Analytical Hierarchy Process (AHP)

Pair-wise Comparison Using Trapezoidal Fuzzy sets:

$$A = \begin{array}{c} \\ \text{apple} \\ \text{banana} \\ \text{cherry} \\ \text{Sum:} \end{array} \begin{array}{c} \begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \end{array} \\ \begin{bmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{bmatrix} \\ \begin{array}{ccc} \frac{21}{5} & \frac{31}{21} & 13 \end{array} \end{array}$$

Converting matrix A to trapezoidal fuzzy matrix A:

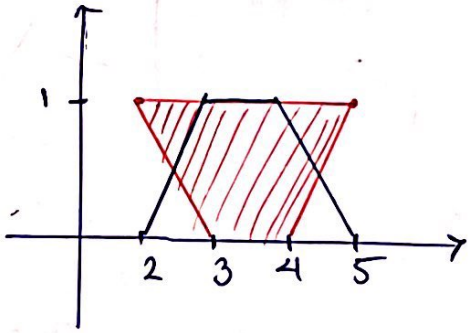
$$A = \begin{bmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{bmatrix} = \begin{bmatrix} (1,1,1,1) & 1/3 & (4,5,6,7) \\ (2,3,4,5) & (1,1,1,1) & (6,7,8,9) \\ 1/5 & 1/7 & (1,1,1,1) \end{bmatrix}$$

So, what about fractional numbers?:

$a=3 \rightarrow (2,3,4,5) \rightarrow a^{-1} = (3,2,5,4)$

$b=5 \rightarrow (4,5,6,7) \rightarrow b^{-1} = (5,4,7,6)$

$c=7 \rightarrow (6,7,8,9) \rightarrow c^{-1} = (7,6,9,8)$



$$\Rightarrow A = \begin{bmatrix} (1,1,1,1) & (3,2,5,4) & (4,5,6,7) \\ + & & \\ (2,3,4,5) & (1,1,1,1) & (6,7,8,9) \\ + & & \\ (5,4,7,6) & (7,6,9,8) & (1,1,1,1) \end{bmatrix}$$

Sum: $(8,8,12,12)$ $(11,9,15,13)$ $(11,13,15,17)$

now we should normalize our matrix:

$$A = \begin{bmatrix} \left(\frac{1}{8},\frac{1}{8},\frac{1}{12},\frac{1}{12}\right) & \left(\frac{3}{11},\frac{2}{9},\frac{4}{13},\frac{4}{13}\right) & \left(\frac{4}{11},\frac{5}{13},\frac{2}{5},\frac{7}{17}\right) \\ \left(\frac{1}{4},\frac{3}{8},\frac{1}{3},\frac{5}{12}\right) & \left(\frac{1}{11},\frac{1}{9},\frac{1}{15},\frac{1}{13}\right) & \left(\frac{6}{11},\frac{7}{13},\frac{8}{15},\frac{9}{17}\right) \\ \left(\frac{5}{8},\frac{1}{2},\frac{7}{12},\frac{1}{2}\right) & \left(\frac{7}{11},\frac{2}{3},\frac{3}{5},\frac{8}{13}\right) & \left(\frac{1}{11},\frac{1}{13},\frac{1}{15},\frac{1}{17}\right) \end{bmatrix}$$

Now we should come to priority vector W:

$$W = \frac{1}{8} \times \begin{bmatrix} (\frac{1}{8},\frac{1}{8},\frac{1}{12},\frac{1}{12}) + (\frac{3}{11},\frac{2}{9},\frac{1}{3},\frac{4}{13}) + (\frac{4}{11},\frac{5}{13},\frac{2}{5},\frac{7}{17}) \\ (\frac{1}{4},\frac{3}{8},\frac{1}{3},\frac{5}{12}) + (\frac{1}{11},\frac{1}{9},\frac{1}{15},\frac{1}{13}) + (\frac{6}{11},\frac{7}{13},\frac{8}{15},\frac{9}{17}) \\ (\frac{5}{8},\frac{1}{2},\frac{7}{12},\frac{1}{2}) + (\frac{7}{11},\frac{2}{3},\frac{3}{5},\frac{8}{13}) + (\frac{1}{11},\frac{1}{13},\frac{1}{15},\frac{1}{17}) \end{bmatrix}.$$

$$W = \begin{bmatrix} (\frac{67}{264},\frac{685}{2808},\frac{49}{180},\frac{2129}{7956}) \\ (\frac{13}{44},\frac{959}{2808},\frac{14}{45},\frac{2713}{7956}) \\ (\frac{119}{264},\frac{97}{134},\frac{5}{12},\frac{173}{442}) \end{bmatrix}$$

Difuzzification of Matix W using Centroid method:

Formula: if $A=(a,b,c,d) \xrightarrow[\text{Method}]{\text{Centroid}} A=(a+2b+2c+d)/6$

$$\Rightarrow W = \begin{bmatrix} 0.258953 \\ 0.323621 \\ 0.417426 \end{bmatrix} \Rightarrow \lambda_{max} = \frac{21}{5}(0.2590) + \frac{31}{21}(0.3236) + 13(0.4174)$$

$$\rightarrow \lambda_{max} = 6.99170$$

$$\rightarrow CI = \frac{\lambda_{max} - n}{n-1} = \frac{6.99170-3}{3-1} = \frac{3.9917}{2} = 1.99585$$

$$\rightarrow CR = \frac{CI}{RI} = \frac{1.99585}{0.58} = 344.1\% \gg 10\%$$

Difuzzification using Center of Sums (COS)

Formula: if $A=(a,b,c,d) \xrightarrow[\text{Sums}]{\text{Center of}} A=(a+b+c+d)/4$

$$\Rightarrow W = \begin{bmatrix} 0.25938825 \\ 0.3222725 \\ 0.4183395 \end{bmatrix} \Rightarrow \lambda_{max} = \frac{21}{5}(0.2593883) + \frac{31}{21}(0.3222725) + 13(0.4183395)$$

$$\rightarrow \lambda_{max} = 7.00358$$

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{7.00358-3}{3-1} = \frac{4.00358}{2} = 2.0018$$

$$\rightarrow CR = \frac{CI}{RI} = \frac{2.0018}{0.58} = 3.451 \times 100 = 345.1\% \gg 10\%$$