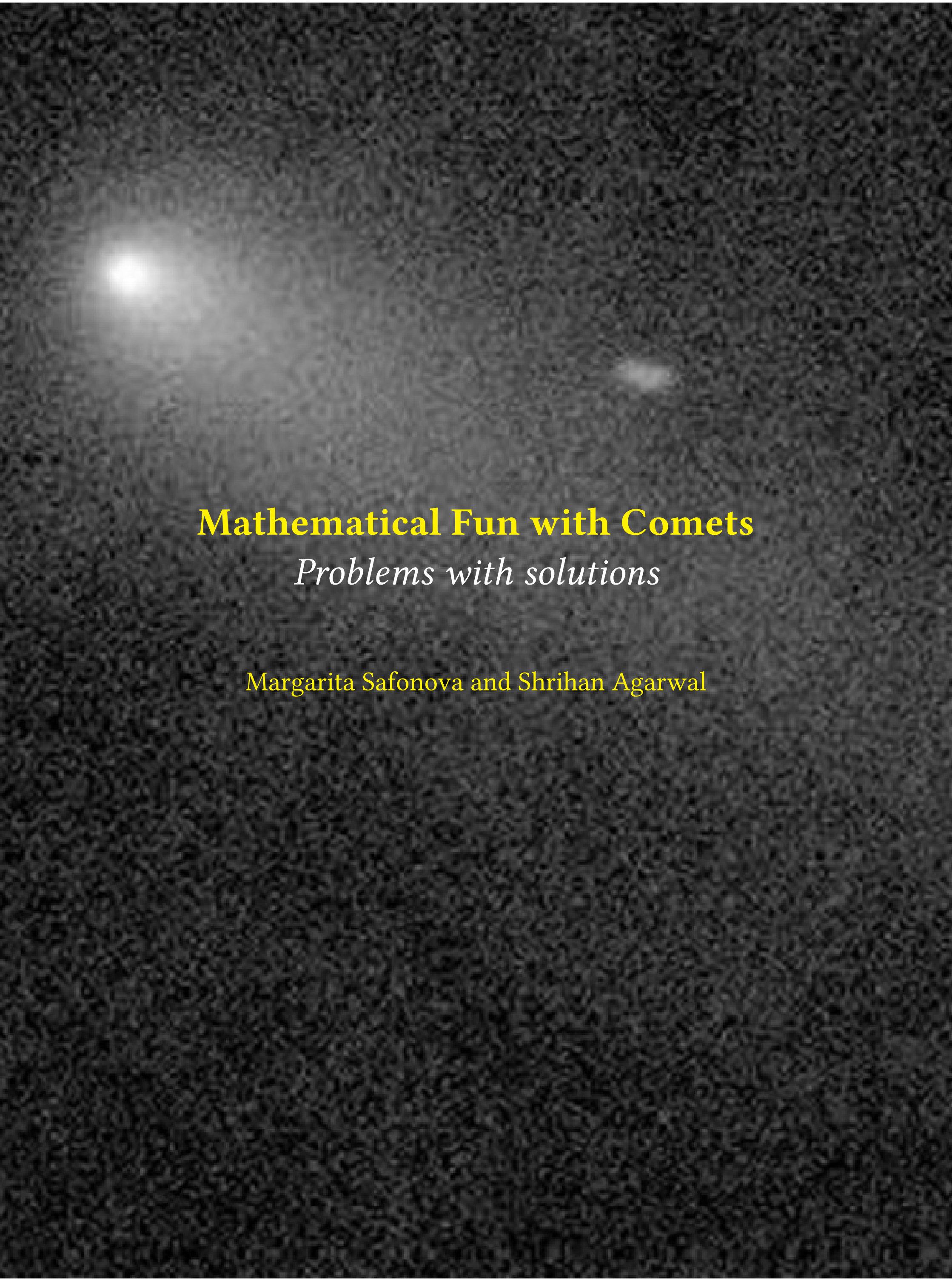

Mathematical Fun with Comets

Problems with solutions

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MATHEMATICAL FUN WITH COMETS
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Cover Images

Front cover: Comet ISON in September 2013. Comet C/2012 S1, also known as comet ISON, was a sungrazing Oort cloud comet discovered on 21 September 2012 by Vitali Nevski and Artyom Novichonok in Russia. The 300-sec image is taken in the I-band filter on Himalayan Chandra Telescope (HCT) of the Indian Astrophysical Observatory (IAO), Hanle, Leh, by Dr. Margarita Safonova, Indian Institute of Astrophysics (IIA).
Back cover: Comet 21P/Giacobini-Zinner – a periodic Jupiter family comet with period of ∼6.54 years. It is the parent to the annual October Draconid meteor shower. This RGB image is a combination of 20-sec frames, taken in R, V and B Bessel filters (on HCT, Hanle) each of 20-sec exposure, aligned on the comet's optocentre, and co-added. The image was taken by Dr. Margarita Safonova, and the RGB composition was done by Dr. Sujatha (MPBIFR).

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PREFACE

The M. P. Birla Institute of Fundamental Research (MPBIFR) has been established as a Society for undertaking research in streams of natural and applied sciences in various disciplines of science, engineering and technology. The Society, registered under the West Bengal Societies Act 1961, has its Head Office at Kolkata.

With a view to broaden the scope of its activities in other parts of the country, the Society opened a Center at Bangalore in the year 2000 under the name of 'M.P. Birla Institute of Fundamental Research, Bangalore' with the main aim of teaching and conducting research work in Astronomy, Astrophysics, and other related fields.

This Institute is recognized as a Centre for Research for conducting programs leading to the Ph.D. degree in Physics (with specialization in Astrophysics) under the jurisdiction of Bangalore University, as well as the Jain University. The Institute is also recognized as a research center by the Dept. of Science and Industrial Research (DSIR), Govt. of India.

The Institute is conducting various academic programs throughout the year, aimed at different age and education level groups and open for everyone.

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- Support for research programs in Astrophysics leading to the Ph. D. degree.

Contents

Chapter 1

Small Introduction

Comets are fun! In fact, the famous comet hunter David Levy once said: "*Comets are like cats: they have tails and do exactly what they want.*" NASA estimates that every day, about 100 tonnes of cosmic dust falls on Earth. Most of it is the remnants of comets and asteroids: 70% of space dust that is found between Sun and Mars comes from the comets, 22% from asteroids, and about 7.5% reaches us from the interstellar space. Comets are essentially the dirty snowballs: albeit the snow is not only made of water, but of frozen carbon dioxide, carbon monoxide, methane, and ammonia too. Some comets even found to have aminoacids: in comet Wild 2 the spacecraft *Deep Impact* has found glycine!

Comets have two dwelling places in our Solar System: long-period comets come from the Oort cloud (the best known example till 2013 was comet Hale-Bopp), short-period – from Kuiper belt (the best known example is Halley's comet). In 2013, new Oort cloud comet – comet ISON – has arrived to the inner Solar System and ... came so close to the Sun that got itself destroyed at the time of perihelion.

Just recently, in August 2019, humanity discovered the interstellar comets – the comet, now called Comet 2I/Borisov after its discoverer, came from out of the Solar System entirely. Now we know that other stars have not only planets but comets and asteroids too: the interstellar asteroid named Oumuamua visited us in 2017 only to never return again. How do we know they are from interstellar space? Read on – that's where mathematical fun lies!

Oort cloud is very far from the Sun – it starts at ~ 2000 A.U. (astronomical unit $\simeq 150$ million km) from the Sun; while Kuiper belt is nearby – just beyond the Neptune's orbit. Pluto's orbit is within the Kuiper Belt, and at that distance the temperature is about $-233°$C, so everything is frozen solid and we cannot distinguish a comet from an asteroid.

Closer to the Sun, at about Saturn's distance, the sunlight is strong enough to begin boiling frozen CO_2 ices in the comet. At Jupiter's distance, the heat from the Sun is enough to start boiling the water ices, so the comet's coma – atmosphere – forms.

But then, why the tail? After all, Earth is close to the Sun and does not have the tail – it does not leave our atmosphere behind as we move around the orbit. Well, if the Earth's gravity were as weak as the comet's, it would! Actually, comets even have two tails. As mentioned earlier, comets are dirty snowballs, in other words, made out of dirt – dust, as astronomers call it, and frozen gases – ices.

Dust is heavier than gases, so when they are heated, they separate. Radiation pressure from the sunlight pushes the solid dust particles away from the coma (because comet's gravity is weak), and solar wind, consisting of the superfast charged particles – protons and electrons – pushes the gas ions away from coma. Because ions are lighter, they align away from the Sun at nearly straight line. But dust particles are heavier, and as the comet moves along the orbit, the dust tail curves (Fig. 1).

Figure 1.1: Two tails of comet Hale-Bopp. Credit: Dr. Chuck Claver (NOAO/LSST Corp.), USA, April 12, 1997.

As comets shed their stuff going around the Sun, they leave behind trails of dust and gasses. Their remains give us the beautiful phenomena observable from the ground: meteor showers, noctilucent clouds and zodiacal light (Fig. 1.2).

Meteor showers are the result of the Earth's passage through the debris left by the comets. And there is a LOT of cosmic dust: for example, in the last 24 hrs, millions of meteors with sizes from grains of sand to basketballs fell on Earth. Annual Orionids meteor showers in October and Eta Aquarids in May are the meteor showers created by debris from the Halley comet; though last time it had visited us in 1986 and the next will be only in 2061, we pass through its orbital path, and what it has left behind, twice a year: every May and October. Perseid meteor showers (in August) are the remains of the comet Swift-Tuttle, which returns to the Sun every 133 years, and Leonids (in November) are dust remains of the comet Tempel-Tuttle, whose period is 33 years.

Figure 1.2: *Left*: Perseid meteor shower. Credit: Jeremy Perez. *Middle*: Zodiacal light over Paranal – mountain in the Atacama Desert of northern Chile, the home of the Paranal Observatory, site of the Very Large Telescope and the VLT Survey Telescope. Credit: ESO/Y, Beletsky. *Right*: Noctilucent clouds near Edmonton, Alberta, Canada on July 2, 2011. Credit: NASA (http://spaceweather.com/nlcs/gallery2011.htm).

Oort cloud comets, like the comet ISON in 2013, can only cause the phenomenon of **noctilucent clouds** – the wispy, glowing, mesosphere (80 km altitude) clouds made of water-ice crystals seeded by comet's dust.

Zodiacal light is a glow in the sky produced by the reflection of the sunlight by cometary dust, and dust generated by broken parts of asteroids, aggregated in the interplanetary dust cloud.

But fun with comets does not just stop at their pretty images. Their behaviour can be described mathematically.

Chapter 2

Mathematical problems

2.1 How fast comets move?

1. Escape velocity

The comet is coming from infinity, therefore it is in a free-fall towards the Sun. Its speed at the surface of the Sun will be equal (or more) to the velocity it needs to escape the Sun's gravity, v_e. For a body mass m to escape the gravity of a larger body of mass M, its kinetic energy has to be equal or larger than its potential energy. Let initial (at the surface) kinetic K and potential U energies of the comet be

$$K_i = \frac{mv_e^2}{2}\,, \quad \text{and} \quad U_i = -\frac{GMm}{r}\,. \tag{2.1}$$

Kinetic and potential energies of a body m at infinity are: $K_f = 0$, since speed goes to zero at infinity, and $U_f = 0$, since gravity due to the body M is zero at infinity. Due to the conservation of energy,

$$(K+U)_i = (K+U)_f = 0\,, \tag{2.2}$$

therefore

$$K_i = -U_i\,, \tag{2.3}$$

and hence

$$\frac{mv_e^2}{2} = \frac{GMm}{r}\,, \tag{2.4}$$

and therefore the escape velocity (speed actually, as it does not depend on the direction) is

$$v_e = \sqrt{\frac{2GM}{r}}\,, \tag{2.5}$$

where $r = 700,000$ km is the radius of the Sun. For $M_\odot = 1.9 \times 10^{30}$ kg and $G = 6.67 \times 10^{-11}$ m^3/kg/sec^2, escape velocity from the surface of the Sun is $v_e \simeq 600$ km/sec.

2. Speed of the comet at perihelion

The comet, however, is not touching the surface of the Sun; its closest approach is ~ 3 solar radii, or $h_{\text{perihelion}} = 1.8 \times 10^6$ km. The farther from the centre, the weaker the gravity and less speed is needed to overcome it. Thus, the speed of the comet at the time of perihelion is, at least,

$$v_e = \sqrt{\frac{2GM}{h_{\text{perihelion}}}} \simeq 360\,\text{km/sec}\,. \tag{2.6}$$

It is very fast — how fast? What percentage of the speed of light it is? Speed of light is $c \approx 300,000$ km/sec, therefore

$$\begin{aligned} v_{\text{comet}}^{\text{perihelion}} &= 100\%\,\frac{v_e}{c} = 100\%\,\frac{360}{300,000} \\ &\simeq 0.12\%\ \text{of speed of light!} \end{aligned} \tag{2.7}$$

Note: According to the measured data, comet ISON had a perihelion speed of ≈ 375 km/sec.

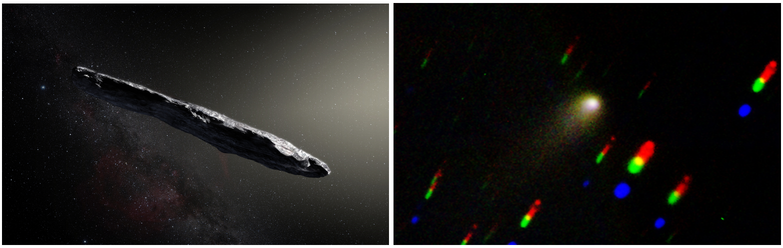


Figure 2.1: First interstellar objects to visit inner Solar System. *Left*: An artist's impression of the interstellar asteroid Oumuamua, discovered on October 19, 2017. Credit: ESO/M. Kornmesser. *Right*: Interstellar comet 2I/Borisov colour image, composed from B, V and R images, taken on HCT, IIA, Hanle, on December 7, 2019. Credit: IIA/B. Chandra.

3. Interstellar comets

Till 2017, we knew only of the comets and asteroids that belong to the Solar System, though existence of such objects in other stellar systems was hypothesized nearly 20 years ago. In 2017, the world was startled to discover an object that was obviously not ours! How did we known that?

Objects belonging in our Solar System move in elliptical orbits around the Sun – they are gravitationally bound – thus they move with velocities below escape velocity at their respective distances. For stable orbits, the closer you are to the Sun, the faster you have to move to stay in Sun's orbit – Mercury's average speed is 48 km/sec. Objects in far away Kuiper belt, or the Oort cloud, have speeds of about 4 km/sec (inner Kuiper belt), and just a few hundred mtrs/sec (in the Oort cloud). To escape the Solar System, an object needs to be imparted the velocity greater than the escape velocity – for example, Voyager 1 is the first man-made object to leave the Solar System completely; it is now (at the distance of 145 AU) moving at the speed of 16.9 km/sec (calculate the escape velocity at that distance using Eq. 2.5). Sun constitutes 99.8% of the total mass of the Solar System, so assume only the Sun's mass in the escape velocity calculations.

But what about objects coming into the inner Solar System? If an object coming towards the central body has more than enough speed to escape the central body's gravitational pull, the trajectory is called hyperbolic, because such orbit has the shape of a hyperbola. Such orbits are gravitationally unbound, unlike the elliptical orbits.

This was what has given away the Oumuamua[1] (Fig. 2.1, *Left*) – its speed was $\sim$50 km/sec when it was detected at the distance of 0.22 AU from Earth 40 days after the perihelion, already on the way out of the Solar System. One month later it was moving at 44 km/sec, speeding towards the constellation Pegasus. The calculated perihelion speed was 87.71 km/sec (see Sec. 2.1.2). It did not show any cometary-like actvity – tails, coma, outbursts; thus it was assumed that it was an interstellar asteroid.

Just 2 years later, on September 13, 2019, NASA's Jet Propulsion Laboratory website[2], which generates ephemerides (data on orbits and positions) for all Solar System bodies, provided the following data for the new Borisov comet, discovered less than a month before that – on 31st August 2019 (Fig. 2.1, *Right*):

Distance from the Sun (AU)	Velocity w.r.t. Sun (km/sec)
2.754358	41.05

[1]Hawaiian for 'first distant messenger'; name given by the discovery team.
[2]http://ssd.jpl.nasa.gov/horizons.cgi#top

To compare its speed with the escape velocity at that distance, we use Eq. 2.5, where r now is the distance from the Sun (1 AU is equal to 149,598,000 km),

$$v_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \cdot 1.9 \times 10^{30} \cdot 6.67 \times 10^{-11}}{2.754358 \cdot 1.49598 \times 10^8 \cdot 10^3}} =$$

$$= 24801\,\text{m/sec} \equiv 24.8\,\text{km/sec}\,.$$

The comet is already moving much faster than the escape velocity at that distance, which means that it is not gravitationally bound to the Sun. This high speed cannot be due to the acceleration by the Sun's gravity alone; it must have entered the Solar System with already high initial velocity – thus, we must assume it has come from outside the Solar System. The relative velocities of stars in the solar neighbourhood are roughly 10–30 km/sec, and that is how fast the interstellar visitors are likely to move.

2.2 How much mass comets have?

Assume a typical size of the nucleus 5 km (comets could be 3 to 100 km in size; Halley's comet nucleus is 15-km long potato shape, size of ISON comet nucleus was about 5 km.) We have to assume it to be a sphere for easy calculations; note, however, that cometary nuclei are generally not spherical – Fig. 2.2.

The volume of the nucleus is then

$$\mathcal{V}_{\text{comet}} = \frac{4}{3}\pi r^3 \simeq 520\,\text{km}^3 = 5.2 \times 10^{17}\text{cm}^3\,. \qquad (2.8)$$

Comets mostly consist of water (ice) and dust; by some estimates their composition can be up to 80% is water. We can assume the density of a comet to be that of water $\rho_{\text{water}} = 1$ g/cm^3. Real

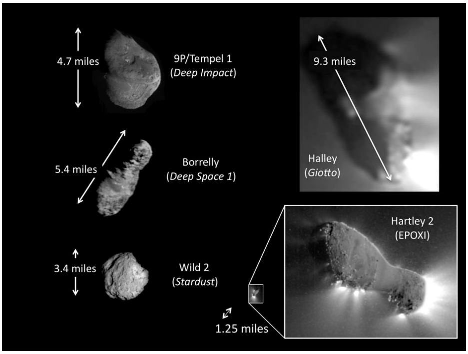


Figure 2.2: Sizes of five comets visited and photographed with spacecraft. Credit: NASA/JPL-Caltech/UMD.

comets may have densities of 0.3 to 0.6 g/cm^3, more like 0.6 g/cm^3; however, 1 g/cm^3 is a good assumption. Therefore, comet's mass is

$$\begin{aligned} \mathcal{M}_{\mathrm{comet}} = \rho\dot{\mathcal{V}} &= 1\,\mathrm{g/cm}^3 \times 5.2 \times 10^{17}\mathrm{cm}^3 \\ &= 5.2 \times 10^{17}\mathrm{g} \\ &= 5.2 \times 10^{14}\,\mathrm{kg}\,. \end{aligned} \tag{2.9}$$

But one may ask – what about the comet's tail; after all they can be as long as hundreds of thousands of kilometers[3]? How much mass is there? Comet's tail is essentially vacuum. The density of atoms in a typical comet tail is $\sim 50,000 \equiv 5 \times 10^4$ atoms/cm^3. Compare it with the density of air at sea level: 10^{19} (atoms/molecules)/cm^3.

[3]Or even millions! Comet Hyakutake's tail is known to have been at least 570 million km, or 3.8 AU

2.3 How much mass comets lose during the inner Solar System passage?

Let us take as an example the Halley's comet (a well-studied short-period comet that comes every 76 years). The peak mass loss rate for Halley's comet was 30 tons of dust every second and 24 tons of gas (ice) every second at the Earth's distance from the Sun – 1 A.U. Therefore, the total mass loss rate is

$$\begin{aligned} 54\,\text{tons/sec} \times 1000\,\text{kg/ton} &= 54,000\,\text{kg/sec} \\ &\equiv 5.4 \times 10^4\,\text{kg/sec}\,. \end{aligned} \tag{2.10}$$

Typical time spent by short-period comets in the inner Solar System is about $1/3$ year, which is $\frac{1}{3} \times 3.15 \times 10^7 = 1.05 \times 10^7$ sec. Why the inner Solar System is important? – this is where the heat of the Sun starts evaporating the comets! Far away from the Sun, beyond Saturn's orbit, comets freeze back into ice and do not lose mass.

Thus, the mass loss of Halley's comet per passage is

$$\begin{aligned} \text{Mass loss} &= 5.4 \times 10^4\,\text{kg/sec} \times \frac{1}{3} \times 3.15 \times 10^7\,\text{sec} \\ &\approx 5.7 \times 10^{11}\,\text{kg} \\ &= 5.7 \times 10^8\,\text{tons}\,. \end{aligned} \tag{2.11}$$

Comet ISON was observed by the *Hubble Space Telescope* in January 2013. Though it was still far from the Sun at $\sim 740 \times 10^6$ km – somewhere between Jupiter and Saturn – it had already developed a tail 65,400-km long. *Hubble* estimated that it was losing roughly 51,000 kg of dust and 60 kg of water every minute. How much mass it had lost between the Jupiter's orbit (740×10^6 km from the Sun) and the Earth's orbit (150×10^6 km from the Sun)? Assume the constant mass loss rate and the same speed (in January it was travelling at 21 km/sec).

The total distance travelled is $740 \times 10^6 - 150 \times 10^6 = 590 \times 10^6$ km. At a speed of 21 km/sec, it takes

$$\frac{590 \times 10^6}{21} \approx 28 \times 10^6 \text{sec} \approx 470,000 \, \text{minutes} \,. \tag{2.12}$$

At a rate of $51,060$ kg/minute, it had lost $51,060 \times 470,000 = 2.4 \times 10^{10}$ kg. This looks like quite a lot! But what percentage of its total mass it represents?

$$P = 100\% \times \frac{2.4 \times 10^{10}}{5.2 \times 10^{14}} \simeq 0.005\% \,. \tag{2.13}$$

2.4 How many passages can Halley's comet survive?

This we can find by dividing the total mass of the comet by the mass loss per passage

$$\text{maximum passages} = \frac{\text{mass of the comet}}{\text{mass loss rate}} \,.$$

Putting in the numbers found in previous sections, we find

$$\text{maximum passages} = \frac{5.2 \times 10^{14} \, \text{kg}}{5.7 \times 10^{11} \, \text{kg/passage}}$$

$$\approx 912 \, \text{passages} \,. \tag{2.14}$$

2.5 Maximum lifetime of a typical short-period comet

The period of Halley's comet is 76 years. Therefore, assuming the mass loss rate does not change, it will survive for

$$T_{\mathrm{Halley}} = 912\,\mathrm{passages} \times 76\,\mathrm{years} \simeq 69,000\,\mathrm{years!} \tag{2.15}$$

However, with every passage, more water and ice are lost and the comet darkens – develops the dark dust crust, which is less heated – a sort of insulation, and as a result, it loses less mass, thus surviving for longer. It can even turn completely black and stop looking like a comet, more like an asteroid.

2.6 How much mass is there in comets?

Scientists estimate that there are trillions of comets (10^{13}) in the outer Solar System – in the Oort cloud. If each of them has $\sim 10^{14}$ kg of dust and ice, then the total mass in comets is

$$\left(10^{13}\mathrm{comets}\right) \times \left(10^{14}\,\mathrm{kg}\right) = 10^{27}\,\mathrm{kg}\,. \tag{2.16}$$

How much mass is it? Mass of the Earth is 6×10^{24} kg, thus total mass in comets is

$$\frac{10^{27}}{6 \times 10^{24}} = 166\,\textit{Earths}, \quad \text{or ONLY...} \simeq \frac{1}{2}\,\textit{Jupiter}\,. \tag{2.17}$$

2.7 Comets as source of water on Earth

It is believed that all water on Earth was brought in by comets after the Earth's formation and cooling down — is it possible? How many comets it would have taken?

Earth's oceans contain $\sim 10^{21}$ kg of water. How many Halley's comets are needed to fill the oceans? Remember, the total mass of a typical comet is 5.2×10^{14} and comets are, say, 80% water.

$$N_{\text{Halley}} = \frac{10^{21}}{0.8 \times 5.2 \times 10^{14}} \sim 2.5{\times}10^{6}\text{– two million Halley's comets}\,. \tag{2.18}$$

But Oort cloud has trillions of comets! So, it is possible.

Chapter 3

Can you find the speed of the comet yourself?

3.1 Comet in images

The comet ISON – the proper name is comet C/2012 S1 (ISON), but everyone now knows it as simply ISON – was observed from January to October 2013 with the Himalayan Chandra Telescope (HCT) of the Indian Institute of Astrophysics, located in Himalayas at the altitude of 4.5 km (near Hanle, Leh, Ladakh). In these images, the comet is seen moving through the background of fixed stars (see sequential frames in Fig. 3.1). These images are in what astronomers call FITS format, because this format preserves the information of the image in the so-called header: the information on the date and exact time of observation, exposure time – the duration of observation, filter in which it was observed, and a lot of other useful data.

If we know the position of the comet on the sky and the time the first image was taken, and the position and the time the next (or the last) image was taken, we can calculate the angular distance on the sky the comet moved in that time period. Since we know the distance to the comet by other means – it was measured by the space telescopes (in this instance, NASA's *SWIFT* and *Hubble*) – we can calculate the physical distance (for example, in km) the comet travelled between the exposures. Knowing the physical distance and the time between the exposures, we can calculate the speed of the comet. In the following paragraphs we describe the steps one needs to perform in order to do that. To obtain the FITS images, please contact the Editor by E-mail: **info@mpbifr-blr.in**.

3.1.1 Finding the celestial coordinates of the comet in FITS images

Aladin is an interactive software sky atlas allowing the user to visualize digitized astronomical images, superimpose entries from astronomical catalogues or databases, and interactively access related data and information from the astronomical databases and archives for all known sources in the image. It can be downloaded from the Aladin website[1] and installed on the Windows (or MacOS) machine.

To open your FITS image in Aladin, click on option "File" on the top taskbar, then on "Open local file". Find your directory and the first image and click "OK". Repeat this for the subsequent images. In this case, we will be using the equatorial coordinate system, where the position of an object on the sky is given as Right Ascension (RA) and Declination (Dec), expressed as hours:minutes:seconds (HH:MM:SS) of the day (from 0 hours to 24 hours), and DEG:MIN:SEC (DD:MM:SS) of the circle (there are 60 arcmin in a degree, and 60 arcsec in an arcmin), respectively. Declinations lie in the range between $-90°$ and $+90°$.

[1]http://aladin.u-strasbg.fr/java/nph-aladin.pl?frame=downloading

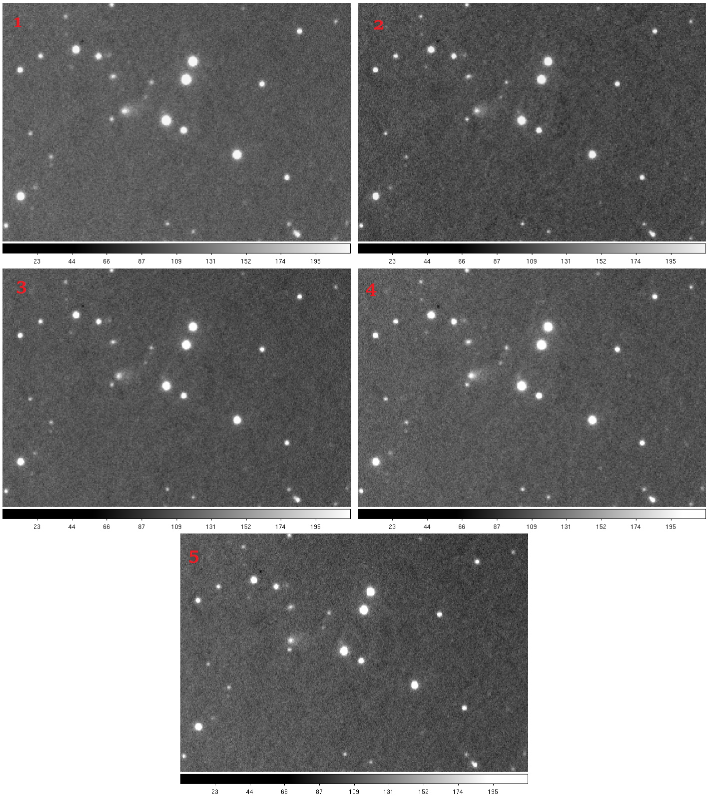


Figure 3.1: February 2013 images of comet ISON moving through the stellar field (1 - 5).

RA and Dec are to the sky what longitude and latitude are to the surface of the Earth. RA corresponds to East/West direction (like the longitude), while Dec measures North/South directions (like the latitude). 0 hours RA is by convention the RA of the Sun on the vernal equinox, March 21. The celestial equator is the origin of the declination system at exactly $0°$. The celestial equator is the part of the sky which is directly overhead the equator of the Earth.

The north celestial pole is at $+90°$, while the south celestial pole would be at $-90°$, just like the latitudes on Earth.

To find the equatorial coordinates RA/Dec of object in your images in Aladin, to the right of the window "Location" on the taskbar put the option "J2000" and point a cursor at an object and click left mouse button. The red cross mark will appear at that position. Since the comet is an extended object, try to find the brightest spot. You may zoom into the image using the zoom bar on the right hand side in order to get more accurate coordinates, and be able to select the brightest position/pixel in the image. Furthermore, a tool labeled "HDR" (High Dynamic Pixel Range) in the bottom portion of Aladin can be used to provide more contrast between the pixels, helping you select the brightest pixel. The coordinates of the position of the cursor will be shown in the window "Location" as HH:MM:SS and ±DD:MM:SS and at the bottom of the Aladin window as "Reticle location => HH:MM:SS ±DD:MM:SS". These are the needed coordinates. Record and repeat this for each image (or first and last, for example).

Note: These images have to be already *astrometrically* calibrated, which means that every x, y on the image (CCD) plane is associated with the coordinates on the sky plane: RA and Dec. Please see the Appendix on how to calibrate the images yourself.

3.1.2 Finding the angular distance between two positions

RA and Dec coordinates are just like vectors, and finding the difference between them is like finding the distance between two vectors. Calculating angular distances on the celestial sphere involves no more than intermediate-level geometry. First, the RA and

Dec found from Aladin have to be converted to decimal values:

$$\text{Dec (decimal degrees)} = \pm(\text{DD} + \text{MM}/60 + \text{SS}/3600)\,, \qquad (3.1)$$

and

$$\text{RA (degrees)} = 15 \times (\text{HH} + \text{MM}/60 + \text{SS}/3600)\,, \qquad (3.2)$$

using the fact that 1 hour in RA represents 15° on the celestial sphere. An easy alternative to this is to select "Frame" in Aladin as "J2000d" instead of "J2000" when taking down the coordinates, allowing Aladin to convert the coordinates to decimals for you. *This can also serve as a double-check for your calculations.* The angular distance r in degrees between the two positions, 1 and 2, is determined by the following formula:

$$r = \arccos\left[\sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \times \cos\left(\alpha_1 - \alpha_2\right)\right]\,, \quad (3.3)$$

where (α_1, δ_1) and (α_2, δ_2) are RA and Dec values of the first and last positions of the comet. Then, the angle in degrees is converted to arcsecond after multiplying it by 3600.

Now the angular distance covered by the comet is known. You can double-check this calculation by using the distance tool provided in Aladin, in the panel on the right hand side of the image.

3.1.3 Finding the time when the images were taken

To find the time at which these images were taken, in the same Aladin window, click the option "Edit" on the top taskbar, and then on "fits header". The window will open displaying the information writen in the text format about the image. Find the field called "UT" – Universal Time.

UT is a time standard, based on the rotation of the Earth. It is a modern continuation of the Greenwich Mean Time (GMT), i.e., the mean solar time on the Prime Meridian at Greenwich, and is used in astronomy to standardize the observations taken all around the world. It is usually given in either a decimal format in hours, or as HH/MM/SS, or as HH:MM:SS. Precise format is not important as long as it is the same in all the images that are used – because the time span is what is needed to calculate the speed. Find the time span between these subsequent images in seconds.

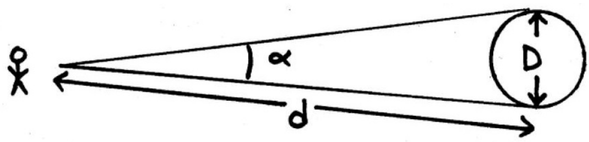
3.1.4 Finding the physical distance in km from angular distance in arcseconds

Now that the angular distance covered by the comet and the time it took to cover that distance are known, we need to know the distance to the comet from the Earth. This distance can be found from the NASA's Jet Propulsion Laboratory website. For example, in January and February the comet ISON was at 4.0902 A.U. and 4.0138 A.U., respectively. For small angles (actually, less than $10°$), the angular distance, the physical distance the comet moved, and the distance to the comet are related as

$$\text{angular distance (arcsec)} \approx \frac{\text{physical distance (km)} \times 206,265}{\text{distance to comet (km)}}, \tag{3.4}$$

where $206,265$ is the conversion factor from radians to arcseconds. Distances need not be in km, but must be in same units. This is derived from simple trigonometry — see Fig. 3.2 for illustration.

Knowing the distance at which the comet was away from the Earth in January and in February (or any other month for which you have the images) and the angular distance it travelled in these months, you can find the physical distance it travelled in km.



If the angle is expressed in arcseconds, then the small angle approximation becomes

$$D = \frac{\alpha d}{206,265}$$

Figure 3.2: Diagram for the formula of angular distance.

3.1.5 Finding the speed

Now you have everything you need to find the speed:

$$\text{speed (km/sec)} = \frac{\text{physical distance (km)}}{\text{time (sec)}} \,. \tag{3.5}$$

Now, this is the comet speed projected on the sky as we (observers) see it — a transverse component of the comet's velocity: the velocity component parallel to the plane of the sky. Comet's orbit is usually at an angle to us – the observers (Fig. 3.3).

To find its space speed, or rather velocity now, we need to know the radial component. Radial velocity is the velocity along the line of sight away from (considered a positive velocity) or toward (negative velocity) the observer. Radial velocity is determined from the Doppler effect in the spectra. The same JPL Horizons website also provides what is called the line-of-sight velocity: delta-dot – or the radial component of the velocity. Using the usual Pythagoras theorem, we find the true – space – velocity of the comet.

One can check the correctness of the assumptions and calculations: JPL Horizons webpage also provides the velocities (in km/sec): "Magnitude of target center velocity w.r.t. Sun (VmagSn)

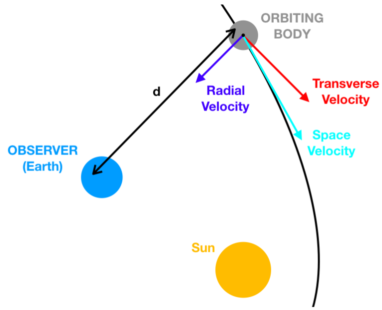


Figure 3.3: Radial, transverse and space velocity of a sun-orbiting body as observed from the Earth.

and the observer (VmagOb)" for that particular time interval, which can be compared with the calculated one.

```
*******************************************************************************
Initial IAU76/J2000 heliocentric ecliptic osculating elements (au, days, deg.):
  EPOCH=  2456630.5 ! 2013-Dec-04.0000000 (TDB)    RMSW= n.a.
   EC= 1.000201003833968   QR= .01245259242960607  TP= 2456625.2786582736
   OM= 295.65203155196     W= 345.5312406205832    IN= 62.40397752235779
  Equivalent ICRF heliocentric equatorial cartesian coordinates (au, au/d):
   X=-2.923813363145732E-02  Y= 1.775530915125493E-01  Z= 2.637915950074458E-01
  VX=-8.923607962793803E-03 VY= 2.889668669261424E-02 VZ= 3.071661566610723E-02
Comet physical (GM= km^3/s^2; RAD= km):
   GM= n.a.                RAD= n.a.
   M1=  8.        M2=  n.a.     k1=  8.     k2= n.a.     PHCOF= n.a.
Comet non-gravitational force model (AMRAT=m^2/kg;A1-A3=au/d^2;DT=days;R0=au):
   AMRAT=  0.                                      DT=  0.
   A1= 9.031169891357E-8   A2= 5.558424592018E-9   A3= 0.
 Standard model:
   ALN=  .1112620426   NK=  4.6142   NM=  2.15     NN=  5.093    R0=  2.808
**********************************************************************************************************
 Date__(UT)__HR:MN     R.A._(ICRF/J2000.0)_DEC  T-mag  N-mag            delta      deldot   VmagSn   VmagOb
**********************************************************************************************************
$$SOE
 2013-Jul-12 00:00     07 37 00.00 +26 14 12.4  14.75   n.a. 3.96251292311360 -23.4523350 24.52553 37.92943
 2013-Jul-22 00:00     07 48 08.48 +25 37 09.8  14.49   n.a. 3.81489506582821 -27.6378350 25.14206 41.37823
 2013-Aug-01 00:00     07 59 59.58 +24 55 41.5  14.21   n.a. 3.64347595171045 -31.7031466 25.82569 44.72551
 2013-Aug-11 00:00     08 12 35.46 +24 08 53.8  13.89   n.a. 3.44891814821965 -35.6321081 26.59032 47.93528
$$EOE
**********************************************************************************************************
```

Figure 3.4: The output of calculated ephemerids from the JPL HORIZONS Web-Interface for comet ISON for 1 month with 10 days interval.

Chapter 4

Appendix: Doing Astrometric Calibration Yourself

You can do astrometric calibration of the images yourself. There are several ways to do it. Doing it yourself is fun! Firstly, the calibration can be done using the same Aladin.

Appendix A. Astrometric calibration using Aladin

Load, say, the first image in the series – this was taken in January 2013 (the comet is a small smudge in the centre). Aladin will respond – "No astrometric reduction". Go to the option "Image", click on "Astrometric Calibration", and when the window opens, choose "By matching stars". You need the image of the same field, opened in another window, that is astrometrically calibrated – an image from any optical mission like *Hubble*, DSS, etc. For that you need to know roughly the coordinates of the field where the comet is –

that we easily find from the same JPL Horizons. Some FITS headers also contain the RA and Dec of the object, or of the image centre. Input this coordinates into Aladin and ask it to open the image.

1. Go to “File: Load Astronomical Image” – click on this.

2. Click on DSS (stands for Deep Sky Survey).

3. Click on DSS from STScI – this is Space Telescope (*Hubble*) Institute site. The window will open, where you input the coordinates of the field, and click on “Submit”.

4. The image will open in Aladin, which will be astrometrically calibrated (with RA/DEC). Now you match visually the stars, and go on clicking first in your image on a star to input its x, y to the table, and then in calibrated image on the same star to input its RA/DEC. Try to do it around the full image to avoid distortions. In the end, click on “Create”. After that check how good is the calibration – whether stars do coincide on both images.

Appendix B. Astrometric calibration using the web

There are also powerful tools available online that can simplify the process of astrometric calibration (albeit you will miss out on the fun!). One of these online tools is on the website of Astrometry.net project[1]. Simply click the upload tab in the menu and upload the file you wish to calibrate. Once the job is completed, usually after 2 minutes of processing time, you may proceed to the results page.

[1]http://nova.astrometry.net. This project is partially supported by the US National Science Foundation, the US National Aeronautics and Space Administration, and the Canadian National Science and Engineering Research Council.

On the results page, you will get all calibration data, as well as the astrometrically calibrated image. To download it, click on “New FITS Image: new-image.fits”.

The download process should begin. This process has its drawbacks however, as the software is not always able to produce a successful calibration. If the result fails to calibrate, you will see “No calibration data available” under Calibration on the results page, and your only option will be to calibrate manually.

Comets are fun! The famous comet hunter David Levy once said: "*Comets are like cats: they have tails and do exactly what they want*". Their remains give us the beautiful phenomena: meteor showers, noctilucent clouds and zodiacal light. But fun with comets does not stop at just their pretty images. Their behaviour can be described mathematically. In this book we present mathematical problems (with solutions) applicable to comets. We also offer the readers opportunity to find the speed of the comet themselves by using real-life telescope images.

About the Authors

Margarita Safonova is an astrophysicist, originally from Russia, and currently working in the Indian Institute of Astrophysics (IIA), Bangalore. Observations of comets and asteroids are some of her interests. Her images and 'movies' of the recent comets are available on the Outreach page of IIA:
www.iiap.res.in/Outreach_IIA/?q=Astronomical_Events.

Shrihan Agarwal at time of writing was a high-school student of the Greenwood High International School, Bangalore. He has joined the University of California, Berkeley, US, for the B.Sc. program, and is aspiring to be an astrophysicist. Shrihan writes about his physics passions in his blogs. The blog about the comet's ISON precise orbital calculations is available at:
https://planetshrihan.wordpress.com/ison/.

Notes

Useful websites for comet hunters

Visual Comets in the Future (Northern Hemisphere)
Information on visibility of known comets from current month till 2024; profile, orbital elements, and finding charts.
http://www.aerith.net/comet/future-n.html

Weekly Information about Bright Comets
Currently observable comets: best times for observation, including comet's azimuth and elevation for an observer at 35 degrees latitude in the Northern Hemisphere.
http://www.aerith.net/comet/weekly/current.html

The "Official Guide" to SOHO Comet Hunting
Over 1800 comets have been discovered within images taken by the joint ESA/NASA mission called *SOHO* (Solar and Heliospheric Observatory"). The majority of these comets have been found by amateur astronomers and enthusiasts around the world who download the latest satellite data from the Internet and search the images for signs of a comet. The websites given here provide freely available data to users in a variety of formats, the most commonly used of which (by comet hunters) being the real-time "gif" images.
https://sungrazer.nrl.navy.mil/index.php?p=guide

Comet Hunting Resources on the Web
Web links to Amateur Comet Hunters and Discoveries, and Professional Surveys and Discoveries.
http://www.comethunter.de/links.html

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