Mispricing of S&P500 Index Options: Methodological Artifact or Robust Market Inefficiency

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Abstract

This study investigates the pricing efficiency of the market for one-month S&P 500 index options using two sets of stochastic arbitrage conditions and a distributionally robust optimization. Additionally, factor regression analysis is employed to test for CBOE option strategies exposures. Non-parametric arbitrage conditions are applied to avoid relying on parametric pricing formulas. Two sets of constraint systems are compared: the sufficient but not necessary stochastic arbitrage conditions of Constantinides et al. (2020) and alternative general necessary conditions of Post and Kovalenko (2025). Out-of-sample tests demonstrate that alternative conditions identify stochastic arbitrage opportunities more completely and faster. Robust optimization is used to vary the market risk premium, market volatility, and conditional skewness of returns within plausible bounds. The out-of-sample results point to robust arbitrage opportunities in high-volatility regimes for a plausible range of distribution estimators. Factor analysis distinguishes between risk-adjusted outperformance and risk premiums arising from hidden risk exposures. Factor analysis suggests that predefined option strategies do not explain the outperformance of the option-enhanced portfolio; instead, the gains are driven by month-by-month optimization of option combinations, notably call front spreads with appropriately chosen strike prices. The optimization model, therefore, identifies mispriced options rather than relying on a fixed strategy.

1 Introduction

The pricing efficiency of equity index options remains a question that leads to divergent findings with respect to the existence of stochastic arbitrage. The parametric option pricing model of Black and Scholes (1973) and Merton (1973) involves a central premise that market is complete and frictionless, and the price of an option can be determined by replicating the option payoff. However, assumptions that make the market complete are violated in reality. First, the underlying security's price excludes jumps. Secondly, there are no trading costs. And, thirdly, the volatility of a stock is constant. However, stock market crashes, such as the October 1987 and the 2007 financial crisis, challenge those assumptions (Constantinides et al., 2020).

Recent methodological improvements in stochastic dominance theory enabled the test of option market efficiency without relying on parametric models introduced by, for example, Black-Scholes-Merton. Stochastic arbitrage (SA) allows risk-averse investors to improve their welfare by providing the ability to construct a zero-net-cost option overlay that stochastically dominates a passive index investment, which is a direct violation of market efficiency. The evidence of the SA, however, remains inconsistent.

The study by Constantinides et al. (2020) argued that the optimal portfolio of a risk-averse utility-maximizing investor includes zero-net-cost options with higher mean return and lower volatility as opposed to simply holding an index-tracking portfolio. They find particularly strong mispricing in short-maturity (7, 14 days) and high volatility periods. They assume lognormal index return distribution, and constant market risk premium (MRP). However, Beare et al. (2025) find no evidence of the existence of the systematic stochastic arbitrage opportunities when using the skewed generalized t-distribution with a constant MRP.

Post & Kovalenko, however, find that identifying stochastic arbitrage heavily relies on the completeness of the SA system and the distribution's underlying modeling assumptions. By using the skewed-generalized t-distribution, first adopted by Beare (2025), a conditional on volatility MRP, and introducing more precise sufficient but necessary SA conditions, they find mispricing of options in high volatility regimes. Thus, the contradictory results pose a fundamental question: are found stochastic arbitrage opportunities genuine market inefficiencies, or are they methodological artifacts arising from favorable choices of distribution parameters and SA system?

Resolving the question requires addressing two methodological challenges. First, identifying SA opportunities depends critically on the constraint system. Constantinides et al. (2020) employ sufficient but not necessary conditions based on a single-crossing rule property: the option payoff is negative in good states of index, and positive in bad states, given that the options are zero-net-cost. Post & Kovalenko (2025), on the other hand, employ the alternative system of general sufficient but necessary constraints for identifying SA opportunities, which potentially identifies more opportunities potentially missed in the former system.

Secondly, the distribution of index returns plays a pivotal role in constructing possible index outcomes, or atoms. The distribution that better captures the reality helps the linear program (LP) optimization identify the SA opportunities. However, the parameters used to model the distribution are subject to uncertainty and misspecification, and the robustness of arbitrage opportunities to such misspecifications remains unexplored.

Post (2025) found that pre-defined CBOE option-enhanced strategies substantially underperform the optimized option-combinations in achieving risk-reduction that also account for bid-ask spread. However, the question remaining is to what extent the optimized option-combinations are unique to the pre-defined strategies.

This paper, thus, investigates whether stochastic arbitrage opportunities in S&P 500 index options represent robust market inefficiency or methodological artifacts. The analysis makes three contributions to the literature. First, the distributionally robust optimization (DRO) is implemented to account for plausible parameter ranges of MRP, volatility, and skewness. DRO requires the active option-enhaced portfolio to satisfy arbitrage conditions across those ranges jointly. Secondly, Constantinides et al. (2020) sufficient but not neccesary SA conditions are compared with the Post & Kovalenko (2025) alternative general sufficient but necessary conditions under identical data, distribution, and objective functions assumptions, isolating the effect to just their formulations of SA conditions. Lastly, in order to additionally clarify the source of mispricing, the factor regression analysis is conducted, regressing the excess return of an active portfolio on CBOE option strategies.

Three main findings emerged. First, DRO strengthens rather than eliminates arbitrage opportunities: it identifies more SA opportunities, achieved higher realized return, and delivers better outcome for risk-averse investors. Secondly, the general out-of-sample pattern of both SA systems remains, meaning that option mispricing does violate market efficiency. However, the alternative system finds more complete solutions and 3.6 times faster. Lastly, the factor analysis reveals that pre-defined CBOE option strategies explain less than 5% of variance in optimized portfolio excess returns, signifying that optimization provides a unique option combination strategy.

These findings provide robust evidence of S&P 500 option market inefficiency during high-volatility periods. The consistency of results across fundamentally different constraint formulations, persistence under distributional uncertainty, and failure of attribution to predefined strategies collectively indicate genuine mispricing rather than methodological artifact. Low and medium VIX regimes show no evidence of arbitrage with active portfolios generating losses losses, indicating pricing efficiency violations are regime-specific rather than pervasive.

The remainder of the paper is organized as follows. Section 2 provides the theoretical framework for stochastic arbitrage and describes the two alternative constraint systems. Section 3 describes the data and filtering procedures. Section 4 presents the methodology. Section 5 reports empirical results. Section 6 concludes.

2 Theory

2.1 Stochastically Dominating Portfolios

Stochastic arbitrage (SA) is a weaker form of arbitrage opportunity that violates the efficient market hypothesis. Unlike classical arbitrage, stochastic arbitrage occurs when an investor can construct an active portfolio that provides a zero or positive net premium from writing and buying options, and produces payoff that stochastically dominates the benchmark passive index-tracking portfolio (Beare et al. (2025). SA involves probabilistic dominance, which means it is preferred by all risk-averse investors, even though it may underperform in some states of the world. Essentially, in an environment of decreasing utility function, SA allows investors insure against bad states at no cost. An investor, who holds an active portfolio, holds a passive index portfolio S plus a zero-net cost portfolio with a payoff $X = \mathbf{x}'\boldsymbol{\alpha} - \mathbf{x}'\boldsymbol{\beta}$ thus, option-enhanced return is Z = X + S. In Post&Kovalenko(2025), they provided a precise definition of stochastic dominance in terms of the lower partial moment function (LPM) $\mathcal{L}(Z \mid t) = E_{\mathcal{F}}[(t - Z)\mathcal{I}(Z \leq t)] = \sum_{j=1}^{N} p_j(t - Z_j)\mathcal{I}(Z_j \leq t)$, where $\mathcal{I}(\cdot)$ is a binary indicator of whether the conditions within the brackets are true or not. Active payoff, Z, which is payoff from options and index return, second-order stochastically dominates index return if and only if it is self-financing and at every possible threshold t the active portfolio Z has less downside risk than the passive index portfolio S, mathematically denoted as $\mathcal{L}(Z \mid t) \leq \mathcal{L}(S \mid t)$, for all $t = S_j, j = 1, ..., N$.

Constantinides et al. (2020) provide a utility-theoretic interpretation of SA. They argue that if an investor holds an index portfolio with a "no-transaction region" (where due to transaction costs rebalancing is suboptimal, and because the option maturity is short), the marginal utility of index hodlings is monotone decreasing with S_t. Thus, if there is a zero-cost option overlay with single-crossing payoff structure - positive at low index values, and negative at high index values, as well as a positive expected payoff, it increases the utility of a risk-averse investor.

Those two theoretical interpretations provide conditions for SA, however, Post & Kovalenko requires conditions to be necessary, while Constantinides et al. (2025) are based on the premise of sufficiency but not necessity, which might lead to exclusion of some valid dominating portfolios.

2.2 Stochastic Arbitrage System

2.2.1 General assumptions and preliminaries

For both SA systems, general assumptions hold. Risk-free rate and index portfolio are assumed to have zero bid-ask spread. Options' contingent payoffs are $X_i = max(S - K_i, 0)$ for calls, and $X_i = max(K_i - S, 0)$ for puts, where K_i is a strike price. Option payoffs, $(X_{3,j}, ..., X_{M,j})'$ are contingent upon the index value S_j, making the SA system a single factor system. Thus, the joint distribution of option payoffs, $x := (X_i, ..., X_M)'$ relies solely on the distribution of index, $\mathcal{F}$. In order to construct optimal option combinations and test the pricing efficiency of index options, it is necessary to know the distribution $\mathcal{F}$, although the unique distribution does not exist. Both SA approaches adopt a discrete representation of the distribution to allow numerical optimization, where S_j can be modeled as atoms $S_1 < ...S_N$ with corresponding probabilities $p_j > 0$ for $j = 1, ..., N$. However, the choice of the shape of the distribution and market risk premium (MRP) greatly affects the out-of-sample outcome of the chosen option set. While Constantinides et al. (2020) chose a simple lognormal assumption of the index distribution and constant MRP of 4%, Post & Kovalenko (2025) found that negative skewness and conditional MRP produce a better out-of-sample active portfolio performance. Therefore, in this paper, the Skewed Generalized t-distribution (SGT) and conditional MRP are used to model S_t, but the performance of two SA systems is compared on the premise of different SA conditions formulations.

Both approaches assume co-monotonicity, ensuring that option overlay is a modest adjustment, and not an independent leveraged speculation. An investor holds one unit of index. $w_i \geq 0, i = 1, ..., 2n$ denote the number of both call and put options from the set of available realistic options. The total option position is one option per index, such that:

$$0 \leq \sum_{i=1}^{2n} w_i \leq 1, \quad \Pi = \sum_{i=1}^{2n} w_i c_i \tag{1}$$

This position limit allows for fractional option positions, such that common combinations can be formed such as covered calls or protective puts (Post&Kovalenko, 2025). Stochastic arbitrage opportunity is an option combination, such as call front spread, that generates a positive payoff when the market is bearish, but a negative payoff if the market is bullish. Figure 1 demonstrates the contingent payoff diagram of a call front spread. In a call front spread, probability of higher state outcomes is shifted toward lower state outcomes, which is appealing to risk-averse investors.

2.2.2 Sufficient but not Necessary Conditions System

$V(x_t, y_t, t)$ is an indirect utility function where (x_t, y_t) are cash and index holdings, respectively. $V(x_t, y_t, t)$ is strictly increasing and concave in (x_t, y_t), but monotone decreasing with respect to y_t, which assumes $V(x_t, y_t, t + 1)$ is monotone decreasing in the stock price (S_{t+1}) (Constantinides et.al, 2020). $A(S_{t+1})$ represents the payoff of the zero-net-cost portfolio at t+1. If:

$$E_t \left[\Delta(S_{t+1}) V_y(x_{t+1}, y_{t+1}, t + 1)\right] > 0, \tag{2}$$

the investor increases the utility by overlaying zero-net-cost option-enhanced portfolio over passive index-tracking portfolio. The following sufficient conditions ensure Equation (1) holds:

Lemma. *A sufficient condition for Equation (1) to hold is that (a)* $E_t[\Delta(S_{t+1})] \geq 0$ *and (b) there exists a number* $\hat{S}$ *such that* $\Delta(S_{t+1}) > 0$ *for* $S_{t+1} \leq \hat{S}$ *and* $\Delta(S_{t+1}) \leq 0$ *for* $S_{t+1} > \hat{S}$ (Constantinides et.al, 2020). The methodology involves two steps. First, an active option-enhanced portfolio constructed each month by overlaying the passive portfolio with a zero-net-cost option portfolio with a payoff $A(S_{t+1})$. The set should contain options that can be traded in a realistic environment. Thus, the option filter removes obvious errors, checks put-call-parity, and synchronizes option prices to their strike prices. The second step involves verifying out-of-sample performance of an active portfolio relative to the passive index portfolio. The comparison is done at option expiration date corresponding to an index value at $t + 1$.

Given Lemma, at $t = 0$, firstly, the grid of feasible values of $\hat{S}$ is built, which is used to find zero-net-cost options combination from a pool of available options. The contraints are:

$$A(S_{t+1}) > 0 \text{ for } 0.6S_t \leq S_{t+1} \leq \hat{S}; \text{ and } A(S_{t+1}) \leq 0 \text{ for } S_{t+1} > \hat{S}. \text{ Lastly },E_t[A(S_{t+1})] > 0. \tag{3}$$

The active option-enhanced portfolio is found by solving the following Linear Program (LP):

$$max_{w_i} E[A(S_{t+1})] \text{ given } \hat{S}, \tag{4}$$

The maximum value of $\hat{S}$ is restricted to $1.15 \times S_t$. As such, the grid of values among which the optimal portfolio is found is $[S_t; 1.15S_t]$. After maximizing option payoff $E[A(S_{t+1})]$ at every possible $\hat{S}$, the optimal portfolio is chosen which is the one that achieves the best possible value of the selection criterion among all feasible alternatives. The main selection criterion is the maximum expected return of an active portfolio, because it directly compares to the alternative constraints of Post & Kovalenko (2025), although Constantinides et al. (2020) use other selection criteria such as the Sharpe Ratio, Sortino Ratio, and gain/loss ratio, which produce similar results.

2.2.3 Alternative System: General Necessary Conditions

The algorithm for computing the second-order stochastic arbitrage opportunity is a linear program. Checking LPM at every threshold t would be computationally expensive, therefore, Post&Kovalenko (2025) adopt a co-moments constraints approach. Their general definition of SA opportunity is a zero-net-cost overlay portfolio that dominates a passive index portfolio by Second-degree Stochastic Dominance (SSD):

$$\begin{gathered} \mathbf{x}'\boldsymbol{\alpha} - \mathbf{x}'\boldsymbol{\beta} + S \succeq_F S; \\ \mathbf{a}'\boldsymbol{\alpha} - \mathbf{b}'\boldsymbol{\beta} < 0; \\ (\boldsymbol{\alpha}, \boldsymbol{\beta}) \in \Omega. \end{gathered} \tag{5}$$

Such definition takes into account bid-ask spread, no portfolio re-balancing or dynamic hedging, or assumptions about risk attitudes.

The alternative system features an overlay portfolio with positive net premium from writing and buying options, $(a'\alpha - b'\beta)$, unlike Constantinides et al. (2020), which requires zero-net-cost portfolio. Also, it requires non-negative partial co-moments $C'_j(\alpha - \beta) := C(x'(\alpha - \beta)|S_j), j = 1, ..., N,$ where $C(x|t) := E_{\mathcal{F}}[x\mathcal{I}(S \leq t)] = \sum_{j=1}^{N} p_j X_j \mathcal{I}(S_j \leq t)$. C'_j represents the payoff of the option overlay portfolio at expiration. The co-moment C'_j measures the expected option payoff conditional on the index value falling below threshold S_j. When the co-moment is non-negative for all thresholds, the active option portfolio payoff provides downside protection by ensuring positive expected payoff during adverse market outcomes. The co-moments. thus, measures the reduction of the downside risk, provided by the LPM, across all threshold by overlaying option combinations with indexed portfolio. As such, SSD requires $\mathcal{L}(Z \mid t) \leq \mathcal{L}(S \mid t)$, for all $t = S_j, j = 1, ..., N$. By enforcing these constraints to be non-negative, the Alternative System ensures that overlaying options reduces LPMs at all atoms of the discretized distribution, thereby establishing SSD in a computationally tractable manner. The alternative system as a result only consists of 2M model variables, and N+1 linear constraints, where M is the number of options and N is the number of scenarios, unlike $(N^2 + 2M)$ variables and $(N^2 + N + L + 1)$ constraints in Post&Longarela (2021) or the $(N^2 + 2M)$ variables and $(4N + L + 1)$ constraints in Beare et al. (2025).

The alternative system can be summarized as follows:

$$\begin{aligned} \sum_{i=1}^{M} C(X_i|S_j)\alpha_i - \sum_{i=1}^{M} C(X_i|S_j)\beta_i \geq 0, \quad j = 1, \ldots, N; \\ \sum_{i=1}^{M} A_i\alpha_i - \sum_{i=1}^{M} B_i\beta_i < 0; \\ \alpha_i, \beta_i \geq 0, \quad i = 1, \ldots, M. \end{aligned} \tag{6}$$

3 Data

The dataset was obtained from the OptionMetrics IvyDB US database and contains 325 months for the period January 1996 to February 2023. The dataset is cleaned for obvious data errors, such as bid prices greater than ask pricing, missing or multiple same values. The data is filtered according to put-call parity involving a one-way transaction cost set at 10 basis points (Constantinides et al., 2020). Following Constantinides et al., liquidity filters are applied to ensure that only options that are tradable under realistic conditions are admitted to the pool. Call options are admitted based on bid prices of at least 0.15 dollars and moneyness range 0.96-1.08. For put options, however, moneyness range is 0-1.04, and bid prices of at least 0.15. The reason for asymmetric filters for calls and puts is greater liquidity of OTM puts. Lastly, the synchronization of spot prices was applied, obtaining the moneyness hat used for moneyness filters. For synchronization, put-call parity was used by using midpoint option prices of a pair of puts and call with the same stike price $K_i = K_j$ Then, $\hat{S}_{0,i} := (C_i - P_i + R_B^{-1}K_i + R_B^{-1}D)$ is computed for all matched pairs, where the midpoint of the call is $C_i := \frac{1}{2}(A_i + B_i)$ and the midpoint of a put is $P_i := \frac{1}{2}(A_j + B_j)$, and $\hat{S}_{0,i}$ is averaged out to obtain a synchronized S_0.

In order to compute Mincer-Zarnowitz coefficients and three parameters for the generalized skewed distribution, historical SPX monthly and daily prices data was obtained from the CRSP database, respectively. Furthermore, the VIX index dataset was obtained from the CBOE website - both archive dataset from 1986 to 1990, and the newest dataset from 1990 to 2023. Lastly, the Strategy Benchmark Indices were obtained from the CBOE website for conducting the factor analysis.

For every month in the sample, 29 days to maturity (DTM) options are chosen as priority in the evaluation. If the 29 DTM is not available, the 28 DTM options are chosen. To avoid forward-looking bias by replacing 29 DTM options with better 28 DTM options, the decision to trade certain options is assumed to be made in advance.

4 Methodology

4.1 Distribution estimation and Distributionally Robust Optimization

4.1.1 Market Risk Premium

In Constantinides et al. (2020), Beare et al. (2025), and Post & Longarela (2021), MRP is assumed to be constant across all formation dates. However, Post & Kovalenko (2025) introduce the time-varying MRP that increases with variance. Post & Kovalenko (2025) specify $\lambda = \gamma\sigma^2$, where γ is a coefficient of relative risk aversion (RRA), and λ is implied Equity Risk Premium. RRA is assumed to be relatively stable. The authors employ an expanding window from 1960 to 2023 to first estimate $\hat{\lambda}$ and $\hat{\sigma^2}$. Then, using the estimated values, the authors obtain median values and estimate $\hat{\gamma} = \tilde{\hat{\lambda}}/\tilde{\hat{\sigma}}$. The value of $\hat{\gamma}$ is constant and is equal to 3.25. Thus, in the projection of returns, MRP changes for each formation date with the constant $\hat{\gamma} = 3.25$ in the baseline. The authors report that empirical estimated of γ range from 2 to 5, and 3.25 falls to the midpoint. Although the authors report the standard error of $\hat{\lambda}$ to be only 0.19%, the true value is still subject to parameter uncertainty. Thus, in the Distributionally Robust Optimization (DRO) specification, the plausible range of $\gamma \in [2, 5]$ is considered to examine the sensitivity of the LP optimization set up to the parameter uncertainty.

4.1.2 Volatility

It is argued that option-implied measures of volatility such as VIX tend to overestimate realized volatility (Beare et al., 2025), therefore, it is important to deflate it. Post & Kovalenko (2025) use the Mincer-

Zarnowitz forecast evaluation regression model in the form $\hat{\sigma_t} = c_0 + c_1 VIX_{t-1}$ to correct the overestimation. $\hat{\sigma_t}$ is the 21-day volatility after the formation date using the VIX data, as well as the daily SPX returns. The coefficients c_o and c_1 are also re-estimated at each formation date using an expanding window from 1986 to 2023. The median estimates are $\tilde{\hat{c_0}} = -0.0014$ and $\tilde{\hat{c_1}} = 0.7833$, which means the realized volatility is typically about 78% of the VIX level.

The estimated volatility is also a subject of parameter uncertainty. Thus, the ambiguity set is constructed $\sigma_{DRO} \in [0.9 \times \hat{\sigma_t}, 0.9 \times \hat{\sigma_t}]$. The $\pm 10\%$ range is arbitrary and represents plausible deviations from the point of forecast.

4.1.3 Shape

The standard normal distribution fails to capture the asymmetry of the return distribution, it is necessary to model skewness and apply dynamic configurations. Thus, SGT, introduced first by Theodossiou (1998) provides a better fit for the data (Beare, 2025). The SGT distribution involves three parameters: 1) shape, $\hat{\theta_1}$; 2) degrees of freedom, $\hat{\theta_2}$; 3) and asymmetry, $\hat{\theta_3}$. Those parameters are re-estimated as maximum likelihood estimates at each option formation date using an expanding window approach. The median estimates are 1.82 for shape, 3.61 for degrees of freedom, and -0.24 for asymmetry. Thus, following Beare et al. (2025) and motivated by the findings of Cont (2001), distribution has high tail thickness, as indicated by low $\hat{\theta_1}$ and $\hat{\theta_2}$.

To generate a discrete approximation of the SGT distribution suitable for LP, the Cox et al. (1979) binomial tree is employed to construct a probability vector for each modeled month. The standardized return scenarios are obtained by inverting the SGT cumulative distribution function. Post & Kovalenko (2025) median parameter estimates are $\hat{\theta_1} = 2.07$ $\hat{\theta_2} = 5.00$ $\hat{\theta_3} = -0.26$, which deviates from the median estimates found in this paper. Therefore, shape parameters are also subject to parameter uncertainty. Post & Kovalenko (2025), instead of assuming lognormality, improve on Constantinides et al. (2020) by using a more flexible, negatively skewed distribution which better reflects real distribution. Thus, skewness seems to be a natural candidate for testing under the DRO. As such, the ambiguity set for the skewness parameter is $\in [0.9 \times \widehat{skew}_t, 1.1 \times \widehat{skew}_t]$.

4.2 Comparison of Stochastic Arbitrage Systems

A central methodological question in stochastic dominance-based option portfolio optimization concerns computational tractability and completeness of the set of constraints used to identify SA opportunities. Those two alternative formulations of the set of SA conditions are compared using the same assumptions about the distribution, identical SGT parameter estimates, and an objective function. One of important constraints used in the Constantinides et al. approach is that the portfolio should be zero-net-cost, while in Post&Kovalenko approach involves negative net cost (positive net premium: $a'\alpha - b'\beta < 0$, which is used as an objective to be maximized in a baseline optimization). Thus, in order to compare those two approaches, it is necessary to set the same objective for maximization. Expected active portfolio return is used as an objective, since it is one of the alternative objectives that produce similar robust results. Therefore, under this structure the effect of the constraint structure is isolated to observe the performance OOS.

Constantinites et al. (2020) derive sufficient but not necessary conditions for a SA based on a single-crossing property. For each candidate $\hat{S}$ a separate LP is solved with the optimal solution being the one that yields the highest expected return among all feasible $\hat{S}$ values. The search over the grid of $\hat{S}$ substantially increases the computational burden, which requires solving the LP for each value in a grid per formation date rather than one single LP. To reduce computational burden, 60 values were chosen.

Under Post&Kovalenko system, no grid search is required, and under co-monotonicity assumptions, one LP is solved per formation date. The system provides exact definition of the SA opportunities - any portfolio that satisfies position limit, co-moment constraints and is self-financing dominates the passive index portfolio by SSD.

The distribution was divided into 5000 atoms and truncated by removing states with cumulative probabilities smaller than 1×10^{-13} and larger than (1-1 $\times 10^{-13}$ for numerical stability.

4.3 Evaluation Method

The optimized option combination is constructed at the option formation date and the options are bought and written at corresponding bid and ask prices. Options are held to maturity and both 28 and 29 DTM options expire on the same date. For the baseline option-enhanced portfolio, the maximization of net-premium constraints is used, and the DRO option-enhaced portfolio with the same objective is compared to it. For the two SA systems, the maximization objective is set to the expected active option-enhanced portfolio return. The performance of option-enhanced portfolios is reported in tables for all and each of the three VIX regimes (table 1-table 4). The composition of the option combinations is reported as medians with inter-quartile ranges in the square brackets of bought and written calls and puts. The moneyness and implied volatility are reported as median weighted averages, computed only from months where the optimization finds a non-zero solution to avoid inflation with zeros. The projected distribution panel in the tables reports annualized statistics that is based on the distributional model (SGT). Mean, standard deviation, skewness, and Conditional Value-at-Risk (at 99% confidence level) are reported. In addition net premium is reported as a percentage of the index at the time of formation date.

The improvements achieved by the active portfolio are described by annualized mean and ccertainty-equivalent returns (CERs) with RRA values of 1, 4, and 10. Statistical significance is reported with p-values.

Last but not least, Empirical Likelihood Ratio test, first proposed by Arvantis and Post (2024) is used to assess if the optimized option combination satisfies the dominance constraints out of sample. The p-value is reported in round brackets. The null hypothesis is that the active portfolio dominates the passive index portfolio by SSD. The ELRT value indicates the strength of the possible violations of stochastic dominance, with a p-value close to 1 corresponding to the failure to reject the null of dominance. The CERs, on the other hand, evaluate how strong dominance relationship is, and a corresponding low p-value indicates statistical significance.

4.4 Comparison with Standard Option Strategies

Post & Kovalenko (2025) find that the strongest outperformance is in high VIX regime that is achieved by optimized combination of options rather than by a pre-defined option strategy. The most frequent combination, though, resembles call-front spread. However, to what extent this call-front spread and other option combinations resemble pre-defined strategies is not discussed.

In order to investigate whether the outperformance generated in the high VIX regime can be explained by the exposure to existing option strategies, the comparison with CBOE option strategies is conducted. For that purpose a simple multi-factor regression model is used. The regression is specified as:

$$R_{OT,t} - R_{IT,t} = \alpha + \sum_{j=4}^{4} \beta_j F_{j,t} + \epsilon_t \tag{7}$$

Regression is performed upon excess return rather than absolute return to isolate the contribution of the overlay portfolio from the dominant passive index return component. The common index exposure would dominate any regression of absolute returns, obscuring similarities or differences in the option strategies themselves. Factor loadings (β_j) indicate the degree of correlation with each benchmark strategy. The null hypothesis is $\alpha = 0$ - outperformance can be explained by the exposure to the CBOE benchmark option strategies. Rejection of the null indicates that the LP-optimized option strategies are able to generate returns not replicable by passive exposure to those strategies, consistent with option mispricing. However, the purpose of this analysis is not to explain the return, but rather to find a hidden factor exposure. A potential alternative explanation for finding SA opportunity is that the LP-optimized option combinations just merely replicate some pre-defined strategies. If true, then SA would simply be a compensation for bearing systematic option-strategy risk.

CBOE option strategies are subject to multicollinearity, therefore VIF is computed for all strategies to select 4 strategies with low cross-correlation (VIF ¡ 3.5).

5 Results

5.1 Distributionally Robust Optimization

DRO is used to address misspecification of estimated parameters for the distribution, in particular, the Mincer-Zarnowitz bias-correction coefficients (c_0, c_1), the skewness parameter (θ_3), and the aggregate relative risk aversion coefficient ($\gamma = 3.25$). Rather than optimizing against a single projected distribution with certain estimated parameters, DRO requires the LP to satisfy arbitrage conditions across $2\times2\times2$=6 different scenarios (γ : 2 values $\times$ vol: 2 values $\times$ skew: 2 values . The DRO optimization follows the same procedure for satisfying SA conditions by generating non-negative co-moments and maximizing the objective (net premium). Computationally, this requires generating 9 separate projected distributions and 9 corresponding payoff matrices, although the tractable formulation still allows for faster execution. For the DRO approach, the number of grid atoms used is 150, and for the baseline it is 1000. The difference is because the solver has to find 150×9 solutions, which is already computationally heavy for the hardware at use.

Table 1 presents the baseline results, while table 2 presents results obtained using DRO. Overall, the DRO approach yields results that are similar to the baseline results, however exhibit improvements. First, DRO identifies more non-zero solutions: 265 against 282 for the baseline, and DRO, respectively. Strike selection of written calls appears to be more selective as demonstrated by a decreased IQR range. In the projected distribution section, both passive and active projected returns are higher across all regimes, as well as lower projected volatility, resulting in improved risk-adjusted performance. Furthermore, the higher premium income across all regimes suggests that the robust optimization shifts more towards extracting value through selling options, rather than relying on future performance from directional bets and precise forecasts.

The DRO approach also produces slightly higher realized active portfolio outperformance in all VIX regimes (0.10 vs 0.08), as well as higher returns in low and high VIX regimes. For more risk-averse investors, the gains from the robust optimization are the most pronounced in the high VIX regime (2.98 vs 2.51). The most striking difference is the reduction in losses from BuyPuts (-0.09% vs -0.45%), suggesting that DRO better identifies when long puts is an expensive insurance, and when it is more cost-effective. Across all VIX regimes CER statistics also demonstrates improvements across all types of risk-aversion.

In medium VIX regimes, DRO worsens performance even more with larger losses and stronger rejection of dominance. The reason might be robust optimization shifting to finding SA in more volatile market, rather than in transitional market conditions.

Such regime-specific performance reinforces that parameter uncertainty is problematic, but more in high VIX regime, where distributional features change more rapidly, consistent with earlier findings that options in high-volatility environments exhibit mispricing, violating the efficiency of the market. In calmer markets precise point estimates seem to be more reliable.

The ELR t-test suggests that SSD remains perfect out-of-sample. That confirms that accounting for parameter uncertainty does not divert arbitrage opportunities in high VIX regime, but even strengthens them.

The DRO framework demonstrates that results of the optimization to the alternative framework of general necessary constraints was not the result of overfitting to certain parameters, but rather is a robust implementation, consistent with earlier mispricing of high VIX options conclusions.

5.2 Comparative Analysis of Two Sets of Stochastic Arbitrage Systems

Table 3 summarizes the results obtained from the SA sufficient, but not necessary conditions. Table 4 summarizes results based on the alternative set of general necessary conditions. First, the number of non-zero solutions in the alternative system is higher across all regimes (255 vs 240 in the all scenario; 90 vs 86 in low VIX; 88 vs 84 in medium VIX, and 77 vs 70 in high VIX), suggesting that single-crossing rule may exclude some feasible solutions. Both approaches, however, yield similar pattern in portfolio construction where writing calls dominate.

Projected risk-adjusted performance is similar in both cases, with PK(2025) exhibiting higher projected returns in active portfolios and slightly lower volatility.

The critical test is whether the additional non-zero solutions detected using the alternative general approach translate into better OOS realized returns. The general approach shows consistent modest advantages in realized returns (0.13% vs 0.08% across all VIX regimes) that strengthen for more risk-averse investors. In high VIX regimes realized returns are almost identical, with the alternative approach showing modest advantage for risk-averse investors (CER_{10} 2.29 vs 2.07).

The computational burden differs dramatically between two approaches. System specs of the PC are: 2.50 GHz CPU and 16.0 GB RAM. The software used is Python with a SCS solver. under the sufficient only approach, the total solution time is 18 minutes and 30 seconds across all 319 formation dates, while for the general approach, the solution time drops more than 3 times to 5 minutes.

In conclusion, both methods identify option mispricing in high VIX regime under a more theoretically justified distribution assumptions. Empirical results, however, validate three theoretical advantages of the alternative SA system. First, it is more complete, as demonstrated by more 15 more non-zero solutions. Secondly, it is much more computationally efficient (3.7× faster). Thirdly, it allows for scalability, because it only requires solving one LP, not the grid. Lastly, it produces a higher OOS realized return. Those factors confirm that the framework of Constantinides et al. (2020) of sufficient only conditions is incomplete and exclude some valid SSD portfolios. Both approaches, however, yield a similar general pattern, reinforcing that core SA opportunities are robust, and the market is inefficient during the high volatility environment.

5.3 Comparison with CBOE otpion strategies

Table 4 demonstrates the result of the regression analysis comparing the realized LP-optimized option combinations with pre-defined CBOE option strategies. There is a severe multicollinarity among covered call strategies (BuyWrite, OTM BuyWrite, and Conditional BuyWrite exhibit correlations exceeding 0.98). Thus, 4 factors are selected with acceptable Variance Inflation Factors: Put Protection (VIF = 3.27), Iron Condor (VIF = 2.83), Iron Butterfly (VIF = 1.65), and Conditional BuyWrite (VIF = 4.30).

The 4 CBOE strategies collectively explain less than 5% of the variance in option-enhanced portolio's excess return. Among 4 factors only IronCondor strategy exhibits statistically significant loading ($\beta = 0.11$, p=0.02). At first, it appears paradoxical, because IronCondor strategies are usually employed in low volatility environments. However, the positive loading may be explained by structural similarity rather than strategic equivalence - just like a covered call strategy, it involves short-selling the OTM calls and limited upside participation when the index rallies strongly. The Conditional BuyWrite exposure is insignificant, which is counterintuitive. The likely explanation, however, is that Conditional BuyWrite implements tactical timing by switching between just holding a passive index and BuyWrite strategy depending on market conditions, while the Iron Condor strategy is more complex and involves diversification across strikes. Ultimately, economic significance of both factors is still negligible.

Table 1, consistent with Post & Kovalenko (2025) shows that LP solver predominantly employs covered call strategies. However, according to table 5, there is no significant loading on a Conditoinal BuyWrite strategy, which indicates that LP optimized strategy differs from a CBOE covered call approach.

6 Conclusion

This paper used tractable conditions for SA opportunities, derived by Post & Kovalenko (2025), which include small set of linear constraints and embed bid-ask spreads. Furthermore, the conditional distribution with the time-varying MRP is used for a better fit of the index returns. Those conditions are applied to the optimization problem for the S&P 500 index options. The baseline model, consistent with the findings of Post & Kovalenko, yields statistically significant mispricing of the equity options in the high VIX regime. In order to assess the robustness of the baseline model, two contributions were made; Distributionally Robust Optimization, and the comparison of the alternative tractable conditions with the widely recognized sufficient but not neccessary conditions proposed by Constantinides et al. (2020). The analysis provides robust evidence that S&P500 options are systemically mispriced, and not subject to spurious parameter estimates, or artifacts of plausibly set up conditions. The found evidence rejects pricing efficiency in high VIX regimes. Active portfolios achieve 2.41%- 2.77% realized outperformance - the results that persist across two different SA constraint formulations and parameter uncertainty bounds. In constrast, low and medium VIX regimes show no evidence of arbitrage opportunities, yielding losses in active portfolios relate to the passive portfolios. Complementary factor regression model confirms that the optimized option strategies generate statistically significant alpha in high VIX regime, implying that the strategy is unique and is not explained by CBOE pre-defined strategies. The gains rather stem from identifying mispriced options thorugh LP optimization, rather than exposure to a static strategy.

The study demonstrated that: 1) neccesary and sufficient conditions provide more complete SA opportunities, and 3.7× computational efficiency, while delivering a slightly higher outperformance; 2) SA opportunities subject to distributional uncertainty persist in volatile regimes, although the negative conditional skewness and the volatility dependence of the market risk premium should be taken into account; 3) the optimized option strategy in high VIX regime is not explained by any pre-defined option strategy, suggesting the LP identifies mispriced options.

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Table 1: Performance of a Baseline Optimized Option Combination with the alternative system restrictions and net premium as an objective

Section	Metric	All VIX		Low VIX		Mid VIX		High VIX	
Restrictions	Filters	C96/108; P0/104		C96/108; P0/104		C96/108; P0/104		C96/108; P0/104	
	Dates	319		107		106		106	
	Non-zero	265		90		92		83	
Composition	BuyCall	0.12	[0.22]	0.12	[0.28]	0.13	[0.16]	0.13	[0.19]
	BuyPut	0.00	[0.21]	0.10	[0.28]	0.00	[0.00]	0.00	[0.00]
	WriteCall	0.65	[0.74]	0.23	[0.74]	0.81	[0.70]	0.79	[0.84]
	WritePut	0.00	[0.20]	0.10	[0.28]	0.00	[0.00]	0.00	[0.00]
Moneyness	BuyCall	96.77	[6.48]	101.46	[6.65]	96.61	[6.94]	96.71	[2.13]
	BuyPut	97.74	[1.91]	97.95	[1.36]	97.34	[2.43]	96.04	[15.80]
	WriteCall	102.11	[3.31]	100.47	[3.29]	102.23	[2.51]	103.51	[2.68]
	WritePut	97.57	[2.02]	97.78	[1.42]	97.34	[2.43]	96.04	[10.36]
Implied Vol	BuyCall	0.17	[0.10]	0.09	[0.05]	0.17	[0.07]	0.24	[0.06]
	BuyPut	0.14	[0.06]	0.13	[0.02]	0.18	[0.03]	0.26	[0.25]
	WriteCall	0.15	[0.07]	0.12	[0.02]	0.16	[0.03]	0.21	[0.05]
	WritePut	0.14	[0.06]	0.13	[0.02]	0.18	[0.03]	0.27	[0.25]
Projected Distribution	Passive Mean	6.83	[7.31]	2.92	[2.84]	6.70	[3.21]	13.16	[6.44]
	Passive Std	13.65	[6.07]	10.07	[1.62]	13.65	[2.09]	18.94	[5.66]
	Passive Skew	−0.82	[0.22]	−0.78	[0.11]	−0.83	[0.27]	−0.86	[0.29]
	Passive CVaR	−57.50	[25.41]	−40.18	[7.00]	−57.52	[7.78]	−77.44	[21.55]
	Premium Inc.	0.44	[1.28]	0.24	[0.46]	0.71	[1.42]	0.89	[2.24]
	Active Mean	7.62	[8.83]	3.08	[3.02]	7.34	[4.58]	15.45	[7.00]
	Active Std	12.60	[5.76]	9.01	[1.90]	12.61	[2.34]	17.91	[6.04]
	Active Skew	−1.35	[0.94]	−1.08	[0.81]	−1.45	[1.02]	−1.48	[1.10]
	Active CVaR	−57.00	[25.61]	−40.05	[7.22]	−57.04	[7.04]	−77.25	[21.13]
Realized Performance	Passive Mean	8.53		3.42		8.71		13.52	
	Active Mean	8.61		2.57		7.40		15.93	
Improvements	BuyCall Mean	−0.15	(1.28)	0.05	(0.55)	0.09	(1.46)	−0.60	(1.58)
	BuyPut Mean	−0.45	(1.07)	−0.62	(1.12)	−0.28	(1.25)	−0.46	(0.82)
	WriteCall Mean	0.40	(3.26)	−0.72	(2.23)	−1.23	(3.89)	3.15	(3.31)
	WritePut Mean	0.29	(0.93)	0.43	(0.79)	0.12	(1.16)	0.32	(0.81)
	Mean	0.08	(0.87)	−0.85	(0.26)	−1.31	(0.18)	2.41	(0.01)
	CER1	0.19	(0.71)	−0.75	(0.32)	−1.12	(0.26)	2.46	(0.01)
	CER4	0.55	(0.31)	−0.39	(0.56)	−0.42	(0.58)	2.45	(0.00)
	CER10	1.26	(0.02)	0.30	(0.80)	0.90	(0.54)	2.51	(0.00)
	ELRT	0.00	(0.99)	0.36	(0.55)	8.39	(0.00)	0.00	(1.00)

Note: This table reports annualized descriptive statistics for optimized option-enhanced portfolios over the full sample and three volatility regimes. Optimization uses an SGT distribution, CAPM MRP, moneyness filters [0.96, 1.08] for calls and [0, 1.04] for puts, and maximization of net option premium. The top panel shows the number of formation dates and non-zero solutions. The next three panels report the median and inter-quartile range (in square brackets) of the composition (number of options per index unit), weighted-average moneyness, and implied volatility for bought and written options. The Projected Distribution panel summarizes statistics for the passive index and active enhanced portfolio: median and IQR of mean, standard deviation, skewness, CVaR (99%), and net premium income. The Realized Performance Improvements panel shows means of bought/written options and CER values with mean changes and p-values in round brackets. Last, ELRT for SSD of the index with a p-value.

Table 2: Performance of a DRO portfolio with the alternative system restrictions and net premium as an objective

Section	Metric	All VIX		Low VIX		Mid VIX		High VIX	
Restrictions	Filters	C96/108; P0/104		C96/108; P0/104		C96/108; PVI04		C96/108; P0/104	
	Dates	319		107		106		106	
	Non-zero	282		100		102		80	
Composition	BuyCall	0.11	[0.14]	0.09	[0.17]	0.12	[0.10]	0.12	[0.13]
	BuyPut	0.00	[0.00]	0.00	[0.27]	0.00	[0.00]	0.00	[0.00]
	WriteCall	0.83	[0.57]	0.51	[0.62]	0.86	[0.13]	0.83	[0.71]
	WritePut	0.00	[0.00]	0.00	[0.27]	0.00	[0.00]	0.00	[0.00]
Moneyness	BuyCall	96.35	[1.53]	96.28	[5.33]	96.39	[0.91]	96.45	[1.42]
	BuyPut	98.50	[1.49]	98.60	[1.27]	98.53	[1.93]	95.68	[41.06]
	WriteCall	101.96	[2.61]	100.89	[1.58]	102.12	[1.99]	103.56	[2.54]
	WritePut	98.46	[1.43]	98.56	[1.18]	98.53	[1.93]	95.68	[22.28]
Implied Vol	BuyCall	0.18	[0.08]	0.14	[0.06]	0.19	[0.05]	0.25	[0.06]
	BuyPut	0.14	[0.05]	0.12	[0.02]	0.17	[0.03]	0.25	[0.65]
	WriteCall	0.15	[0.07]	0.11	[0.02]	0.15	[0.03]	0.21	[0.04]
	WritePut	0.14	[0.05]	0.12	[0.02]	0.17	[0.03]	0.26	[0.52]
Projected Distribution	Passive Mean	8.49	[8.50]	3.78	[3.07]	8.49	[3.38]	16.25	[7.29]
	Passive Std	12.50	[5.49]	9.05	[1.45]	12.52	[1.80]	17.18	[4.97]
	Passive Skew	−0.74	[0.20]	−0.71	[0.10]	−0.76	[0.25]	−0.78	[0.27]
	Passive CVaR	−50.99	[22.61]	−35.87	[5.98]	−51.06	[7.52]	−67.67	[18.26]
	Premium Inc.	0.86	[1.63]	0.54	[0.72]	1.10	[1.59]	1.58	[2.94]
	Active Mean	9.60	[10.14]	4.34	[3.21]	9.55	[5.11]	18.95	[8.34]
	Active Std	11.34	[5.60]	7.78	[1.59]	11.34	[2.06]	15.86	[4.97]
	Active Skew	−1.62	[0.97]	−1.74	[0.94]	−1.62	[0.55]	−1.50	[1.19]
	Active CVaR	−50.59	[22.41]	−35.13	[5.99]	−50.60	[6.98]	−67.39	[18.23]
Realized Performance	Passive Mean	8.53		3.42		8.71		13.52	
	Active Mean	8.64		2.84		6.84		16.29	
Improvements	BuyCall Mean	−0.03	(1.36)	−0.04	(0.74)	0.35	(1.58)	−0.38	(1.58)
	BuyPut Mean	−0.54	(0.77)	−0.88	(0.90)	−0.65	(0.66)	−0.09	(0.70)
	WriteCall Mean	0.22	(3.63)	−0.33	(2.69)	−2.20	(4.33)	3.21	(3.56)
	WritePut Mean	0.45	(0.65)	0.67	(0.73)	0.63	(0.44)	0.03	(0.74)
	Mean	0.10	(0.85)	−0.59	(0.48)	−1.87	(0.09)	2.77	(0.00)
	CER1	0.23	(0.68)	−0.48	(0.56)	−1.64	(0.14)	2.84	(0.00)
	CER4	0.65	(0.27)	−0.14	(0.83)	−0.79	(0.39)	2.85	(0.00)
	CER10	1.47	(0.01)	0.53	(0.59)	0.83	(0.68)	2.98	(0.00)
	ELRT	0.00	(0.97)	0.00	(1.00)	29.33	(0.04)	0.00	(1.00)

Note: This table reports annualized descriptive statistics for a distributional robust optimization option-enhanced portfolios over the full sample and three volatility regimes. Optimization uses an SGT distribution with plausible parameter ranges (CAPM MRP $\in [2, 5]$, Vol. $\in [\times 0.9, \times 1.1]$, and skew $\in [\times 0.9, \times 1.1]$. Moneyness filters [0.96, 1.08] for calls and [0, 1.04] for puts, and maximization of net option premium. The top panel shows the number of formation dates and non-zero solutions. The next three panels report the median and inter-quartile range (in square brackets) of the composition (number of options per index unit), weighted-average moneyness, and implied volatility for bought and written options. The Projected Distribution panel summarizes statistics for the passive index and active enhanced portfolio: median and IQR of mean, standard deviation, skewness, CVaR (99%), and net premium income. The Realized Performance Improvements panel shows means of bought/written options and CER values with mean changes and p-values in round brackets. Last, ELRT for SSD of the index with a p-value.

Table 3: The sufficient but not necessary constraints system table with the active portfolio return as an objective

Section	Metric	All VIX		Low VIX		Mid VIX		High VIX	
Restrictions	Filters	C96/108; P0/104		C96/108; P0/104		C96/108; P0/104		C96/108; P0/104	
	Dates	319		107		106		106	
	Non-zero	240		86		84		70	
Composition	BuyCall	0.11	[0.18]	0.09	[0.15]	0.10	[0.17]	0.13	[0.19]
	BuyPut	0.00	[0.02]	0.03	[0.33]	0.00	[0.00]	0.00	[0.00]
	WriteCall	0.64	[0.78]	0.23	[0.71]	0.78	[0.78]	0.77	[0.84]
	WritePut	0.00	[0.02]	0.03	[0.33]	0.00	[0.00]	0.00	[0.00]
Moneyness	BuyCall	96.71	[5.60]	101.38	[6.17]	96.54	[5.06]	96.68	[1.45]
	BuyPut	98.02	[1.55]	98.02	[1.47]	98.14	[1.76]	98.17	[2.24]
	WriteCall	102.07	[3.48]	100.41	[3.03]	102.14	[2.57]	103.44	[2.76]
	WritePut	97.85	[1.54]	97.98	[1.46]	97.76	[2.12]	97.62	[2.33]
Implied Vol	BuyCall	0.17	[0.10]	0.09	[0.06]	0.18	[0.06]	0.25	[0.07]
	BuyPut	0.14	[0.05]	0.13	[0.02]	0.18	[0.04]	0.24	[0.05]
	WriteCall	0.15	[0.07]	0.11	[0.02]	0.16	[0.03]	0.21	[0.05]
	WritePut	0.14	[0.06]	0.13	[0.02]	0.18	[0.03]	0.26	[0.06]
Projected Distribution	Passive Mean	6.82	[7.31]	2.90	[2.84]	6.68	[3.21]	13.13	[6.43]
	Passive Std	13.69	[6.09]	10.10	[1.63]	13.70	[2.11]	19.00	[5.68]
	Passive Skew	−0.82	[0.22]	−0.79	[0.11]	−0.83	[0.28]	−0.86	[0.30]
	Passive CVaR	−53.08	[23.49]	−37.71	[6.50]	−53.08	[6.51]	−72.55	[18.22]
	Premium Inc.	0.00	[0.00]	0.00	[0.00]	0.00	[0.00]	0.00	[0.00]
	Active Mean	7.57	[8.68]	3.06	[2.93]	7.21	[4.56]	15.28	[6.90]
	Active Std	12.93	[5.89]	9.08	[1.99]	12.95	[2.35]	18.39	[6.22]
	Active Skew	−1.34	[0.89]	−1.20	[0.73]	−1.41	[1.00]	−1.39	[0.99]
	Active CVaR	−53.08	[23.54]	−37.42	[6.71]	−53.08	[6.36]	−72.55	[18.19]
Realized Performance	Passive Mean	8.53		3.42		8.71		13.52	
	Active Mean	8.60		2.48		7.40		15.98	
Improvements	BuyCall Mean	−0.25	(1.42)	0.11	(0.56)	−0.24	(1.65)	−0.64	(1.75)
	BuyPut Mean	−0.39	(1.14)	−0.64	(1.24)	−0.46	(1.26)	−0.08	(0.88)
	WriteCall Mean	0.53	(3.11)	−0.74	(2.17)	−0.88	(3.72)	3.24	(3.13)
	WritePut Mean	0.18	(0.98)	0.33	(0.95)	0.26	(1.09)	−0.05	(0.90)
	Mean	0.07	(0.88)	−0.94	(0.19)	−1.31	(0.13)	2.47	(0.00)
	CER1	0.15	(0.74)	−0.84	(0.24)	−1.15	(0.19)	2.48	(0.00)
	CER4	0.43	(0.39)	−0.50	(0.44)	−0.56	(0.44)	2.31	(0.00)
	CER10	0.97	(0.05)	0.16	(0.94)	0.58	(0.73)	2.07	(0.00)
SSD test	ELRT	0.00	(0.99)	1.19	(0.28)	6.09	(0.01)	0.00	(1.00)

Note: This table reports annualized descriptive statistics for an optimized option-enhanced portfolios over the full sample and three volatility regimes following the methodology of Constantinides el al. (2025) for SA constraints. Optimization uses an SGT distribution, CAPM MRP, moneyness filters [0.96, 1.08] for calls and [0, 1.04] for puts, and maximization of net option premium. The top panel shows the number of formation dates and non-zero solutions. The next three panels report the median and inter-quartile range (in square brackets) of the composition (number of options per index unit), weighted-average moneyness, and implied volatility for bought and written options. The Projected Distribution panel summarizes statistics for the passive index and active enhanced portfolio: median and IQR of mean, standard deviation, skewness, CVaR (99%), and net premium income. The Realized Performance Improvements panel shows means of bought/written options and CER values with mean changes and p-values in round brackets. Last, ELRT for SSD of the index with a p-value.

Table 4: The alternative system constraints table with the active portfolio return as an objective

Section	Metric	All VIX		Low VIX		Mid VIX		High VIX	
Restrictions	Filters	C96/108; P0/104		C96/108; P0/104		C96/108; P0/104		C96/108; P0/104	
	Dates	319		107		106		106	
	Non-zero	255		90		88		77	
Composition	BuyCall	0.13	[0.29]	0.11	[0.53]	0.13	[0.18]	0.13	[0.19]
	BuyPut	0.00	[0.12]	0.07	[0.17]	0.00	[0.07]	0.00	[0.00]
	WriteCall	0.57	[0.78]	0.17	[0.77]	0.80	[0.78]	0.78	[0.86]
	WritePut	0.00	[0.12]	0.07	[0.17]	0.00	[0.07]	0.00	[0.00]
Moneyness	BuyCall	96.87	[6.90]	101.54	[6.74]	96.62	[7.15]	96.72	[2.75]
	BuyPut	97.94	[1.77]	98.07	[1.48]	97.78	[2.31]	96.17	[15.74]
	WriteCall	102.02	[3.74]	100.41	[3.27]	102.09	[2.89]	103.38	[2.91]
	WritePut	97.59	[2.09]	97.83	[1.58]	97.69	[2.57]	96.17	[11.01]
Implied Vol	BuyCall	0.17	[0.11]	0.09	[0.06]	0.17	[0.07]	0.25	[0.07]
	BuyPut	0.14	[0.07]	0.13	[0.02]	0.17	[0.04]	0.26	[0.22]
	WriteCall	0.15	[0.07]	0.12	[0.02]	0.16	[0.03]	0.21	[0.05]
	WritePut	0.15	[0.07]	0.13	[0.02]	0.19	[0.04]	0.27	[0.22]
Projected Distribution	Passive Mean	6.82	[7.31]	2.90	[2.84]	6.68	[3.21]	13.13	[6.43]
	Passive Std	13.69	[6.09]	10.10	[1.63]	13.70	[2.11]	19.00	[5.68]
	Passive Skew	−0.82	[0.22]	−0.79	[0.11]	−0.83	[0.28]	−0.86	[0.30]
	Passive CVaR	−53.08	[23.49]	−37.71	[6.50]	−53.08	[6.51]	−72.55	[18.22]
	Premium Inc.	0.00	[0.59]	0.00	[0.01]	0.00	[1.44]	0.00	[1.80]
	Active Mean	7.62	[8.78]	3.36	[3.15]	7.32	[4.46]	15.52	[6.83]
	Active Std	12.91	[5.86]	9.24	[1.88]	12.96	[2.42]	18.21	[6.25]
	Active Skew	−1.33	[1.03]	−0.85	[1.01]	−1.44	[1.03]	−1.44	[1.15]
	Active CVaR	−53.06	[23.26]	−37.48	[6.93]	−53.07	[6.40]	−72.24	[18.20]
Realized Performance	Passive Mean	8.53		3.42		8.71		13.52	
	Active Mean	8.68		2.61		7.53		15.96	
Improvements	BuyCall Mean	−0.12	(1.36)	0.14	(0.69)	−0.12	(1.56)	−0.37	(1.64)
	BuyPut Mean	−0.33	(0.92)	−0.53	(0.89)	−0.12	(1.22)	−0.36	(0.54)
	WriteCall Mean	0.41	(3.20)	−0.74	(2.19)	−0.91	(3.84)	2.88	(3.25)
	WritePut Mean	0.19	(0.82)	0.32	(0.65)	−0.03	(1.16)	0.29	(0.50)
	Mean	0.15	(0.76)	−0.82	(0.25)	−1.18	(0.22)	2.45	(0.00)
	CER1	0.24	(0.63)	−0.73	(0.30)	−1.01	(0.29)	2.48	(0.00)
	CER4	0.53	(0.30)	−0.43	(0.50)	−0.39	(0.60)	2.38	(0.00)
	CER10	1.10	(0.03)	0.14	(0.95)	0.81	(0.58)	2.29	(0.00)
SSD test	ELRT	0.00	(0.95)	0.16	(0.69)	7.60	(0.00)	0.00	(1.00)

Note: This table reports annualized descriptive statistics for optimized option-enhanced portfolios over the full sample and three volatility regimes. Optimization uses an SGT distribution, CAPM MRP, moneyness filters [0.96, 1.08] for calls and [0, 1.04] for puts, and maximization of expected active portfolio return. The top panel shows the number of formation dates and non-zero solutions. The next three panels report the median and inter-quartile range (in square brackets) of the composition (number of options per index unit), weighted-average moneyness, and implied volatility for bought and written options. The Projected Distribution panel summarizes statistics for the passive index and active enhanced portfolio: median and IQR of mean, standard deviation, skewness, CVaR (99%), and net premium income. The Realized Performance Improvements panel shows means of bought/written options and CER values with mean changes and p-values in round brackets. Last, ELRT for SSD of the index with a p-value.

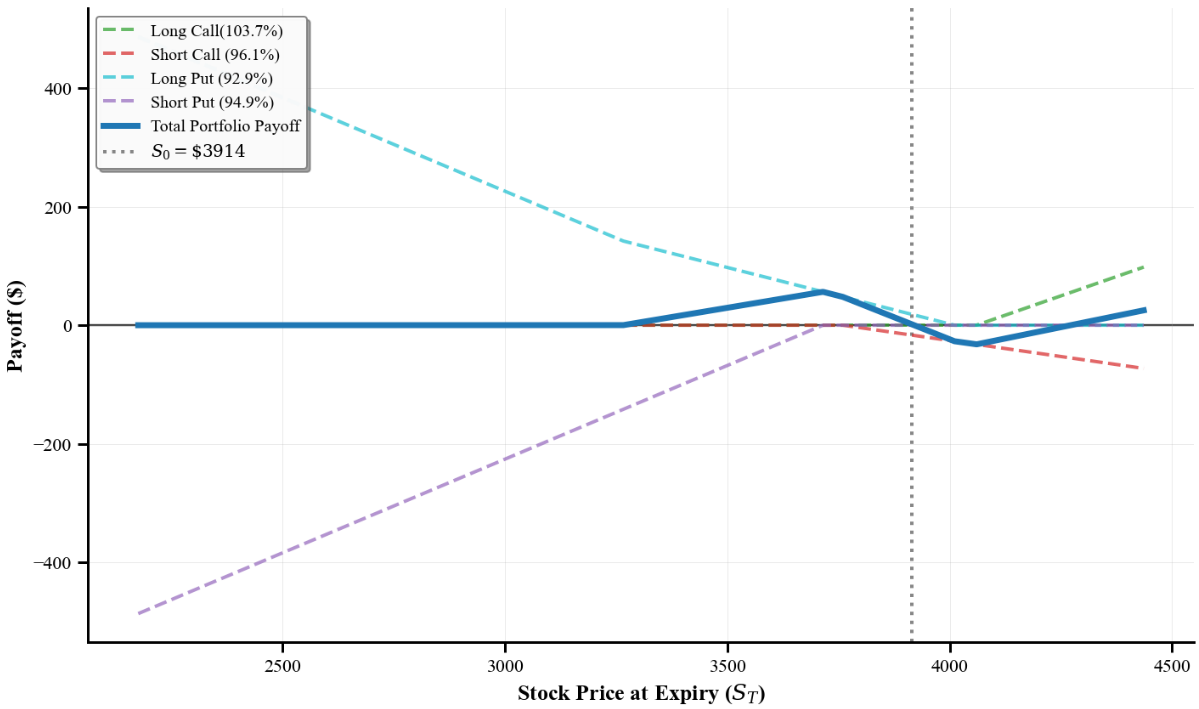

Figure 1: Call front spread. Option combination diagram (High VIX regime). Long call: 0.26— Short call: 0.11 — Long put: 0.32 — Short put: 0.32

Table 5: Style Analysis: High VIX Regime Factor Loadings

Factor	**Beta**	**t-statistic**	**p-value**
Put Protection	0.0088	0.35	0.7303
Iron Condor	0.1105**	2.39	0.0188
Iron Butterfly	−0.0017	−0.07	0.9420
Conditional BuyWrite	−0.0359	−1.05	0.2983
Regression Statistics			
$N = 106$ $R^2 = 0.078$ Adj. $R^2 = 0.042$			
Annual Alpha = 2.17%*** ($t = 2.54$, $p = 0.0125$)			

Notes: This table reports the results of a style analysis regression for the optimized option portfolio in the High VIX regime. The dependent variable is the monthly excess return of the optimized portfolio. Independent variables are returns from benchmark option strategies. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively.