- **Systems Analysis and Design**
- **Course Code: CSE 305**

Instructor:
Dr. Shah Murtaza Rashid Al Masud
Assoc. Prof. CSE Dept., UAP

PROJECT MANAGEMENT

CPM & PERT TECHNIQUES

Project Management

Project

- A project is an interrelated set of activities that has a definite starting and ending point and that results in a unique product or service.

Project management

- Project management is a scientific way of planning, implementing, monitoring & controlling the various aspects of a project such as time, money/cost, SW HW materials, manpower & other resources.

Project Scheduling and Tracking

- **Scheduling/time** is the process of deciding:
  - In what sequence a set of activities will be performed.
  - When they should start and be completed/end.
- **Tracking/monitoring** is the process of determining how well you are sticking to the cost estimate and schedule.

Project Scheduling Network Diagram

Project schedule network diagrams show the order in which activities should be scheduled to address logical relationships between these activities.

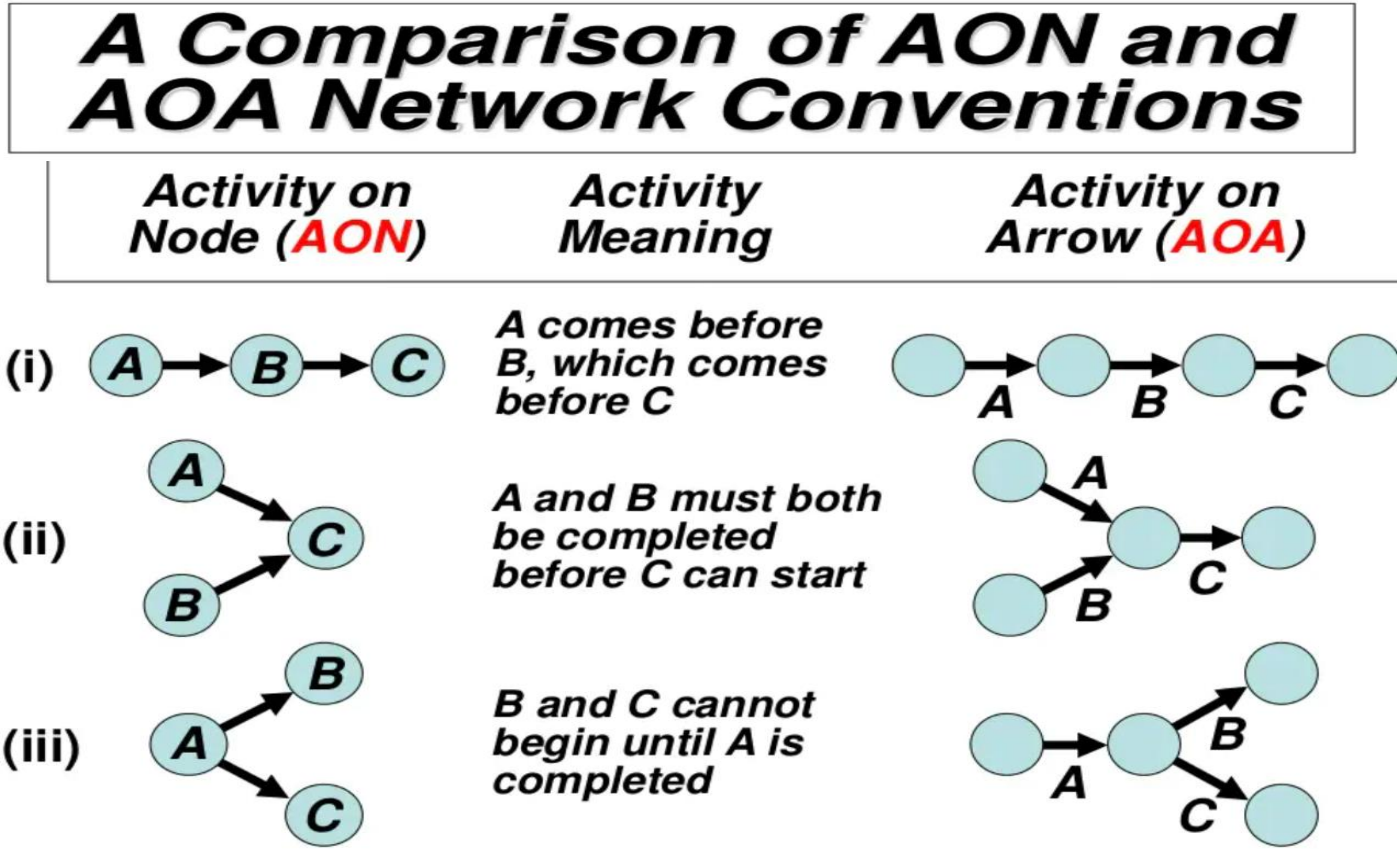
It typically comprises of nodes that represent activities and arrows that show the sequence and dependencies.

Drawing the Project Network

- **AOA – Activity on Arrow:** networks show each activity as an arrow, and the nodes represent the starting and ending points

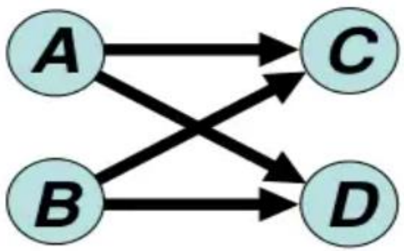
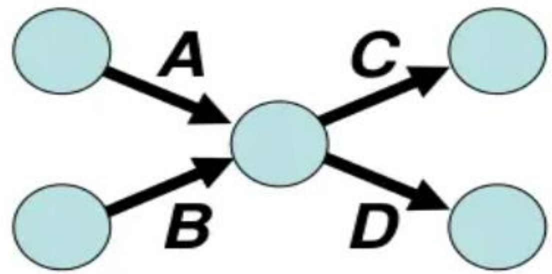
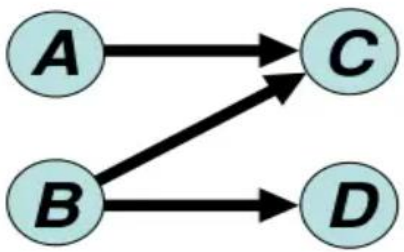
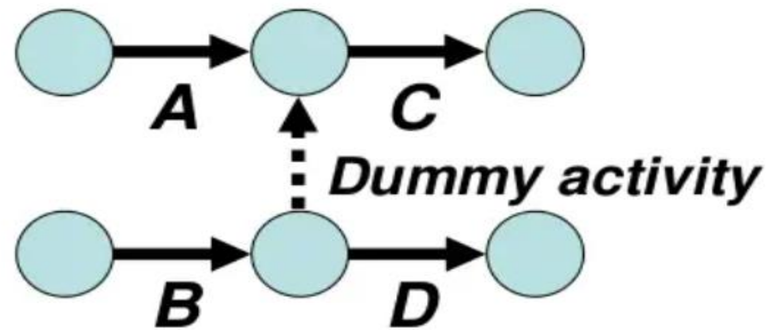
- **AON – Activity on Node:** networks show each activity as a node and arrows show the immediate predecessor activities

Guidelines for network diagram



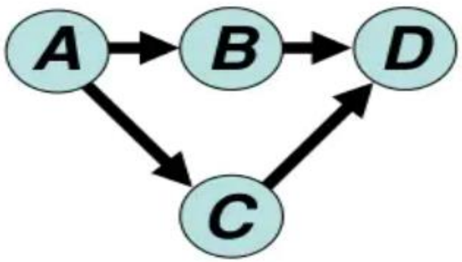
Guidelines for network diagram

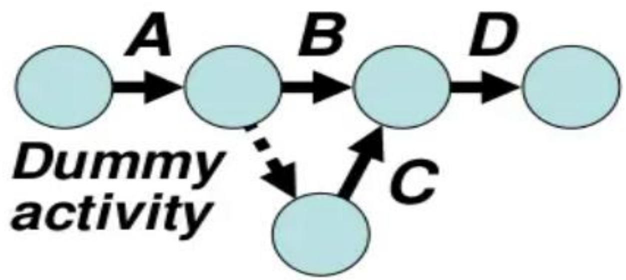
A Comparison of AON and AOA Network Conventions

	Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(iv)		C and D cannot begin until both A and B are completed	
(v)		C cannot begin until both A and B are completed; D cannot begin until B is completed. A dummy activity is introduced in AOA	

A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(vi)	B and C cannot begin until A is completed. D cannot begin until both B and C are completed. A dummy activity is again introduced in AOA.	



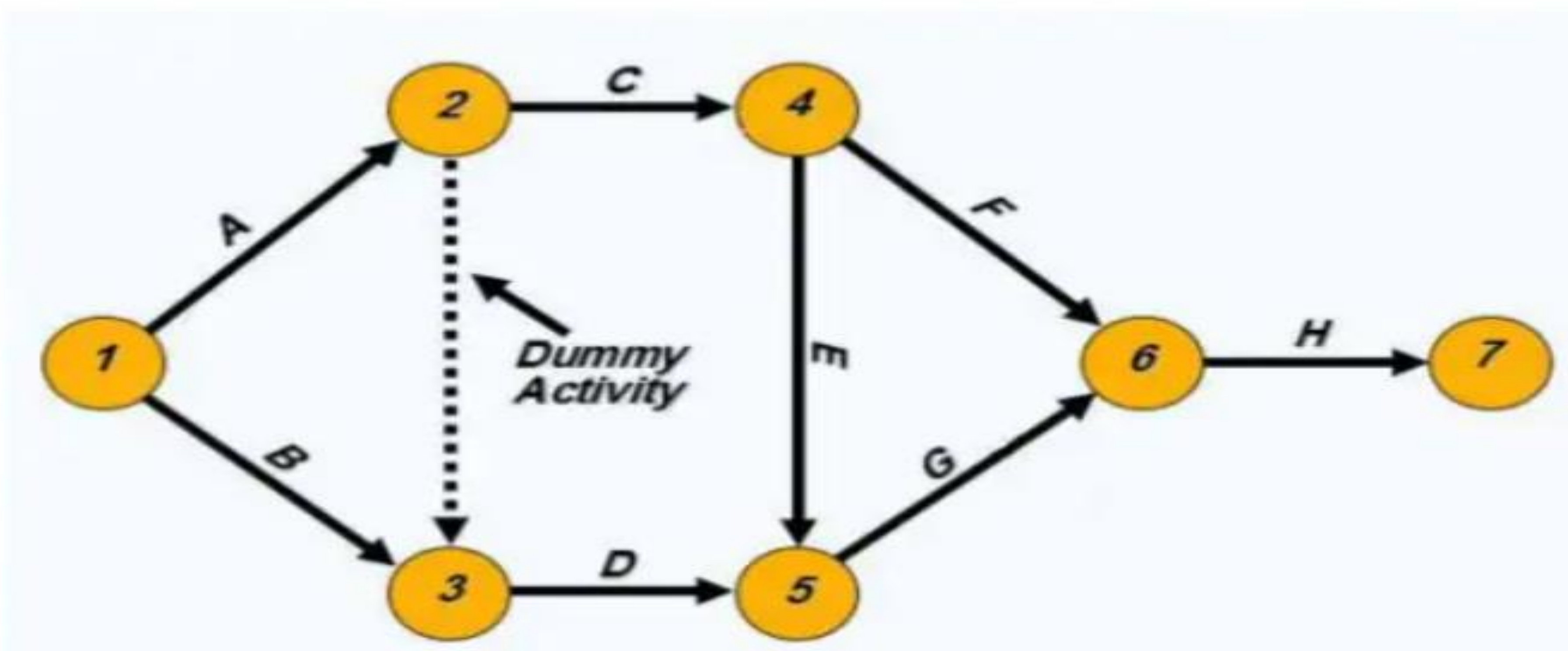


Guidelines for network diagram

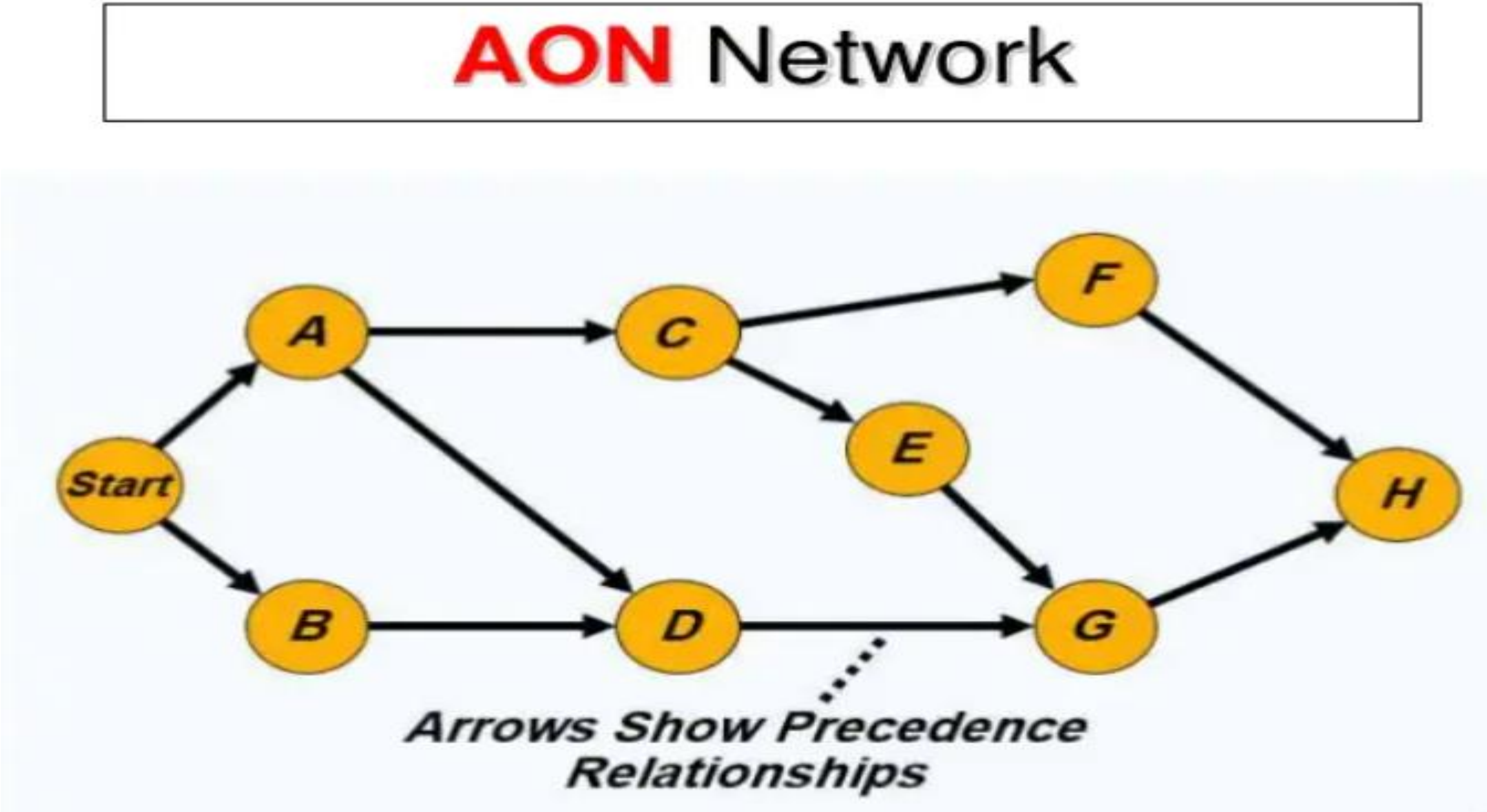
Activity	Predecessors
A	—
B	—
C	A
D	A, B
E	C
F	C
G	D, E
H	F, G

Guidelines for network diagram

AOA Network



Guidelines for network diagram



Network Planning Methods

- Methods used for network planning are:

  CPM (**Critical Path Method**)

  PERT (**Program evaluation and review technique**)

Managing a project with network planning methods involves **four steps:**

Describing the Project.

Diagramming the Network.

Estimating time of completion.

Monitoring Project Progress.

PERT Analysis

0 **PERT =** Program Evaluation and Review Technique

❖ A PERT **chart** shows the sequence in which **tasks** must be

PERT is a technique that uses **Optimistic time (O), Pessimistic time (P)** and **Most likely/ Realistic Time (R)** estimates to calculate the **Expected Time (ET)** or a particular task.

The **Optimistic time (t_o)** reflects the maximum possible periods of time for an activity to be completed OR it is a duration of the activity when everything goes well. It is assumed that such performance can be improved only in about 1% of the cases. While the **Pessimistic time (t_p)** is the longest duration expected under the assumption that every thing goes wrong. There is only 1 % chance that the activity will extend beyond this value.

PERT Analysis

The **Realistic time (R)** OR the **Most likely time (tm)** reflects the Project manager's **Best Guess** of the amount of time required for a task completion.

It is a normal time for the activity which would occur most often if the activity was to be repeated several times under the same conditions.

Compute Expected Completion Time (ET)/ Mean Completion Time

The expected Completion time should be closer to the realistic time (r), it is typically weighed Four times more than the Optimistic time (o) and the Pessimistic time (p). Once these values are added together , it must be divided by 6 to determine the Expected Time for a task.

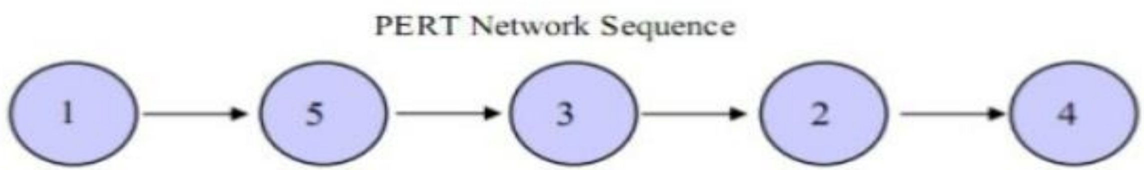
$$ET = \frac{O + 4r + p}{6}$$

PERT Analysis

Program Evaluation & Review Technique (PERT)

PERT Network

- Events must take place in a logical order.
- Activities represent the time and the work it takes to get from one event to another.
- No event can be considered reached until all activities leading to the event are completed.
- No activity may be begun until the event preceding it has been reached.



PERT Analysis

Program Evaluation & Review Technique (PERT)

Compute Expected Completion Time (ET)/ Mean Completion Time

$$ET = \frac{O + 4r + p}{6}$$

The above equation is a weighted average where the Most Likely/ Realistic Time estimate is weighted 4 times more heavily than the optimistic and pessimistic estimations.

Example:

A project manager estimate that the most likely time to complete a project is 12 days, where as the optimistic duration is 10 days, and pessimistic duration is 17 days.

$$ET = \frac{10 + 4(12) + 17}{6} = 12.5 \text{ days}$$

PERT Analysis

Program Evaluation & Review Technique (PERT)

Working project example

Activity	predecessor	Optimistic	Most Likely	Pessimistic
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B, C	3	4	5
G	B, C	1	1.5	5
H	E, F	5	6	7
I	E, F	2	5	8
J	D, H	2.5	2.75	4.5
K	G, I	3	5	7

PERT Analysis

Program Evaluation & Review Technique (PERT)

Working project example Cont…

- What is the expected time to complete the project..?
- Compute the variance.
- Go for ES, EF and LS, LF time for each activity.
- Determine the critical path and project completion time..?

PERT Analysis

Program Evaluation & Review Technique (PERT)

Working project example **Cont…**

Sol: 1. The expected time $ET = \frac{O + 4r + p}{6}$

2. The variance $\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$

Activity	ET	Variance (σ^2)
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

3. The ES, EF and LS, LF with slack

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0
B	0	4	5	9	5
C	6	9	6	9	0
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0
J	19	22	20	23	1
K	18	23	18	23	0

The Project Network
Activity Scheduling in Activity-on-Node Configuration

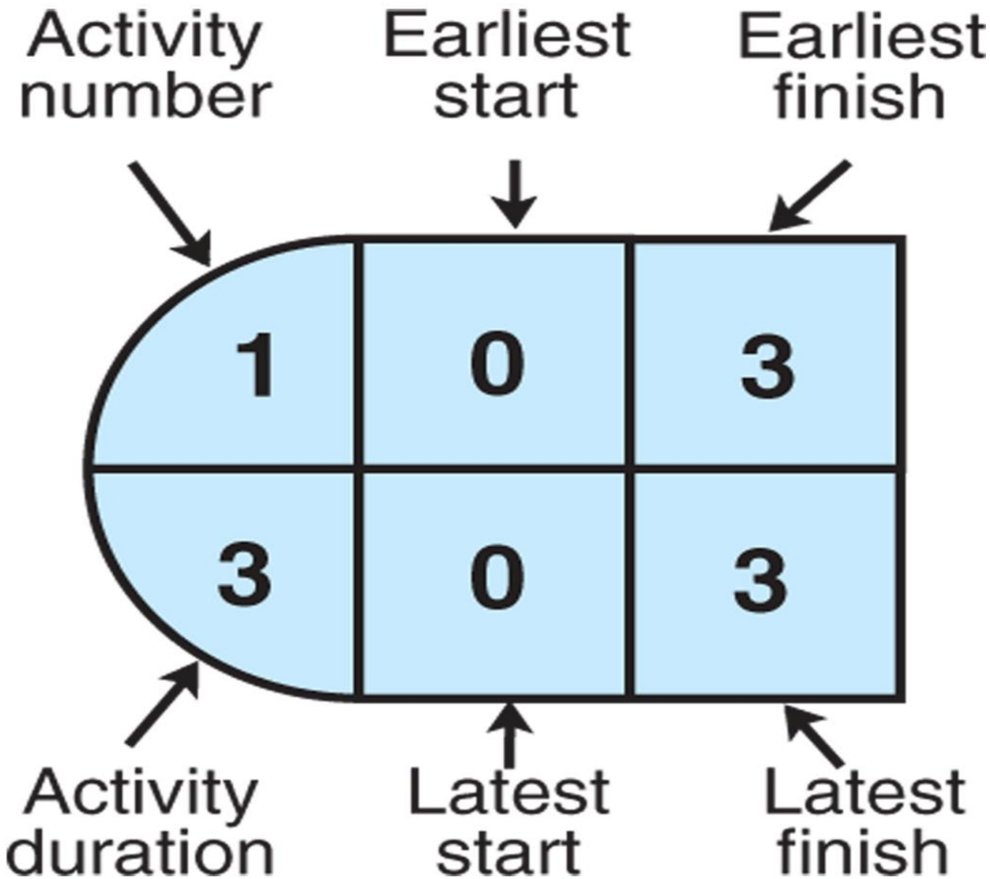


Figure: Activity-on-node configuration

ESTIMATING TIME OF COMPLETION

Planning the schedule of the project

Time estimates include:

1) Total time for completion.
2) ES- Earliest start time: the earliest time at which the activity can start given that its precedent activities must be completed first.
3) EF-Earliest finish time: equals to the earliest start time for the activity plus the time required to complete the activity.
4) LF- Latest finish time: the latest time in which the activity can be completed without delaying the project.
5) LS- Latest start time: equal to the latest finish time minus the time required to complete the activity.

6) FORWARD PASS:

The early start and early finish times are calculated by moving forward through the network and considering the predecessor activities Considers maximum

7) BACKWARD PASS:

The latest start and finish times are calculated by moving backward through the network.

Considers minimum

8) SLACK TIME:

Slack time for an activity is the difference between its earliest and latest start time or between the earliest and latest finish time.

Critical path is the path of activities having zero Slack time.

PERT Analysis

Program Evaluation & Review Technique (PERT)

Working project example

Activity	predecessor	Optimistic	Most Likely	Pessimistic
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B, C	3	4	5
G	B, C	1	1.5	5
H	E, F	5	6	7
I	E, F	2	5	8
J	D, H	2.5	2.75	4.5
K	G, I	3	5	7

The Project Network

Activity Scheduling : Earliest Times

- ES is the earliest time an activity can start:

$$ES = Maximum\{EF \ \ immediate \ predecessors\}$$

- EF is the earliest start time plus the activity time: $EF = ES + t$

Activity	ET	Variance (σ^2)
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0
B	0	4	5	9	5
C	6	9	6	9	0
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0
J	19	22	20	23	1
K	18	23	18	23	0

The Project Network

Activity Scheduling : Latest Times

- LS is the latest time an activity can start without delaying critical path time: $LS = LF - t$

- LF is the latest finish time:

$$LF = Minimum\{LS\ following\ activities\}$$

Activity	ET	Variance (σ^2)
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0
B	0	4	5	9	5
C	6	9	6	9	0
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0
J	19	22	20	23	1
K	18	23	18	23	0

The Project Network
Activity Slack Time (1 of 2)

- ***Slack*** is the amount of time an activity can be delayed without delaying the project: S = LS – ES = LF - EF
- ***Slack Time*** exists for those activities not on the critical path for which the earliest and latest start times are not equal.

Critical path is the path of activities having zero Slack time.

Activity	ET	Variance (σ^2)
A	6	4/9
B	4	4/9
C	3	0
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F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0
B	0	4	5	9	5
C	6	9	6	9	0
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0
J	19	22	20	23	1
K	18	23	18	23	0

PERT Analysis

Program Evaluation & Review Technique (PERT)

Working project example **Cont…**

4. The Critical path and project completion time

Hence A – C – F – I – K is a critical path and the project completion Time will be 6+3+4+5+5 = **23 Days**

A **critical path** is the longest path in the network. Each node which falls under critical path has zero or negative float (Slack).

The Project Network

House Building Project Data (**another example**)

Number	Activity	Predecessor	ET Duration
1	Design house and obtain financing	--	3 months
2	Lay foundation	1	2 months
3	Order and receive materials	1	1 month
4	Build house	2,3	3 months
5	Select paint	2, 3	1 month
6	Select carper	5	1 month
7	Finish work	4, 6	1 month

The Project Network
AON Network for House Building Project

Activity-on-Node (AON) Network

- A node represents an activity, with its label and time shown on the node
- The branches show the precedence relationships

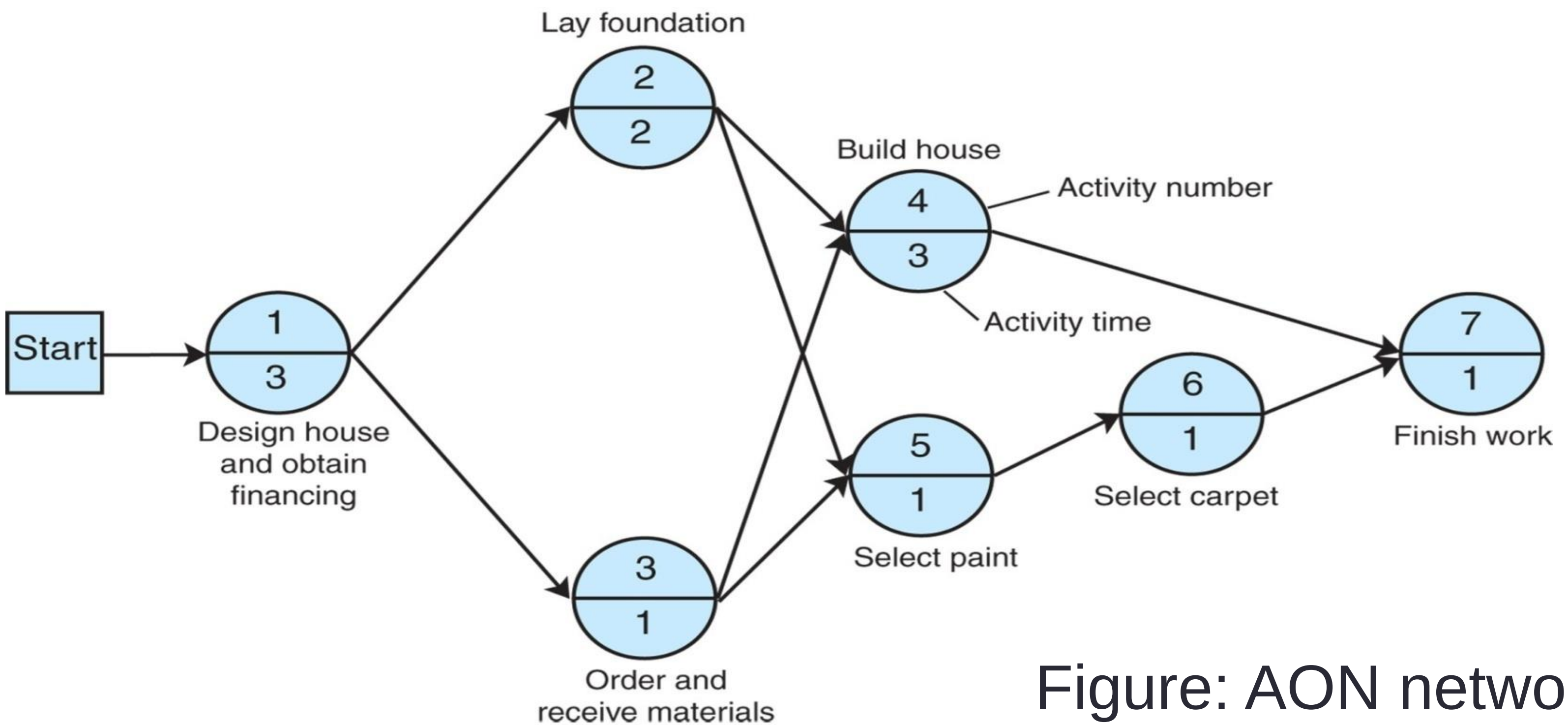

Figure: AON network

The Project Network
Paths Through a Network

Path	Events
A	1→2→4→7
B	1→2→5→6→7
C	1→3→4→7
D	1→3→5→6→7

Table:
Paths through the house-building network

The Project Network
The Critical Path

The ***critical path*** is the longest path through the network; the minimum time the network can be completed. From Figure :

Path A: $1 \rightarrow 2 \rightarrow 4 \rightarrow 7$	$3 + 2 + 3 + 1 = 9$ months
Path B: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 7$	$3 + 2 + 1 + 1 + 1= 8$ months
Path C: $1 \rightarrow 3 \rightarrow 4 \rightarrow 7$	$3 + 1 + 3 + 1 = 8$ months
Path D: $1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 7$	$3 + 1 + 1 + 1 + 1 = 7$ months

The Project Network
Activity Start Times

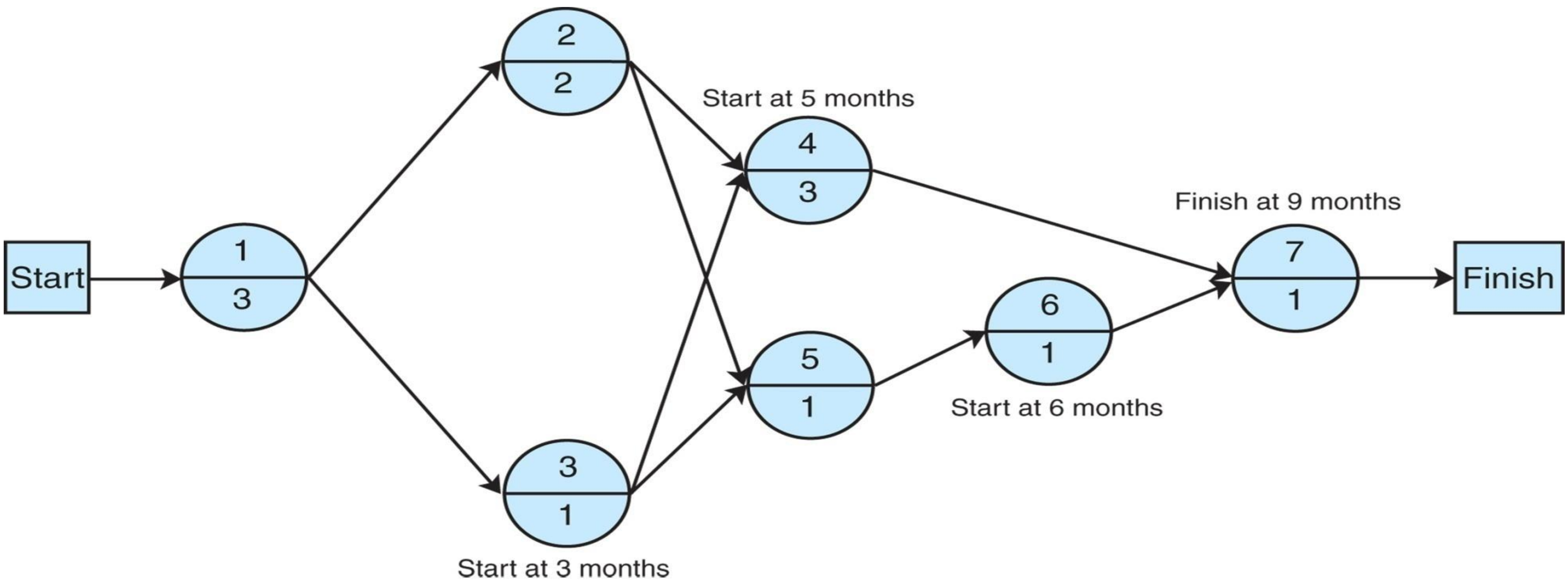

Figure: Activity start time

The Project Network

Activity Scheduling in Activity-on-Node Configuration

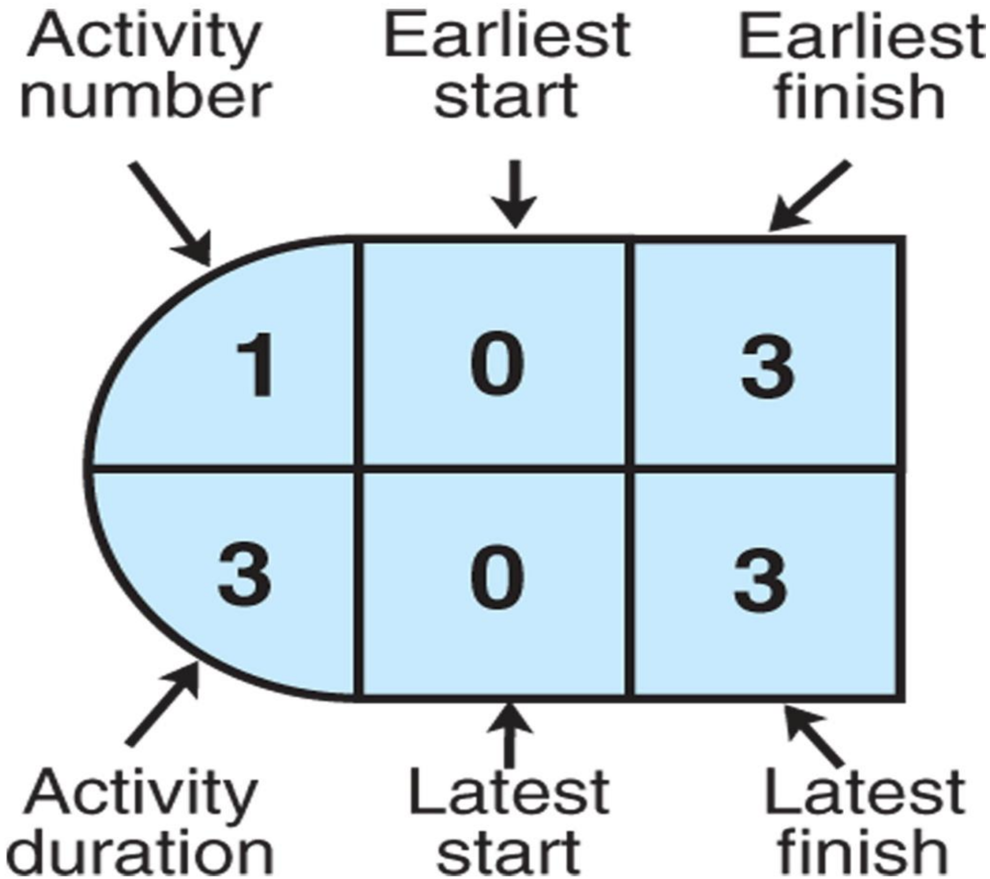

Figure: Activity-on-node configuration

ESTIMATING TIME OF COMPLETION

Planning the schedule of the project

Time estimates include:

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Critical path is the path of activities having zero Slack time.

The Project Network

Activity Scheduling : Earliest Times

- ES is the earliest time an activity can start:

$$ES = Maximum\{EF \;\; immediate \;\; predecessors\}$$

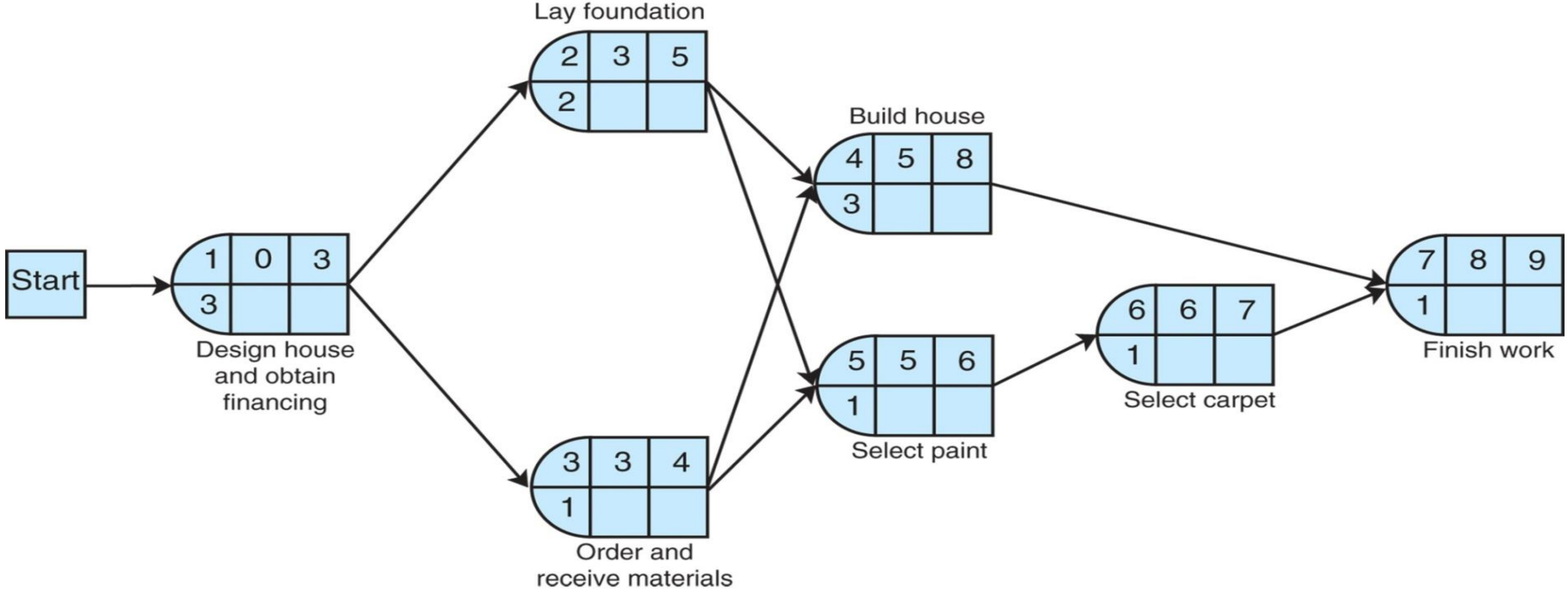


Figure: Earliest activity start and finish times

$$EF = ES + t$$

- EF is the earliest start time plus the activity time:

The Project Network

Activity Scheduling : Latest Times

- LS is the latest time an activity can start without delaying critical path time: $LS = LF - t$

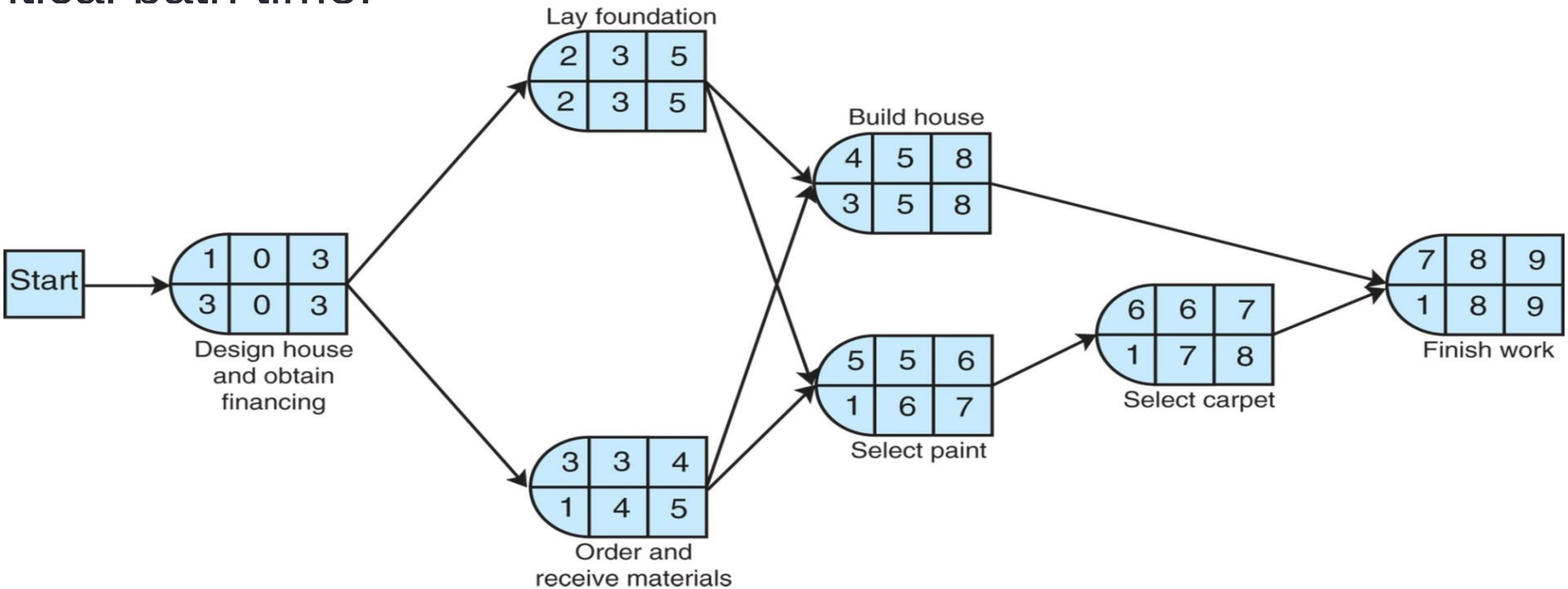


Figure: Latest activity start and finish times

- LF is the latest finish time: $LF = Minimum\{LS \ following \ activities\}$

The Project Network

Activity Slack Time (1 of 2)

- ***Slack*** is the amount of time an activity can be delayed without delaying the project: S = LS – ES = LF - EF
- ***Slack Time*** exists for those activities not on the critical path for which the earliest and latest start times are not equal.

Activity	LS	ES	LF	EF	Slack, S
*1	0	0	3	3	0
*2	3	3	5	5	0
3	4	3	5	4	1
*4	5	5	8	8	0
5	6	5	7	6	1
6	7	6	8	7	1
*7	8	8	9	9	0

*Critical path

The Project Network
Activity Slack Time (2 of 2)

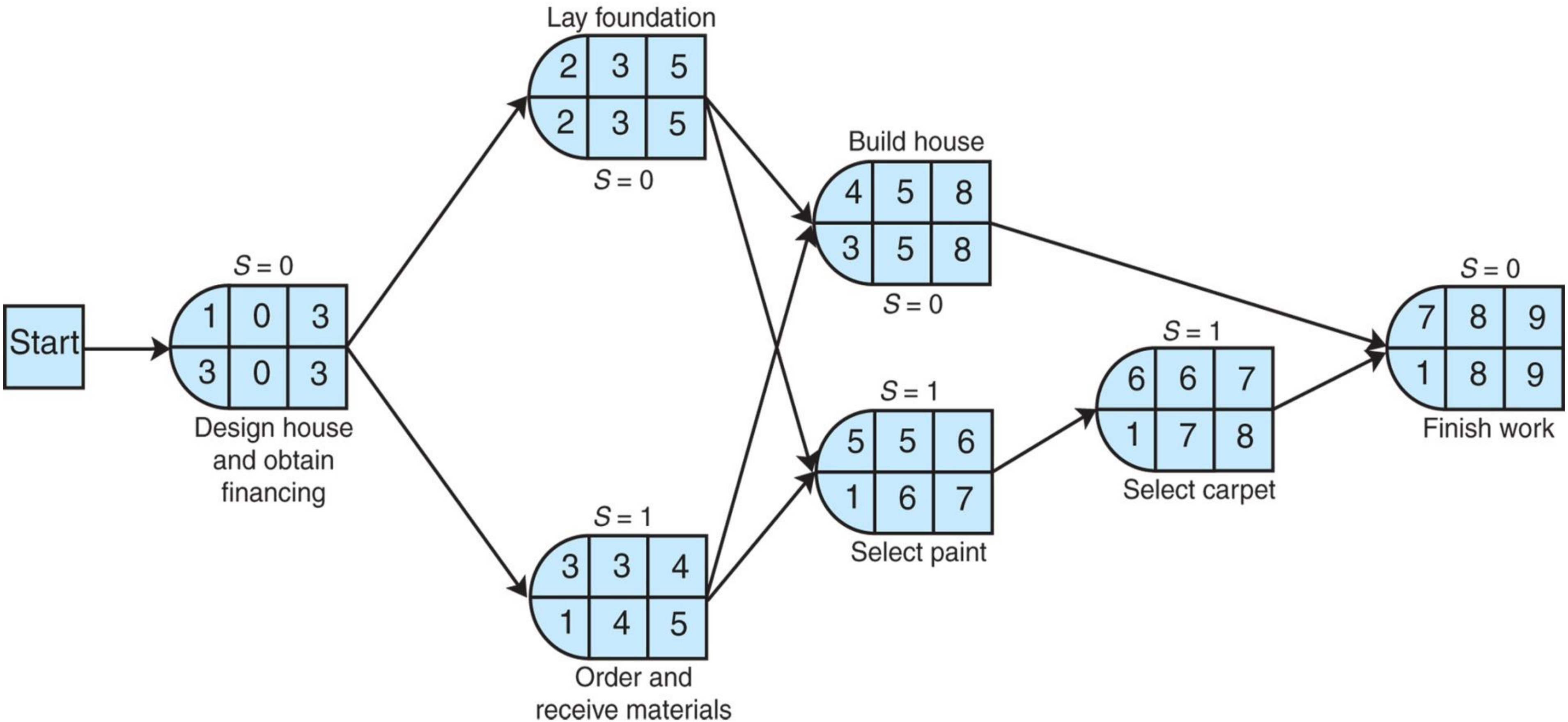


Figure: Activity slack

Example 2

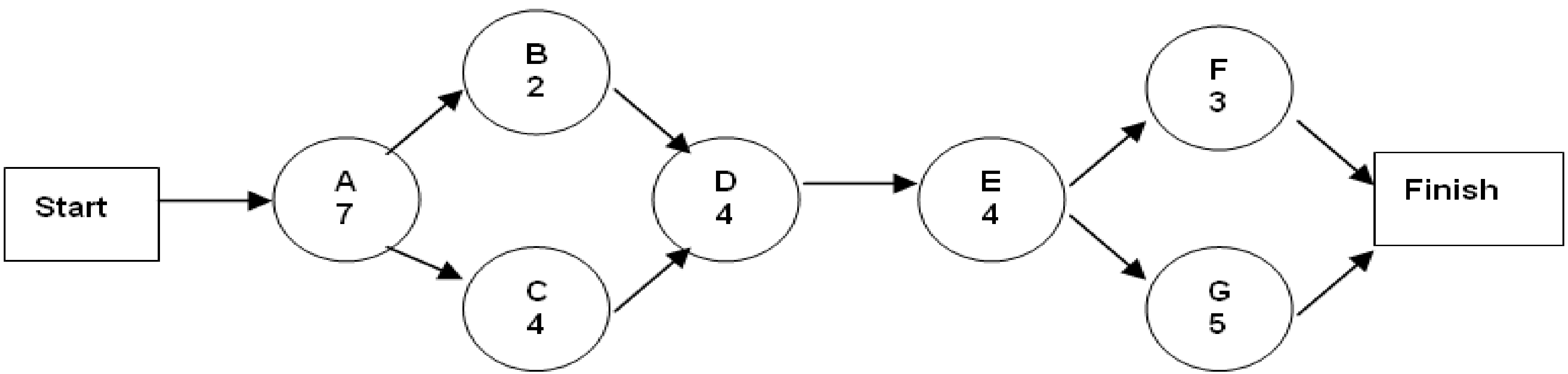
The following information is known about a project

Activity	Activity Time (days)	Immediate Predecessor(s)
A	7	--
B	2	A
C	4	A
D	4	B, C
E	4	D
F	3	E
G	5	E

Draw the AON network for this project.
What is the Critical Path and Project Duration?

Example 2 - Solution

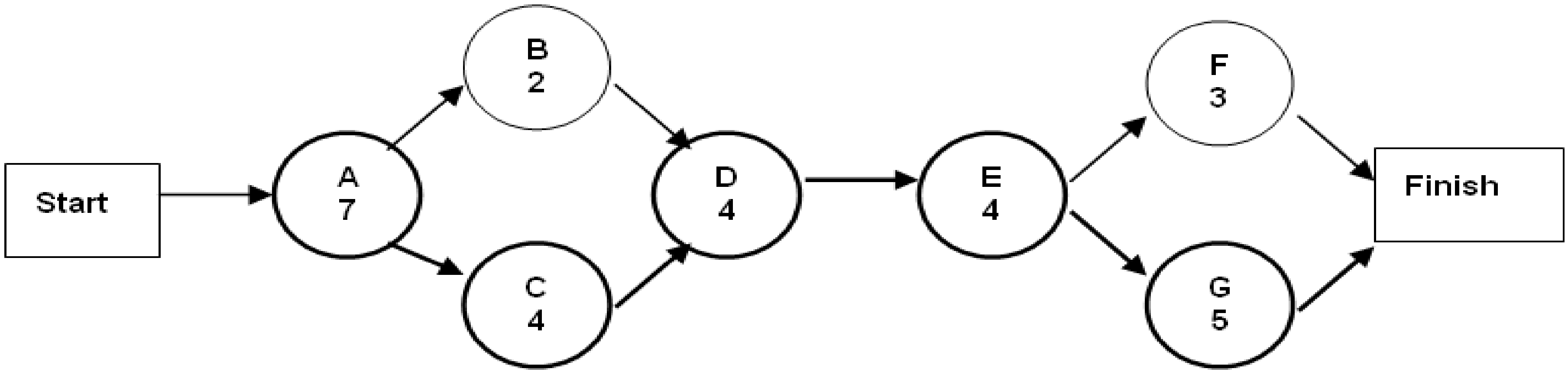
Activity	Activity Time (days)	Immediate Predecessor(s)
A	7	--
B	2	A
C	4	A
D	4	B, C
E	4	D
F	3	E
G	5	E



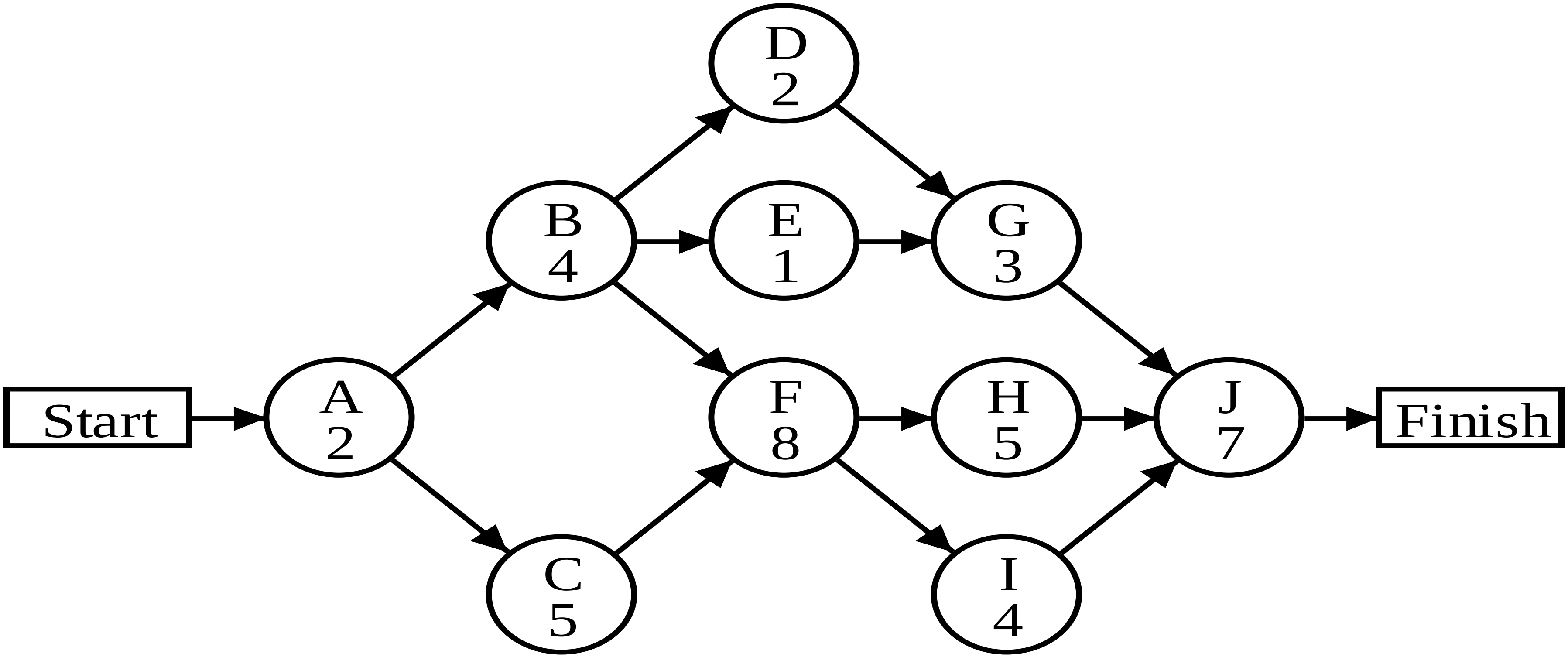
Problem 2 - Critical Path and Project Duration

Activity	Duration	Earliest Start (ES)	Latest Start (LS)	Earliest Finish (EF)	Latest Finish (LF)	Free Slack (LS-ES)	On the Critical Path?
A	7	0	0	7	7	0-0=0	Yes
B	2	7	9	9	11	9-7=2	No
C	4	7	7	11	11	7-7=0	Yes
D	4	11	11	15	15	11-11=0	Yes
E	4	15	15	19	19	15-15=0	Yes
F	3	19	21	22	24	21-19=2	No
G	5	19	19	24	24	19-19=0	Yes

The critical path (bold path) is A-C-D-E-G with project duration of 24 days.



Problem 3 – Consider the following project network.



Determine the critical path and the project duration.

Problem 3 – Solution:

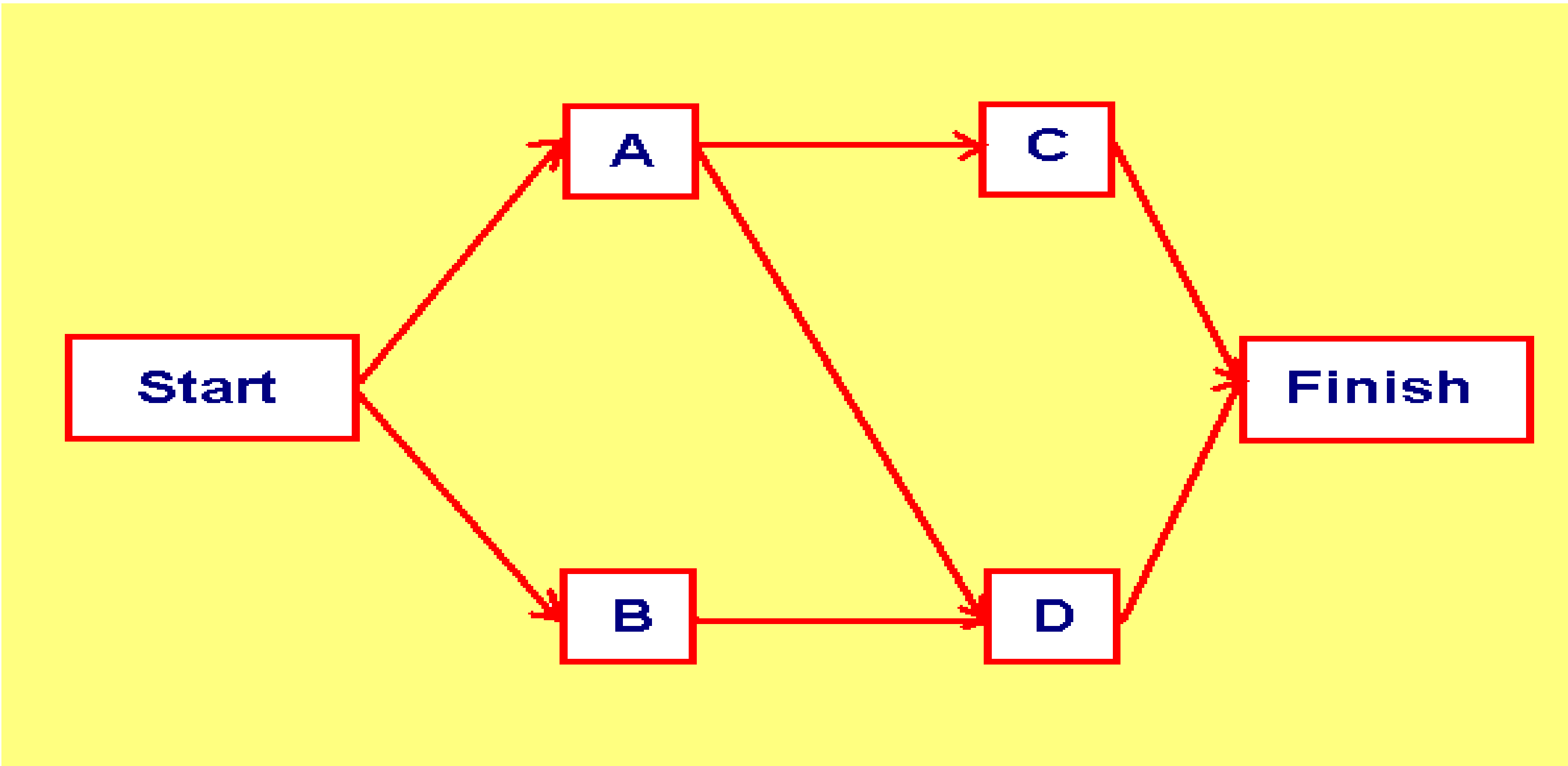
The critical path is A–C–F–H–J with a completion time of 27 days.

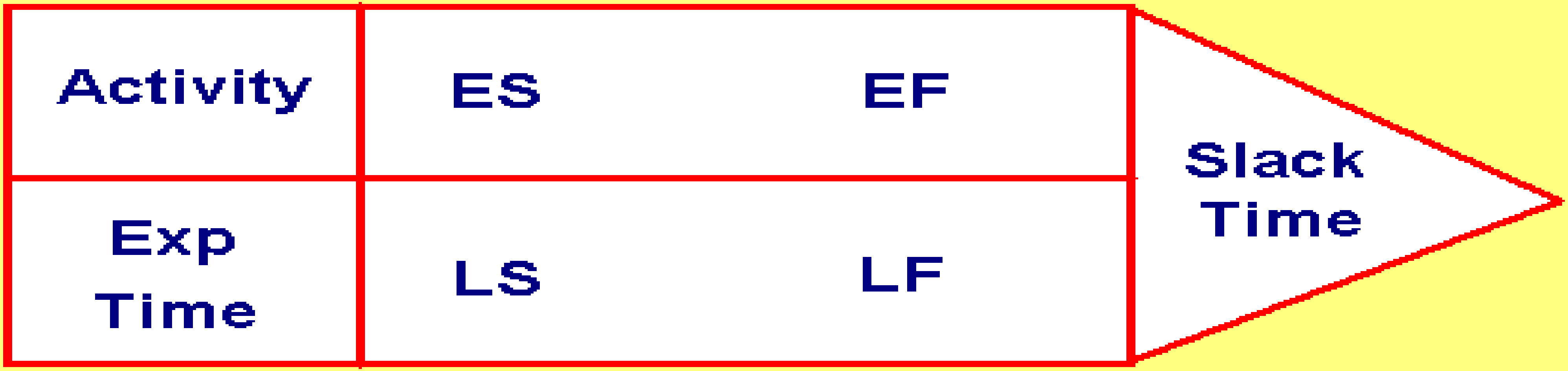
Activity	Duration	Earliest Start	Latest Start	Earliest Finish	Latest Finish	Total Slack	On Critical Path?
A	2	0	0	2	2	0	Yes
B	4	2	3	6	7	1	No
C	5	2	2	7	7	0	Yes
D	2	6	15	8	17	9	No
E	1	6	16	7	17	10	No
F	8	7	7	15	15	0	Yes
G	3	8	17	11	20	9	No
H	5	15	15	20	20	0	Yes
I	4	15	16	19	20	1	No
J	7	20	20	27	27	0	Yes

A Simple Project

Activity	Immediate Predecessor	Expected Time
A	-	5
B	-	6
C	A	4
D	A, B	2

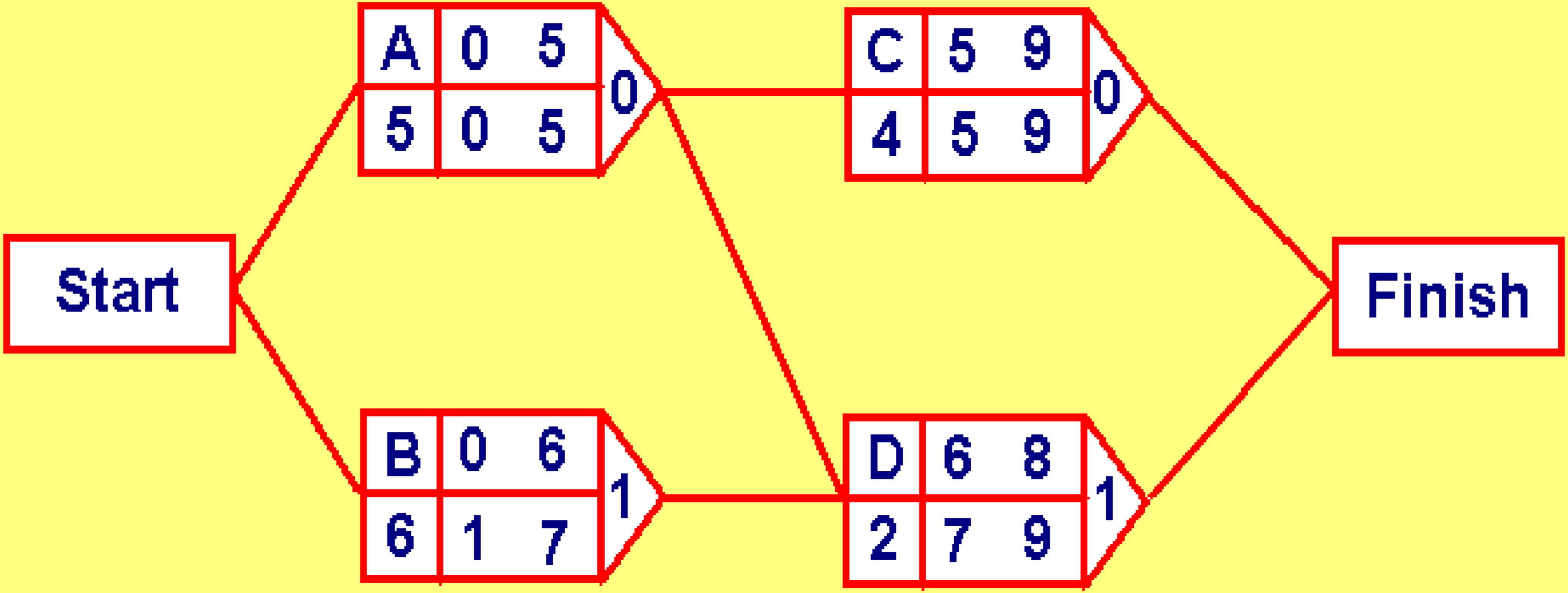
Precedence Diagram





ES	**Earliest Starting (time)**
EF	**Earliest Finishing**
LS	**Latest Starting**
LF	**Latest Finishing**
Slack	**Difference Time**

$$\text{Slack} = LF - EF = LS - ES$$



$$ES[D] = \max\{ EF[A], EF[B] \}$$

$$LF[A] = \min\{ LS[C], LS[D] \}$$

CRITICAL PATH METHODS(CPM)

A **critical path** is the longest path in the network. Each node which falls under critical path has zero or negative float (Slack).

There are 3 steps to calculate CPM:

1. Forward Pass - To calculate the Early Start(ES) and Early Finish(EF) of node.
2. Backward Pass - To calculate Late Start (LS) and Late Finish(LF) of node.
3. Calculate Float and Thus CPM.

critical path of the previous example is A-C

CRITICAL PATH METHODS(CPM)

HISTORY :

It was developed by J.E.KELLY of REMINGTON-RAND and M.R.WALKER of DU PONT and the emphasis was on the trade-off between the cost of project and its overall completion time. The first test was made in 1958,when CPM was applied to the construction of a new chemical plant.

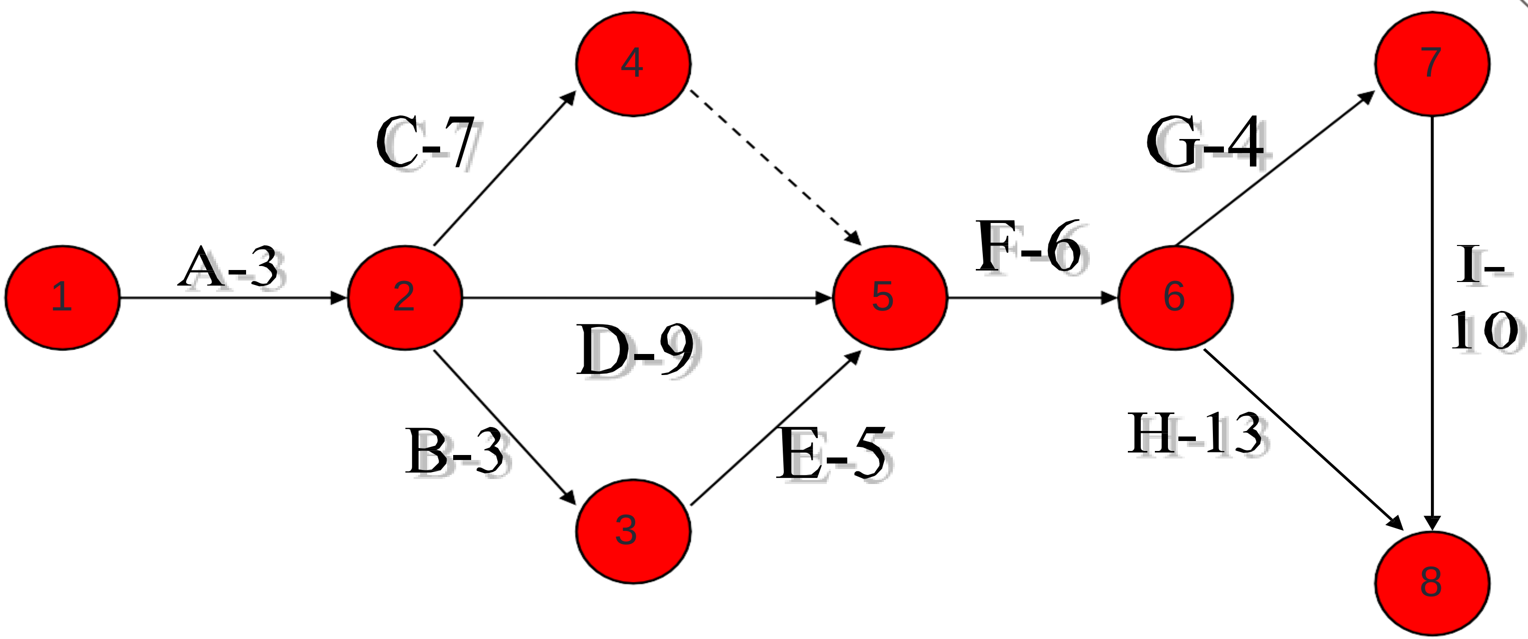
DEFINITION:

Critical path is the sequence of activities between a project's start and finish that takes the longest time to complete.

STEPS IN DETERMINING CRITICAL PATH

- Specify the individual activities.
- Determine the sequence of the activities.
- Draw the network diagram.
- Estimate the activity completion time.
- Identify the critical path.
- Update the CPM diagram.

Activity	Precedence	Normal time (week)	Normal Cost
A	-	3	300
B	A	3	30
C	A	7	420
D	A	9	720
E	D	5	250
F	B,C,E	6	320
G	F	4	400
H	F	13	780
I	G	10	1000
Total			4220



1
A-3
2
C-7
4
D-9
B-3
3
E-5
5
F-6
6
G-4
7
H-13
I-
10
8

Overhead cost as per the given data- Tk.50

Paths in the network diagram :

A-D-F-G-I = 32

A-D-F-H = 31

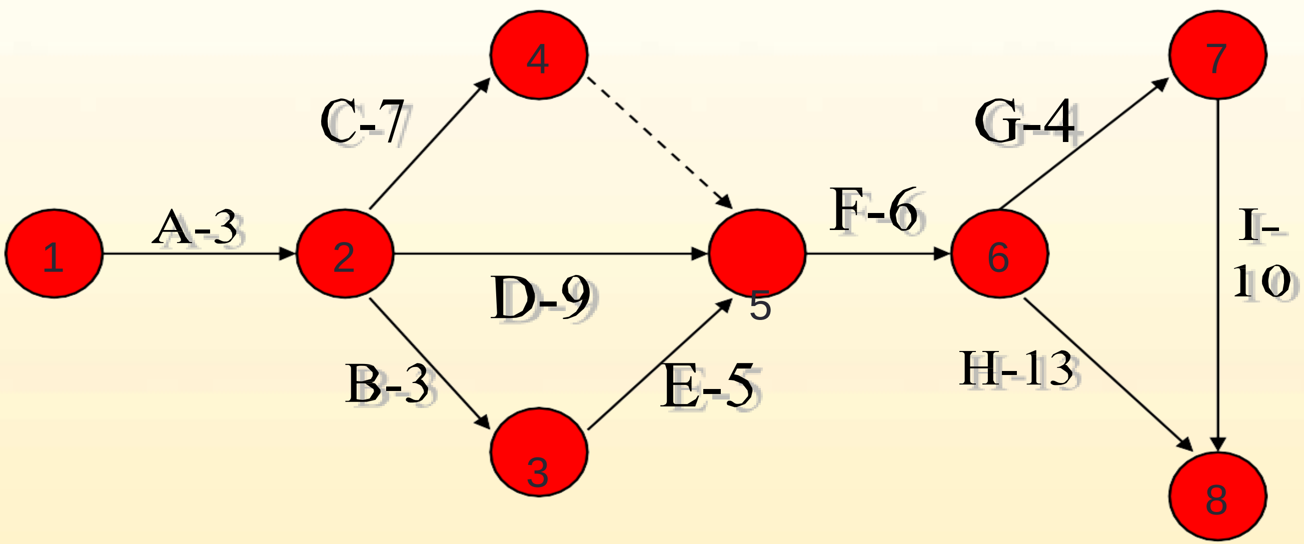
A-C-F-H = 29

A-C-F-G-I = 30

A-B-E-F-H = 30

A-B-E-F-G-I = 31

Critical path – A-D-F-G-I = 32



TIME ESTIMATES

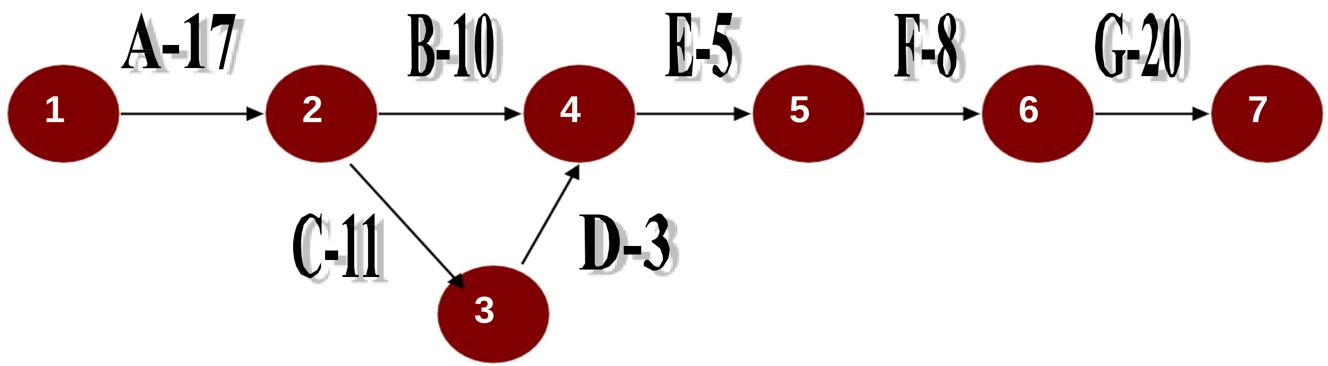
- Optimistic time (to) – It is the shortest time in which the activity can be completed.
- Most likely time (tm) – It is the probable time required to perform the activity.
- Pessimistic time (tp) – It is the longest estimated time required to perform an activity.
- Expected time

$$te = \frac{to + 4tm + tp}{6}$$

STEPS IN PERT

1. Identify the specific activities.
2. Determine proper sequence of the activities.
3. Construct the network diagram.
4. Estimate the time required for each activity.
5. Determine the critical path.
6. Update the PERT chart.

Activity	Description	Precedence	Optimistic time	Most Likely time	Pessimistic time	Expected time
A	Initial design	-	12	16	26	17
B	Survey market	A	6	9	18	10
C	Build prototype	A	8	10	18	11
D	Test prototype	C	2	3	4	3
E	Redesigning	B,D	3	4	11	5
F	Market testing	E	6	8	10	8
G	Set up production	F	15	20	25	20



A-B-E-F-G = 60

A-C-D-E-F-G = 64 (CRITICAL PATH)

Advantages of PERT

- Expected project completion time.
- Probability of completion before a specified date.
- The critical path activities that directly impact the completion time.
- The activities that have slack time and that can lend resources to critical path activities.
- Activity start and end dates.

LIMITATIONS

- The PERT Formula Requires Too Much Work.
- The network charts tend to be large and unwieldy.
- Calculating the time estimates is very complex for all the activities.
- Updating of the project is time consuming and requires high costs.
- Emphasis is laid only on time factors and cost factors are neglected.

Difference between CPM & PERT

CPM	PERT
• CPM works with fixed deterministic time	• PERT works with probabilistic time
• CPM is useful for repetitive and non complex projects with a certain degree of time estimates.	• PERT is useful for non repetitive and complex projects with uncertain time estimates.
• CPM includes time-cost trade off.	• PERT is restricted to time variable.
• CPM- for construction projects.	• PERT- used for R&D programs.

Thank You…