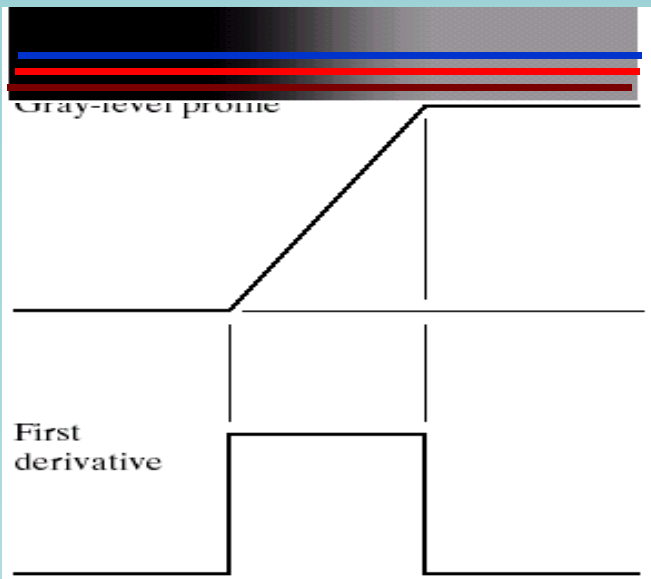
00000111100000



0000010000-100000



Edge Modeling



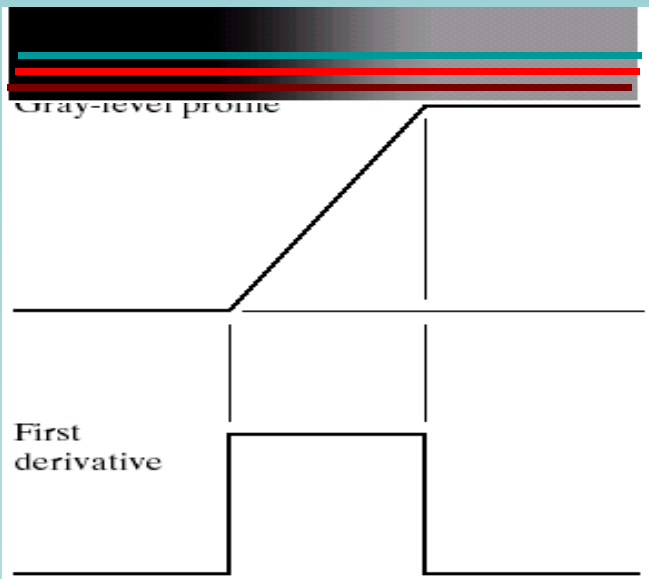
000001111100000
 000001111100000
 000001111100000



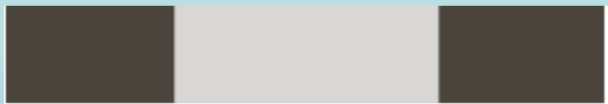
0000010000-100000
 0000010000-100000
 0000010000-100000



Edge Modeling



000001111100000
 000001111100000
 000001111100000

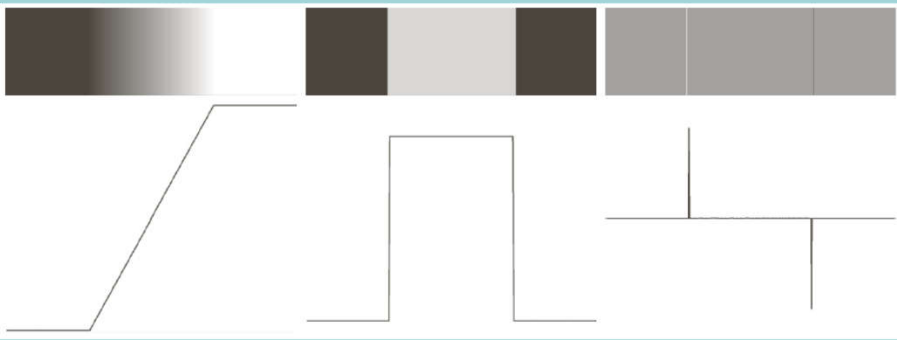


0000010000-100000
 0000010000-100000
 0000010000-100000



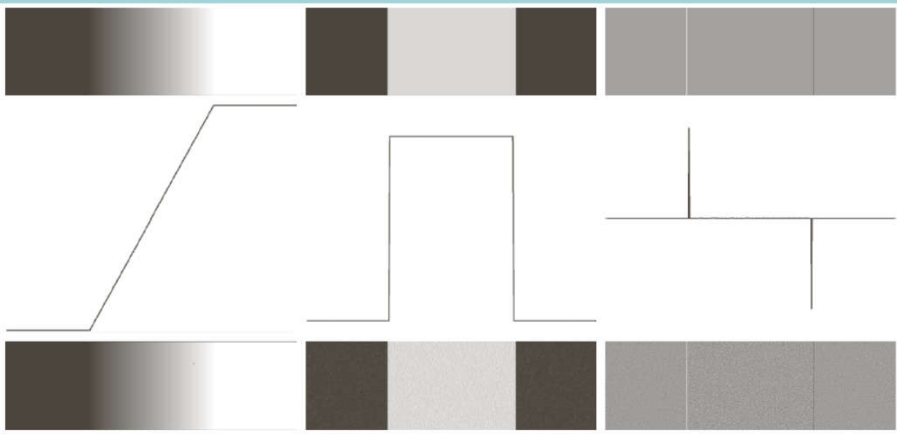
Edge Modeling in noisy images

Gray profile 1st derivative 2nd derivative



Edge Modeling in noisy images

Gray profile 1st derivative 2nd derivative



Noisy images: corrupted
by Gaussian noise with

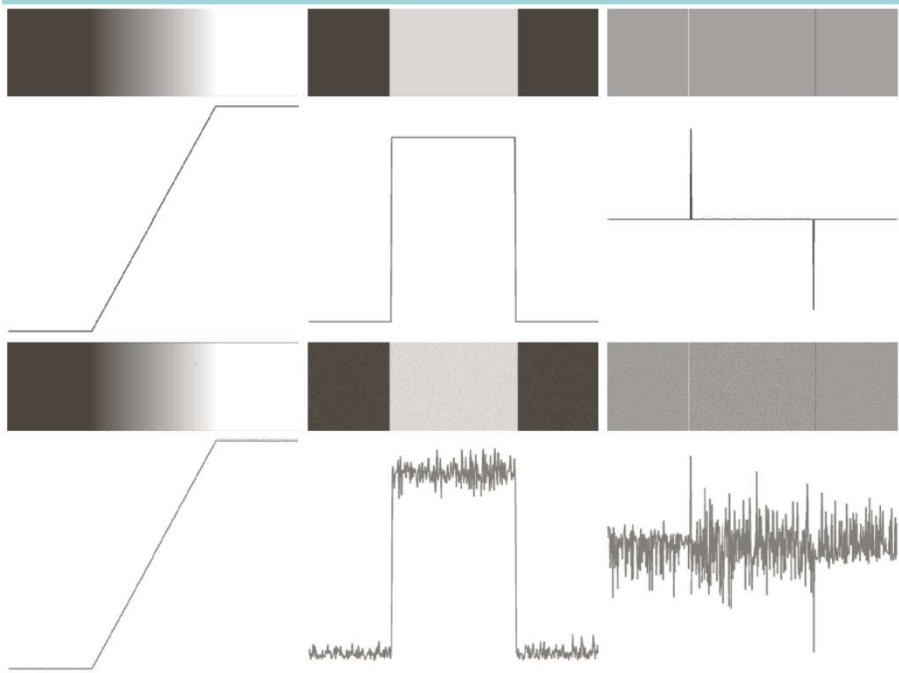
$\mu=0$ and $\sigma=0.1$



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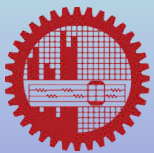
Edge Modeling in noisy images

Gray profile 1st derivative 2nd derivative



Noisy images: corrupted
by Gaussian noise with

$\mu=0$ and $\sigma=0.1$



CSE-BUET

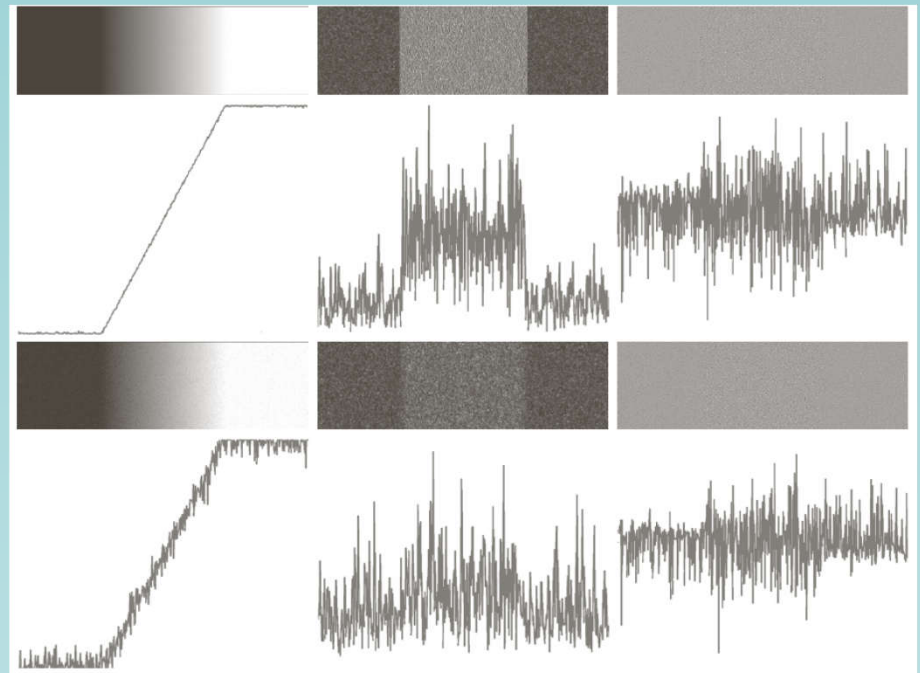
Edge Modeling in noisy images

Noisy images: corrupted
by Gaussian noise with

$$\mu=0, \sigma=1.0$$

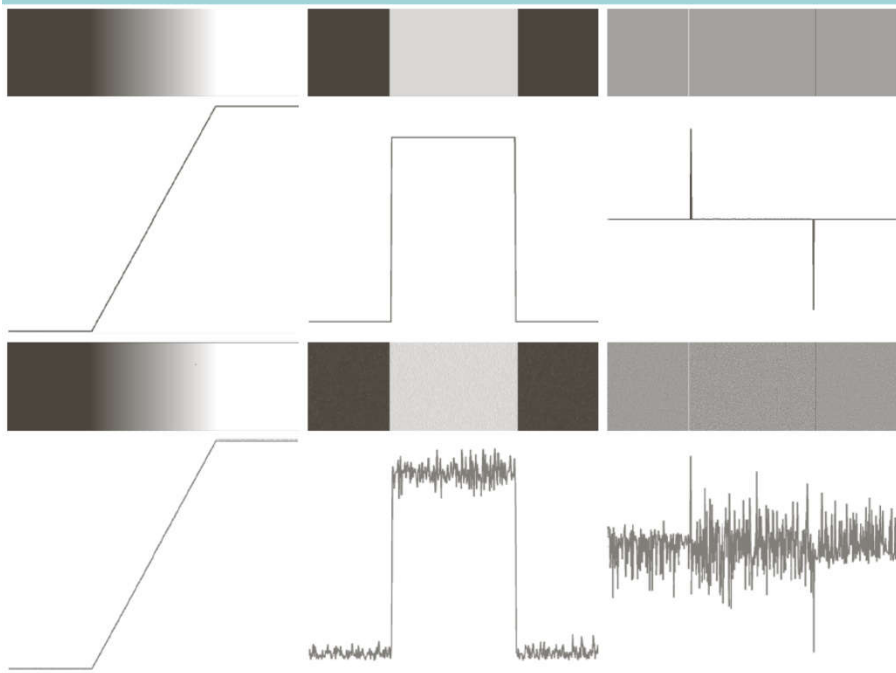
$$\mu=0, \sigma=10$$

Gray profile 1st derivative 2nd derivative

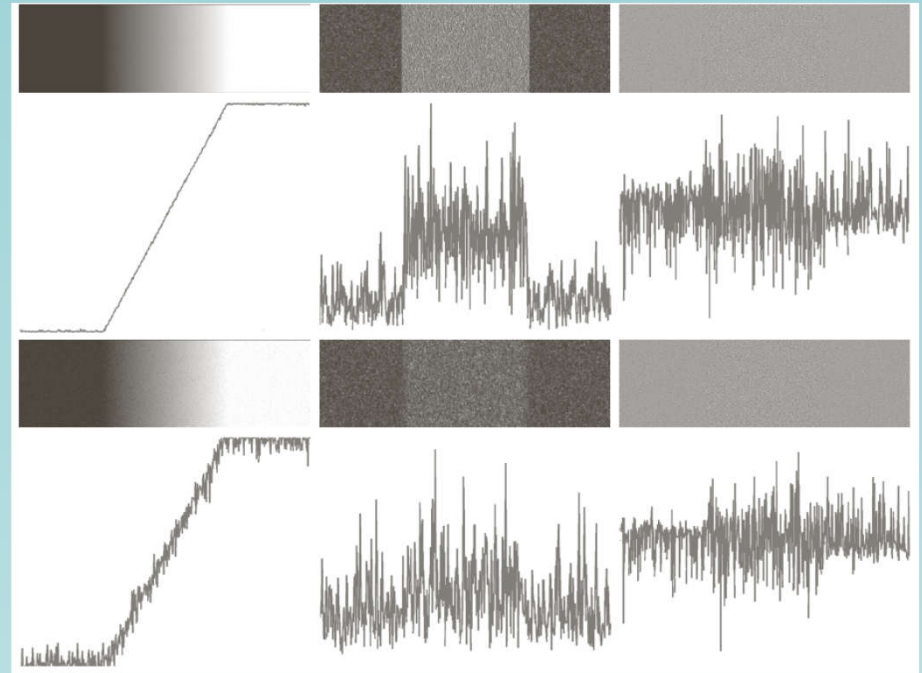


Edge Modeling in noisy images

Gray profile 1st derivative 2nd derivative



Gray profile 1st derivative 2nd derivative



Noisy images: corrupted by Gaussian noise with

$\mu=0$ and

$\sigma = 0, 0.1, 1.0, 10$



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Edge Modeling

- Transition of gray level in edge point should be *significantly stronger* than that in background
- Thresholding can be used to determine *this transition*
- 1st order derivative should be greater than the threshold



Review of 1st Order Derivative – The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\begin{aligned} \nabla f = \text{mag}(\nabla \mathbf{f}) &= \left[G_x^2 + G_y^2 \right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \end{aligned}$$

The
Gradient



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Review of 1st Order Derivative – The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The
Gradient

Misnomer for
gradient

$$\begin{aligned} \nabla f &= \text{mag}(\nabla \mathbf{f}) = [G_x^2 + G_y^2]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \end{aligned}$$



Direction of The Gradient Vector

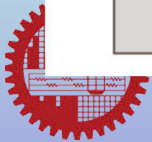
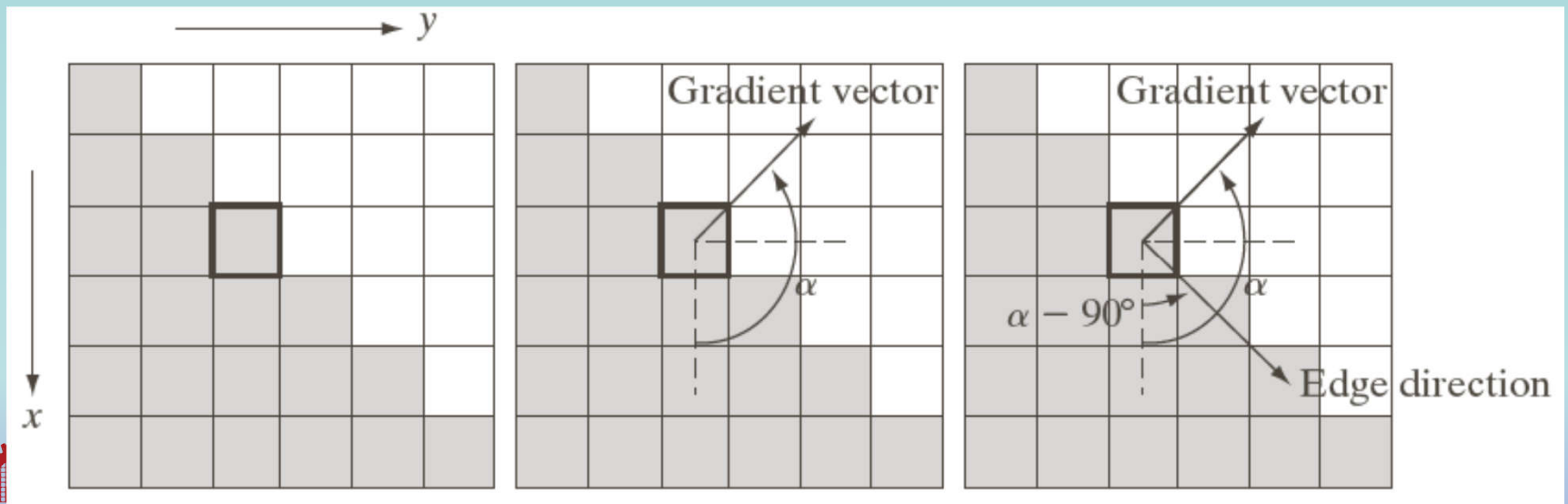
$$\alpha(x, y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

- The edge is perpendicular to α



Direction of The Gradient Vector


$$\alpha(x, y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$



Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0	0	-1
0	1	1	0



$$Z_9 - Z_5$$

$$Z_8 - Z_6$$

- Many implementations

$$G_x = Z_9 - Z_5$$

$$G_y = Z_8 - Z_6$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

Robert's Operator

Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Sobel Operators

$$\nabla f = \left| (Z_7 + 2Z_8 + Z_9) - (Z_1 + 2Z_2 + Z_3) \right|$$

$$+ \left| (Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7) \right|$$



Digital Approximation of The Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt Operators

$$\nabla f = \left| (Z_7 + Z_8 + Z_9) - (Z_1 + Z_2 + Z_3) \right| \\ + \left| (Z_3 + Z_6 + Z_9) - (Z_1 + Z_4 + Z_7) \right|$$



Prewitt and Sobel Operators for Diagonal Edges

0	1	1	-1	-1	0
-1	0	1	-1	0	1
-1	-1	0	0	1	1

Prewitt Operators

Sobel Operators

0	1	2	-2	-1	0
-1	0	1	-1	0	1
-2	-1	0	0	1	2



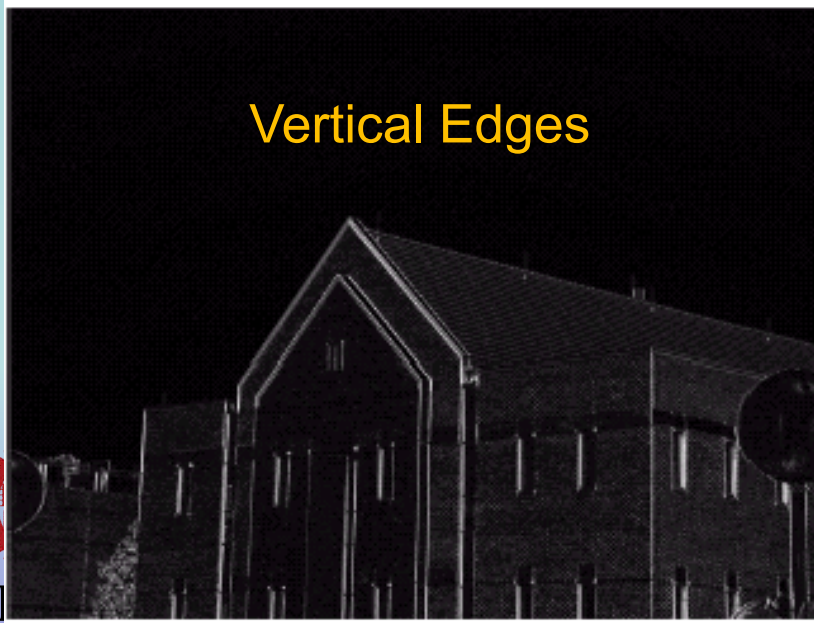
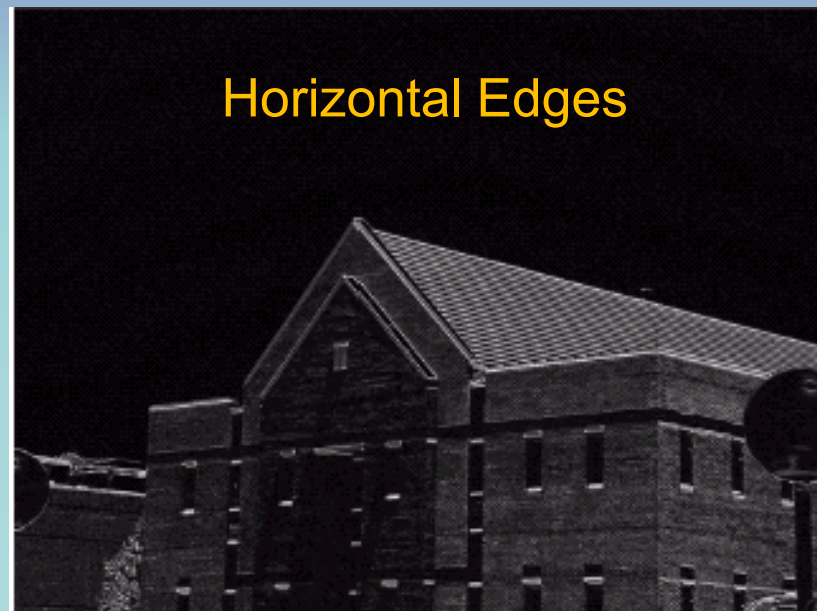
Horizontal and Vertical Edge Detection



Original Image

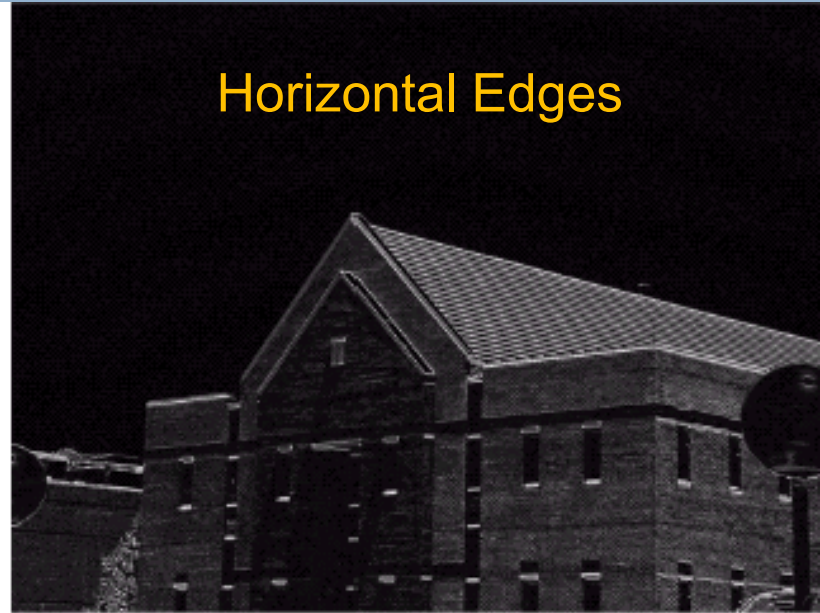
- Sobel operator will be used



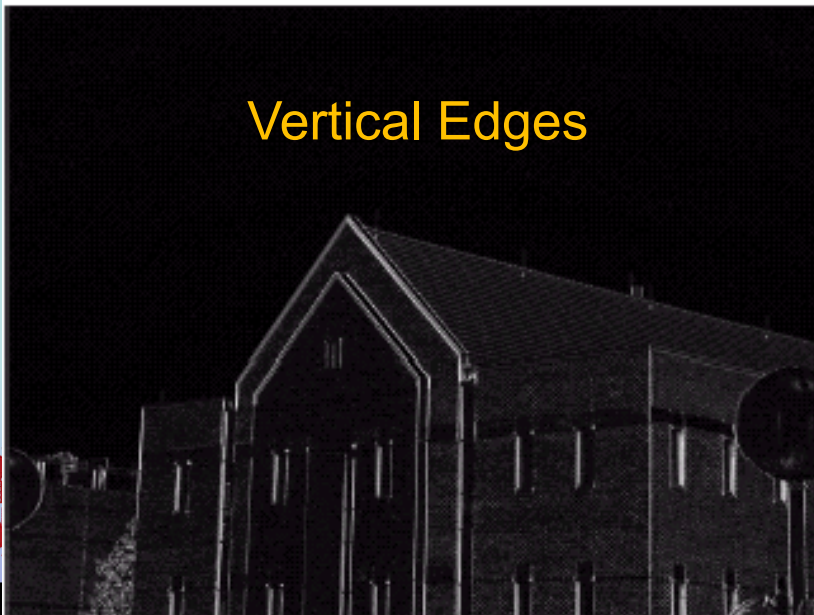




Horizontal Edges



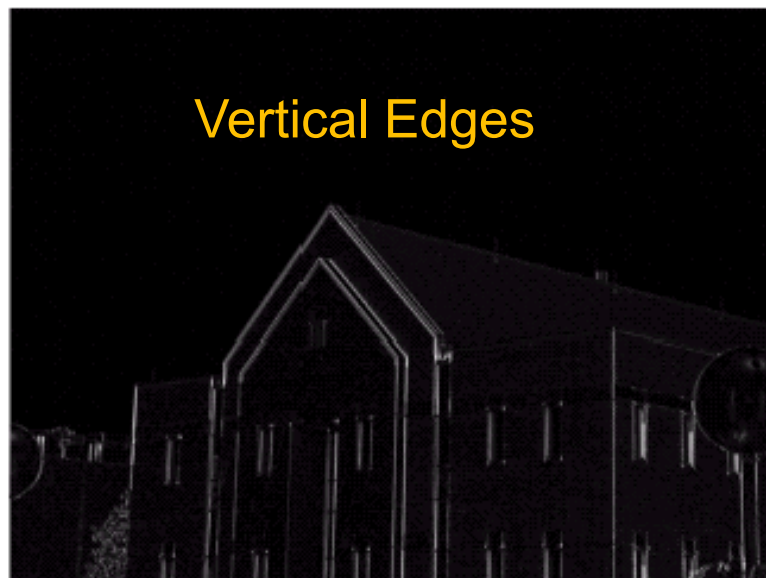
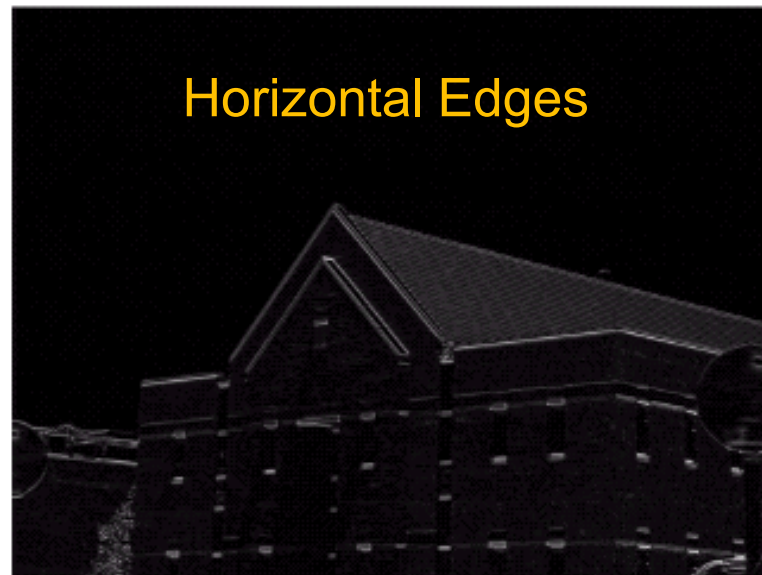
Vertical Edges



Added together



Hor/Vert Edge Detection after Smoothing

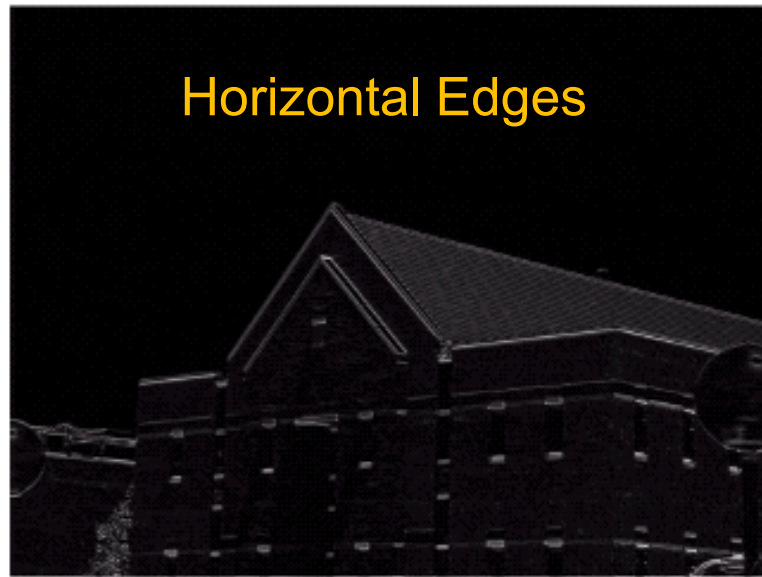


Hor/Vert Edge Detection after Smoothing

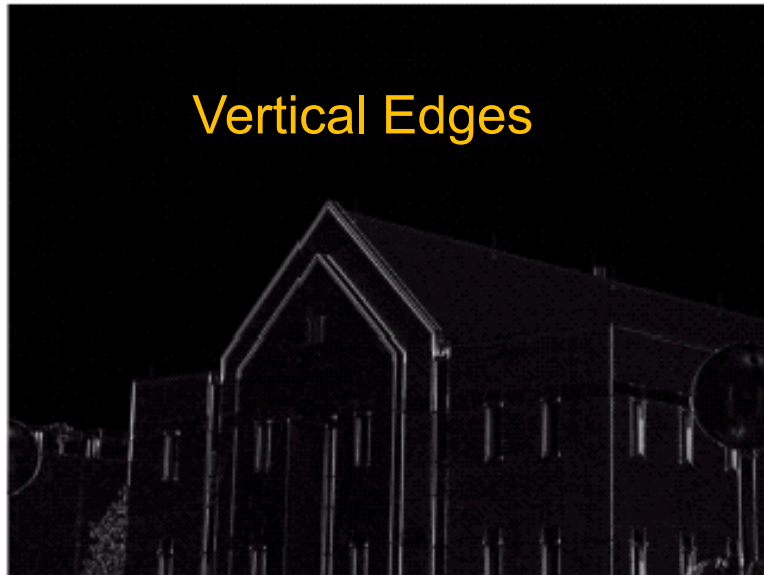
- Unnecessary details removed
- No contribution due to bricks



Horizontal Edges



Vertical Edges



Added together



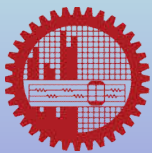
Diagonal Edge Detection



Off diagonal Edges



Diagonal Edges



Review of 2nd Order Derivative- The Laplacian Operator

- Laplacian 2nd order derivative
 - Rotation invariant or isotropic

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



2nd Order Derivative

$$\begin{aligned}\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \\ &= 4f(x, y) \\ &\quad - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)]\end{aligned}$$



Digital Approximation of The Laplacian

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$\nabla^2 f = 4Z_5 - (Z_2 + Z_4 + Z_6 + Z_8)$$



Digital Approximation of The Laplacian

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

0	-1	0
-1	4	-1
0	-1	0

$$\nabla^2 f = 4Z_5 - (Z_2 + Z_4 + Z_6 + Z_8)$$

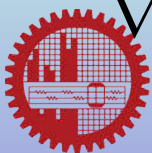


Digital Approximation of The Laplacian

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	-1	-1
-1	8	-1
-1	-1	-1

$$\nabla^2 f = 8Z_5 - (Z_1 + Z_2 + Z_3 + Z_4 + Z_6 + Z_7 + Z_8 + Z_9)$$



Properties of 2nd order derivative

- Sensitive to noise
- Double response to edges
- However, has zero crossing property



Smoothing before Laplacian (2nd order derivative)

- Sensitive to noise
 - Smoothing will remove noise sensitivity



Smoothing before Laplacian (2nd order derivative)

- Sensitive to noise
 - Smoothing will remove noise sensitivity

$$h(r) = e^{-\frac{r^2}{2\sigma^2}}, \text{ where } r^2 = x^2 + y^2$$

Gaussian
Smoothing
Function



Smoothing before Laplacian (2nd order derivative)

- 2nd order derivative of Gaussian (LoG)
 - Marr-Hildreth Edge Detector

$$h(r) = e^{-\frac{r^2}{2\sigma^2}}$$

Gaussian
Smoothing
Function

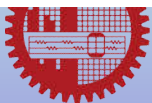
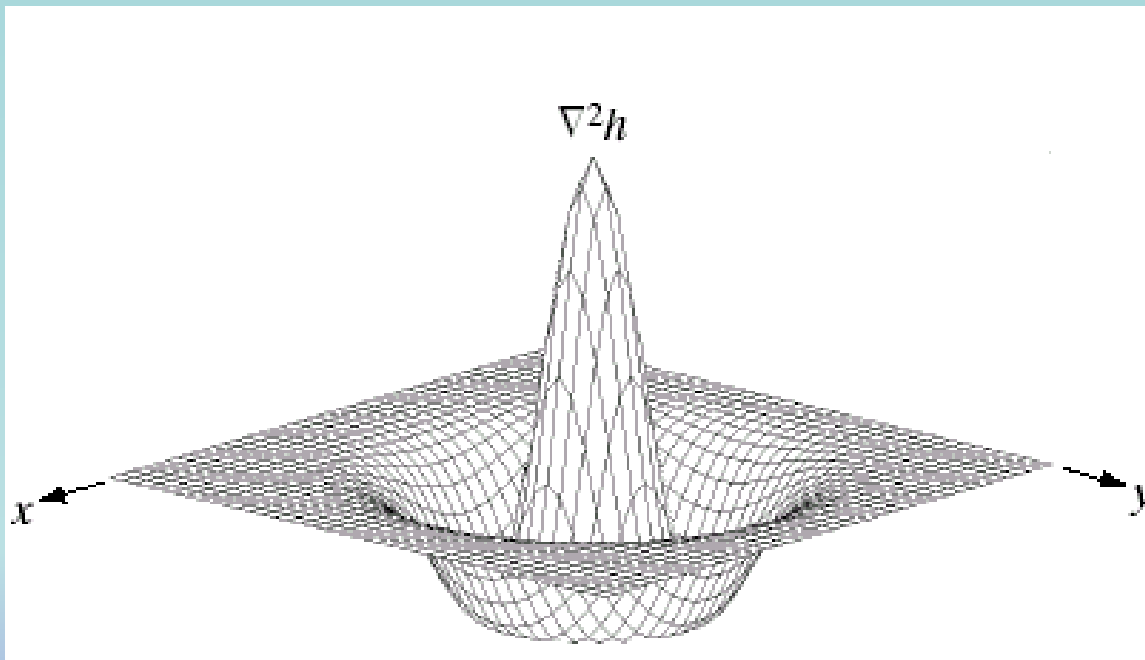
$$\nabla^2 h(r) = \left[\frac{r^2 - 2\sigma^2}{\sigma^4} \right] e^{-\frac{r^2}{2\sigma^2}}$$

Gaussian
Smoothing
Function



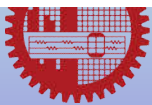
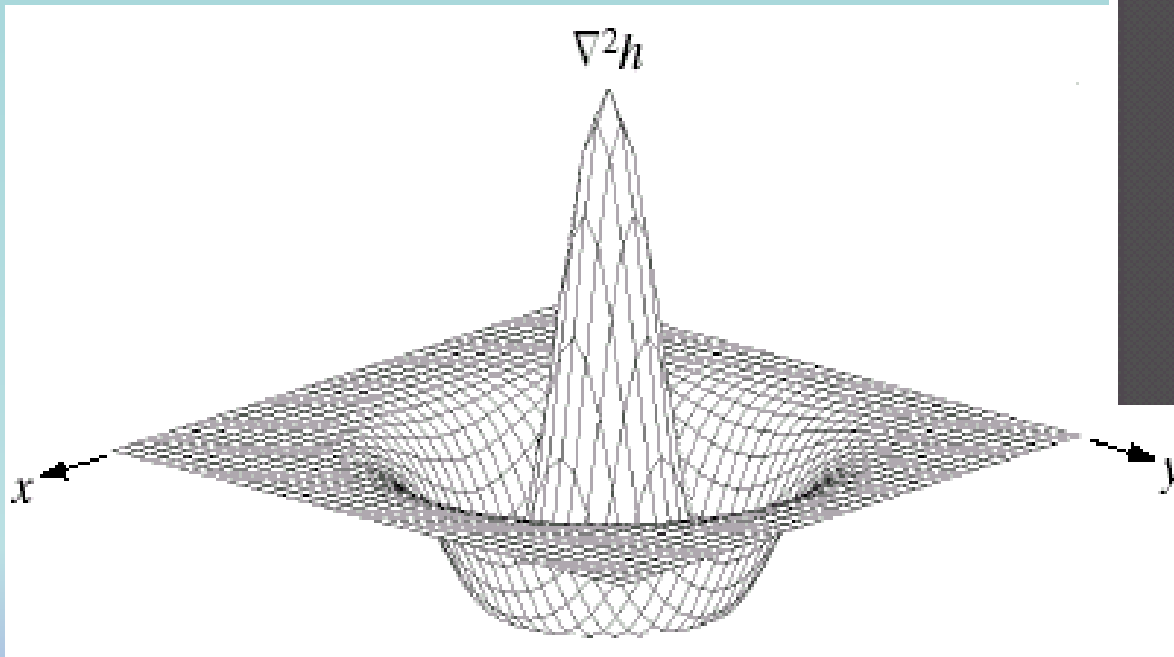
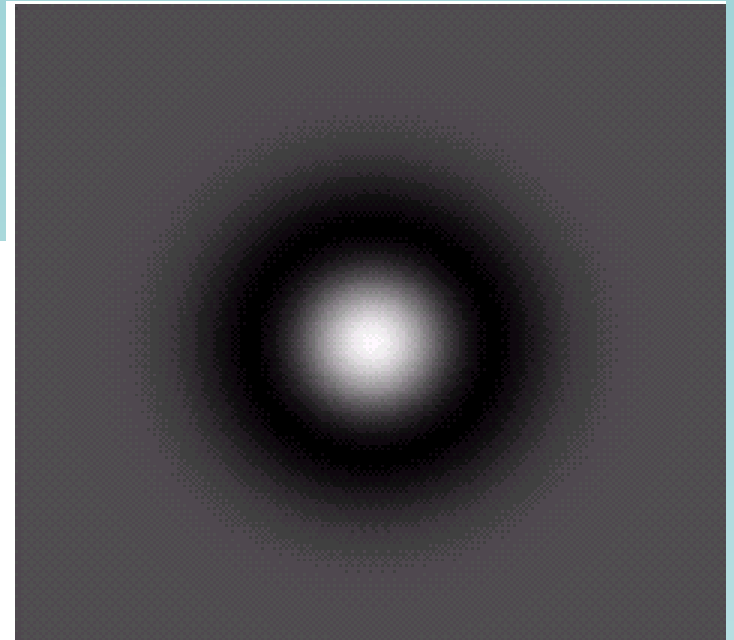
Smoothing before Laplacian (2nd order derivative)

$$\nabla^2 h(r) = \nabla^2 h(x, y) = \left[\frac{r^2 - 2\sigma^2}{\sigma^4} \right] e^{-\frac{r^2}{2\sigma^2}}$$



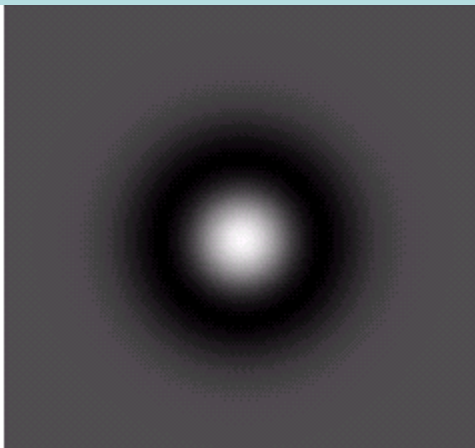
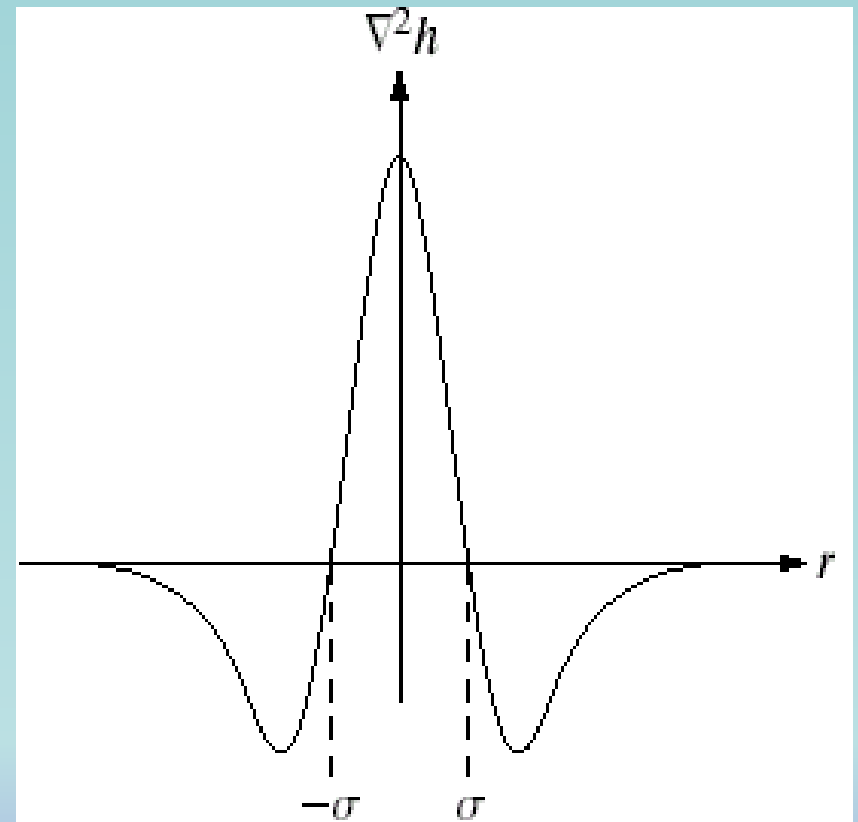
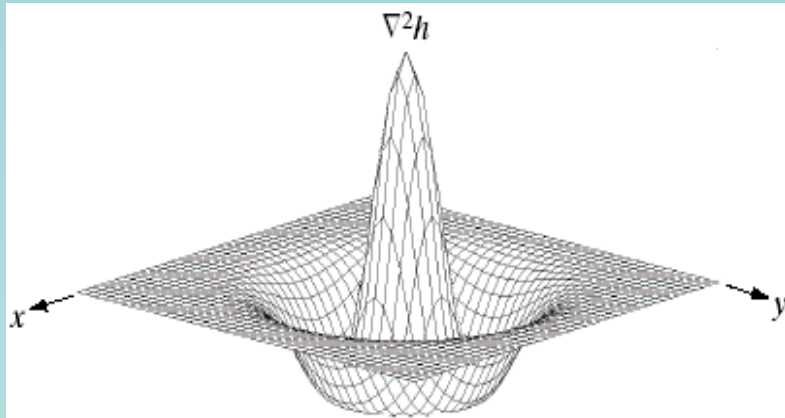
Smoothing before Laplacian (2nd order derivative)

$$\nabla^2 h(r) = \nabla^2 h(x, y) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

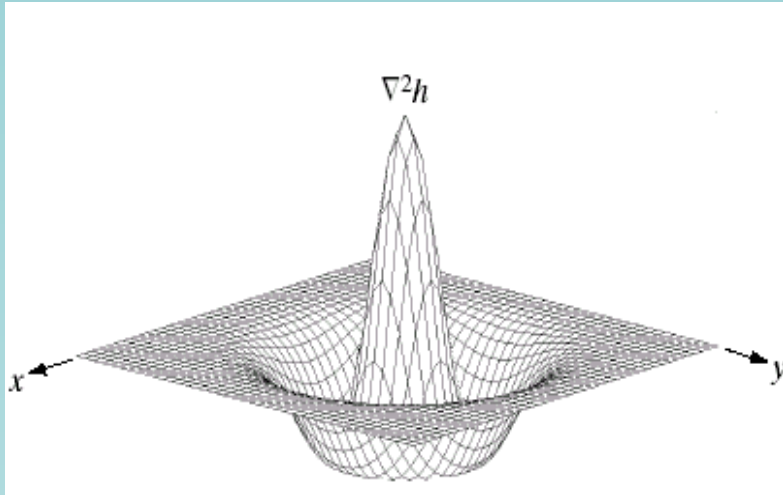


Smoothing before Laplacian (2nd order derivative)

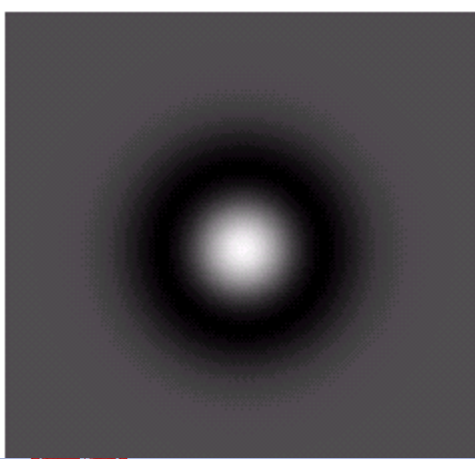
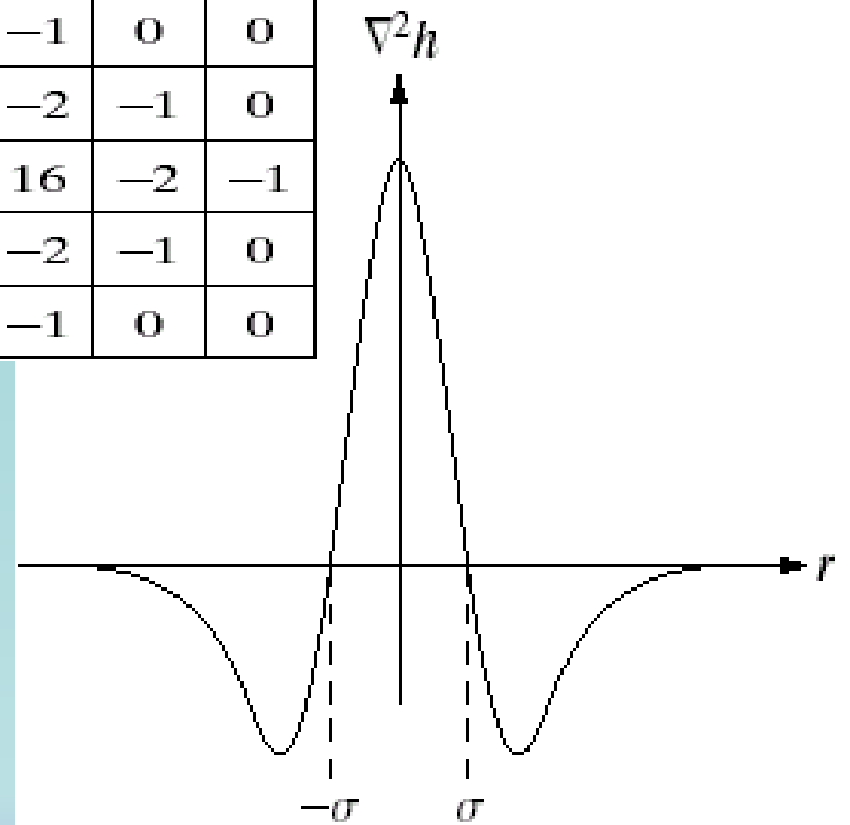
$$\nabla^2 h(r) = \nabla^2 h(x, y) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$



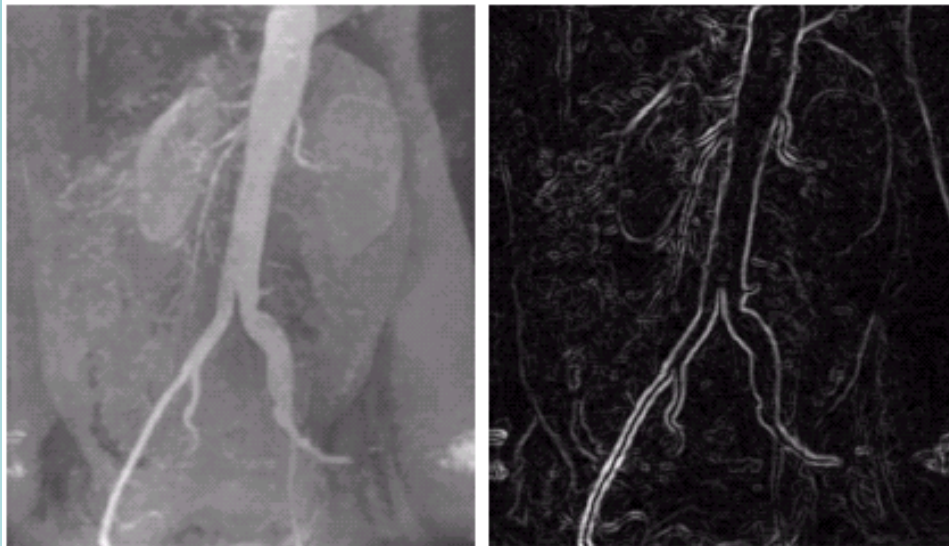
Smoothing before Laplacian (2nd order derivative)



0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0



Example of Laplacian Edge Detection (1)

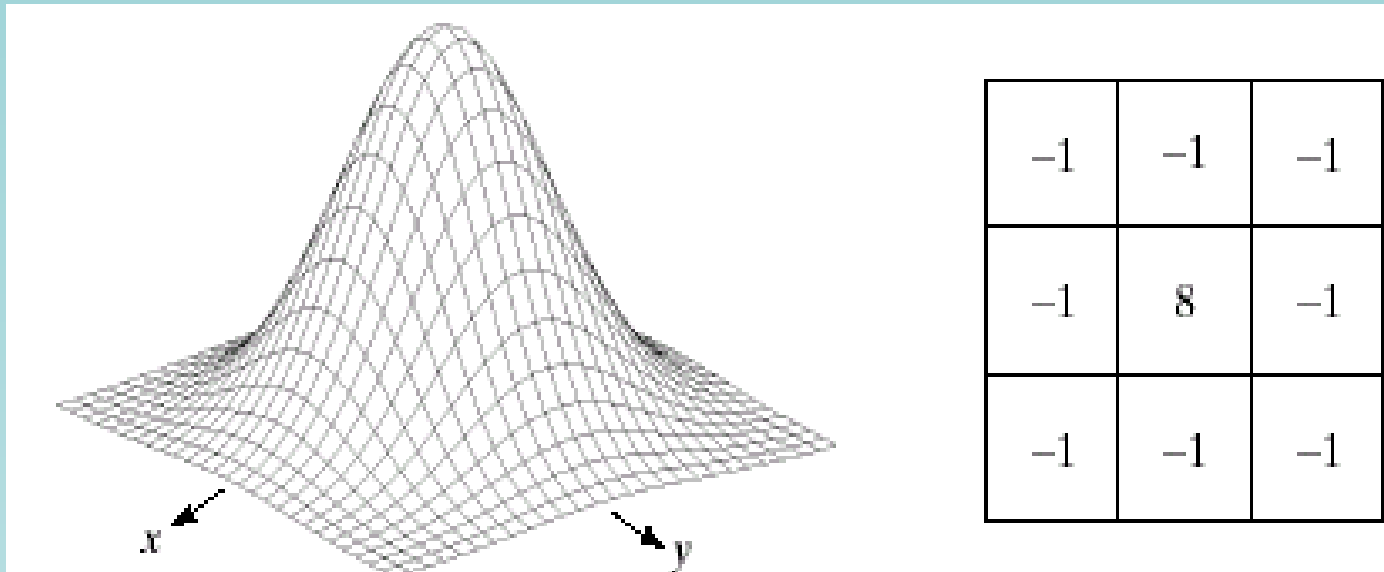


Original Image

After Sobel-ing



Example of Laplacian Edge Detection (1)

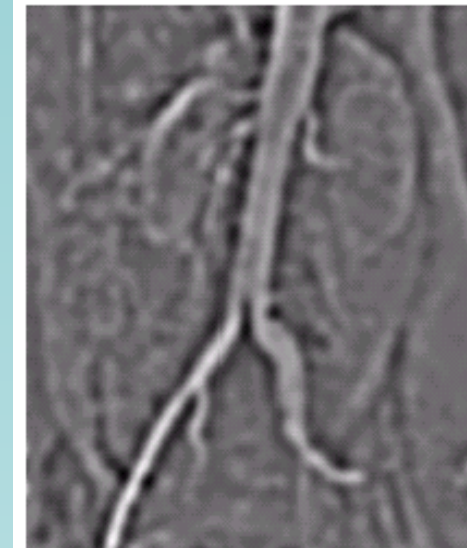


Gaussian function for
smoothing

Laplacian for edging



Example of Laplacian Edge Detection (1)



Applying 1) LoG

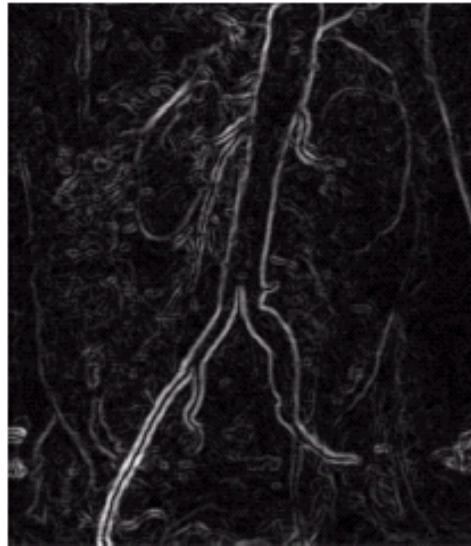
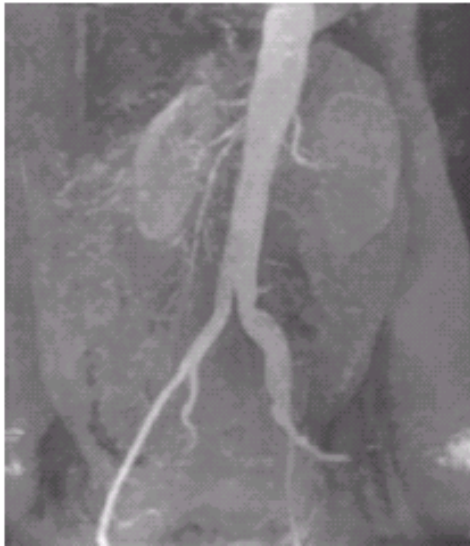
Or

2) Gauss, then Laplacian

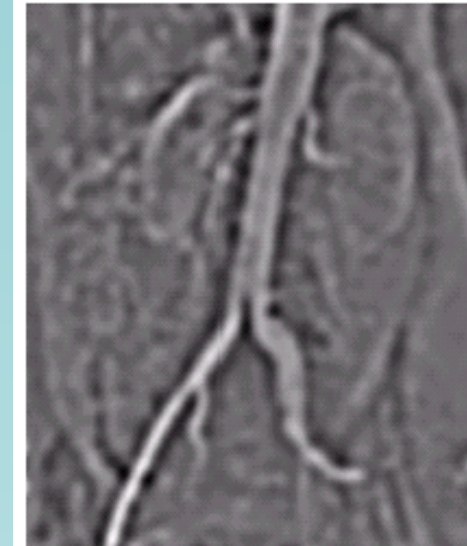


CSE-BUET

Example of Laplacian Edge Detection (1)



With Sobel



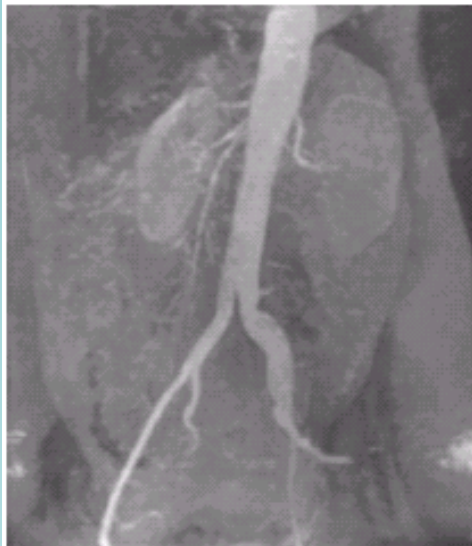
Applying 1) LoG

Or

2) Gauss, then Laplacian



Example of Laplacian Edge Detection (1)



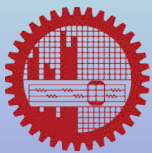
With Sobel



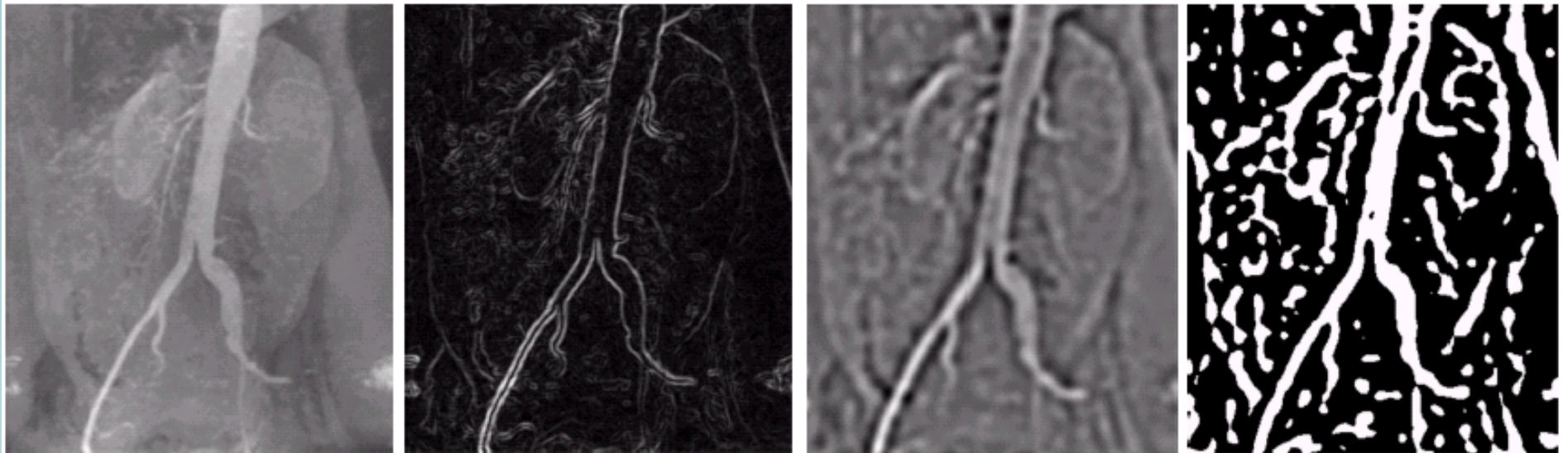
With LoG



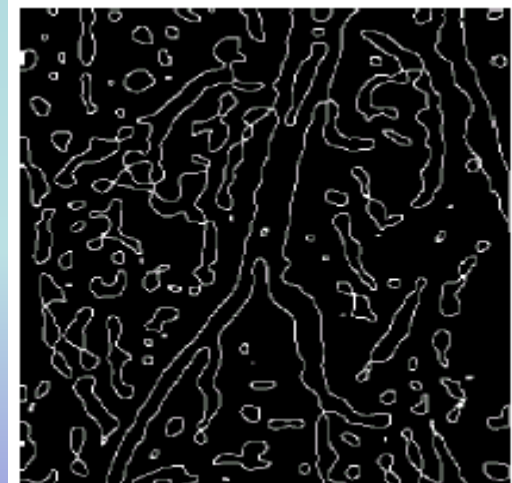
Thresholded



Example of Laplacian Edge Detection (1)



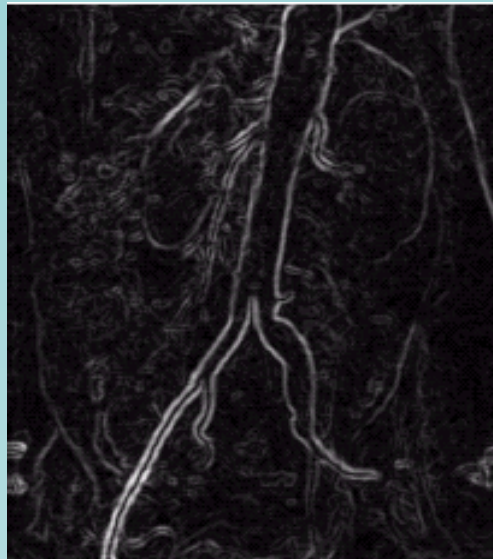
Zero crossing



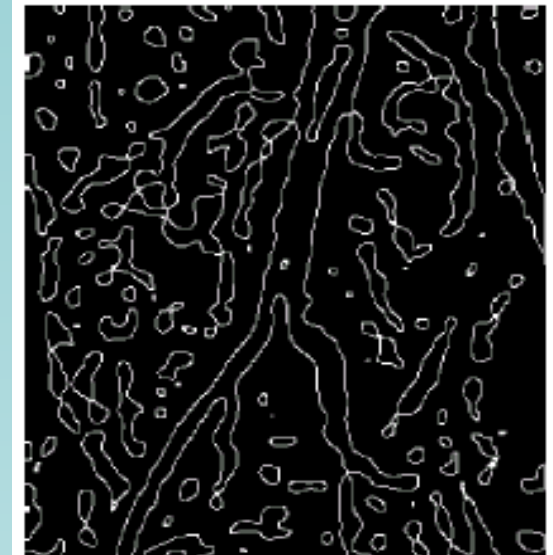
Example of Laplacian Edge Detection (1)



Original
Image



Using Sobel

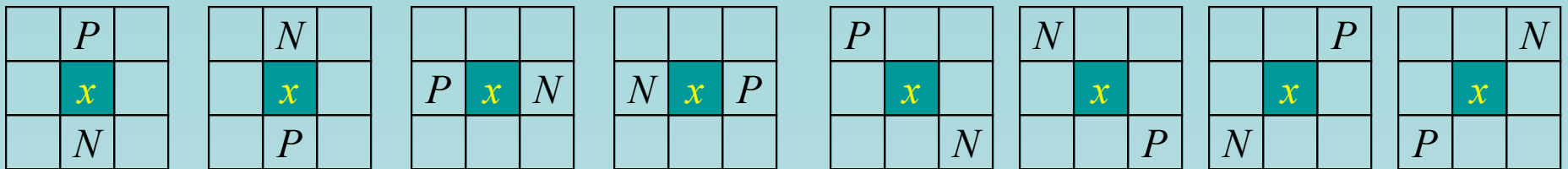


Zero crossing



Improvement of Laplacian Edge Detection

- Simultaneous Thresholding and Detection of Zero crossing



Probable Zero Crossing Pixel

- x is a *zero crossing* pixel if
 - $\text{Abs}(N - P) > T$
 - $\text{Gary vale}(x) > T$

P: Positive LoG value
N: Negative LoG value

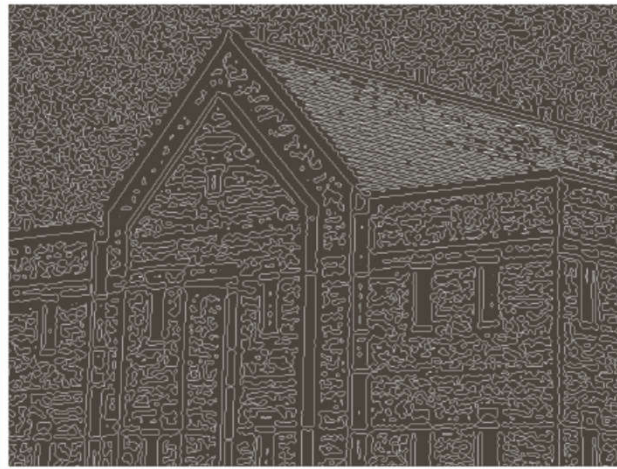
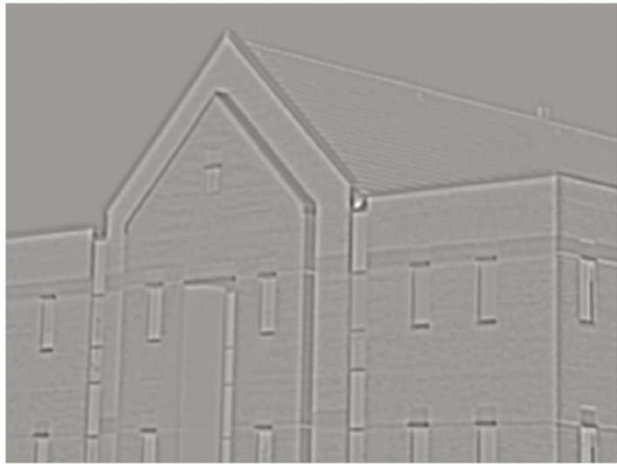


Example of Laplacian Edge Detection (2)

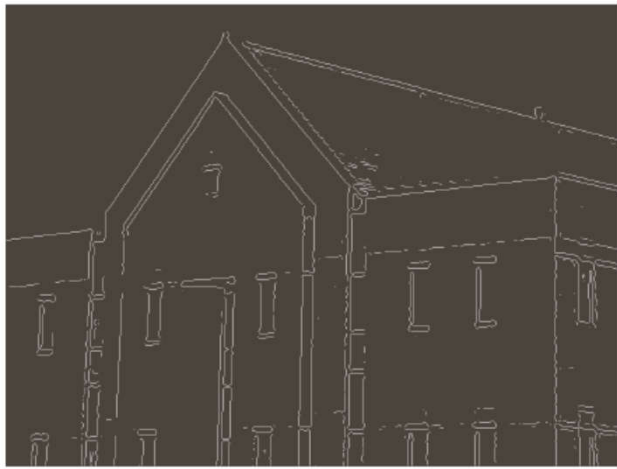
Original
Image



LoG Image



Zero crossing with
 $T = 0$



Zero crossing with
 $T = 4\%$ of Max

