Effect of Signal Sparsity, Signal Density and Noise Level on Rotational Equivariant Features for Image Classification

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Abstract

Current convolution methods for feature learning are already translation equivariant i.e. translation in input image produces a proportionate translation in feature maps. However, this is not true for rotational equivariance. A lot of recent research has been focussed on ensuring rotational equivariance for the same. Our research focusses on characterizing effect of different input image parameters, like sparsity, density and noise, on effectiveness of the learnt rotational equivariant features proposed in the recent work of Harmonic Convolutions. Feature maps learnt using Harmonic Convolutions exhibit equivariance to patch-wise translation and 360 degree rotation. These variant of normal convolutions use parameter-efficient and low computational complexity representation, thereby encoding complicated rotational equivariance within the network. In this paper, we show the effectiveness of rotational equivariance features for image classification as the sparsity, density and noise levels of the input image vary.

1. Introduction

2. Related Work

3. Problem Analysis

Harmonic Convolutions hard bake 360 degree rotational equivariance into their feature representations by restricting the convolution filters to be from the circular harmonics family. In the following sections, we will discuss the properties of circular harmonics which help the network learn rotation equivariant features. To reiterate, rotational equivariance implies that a particular rotation in input image produces a proportionate rotation in feature maps.

3.1. Circular Harmonics Equivariance

We describe an image using polar co-ordinates r and ϕ as F(r , ϕ). This can be understood better by looking at Figure 1. Please note that this is just to depict the polar co-ordinates of a filter. In actual setting, the filter will be applied patch-wise to the image and therefore, a single image patch will be described using this method.
We will now show that there exists a filter W_m such that the cross-correlation of F with W_m yields a rotationally equivariant feature map. This condition is satisfied when W_m is a circular harmonic of the form W_m = R(r) $e^{i(m\phi+\beta)}$ for some m belonging to integers. Consider the rotation of original image by θ which leads to a new image F(r, $\phi-\theta$). The cross-correlation for the rotated image is

$$\begin{aligned}[\mathrm{W} * \mathrm{F}(\mathrm{r}, \phi-\theta)] &= \int \mathrm{W}(\mathrm{r}, \phi)\, \mathrm{F}(\mathrm{r}, \phi-\theta)\, \mathrm{dr}\, \mathrm{d}\phi \\ &= \int \mathrm{W}(\mathrm{r}, \phi' + \theta)\, \mathrm{F}(\mathrm{r}, \phi')\, \mathrm{dr}\, \mathrm{d}\phi'\end{aligned}$$

where $\phi' = \phi-\theta$. If we replace W_m to be of the form R(r) $e^{i(m\phi+\beta)}$, then the integral transforms as:

$$\begin{aligned}[\mathrm{W} * \mathrm{F}(\mathrm{r}, \phi-\theta)] &= \int \mathrm{R}(\mathrm{r})\, e^{i(m(\phi'+\theta)+\beta)}\, \mathrm{F}(\mathrm{r}, \phi')\, \mathrm{dr}\, \mathrm{d}\phi' \\ &= e^{im\theta} \int \mathrm{R}(\mathrm{r})\, e^{i(m\phi'+\beta)}\, \mathrm{F}(\mathrm{r}, \phi')\, \mathrm{dr}\, \mathrm{d}\phi'\end{aligned}$$

When rotation θ = 0, then $\phi = \phi'$. Therefore, the above equation can be written as

$$\begin{aligned}&= e^{im\theta} \int \mathrm{R}(\mathrm{r})\, e^{i(m\phi+\beta)}\, \mathrm{F}(\mathrm{r}, \phi)\, \mathrm{dr}\, \mathrm{d}\phi \\ &= e^{im\theta}\, [\mathrm{W} * \mathrm{F}(\mathrm{r}, \phi)]\end{aligned}$$

Hence, we observe that the cross-correlation of the rotated signal F(r, $\phi-\theta$) with harmonic filter W_m = R(r) $e^{i(m\phi+\beta)}$ is equal to the response at 0 rotation [W * F(r, ϕ)] , multiplied by a complex phase shift $e^{im\theta}$. Thus, we have shown that cross-correlation with W_m yields a rotationally equivariant feature mapping when W_m is a circular harmonic.

3.2. Chain rule for cross-correlation of circular harmonics

In this section, we will show that the rotation order of a feature map, that we obtain after subsequent cross-correlations in each layer, is equal to the sum of the rotation orders of

Figure 1. An image describing polar co-ordinates with image center as origin

the filters in the chain. Let us consider cross-correlation of a θ rotated image F(r, $\phi - \theta$) with a filter W_m and then followed by another cross-correlation with W_n. As shown previously, we write the response of first cross-correlation as $e^{im\theta}$[W * F(r, ϕ)] . Then we can write the chain rule of harmonic cross-correlation as:

$$\begin{aligned} W_n * [W_m * \mathrm{F}(r, \phi - \theta)] &= W_n * e^{im\theta}[W_m * \mathrm{F}(r, \phi)] \\ &= e^{im\theta}(W_n * [W_m * \mathrm{F}(r, \phi)]) \\ &= e^{im\theta} e^{in\theta} [\ W_n * [\ W_m * \mathrm{F}]] \\ &= e^{i(m+n)\theta} [\ W_n * [\ W_m * \mathrm{F}]] \end{aligned}$$

Therefore, we see that chained cross-correlation results in a summation of the rotation orders of the individual filters W_m and W_n.

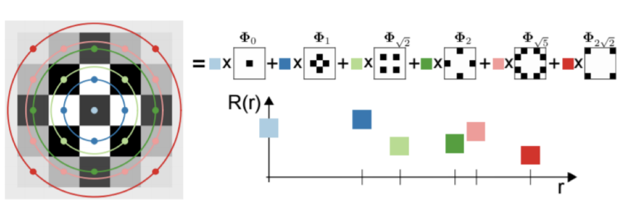

Figure 2. Filter Architecture is defined as linear combination of Radial profile multiplied by corresponding concentric cells in the sampling matrix

3.3. Circular harmonic filter architecture

4. Experiments

5. Conclusion

6. Future Work

7. References

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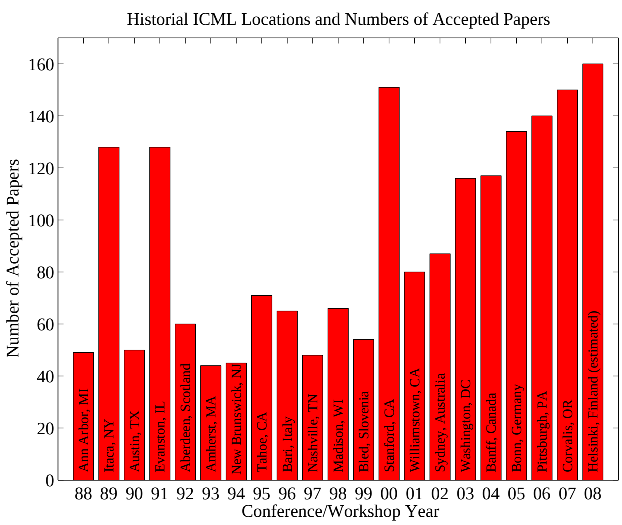

Figure 3. Historical locations and number of accepted papers for International Machine Learning Conferences (ICML 1993 – ICML 2008) and International Workshops on Machine Learning (ML 1988 – ML 1992). At the time this figure was produced, the number of accepted papers for ICML 2008 was unknown and instead estimated.

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Algorithm 1 Bubble Sort

Input: data x_i, size m
repeat
  Initialize $noChange = true$.
  for $i = 1$ **to** $m - 1$ **do**
    if $x_i > x_{i+1}$ **then**
      Swap x_i and x_{i+1}
      $noChange = false$
    end if
  end for
until $noChange$ is $true$

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Table 1. Classification accuracies for naive Bayes and flexible Bayes on various data sets.

DATA SET	NAIVE	FLEXIBLE	BETTER?
BREAST	95.9± 0.2	96.7± 0.2	$\surd$
CLEVELAND	83.3± 0.6	80.0± 0.6	$\times$
GLASS2	61.9± 1.4	83.8± 0.7	$\surd$
CREDIT	74.8± 0.5	78.3± 0.6	
HORSE	73.3± 0.9	69.7± 1.0	$\times$
META	67.1± 0.6	76.5± 0.5	$\surd$
PIMA	75.1± 0.6	73.9± 0.5	
VEHICLE	44.9± 0.6	61.5± 0.4	$\surd$

to a column's top or bottom, and set wide tables across both columns, but place two-column tables at the top or bottom of the page.

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