Propositional Logic

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Logical Equivalence

- Two logical statements P and Q are said to be logically equivalent if, and only if, they have identical truth values in every possible interpretation.
- This means that P $\leftrightarrow$ Q (P if and only if Q) is always true, regardless of the truth values of the individual components.

In symbolic terms, $P \equiv Q$

P≡Q indicates that P and Q are logically equivalent.

Logical Equivalence

Can we say **$p \rightarrow q \equiv \neg q \rightarrow \neg p$? YES**

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Logical Equivalence

Prove or disprove the following equivalence using truth table:

$p \rightarrow q \equiv \neg p \vee q$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	**T**	**T**	**F**	**T**
T	**F**	**F**	**F**	**F**
F	**T**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**

Logical Equivalence Law

Law	Representation
Identity	$p \land T \equiv p$ $p \lor F \equiv p$
Domination	$p \lor T \equiv T$ $p \land F \equiv F$
Idempotent	$p \land p \equiv p$ $p \lor p \equiv p$
Negation	$p \land \neg p \equiv F$ $p \lor \neg p \equiv T$
Double negation	$\neg(\neg p) \equiv p$
Absorption	$p \land (p \lor q) \equiv p$ $p \lor (p \land q) \equiv p$

Logical Equivalence Law

Law	Representation
Commutative	$p \land q \equiv q \land p$ $p \lor q \equiv q \lor p$
Associative	$(p \land q) \land r \equiv p \land (q \land r)$ $(p \lor q) \lor r \equiv p \lor (q \lor r)$
Distributive	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
De Morgan's Law	$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$

Logical Equivalence Law

Law	Representation
Conditional	$p \rightarrow q \equiv \neg p \vee q$
Contrapositive	$p \rightarrow q \equiv \neg q \rightarrow \neg p$
Biconditional	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
Exclusive OR	$(p \oplus q) \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$

Propositional Equivalences

Tautology: A compound proposition that is always true no matter what the truth values of the propositions that occur in it is called tautology. Ex:- $P \vee \neg P$

The light is either on, or the light is not on.

Contradiction: A compound proposition that is always false is called contradiction. Ex:- $P \wedge \neg P$

The light is on and the light is not on at the same time.

Logical Equivalence: Compound propositions that have the same truth values in all possible cases are called logically equivalent.

Ex:- $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Propositional Equivalences

Prove that ¬(p→q)≡p∧¬q

$\neg(p \rightarrow q)$

$\equiv \neg(\neg p \vee q)$ [Conditional]

$\equiv \neg(\neg p) \wedge \neg q$ [De Morgan's Law]

$\equiv p \wedge \neg q$ [Double Negation]

Propositional Equivalences

Prove that $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$

$\equiv \neg(p) \land \neg(\neg p \land q)$ [De Morgan's Law]

$\equiv \neg p \land [\neg(\neg p) \lor \neg q]$ [De Morgan's Law]

$\equiv \neg p \land (p \lor \neg q)$ [Double Negation]

$\equiv (\neg p \land p) \lor (\neg p \land \neg q)$ [Distributive Law]

$\equiv F \lor (\neg p \land \neg q)$ [Negation Law]

$\equiv (\neg p \land \neg q)$ [Identity Law]

Propositional Equivalences

Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg(p \wedge q) \vee (p \vee q)$ [Conditional]

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ [De Morgan's Law]

$\equiv (\neg p \vee p) \vee (q \vee \neg q)$ [Commutative & Associative Law]

$\equiv T \vee T$ [Negation Law]

$\equiv T$

Propositional Equivalences

Prove that (p→q)∧(p→r)≡p→(q∧r)

$(p \to q) \land (p \to r)$

$\equiv (\neg p \lor q) \land (\neg p \lor r)$ [Conditional Law]

$\equiv \neg p \lor (q \land r)$ [Distributive Law]

$\equiv p \to (q \land r)$ [Conditional Law]

Propositional Equivalences

Prove that ¬p→(q→r)≡q→(p∨r)

$\neg p \to (q \to r)$

$\equiv \neg(\neg p) \vee (q \to r)$ [Conditional]

$\equiv p \vee (q \to r)$ [Double Negation]

$\equiv p \vee (\neg q \vee r)$ [Conditional]

$\equiv p \vee \neg q \vee r$

$\equiv \neg q \vee p \vee r$ [Commutative]

$\equiv \neg q \vee (p \vee r)$

$\equiv q \to (p \vee r)$ [Conditional]

Propositional Equivalences

Prove that ¬(p↔q)≡p↔¬q

$\neg(p \leftrightarrow q)$	
$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$	[Biconditional]
$\equiv \neg(p \rightarrow q) \vee \neg(q \rightarrow p)$	[De Morgan's]
$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$	[Conditional]
$\equiv (\neg(\neg p) \wedge \neg q) \vee (\neg(\neg q) \wedge \neg p)$	[De Morgan's]
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$	[Double Negation]
$\equiv (p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p)$	[Distributive]
$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$	[Negation, Identity]
$\equiv (q \vee p) \wedge (\neg p \vee \neg q)$	[Commutative]
$\equiv (\neg(\neg q) \vee p) \wedge (\neg p \vee \neg q)$	[Double Neg]
$\equiv (\neg q \rightarrow p) \wedge (p \rightarrow \neg q)$	[Implication]
$\equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$	[Commutative]
$\equiv (p \leftrightarrow \neg q)$	[Biconditional]

Propositional Equivalences

Prove that (p→q)∧(q→r)→(p→r) is a tautology

$(p \to q) \land (q \to r) \to (p \to r)$	
$\equiv \neg((p \to q) \land (q \to r)) \lor (p \to r)$	[Conditional]
$\equiv \neg(p \to q) \lor \neg(q \to r) \lor (p \to r)$	[De Morgan's]
$\equiv \neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor \neg p \lor r$	[Conditional]
$\equiv (p \land \neg q) \lor (q \land \neg r) \lor \neg p \lor r$	[De Morgan's]
$\equiv ((p \land \neg q) \lor \neg p) \lor ((q \land \neg r) \lor r)$	[Commutative & Associative]
$\equiv ((p \lor \neg p) \land (\neg q \lor \neg p)) \lor ((q \lor r) \land (\neg r \lor r))$	[Distributive Law]
$\equiv (T \land (\neg q \lor \neg p)) \lor ((q \lor r) \land T)$	[Negation Law]
$\equiv (\neg q \lor \neg p) \lor (q \lor r)$	[Identity Law]
$\equiv \neg p \lor (\neg q \lor q) \lor r$	[Associative]
$\equiv \neg p \lor T \lor r$	[Negation Law]
$\equiv T$	[Domination]


THANK YOU