CSE6706:
Advanced Digital Image Processing

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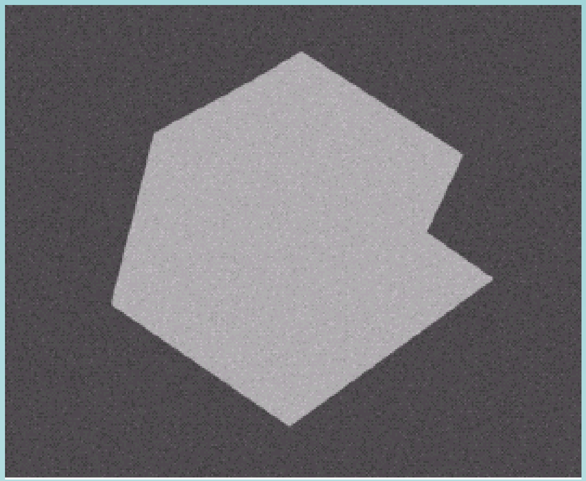
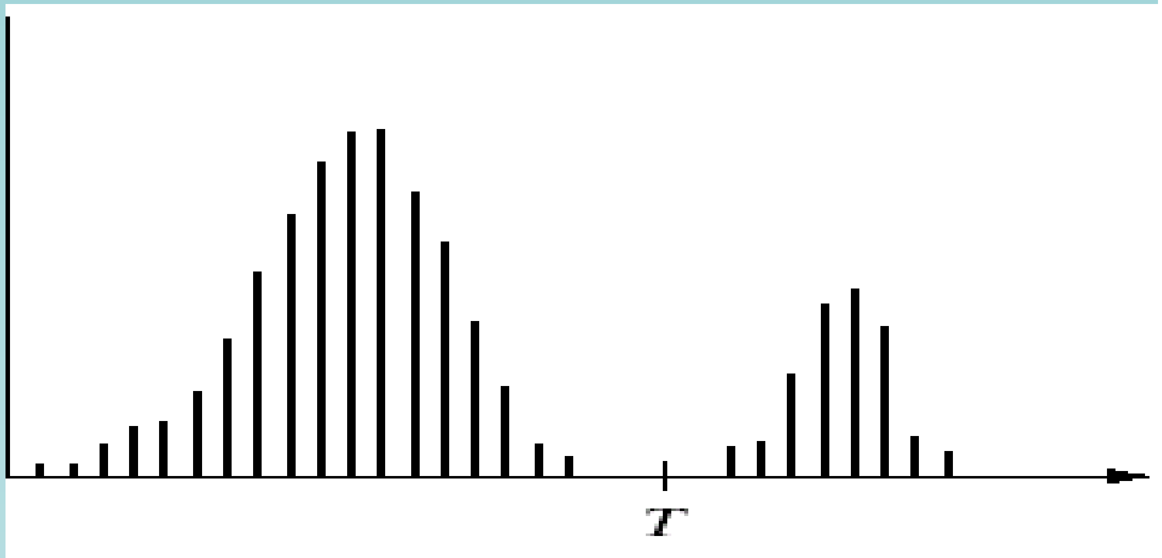
CSE-BUET

Thresholding





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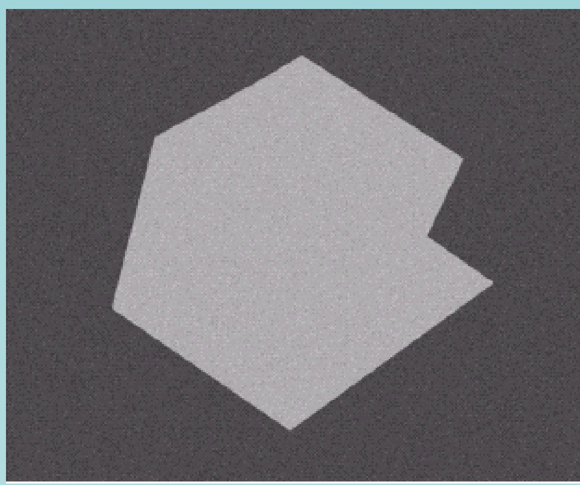
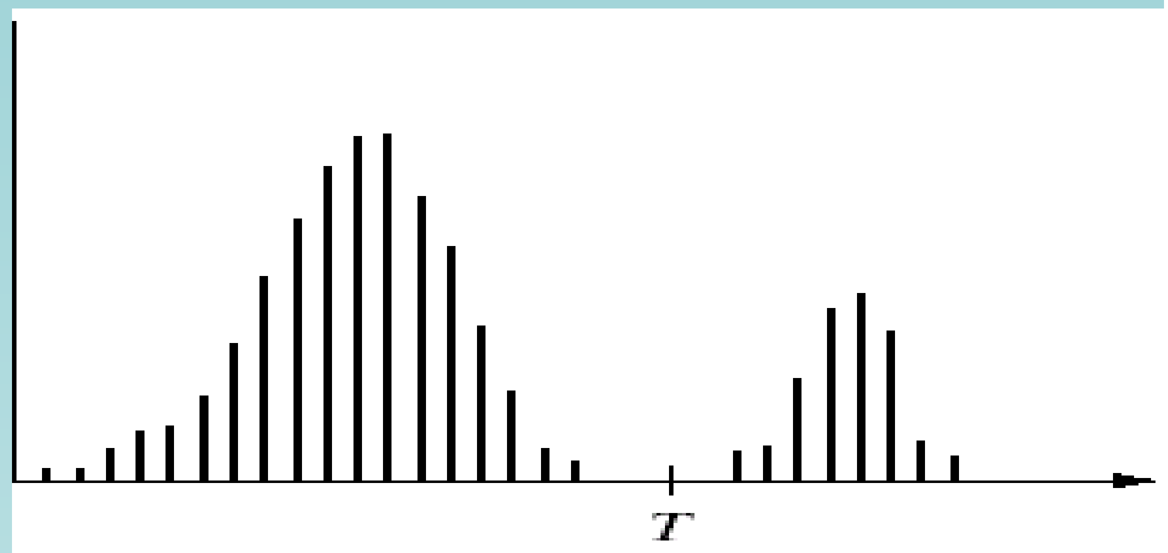
Thresholding





Segment the image with the function

f(*x*, *y*) > *T*: Object

f(*x*, *y*) <= *T*: Background

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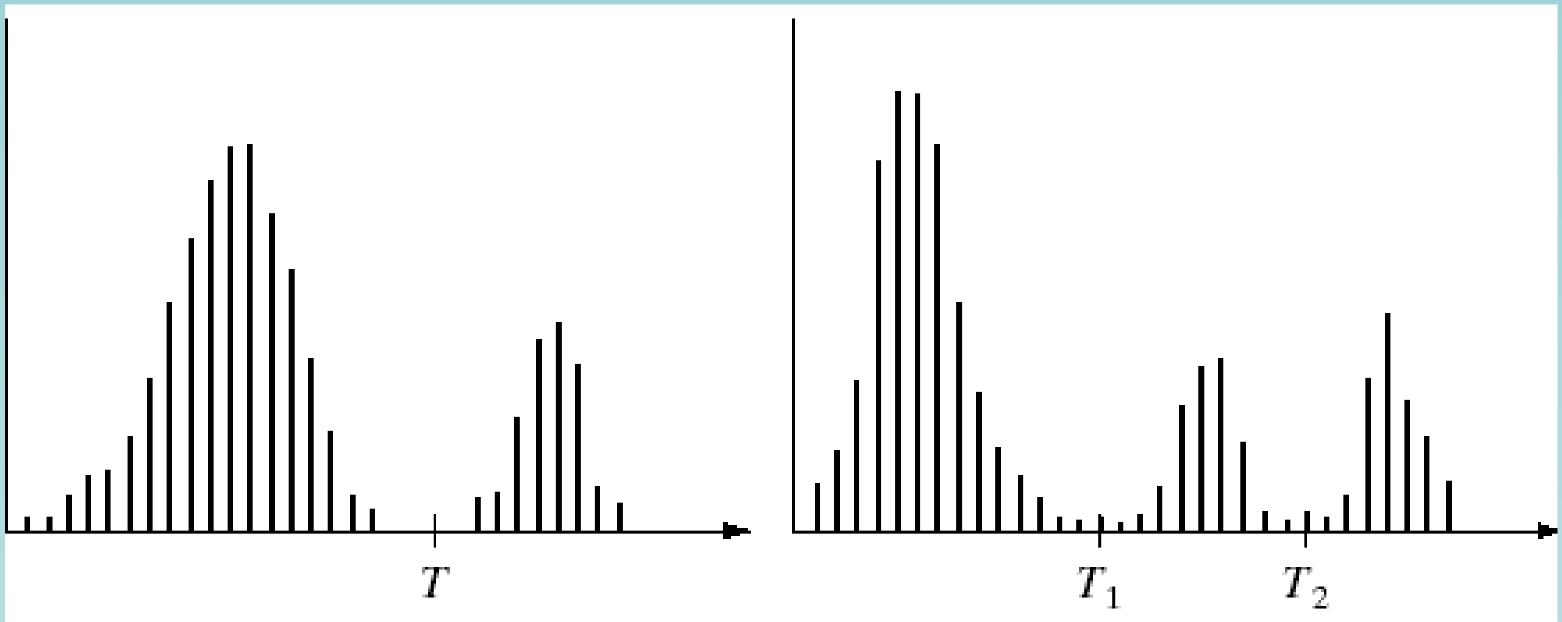
Thresholding



Multi-level thresholding:

$T_1 < f(x, y) < T_2$: Object 1

$f(x, y) > T_2$: Object 2

Otherwise: Background

Generalized Thresholding Equation

$$T = T[x, y, p(x, y), f(x, y)]$$

$$g(x, y) = \begin{cases} 1 & \text{if } \; f(x, y) > T \\ 0 & \text{if } \; f(x, y) \le T \end{cases}$$

where,

f(*x*, *y*): gray level value at pixel (*x*, *y*)

p(*x*, *y*): neighborhood properties

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Types of Thresholding

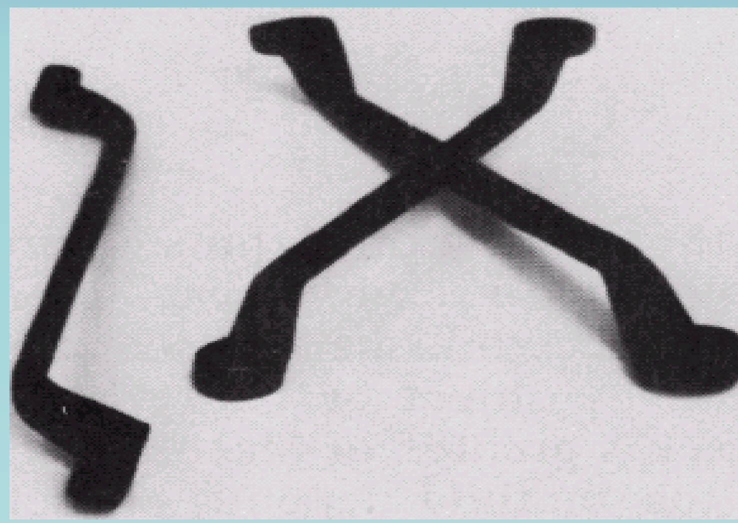
$$T = T[x, y, p(x, y), f(x, y)]$$

$$g(x, y) = \begin{cases} 1 & \text{if } \; f(x, y) > T \\ 0 & \text{if } \; f(x, y) \leq T \end{cases}$$

- Global: *T* depends on only $f(x,y)$
- Local: *T* depends on $f(x,y)$ and $p(x,y)$
- Adaptive: *T* depends on x, y, and $f(x,y)$ and $p(x,y)$

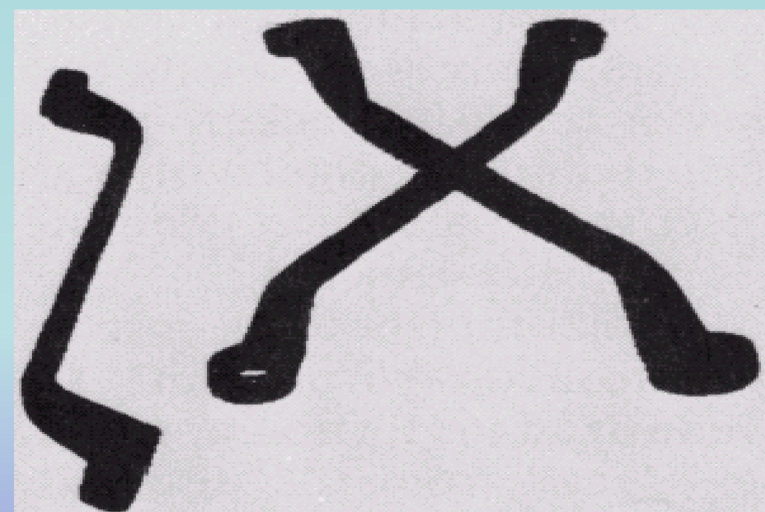
Global Thresholding

Original Image

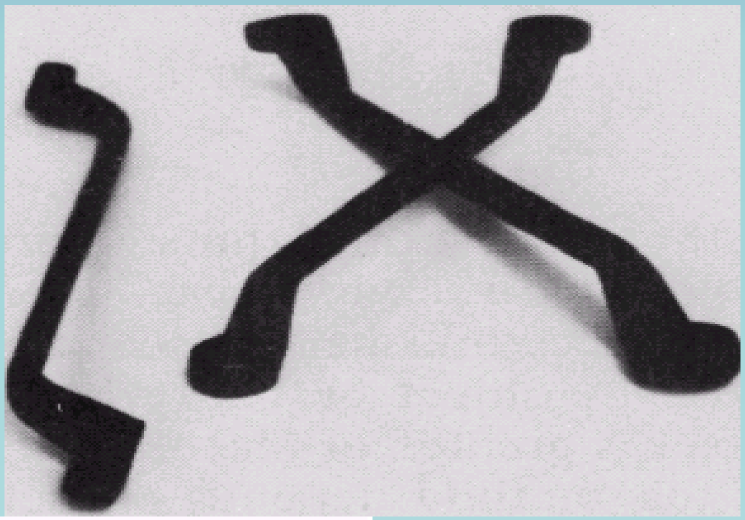


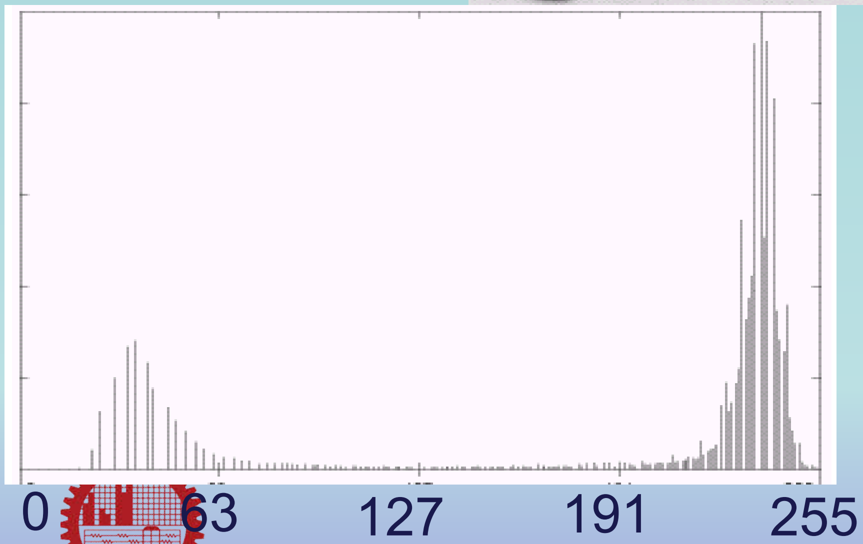
Thresholding

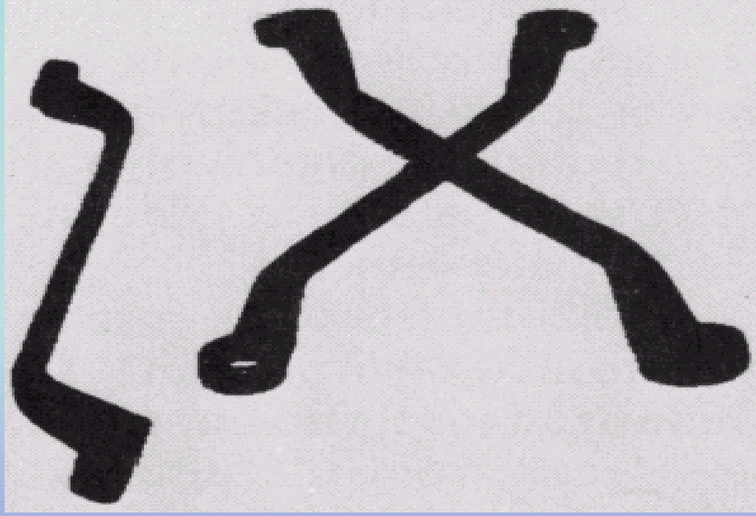
Shadow disappears



Global Thresholding







- *T* is mid point between max and min gray value
- Visual Inspection

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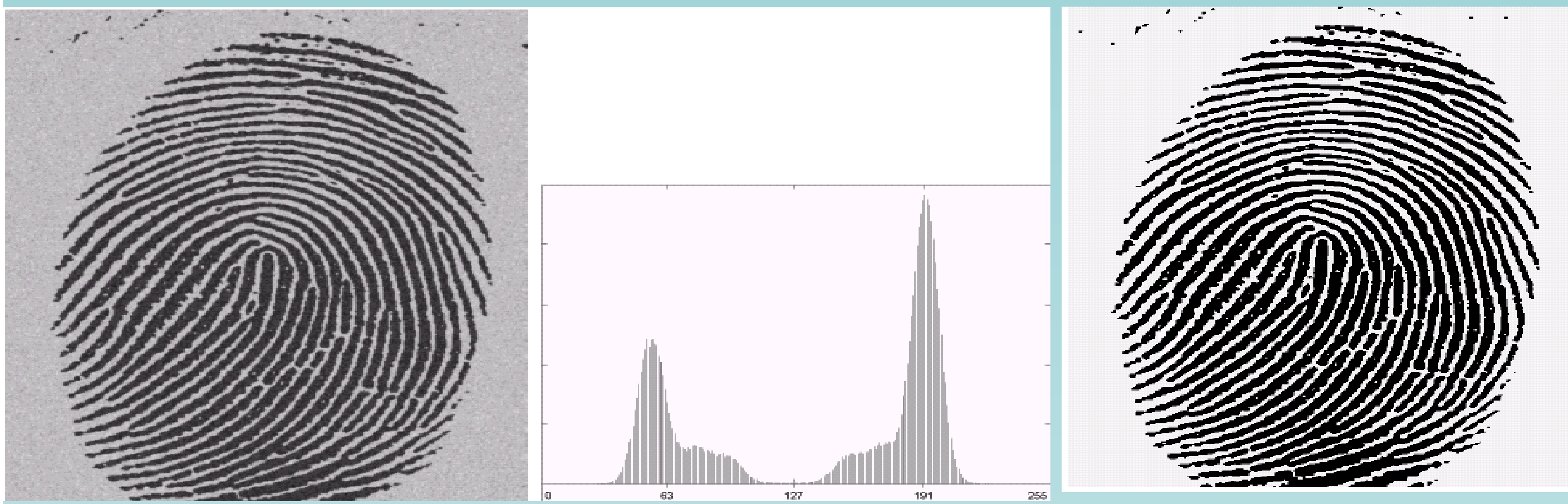
Automatic Determination of Threshold, *T*

1. Assume an initial T
2. Segment image using T
   - Produce two groups of pixels: G_1 and G_2
3. Find avg gray values in G_1 and G_2. let them be μ_1 and μ_2
4. Update T as $T = (\mu_1 + \mu_2)/2$
5. Repeat 2 to 4 until no more significant change of T

Automatic Determination of Threshold, *T*

- Choice of initial *T*
  - Avg gray level
  - Mid point betn max and min gray level

Global Thresholding



T =125.4

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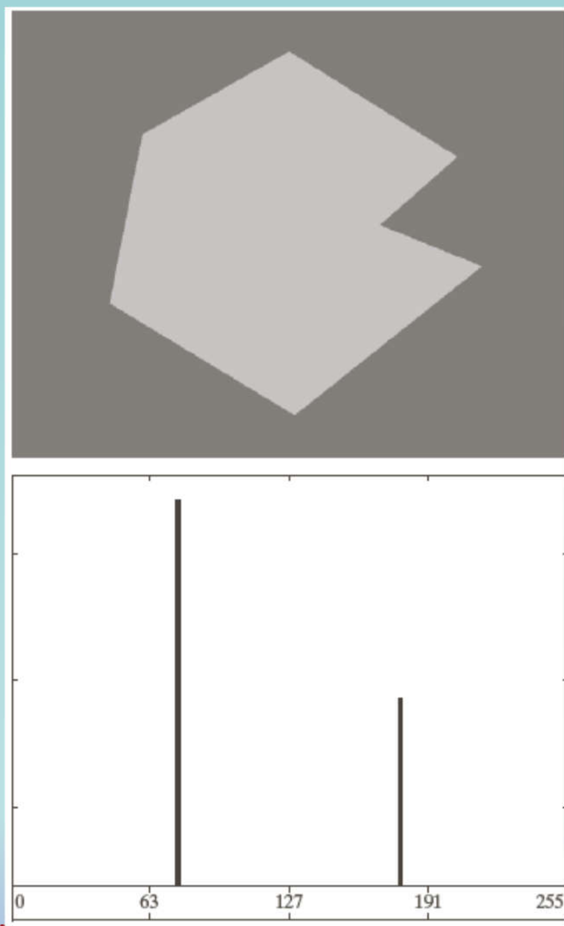
Limitation of Global Thresholding

- Works when background can be controlled
- Fixed illumination
- Low Noise
- Example: Industrial inspection

Role of Noise

No Noise



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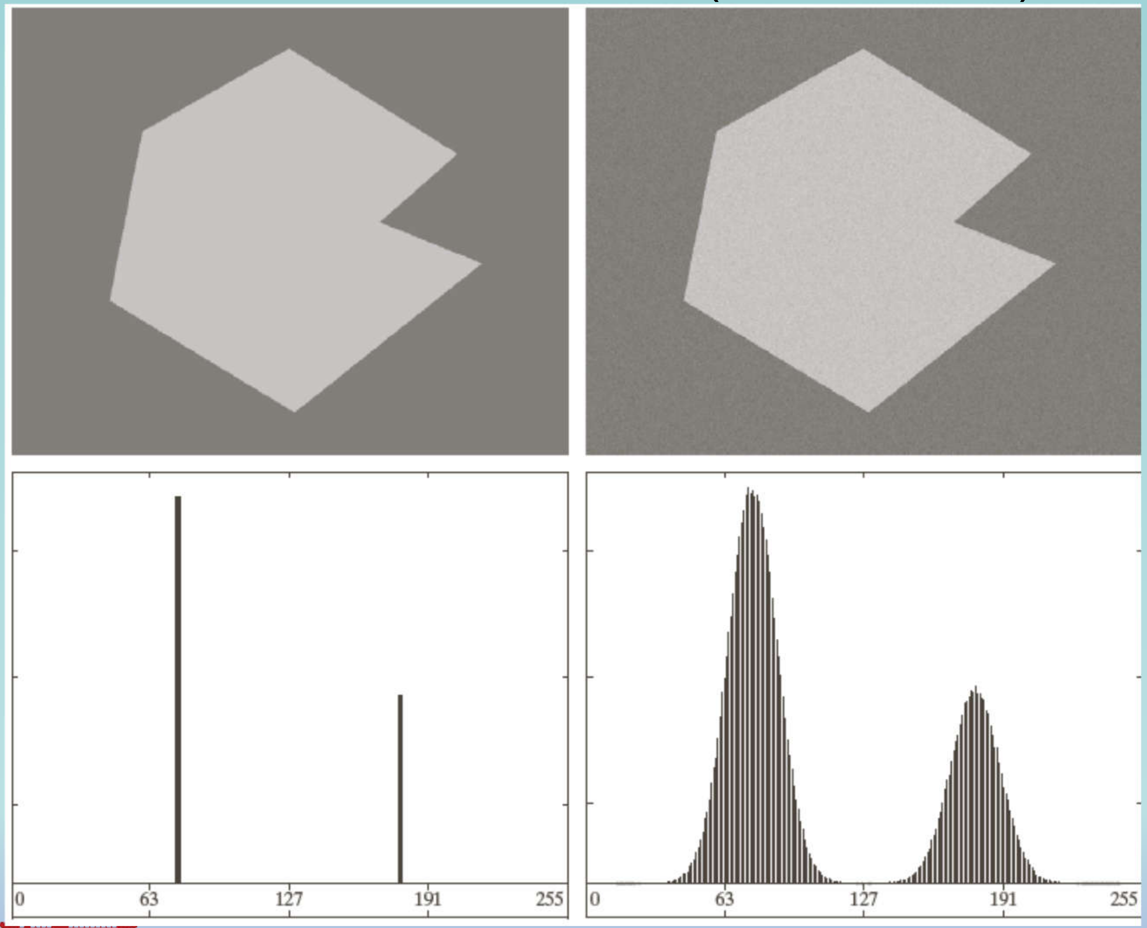
Role of Noise

No Noise

Added Gauss Noise
(m=0, std=10)



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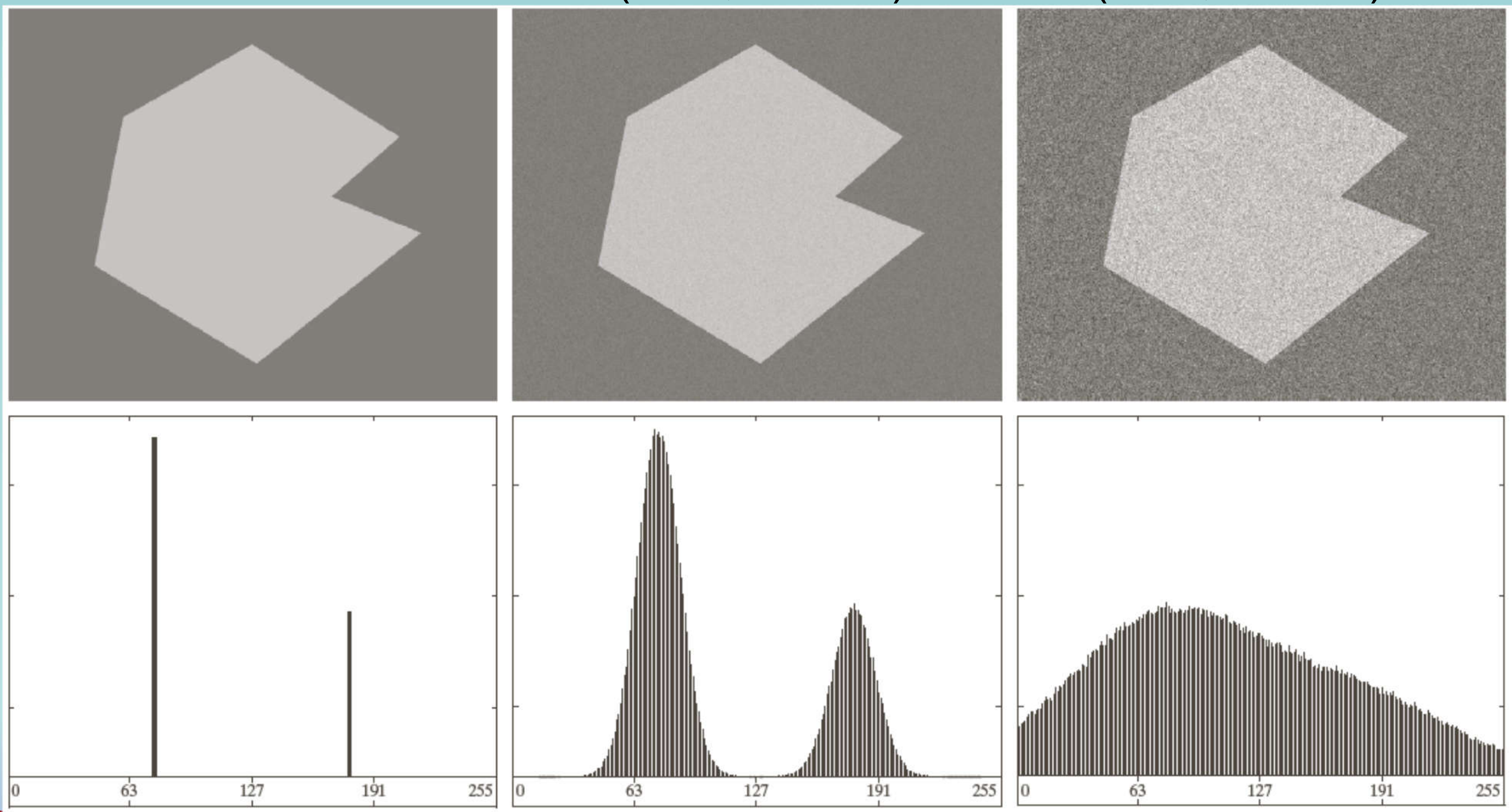
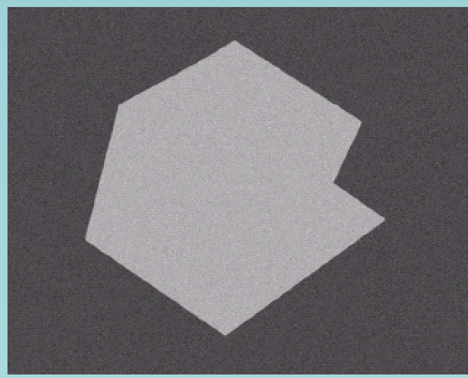
Role of Noise

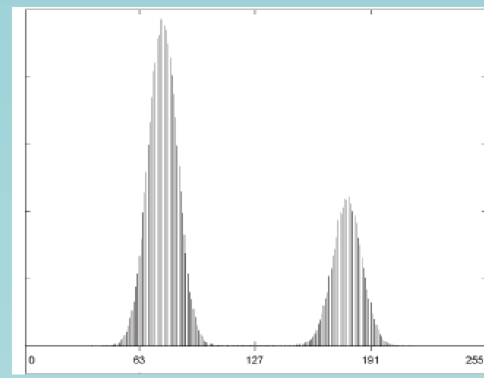
No Noise | Added Gauss Noise (m=0, std=10) | Added Gauss Noise (m=0, std=50)



Role of Illumination

Original image





Original histo.

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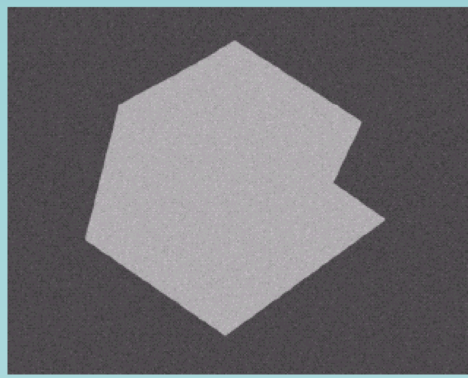
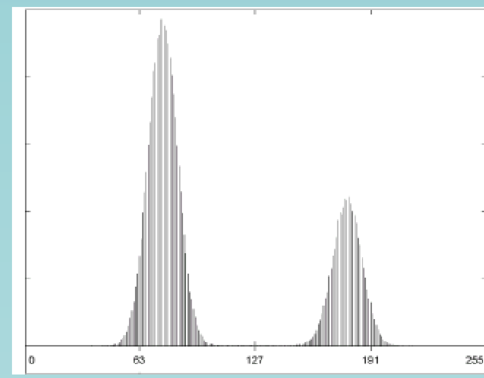
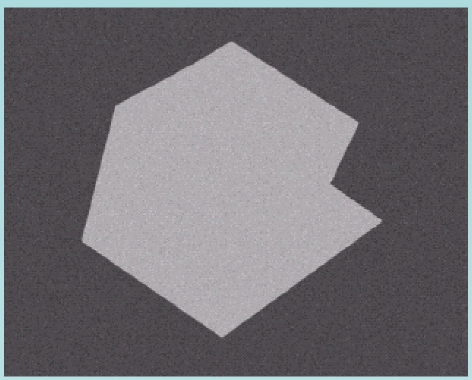
Role of Illumination

Original image

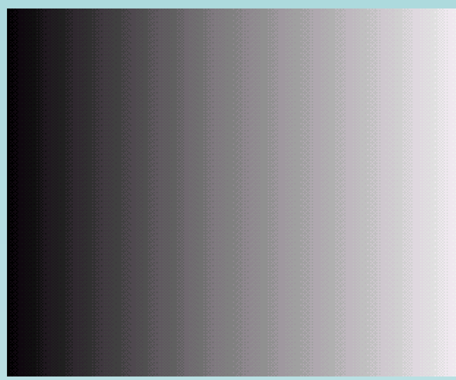




Original histo.



X



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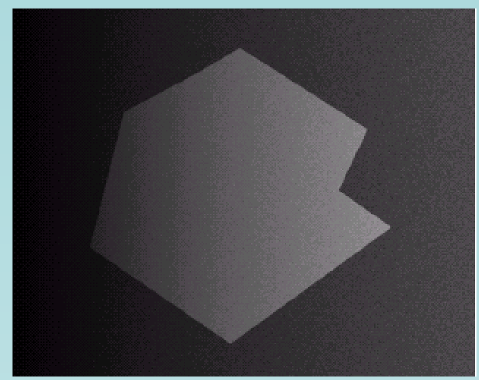
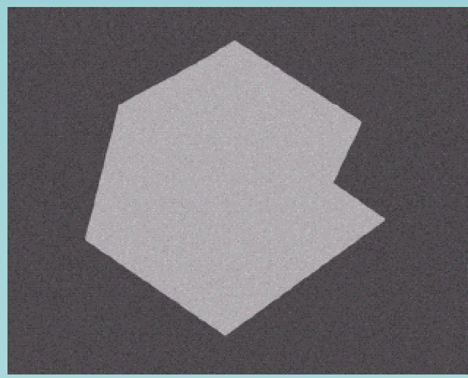


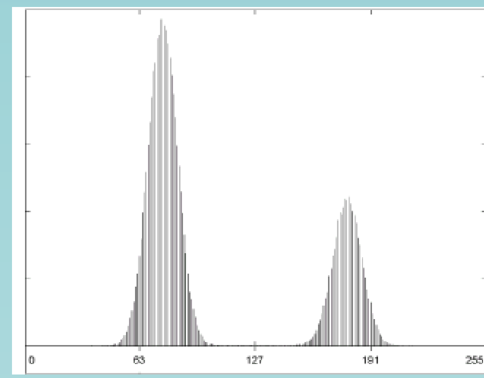
Original image

Varying illumination

Role of Illumination

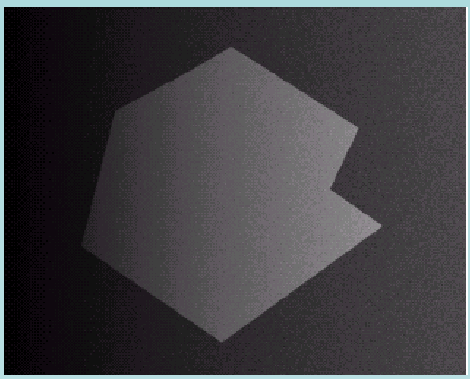
Original image

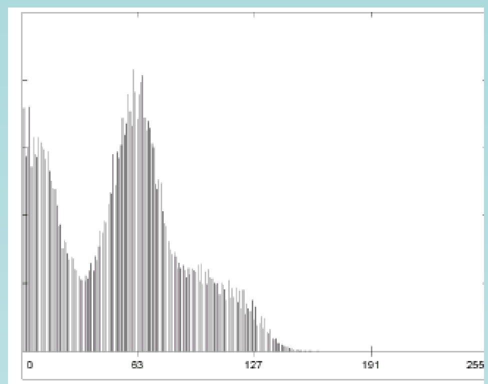




Original histo.

Image with variable illumination





Changed histogram

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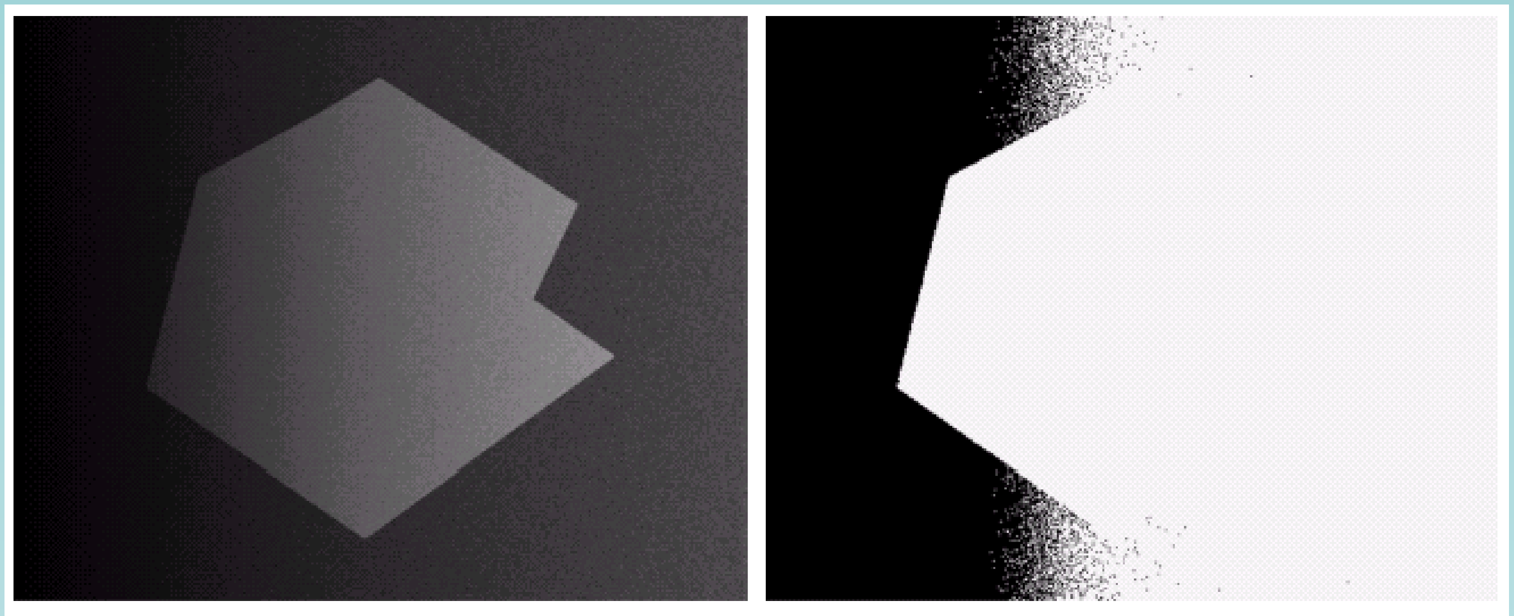
Global Thrsholding of image with non-uniform illumination



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Eliminating Illumination Effect

- Illumination normalization
- Adaptive thresholding

Eliminating Illumination Effect by Normalization

- We know from image formation model,

$$f(x, y) = i(x, y)r(x, y)$$

Eliminating Illumination Effect by Normalization

- We know from image formation model,

$$f(x, y) = i(x, y)r(x, y)$$

- If illumination source is accessible
  - Project the source on a constant white surface to generate

$$g(x, y) = ki(x, y)$$

Eliminating Illumination Effect by Normalization

- We know from image formation model,

$$f(x, y) = i(x, y)r(x, y)$$

- If illumination source is accessible
  - Project the source on a constant white surface to generate

$$g(x, y) = ki(x, y)$$

- Normalize the image, $h(x, y) = f(x, y) / g(x, y) = r(x, y) / k$

Eliminating Illumination Effect by Normalization

- We know from image formation model,

$$f(x,y)=i(x,y)r(x,y)$$

- If illumination source is accessible
  - Project the source on a constant white surface to generate

$$g(x,y)=ki(x,y)$$

- Normalize the image, $h(x,y)=f(x,y)/g(x,y)=r(x,y)/k$

- If $r(x,y)$ can be segmented by a global threshold, so we can $h(x,y)$

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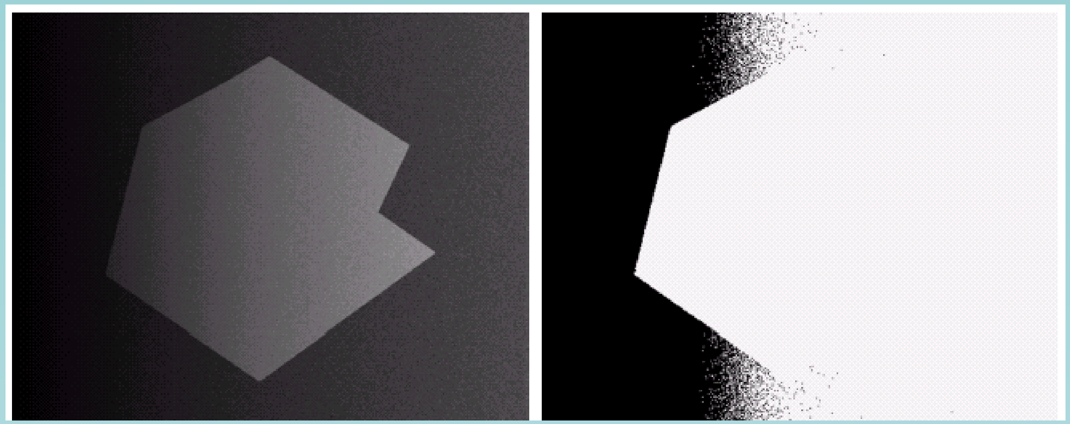
Adaptive Thresholding



- Divide the image into sub-regions
- Apply thresholding separately in each region

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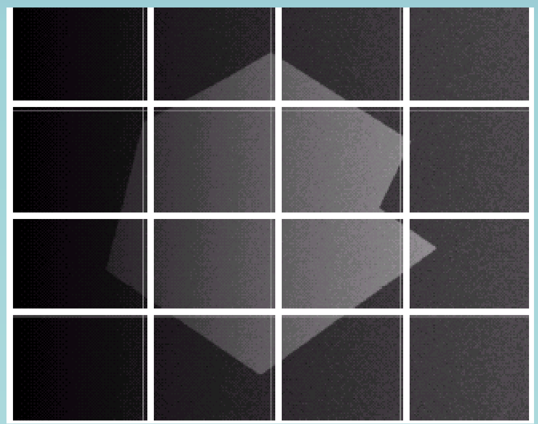
Adaptive Thresholding



- Some regions: no distinct boundary btn object and background
- most others: have distinct boundary

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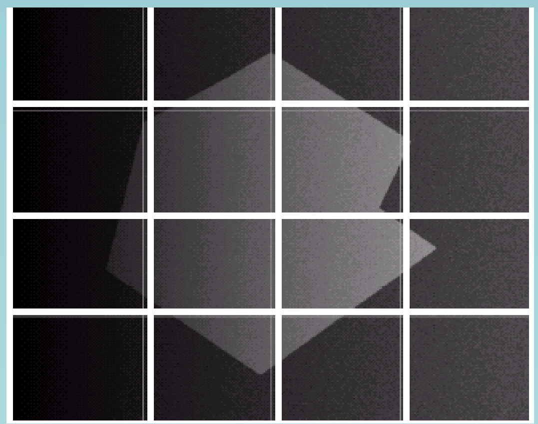
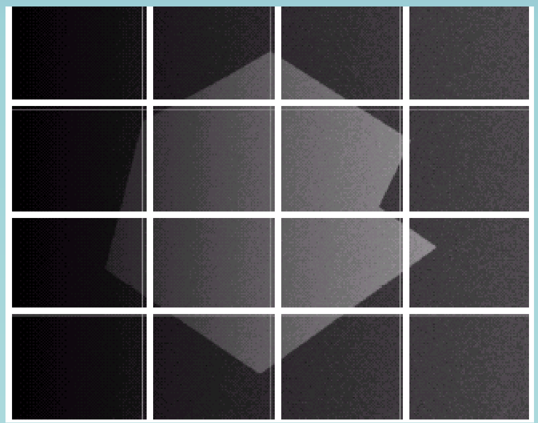
Adaptive Thresholding



- Some regions: no distinct boundary btn object and background - gray level variance ~75
- most others: have distinct boundary - gray level variance >100

Adaptive Thresholding



- Regions with gray level variance >100:
  - clearly have bimodal histograms
  - Find threshold separately for each region
  - Segment each of them separately

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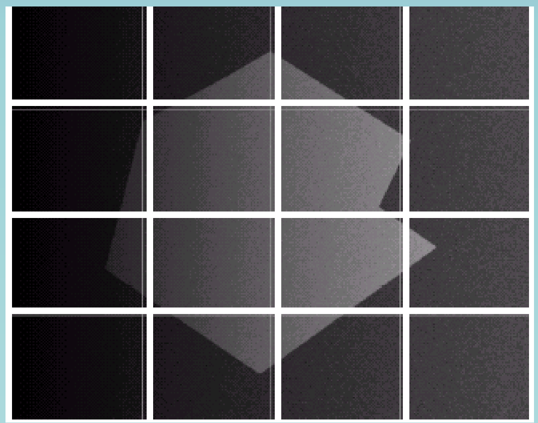
Adaptive Thresholding



- Regions with gray level variance <100:
  - have unimodal histograms
  - Combine all regions
  - Find a single threshold for all
  - Segment them with the threshold

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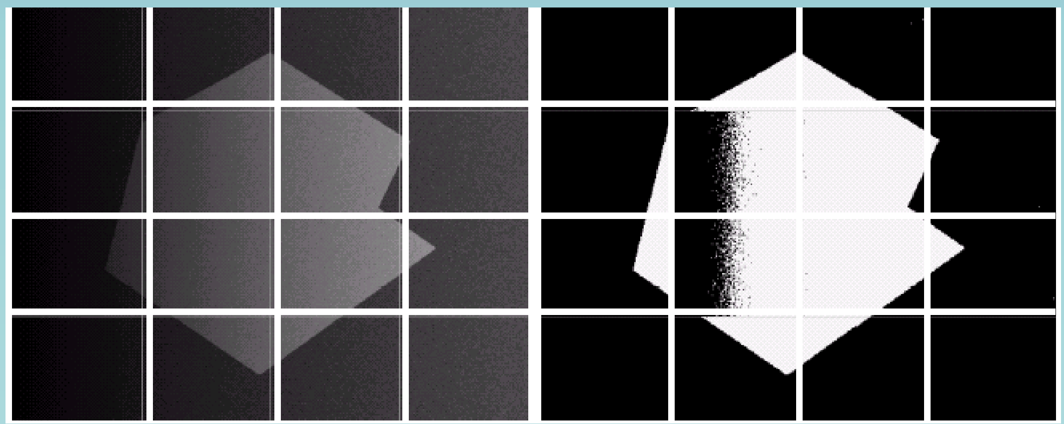
Adaptive Thresholding



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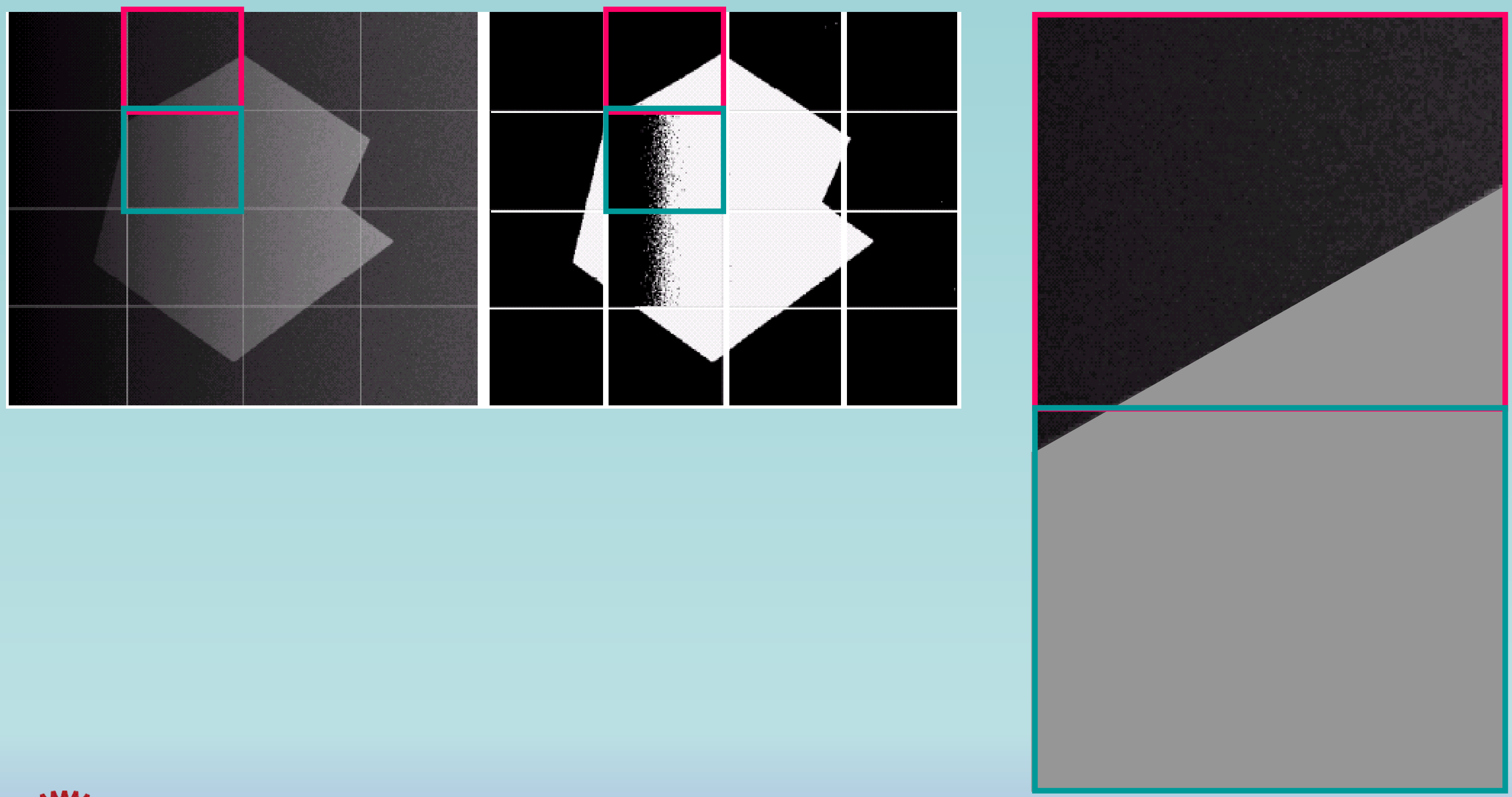
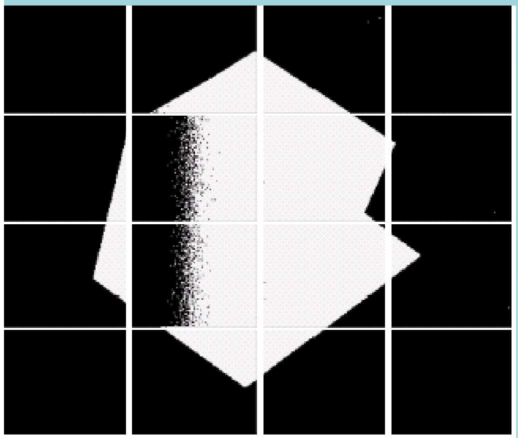
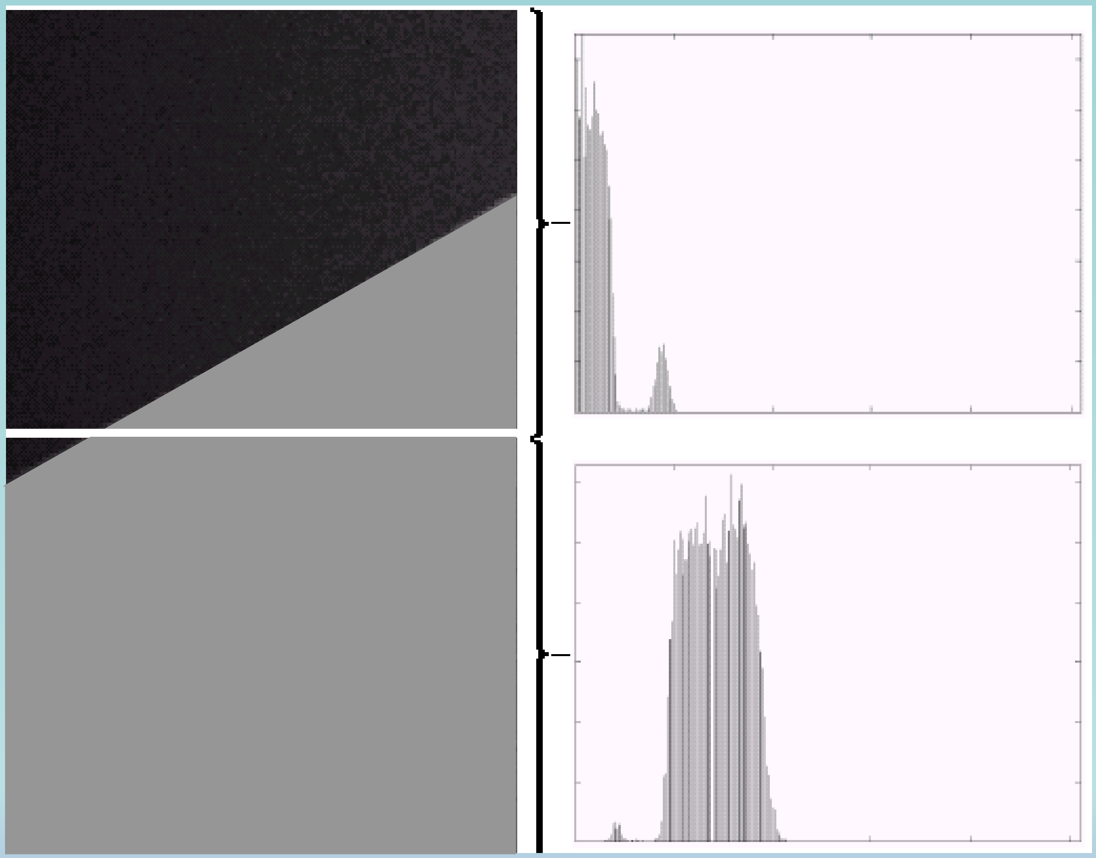
Adaptive Thresholding



Adaptive Thresholding





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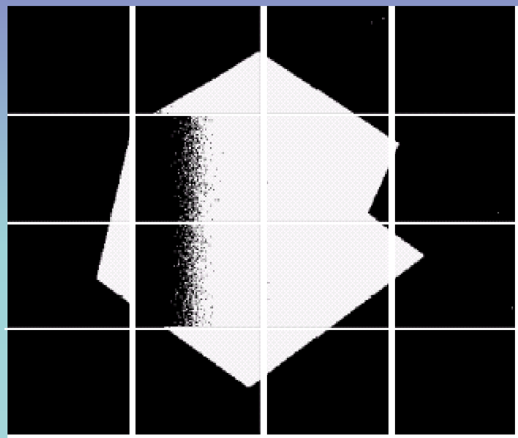
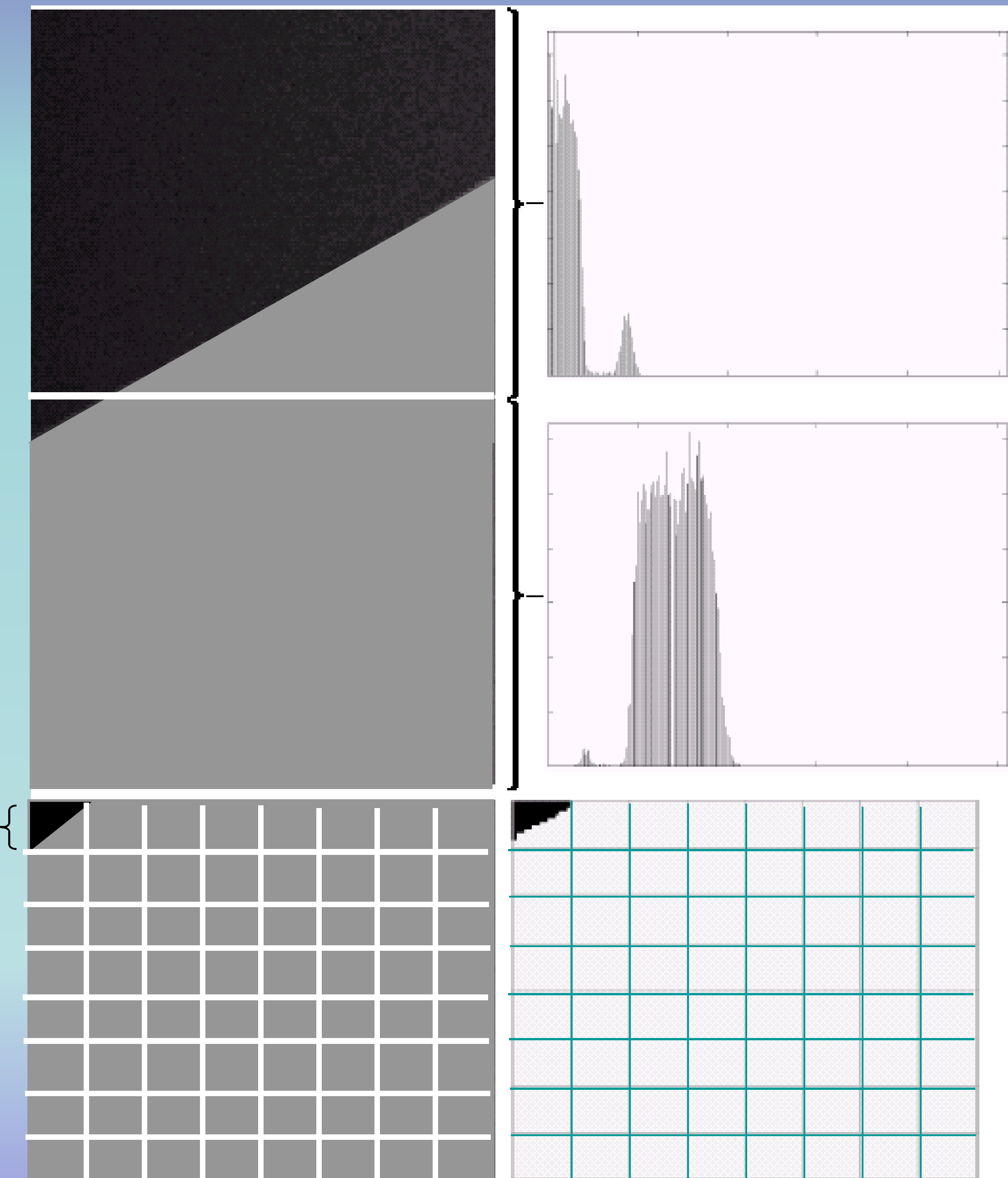
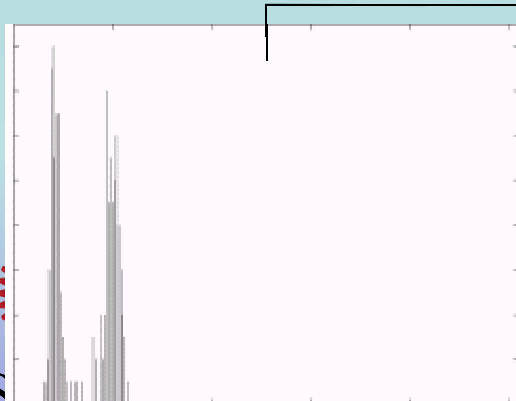






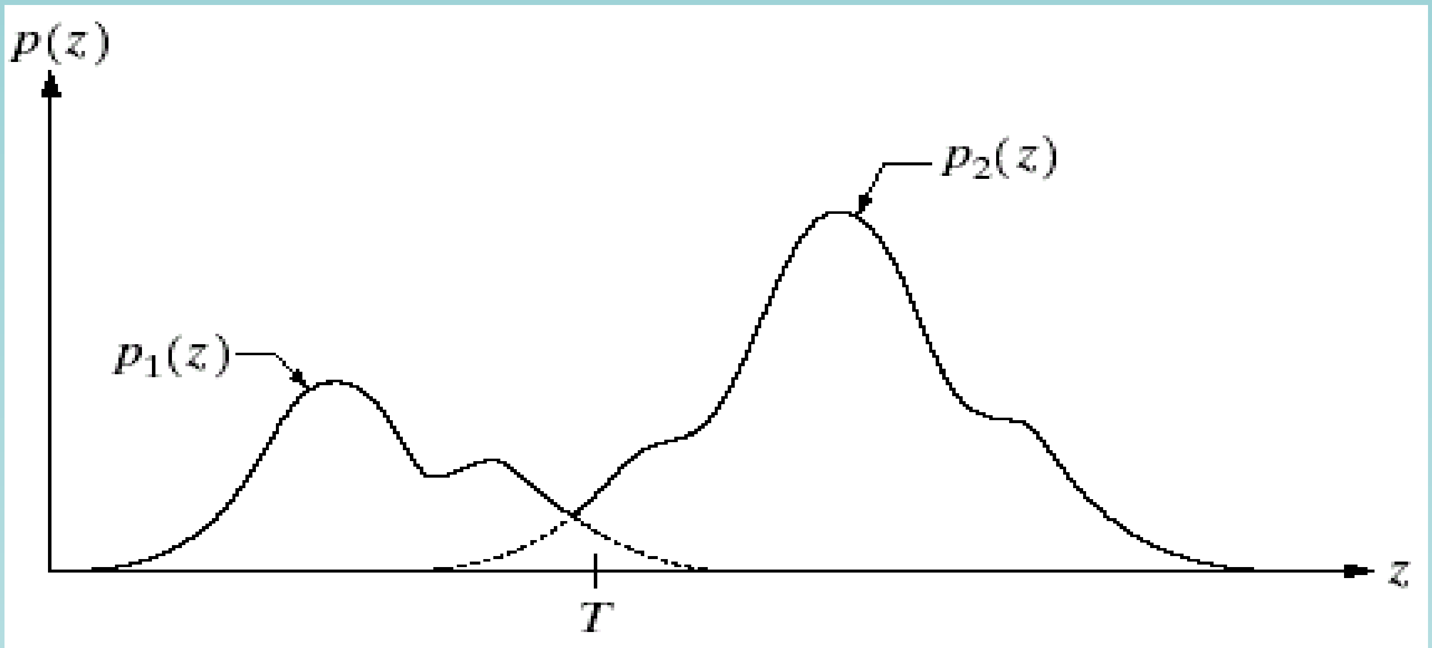
CS

Optimal Thresholding

- Objective: Determine the optimal adaptive threshold which minimizes the segmentation error

- Background:
  - Gray level z can be assumed a random variable
  - Histogram $p(z)$ or $h(z)$ can assumed as *pdf* of z

Optimal Thresholding



- Two histograms in an image
  - $p_2(z)$: for background
  - $p_1(z)$: for object
  - $p(z)$: for overall pdf, $p(z) = P_1 p_1(z) + P_2 p_2(z)$

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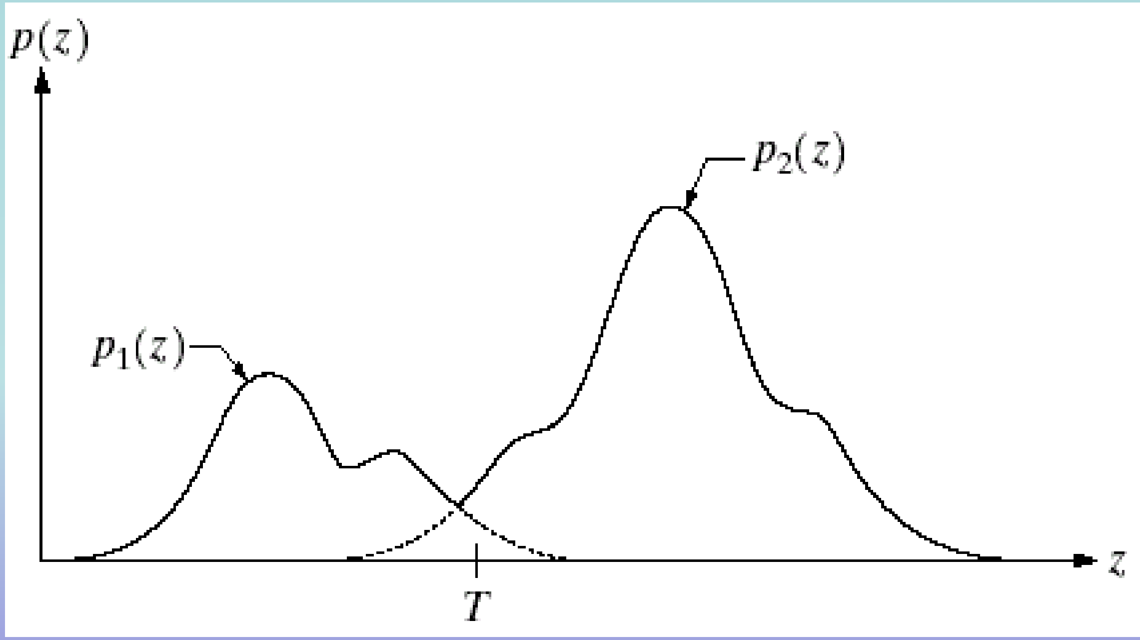
Optimal Thresholding

- If we know the form of $p_1(z)$ and $p_2(z)$, we can find optimal T that can segment the image



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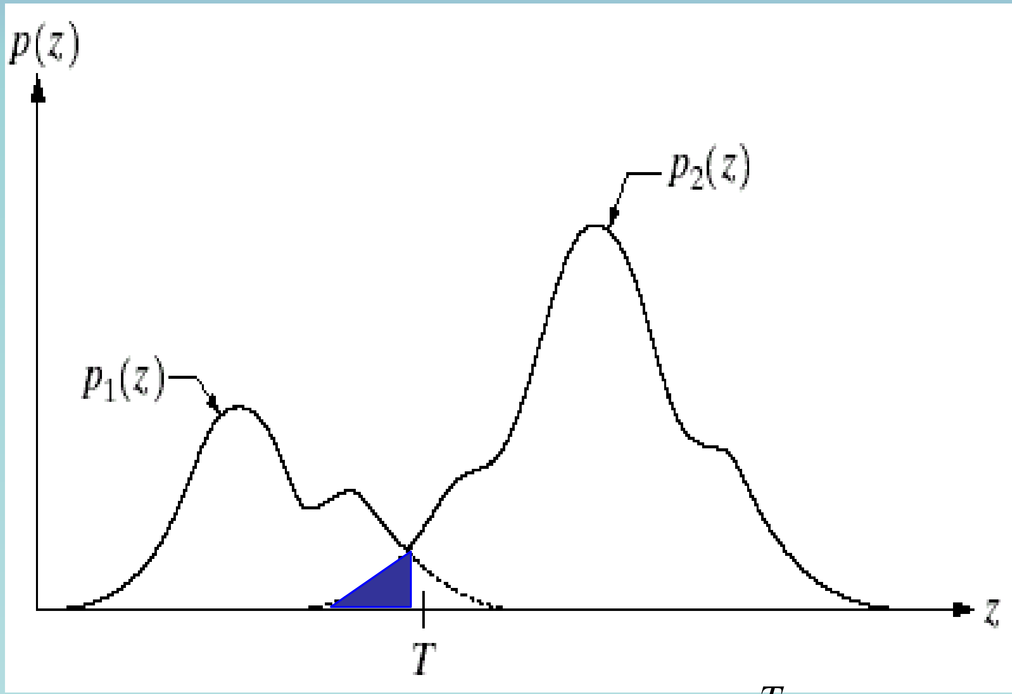
Minimizing segmentation Error



- Error1:
  - background classified as Object: $E_1(T) = \int_{-\infty}^{T} p_2(z)dz$

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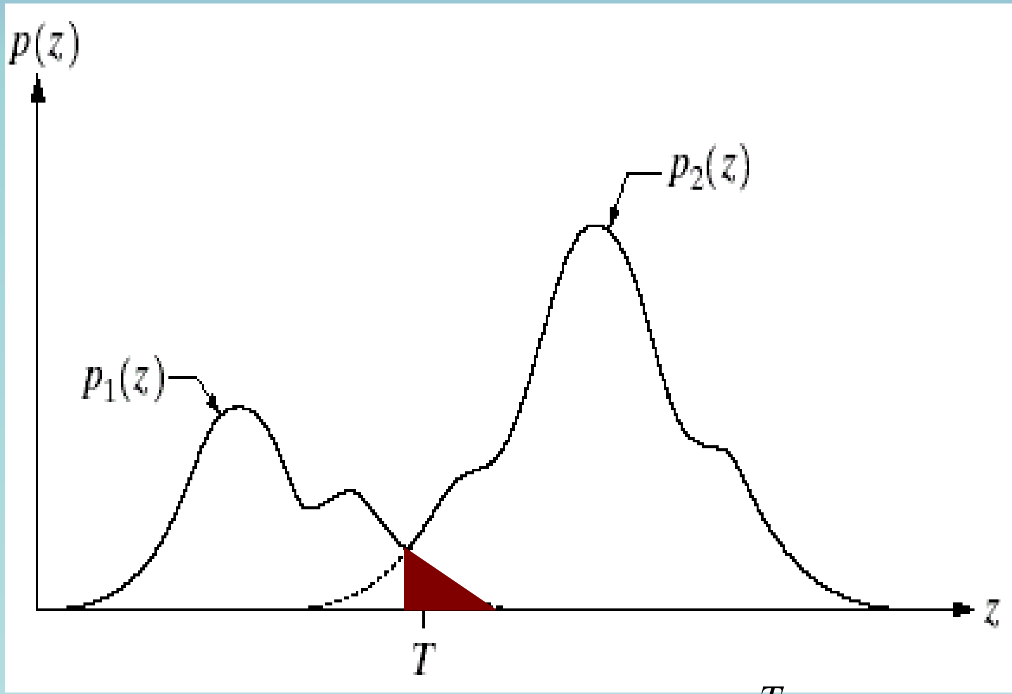
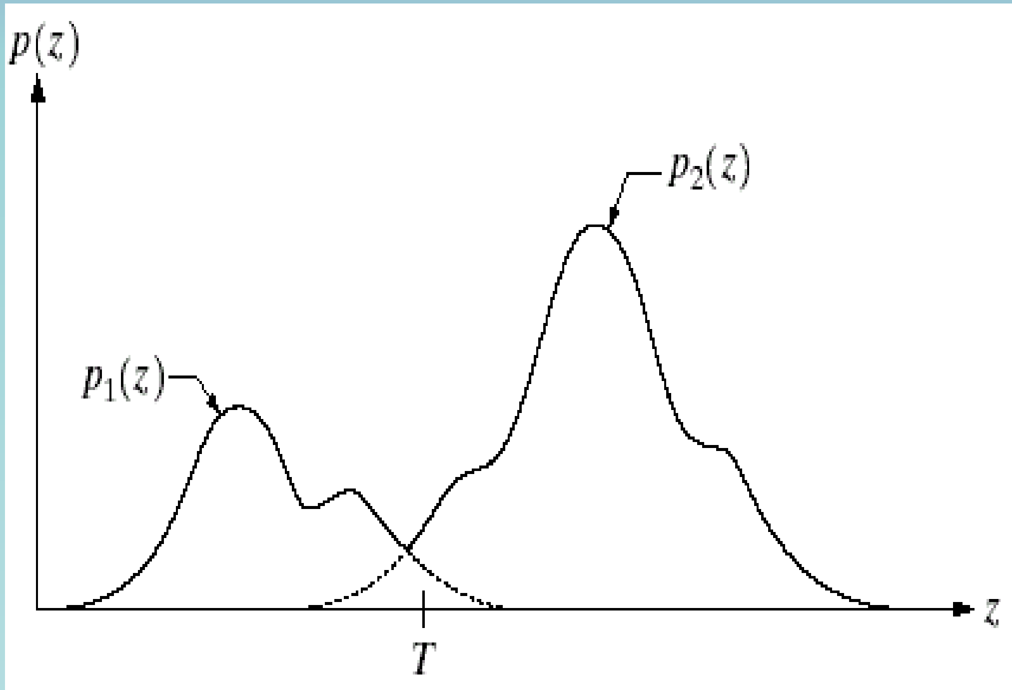
Minimizing segmentation Error



- Error1:
  - background classified as Object: $E_1(T) = \int_{-\infty}^{T} p_2(z)dz$
- Error2:
  - object classified as background: $E_2(T) = \int_{T}^{\infty} p_1(z)dz$

Minimizing segmentation Error



- Overall Error:

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$
$$= \int_{-\infty}^{T} P_2 p_2(z) dz + \int_{T}^{\infty} P_1 p_1(z) dz$$

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Minimizing segmentation Error

- Overall Error:

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$
$$= \int_{-\infty}^{T} P_2 p_2(z) dz + \int_{T}^{\infty} P_1 p_1(z) dz$$

- Minimize $E(T)$ by Leibniz's formula:

$$P_1 p_1(T) = P_2 p_2(T)$$

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Minimizing segmentation Error

- If we know the form of p1(z) and p2(z)
  - Let they are Gaussian

$$p_1(z) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(z-\mu_1)^2}{2\sigma_1^2}} \quad \text{and} \quad p_2(z) = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(z-\mu_2)^2}{2\sigma_2^2}}$$

- The formula: $P_1 p_1(T) = P_2 p_2(T)$ turns into

$$AT^2 + BT + C = 0$$

$$A = \sigma_1^2 - \sigma_2^2$$

$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$

$$C = \mu_2^2\sigma_1^2 - \mu_1^2\sigma_2^2 + 2\sigma_1^2\sigma_2^2 \ln\left(\sigma_2 P_1 / \sigma_1 P_2\right)$$

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Minimizing segmentation Error

- We will find *two* *T* from

$$AT^2 + BT + C = 0$$

$$A = \sigma_1^2 - \sigma_2^2$$
$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$
$$C = \mu_2^2\sigma_1^2 - \mu_1^2\sigma_2^2 + 2\sigma_1^2\sigma_2^2 \ln\left(\sigma_2 P_1 / \sigma_1 P_2\right)$$

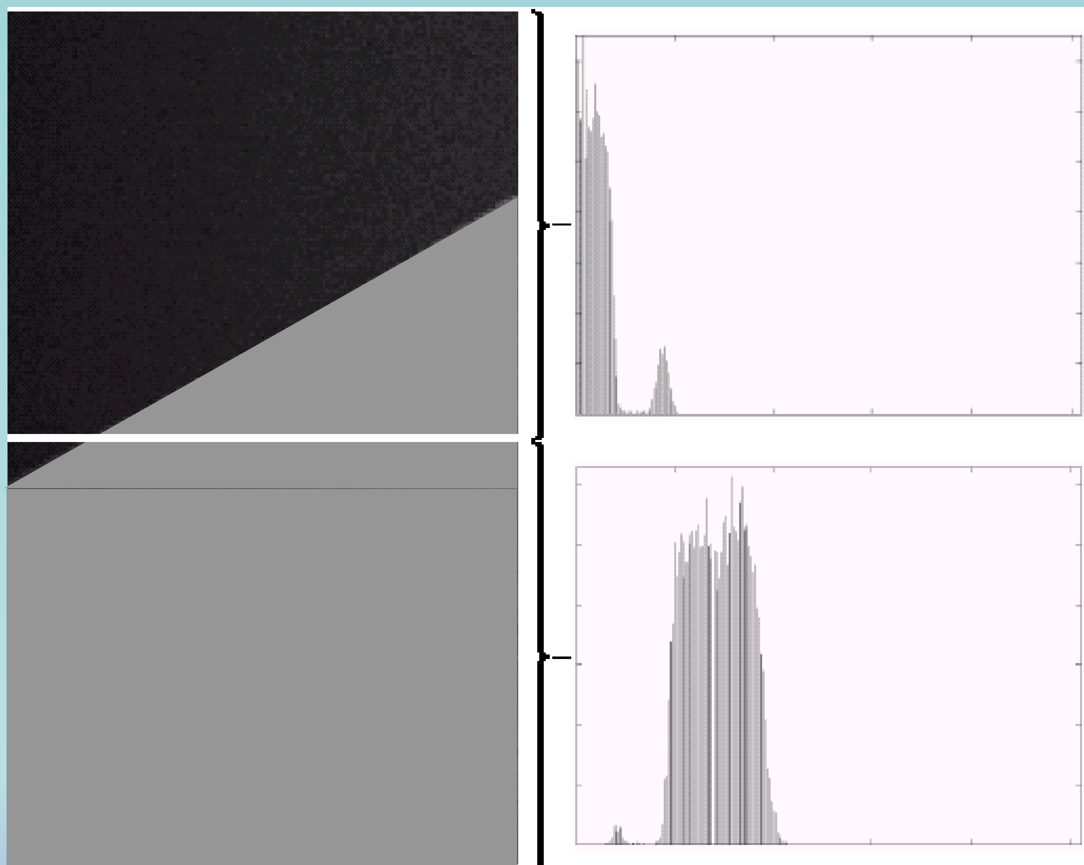
- If $\sigma_1^2 = \sigma_2^2 = \sigma^2$, only a single T is sufficient

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2}\ln\left(P_2 / P_1\right)$$

Good Threshold: When Possible?

- When histogram peaks are
  - tall
  - narrow
  - Symmetric and
  - separated by deep valleys

Good Threshold: When Possible?



Sizes of object/background dominate the histogram

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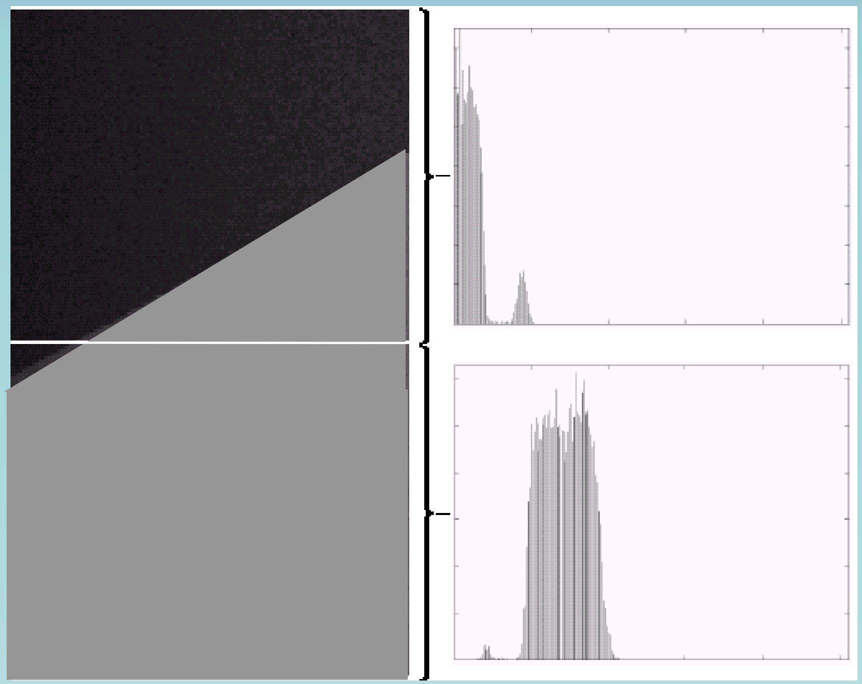
Good Threshold: When Possible?



- Histogram should be less dependant on relative sizes of object and background

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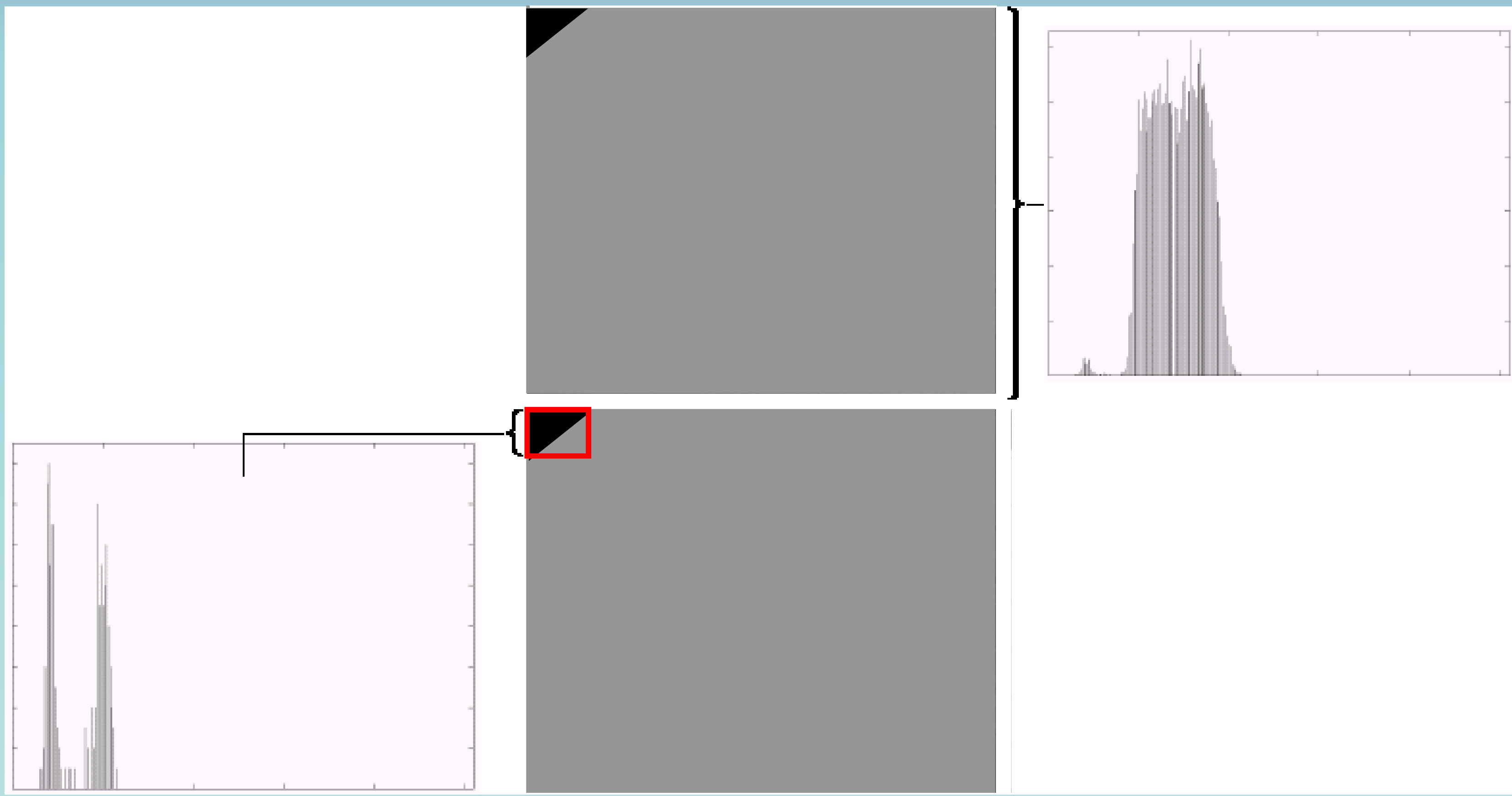
Good Threshold: When Possible?



- Histogram from pixels near the edge can reduce the dominance

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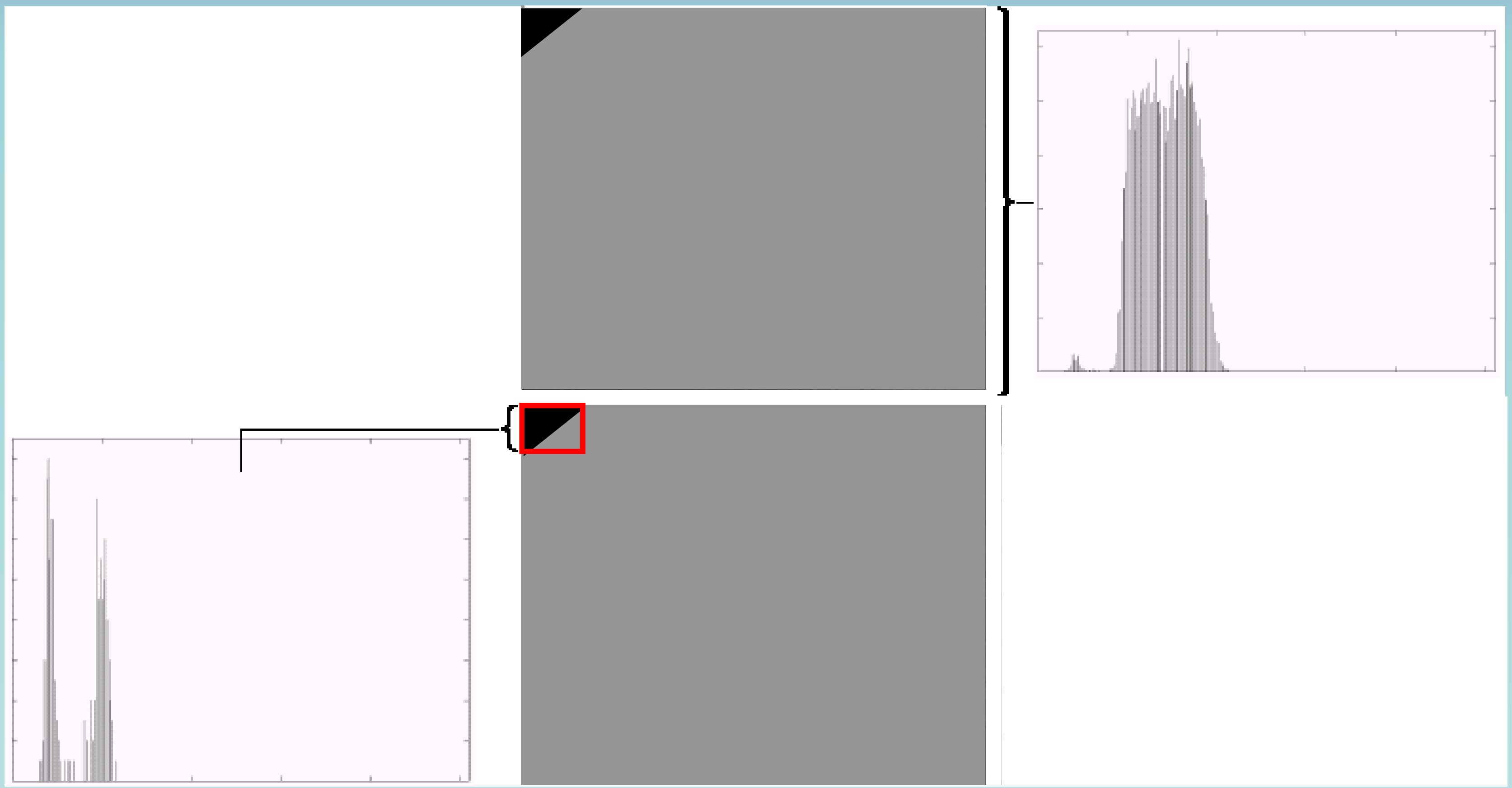
Good Threshold: When Possible?



- More symmetric histogram peaks
- Deep valley

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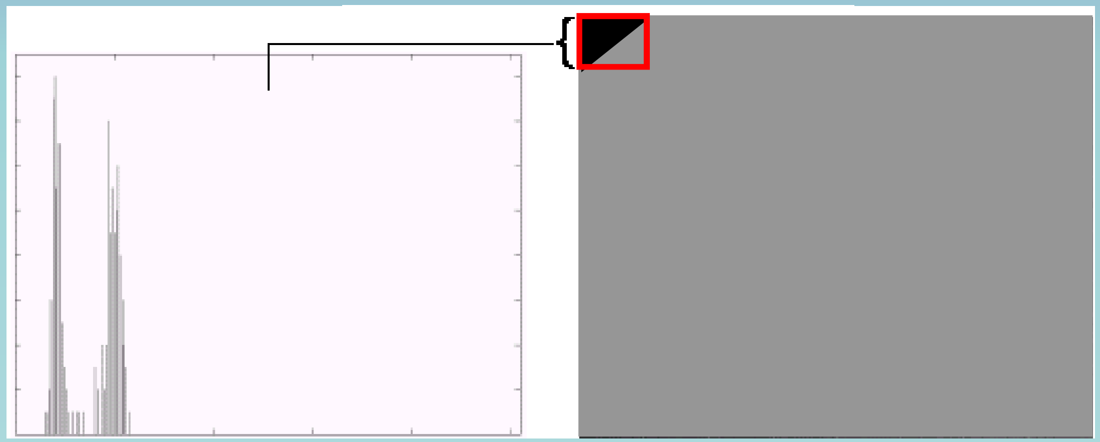
Issues Related to Pixels Near Edges



- Can we predict edges before segmentation?

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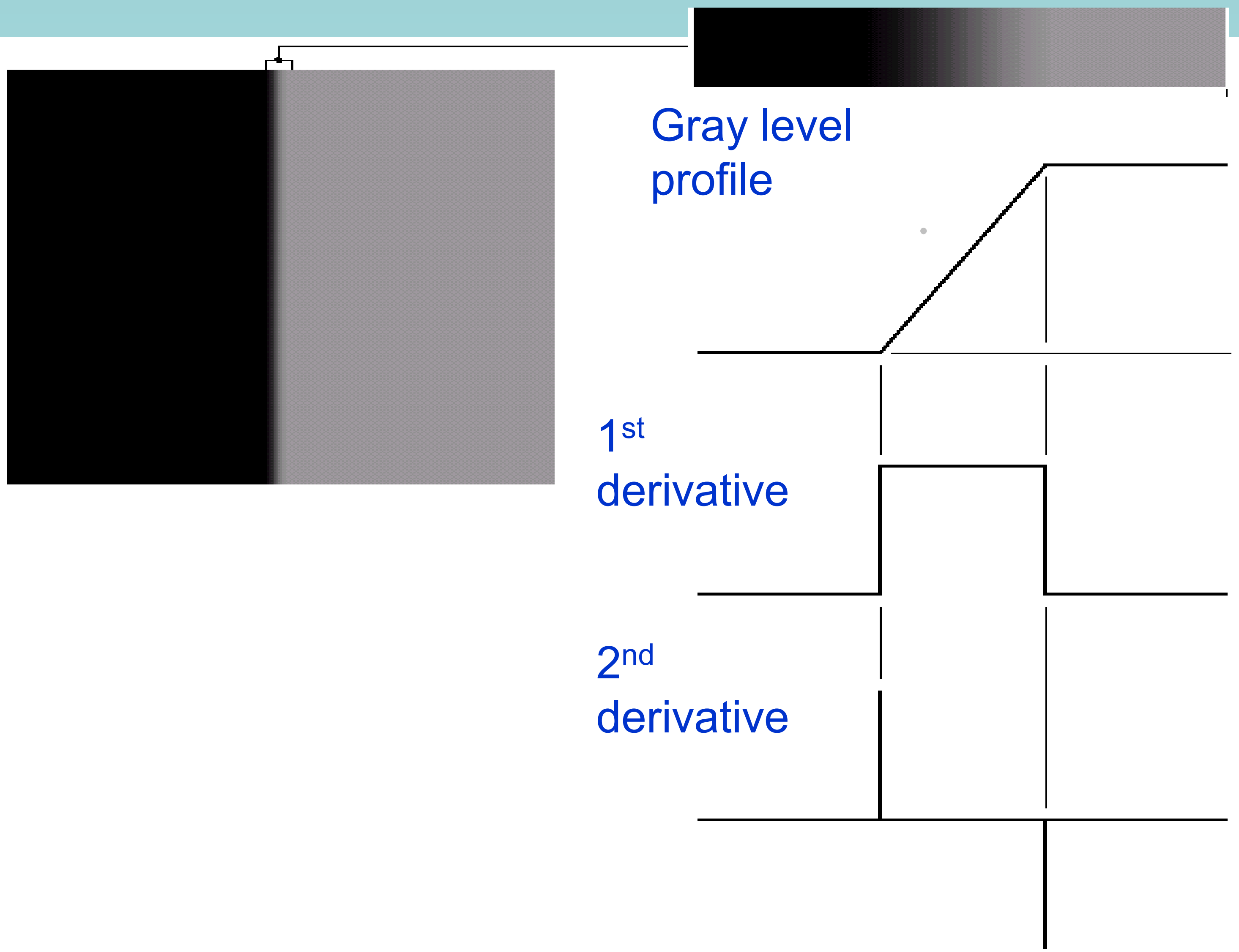
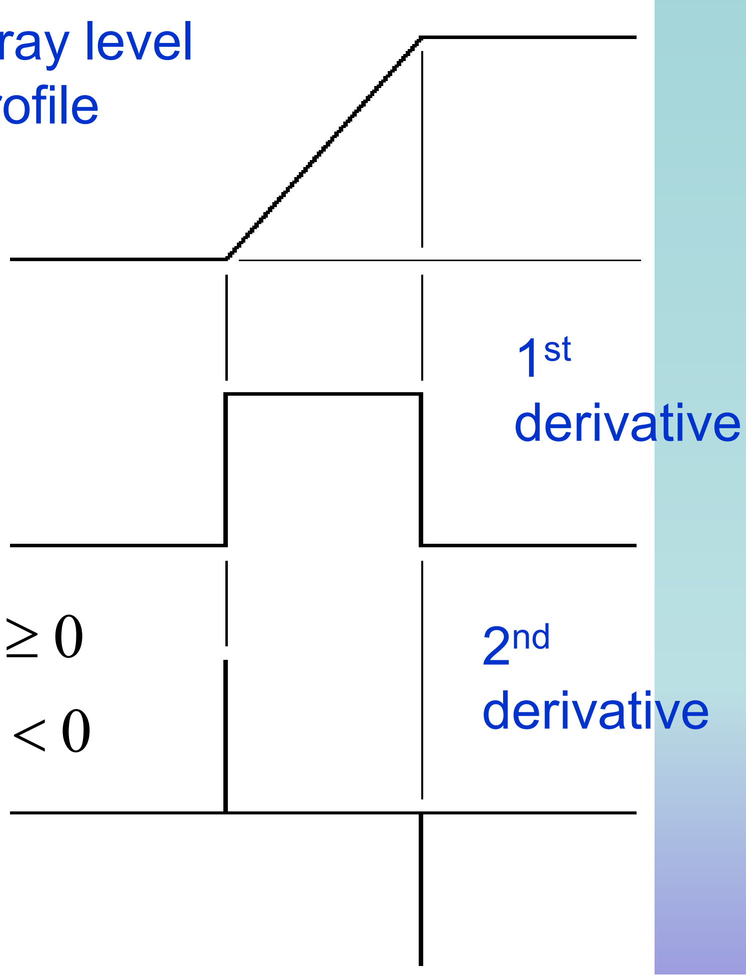
Alternate Solution Through Gradient/Laplacian



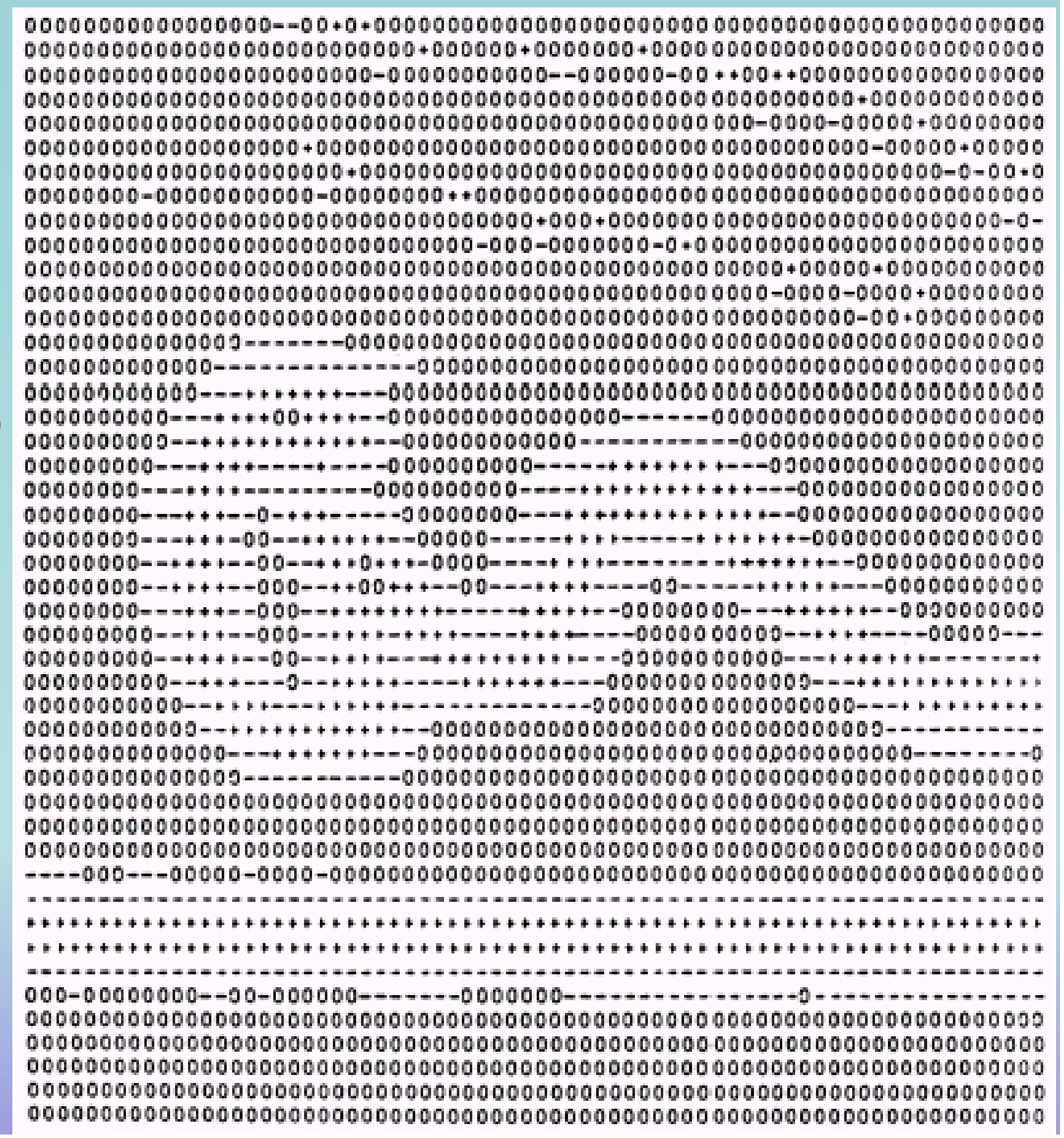
Alternate Solution Through Gradient/Laplacian



$$s(x,y)=\begin{cases} 0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f \geq T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f \geq T \text{ and } \nabla^2 f < 0 \end{cases}$$

Alternate Solution Through Gradient/Laplacian

$$s(x,y)=\begin{cases}0 & \text{if } \nabla f < T\\ + & \text{if } \nabla f \ge T \text{ and } \nabla^2 f \ge 0\\ - & \text{if } \nabla f \ge T \text{ and } \nabla^2 f < 0\end{cases}$$



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Alternate Solution Through Gradient/Laplacian

$$s(x,y)=\begin{cases}0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f \geq T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f \geq T \text{ and } \nabla^2 f < 0\end{cases}$$

Object

starts ends

$(\cdots)$ (-,+) (0 or +) (+,-) $(\cdots)$

interior

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Alternate Solution Through Gradient/Laplacian

$$s(x,y) = \begin{cases} 0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f \geq T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f \geq T \text{ and } \nabla^2 f < 0 \end{cases}$$

Object

starts ends

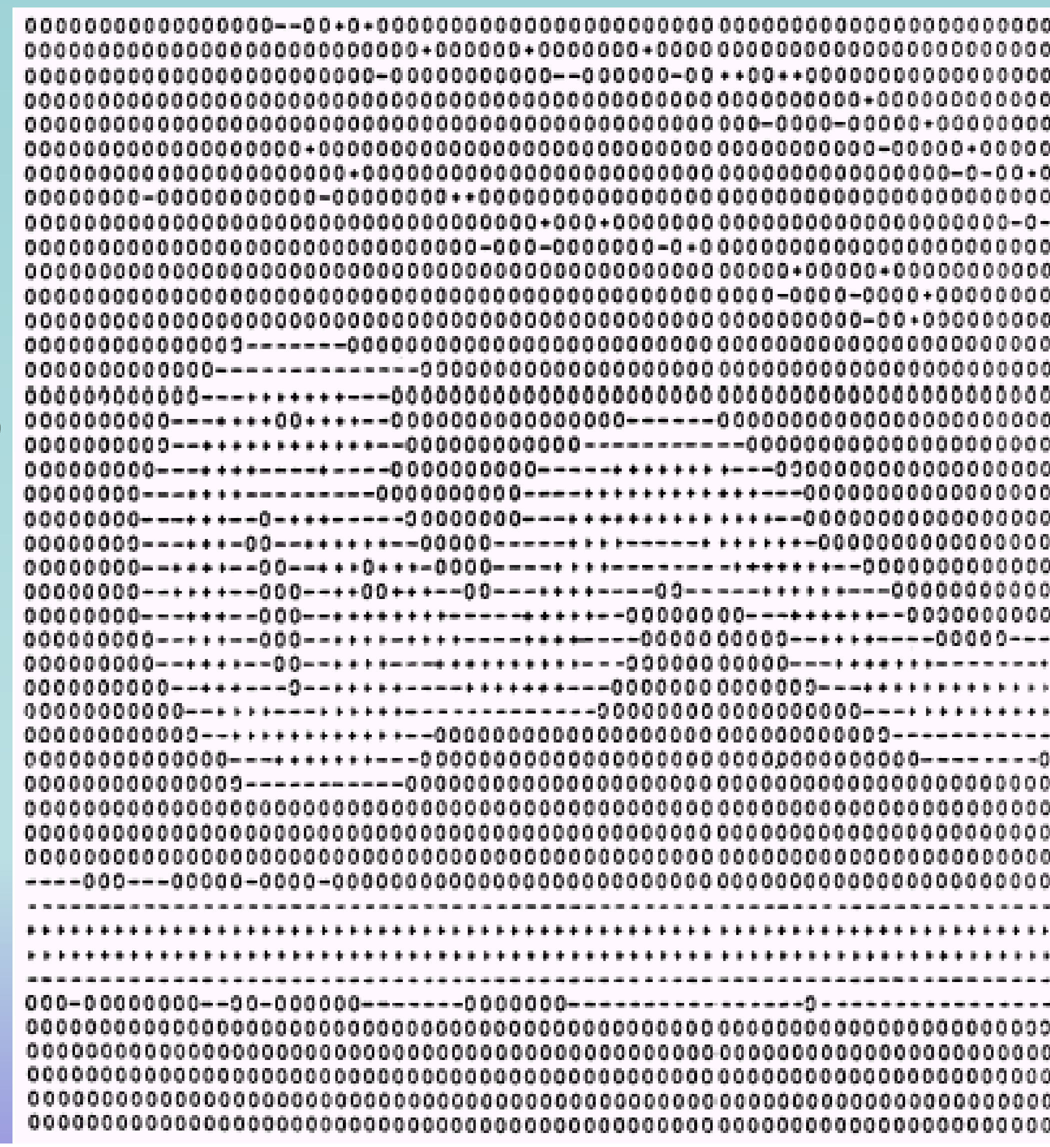
$(\cdots)$ (-,+) (0 or +) (+,-) $(\cdots)$

interior

Segmentation:

1: Internal

0: Other



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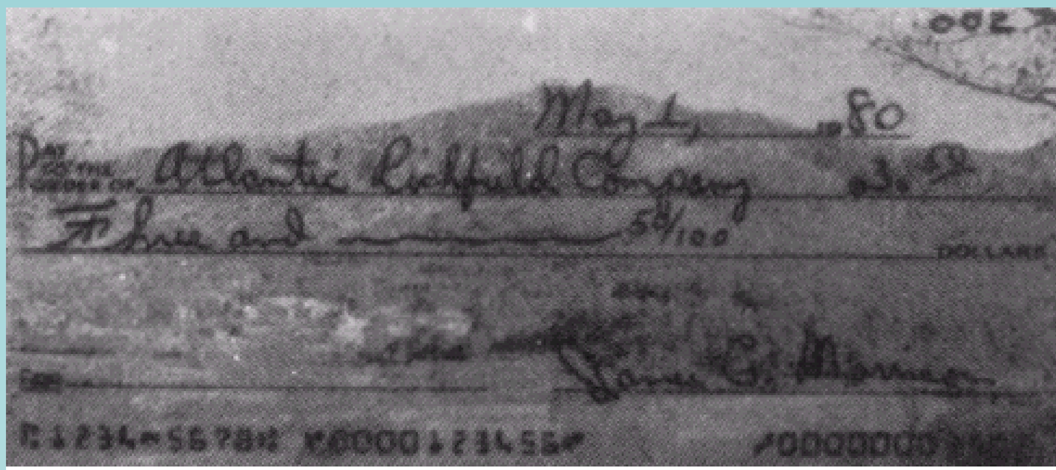
Alternate Solution Through Gradient/Laplacian



A Bank Check

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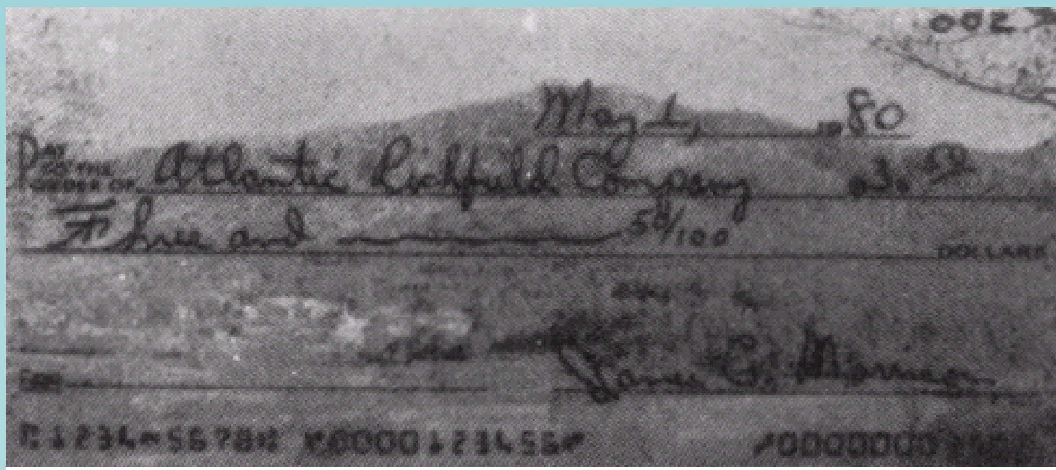
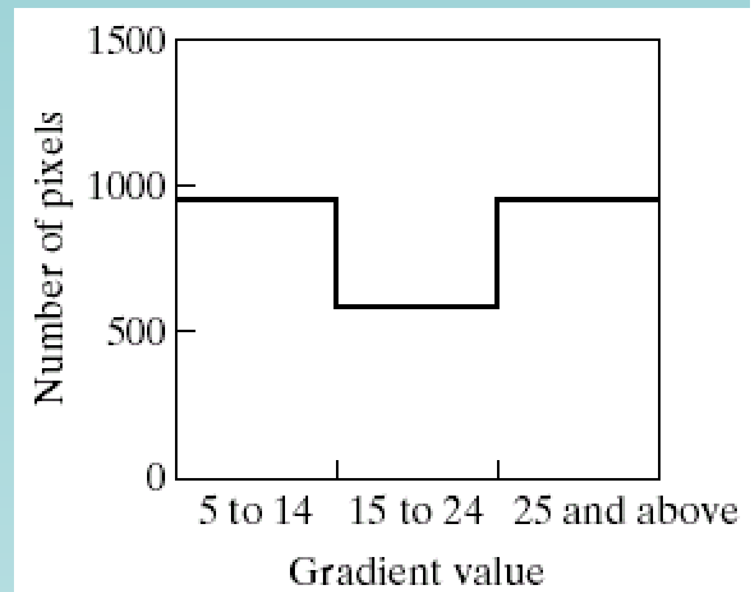
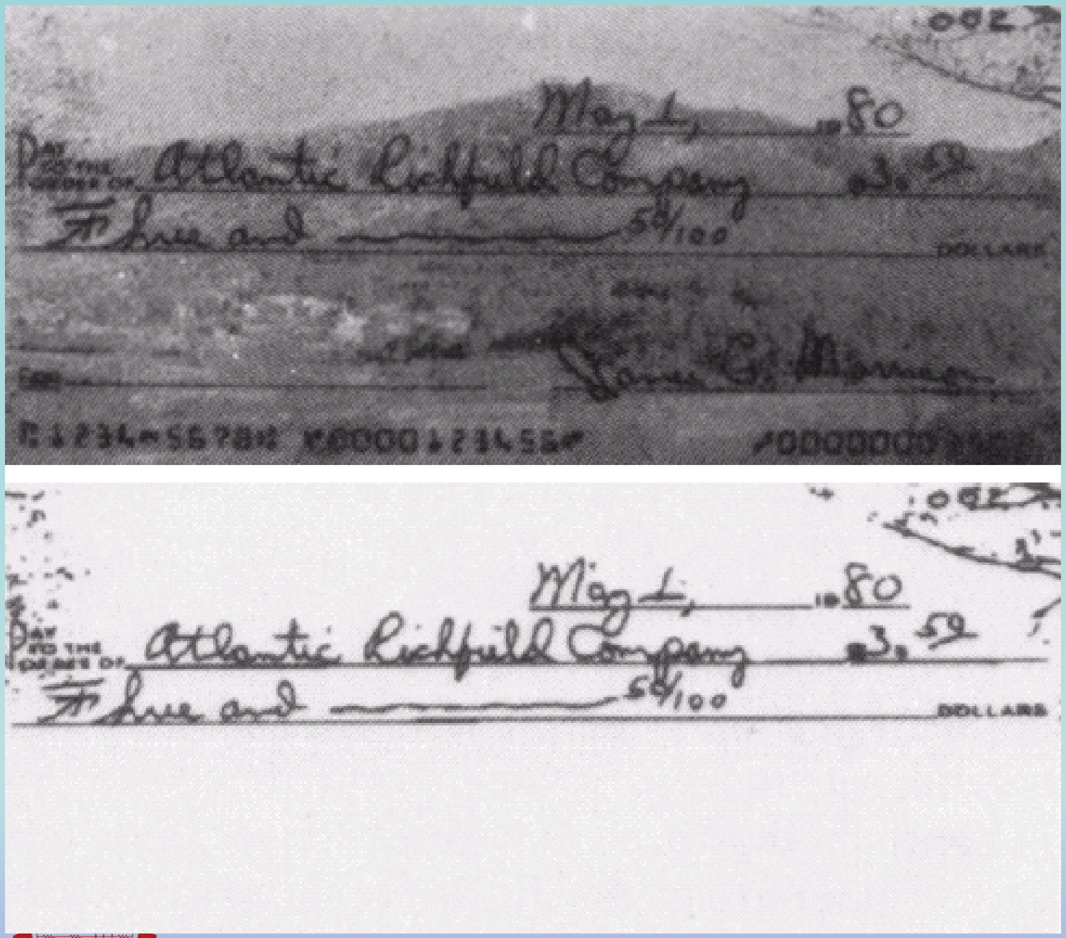
Alternate Solution Through Gradient/Laplacian

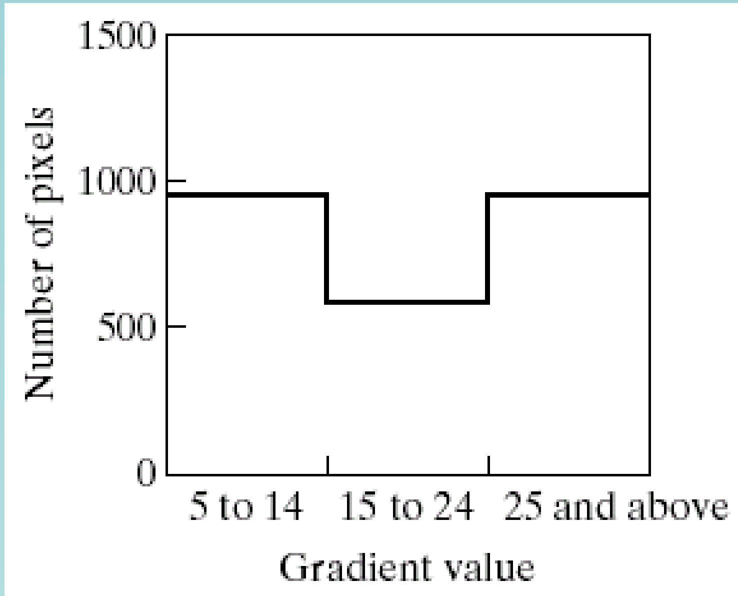


A Bank Check



Alternate Solution Through Gradient/Laplacian





Region Based Segmentation

- Let R = Entire image
- Objective: partition R into R_1, R_2,. . ., R_n so that

(a) $\bigcup_{i=1}^{n} R_i = R$

(b) R_i is a connected region

(c) $R_i \cap R_j = \varnothing$

(d) $P(R_i) = \text{TRUE}$ for $i = 1,2,3,\ldots,n$

(e) $P(R_i \cup R_j) = \text{FALSE}$ for $i \neq j$

Types of Region Based Segmentation

- Region growing
- Region splitting and merging

Region Growing

- *Region* grows from *seed points*
- *Append* to *growing region* those *neighbors* which are *similar* to the seed

Issues in Region Growing

- *seed points:*
  - How to select
  - Based on which property?
  - Any prior knowledge helpful?
  - What to do, if no prior knowledge available?
- *Similarity:*
  - Color, texture, spatial, gray level values?

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Issues in Region Growing

- *Is connectivity an issue?*
  - Needed to avoid a distorted set of pixels with similar values
- *When to stop:*
  - How and when to stop?
  - Any history of the prev data?
  - Size of the region so far grown
  - likeness of the new pixel
  - shape

Example: Region Growing (1)

Welding with breaks

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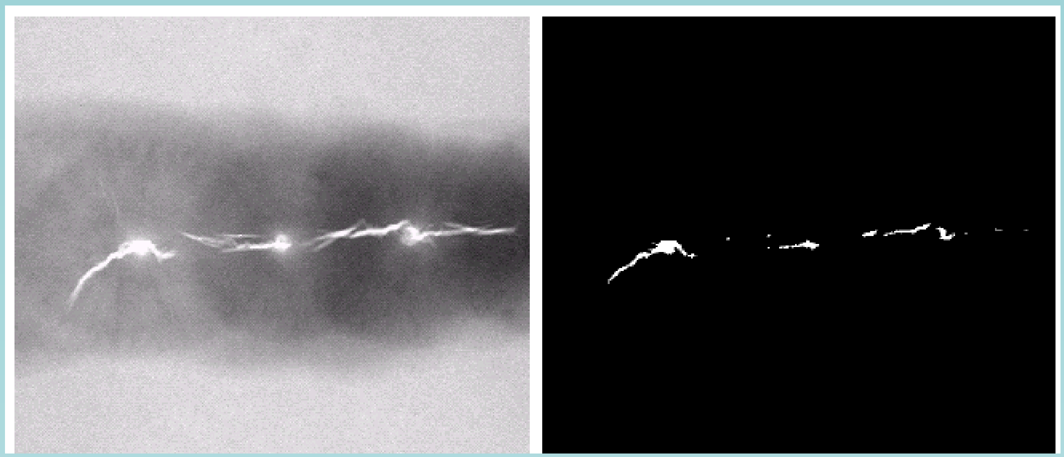
Example: Region Growing (1)



Probable seed points: gray level 255

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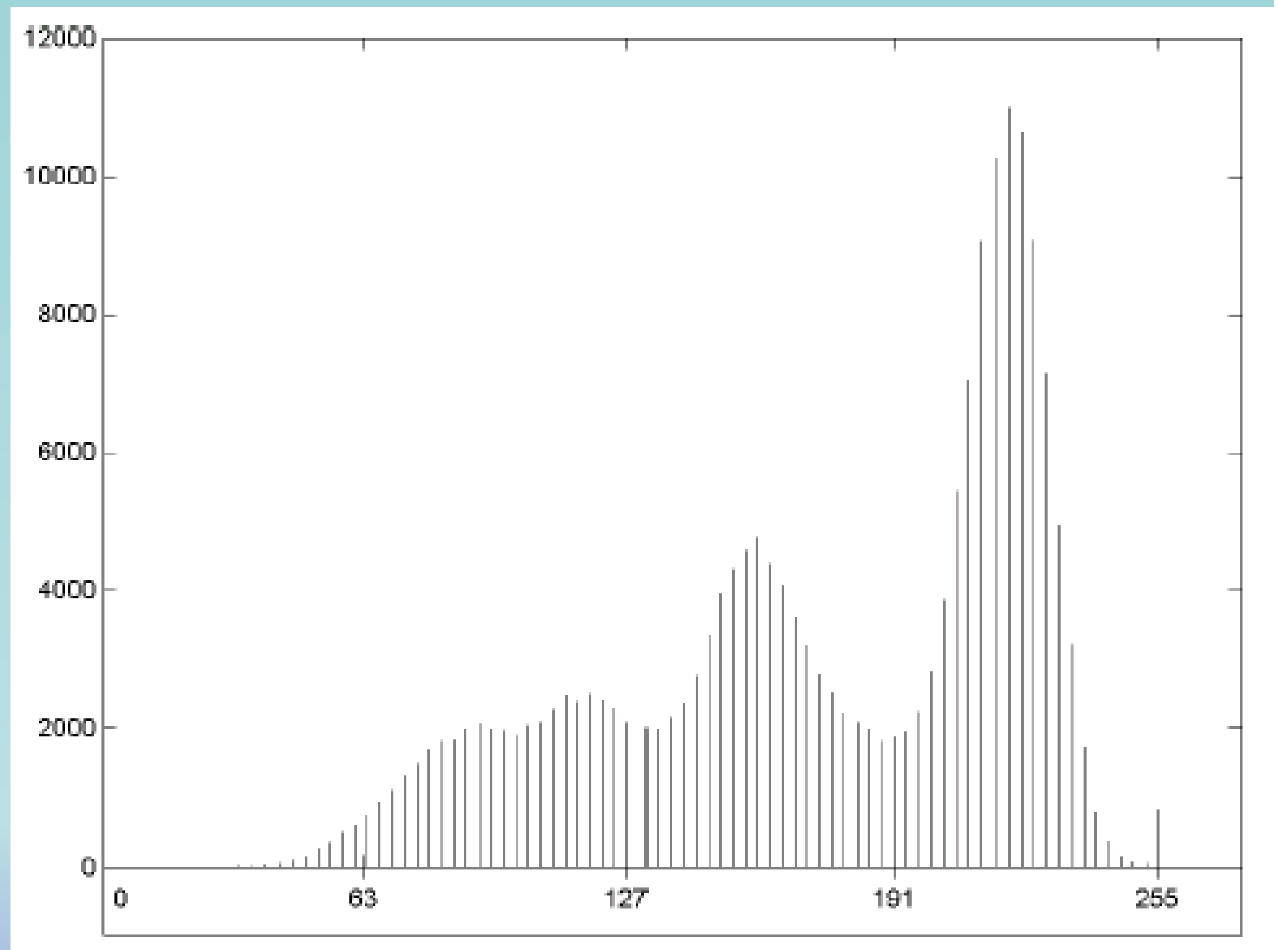
Example: Region Growing (1)

Cr1. gray level
diff <65



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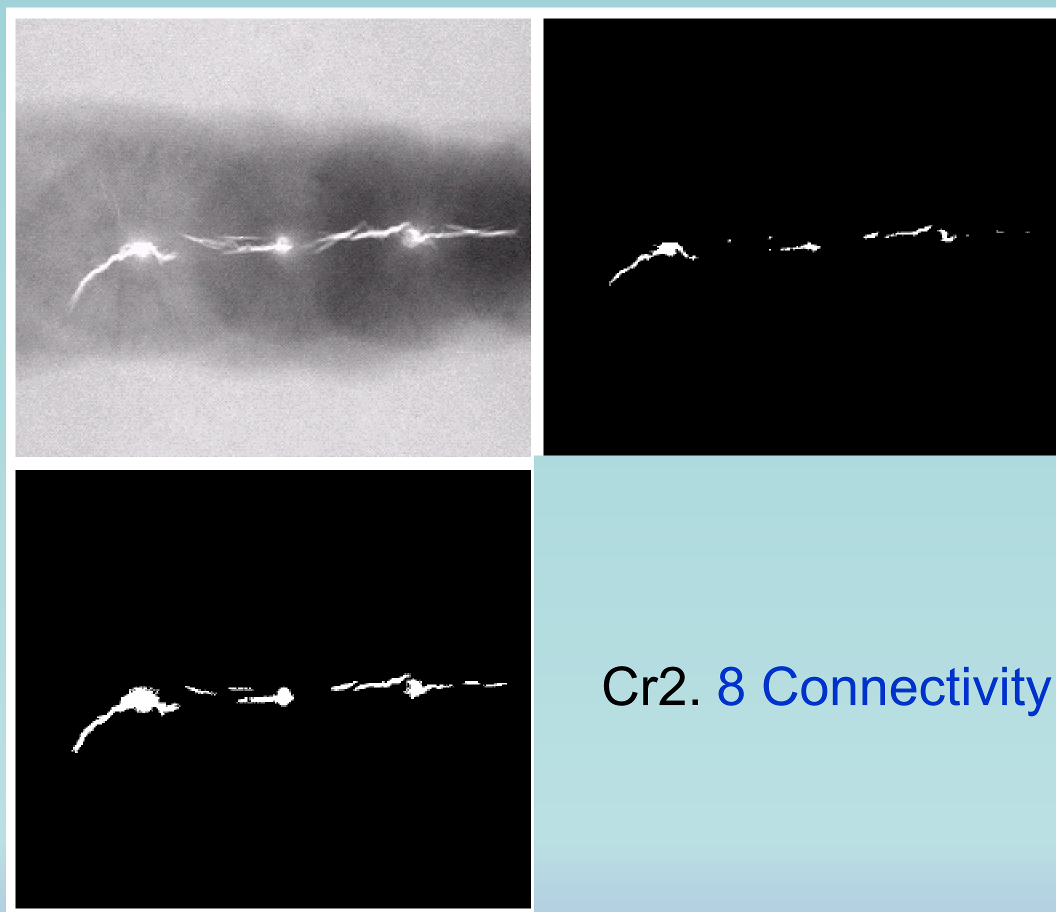
Example: Region Growing (1)



Cr2. 8 Connectivity

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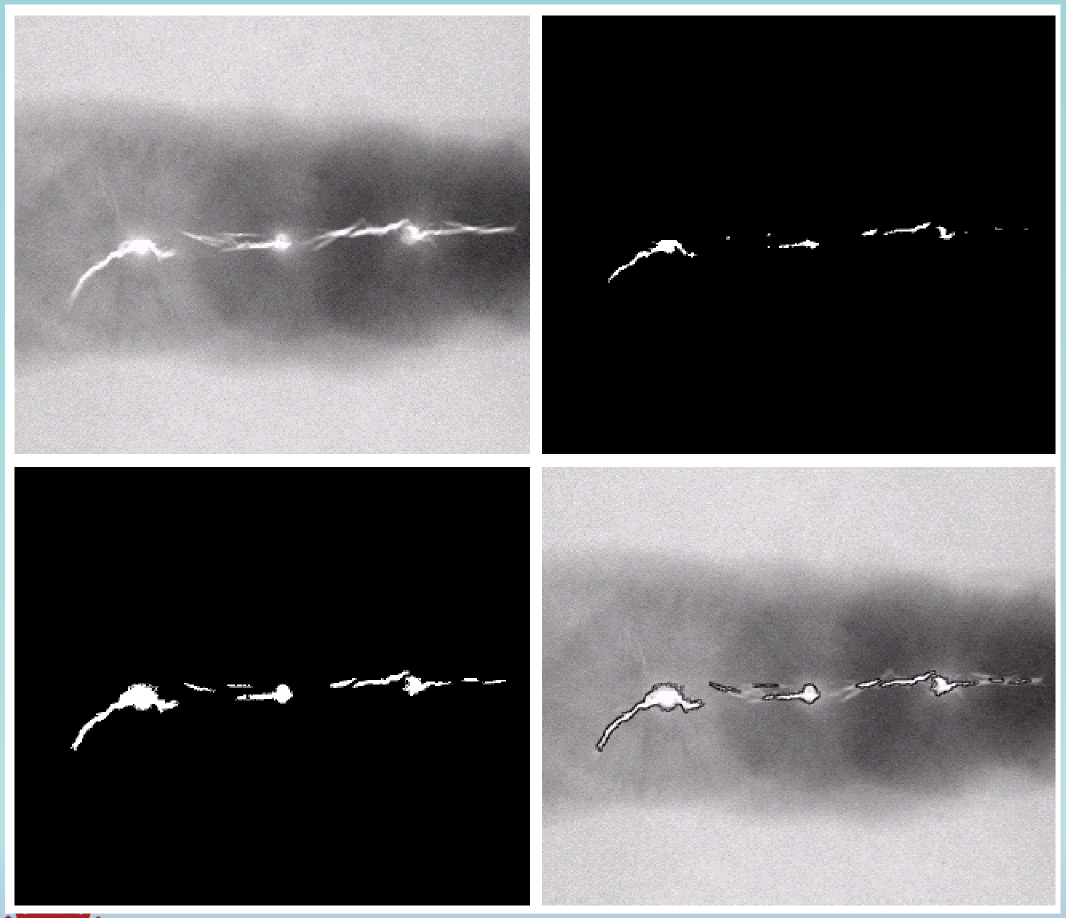
Example: Region Growing (1)



Superimposed
on the original

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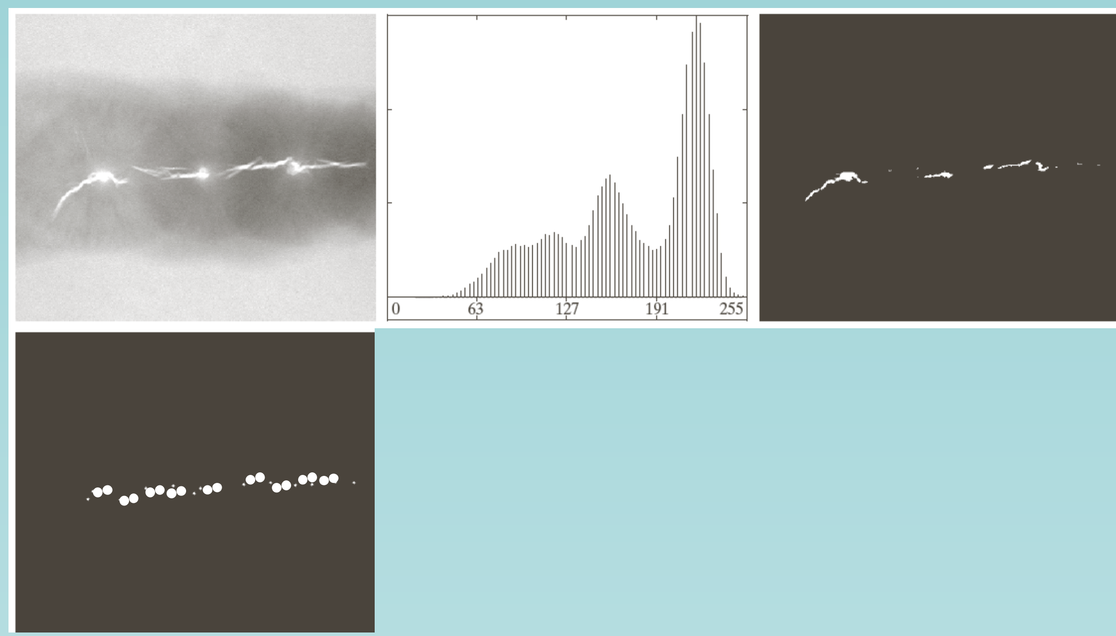
Example: Region Growing (2)



Initial seed points

Final seed points after morph. eroding

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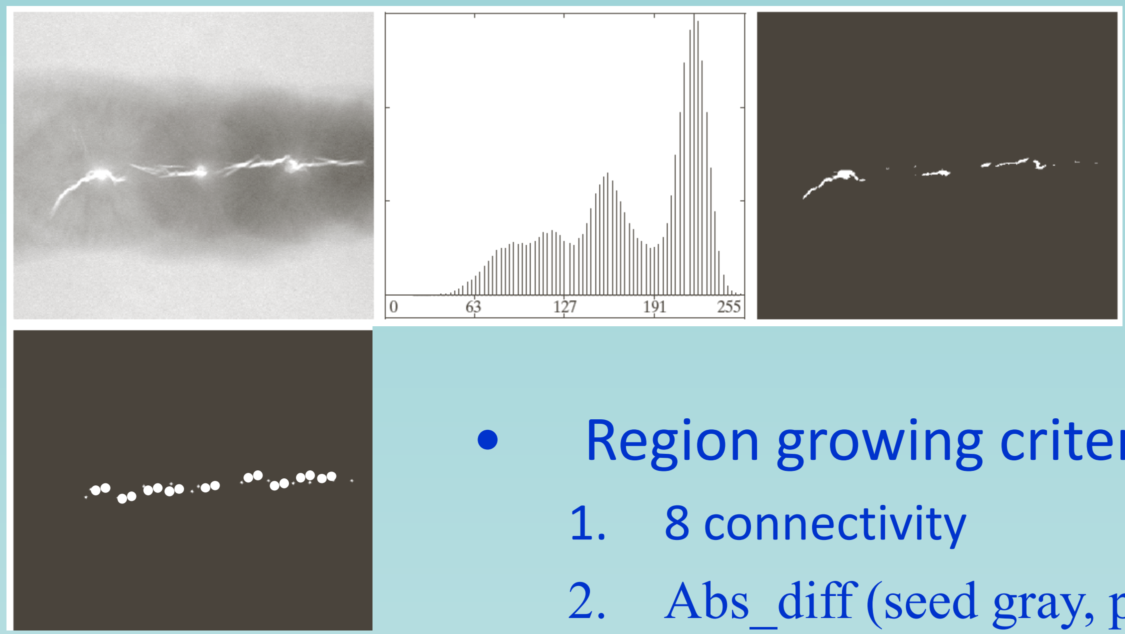
Example: Region Growing (2)



- Region growing criteria:
  1. 8 connectivity
  2. Abs_diff (seed gray, pixel gray) <= T

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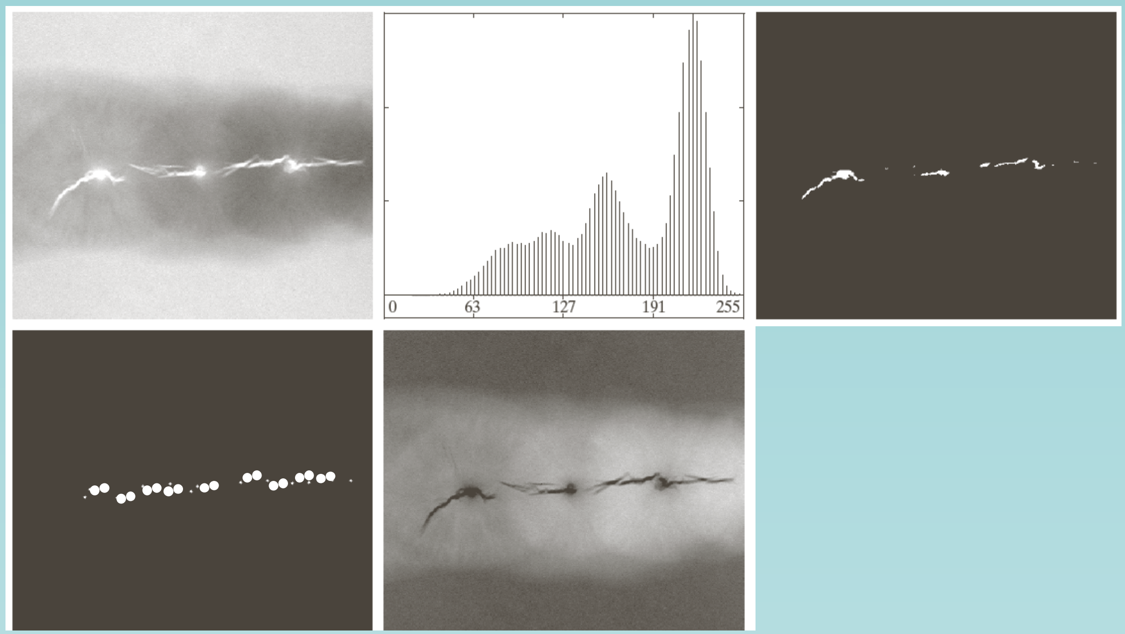
Example: Region Growing (2)



Diff (255, original image)

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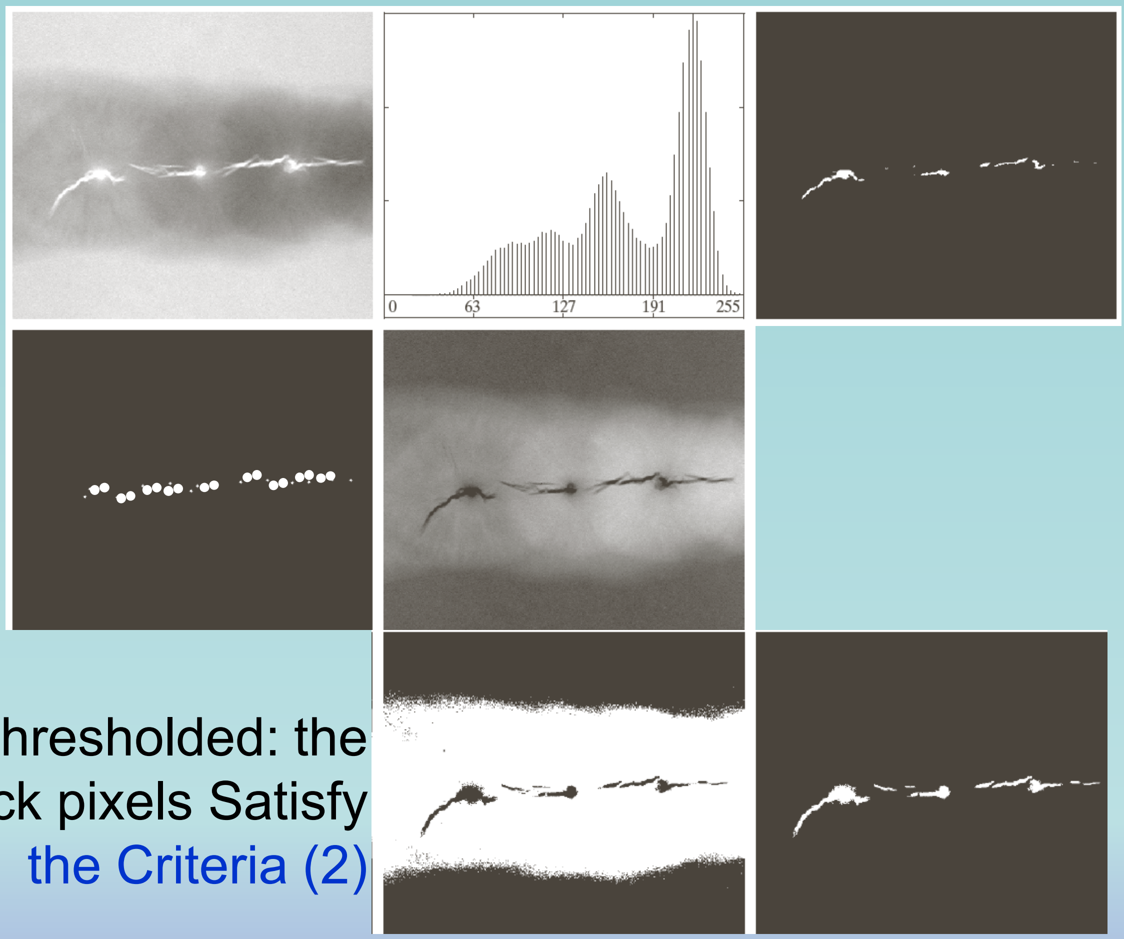
Example: Region Growing (2)



Thresholded: the
Black pixels Satisfy
the Criteria (2)

Final Result

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