Predicates and Quantifiers

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Proposition

x+y=10 – true or false?

A proposition with gap(s)(Number of gaps = number of parameters)

When all the gaps are filled up (directly or indirectly), we get a proposition.

- For x=5 and y=5, x+y=10 (or, 5+5=10) – a proposition
- For x=2 and y=6, x+y=10 (or, 2+6=10) – a proposition?

Predicates

- A predicate is a function that returns a truth value (true or false) based on the input values.

- A predicate is a proposition function
  - Takes as input one or more objects
  - Returns a proposition

A predicate does not have a truth value

$P(x) = x > 3$

" x is greater than 3".

Predicates

We use function-like symbols to represent a predicate

$$P(x,y) : x+y=10$$

- $P(x,y)$ denotes the statement, $x+y=10$
- A predicate with two parameters

    Can there be more parameters?

- $P(5,5)$ is a proposition
- $P(6,4)$ is a proposition

Predicates

1. Let $P(x)$ denote the statement "$x > 3$." What are the truth values of P(4) and P(2)?

2. Let $Q(x, y)$ denote the statement "$x = y + 3$." What are the truth values of the propositions Q(1, 2) and Q(3, 0)?

3. Let $R(x, y, z)$ denote the statement "$x + y = z$." What are the truth values of the propositions R(1, 2, 3) and R(0, 0, I)?

Predicates

- More general than simple propositions
- We can **quantify** objects

……wait, quantify?

Quantifier

Other than putting specific values in a predicate, we can also declare that the predicate is true for every or some values in the domain

Quantification expresses the extent to which a predicate is true over a **quantity** of elements.

Example: For every value of x, $x + 1 > x$

Example: For some value of x, $x < 2$

Symbols that are used to represent this sort of relation is called a quantifier

- For every object – Universal quantifier
- For some object – Existential quantifier

Universal Quantifier

A universal quantifier is a type of quantifier used to express that a predicate applies to all members of a specified set. It is commonly denoted by the symbol ∀ (an inverted A) and is read as "for all" or "for every."

- Let P(x) denote the statement $x+1>x$

For every value of x, $x+1>x$

We can write $\forall x\, P(x)$

∀ is called the universal quantifier

Existential Quantifier

An existential quantifier is used to indicate that a predicate is true for at least one member of a specified set. It is commonly denoted by the symbol ∃ (a rotated E) and is read as "there exists" or "there is at least one.“

- Let Q(x) denote the statement x<2

For some value of x, x<2

We can write ∃x Q(x)

∃ is called the existential quantifier

Exercise

- Let *P(x)* be the statement *"x* + 1 > *x."* What is the truth value of the quantification $\forall x\ P(x)$ where the domain consists of all real numbers?

- Let *Q(x)* be the statement *"x* < 2." What is the truth value of the quantification $\forall \boldsymbol{x}$ *Q(x),* where the domain consists of all real numbers?

Exercise

- What is the truth value of $\forall x\ P(x)$, where P(x) is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

X={1,2,3,4}

P(4) is false

Exercise

- Let $P(x)$ denote the statement "$x > 3$." What is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

- What is the truth value of $\exists x\, P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the domain consists of the positive integers not exceeding 4?

Exercise

C(x)≡x has a cat

D(x)≡x has a dog

F(x)≡x has a ferret

- Express the sentences using these predicates, appropriate quantifiers and logical connectives.

The domain of the variables is the set of all students in your class.

1. A student in your class has a cat, a dog and a ferret
2. All students in your class has a cat, a dog or a ferret
3. No student in your class has a cat, a dog and a ferret

$\neg \exists x (C \wedge D \wedge F)$

$\equiv \forall x (\neg C \vee \neg D \vee \neg F)$


THANK YOU